

Quantum Limits (Noise, Measurement, Metrology, and Information)

Compiled by Onri Jay Benally

December 2025

Conventions and scope. Unless otherwise stated, $S(\omega)$ denotes a (classical) one-sided power spectral density when used in RMS-in-bandwidth formulas (e.g., $\Delta I_{\text{rms}} = \sqrt{S_I \Delta f}$), while $\bar{S}(\omega)$ denotes a *symmetrized* (quantum) noise spectral density. When comparing to one-sided engineering conventions for stationary noise, a common conversion is $S^{(1)}(\omega > 0) = 2\bar{S}(\omega)$ (and similarly for cross-spectra). Several entries are *standard quantum limits* (SQLs): they are rigorous under stated assumptions (e.g., coherent probes, phase-preserving detection, linear response), and they can be surpassed by changing the assumptions (e.g., squeezed states, entanglement, quantum nondemolition measurement, or back-action evasion).

#	Limit/ bound name	Equation (LaTeX)	Category/ context	Key symbols, assumptions, and “how to beat”	Primary ref.
1	Poisson shot-noise current (RMS in bandwidth Δf)	$\Delta I_{\text{rms}} = \sqrt{2qI \Delta f}$	Electronic transport; photon/electron counting; Poisson statistics	I average current, q carrier charge, Δf measurement bandwidth. Equivalent one-sided current-noise PSD: $S_I^{(1)} = 2qI$. <i>Not an ultimate limit</i> : sub-Poisson sources (Fano factor < 1) reduce it.	(standard)
2	Coherent-beam number–phase noise constraint (optical shot noise/ SQL for phase readout)	$S_{\dot{N}\dot{N}}(\omega) S_{\phi\phi}(\omega) \geq \frac{1}{4},$ $S_{\phi\phi}^{\text{coh}}(\omega) = \frac{1}{4\dot{N}} = \frac{\hbar\omega}{4\bar{P}}$	Quantum optics; homodyne/heterodyne phase sensing; interferometry	\dot{N} photon flux, $\bar{P} = \hbar\omega\dot{N}$ optical power, ϕ phase. <i>Beatable</i> : phase squeezing (reduce $S_{\phi\phi}$) at cost of increased conjugate-quadrature noise; see squeezed-input interferometry.	[5, 4]
3	Phase-preserving amplifier quantum limit (Haus–Caves)	$n_{\text{add}} \geq \frac{1}{2} (G \gg 1),$ $T_N \gtrsim \frac{\hbar f}{2k_B}$	Linear amplification with equal treatment of both quadratures (“phase preserving”)	n_{add} added quanta referred to the input; T_N input-referred noise temperature; f signal frequency ($\omega = 2\pi f$). The second line matches the SQL statement written as Eq. (143) in [11]. <i>Assumption</i> : phase-preserving, linear, bosonic mode, high gain. <i>Evasion</i> : phase-sensitive amplification can add ≈ 0 quanta to <i>one</i> quadrature.	[3, 5, 11]
4	Free-mass standard quantum limit (SQL) for displacement monitoring (time-domain form)	$(\Delta x)_{\text{SQL}} = \sqrt{\frac{2\hbar\tau}{m}}$	Continuous position readout of a free mass; gravitational-wave interferometers	m test-mass, τ averaging/integration time. <i>Interpretation</i> : optimal balance of measurement imprecision and radiation-pressure back-action in a standard (phase-preserving) readout. <i>Beatable</i> : squeezing, variational readout, speed-meters, QND strategies.	[4, 1]
5	Imprecision–back-action (Heisenberg) product for a linear continuous measurement (simplest form)	$\bar{S}_{xx}^{\text{imp}}(\omega) \bar{S}_{FF}^{\text{ba}}(\omega) \geq \frac{\hbar^2}{4}$	General linear measurement; quantum noise tradeoff	$\bar{S}_{xx}^{\text{imp}}$ imprecision noise, \bar{S}_{FF}^{ba} back-action force noise. <i>Assumption</i> : no useful imprecision–back-action correlations are exploited (cf. row 9). <i>Beatable in a narrow sense</i> : correlations and/or QND observables reshape total noise.	[1, 5, 10]
6	Heisenberg scaling for single-parameter phase estimation (entanglement-enabled)	$\Delta\phi \geq \frac{1}{\sqrt{\nu} N}$	Quantum metrology (unitary parameter encoding)	N quanta per probe state; ν independent repetitions. Often contrasted with shot-noise scaling $\Delta\phi \sim 1/\sqrt{\nu N}$. <i>Caveat</i> : decoherence/noise can destroy N -scaling, restoring shot-noise scaling in many realistic settings.	[6]
7	Shot-noise limit (SNL) for phase estimation (separable probes)	$\Delta\phi \geq \frac{1}{\sqrt{\nu} N}$	Classical-like metrology with independent probes (no entanglement)	N uncorrelated particles/photons per repetition; ν repetitions. <i>Beatable</i> : entanglement (Heisenberg scaling) or squeezing (improved constant factor).	[6]

#	Limit/ bound name	Equation (LaTeX)	Category/ context	Key symbols, assumptions, and “how to beat”	Primary ref.
8	Quantum Cramér–Rao bound (QCRB)	$\text{Var}(\hat{\theta}) \geq \frac{1}{\nu F_Q(\theta)},$ $\Delta\theta \geq \frac{1}{\sqrt{\nu F_Q(\theta)}}$	Ultimate single-parameter estimation bound; quantum Fisher information (QFI)	$F_Q(\theta)$ quantum Fisher information of ρ_θ ; ν repetitions. Shot-noise and Heisenberg scalings are corollaries of upper bounds on F_Q under different resource assumptions.	[2, 6]
9	General quantum noise inequality with imprecision–back-action correlations	$\bar{S}_{xx}^{\text{imp}}(\omega) \bar{S}_{FF}^{\text{ba}}(\omega) - \bar{S}_{xF}(\omega) ^2 \geq \frac{\hbar^2}{4}$	Linear-response detector theory; correlated quantum noise	\bar{S}_{xF} is the (symmetrized) cross-spectrum. <i>Meaning:</i> correlations can reduce total added noise at selected frequencies (variational measurement), though the inequality still constrains what is possible.	[5]
10	Quantum nondemolition (QND) measurement efficiency bound (measurement vs dephasing)	$\eta \equiv \frac{\Gamma_{\text{meas}}}{\Gamma_\varphi} \leq 1$	Continuous QND qubit readout; measurement chains	Γ_{meas} measurement rate; Γ_φ measurement-induced dephasing rate. $\eta = 1$ is the quantum-limited (ideal) case; technical loss/noise yields $\eta < 1$.	[5]
11	Landauer bound (bit erasure cost)	$Q_{\text{diss}} \geq k_B T \ln 2$	Information thermodynamics; quantum/classical computing hardware	Q_{diss} heat dissipated to a bath at temperature T by logically irreversible erasure. <i>Not “purely quantum”:</i> thermodynamic, but fundamental; relevant to quantum information engines and control costs.	[9]
12	Holevo bound (accessible classical information from a quantum ensemble)	$I_{\text{acc}}(X:Y) \leq \chi \equiv S(\rho) - \sum_x p_x S(\rho_x),$ $\rho = \sum_x p_x \rho_x$	Quantum Shannon theory; communication; measurement limits	$S(\rho) = -\text{Tr}(\rho \log \rho)$ is von Neumann entropy. <i>Beatable?</i> Not in general without changing the task: χ is the task-dependent upper bound on accessible information for the ensemble.	[8, 12]
13	Helstrom bound (minimum error for binary quantum state discrimination)	$P_{e,\text{min}} = \frac{1}{2} (1 - \ p_0 \rho_0 - p_1 \rho_1\ _1)$	Quantum hypothesis testing; optimal measurements	$\ \cdot\ _1$ trace norm; priors p_0, p_1 ; states ρ_0, ρ_1 . <i>Meaning:</i> ultimate performance of any measurement for binary discrimination.	[7, 12]

Acronym Glossary

Acronym	Expansion
RMS	Root-mean-square
PSD	Power spectral density
SQL	Standard quantum limit
SNL	Shot-noise limit
QCRB	Quantum Cramér–Rao bound
QFI	Quantum Fisher information
QND	Quantum nondemolition
TLS	Two-level system
ZPF	Zero-point fluctuations

References

- [1] Vladimir B. Braginsky and Farid Ya. Khalili. *Quantum Measurement*. Cambridge University Press, 1992. ISBN: 9780521419284. URL: <https://doi.org/10.1017/CB09780511622748>.
- [2] Samuel L. Braunstein and Carlton M. Caves. “Statistical distance and the geometry of quantum states”. In: *Physical Review Letters* 72.22 (1994), pp. 3439–3443. DOI: 10.1103/PhysRevLett.72.3439. URL: <https://doi.org/10.1103/PhysRevLett.72.3439>.
- [3] Carlton M. Caves. “Quantum limits on noise in linear amplifiers”. In: *Physical Review D* 26.8 (1982), pp. 1817–1839. DOI: 10.1103/PhysRevD.26.1817. URL: <https://doi.org/10.1103/PhysRevD.26.1817>.
- [4] Carlton M. Caves. “Quantum-mechanical noise in an interferometer”. In: *Physical Review D* 23.8 (1981), pp. 1693–1708. DOI: 10.1103/PhysRevD.23.1693. URL: <https://doi.org/10.1103/PhysRevD.23.1693>.
- [5] A. A. Clerk et al. “Introduction to quantum noise, measurement, and amplification”. In: *Reviews of Modern Physics* 82.2 (2010), pp. 1155–1208. DOI: 10.1103/RevModPhys.82.1155. URL: <https://doi.org/10.1103/RevModPhys.82.1155>.
- [6] Vittorio Giovannetti, Seth Lloyd, and Lorenzo Maccone. “Quantum Metrology”. In: *Physical Review Letters* 96.1 (2006), p. 010401. DOI: 10.1103/PhysRevLett.96.010401. URL: <https://doi.org/10.1103/PhysRevLett.96.010401>.
- [7] Carl W. Helstrom. *Quantum Detection and Estimation Theory*. Academic Press, 1976. ISBN: 9780123400505.
- [8] A. S. Holevo. “Bounds for the quantity of information transmitted by a quantum communication channel”. In: *Problems of Information Transmission* 9.3 (1973), pp. 177–183.
- [9] R. Landauer. “Irreversibility and Heat Generation in the Computing Process”. In: *IBM Journal of Research and Development* 5.3 (1961), pp. 183–191. DOI: 10.1147/rd.53.0183. URL: <https://doi.org/10.1147/rd.53.0183>.
- [10] Vivishek Sudhir. *Quantum Limits on Measurement and Control of a Mechanical Oscillator*. Springer International Publishing, 2018. DOI: 10.1007/978-3-319-69431-3. URL: <https://doi.org/10.1007/978-3-319-69431-3>.
- [11] Florian Vigneau et al. “Probing quantum devices with radio-frequency reflectometry”. In: *Applied Physics Reviews* 10.2 (2023), p. 021305. DOI: 10.1063/5.0088229. URL: <https://doi.org/10.1063/5.0088229>.
- [12] Mark M. Wilde. *Quantum Information Theory*. Cambridge University Press, 2013. ISBN: 9781107034259.