Onri's Quantum Plots

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Introduction

This document presents a speculative overview of the fractal time-dependent Schrödinger equation and its derived solutions, including fractal probability densities for given fractal dimensions and, eventually, the wavefunctions for the ground and first three excited states.

Quantum Anharmonic Oscillator with Cosine Potential

The potential $V(x)$ for a quantum anharmonic oscillator can be represented as a cosine function:

$$V(x) = \cos(x)$$

This can be plotted as follows:

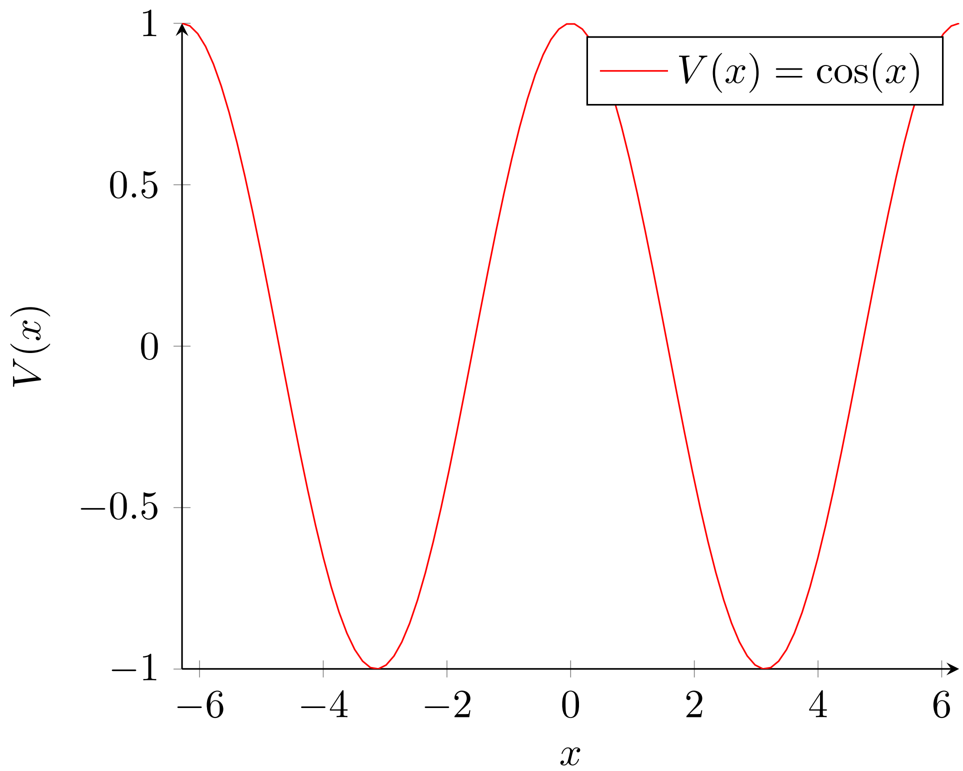

Figure 1: Plot of the anharmonic oscillator potential as a cosine function.

Anharmonic Oscillation Equation

The quantum anharmonic oscillator can be described by the following potential energy equation:

$$V(x) = \frac{1}{2}m\omega^2 x^2 + \lambda x^4 \tag{1}$$

where m is the mass of the particle, ω is the angular frequency, and λ is the anharmonicity constant.

Energy Potential Plot

The plot for the anharmonic potential $V(x)$ is shown below:

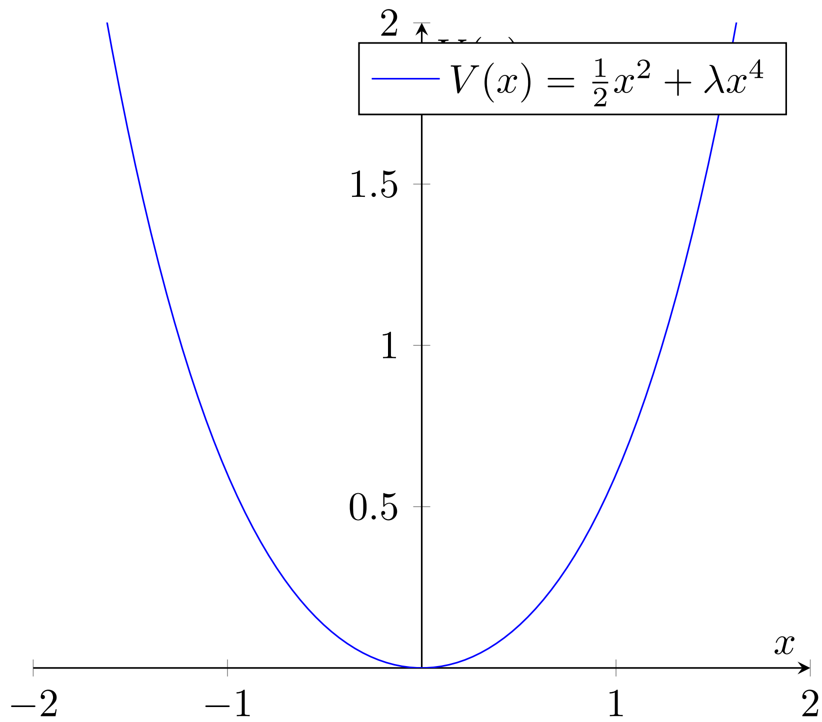

Figure 2: Plot of the energy potential of the anharmonic oscillator.

Hamiltonian of the Anharmonic Oscillator

The Hamiltonian for a one-dimensional quantum anharmonic oscillator can be represented as:

$$\hat{H} = \frac{\hat{p}^2}{2m} + \frac{1}{2}m\omega^2\hat{x}^2 + \lambda\hat{x}^4 \tag{2}$$

where $\hat{p}$ is the momentum operator, m is the mass, ω is the angular frequency, and λ is the anharmonicity constant.

Energy Potential Plot with Discrete Energy Levels

The plot for the anharmonic potential $V(x) = \frac{1}{2}m\omega^2x^2 + \lambda x^4$ with discrete energy levels is shown below:

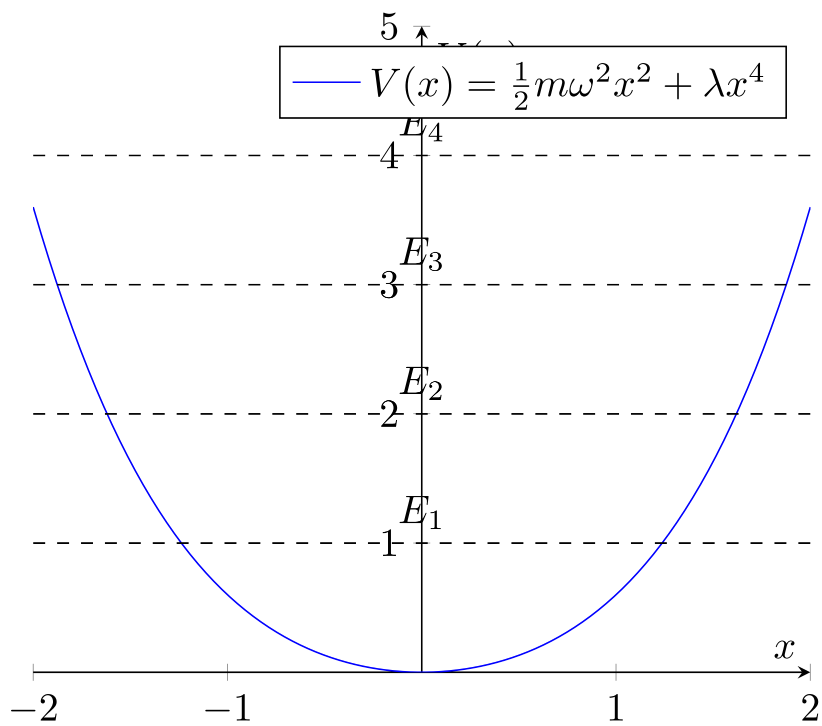

Figure 3: Plot of the energy potential of the anharmonic oscillator with discrete energy levels E_1, E_2, E_3, and E_4.

Energy Potential of a Superconducting Qubit

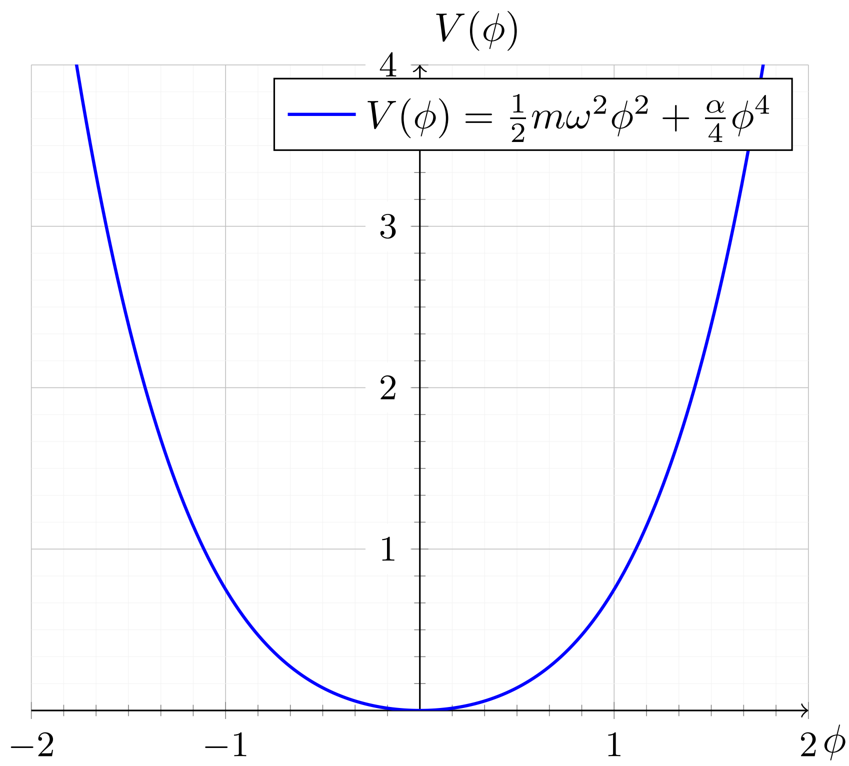

Figure 4: Plot of the anharmonic quantum oscillator energy potential of a superconducting qubit.

Damped Harmonic Oscillator

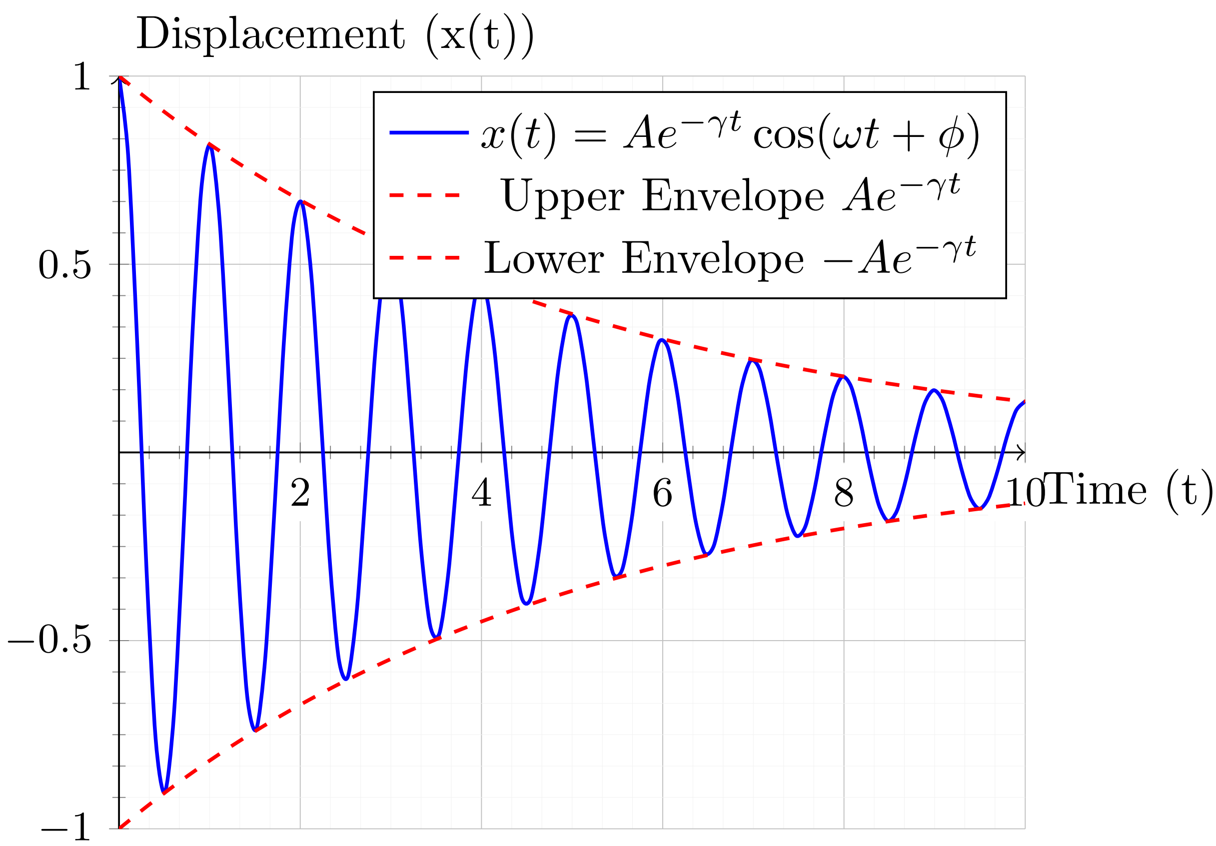

Figure 5: Plot of a damped harmonic oscillator with exponential decay envelopes.

Fractal Schrödinger Equation (Anharmonic Case)

While a "fractal" Schrödinger equation is non-standard, an anharmonic oscillator's potential introduces non-equidistant energy levels. The Schrödinger equation for such a system is:

$$\hat{H}\psi(x) = \left[-\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + V(x)\right]\psi(x) = E\psi(x) \tag{3}$$

where $V(x) = \frac{1}{2}m\omega^2x^2 + \lambda x^4$ represents the anharmonic potential.

Wavefunction Solutions

The wavefunctions $\psi_n(x)$ are the solutions to the Schrödinger equation, where n denotes the quantum number corresponding to different energy levels. These solutions are not simple sinusoids like the harmonic oscillator but require numerical methods to solve.

The Schrödinger Equation in a Cosine Potential Well

The time-independent Schrödinger equation for a particle in a potential well is given by:

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + V(x)\psi(x) = E\psi(x) \tag{4}$$

where $\psi(x)$ is the wave function, $V(x)$ is the potential energy, E is the energy of the particle, m is its mass, and $\hbar$ is the reduced Planck's constant.

For a particle in a cosine potential well with anharmonicity, the potential can be expressed as:

$$V(x) = V_0(1 - \cos(2\pi x/a)) \tag{5}$$

where V_0 is the depth of the well and a is the period of the cosine potential.

Solution to the Schrödinger Equation

The exact solution to the Schrödinger equation with a cosine potential is complex and often requires numerical methods. However, qualitative features of the solution can be discussed, such as band formation due to the periodic potential and the presence of band gaps.

Plot of Discrete Energy Levels

The discrete energy levels in a cosine potential well with anharmonicity are depicted below. The energy levels are not equally spaced, and their spacing can be calculated numerically.

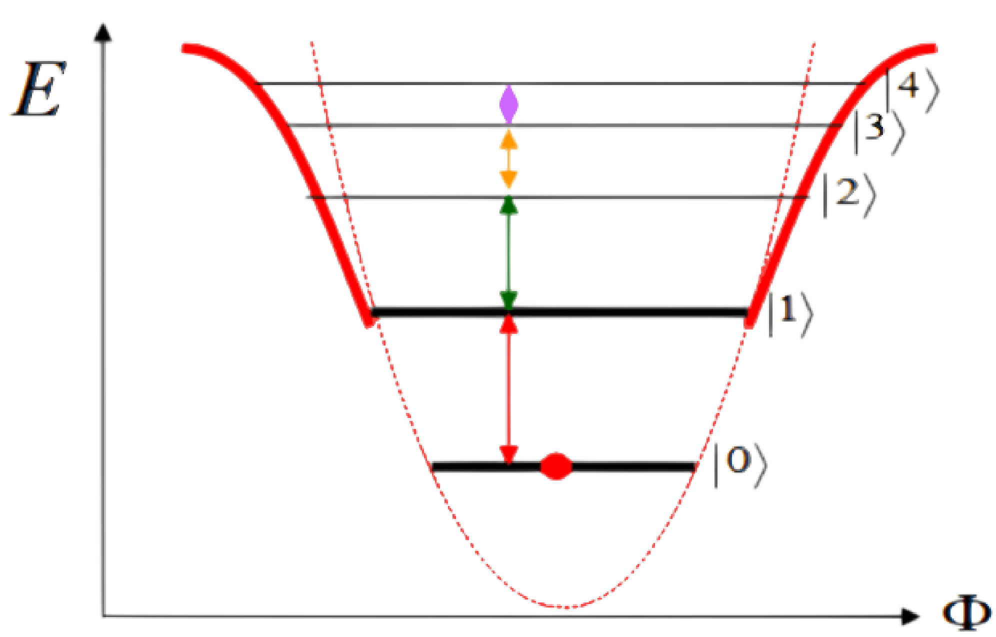

Figure 6: Discrete energy levels in a cosine potential quantum well.

Plot of Discrete Energy Levels for a Transmon

The discrete energy levels in a cosine potential well with anharmonicity are depicted below. The energy levels are not equally spaced, and their spacing can be calculated numerically. Real values extracted from an IBM Quantum computer.

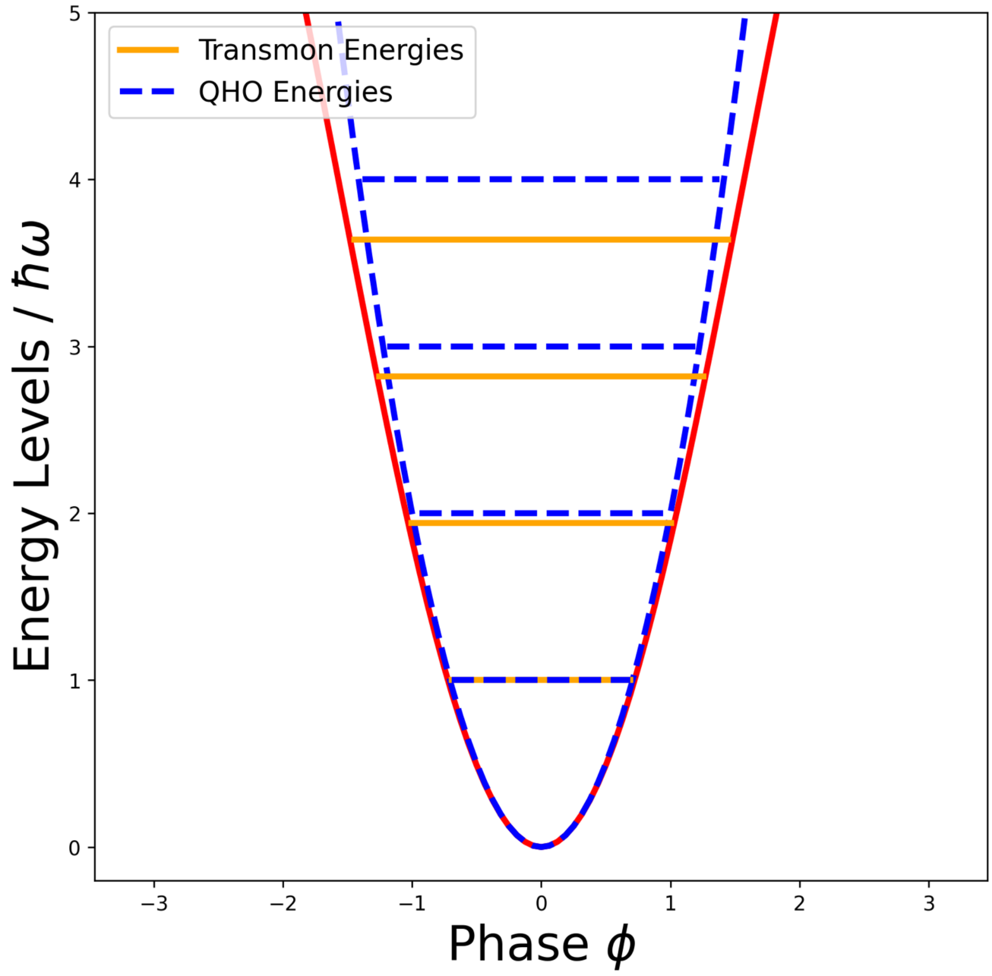

Figure 7: Discrete energy levels in a cosine potential quantum well from a transmon in the IBM Quantum System One.

Fractal Time-Dependent Schrödinger Equation

The standard time-dependent Schrödinger equation in N-dimensions is given by:

$$i\hbar \frac{\partial}{\partial t}\Psi(\vec{r},t) = \left[-\frac{\hbar^2}{2m}\nabla^2 + V(\vec{r},t)\right]\Psi(\vec{r},t) \tag{6}$$

where $\Psi(\vec{r},t)$ is the wave function, $V(\vec{r},t)$ is the potential energy, m is the particle mass, and $\hbar$ is the reduced Planck constant. The operator ∇^2 is the Laplacian, which in fractal dimensions is not well-defined and may require alternative forms based on the concept of fractional calculus or other generalizations suitable for non-integer dimensions.

Derived Solutions & Fractal Probability Density

Considering fractal dimensions D, the fractal probability density $\rho(\vec{r},t)$ for the wavefunction $\Psi(\vec{r},t)$ in dimension D can be postulated as:

$$\rho(\vec{r},t) = |\Psi(\vec{r},t)|^2 \tag{7}$$

where the modulus squared of the wavefunction gives the likelihood of finding a particle at position $\vec{r}$ at time t.

For fractal dimensions of 2.141516, 1.154740, and 2.72727, we define hypothetical solutions and densities as follows:

Wavefunctions & Probability Densities for a Harmonic Oscillator

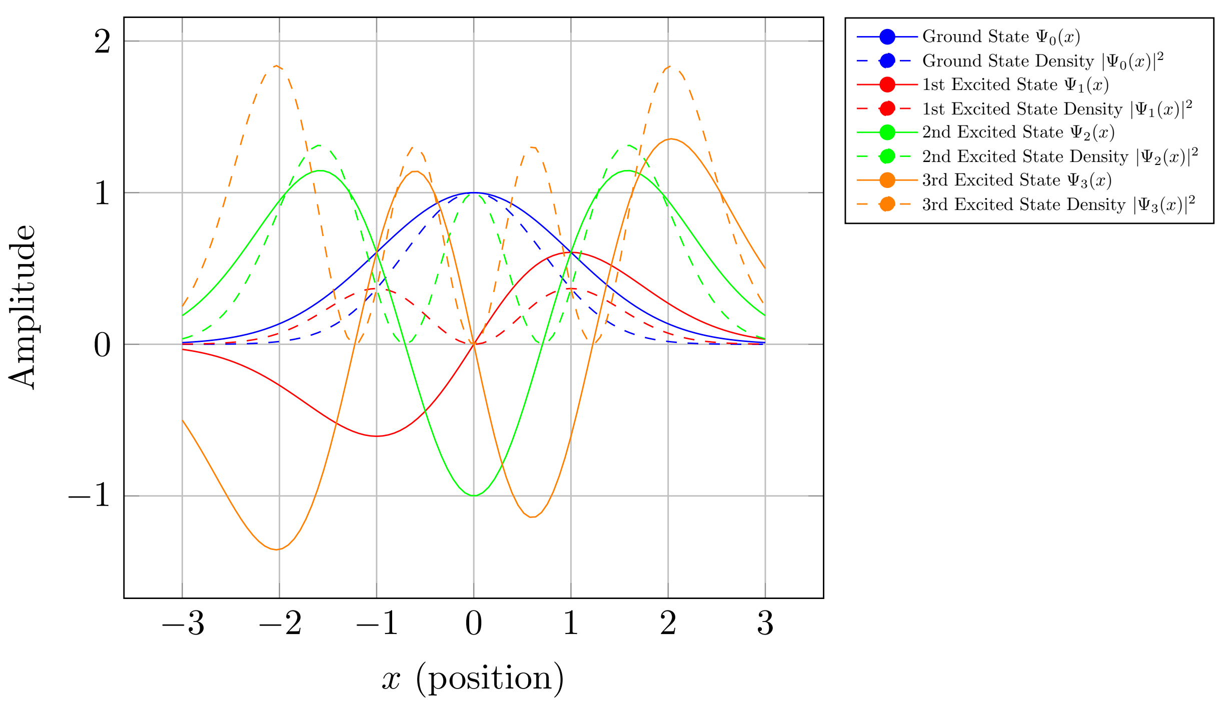

Figure 8: Plots of the wavefunctions and probability densities for the ground state and the first three excited states of a quantum harmonic oscillator. Solid lines represent wavefunctions, and dashed lines represent probability densities.

Wavefunctions & Probability Densities for an Anharmonic Oscillator

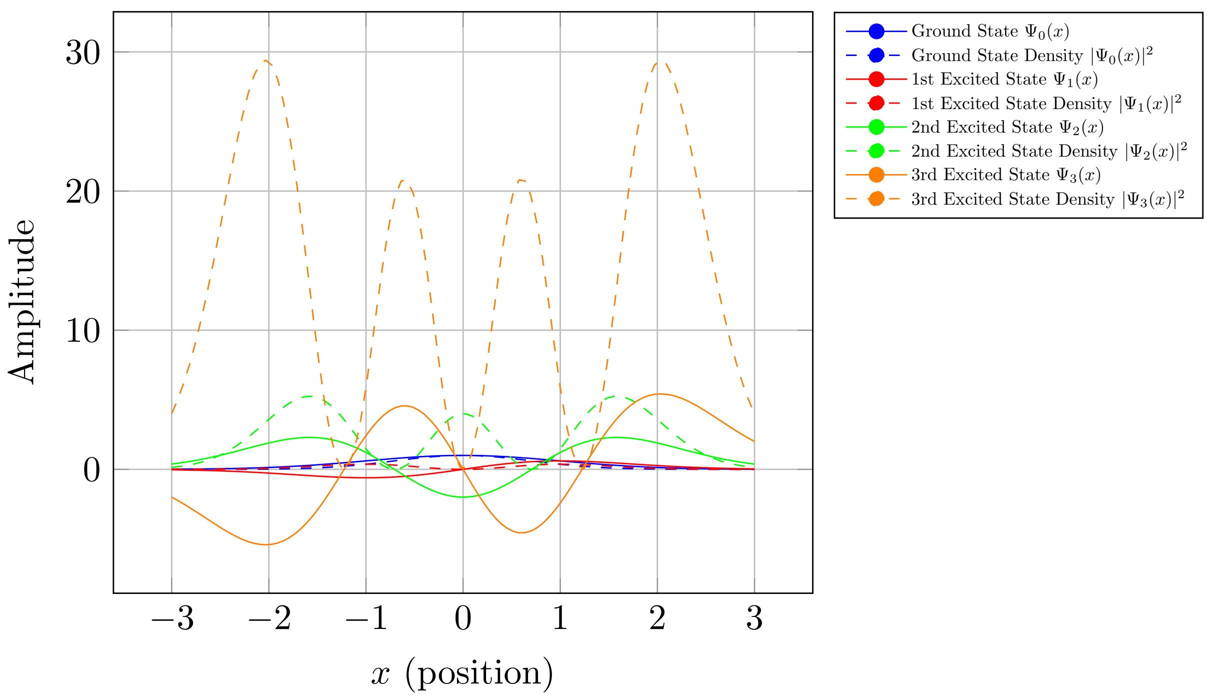

Figure 9: Theoretical plots of the wavefunctions and probability densities for the ground state and the first three excited states of an anharmonic quantum oscillator. Solid lines represent wavefunctions, and dashed lines represent probability densities.

Conclusion

This theoretical framework provides a starting point for considering quantum mechanics in non-integer dimensions. The actual implementation of such a theory would require extensive mathematical development beyond the scope of this document.