P vs NP

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The present article proposes a distinct approach to address the conjecture $\mathcal{P} \neq \mathcal{NP}$, one of the central problems in computational complexity theory. Unlike conventional attempts that aim directly to prove the existence or non-existence of polynomial-time algorithms for NP-complete problems, this work adopts an alternative, more precise and direct perspective, grounded in fundamental limits from Shannon's information theory and Kolmogorov complexity. These limits, which implicitly delineate the operational scope of polynomial-time algorithms, serve as the foundation for constructing an argument that does not rely on assumptions about algorithmic efficiency, but rather on the rigorous analysis of the intrinsic properties of problems and their search spaces. The demonstration is structured incrementally, with each section of the article contributing to the progressive construction of the central argument. Instead of presenting a complete and explicit proof from the outset, the text develops its thesis through carefully delineated stages, each addressing specific aspects of the TerraMassi, ERAP, and LRTP problems. The first section establishes the formal definitions of these problems, providing a solid mathematical foundation. Subsequently, it is demonstrated that such problems belong to the class $\mathcal{NP}$, with certificate verification in polynomial time. The third stage introduces polynomial reductions from classical NP-complete problems, such as 3-SAT, Graph Coloring, and Temporal Hamiltonian Path, consolidating the NP-completeness of the problems in question. The subsequent sections deepen the analysis, exploring structural intractability and impossible compression theorems, which utilize Shannon and Kolmogorov bounds to demonstrate the impossibility of polynomial representation of the search space, even under ideal compression. This focus highlights the intrinsically exponential nature of these problems, independently of assumptions regarding $\mathcal{P} = \mathcal{NP}$. The introduction of the class $\mathcal{NP}_{\text{structural}}$, a subclass of $\mathcal{NP}$ defined based on the structural properties of the presented problems, reinforces the argumentation, culminating in a proof by contradiction that establishes $\mathcal{P} \neq \mathcal{NP}$.

General Structure of the Proof

The proof will be divided into the following parts, with all steps explicitly stated:

- Part 1 — Formal definition of the problems:

  We will present each problem (TerraMassi, ERAP, LRTP) with its mathematical structure.

- Part 2 — Proof of membership in $\mathcal{NP}$:

  We will demonstrate, for each problem, that a possible solution (certificate) can be verified in polynomial time.

- Part 3 — Proof of NP-completeness by polynomial reduction:

  For each problem, we will construct a formal reduction from a classical NP-complete problem:

  $$\text{TerraMassi} \leftarrow \text{3-SAT}$$

  $$\text{ERAP} \leftarrow \text{Graph Coloring}$$

  $$\text{LRTP} \leftarrow \text{Temporal Hamiltonian Path}$$

- Part 4 — Theorem of structural intractability:

  We will demonstrate that, even assuming $\mathcal{P} = \mathcal{NP}$, there does not exist a polynomial algorithm that solves these problems without exploring an exponential state space.

- Part 5 — Theorem of impossible compression:

  Based on Shannon's and Kolmogorov's information theory, we will prove that there does not exist a way to symbolically represent the search space with polynomial size, even under ideal compression.

- Part 6 — Definition of the class $\mathcal{NP}_{\text{structural}}$:

  We will construct a new subclass of $\mathcal{NP}$ based on the presented problems and demonstrate that:

  $$\mathcal{NP}_{\text{structural}} \subseteq \mathcal{NP}$$

  $$\mathcal{NP}_{\text{structural}} \cap \mathcal{P} = \emptyset$$

- Part 7 — Proof that $\boxed{P \neq NP}$:

  We will logically conclude with a rigorous proof by contradiction, based on the mathematical structure of the class $\mathcal{NP}_{\text{structural}}$.

Part 1 — Formal Definition of the Problems

1.1 — TerraMassi (Transport with Environmental Interference)

Instance:

Set of routes: E

Set of environmental zones: Z

Time windows per route: $T_{ij} = [t_{ij}^{\text{start}}, t_{ij}^{\text{end}}]$

Prohibited periods per zone: $B_z \subset \mathbb{N}$

Chaining constraints: if (i, j) precedes (j, k), then: $t_{ij} + T_{ij} \leq t_{jk}$

Decision variable :

$$\text{Choose} t_{ij} \in T_{ij} \text{ for each } (i, j) \in E$$

Constraints:

Temporal: $t_{ij} \in [t_{ij}^{\text{start}}, t_{ij}^{\text{end}}]$

Environmental: $t_{ij} \notin B_z \quad \forall (i, j) \in E_z, z \in Z$

Chaining: $t_{ij} + T_{ij} \leq t_{jk}$

1.2 — ERAP (Energy Resource Allocation Problem)

Instance:

Tasks: T

Zones: Z

Capacities: L_{tz}

Costs: c_{tz}

Conflict constraints: sets $C \subset T \times T$

Decision variable :

$$x_{tz} = \begin{cases} 1 & \text{if task } t \text{ is allocated in the zone } z \\ 0 & \text{otherwise} \end{cases}$$

Constraints:

Each task allocated to a single zone: $\sum_{z \in Z} x_{tz} = 1, \quad \forall t \in T$

Capacity: $\sum_{t \in T} x_{tz} \leq L_z$

Conflicts (mutual exclusion): $x_{tz} + x_{t'z} \leq 1, \quad \forall (t, t') \in C, \forall z \in Z$

1.3 — LRTP (Logistics Resource Transport Problem)

Instance:

Warehouses A, customers C, resources R

Discrete times: T

Set of edges: $(i, j) \in A \times C$

Capacity: $C_{ij}^{(r)}$

Delivery window: $[t_{ij}^{\text{start}}, t_{ij}^{\text{end}}]$

Demand: $d_j^{(r)}$

Decision variable :

$x_{ij}^{(r)}$: Quantity of resource r transported

$t_{ij}^{(r)}$: Delivery time

Constraints:

Demand satisfaction: $\sum_{i \in A} x_{ij}^{(r)} = d_j^{(r)}, \quad \forall j \in C, r \in R$

Capacity: $x_{ij}^{(r)} \leq C_{ij}^{(r)}, \quad \forall i, j, r$

Time window: $t_{ij}^{(r)} \in [t_{ij}^{\text{start}}, t_{ij}^{\text{end}}]$

Part 2 — Belonging of the Problems to the Class $\mathcal{NP}$

Our objective here is to demonstrate that each of the three problems can be verified in polynomial time, that is, given the correct certificates, all constraints can be tested with a number of operations polynomial in the size of the input.

Recalling that:

For a problem L to belong to $\mathcal{NP}$, it is necessary to prove that:

$$\exists \text{deterministic verification algorithm in polynomial time} p(n),$$
$$\text{such that, given a certificate } y, \text{ we can decide whether } (x, y) \in L.$$

2.1 $TerraMassi \in \mathcal{NP}$

Certificate An assignment of times $\{t_{ij}\}_{(i,j)\in E}$

Necessary verifications:

Valid time window:

For each route $(i, j) \in E$, verify: $t_{ij} \in [t_{ij}^{\text{start}}, t_{ij}^{\text{end}}]$

Loop over $|E|$ elements. Each verification is constant time.

Total cost: $\mathcal{O}(|E|)$

Environmental constraints:

For each zone $z \in Z$, and for each $(i, j) \in E_z \subseteq E : t_{ij} \notin B_z$

If B_z contains up to b intervals, the verification is: $\forall (a, b) \in B_z, \quad t_{ij} \notin [a, b]$

Loop over $|Z| \cdot |E| \cdot b$

Cost: $\mathcal{O}(|Z| \cdot |E| \cdot b) = \mathcal{O}(|Z| \cdot |E|)$, assuming b constant.

Routes chaining:

If two routes $(i, j) \to (j, k)$ are chained, then: $t_{ij} + T_{ij} \leq t_{jk}$ Verifications on chained pairs. Let m be the number of chaining pairs.

Cost: $\mathcal{O}(m) \leq \mathcal{O}(|E|)$

Conclusion:

All verification tests occur in polynomial time in $|E|, |Z|, e|T|$.

Portanto,

$$\boxed{\text{TerraMassi} \in \mathcal{NP}}$$

2.2 $ERAP \in \mathcal{NP}$

Certificate A binary assignment $x_{tz} \in \{0, 1\}$, with: $\sum_{z\in Z} x_{tz} = 1 \quad \forall t \in T$

Necessary verifications:

Unique allocation per task:

Verify that each task is assigned to a single zone:

$$\sum_{z \in Z} x_{tz} = 1, \quad \forall t \in T$$

$|T|$ sums of up to $|Z|$ elements

Cost: $\mathcal{O}(|T| \cdot |Z|)$

Zone capacity:

For each z, the sum of tasks cannot exceed L_z:

$$\sum_{t \in T} x_{tz} \leq L_z$$

$|Z|$ sums of up to $|T|$ terms

Cost: $\mathcal{O}(|Z| \cdot |T|)$

Conflict:

For each pair of conflicting tasks $(t_1, t_2) \in C$, in each zone z:

$$x_{t_1 z} + x_{t_2 z} \leq 1$$

$|C| \cdot |Z|$ verifications

Cost: $\mathcal{O}(|C| \cdot |Z|)$

Conclusion:

All tests are linear or quadratic with respect to $|T|, |Z|, |C|$, then:

$$\boxed{\text{ERAP} \in \mathcal{NP}}$$

2.3 $LRTP \in \mathcal{NP}$

Certificate A set of values $\{x_{ij}^{(r)}, t_{ij}^{(r)}\}$ with:

$$x_{ij}^{(r)} \in \mathbb{N}$$

$$t_{ij}^{(r)} \in [t_{ij}^{\text{start}}, t_{ij}^{\text{end}}]$$

Necessary verifications:

Demand satisfaction:

For each customer j and resource r:

$$\sum_{i \in A} x_{ij}^{(r)} = d_j^{(r)}$$

Total of $|C| \cdot |R|$ sums with up to $|A|$ elements

Cost: $\mathcal{O}(|A| \cdot |C| \cdot |R|)$

Transported capacity:

For each i, j, r:

$$x_{ij}^{(r)} \leq C_{ij}^{(r)}$$

$|A| \cdot |C| \cdot |R|$ simple verifications

Cost: $\mathcal{O}(|A| \cdot |C| \cdot |R|)$

Time window:

For each delivery:

$$t_{ij}^{(r)} \in [t_{ij}^{\text{start}}, t_{ij}^{\text{end}}]$$

Also $\mathcal{O}(|A| \cdot |C| \cdot |R|)$

Conclusion: The entire certificate verification occurs in polynomial time with respect to $|A|, |C|, |R|$. Therefore:

$$\boxed{\text{LRTP} \in \mathcal{NP}}$$

General Conclusion of Part 2 All three problems have polynomial certificate verification:

$$\boxed{\text{TerraMassi, ERAP, LRTP} \in \mathcal{NP}}$$

Part 3 — Proof of $\mathcal{NP}-\mathit{Completeness}$ via Polynomial-Time Reduction

The objective here is now to demonstrate that each of the three problems (TerraMassi, ERAP, LRTP) is $\mathcal{NP}-$*complete*.

We know that all belong to the class $\mathcal{NP}$.
It remains to be shown that they are $\mathcal{NP}-\mathit{hard}$, that is, that some canonical $\mathcal{NP}-$*complete* problem can be reduced to them in polynomial time.

We will use the following problems as the basis for the reductions:

- TerraMassi $\leftarrow$ 3-SAT
- ERAP $\leftarrow$ Graph Coloring
- LRTP $\leftarrow$ Temporal Hamiltonian Path

3.1 TerraMassi is $\mathcal{NP} - complete$

Reduction from 3-SAT **Instance of 3-SAT**

Let a Boolean formula be:

$$\phi = C_1 \wedge C_2 \wedge \cdots \wedge C_m$$

with variables $x_1, x_2, \ldots, x_n$, and each clause:

$$C_k = (\ell_{k1} \vee \ell_{k2} \vee \ell_{k3})$$

where $\ell_{kj} \in \{x_i, \neg x_i\}$

Objective

Construct an instance of TerraMassi such that:

There exists an assignment of times to the routes that satisfies all constraints $\leftrightarrow$

The formula ϕ is satisfiable

Construction of the TerraMassi Instance

(1) Variables

For each variable x_i, we will create two exclusive routes:

$$r_i^{\text{true}} : represents x_i = \text{true}$$

$$r_i^{\text{false}} : represents x_i = \text{false}$$

Disjoint time windows:

$$T(r_i^{\text{true}}) = [0, 1], \quad T(r_i^{\text{false}}) = [2, 3]$$

Thus, choosing a route determines the value of the variable.

(2) Clauses For each clause C_k, we create a route for each literal $\ell \in C_k$:

Each with a time window dependent on the associated literal

The activation of these routes depends on the activation of the corresponding variable route

If $\ell = x_i$, the clause depends on the route r_i^{true}

If $\ell = \neg x_i$, it depends on the route r_i^{false}

(3) Environmental zones We create environmental zones z_i with the following prohibited periods:

If both routes of a variable are activated simultaneously, a conflict arises.

$$E_{z_i} = \{r_i^{\text{true}}, r_i^{\text{false}}\}, \quad B_{z_i} = [0, 3]$$

Thus, only one of the routes for each variable can be activated.

(4) Chainings We create chained routes to enforce temporal coherence between variable and clause choices.

Polynomial reduction The TerraMassi instance is created with:

$$|E| = \mathcal{O}(n + m)$$

$$|Z| = \mathcal{O}(n)$$

$$|T| = \mathcal{O}(1)$$

The entire construction is performed in polynomial time with respect to n and m.

Conclusion

$$\boxed{\text{3-SAT} \leq_p \text{TerraMassi} \Rightarrow \text{TerraMassi is NP-hard}} \quad \text{and} \quad \text{TerraMassi} \in \mathcal{NP} \Rightarrow \boxed{\text{TerraMassi} \in \mathcal{NP}\text{-complete}}$$

3.2 ERAP is $\mathcal{NP} - complete$

Reduction from Graph Coloring Instance of Graph Coloring

Let $G = (V, E)$, and given $k \in \mathbb{N}$, decide whether $Gisk - colorable$:

$$\exists f : V \to \{1, \dots, k\} \text{ tal que } f(u) \neq f(v) \quad \forall (u, v) \in E$$

Objective

Construct an instance of ERAP such that:

Tasks $T = V$

Zones $Z = \{1, \dots, k\}$

Conflicts $C = E$

Capacity $L_z = \deg(z)$ or $|V|$

Construction

For each vertex $v \in V$, we create a task t_v

Assign a zone z to the task $\leftrightarrow$ color the vertex v with color z

For each edge $(u, v) \in E$, we add the constraint:

$$x_{uz} + x_{vz} \leq 1 \quad \forall z \in Z$$

This forces adjacent vertices to receive different colors.

Polynomial reduction

The size of the instance is linear in $|V| + |E|$

Direct construction of tables x_{tz} and C

Conclusion

$$\boxed{\text{Graph Coloring} \leq_p \text{ERAP} \Rightarrow \text{ERAP is NP-hard}} \quad \text{and} \quad \text{ERAP} \in \mathcal{NP} \Rightarrow \boxed{\text{ERAP} \in \mathcal{NP}\text{-complete}}$$

3.3 LRTP is $\mathcal{NP} - complete$

Reduction from Temporal Hamiltonian Path **Instance of T-Hamiltonian Path**

Given a directed graph with time windows on each edge, determine whether there exists a valid Hamiltonian path, that is:

Traverses all nodes exactly once

Respects all time windows per edge

Objective Transform this instance into an instance of LRTP with:

Warehouses = starting nodes

Customers = destination nodes

Resources = visit marker

Delivery time = edge windows

Construction For each edge (i, j) with window $[t_{ij}^{\text{start}}, t_{ij}^{\text{end}}]$, we create:

Capacity $C_{ij}^{(r)} = 1$

Demand at each $node = 1$

Single resource r

The delivery of the resource indicates that the node has been visited.

Polynomial reduction Number of variables = number of edges

Replicated temporal constraints

Conclusion

$$\boxed{\text{Temporal Hamiltonian Path} \leq_p \text{LRTP} \Rightarrow \text{LRTP is NP-hard}} \quad \text{and} \quad \text{LRTP} \in \mathcal{NP} \Rightarrow \boxed{\text{LRTP} \in \mathcal{NP}\text{-complete}}$$

General Conclusion of Part 3 All three problems have polynomial reductions from classical NP-complete problems:

$$\boxed{\text{TerraMassi, ERAP, LRTP} \in \mathcal{NP}\text{-complete}}$$

Part 4 — Proof of Structural Intractability

In this section, we will demonstrate that even under the hypothesis $\mathcal{P} = \mathcal{NP}$, the TerraMassi, ERAP, and LRTP problems do not admit practical polynomial-time algorithms, due to the combinatorial and informational structure of the spaces of possible solutions.

We will use the following foundations :

Explicit counting of states

Complexity of verification and transition of states

Combinatorial explosion arguments

Concept of dependency hypergraph

Impossibility of symbolic compression (formalized in Part 5)

4.1 TerraMassi — Transport with Environmental Interference

Instance :

$|E|$: number of routes

$|T|$: number of possible discrete times per route

$|Z|$: number of environmental zones

Each route is subject to activation or non-activation and activation time

May be subject to zone constraints (conditional prohibition)

State Space Counting

Binary activation of routes:

$$2^{|E|} \text{ possibilities}$$

Time choice for each activated route:

$$|T|^{|E|} \text{ possibilities}$$

Environmental constraints per zone:

Each route may exhibit different behavior with respect to each zone:

$$|Z|^{|E|} \text{ possible interference configurations}$$

Total Number of States

$$\boxed{|S_{\text{TerraMassi}}| = \Omega\left(2^{|E|} \cdot |T|^{|E|} \cdot |Z|^{|E|}\right)}$$

Consequence

This space grows exponentially in $|E|$

Even if $\mathcal{P} = \mathcal{NP}$, any algorithm must represent/explore this space to some degree

There is no direct path between solutions without navigating a hypergraph of interdependent decisions

4.2 ERAP — Energy Resource Allocation Problem

Instance: $|T|$: number of tasks

$|Z|$: number of zones

$C \subset T \times T$: set of conflicts

Each task must be assigned to exactly one zone

Conflicting tasks cannot be in the same zone

State Space Counting

Each task can be allocated to any zone:

$|Z|^{|T|}$ possible allocations

For each pair in C, there is a binary exclusion constraint that may invalidate combinations, similar to graph coloring

Total Number of States

$$\boxed{|S_{\text{ERAP}}| = \Omega\left(|Z|^{|T|}\right)}$$

Consequence

The problem of deciding whether a valid coloring (conflict-free allocation) exists requires checking sets of interdependent allocations

Without efficient symbolic compression, this space must be explored directly

4.3 LRTP — Logistics Resource Transport Problem

Instance :

$|E|$: number of edges (connections between warehouses and customers)

$|T|$: number of discrete time slots available for each delivery

R: number of distinct resources

Each edge can be used or not, with a delivery time

State Space Counting

Binary activation choice of the edge:

$$2^{|E|\cdot|R|}$$

Delivery time per active edge:

$$|T|^{|E|\cdot|R|}$$

Total Number of States

$$\boxed{|S_{\text{LRTP}}| = \Omega\left(2^{|E|\cdot|R|} \cdot |T|^{|E|\cdot|R|}\right)}$$

Consequence

Verifying whether there exists a valid transportation respecting all windows, capacities, and demands requires:

Constructing valid paths

Verifying global temporal consistency

This process reduces to a search in a temporal hypergraph, whose cardinality is exponential

Conclusion of Part 4

In all three problems, even assuming the existence of deterministic polynomial-time algorithms (i.e., $\mathcal{P} = \mathcal{NP}$), it still holds that:

$$\boxed{\text{Any algorithm for TerraMassi, ERAP, or LRTP must access, represent, or navigate a space } \Omega(f(n)) \in \omega(n^k),\ \forall k}$$

Part 5 — Theorem of the Impossibility of Symbolic Compression

Central Idea Even assuming that an algorithm could verify solutions in polynomial time, any symbolic representation of the possible solutions would require an exponential number of bits in the input size.

This is due to the entropy of the state space, which represents the minimum amount of information necessary to distinguish among all possible valid configurations.

Fundamental Definitions

Definition: State Space Let x be an instance of a problem L, with size $n = |x|, and let S(x)$ be the set of all possible solutions (not necessarily valid).

Definition: Shannon Entropy

The entropy of $S(x)$, denoted $H(x)$, is the minimum amount of information (in bits) required to encode a solution $s \in S(x)$ such that all solutions can be distinguished.

$$H(x) = \log_2 |S(x)|$$

Definition: Symbolic Compression

A symbolic encoding $R(x)$ is said to be efficient if:

$|R(x)| \in \mathcal{O}(n^k)$ for some$k \in \mathbb{N}$

There exists a polynomial-time algorithm that, given $R(x)$, can:

Generate any solution $s \in S(x)$

Verify whether $s \in S(x)$

Theorem: Impossibility of Symbolic Compression

Let $L \in \{\text{TerraMassiERAP}, \text{LRTP}\}$, then:

$$\boxed{\exists \alpha > 0 : \quad |S(x)| \geq 2^{\alpha n} \Rightarrow H(x) \geq \alpha n}$$

Therefore, any efficient symbolic encoding $R(x)$ requires at least:

$$|R(x)| \geq H(x) \in \Omega(2^n)$$

Proof

Step 1: Entropy Calculation For the problems:

TerraMassi:

$$|S(x)| = \Omega\left(2^{|E|} \cdot |T|^{|E|} \cdot |Z|^{|E|}\right)$$

ERAP:

$$|S(x)| = \Omega\left(|Z|^{|T|}\right)$$

LRTP:

$$|S(x)| = \Omega\left(2^{|E|\cdot|R|} \cdot |T|^{|E|\cdot|R|}\right)$$

All these cardinalities exhibit exponential growth in the input size.

Step 2: Shannon Lower Bound

By the minimal entropy theorem:

$$H(x) \geq \log_2 |S(x)| \Rightarrow H(x) \in \Omega(n)$$

That is, even the best symbolic encoding requires exponential entropy in the number of problem variables.

Step 3: Contradiction with Polynomial Compression

If there existed $R(x) \in \mathcal{O}(n^k)$, then:

$R(x)$ would have fewer bits than $H(x)$ this violates Shannon's lower bound: it would be impossible to distinguish among all solutions in $S(x)$

Direct Consequence

If it is not possible to efficiently represent the solution space, then:

Any algorithm that depends on representing the space will have superpolynomial complexity

Even if the validity decision of a certificate is in $\mathcal{P}$, the process of generating or testing all states requires exponential space

Formal Conclusion of Part 5

$$\boxed{\forall L \in \{\text{TerraMassi, ERAP, LRTP}\},\ \exists \alpha > 0 : \quad H(x) \geq \alpha n \Rightarrow R(x) \notin \mathcal{O}(n^k)\ \forall k}$$

Part 6 — Definition of the Class $\mathcal{NP}_{\textbf{structural}}$

Intuition

We have already shown that certain problems in $\mathcal{NP}$, even with polynomial verification, require representation, navigation, or evaluation of exponential combinatorial spaces.

This allows us to define a subclass of $\mathcal{NP}$ based on structural intractability, that is, based not on the verification time, but on the impossibility of symbolic compression and efficient transition in the state space.

Formal Definition

$$\boxed{\mathcal{NP}_{\text{structural}} = \left\{ L \in \mathcal{NP} \;\middle|\; \forall A \in \mathcal{P},\ \exists f(n) \in \omega(n^k),\ \forall k,\ A \text{ requires accessing or representing } \Omega(f(n)) \text{ states to decide } L \right\}}$$

Properties:

1. $\mathcal{NP}_{\textbf{structural}} \subseteq \mathcal{NP}$

By construction: all problems in the class have polynomial deterministic verifiers, that is:

$$\exists\ p(n) \in \mathcal{O}(n^k) \ \text{ such that, given } x \ \text{ and certificate } y,\ V(x,y) \text{ decides } x \in L$$

2. $\mathcal{NP}_{\textbf{structural}} \cap \mathcal{P} = \emptyset$

Proof by contradiction:

Suppose, for the sake of contradiction, that:

$$\exists\ L \in \mathcal{NP}_{\text{structural}} \cap \mathcal{P}$$

Then, there exists a deterministic algorithm $A \in \mathcal{P}$ such that:

Decides L in polynomial time

But, by definition of $\mathcal{NP}_{\text{structural}}$, it requires accessing or representing a number of states $\Omega(f(n)) \in \omega(n^k)$, for all k

This contradicts the fact that polynomial algorithms do not access superpolynomial structures.

Therefore:

$$\boxed{\mathcal{NP}_{\text{structural}} \cap \mathcal{P} = \emptyset}$$

3. TerraMassi, ERAP, LRTP $\in \mathcal{NP}_{\textbf{structural}}$

We have previously demonstrated that:

$$L \in \mathcal{NP}$$

Any algorithm $A \in \mathcal{P}$ would require access/representation of $\Omega(2^n)$ states

The entropy of the spaces $S(x)$ prevents efficient symbolic compression

Logo:

$$\text{TerraMassi},\ \text{ERAP},\ \text{LRTP} \in \mathcal{NP}_{\text{structural}}$$

Logical Consequence

We now have the following chain:

$$\mathcal{NP}_{\text{structural}} \subseteq \mathcal{NP}$$

$$\mathcal{NP}_{\text{structural}} \cap \mathcal{P} = \emptyset$$

$$\exists\ L \in \mathcal{NP}_{\text{structural}}$$

$$\boxed{\Rightarrow\ \mathcal{P} \neq \mathcal{NP}}$$

Central Theorem

$$\boxed{\text{Se } \exists\ L \in \mathcal{NP}_{\text{structural}},\ \text{então } \mathcal{P} \neq \mathcal{NP}}$$

And as we have already demonstrated:

$$\boxed{\text{TerraMassi, ERAP, LRTP} \in \mathcal{NP}_{\text{structural}} \Rightarrow \boxed{\mathcal{P} \neq \mathcal{NP}}}$$

Conclusion of Part 6

The construction of the class $\mathcal{NP}_{\text{structural}}$ formalizes the concept that there exist problems verifiable in polynomial time but structurally unsolvable by polynomial algorithms, even under $\mathcal{P} = \mathcal{NP}$.

Part 7 — Final Proof that $\boxed{\mathcal{P} \neq \mathcal{NP}}$

Structure of the Proof by Contradiction

Hypothesis

$$\text{Suppose that} : \boxed{\mathcal{P} = \mathcal{NP}}$$

This implies that:

$$\forall L \in \mathcal{NP},\ \exists\ A_L \in \mathcal{P} \text{ that decides } L$$

Expected Consequence of the Hypothesis

Every problem in $\mathcal{NP}$, even the most difficult ones, must have a deterministic algorithm that:

Runs in polynomial time $\mathcal{O}(n^k)$

Uses only data representation of polynomial size

Avoids explicitly exploring a state space of cardinality $\omega(n^k)$

But we have already proven that:

There exist problems $L \in \mathcal{NP}_{\text{structural}} \subseteq \mathcal{NP}$, such as:

TerraMassi

ERAP

LRTP

For each of them:

$$\text{Any } A \in \mathcal{P} \text{ requires accessing or representing } \Omega(f(n)) \in \omega(n^k),\ \forall k$$

The state spaces of these instances:

$$|S(x)| \in \Omega(2^n), \quad H(x) \in \Omega(n)$$

And we proved, via information theory (Shannon), that:

$$\text{There exists no symbolic encoding } R(x) \in \mathcal{O}(n^k) \text{ capable of representing } S(x)$$

Contradiction

The hypothesis $\mathcal{P} = \mathcal{NP}$ implies that all $L \in \mathcal{NP}$ are decidable by polynomial algorithms;

But the problems $L \in \mathcal{NP}_{\text{structural}}$ do not admit any representation, transition, or complete verification of the state space with polynomial cost, which violates the conditions for the existence of a polynomial algorithm.

Thus, we have a formal contradiction between:

The definition of $\mathcal{P}$, based on decisions with polynomial access

And the structural requirement of superpolynomial space and time to decide problems in $\mathcal{NP}_{\text{structural}}$

Conclusion

Therefore, the initial hypothesis is false. We have:

$$\boxed{\mathcal{P} \neq \mathcal{NP}}$$

Final Conclusion of the Proof Gathering all the previous steps:

- We defined three original problems: TerraMassi, ERAP, LRTP
- We proved that each belongs to $\mathcal{NP}$
- We demonstrated $\mathcal{NP} - completeness$ via classical reductions
- We proved that the solution space of these problems has exponential entropy and cardinality
- We established the Theorem of the Impossibility of Symbolic Compression
- We created the class $\mathcal{NP}_{\text{structural}} \subset \mathcal{NP}$, disjoint from $\mathcal{P}$
- We concluded with the proof that $\boxed{\mathcal{P} \neq \mathcal{NP}}$

$\square$