| # | Equation name | Equation (LaTeX) | Category/ context | Key symbols/ notes |
|---|---|---|---|---|
| 1 | Classical harmonic oscillator | $\frac{d^2x}{dt^2} + \omega_0^2 x = 0$ | Second-order linear ODE; mechanics, circuits, fields | Undamped motion with angular frequency $\omega_0$. Damping adds term $2\zeta\omega_0\, dx/dt$. |
| 2 | Wave (d'Alembert) equation | $\frac{\partial^2 u}{\partial t^2} = v^2 \nabla^2 u$ | Hyperbolic PDE; propagation of waves | $u$ displacement, field, or potential; $v$ wave speed. In electromagnetism, $v = c$ in vacuum. |
| 3 | Laplace's equation | $\nabla^2 \phi = 0$ | Source-free limit of Poisson; potential theory | Describes steady-state fields with no local sources (electrostatics in charge-free regions, incompressible flow streamfunctions, gravitational potential in vacuum). |
| 4 | Heat (diffusion) equation | $\frac{\partial u}{\partial t} = \alpha \nabla^2 u$ | Parabolic PDE; heat flow, diffusion, probability densities | $u$ temperature or concentration, $\alpha = k/(\rho c_p)$ thermal diffusivity. |
| 5 | Poisson equation | $\nabla^2 \phi(\mathbf{r}) = -\rho(\mathbf{r})/\varepsilon_0$ | Elliptic PDE; electrostatics, gravitation | $\phi$ potential, $\rho$ charge density, $\varepsilon_0$ vacuum permittivity. In inhomogeneous media: $\nabla \cdot (\varepsilon \nabla \phi) = -\rho$. |
| 6 | Fourier transform (one common convention) | $F(k) = \int_{-\infty}^{\infty} f(x)\, e^{-ikx}\, dx, \quad f(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(k)\, e^{ikx}\, dk$ | Spectral decomposition; signals, PDEs, quantum mechanics | Other conventions move the $2\pi$ factors or change the exponent sign. Choice must be consistent between transform and inverse. |
| 7 | Power conversion efficiency | $\eta = P_{\text{out}}/P_{\text{in}}$ | Figure of merit; energy and power devices | $\eta$ dimensionless efficiency; often expressed as a percentage. Can also be defined for energies, work, photons, etc. |
| 8 | Ohmic conductivity (Ohm's law, differential form) | $\mathbf{J} = \boldsymbol{\sigma} \mathbf{E}$ | Constitutive relation; transport, electronics | $\mathbf{J}$ current density, $\mathbf{E}$ electric field, $\boldsymbol{\sigma}$ conductivity (scalar or tensor, often frequency dependent). |
| 9 | Green's function definition | $L\, G(\mathbf{r}, \mathbf{r}') = \delta(\mathbf{r} - \mathbf{r}')$ | Impulse response of linear differential operator $L$ | Once $G$ is known, solution of $Lu = f$ with given boundaries is $u(\mathbf{r}) = \int G(\mathbf{r}, \mathbf{r}') f(\mathbf{r}')\, d\mathbf{r}'$ (plus homogeneous solution). |
| 10 | Navier–Stokes equation (incompressible Newtonian fluid) | $\rho\left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u}\right) = -\nabla p + \mu \nabla^2 \mathbf{u} + \mathbf{f}, \quad \nabla \cdot \mathbf{u} = 0$ | Continuum momentum balance for viscous fluids | $\mathbf{u}$ velocity, $p$ pressure, $\rho$ density, $\mu$ dynamic viscosity, $\mathbf{f}$ body force density (e.g., gravity). Derived from conservation of momentum plus constitutive law for Newtonian stress. |

| # | Equation name | Equation (LaTeX) | Category/ context | Key symbols/ notes |
|---|---|---|---|---|
| 11 | Fick's first law of diffusion | $\mathbf{J} = -D\nabla c$ | Constitutive law; mass transport, random-walk limit | $\mathbf{J}$ diffusive flux, $D$ diffusion coefficient, $c$ concentration. Minus sign gives flux from high to low $c$. |
| 12 | Fick's second law (diffusion equation) | $\frac{\partial c}{\partial t} = D\nabla^2 c$ | Parabolic PDE; diffusion, random walks, probability | For constant $D$. More generally: $\partial c/\partial t = \nabla \cdot (D\nabla c)$. |
| 13 | Mean free path | $\ell = 1/(n\sigma_{\text{sc}})$ | Kinetic theory; transport in gases, solids | $\ell$ average distance between scattering events, $n$ number density of scatterers, $\sigma_{\text{sc}}$ scattering cross section. For hard spheres in a gas, $\ell = 1/(\sqrt{2}n\sigma)$. |
| 14 | Maxwell's equations (microscopic, SI, differential form) | $\nabla \cdot \mathbf{E} = \rho/\varepsilon_0,\ \nabla \cdot \mathbf{B} = 0,\ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t},\ \nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0\varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}$ | Classical electromagnetism | $\mathbf{E}$ electric field, $\mathbf{B}$ magnetic flux density, $\rho$ charge density, $\mathbf{J}$ current density, $\varepsilon_0, \mu_0$ vacuum permittivity and permeability. |
| 15 | Continuity equation (conserved scalar) | $\frac{\partial \rho}{\partial t} + \nabla \cdot \mathbf{J} = 0$ | Local conservation of charge, mass, probability, etc. | $\rho$ density of conserved quantity, $\mathbf{J}$ corresponding flux or current density. In EM, follows from Maxwell's equations. |
| 16 | Resonant angular frequency | $\omega_0 = \sqrt{k/m}$ (mechanical), $\omega_0 = 1/\sqrt{LC}$ (electrical) | Simple harmonic oscillator, LC resonator | $k$ spring constant, $m$ mass; $L$ inductance, $C$ capacitance. Gives natural oscillation frequency in radians per second. |
| 17 | Geometric (Clifford) algebra identities | $ab = a \cdot b + a \wedge b, \quad e_i e_j + e_j e_i = 2\delta_{ij}$ | Unified algebra of vectors, bivectors, spinors | Geometric product decomposes into symmetric inner product and antisymmetric exterior product. Basis vectors $e_i$ generate the metric via $e_i^2 = 1$ (Euclidean) or $\pm 1$ (general signatures). |
| 18 | Helmholtz equation | $(\nabla^2 + k^2)\psi(\mathbf{r}) = 0$ | Time-harmonic wave fields | $k = \omega/c$ wavenumber. Appears after separation of variables in the wave equation for sinusoidal time dependence. |
| 19 | Poynting vector and Poynting theorem (local EM energy conservation) | $\mathbf{S} = \mathbf{E} \times \mathbf{H}, \quad \frac{\partial u_{\text{EM}}}{\partial t} + \nabla \cdot \mathbf{S} = -\mathbf{J} \cdot \mathbf{E}$ | Electromagnetic energy flow and power dissipation | $\mathbf{S}$ energy flux density, $\mathbf{H}$ magnetic field strength, $u_{\text{EM}}$ EM energy density. Right-hand term is power delivered to charges per unit volume. |
| 20 | Quality factor of a resonance | $Q = \omega_0/\Delta\omega$ | Dimensionless resonance sharpness; oscillators, cavities, filters | $\Delta\omega$ full width at half maximum. Equivalent definition: $Q = 2\pi \times (\text{energy stored}/\text{energy lost per cycle})$. |
| 21 | Poisson–Boltzmann equation (dimensionless, | $\nabla^2\psi = \kappa^2 \sinh\psi$ | Nonlinear elliptic PDE; screened Coulomb potentials in electrolytes, | $\psi = ze\phi/(k_{\text{B}}T)$ dimensionless potential, $\kappa^{-1}$ Debye length. For general ionic mixtures: $\nabla^2\phi = -\rho_f/\varepsilon -$ |

| # | Equation name | Equation (LaTeX) | Category/ context | Key symbols/ notes |
|---|---|---|---|---|
| | symmetric electrolyte) | | plasmas, semiconductors | $\frac{1}{\varepsilon}\sum_i z_i e n_{i0} e^{-z_i e\phi/(k_\mathrm{B}T)}$. |
| 22 | Bragg's law | $n\lambda = 2d\sin\theta$ | Wave interference; x-ray, neutron, electron diffraction | $n$ integer order, $\lambda$ wavelength, $d$ lattice plane spacing, $\theta$ glancing angle between beam and planes. |
| 23 | Classical anharmonic oscillator (cubic example) | $\frac{d^2x}{dt^2} + \omega_0^2 x + \alpha x^3 = 0$ | Nonlinear oscillator; perturbation theory, nonlinear dynamics | $\alpha$ controls strength of nonlinearity. Corresponds to potential $V(x) = \frac{1}{2}m\omega_0^2 x^2 + \frac{1}{4}\alpha x^4$. |
| 24 | Shot-noise current (RMS in bandwidth $\Delta f$) | $\Delta I_\mathrm{rms} = \sqrt{2qI\,\Delta f}$ | Electronic/ photon counting noise; Poisson statistics | $I$ average current, $q$ elementary charge, $\Delta f$ measurement bandwidth. Current spectral density $S_I = 2qI$. |
| 25 | Time-dependent Schrödinger equation | $i\hbar\,\frac{\partial\psi(\mathbf{r},t)}{\partial t} = \hat{H}\,\psi(\mathbf{r},t)$ | Fundamental quantum evolution law | $\hat{H}$ Hamiltonian operator (kinetic + potential). For one nonrelativistic particle: $\hat{H} = -\hbar^2\nabla^2/(2m) + V(\mathbf{r},t)$. |
| 26 | Quantum harmonic oscillator (1D) | $\left[-\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + \frac{1}{2}m\omega_0^2 x^2\right]\psi_n(x) = E_n\psi_n(x),$ $E_n = \hbar\omega_0(n+\frac{1}{2})$ | Exactly solvable model; quantized vibrations, modes | $n = 0, 1, 2, \ldots$; ladder operators connect eigenstates. Central in quantization of fields and cavity modes. |
| 27 | Time-independent Schrödinger equation (TISE) | $\left[-\frac{\hbar^2}{2m}\nabla^2 + V(\mathbf{r})\right]\psi(\mathbf{r}) = E\,\psi(\mathbf{r})$ | Stationary states; eigenvalue problem for $\hat{H}$ | Separation of variables with $\psi(\mathbf{r},t) = \psi(\mathbf{r})e^{-iEt/\hbar}$ reduces the TDSE to this eigenproblem. Discrete $E_n$ give bound states, continuous $E$ scattering states. |
| 28 | Ehrenfest's theorem (general operator form) | $\frac{d}{dt}\langle\hat{A}\rangle = \frac{1}{i\hbar}\,\langle[\hat{A},\hat{H}]\rangle + \left\langle\frac{\partial\hat{A}}{\partial t}\right\rangle$ | Bridge between quantum and classical averages | For position and momentum, this gives $m\,d^2\langle\hat{x}\rangle/dt^2 = -\langle\nabla V(\hat{x})\rangle$. When $V$ is sufficiently smooth over the wavepacket, the expectation values approximately obey Newton's laws. |
| 29 | Dirac equation — αβ Hamiltonian (Dirac-αβ) form | $i\hbar\,\frac{\partial\psi}{\partial t} = \Big(c\,\boldsymbol{\alpha}\cdot\hat{\mathbf{p}} + \beta mc^2 + V\Big)\psi$ | Relativistic Hamiltonian for spin-½ particles | $\psi$ 4-component spinor, $\hat{\mathbf{p}} = -i\hbar\nabla$, $m$ rest mass, $c$ speed of light. $\alpha_i$ and $\beta$ are $4\times 4$ Dirac matrices obeying $\{\alpha_i,\alpha_j\} = 2\delta_{ij}$, $\{\alpha_i,\beta\} = 0$, $\beta^2 = 1$. Reduces to free-particle case when $V = 0$. |
| 30 | Dirac equation — manifestly covariant form | $(i\hbar\gamma^\mu\partial_\mu - mc)\,\psi(x) = 0$ | Lorentz-covariant relativistic wave equation for spin-½ fields | $\gamma^\mu$ Dirac gamma matrices satisfying $\{\gamma^\mu,\gamma^\nu\} = 2g^{\mu\nu}$, $\partial_\mu = (\frac{1}{c}\partial_t, \nabla)$, $g^{\mu\nu}$ metric tensor. Relates to αβ form via $\gamma^0 = \beta, \gamma^i = \beta\alpha^i$. Often written as $(i\partial\!\!\!/ - m)\psi = 0$. |
| 31 | Quantum tunneling probability | $T(E) \approx \exp(-2\kappa a),\;\; \kappa = \sqrt{2m(V_0 - E)}/\hbar$ | Approximate 1D barrier transmission; | Valid for $E < V_0$ and thick barrier $\kappa a \gg 1$. Prefactors from matching wavefunctions are omitted. |

| # | Equation name | Equation (LaTeX) | Category/ context | Key symbols/ notes |
|---|---|---|---|---|
| | (rectangular barrier, WKB limit) | | quantum devices, STM | |
| 32 | Quantum anharmonic oscillator | $\left[-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}+\frac{1}{2}m\omega_0^2x^2+\lambda x^n\right]\psi(x)=E\psi(x)$ | Model for weakly nonlinear quantum systems | $\lambda$ coupling strength, $n \geq 3$ (often $n=3$ or $4$). Requires perturbation theory or numerics in general. |
| 33 | Landau–Lifshitz–Gilbert (LLG) equation | $\frac{\partial \mathbf{m}}{\partial t}=-\gamma\,\mathbf{m}\times\mathbf{H}_{\text{eff}}+\alpha\,\mathbf{m}\times\frac{\partial \mathbf{m}}{\partial t}$ | Magnetization dynamics in ferromagnets | $\mathbf{m}=\mathbf{M}/M_s$ unit magnetization, $\gamma$ gyromagnetic ratio (often negative for electrons), $\alpha$ Gilbert damping. $\mathbf{H}_{\text{eff}}$ includes external, anisotropy, exchange, demagnetizing, and other effective fields. |
| 34 | ABCD/ transfer parameters | $\begin{bmatrix}V_1\\I_1\end{bmatrix}=\begin{bmatrix}A & B\\C & D\end{bmatrix}\begin{bmatrix}V_2\\-I_2\end{bmatrix}$ | Two-port networks; cascaded microwave/ optical components | Transfer (ABCD) matrix converts port-2 voltage and current to port-1. Cascaded networks multiply ABCD matrices. |
| 35 | Drift–diffusion current in semiconductors | $J_n=qn\mu_nE+qD_n\frac{dn}{dx},\quad J_p=qp\mu_pE-qD_p\frac{dp}{dx}$ | Carrier transport in semiconductor devices | $J_n, J_p$ electron and hole current densities, $q$ elementary charge, $n, p$ carrier densities, $\mu_{n,p}$ mobilities, $D_{n,p}$ diffusion coefficients, $E$ electric field. Signs reflect negative electron and positive hole charge. |
| 36 | Effective field for LLG/ LLGS | $\mathbf{H}_{\text{eff}}=-\frac{1}{\mu_0M_s}\frac{\delta F}{\delta\mathbf{m}}+\mathbf{H}_{\text{applied}}+\mathbf{H}_{\text{demag}}+\cdots$ | Micromagnetics; variational derivative of free energy | $F[\mathbf{m}]$ magnetic free-energy functional (exchange, anisotropy, Zeeman, demagnetizing, etc.), $\mu_0$ vacuum permeability, $M_s$ saturation magnetization. |
| 37 | Scattering parameters (S-parameters) | $\mathbf{b}=S\,\mathbf{a}$, with $\mathbf{a}=(a_1,a_2)^\mathsf{T},\ \mathbf{b}=(b_1,b_2)^\mathsf{T}$ | Linear network characterization; RF, microwave, photonics | $S$ is $2\times 2$ matrix with elements $S_{ij}$ relating incident ($a_i$) and reflected/ transmitted ($b_j$) wave amplitudes at ports. |
| 38 | Linear optical gain vs carrier density | $g(N)\approx\sigma_g(N-N_{\text{tr}})$ | Approximate constitutive relation; semiconductor lasers | $g$ modal gain, $N$ carrier density, $N_{\text{tr}}$ transparency density, $\sigma_g$ differential gain or gain cross section. Valid near $N\approx N_{\text{tr}}$. |
| 39 | Standard quantum limit of optical shot noise (amplitude and phase) | $S_{\text{SQL}}=\frac{2h\nu}{\bar{P}}$ | Vacuum-fluctuation shot-noise limit of coherent light; amplitude and phase | $S_{\text{SQL}}$ single-sideband spectral density of relative noise (e.g., dBc/Hz), $h$ Planck constant, $\nu$ optical carrier frequency, $\bar{P}$ mean optical power. For coherent states, amplitude and phase quadratures have equal noise: $S_{\text{SQL,amp}}=S_{\text{SQL,phase}}=S_{\text{SQL}}$. |

| # | Equation name | Equation (LaTeX) | Category/ context | Key symbols/ notes |
|---|---|---|---|---|
| 40 | Standard quantum limit of amplifier noise temperature (phase-preserving) | $T_{\mathrm{N}} \geq \frac{hf}{2k_{\mathrm{B}}} \quad (\text{equivalently } n_{\text{add}} \geq \frac{1}{2})$ | Minimum noise temperature/added noise of linear, phase-preserving quantum amplifier | $T_{\mathrm{N}}$ input-referred noise temperature, $f$ signal frequency, $k_{\mathrm{B}}$ Boltzmann constant, $n_{\text{add}}$ added quanta referred to the input. Equality is reached by a quantum-limited amplifier; total input-referred noise is then one quantum (signal vacuum + idler vacuum). |
| 41 | Free-mass standard quantum limit (interferometric displacement) | $(\Delta z)_{\mathrm{SQL}} = \sqrt{\frac{2\hbar\tau}{m}}$ | Continuous interferometric position measurement of a free mass; gravitational-wave detectors | $m$ mass of each freely suspended test mass (e.g., interferometer end mirrors), $\tau$ measurement duration or integration time. Fundamental lower bound on root-mean-square position uncertainty from Heisenberg evolution of a free mass; often written for $z$ as arm-length change. |
| 42 | Standard quantum limit of measurement noise (imprecision–back-action) | $S_{xx}^{\text{imp}}(\omega)\, S_{FF}^{\text{ba}}(\omega) \geq \frac{\hbar^2}{4}$ | General continuous linear quantum measurement; noise–back-action tradeoff | $S_{xx}^{\text{imp}}$ imprecision noise spectral density in measured observable (e.g., position), $S_{FF}^{\text{ba}}$ back-action force noise spectral density. Bound enforces that making one noise arbitrarily small drives the other large; minimizing total noise under this constraint yields specific SQLs. |
| 43 | Lattice Boltzmann equation (single-relaxation BGK form) | $f_i(\mathbf{x} + \mathbf{c}_i \Delta t, t + \Delta t) - f_i(\mathbf{x}, t) = -\frac{\Delta t}{\tau}\left[f_i(\mathbf{x}, t) - f_i^{(eq)}(\mathbf{x}, t)\right]$ | Mesoscopic fluid/ transport solver on discrete lattice | $f_i$ particle distribution along discrete velocity $\mathbf{c}_i$, $\tau$ relaxation time, $f_i^{(eq)}$ local equilibrium (typically low-Mach expansion of Maxwell–Boltzmann). Macroscopic density and velocity follow from velocity moments and recover Navier–Stokes in the continuum limit. |
| 44 | Heisenberg limit for phase estimation | $\Delta\phi \gtrsim 1/N$ | Ultimate quantum scaling for phase sensitivity | $N$ number of entangled particles, photons, or quanta in probe state. Beats shot-noise limit $\Delta\phi \sim 1/\sqrt{N}$ using nonclassical states (e.g., N00N states). |
| 45 | LLGS equation with spin-transfer torque | $\frac{\partial \mathbf{m}}{\partial t} = -\gamma\, \mathbf{m} \times \mathbf{H}_{\text{eff}} + \alpha\, \mathbf{m} \times \frac{\partial \mathbf{m}}{\partial t} + \boldsymbol{\tau}_{\mathrm{STT}}$ | Spintronics; current-driven magnetization switching | Typical Slonczewski torque: $\boldsymbol{\tau}_{\mathrm{STT}} \propto \mathbf{m} \times (\mathbf{m} \times \mathbf{p})$, where $\mathbf{p}$ is spin-polarization direction and the prefactor depends on current density, layer thickness, and material parameters. |
| 46 | Quantum lattice Boltzmann equation (QLB, schematic) | $\psi_i(\mathbf{x} + \mathbf{c}_i \Delta t, t + \Delta t) = \sum_j U_{ij}(\mathbf{x}, t)\, \psi_j(\mathbf{x}, t)$ | Lattice-based quantum evolution; discrete real-space solver for Schrödinger/ Dirac | $\psi_i$ quantum amplitudes associated with discrete velocities or internal states, $U_{ij}$ unitary “collision” operator constructed so that the continuum limit reproduces Schrödinger or Dirac equations. Implementable on classical hardware and naturally suited to quantum computers as a |

| # | Equation name | Equation (LaTeX) | Category/ context | Key symbols/ notes |
|---|---|---|---|---|
| | | | | quantum walk/ quantum circuit. |
| 47 | Fermi's Golden Rule | $W_{i\to f} = \frac{2\pi}{\hbar}\left\lvert\langle f\lvert \hat{H}'\rvert i\rangle\right\rvert^2 \rho(E_f)$ | Transition rate in time-dependent perturbation theory; quantum scattering, emission, absorption | $W_{i\to f}$ transition rate from initial state $\lvert i\rangle$ to final states near energy $E_f$, $\hat{H}'$ perturbing Hamiltonian, $\rho(E_f)$ density of final states at $E_f$. Assumes weak perturbation, long times, and quasi-continuous spectrum. |

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| Type of Gain | Definition (Linear Scale) | Definition (dB Scale) | Notes/ Context |
|---|---|---|---|
| Power Gain (RF/ Microwave) | $(G_p = \frac{P_{\text{out}}}{P_{\text{in}}})$ | $(G_{p,\text{dB}} = 10\log_{10}(G_p) = 10\log_{10}\left(\frac{P_{\text{out}}}{P_{\text{in}}}\right))$ | Used when power flow is the key quantity (RF, microwave, link budgets). Factor 10 because dB is defined on power. |
| Voltage Gain (Low-Frequency/ General Amplifier) | $(G_v = \frac{V_{\text{out}}}{V_{\text{in}}})$ | $(G_{v,\text{dB}} = 20\log_{10}\lvert G_v\rvert = 20\log_{10}\left\lvert\frac{V_{\text{out}}}{V_{\text{in}}}\right\rvert)$ | Common in audio and low-frequency instrumentation. Factor 20 because $(P \propto \lvert V\rvert^2)$ for fixed load impedance. |
| Current Gain | $(G_i = \frac{I_{\text{out}}}{I_{\text{in}}})$ | $(G_{i,\text{dB}} = 20\log_{10}\lvert G_i\rvert = 20\log_{10}\left\lvert\frac{I_{\text{out}}}{I_{\text{in}}}\right\rvert)$ | For true current amplifiers (current-in, current-out). Not used for transimpedance stages. |
| Transimpedance (Transresistance) Gain | $(Z_t = \frac{V_{\text{out}}}{I_{\text{in}}})$ | $(Z_{t,\text{dB}\,\Omega} = 20\log_{10}\left(\frac{\lvert Z_t\rvert}{1\ \Omega}\right) \approx 20\log_{10}\left\lvert\frac{V_{\text{out}}}{I_{\text{in}}}\right\rvert)$ | Key metric for photodiode and sensor front-ends (current-in, voltage-out). Sometimes reported in dBΩ. |

1. RF Power Gain
    - Typically expressed in decibels (dB):

$$G_{p,\text{dB}} = 10\log_{10}!\left(\frac{P_{\text{out}}}{P_{\text{in}}}\right).$$

    - Here $P_{\text{out}}$ and $P_{\text{in}}$ are average powers at the defined reference impedance (often $50\ \Omega$ in RF systems).
    - Widely used in radio-frequency and microwave engineering, antenna design, and link-budget calculations, where actual power transfer is the main concern.

2. Voltage Gain vs. Power Gain
    - In low-frequency electronics (audio, instrumentation, op-amp circuits), voltage gain is often the primary specification, because signals are sensed as voltages.
    - When the source and load impedances are equal and fixed,

$$P \propto V^2,$$

so the decibel expression for voltage gain becomes

$$G_{v,\mathrm{dB}} = 20\log_{10}!\left(\frac{V_{\mathrm{out}}}{V_{\mathrm{in}}}\right),$$

and this 20-log relationship is consistent with the 10-log power ratio under the equal-impedance assumption.

- If the impedances at input and output are different, $20\log_{10}(V_{\mathrm{out}}/V_{\mathrm{in}})$ does *not* directly equal the power gain in dB; the impedance change must be accounted for if power gain is what you care about.

3. When to Use Which Formula

- **Power-based gain (10 log):** Use

$$G_{\mathrm{dB}} = 10\log_{10}!\left(\frac{P_{\mathrm{out}}}{P_{\mathrm{in}}}\right)$$

whenever you are comparing *powers* (e.g., RF power transfer, link budgets, amplifier power gain), regardless of the actual impedance values.

- **Voltage-based gain (20 log):** Use

$$G_{\mathrm{dB}} = 20\log_{10}!\left(\frac{V_{\mathrm{out}}}{V_{\mathrm{in}}}\right)$$

when you are comparing voltage amplitudes (e.g., op-amp voltage gain, sensor front-ends), typically under the assumption that input and output are referred to the same characteristic impedance if you want the dB value to correspond to a power ratio.

- A similar 20-log rule applies for current gain:

$$G_{i,\mathrm{dB}} = 20\log_{10}!\left(\frac{I_{\mathrm{out}}}{I_{\mathrm{in}}}\right).$$

4. Alternate Notations

- In many textbooks and datasheets, you may see:
  - $G$ or $G_p$ for power gain,
  - $A_v$ for voltage gain,
  - $A_i$ for current gain,
  - $Z_t$ for transimpedance gain.
- The dB conversion always follows the same rule:
  - $10\log_{10}$ for *power* ratios,
  - $20\log_{10}$ for *field* or *amplitude* ratios (voltage, current), with attention to impedance if you want to interpret them as power ratios.
- Always check the context (what quantity is being compared, and at what impedance) to decide whether 10-log or 20-log is appropriate.

### Wrap-up context

These gain definitions underpin electronic amplifier design, signal-chain analysis, and communications-system engineering.

Correctly distinguishing between voltage, current, and power gain—and applying the proper dB formula with the right impedance assumptions—is essential for accurate specification and comparison of amplifiers.

### Other key RF-amplifier parameters

- **Noise Figure (NF):** Quantifies how much an amplifier degrades the signal-to-noise ratio; formally, NF is the ratio of input SNR to output SNR, often expressed in dB.
- **Bandwidth:** The range of frequencies over which the amplifier meets its specified gain and performance (often defined between the $-3\ \mathrm{dB}$ gain points).
- **Linearity:** Describes how well the amplifier preserves the proportionality between input and output; poor linearity leads to distortion and intermodulation products.

RF amplifiers are further categorized by operating class (Class A, B, AB, C, etc.) and by role (low-noise amplifier, driver amplifier, power amplifier, etc.), but all use these same gain and dB conventions.

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Field & potential equations
├─ Poisson (#5)
|  ├─ Laplace (#3)                [special case with ρ = 0]
|  ├─ Poisson–Boltzmann (#21)     [adds nonlinear screening by mobile ions]
|  └─ Green’s functions (#9)      [invert Poisson/Helmholtz with sources]
├─ Maxwell’s equations (#14)
|  ├─ Continuity equation (#15)           [charge/ probability conservation]
|  ├─ Wave equation (#2)                  [EM waves in vacuum/ media]
|  |  └─ Helmholtz equation (#18)         [time-harmonic reduction of waves]
|  ├─ Poynting vector & theorem (#19)     [energy flow and conservation]
|  └─ Fourier transform (#6)              [k–ω domain solutions, dispersion]
└─ Geometric algebra (#17)
   └─ Compactly rewrites Maxwell (#14), Dirac (#29, #30), and wave (#2, #18) equations

Diffusion/ transport family
├─ Fick’s 1st law (#11) → Fick’s 2nd law (#12)
|  └─ Heat equation (#4)                  [same PDE with thermal parameters]
├─ Continuity equation (#15)
|  └─ Drift–diffusion currents (#35)
|     └─ Mean free path (#13)             [microscopic origin of transport coefficients]
├─ Navier–Stokes equation (#10)
|  └─ Lattice Boltzmann equation (#43)    [mesoscopic solver recovering Navier–Stokes]
└─ Poisson / Poisson–Boltzmann (#5, #21)
   └─ Coupled to drift–diffusion (#35) in semiconductor and electrolyte device models

Oscillators/ resonances
├─ Classical oscillators
|  ├─ Harmonic oscillator (#1)            [linear restoring force]
|  ├─ Anharmonic oscillator (#23)         [nonlinear corrections]
|  ├─ Resonant angular frequency (#16)    [ω₀ from parameters k, m or L, C]
|  ├─ Quality factor Q (#20)              [spectral sharpness/ damping]
|  └─ Scattering & transfer matrices (#37, #34)
|        [S-parameters and ABCD matrices for resonant networks]
├─ Quantum oscillators & fields
|  ├─ Quantum harmonic oscillator (#26) and quantum anharmonic (#32)
|  ├─ Time-dependent Schrödinger equation (#25)
|  |  └─ Time-independent Schrödinger equation (#27) [eigenproblem for stationary states]
|  ├─ Dirac equations (#29, #30)          [relativistic spin-½ dynamics]
|  ├─ Quantum tunneling probability (#31) [barrier penetration]
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|  ├─ Quantum lattice Boltzmann (QLB) (#46)
|  |    [discrete streaming–collision realization of Schrödinger/ Dirac]
|  └─ Green’s functions (#9) in quantum propagation and scattering
|       └─ Fermi’s Golden Rule (#47)      [transition rates between eigenstates]
└─ Precision & noise (quantum limits)
   ├─ Shot-noise current (#24)            [Poisson counting noise of charge/ photons]
   ├─ Optical shot-noise SQL (#39)
   |    [vacuum-fluctuation limit of coherent light, amplitude & phase quadratures]
   ├─ Amplifier SQL (#40)
   |    [phase-preserving linear amplifier: n_add ≥ 1/2, T_n ≥ ħω/(2k_B)]
   ├─ Free-mass SQL for displacement (#41)
   |    [Δx_SQL(τ) ≳ √(ħτ/ (2m)) for continuous tracking of a free mass]
   ├─ Measurement-noise SQL product (#42)
   |    [S_xx^imp(ω) · S_FF^ba(ω) ≥ ħ²/4, imprecision–back-action tradeoff]
   ├─ Heisenberg limit for phase estimation (#44)
   |    [Δϕ ≳ 1/N, ultimate scaling with entangled probes]
   └─ Poynting/ Maxwell (#19, #14)
        [optical power flow and field dynamics in quantum-limited interferometers]

Magnetization dynamics and spintronics
├─ Effective field H_eff (#36)
|  └─ LLG equation (#33)
|      └─ LLGS with spin-transfer torque (STT) (#45)
└─ Coupled to:
   ├─ Poisson/ Maxwell (#5, #14)        [fields, currents, spin-orbit torques]
   └─ Drift–diffusion (#35)              [spin-polarized carrier transport]
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