Practical Superconducting Qubit Design Parameters, OJB

(Found in Chapter 24 of Prof. Hiu Wong's Quantum Computing Architecture and Hardware for Engineers book, see reference section)

Resonator length design equation (57:18)
https://youtu.be/kPrEJo60p5s?si=0uKJz-pZd5CPxZhg&t=3438

Transmon design equations (102:00)
https://youtu.be/kPrEJo60p5s?si=8QZQten1XjQyUCO5&t=3720

Example single qubit gate design equations (102:27)
https://youtu.be/kPrEJo60p5s?si=j1pS4xu31QPxNC_F&t=3747

Translate to Circuit Parameters

QUANTUM WEEK
Tutorial 04

$$\alpha > 0.1\omega_{10}$$

$$\frac{E_J}{E_c} > 50$$

$$R_{n,min} < R_n < R_{n,max}$$

$$\begin{aligned}\alpha &= E_{21} - E_{10},\\ &= -E_c.\end{aligned}$$

$$E_c = \frac{1}{2}\frac{|e|^2}{C}.$$

Get: C_g+C_J+ C_S

$$\begin{aligned}E_{10} &= \hbar\omega_0,\\ &= \sqrt{8E_cE_J} - E_c.\end{aligned}$$

$$E_J = \frac{I_c\Phi_0}{2\pi}.$$

Get: I_c

$$\Delta(T=0) = 1.764kT_c$$

$$I_c = \frac{\pi\Delta}{2R_n|e|}$$

$$I_cR_n = \frac{\pi\Delta}{2|e|}$$

Get: R_n

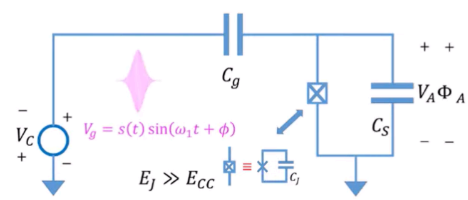


Fig. 21.1 A transmon with AC external voltage for 1-qubit implementation. The circuit is the same as that in Fig. 20.3 except the external voltage only has AC component with an envelope function, $s(t)$.

24.3.2 *Design of Feedline and Resonator*

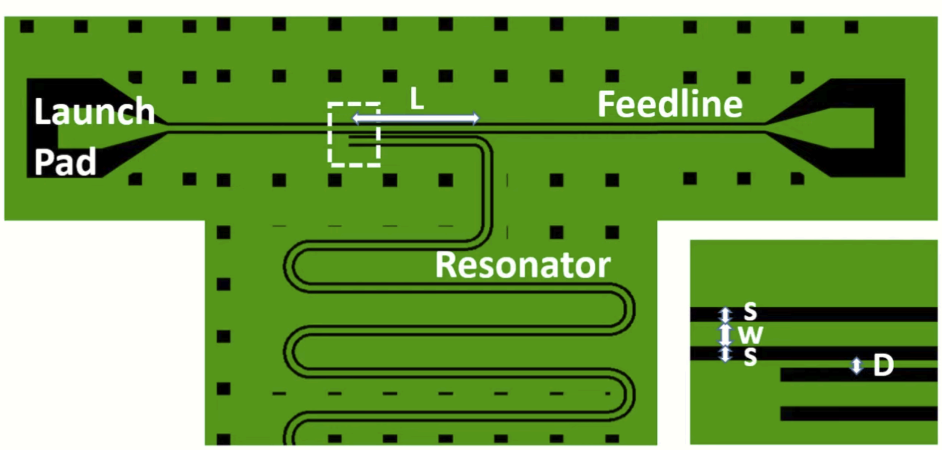
24.3.2.1 CPW Design

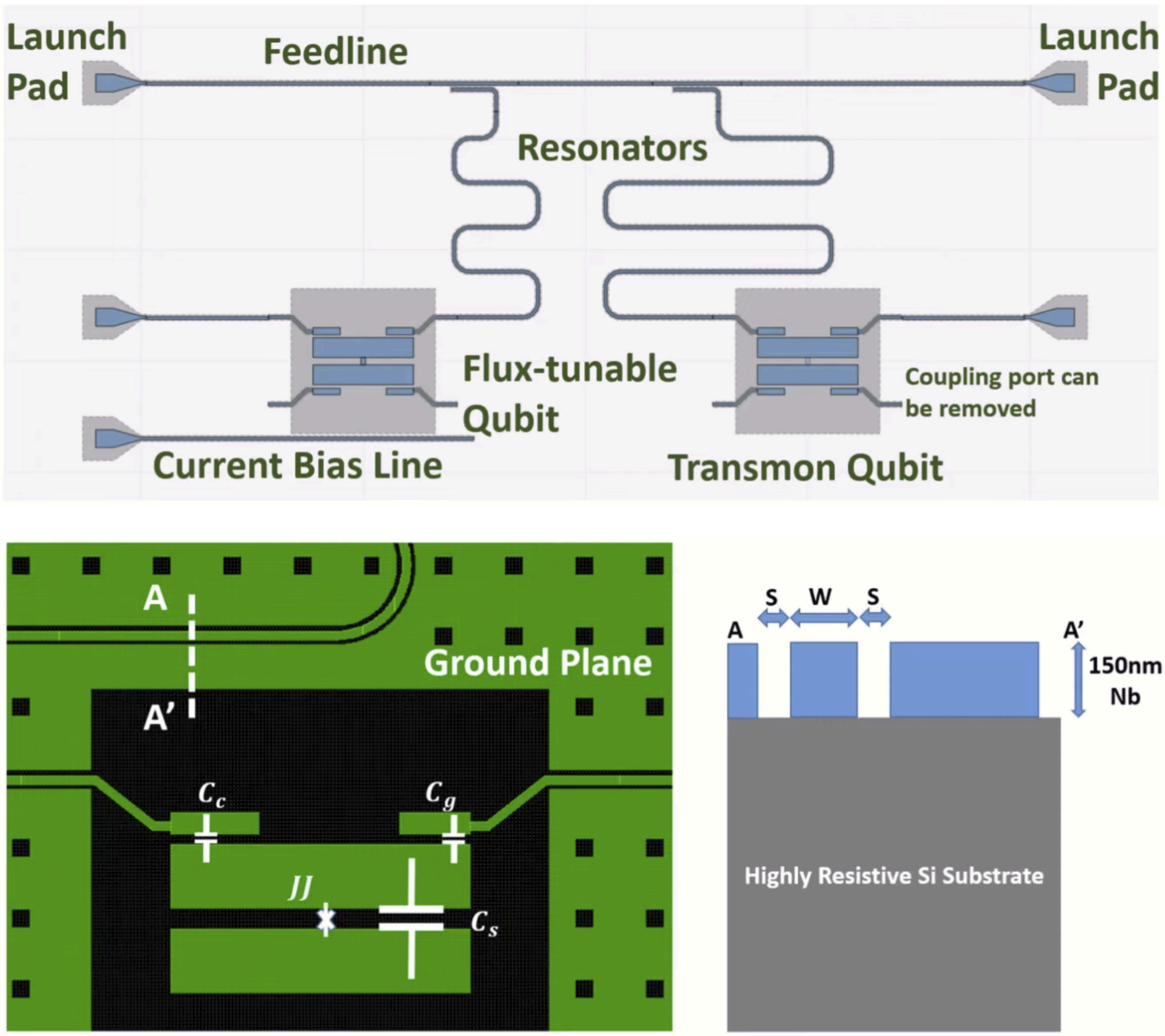
Firstly, we need to design the feedline and the resonator to have the same characteristic impedance, Z_0, as the system has. Here we assume $Z_0 = 50\,\Omega$. To do this, we can go through analytical calculations or simulations. One can find that $s = 5.8\,\mu\text{m}$ and $w = 10\,\mu\text{m}$ will give the required characteristic impedance by using the tools in [5] or [6]. Reference [6] is based on [7].

24.3.2.2 $\lambda/4$-Resonator Design

We need to design a $\lambda/4$-resonator with $\omega_r = 2\pi \times 7\,\text{GHz}$. Since an electromagnetic wave will have a shorter wavelength in matters than in a vacuum, we need to find the effective relative dielectric constant, ϵ_{eff}, so that we can find the wavelength and, thus, the length of the CPW. This can be performed by using simulations. For example, in [8], the effective relative dielectric constant for the EM fields for metals on the top of a silicon substrate is extracted to be 6.1. Therefore, the length of the resonator is found to be

$$
\begin{aligned}
L &= \frac{\lambda\text{matter}}{4}, \\
&= \frac{\lambda\text{vacuum}}{4\sqrt{\epsilon_{eff}}}, \\
&= \frac{c}{4f\sqrt{\epsilon_{eff}}}, \\
&= \frac{3 \times 10^8}{4 \times 7 \times 10^9 \times \sqrt{6.1}}\,\text{m}, \\
&= 4.338\,\text{mm},
\end{aligned}
\tag{24.4}
$$




Launch Pad
Feedline
Launch Pad
Resonators
Flux-tunable Qubit
Coupling port can be removed
Current Bias Line
Transmon Qubit
A
A'
Ground Plane
C_c
C_g
JJ
C_s
S
W
S
A
A'
150nm Nb
Highly Resistive Si Substrate

Additional References

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