# CHAPTER 2 Diode and Zener Diode

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# CHAPTER 2 Diode and Zener Diode

## 2.1 Diode Operation

## 2.2 Voltage-Current (V-I) Characteristic of a Diode

## 2.3 Diode Approximations

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# 2.1 The Diode

- A diode is made from a small piece of semiconductor material, usually silicon, in which half is doped as a *p* region and half is doped as an *n* region with a *pn* junction and depletion region in between.
- The *p* region is called the anode and is connected to a conductive terminal.
- The *n* region is called the cathode and is connected to a second conductive terminal.

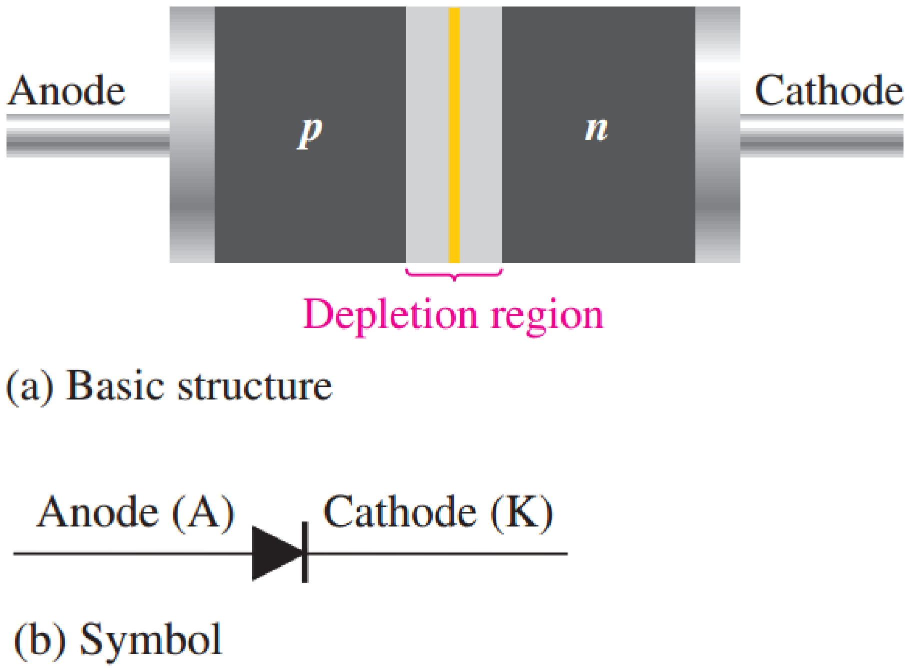



**FIGURE 2-1** The diode

# The Diode

- Typical Diode Packages Several common physical configurations of through-hole mounted diodes are illustrated.
- The anode (A) and cathode (K) are indicated on a diode in several ways, depending on the type of package.
- The cathode is usually marked by a band, a tab, or some other feature. On those packages where one lead is connected to the case, the case is the cathode.

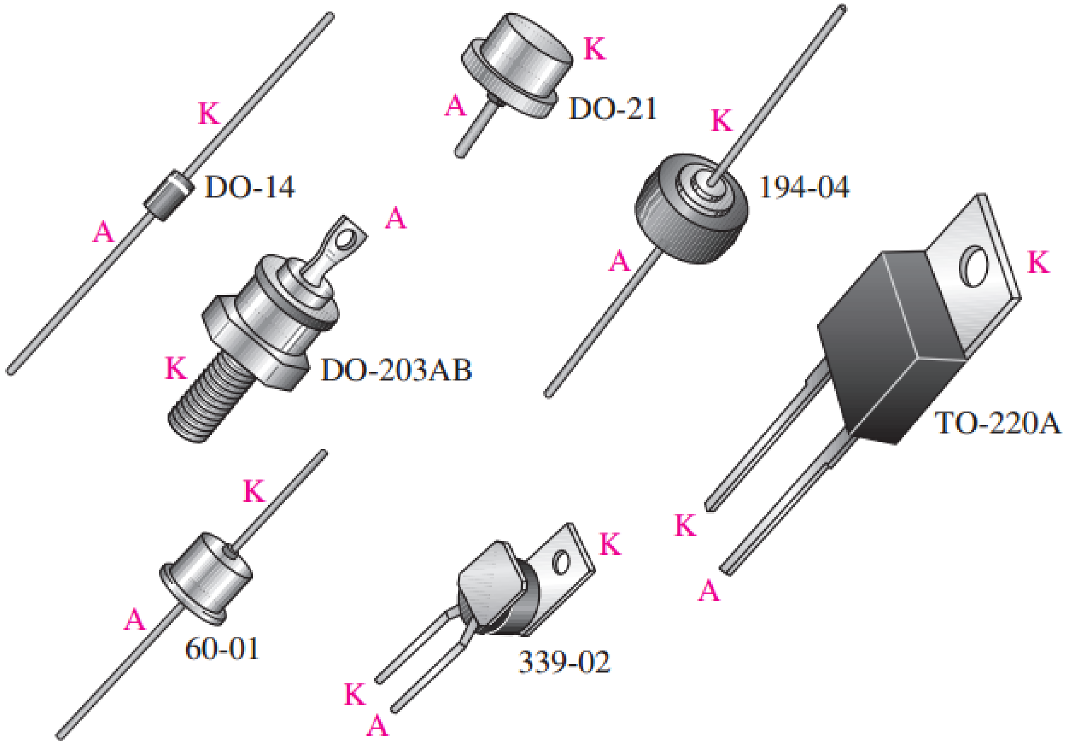



**FIGURE 2-2** Typical diode packages with terminal identification. The letter K is used for cathode to avoid confusion with certain electrical quantities that are represented by C. Case type numbers are indicated for each diode.

# The Diode

- Typical diode packages for surface mounting on a printed circuit board are illustrated.
- The SOD and SOT packages have gull-wing shaped leads. The SMA package has L-shaped leads that bend under the package
- The SOT type is a three-terminal package in which there are either one or two diodes.

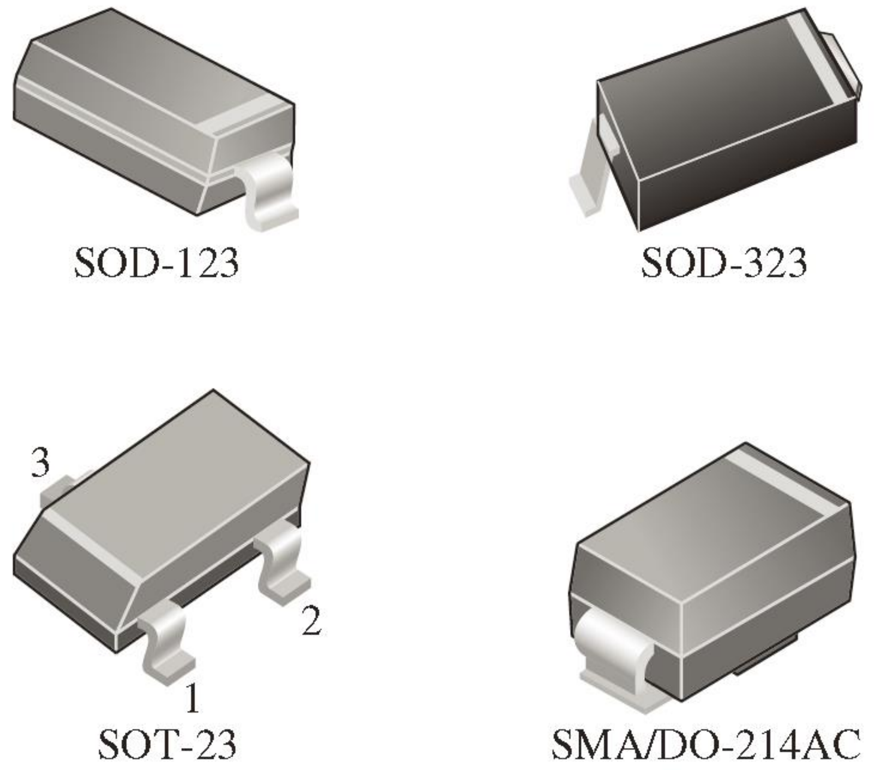



**FIGURE 2-3** Typical diode packages with terminal identification. The letter K is used for cathode to avoid confusion with certain electrical quantities that are represented by C. Case type numbers are indicated for each diode.

# Forward Bias

- To bias a diode, you apply a dc voltage across it. Forward bias is the condition that allows current through the *pn* junction.
- This external bias voltage is designated as $V_{BIAS}$. The resistor limits the forward current to a value that will not damage the diode.
- Forward bias :The negative side of $V_{BIAS}$ is connected to the *n* region of the diode and the positive side is connected to the *p* region.
- The bias voltage, $V_{BIAS}$, <u>must be greater than the barrier potential</u> ($V_B$) such as for Si diode 0.7V .

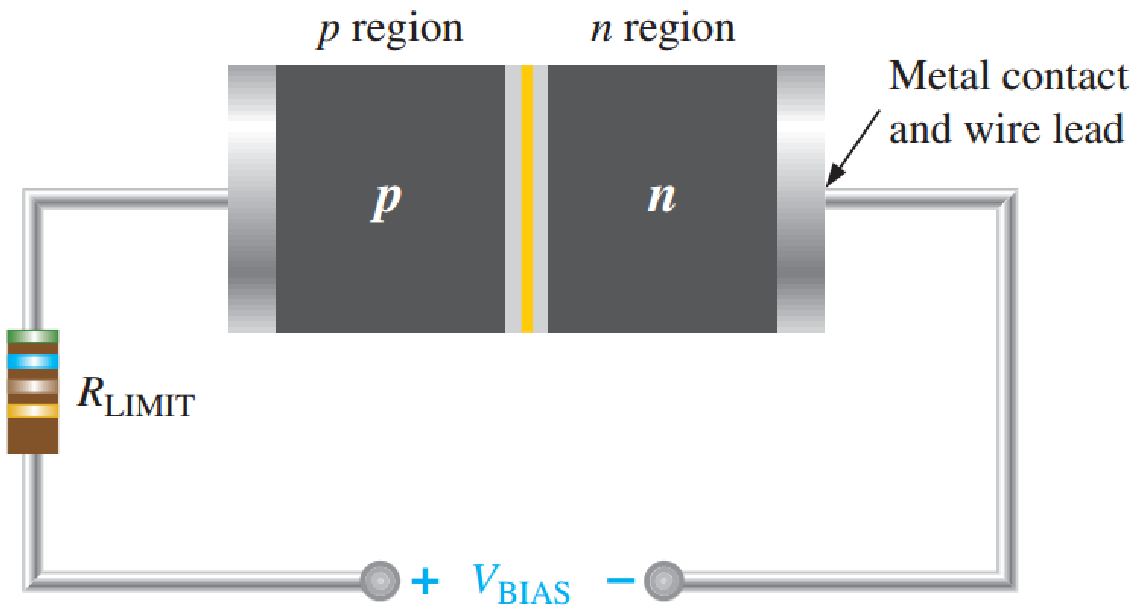



**FIGURE 2-4** A diode connected for forward bias.

# Forward Bias

- Like charges repel, the negative side of the bias-voltage source “pushes” the free electrons, which are the majority carriers in the *n* region, toward the *pn* junction. This flow of free electrons is called electron current (Forward current).

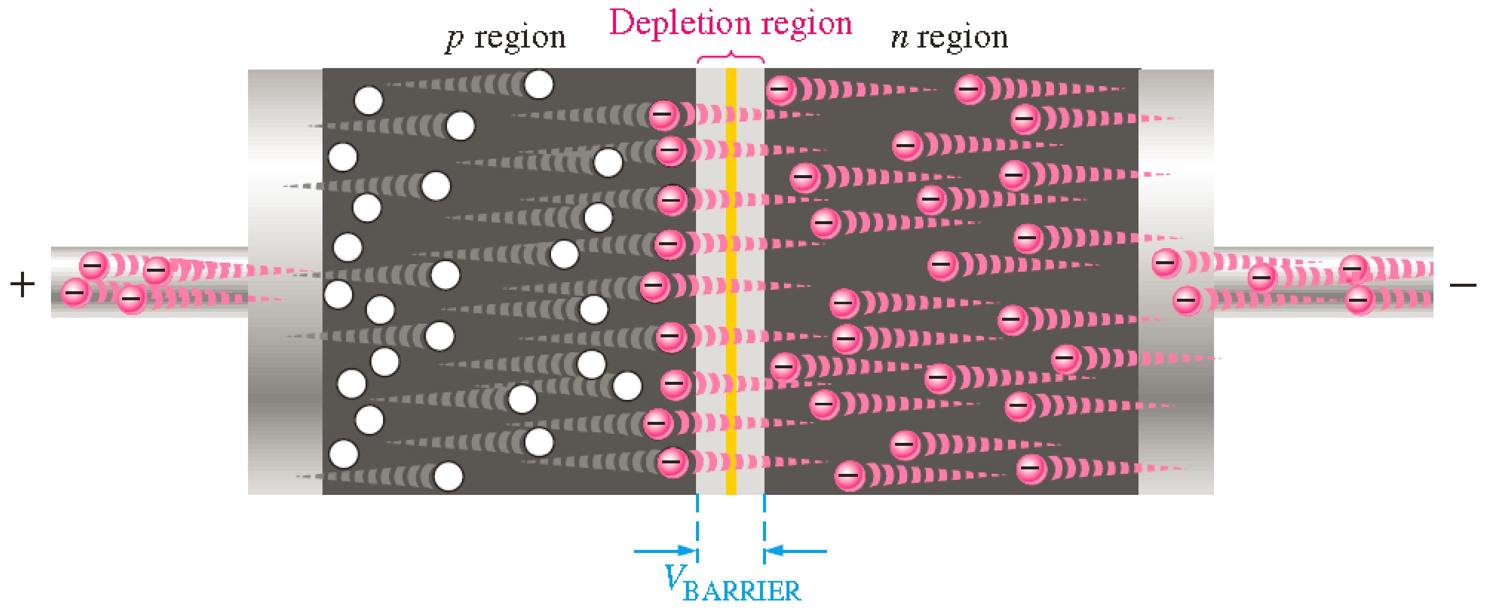



**FIGURE 2-5** A forward-biased diode showing the flow of majority carriers.

# Forward Bias

- As electrons from the *n* side are pushed into the depletion region, they combine with holes on the *p* side, effectively reducing the depletion region. This process during forward bias causes the depletion region to narrow.

- The electrons give up an amount of energy equivalent to the barrier potential when they cross the depletion region. This energy loss results in a voltage drop across the *pn* junction equal to the barrier potential (0.7 V).

- An additional small voltage drop occurs across the *p* and *n* regions due to the internal resistance of the material. For doped semiconductive material, this resistance, called the dynamic resistance, is very small and can usually be neglected.

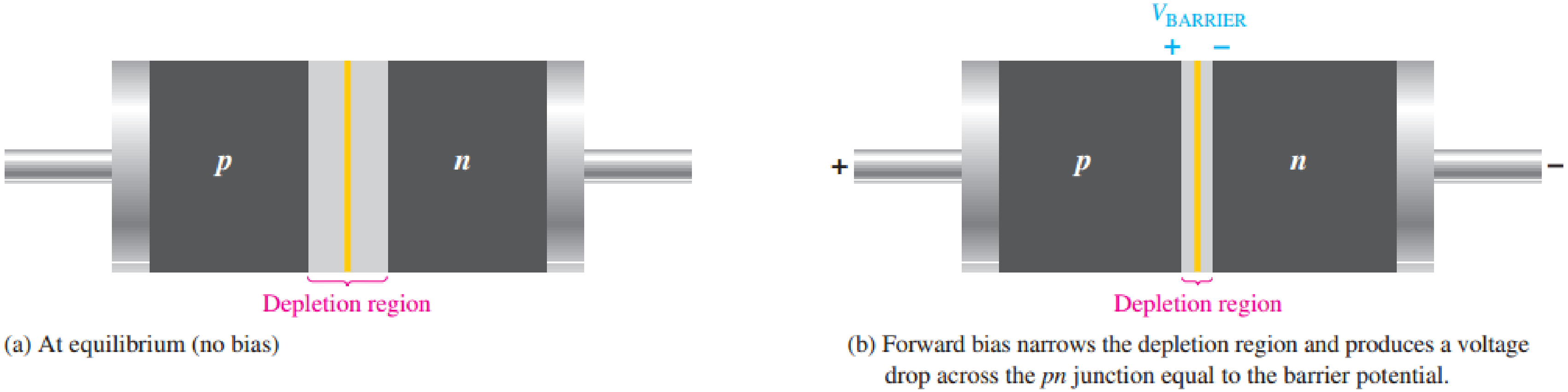



**FIGURE 2-6** The depletion region narrows and a voltage drop is produced across the *pn* junction when the diode is forward-biased.

# Reverse Bias

- Reverse bias is the condition that essentially prevents current through the diode.
- This external bias voltage is designated as $V_{\text{BIAS}}$ just as it was for forward bias.
- Reverse bias : The positive side of $V_{\text{BIAS}}$ is connected to the *n* region of the diode and the negative side is connected to the *p* region.
- <u>The depletion region is shown much wider</u> than in forward bias or equilibrium.

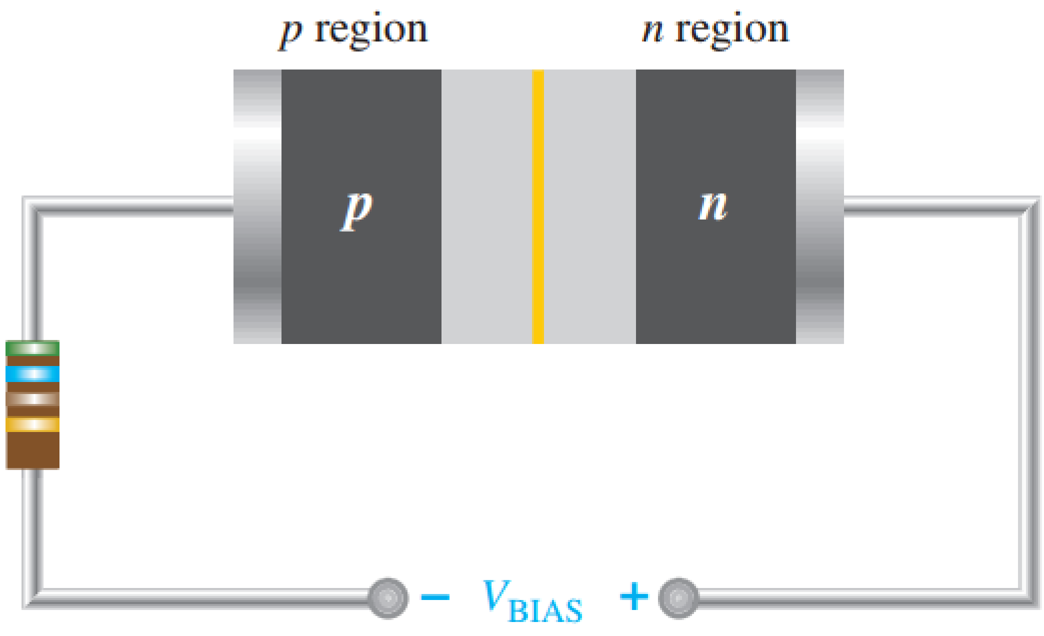



**FIGURE 2-7** A diode connected for reverse bias. A limiting resistor is shown although it is not important in reverse bias because there is essentially no current.

# Reverse Bias

- The extremely small current that exists in reverse bias is caused by the minority carriers in the *n* and *p* regions that are produced by thermally generated electron-hole pairs. The small number of free minority electrons in the *p* region are "pushed" toward the *pn* junction by the negative bias voltage.

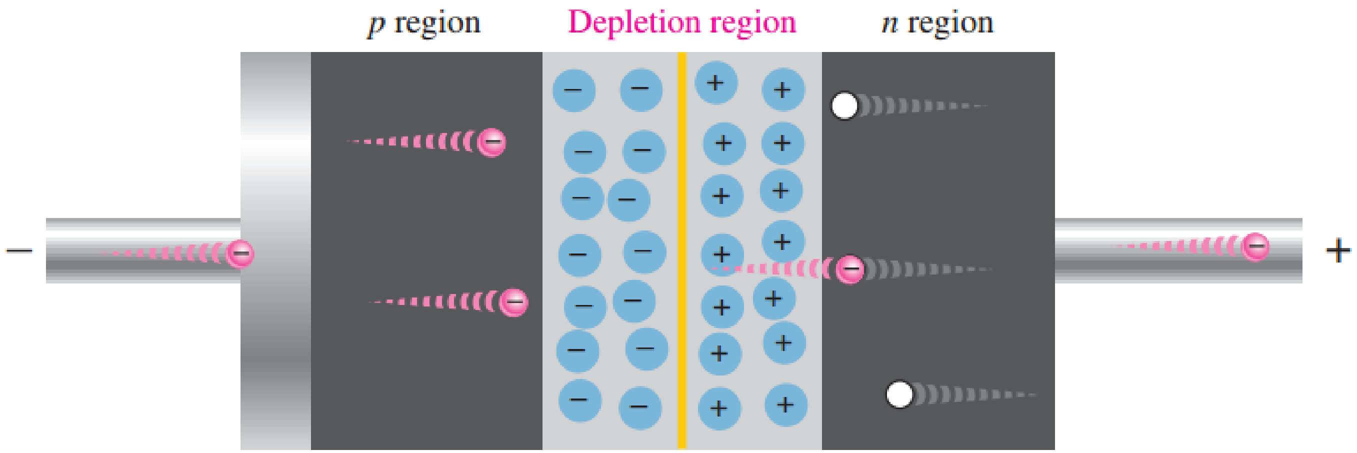



**FIGURE 2-8** The extremely small reverse current in a reverse-biased diode is due to the minority carriers from thermally generated electron-hole pairs.

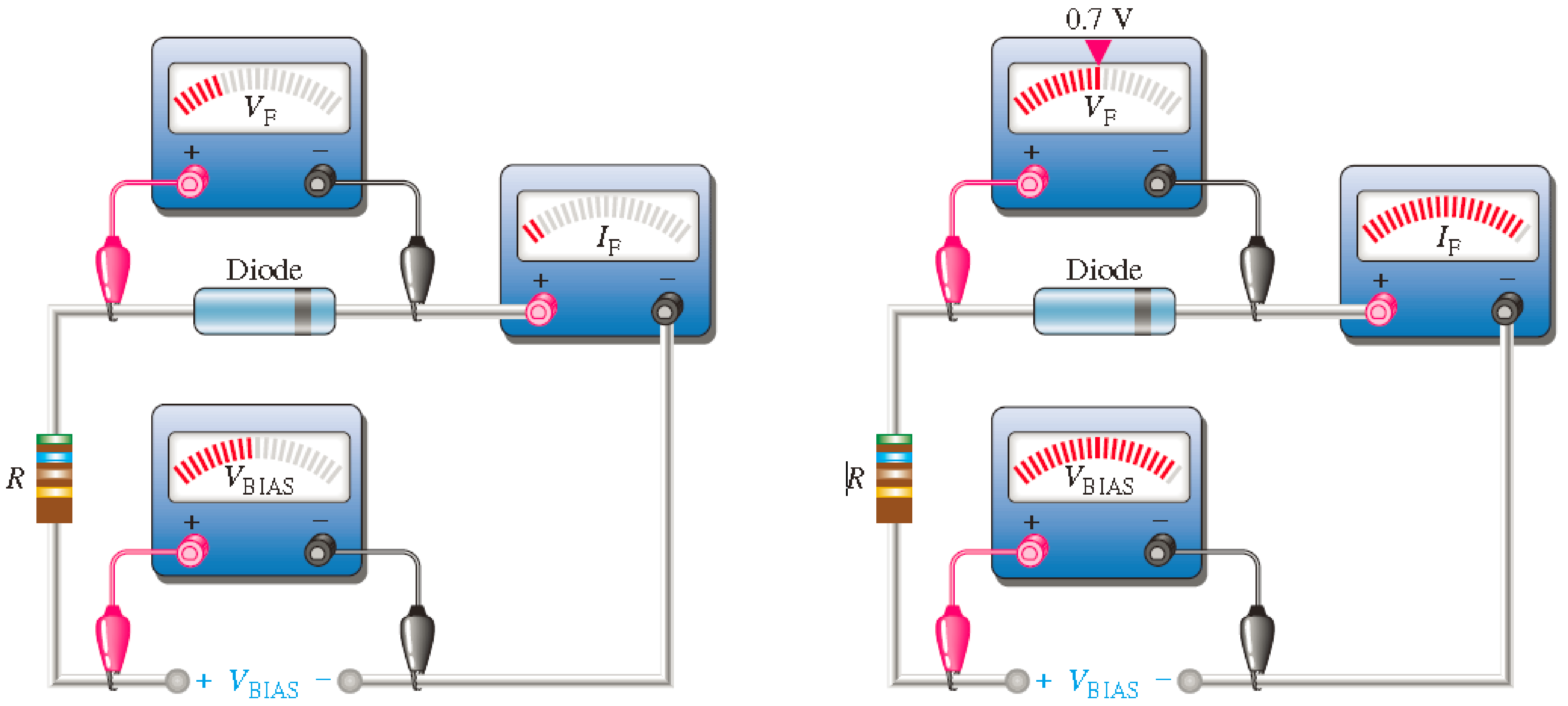
# Reverse Breakdown

- Normally, the reverse current is so small that it can be neglected. However, if the external reverse-bias voltage is increased to a value called the breakdown voltage, the reverse current will drastically increase.

- The high reverse-bias voltage imparts energy to the free minority electrons so that as they speed through the *p* region, they collide with atoms with enough energy to knock valence electrons into the conduction band. The newly created conduction electrons are also high in energy and repeat the process.

- The multiplication of conduction electrons happens, and reverse current can increase dramatically if steps are not taken to limit the current.

# 2.2 VOLTAGE-CURRENT CHARACTERISTIC OF A DIODE




(a) Small forward-bias voltage ($V_F < 0.7$ V), very small forward current.

(b) Forward voltage reaches and remains nearly constant at approximately 0.7 V. Forward current continues to increase as the bias voltage is increased.

**FIGURE 2–9**

Forward-bias measurements show general changes in $V_F$ and $I_F$ as $V_{BIAS}$ is increased.

# V-I Characteristic for Forward Bias

- The forward current increases very little until the forward voltage across the *pn* junction reaches approximately 0.7 V at the knee of the curve.
- After this point, the forward voltage remains nearly constant at approximately 0.7 V, but $I_F$ increases rapidly.
- Point A corresponds to a zero-bias condition. At point B, the forward voltage is less than the potential barrier of 0.7 V. The forward voltage at point C is approximately equal to the potential barrier.
- As the external bias voltage and forward current continue to increase above the knee, the forward voltage will increase slightly above 0.7 V.

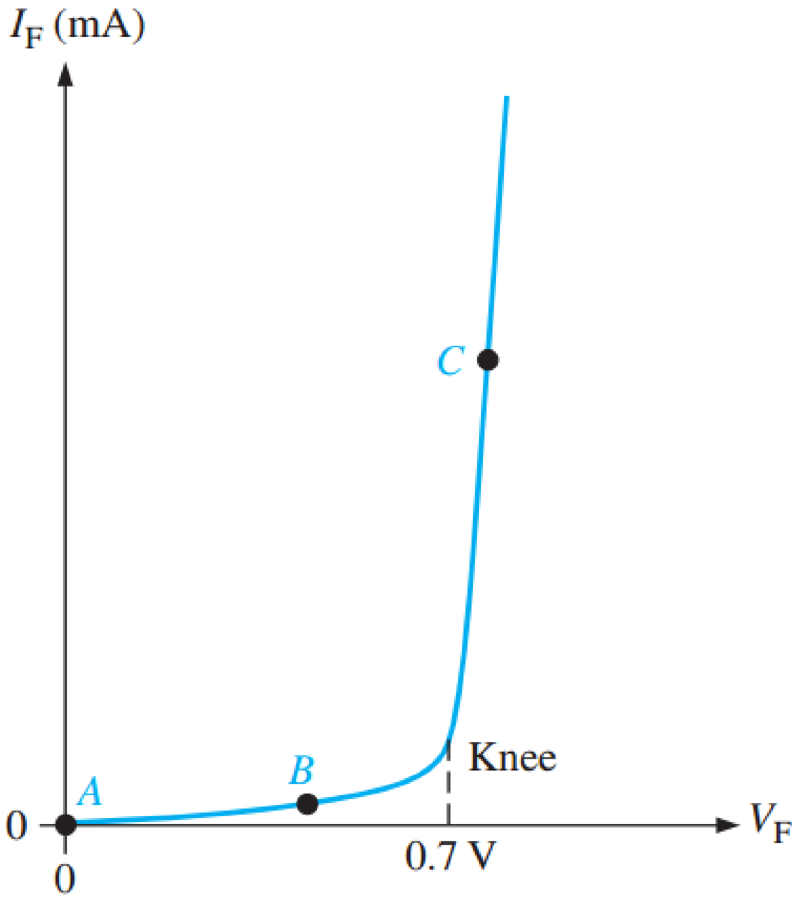



**FIGURE 2-10**
V-I characteristic curve for forward bias.

# V-I Characteristic for Reverse Bias

- When a reverse-bias voltage is applied across a diode, there is only an extremely small reverse current ($I_R$) through the *pn* junction.
- With 0 V across the diode, there is no reverse current.
- As you gradually increase the reverse-bias voltage, there is a very small reverse current and the voltage across the diode increases.
- When the applied bias voltage is increased to a value where the reverse voltage across the diode ($V_R$) reaches the breakdown value ($V_{BR}$), the reverse current begins to increase rapidly.

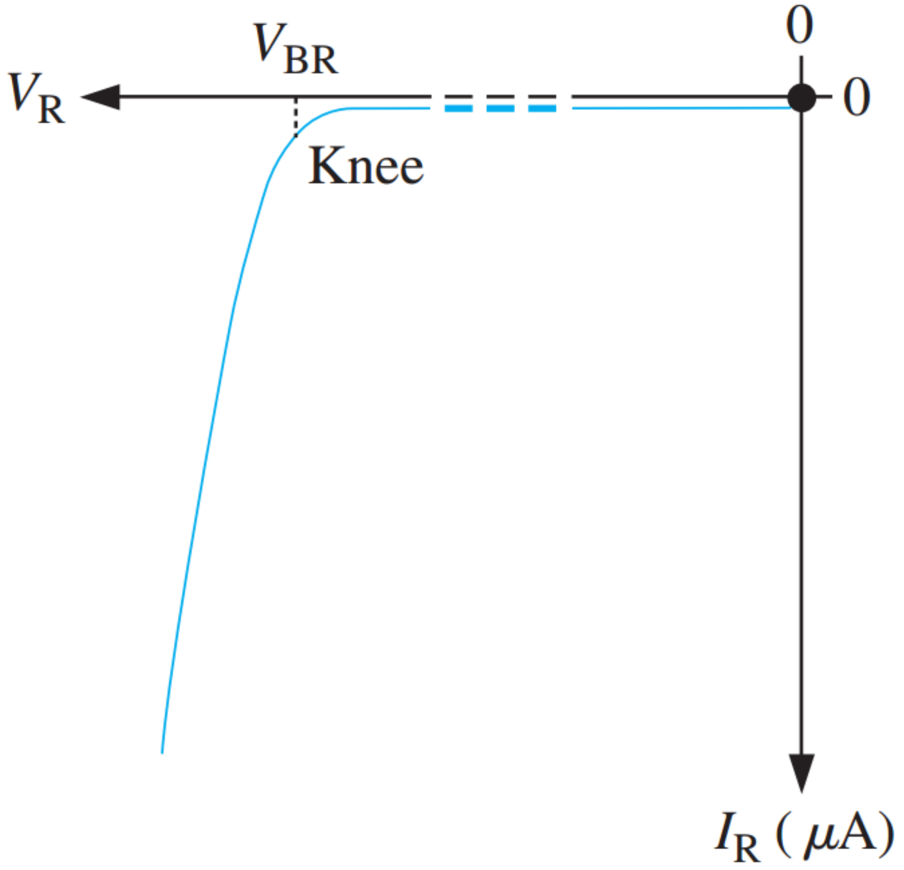



**FIGURE 2-11**
V-I characteristic curve for a reversebiased diode.

# The Complete V-I Characteristic Curve

- For a forward-biased diode, as temperature is increased, the forward current increases for a given value of forward voltage. For a given value of forward current, the forward voltage decreases.

- This is shown with the V-I characteristic curves in Figure 2–12. The barrier potential decreases by 2 mV for each degree increase in temperature.

- For a reverse-biased diode, as temperature is increased, the reverse current increases.

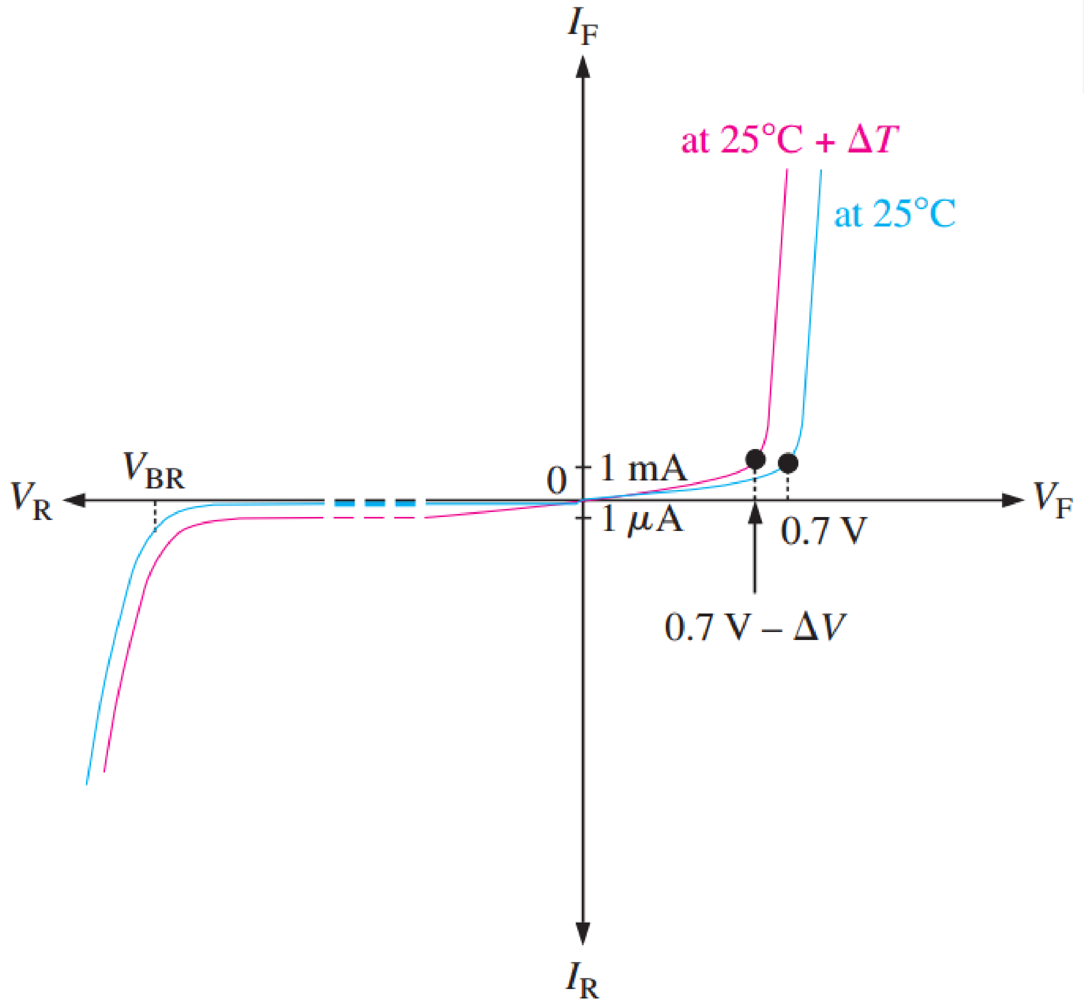



**FIGURE 2-12** Temperature effect on the diode V-I characteristic. The 1 mA and 1 μA marks on the vertical axis are given as a basis for a relative comparison of the current scales.

# 2.3 DIODE APPROXIMATIONS Ideal Model

- The ideal model of a diode is the least accurate approximation and can be represented by a simple switch.
- When the diode is forward-biased, it ideally acts like a closed (on) switch. When the diode is reverse-biased, it ideally acts like an open (off) switch.
- Since the barrier potential and the forward dynamic resistance are neglected, the diode is assumed to have a zero voltage across it when forward-biased.

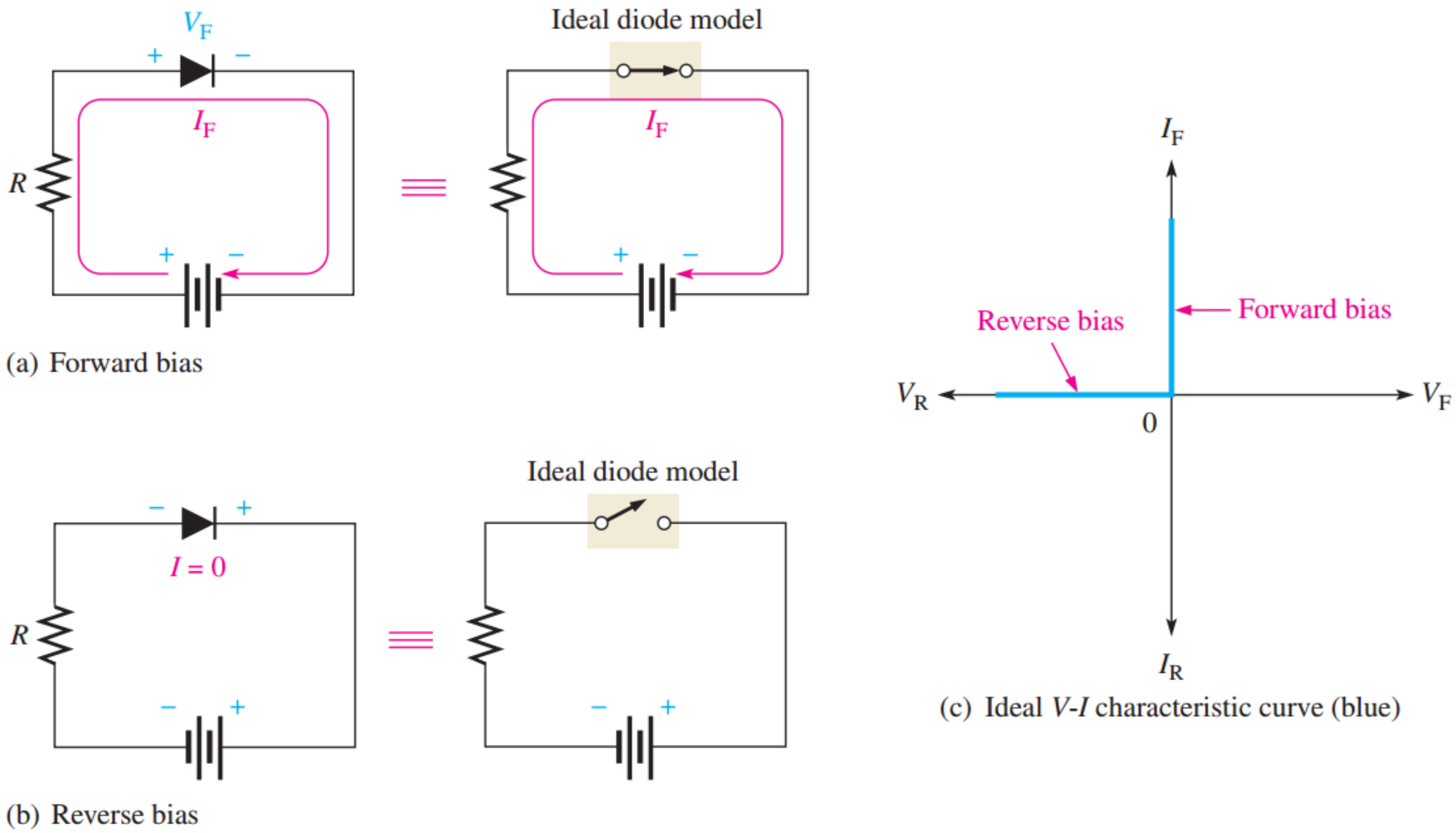


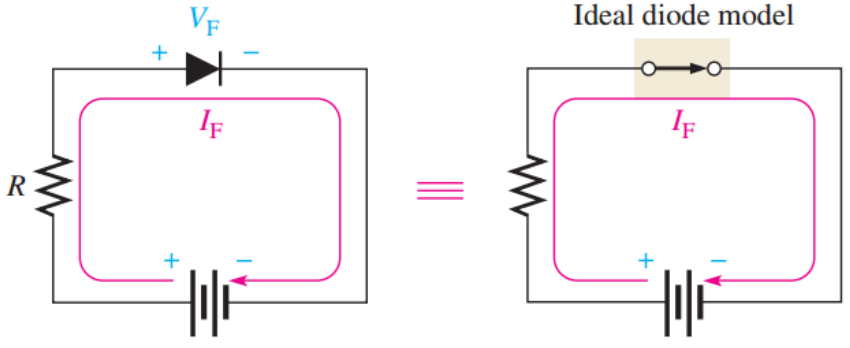

**FIGURE 2-13** The ideal model of a diode.

# Diode Approximations

$$V_F = 0\text{ V}$$

- The forward current is determined by the bias voltage and the limiting resistor using Ohm's law.

$$I_F = \frac{V_{BIAS}}{R_{LIMIT}}$$



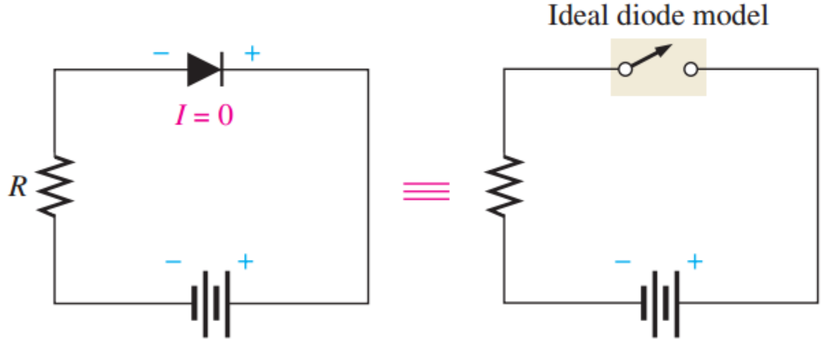
(a) Forward bias

- Since the reverse current is neglected, its value is assumed to be zero.

$$I_R = 0\text{ A}$$

- The reverse voltage equals the bias voltage.

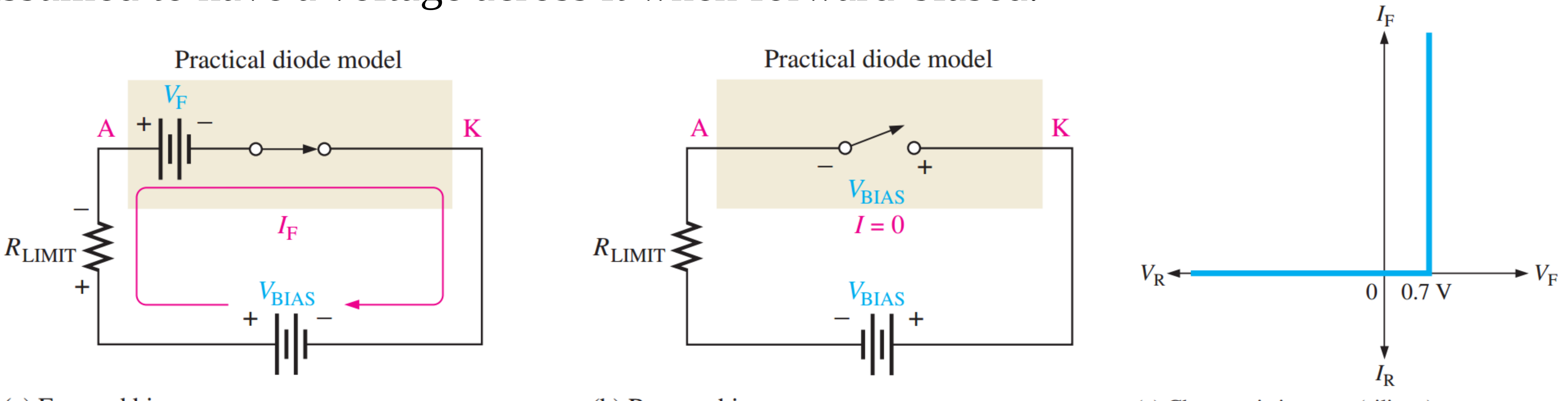
$$V_R = V_{BIAS}$$



(b) Reverse bias

# Diode Approximations  Practical model

- The practical model includes the barrier potential. When the diode is forward-biased, it is equivalent to a closed switch in series with a small equivalent voltage source ($V_F$) equal to the barrier potential (0.7 V) with the positive side toward the anode.
- When the diode is reverse-biased, it is equivalent to an open switch just as in the ideal model.
- Since the barrier potential is included and the dynamic resistance is neglected, the diode is assumed to have a voltage across it when forward-biased.




(a) Forward bias (b) Reverse bias (c) Characteristic curve (silicon)

**FIGURE 2-14** The practical model of a diode.

# Diode Approximations

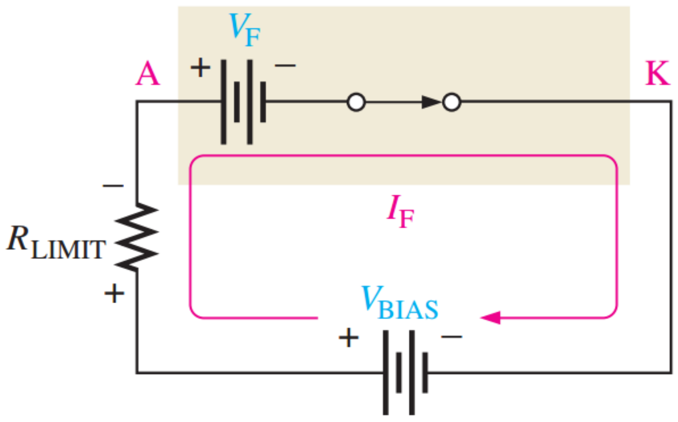
$$V_F = 0.7\ \text{V}$$

- The forward current is determined as follows by first applying Kirchhoff's voltage law.

$$V_{BIAS} - V_F - V_{R_{LIMIT}} = 0$$

$$V_{R_{LIMIT}} = I_F R_{LIMIT}$$

Practical diode model




(a) Forward bias

- Substituting and solving for $I_F$,

$$\boldsymbol{I_F = \frac{V_{BIAS} - V_F}{R_{LIMIT}}}$$

- The diode is assumed to have zero reverse current, as indicated by the portion of the curve on the negative horizontal axis.

$$I_R = 0\ \text{A}$$

$$V_R = V_{BIAS}$$

# Diode Approximations Complete model

- When the diode is forward-biased, it acts as a closed switch in series with the equivalent barrier potential voltage ($V_B$) and the small forward dynamic resistance ($r'_d$), as indicated in Figure 2–15(a). When the diode is reverse-biased, it acts as an open switch in parallel with the large internal reverse resistance ($r'_R$), as shown in Figure 2–15(b).

- Since the barrier potential and the forward dynamic resistance are included, the diode is assumed to have a voltage across it when forward-biased.

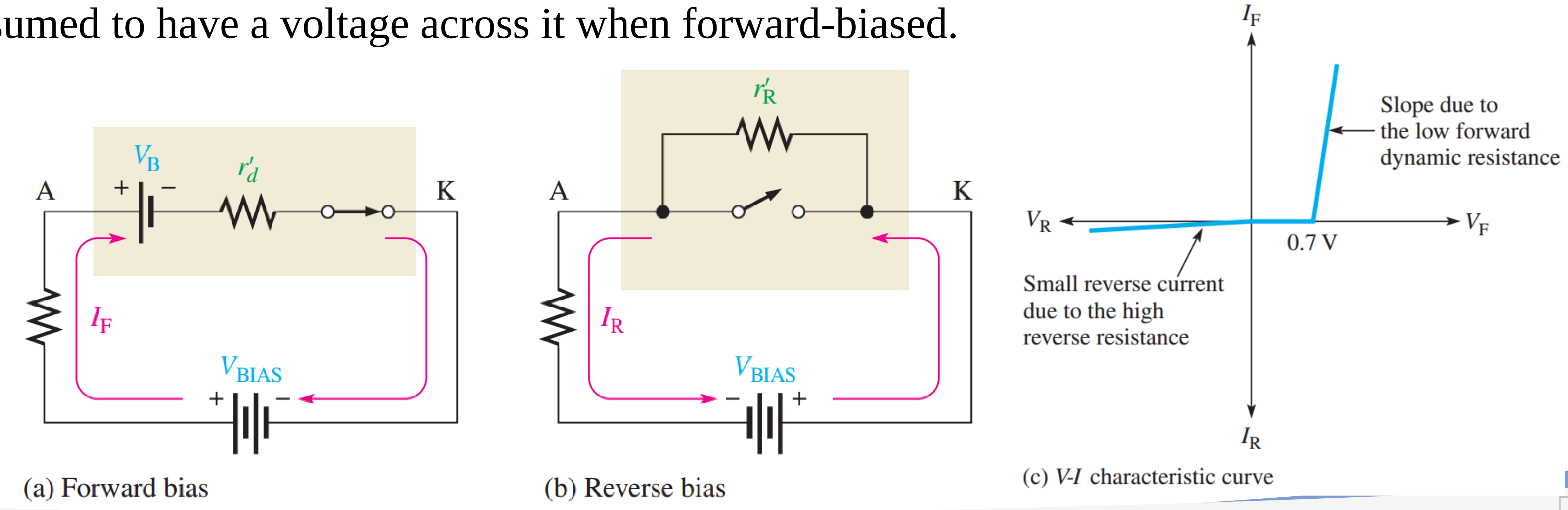



**FIGURE 2-15** The complete model of a diode.

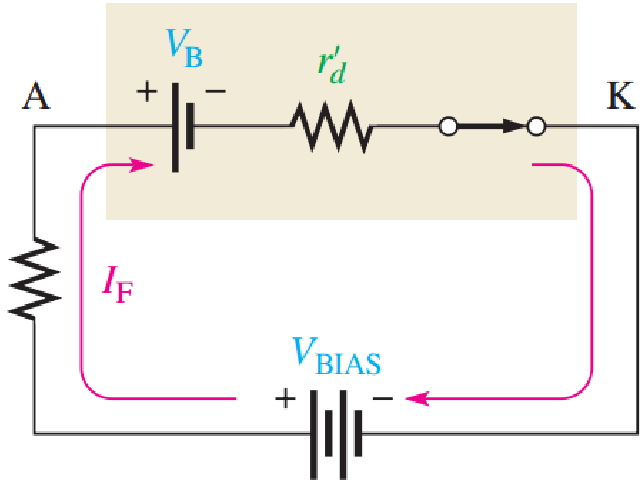
# Diode Approximations

- For the complete model of a silicon diode, the following formulas apply:

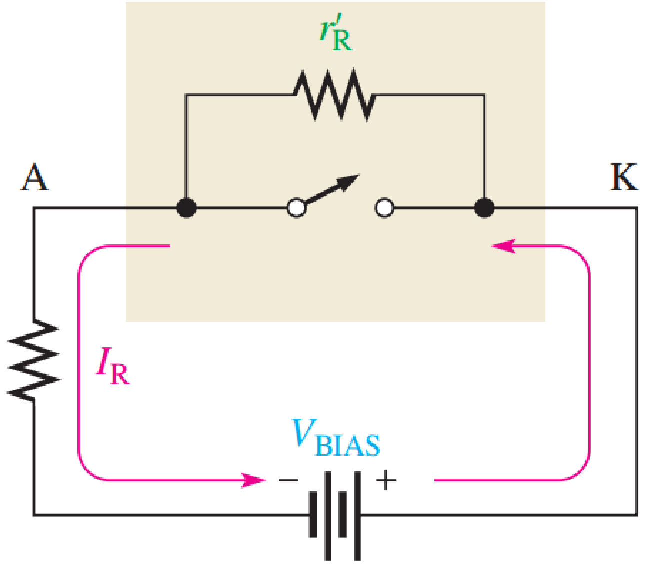
$$V_F = 0.7\text{ V} + I_F r'_d$$

$$I_F = \frac{V_{BIAS} - 0.7\text{ V}}{R_{LIMIT} + r'_d}$$

- The reverse current is taken into account with the parallel resistance and is indicated by the portion of the curve to the left of the origin.



(a) Forward bias



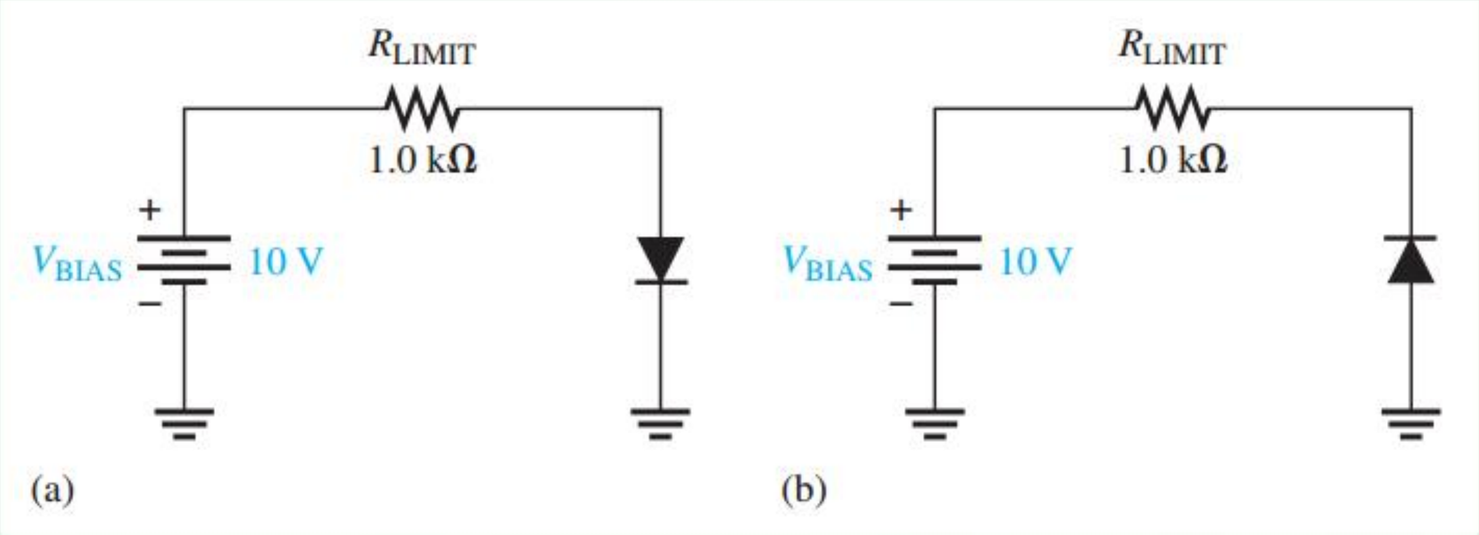
(b) Reverse bias

# Example

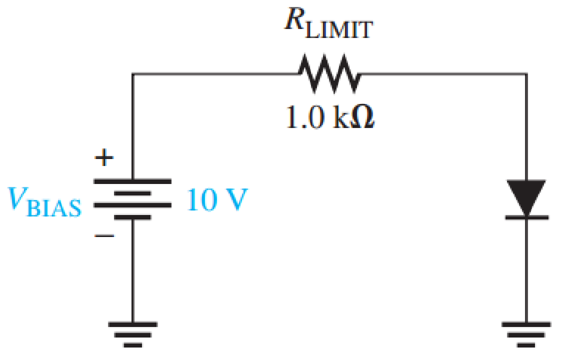
**EXAMPLE 2–1**

**(a)** Determine the forward voltage and forward current for the diode in Figure 2–18(a) for each of the diode models. Also find the voltage across the limiting resistor in each case. Assume $r'_d = 10\ \Omega$ at the determined value of forward current.

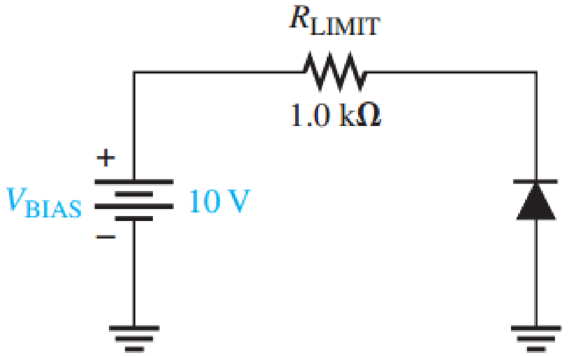
**(b)** Determine the reverse voltage and reverse current for the diode in Figure 2–18(b) for each of the diode models. Also find the voltage across the limiting resistor in each case. Assume $I_R = 1\ \mu A$.



# Example




(a)




(b)

*Solution* **(a)** Ideal model:

$$V_F = \mathbf{0\ V}$$

$$I_F = \frac{V_{BIAS}}{R_{LIMIT}} = \frac{10\text{ V}}{1.0\text{ k}\Omega} = \mathbf{10\ mA}$$

$$V_{R_{LIMIT}} = I_F R_{LIMIT} = (10\text{ mA})(1.0\text{ k}\Omega) = \mathbf{10\ V}$$

Practical model:

$$V_F = \mathbf{0.7\ V}$$

$$I_F = \frac{V_{BIAS} - V_F}{R_{LIMIT}} = \frac{10\text{ V} - 0.7\text{ V}}{1.0\text{ k}\Omega} = \frac{9.3\text{ V}}{1.0\text{ k}\Omega} = \mathbf{9.3\ mA}$$

$$V_{R_{LIMIT}} = I_F R_{LIMIT} = (9.3\text{ mA})(1.0\text{ k}\Omega) = \mathbf{9.3\ V}$$

Complete model:

$$I_F = \frac{V_{BIAS} - 0.7\text{ V}}{R_{LIMIT} + r'_d} = \frac{10\text{ V} - 0.7\text{ V}}{1.0\text{ k}\Omega + 10\ \Omega} = \frac{9.3\text{ V}}{1010\ \Omega} = \mathbf{9.21\ mA}$$

$$V_F = 0.7\ V + I_F r'_d = 0.7\text{ V} + (9.21\text{ mA})(10\ \Omega) = \mathbf{792\ mV}$$

$$V_{R_{LIMIT}} = I_F R_{LIMIT} = (9.21\text{ mA})(1.0\text{ k}\Omega) = \mathbf{9.21\ V}$$

**(b)** Ideal model:

$$I_R = \mathbf{0\ A}$$

$$V_R = V_{BIAS} = \mathbf{10\ V}$$

$$V_{R_{LIMIT}} = \mathbf{0\ V}$$

Practical model:

$$I_R = \mathbf{0\ A}$$

$$V_R = V_{BIAS} = \mathbf{10\ V}$$

$$V_{R_{LIMIT}} = \mathbf{0\ V}$$

Complete model:

$$I_R = \mathbf{1\ \mu A}$$

$$V_{R_{LIMIT}} = I_R R_{LIMIT} = (1\ \mu\text{A})(1.0\text{ k}\Omega) = \mathbf{1\ mV}$$

$$V_R = V_{BIAS} - V_{R_{LIMIT}} = 10\text{ V} - 1\text{ mV} = \mathbf{9.999\ V}$$

# 2.4 THE ZENER DIODE

- The symbol for a zener diode is shown in Figure 2–16. Instead of a straight line representing the cathode, the zener diode has a bent line that reminds you of the letter Z (for zener).

- A zener diode is a silicon *pn* junction device that is designed for operation in the reverse-breakdown region. The breakdown voltage of a zener diode is set by carefully controlling the doping level during manufacture.

- When a diode reaches reverse breakdown, its voltage remains almost constant even though the current changes drastically, and this is the key to zener diode operation.

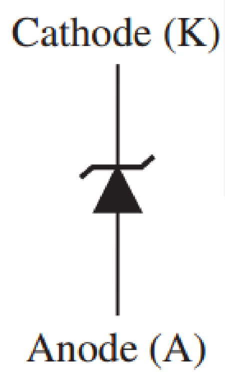



**FIGURE 2-16** Zener diode symbol.

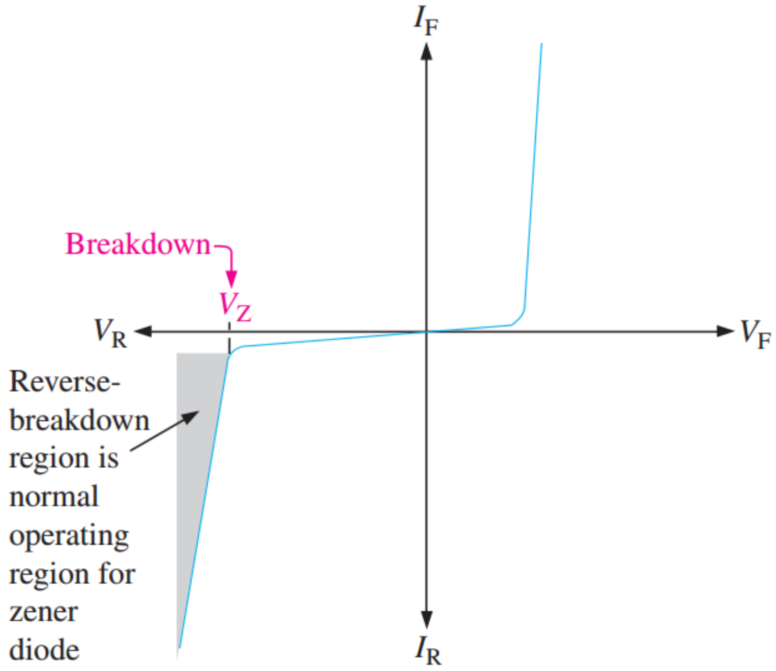



**FIGURE 2-17**
General zener diode V-I characteristic.

# Breakdown Characteristics

- A zener diode operating in breakdown can act as a low-current voltage regulator because it maintains a nearly constant voltage across its terminals over a specified range of reverse-current values.
- A minimum value of reverse current, $I_{ZK}$, must be maintained in order to keep the diode in breakdown for voltage regulation.
- Also, there is a maximum current, $I_{ZM}$, above which the diode may be damaged due to excessive power dissipation.
- Basically, the zener diode maintains a nearly constant voltage across its terminals for values of reverse current ranging from $I_{ZK}$ to $I_{ZM}$ .

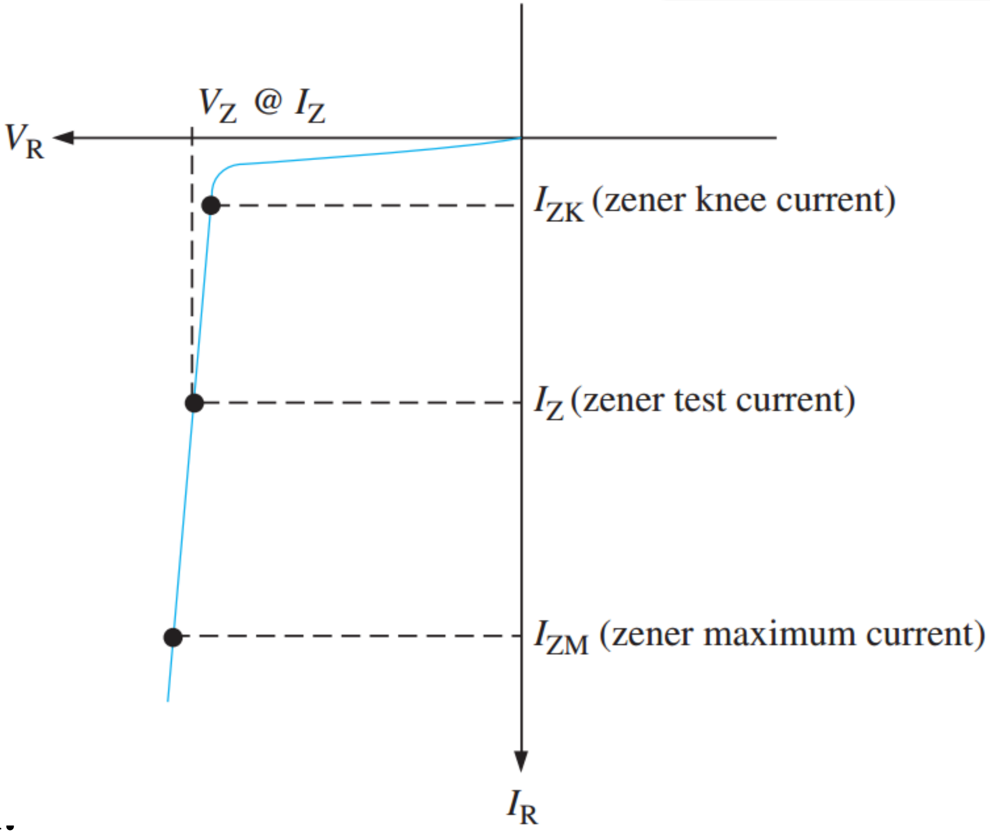



**FIGURE 2-18** Reverse characteristic of a zener diode. $V_Z$ is usually specified at a value of the zener current known as the test current.

# Zener Equivalent Circuits

- Figure 2–19 shows the ideal model (first approximation) of a zener diode in reverse breakdown and its ideal characteristic curve.
- It has a constant voltage drop equal to the nominal zener voltage.

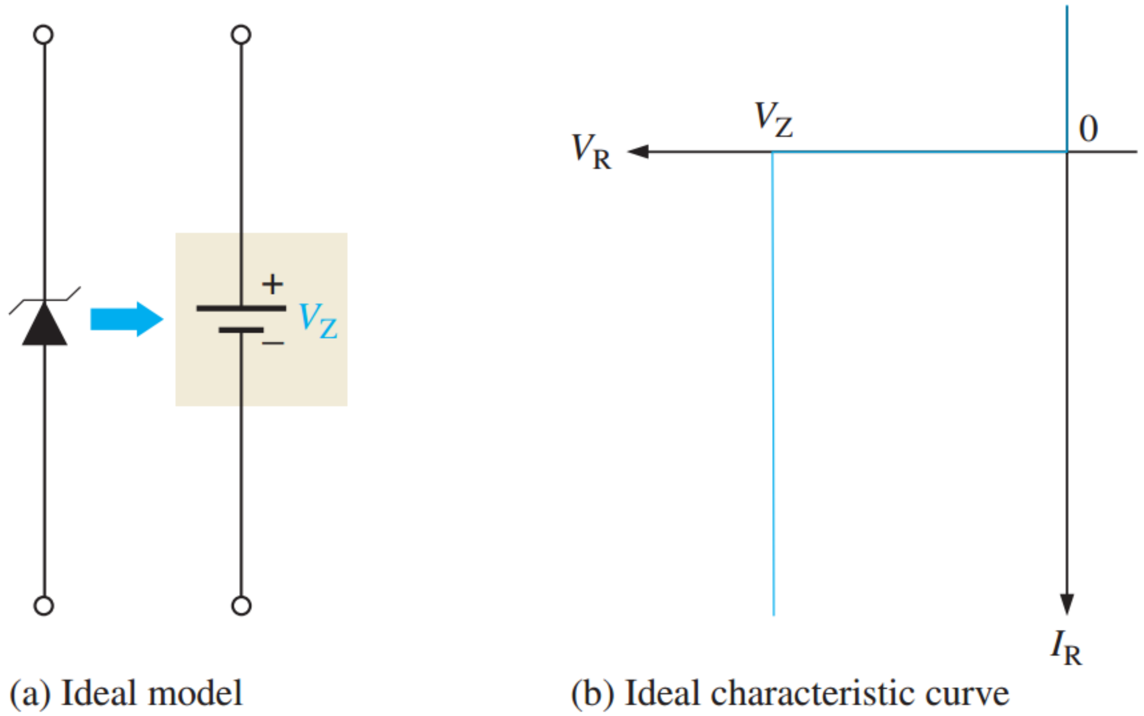



**FIGURE 2-19** Ideal zener diode equivalent circuit model and the characteristic curve.

# Zener Equivalent Circuits

- Figure 2–20(a) represents the practical model of a zener diode, where the zener impedance (resistance), $Z_Z$, is included.
- By Ohm's law, the ratio of zener voltage ($\triangle V_Z$) to zener current ($\triangle I_Z$) is the impedance.
- Because $Z_Z$ is defined as a change in voltage over a change in current, it is a dynamic (or ac) resistance. In most cases, you can assume that $Z_Z$ is a small constant over the full range of zener current values and is purely resistive.

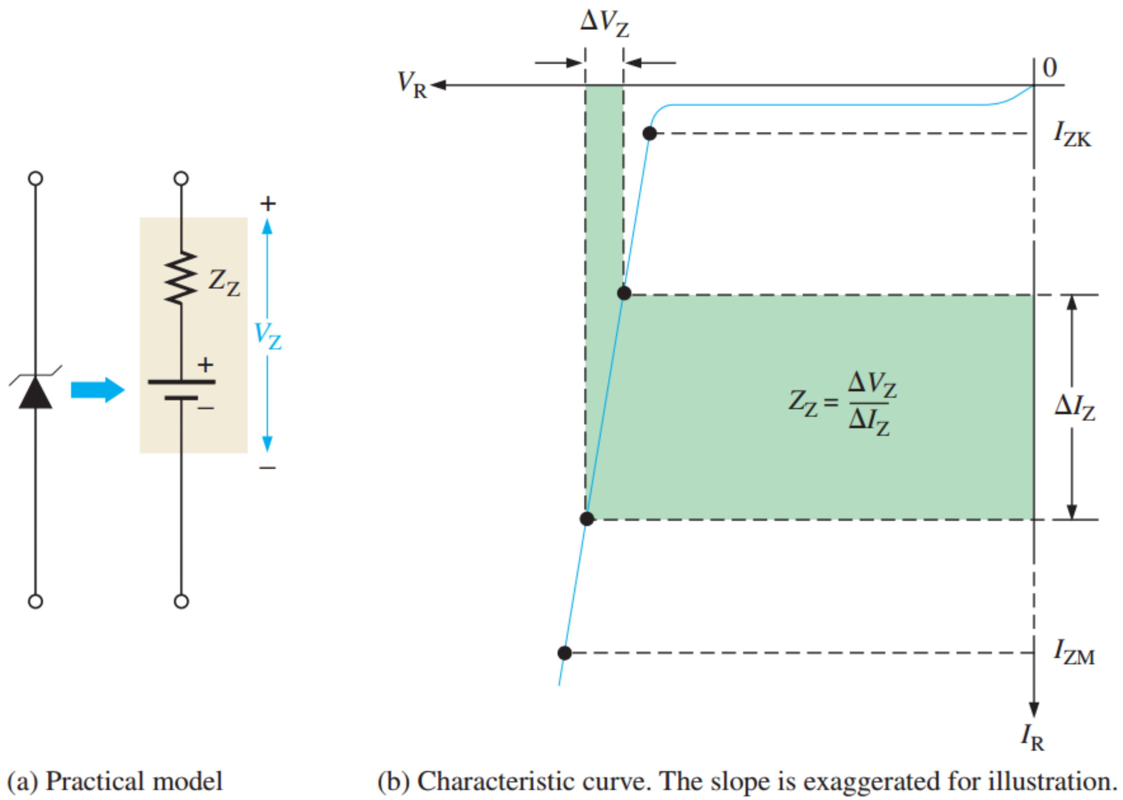


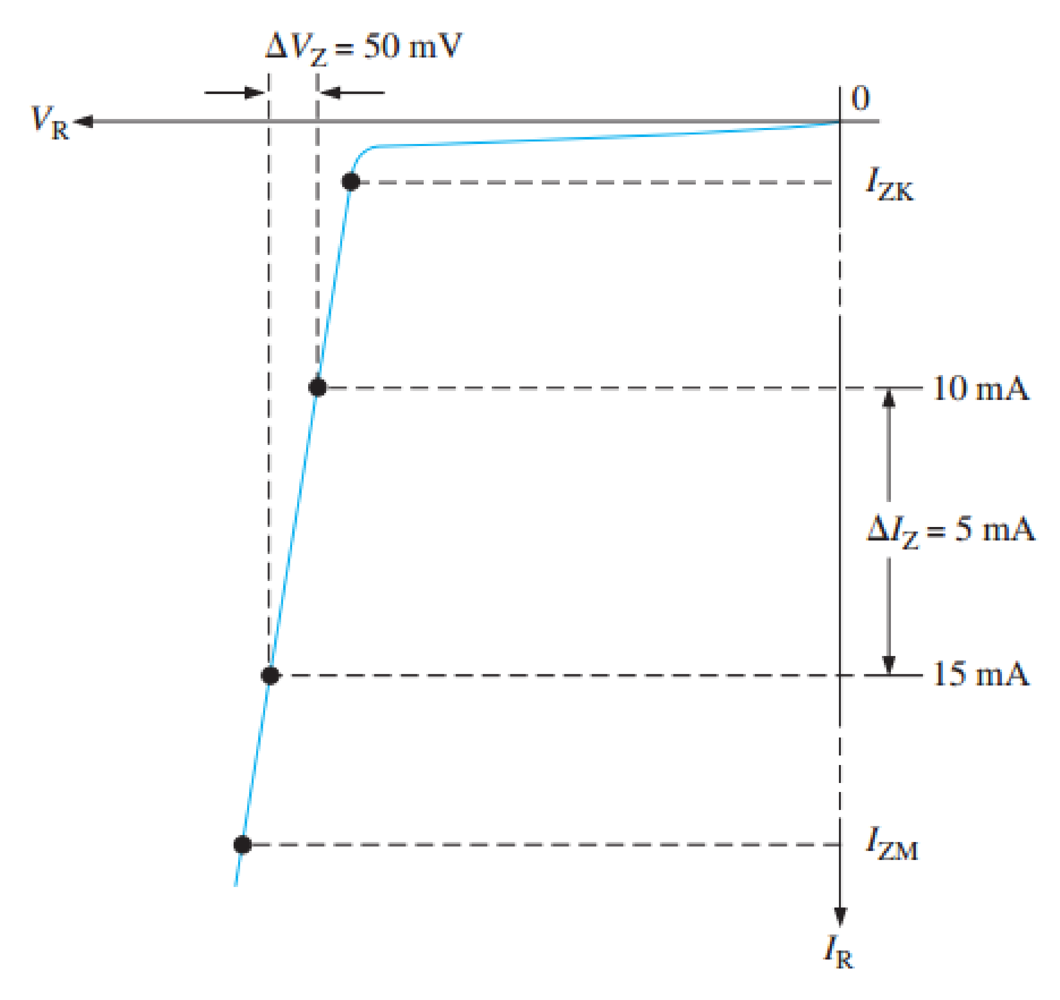

**FIGURE 2-20** Practical zener diode equivalent circuit and the characteristic curve illustrating $Z_Z$.

# Example

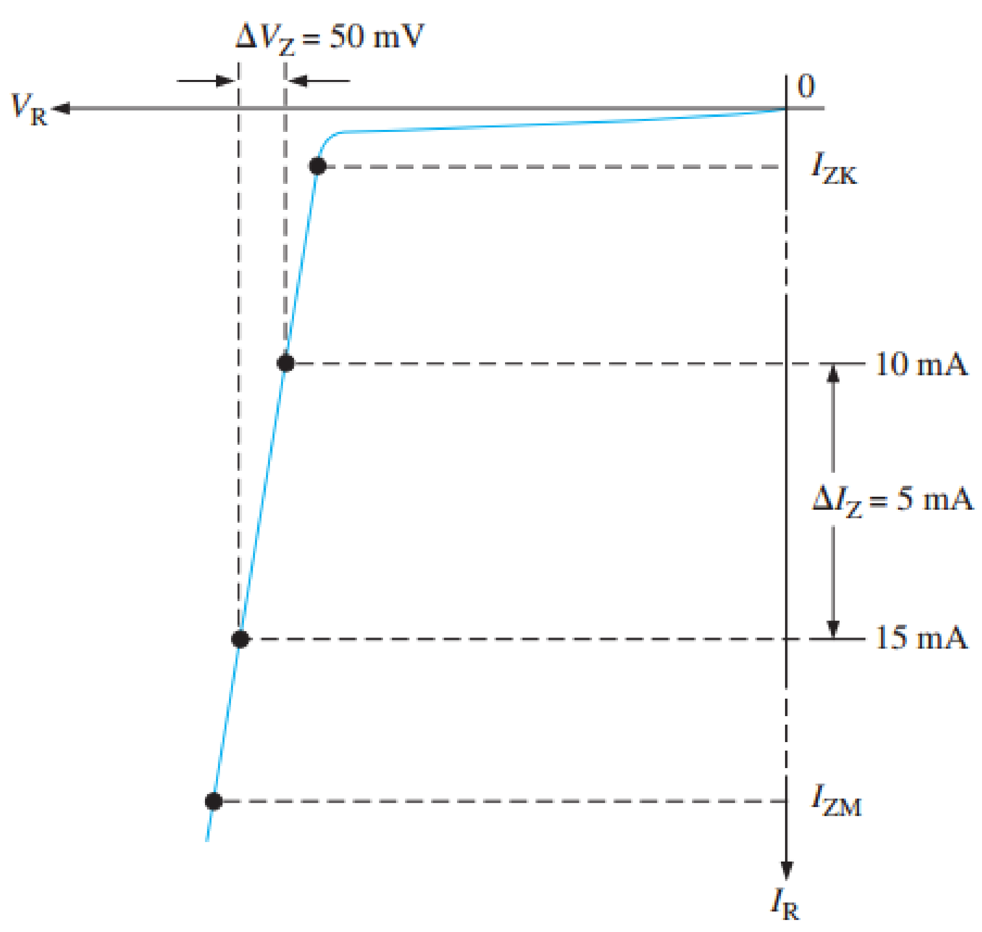
**EXAMPLE 3–1**

A zener diode exhibits a certain change in $V_Z$ for a certain change in $I_Z$ on a portion of the linear characteristic curve between $I_{ZK}$ and $I_{ZM}$ as illustrated in Figure 3–6. What is the zener impedance?

▶ FIGURE 3–6



# Example




*Solution*

$$Z_Z = \frac{\Delta V_Z}{\Delta I_Z} = \frac{50\text{ mV}}{5\text{ mA}} = \mathbf{10\ \Omega}$$

# Zener Power Dissipation and Derating

- For a zener, the voltage drop is $V_Z$ and the current in the device is $I_Z$. The power dissipated is simply $P = V_Z I_Z$.
- The maximum power dissipation of a zener diode is typically specified for temperatures at or below a certain value (50°C, for example). Above the specified temperature, the maximum power dissipation is reduced according to a derating factor.
- The derating factor is expressed in mW/°C. The maximum derated power can be determined with the following formula:

$$P_{D(derated)} = P_D - (\text{mW/°C})\Delta T$$

# Example

A certain zener diode has a maximum power rating of 400 mW at 50°C and a derating factor of 3.2 mW/°C. Determine the maximum power the zener can dissipate at a temperature of 90°C.

# Example

A certain zener diode has a maximum power rating of 400 mW at 50°C and a derating factor of 3.2 mW/°C. Determine the maximum power the zener can dissipate at a temperature of 90°C.

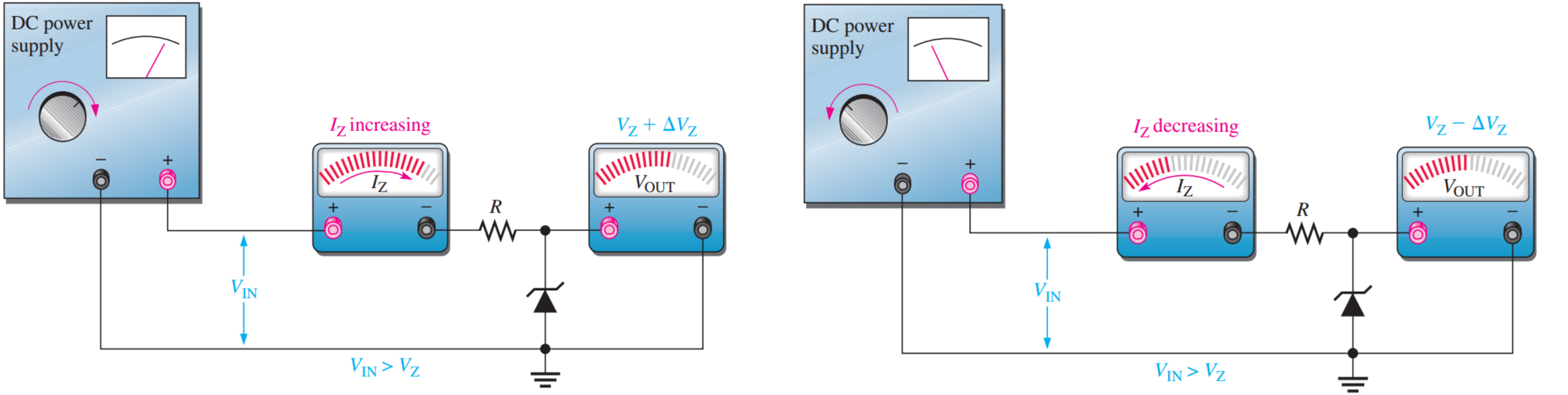
*Solution*

$$
\begin{aligned}
P_{D(\text{derated})} &= P_D - (\text{mW}/^\circ\text{C})\Delta T \\
&= 400\ \text{mW} - (3.2\ \text{mW}/^\circ\text{C})(90^\circ\text{C} - 50^\circ\text{C}) \\
&= 400\ \text{mW} - 128\ \text{mW} = \mathbf{272\ mW}
\end{aligned}
$$

# 2.5 Zener Diode Applications

- As the input voltage varies (within limits), the zener diode maintains a nearly constant output voltage across its terminals.

- However, as $V_{IN}$ changes, $I_Z$ will change proportionally so that the <u>limitations on the input voltage variation are set by the minimum and maximum current values ($I_{ZK}$ and $I_{ZM}$)</u> with which the zener can operate.

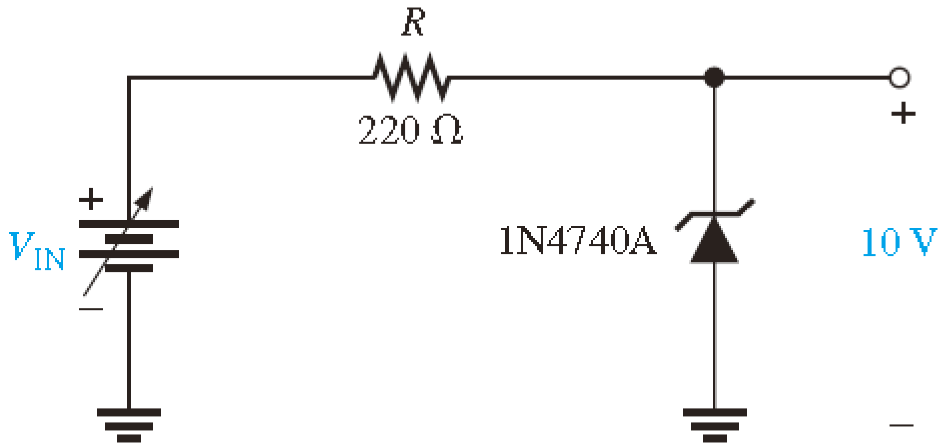



(a) As the input voltage increases, the output voltage remains nearly constant ($I_{ZK} < I_Z < I_{ZM}$).

(b) As the input voltage decreases, the output voltage remains nearly constant ($I_{ZK} < I_Z < I_{ZM}$).

**FIGURE 2-21** Zener regulation of a varying input voltage.

# Example




The absolute lowest current that will maintain regulation is specified at $I_{ZK}$, which for the 1N4740A is 0.25 mA and represents the no-load current. The maximum current is 100mA. Determine the minimum and the maximum input voltage.

For the minimum zener current, the voltage across the 220 Ω resistor is

$$V_R = I_{ZK}R = (0.25\text{ mA})(220\ \Omega) = 55\text{ mV}$$

Since $V_R = V_{IN} - V_Z$,

$$V_{IN(min)} = V_R + V_Z = 55\text{ mV} + 10\text{ V} = 10.055\text{ V}$$

For the maximum zener current, the voltage across the 220 Ω resistor is

$$V_R = I_{ZM}R = (100\text{ mA})(220\ \Omega) = 22\text{ V}$$

Therefore,

$$V_{IN(max)} = 22\text{ V} + 10\text{ V} = 32\text{ V}$$

# Zener Regulation with a Variable Load

- Figure 2–22 shows a zener voltage regulator with a variable load resistor across the terminals. The zener diode maintains a nearly constant voltage across $R_L$ as long as the zener current is greater than $I_{ZK}$ and less than $I_{ZM}$.

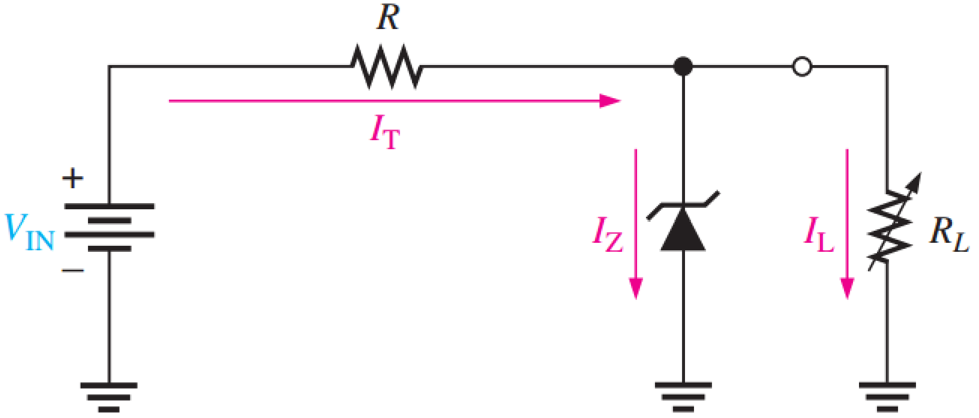
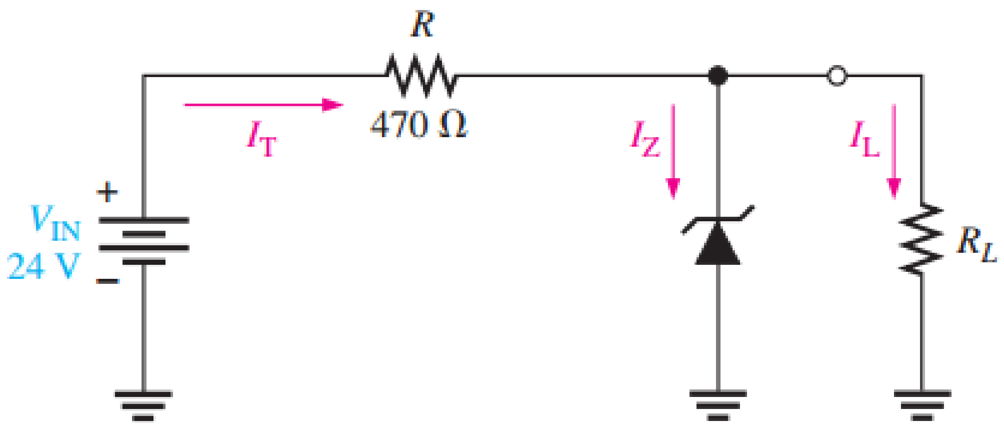



**FIGURE 2-22** Zener regulation with a variable load.

- When the output terminals of the zener regulator are open ($R_L = \infty$), the load current is zero and all of the current is through the zener; this is a no-load condition.
- When a load resistor ($R_L$) is connected, part of the total current is through the zener and part through $R_L$. The total current through $R$ remains essentially constant as long as the zener is regulating.
- As $R_L$ is decreased, the load current, $I_L$, increases and $I_Z$ decreases. The zener diode continues to regulate the voltage until $I_Z$ reaches its minimum value, $I_{ZK}$.

# Example




Determine the minimum and the maximum load currents for which the zener diode in Figure 3–14 will maintain regulation. What is the minimum value of $R_L$ that can be used? $V_Z = 12$ V, $I_{ZK} = 1$ mA, and $I_{ZM} = 50$ mA. Assume an ideal zener diode where $Z_Z = 0\ \Omega$ and $V_Z$ remains a constant 12 V over the range of current values, for simplicity.

# Example

## *Solution*

When $I_L = 0$ A ($R_L = \infty$), $I_Z$ is maximum and equal to the total circuit current $I_T$.

$$I_{Z(max)} = I_T = \frac{V_{IN} - V_Z}{R} = \frac{24\text{ V} - 12\text{ V}}{470\ \Omega} = 25.5\text{ mA}$$

If $R_L$ is removed from the circuit, the load current is 0 A. Since $I_{Z(max)}$ is less than $I_{ZM}$, 0 A is an acceptable minimum value for $I_L$ because the zener can handle all of the 25.5 mA.

$$I_{L(min)} = \mathbf{0\ A}$$

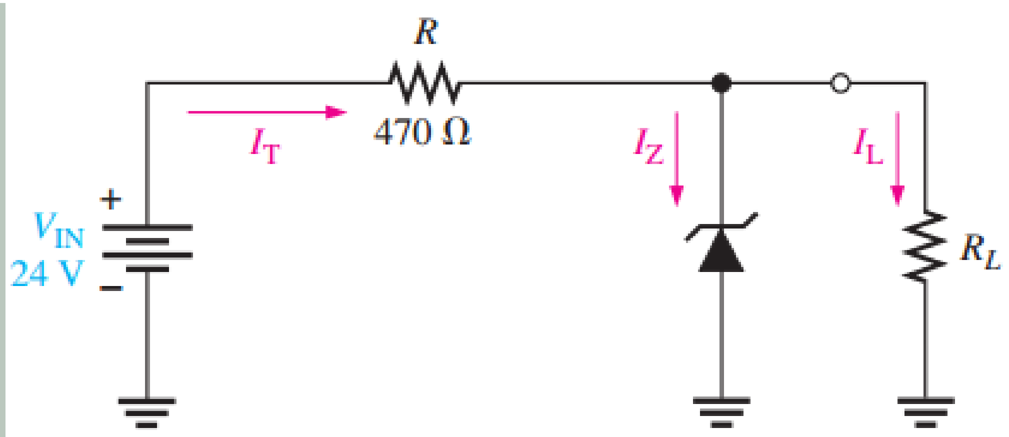
The maximum value of $I_L$ occurs when $I_Z$ is minimum ($I_Z = I_{ZK}$), so

$$I_{L(max)} = I_T - I_{ZK} = 25.5\text{ mA} - 1\text{ mA} = \mathbf{24.5\ mA}$$

The minimum value of $R_L$ is

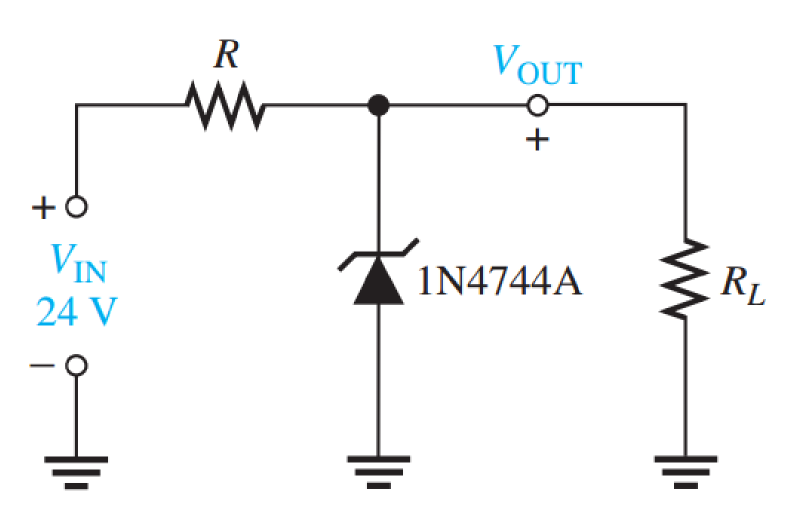
$$R_{L(min)} = \frac{V_Z}{I_{L(max)}} = \frac{12\text{ V}}{24.5\text{ mA}} = \mathbf{490\ \Omega}$$

Therefore, if $R_L$ is less than 490 Ω, $R_L$ will draw more of the total current away from the zener and $I_Z$ will be reduced below $I_{ZK}$. This will cause the zener to lose regulation. Regulation is maintained for any value of $R_L$ between 490 Ω and infinity.



# Example

FIGURE 3–15




For the circuit in Figure 3–15:

**(a)** Determine $V_{OUT}$ at $I_{ZK}$ and at $I_{ZM}$.

**(b)** Calculate the value of $R$ that should be used.

**(c)** Determine the minimum value of $R_L$ that can be used.

# Example

## Solution

The 1N4744A zener used in the regulator circuit of Figure 3–15 is a 15 V diode. The datasheet in Figure 3–7 gives the following information:
$V_Z = 15\text{ V} @ I_Z = 17\text{ mA}$, $I_{ZK} = 0.25\text{ mA}$, and $Z_Z = 14\ \Omega$.

**(a)** For $I_{ZK}$:

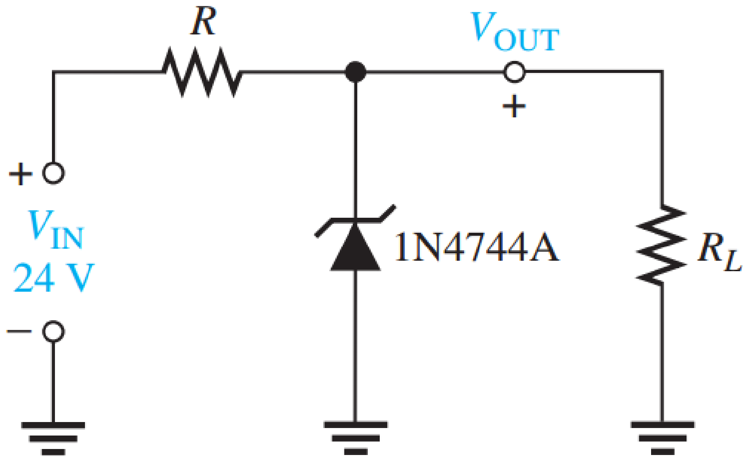
$$V_{OUT} = V_Z - \Delta I_Z Z_Z = 15\text{ V} - \Delta I_Z Z_Z = 15\text{ V} - (I_Z - I_{ZK})Z_Z$$
$$= 15\text{ V} - (16.75\text{ mA})(14\ \Omega) = 15\text{ V} - 0.235\text{ V} = \mathbf{14.76\ V}$$

Calculate the zener maximum current. The maximum power dissipation is 1 W.

$$I_{ZM} = \frac{P_D}{V_Z} = \frac{1\text{ W}}{15\text{ V}} = 66.7\text{ mA}$$

For $I_{ZM}$:

$$V_{OUT} = V_Z + \Delta I_Z Z_Z = 15\text{ V} + \Delta I_Z Z_Z$$
$$= 15\text{ V} + (I_{ZM} - I_Z)Z_Z = 15\text{ V} + (49.7\text{ mA})(14\ \Omega) = \mathbf{15.7\ V}$$



# Example

## *Solution*

**(b)** Calculate the value of $R$ for the maximum zener current that occurs when there is no load as shown in Figure 3–16(a).

$$R = \frac{V_{IN} - V_{OUT}}{I_{ZK}} = \frac{24\text{ V} - 15.7\text{ V}}{66.7\text{ mA}} = 124\ \Omega$$

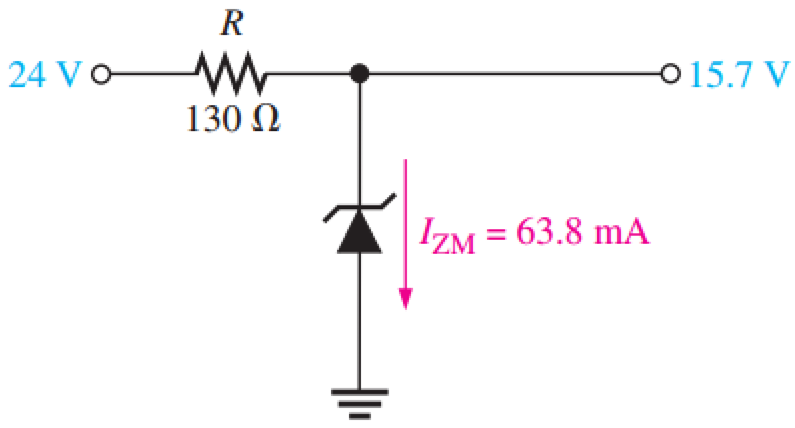
$R = \mathbf{130\ \Omega}$ (nearest larger standard value), which reduces $I_{ZM}$ to 63.8 mA.

**(c)** For the minimum load resistance (maximum load current), the zener current is minimum ($I_{ZK} = 0.25$ mA) as shown in Figure 3–16(b).

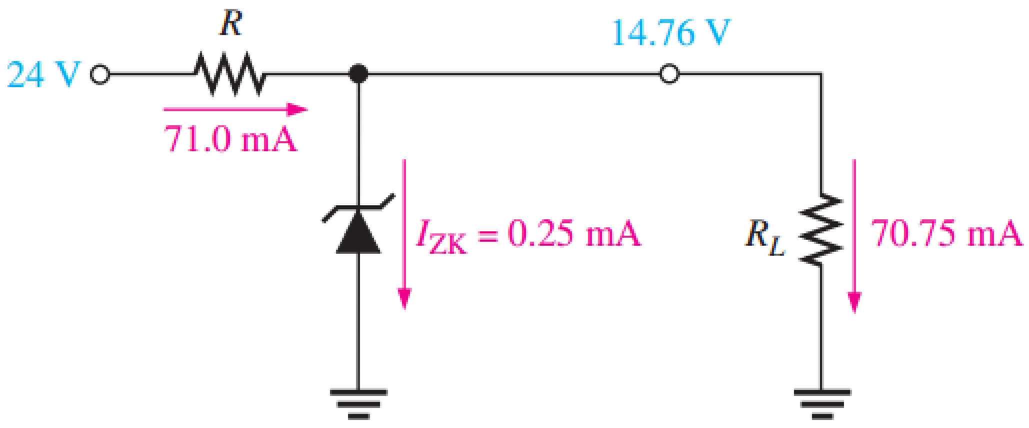
$$I_T = \frac{V_{IN} - V_{OUT}}{R} = \frac{24\text{ V} - 14.76\text{ V}}{130\ \Omega} = 71.0\text{ mA}$$

$$I_L = I_T - I_{ZK} = 71.0\text{ mA} - 0.25\text{ mA} = 70.75\text{ mA}$$

$$R_{L(min)} = \frac{V_{OUT}}{I_L} = \frac{14.76\text{ V}}{70.75\text{ mA}} = \mathbf{209\ \Omega}$$




(a)




(b)