Learn To Think Abstractly

Samir Shukla

Indian Institute of Technology Mandi

February 28, 2023

Why do so many people positively dislike mathematics?

Why do so many people positively dislike mathematics?

- Mathematics and subjects with high mathematical content such as physics, seems to provoke not just indifference but actual antipathy.

Why do so many people positively dislike mathematics?

- Mathematics and subjects with high mathematical content such as physics, seems to provoke not just indifference but actual antipathy.
- What is it that cause many people to give mathematics up as soon as they possibly can and remember them with dread for the rest of their lives ?

Why do so many people positively dislike mathematics?

- Mathematics and subjects with high mathematical content such as physics, seems to provoke not just indifference but actual antipathy.
- What is it that cause many people to give mathematics up as soon as they possibly can and remember them with dread for the rest of their lives ?
- (Probably) it is not so much mathematics itself that people find unappealing.

- Mathematics continuously builds on itself and it is important to keep up when learning it.

- Mathematics continuously builds on itself and it is important to keep up when learning it.
- For example, if you are not reasonably adept at multiplying two-digit numbers together, then you probably won't have good intuitive feel for the distributive law. Without this, you are unlikely to be comfortable in multiplying $(x+2)(x+3)$, and then you will not understand quadratic equation properly.

- Mathematics continuously builds on itself and it is important to keep up when learning it.
- For example, if you are not reasonably adept at multiplying two-digit numbers together, then you probably won't have good intuitive feel for the distributive law. Without this, you are unlikely to be comfortable in multiplying $(x+2)(x+3)$, and then you will not understand quadratic equation properly.
- One of the most misconception is that mathematics is nothing more that a computation. The truth is that in many areas of the subject mathematics has as much to do with computation as writing has to do with typing.

- Mathematics continuously builds on itself and it is important to keep up when learning it.
- For example, if you are not reasonably adept at multiplying two-digit numbers together, then you probably won't have good intuitive feel for the distributive law. Without this, you are unlikely to be comfortable in multiplying $(x+2)(x+3)$, and then you will not understand quadratic equation properly.
- One of the most misconception is that mathematics is nothing more that a computation. The truth is that in many areas of the subject mathematics has as much to do with computation as writing has to do with typing.
- Algorithms, rules are certainly not unimportant, but our mathematical problems result more from insufficient exposure to mathematics as a way of thinking than from an inability to compute.

- Mathematics continuously builds on itself and it is important to keep up when learning it.
- For example, if you are not reasonably adept at multiplying two-digit numbers together, then you probably won't have good intuitive feel for the distributive law. Without this, you are unlikely to be comfortable in multiplying $(x+2)(x+3)$, and then you will not understand quadratic equation properly.
- One of the most misconception is that mathematics is nothing more that a computation. The truth is that in many areas of the subject mathematics has as much to do with computation as writing has to do with typing.
- Algorithms, rules are certainly not unimportant, but our mathematical problems result more from insufficient exposure to mathematics as a way of thinking than from an inability to compute.
- If understanding a mathematical object largely a question of learning rules it obeys rather than grasping its essence, then the distinction between technical fluency and mathematical understanding is less clear-cut than one might imagine.

Are some people just doomed to dislike mathematics at school?

Or might it be possible to teach the subject differently in such a way that far fewer people are excluded from it?

Are some people just doomed to dislike mathematics at school?

Or might it be possible to teach the subject differently in such a way that far fewer people are excluded from it?

- If any child who is given one-to-one tuition in mathematics from an early age by a good and enthusiastic teacher will grow up liking it.

Are some people just doomed to dislike mathematics at school?

Or might it be possible to teach the subject differently in such a way that far fewer people are excluded from it?

- If any child who is given one-to-one tuition in mathematics from an early age by a good and enthusiastic teacher will grow up liking it.
- I do not wish to suggest that one should try to explain to children what the abstract approach is, but merely that teachers should be aware of its implications.

Given that $0 \times 0 = 0$ *and* $1 \times 1 = 1$*, it follows that there are numbers that are their own squares. But then it follows in turn that there are numbers. In a single step of artless simplicity, we seem to have advanced from a piece of elementary arithmetic to a startling and highly controversial philosophical conclusion: that numbers exist. You would have thought that it should have been more difficult.*

A. W. Moore reviewing *Realistic Rationalism*,
by Jerrold J. Katz, in the T.L.S, 11th September 1998.

Given that $0 \times 0 = 0$ *and* $1 \times 1 = 1$*, it follows that there are numbers that are their own squares. But then it follows in turn that there are numbers. In a single step of artless simplicity, we seem to have advanced from a piece of elementary arithmetic to a startling and highly controversial philosophical conclusion: that numbers exist. You would have thought that it should have been more difficult.*

A. W. Moore reviewing *Realistic Rationalism*,
by Jerrold J. Katz, in the T.L.S, 11th September 1998.

Given that the black king, in chess, is sometimes allowed to move diagonally by one square, it follows that there are chess pieces that are sometimes allowed to move diagonally by one square. But then it follows in turn that there are chess pieces.

What is the black king in Chess?

What is the black king in Chess?

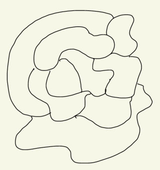
- What can one do to make someone to understand is that point to a chess board and explain the rules of the game, perhaps paying particular attention to the black king.

What is the black king in Chess?

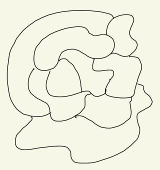
- What can one do to make someone to understand is that point to a chess board and explain the rules of the game, perhaps paying particular attention to the black king.
- What matters about the black king in chess is not its existence, or its intrinsic nature, but the role it plays in the game.

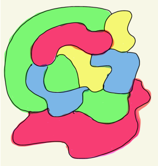
Suppose you are designing a map that is divided into regions, and you wish to choose colours for the regions such that two adjacent regions receive different colours.

Suppose you are designing a map that is divided into regions, and you wish to choose colours for the regions such that two adjacent regions receive different colours.

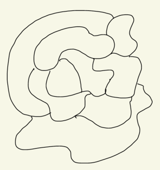


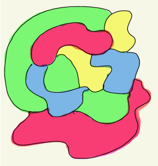
Suppose you are designing a map that is divided into regions, and you wish to choose colours for the regions such that two adjacent regions receive different colours.

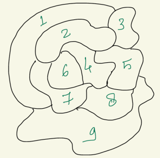




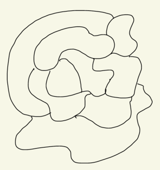
Suppose you are designing a map that is divided into regions, and you wish to choose colours for the regions such that two adjacent regions receive different colours.

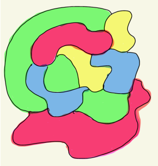


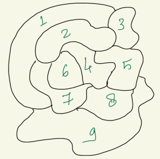


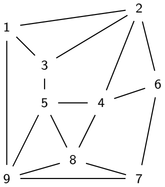


Suppose you are designing a map that is divided into regions, and you wish to choose colours for the regions such that two adjacent regions receive different colours.

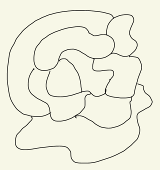


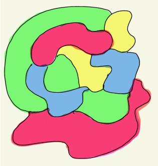


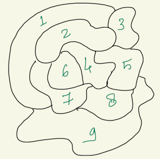


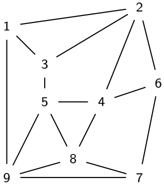


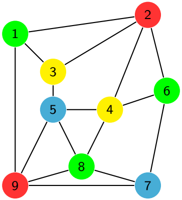
Suppose you are designing a map that is divided into regions, and you wish to choose colours for the regions such that two adjacent regions receive different colours.









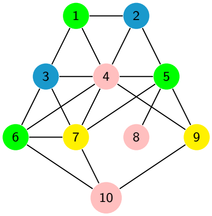


Making Schedule or Time Table

	Lecture Name	clashes with
1	Mathematics	2, 3, 4
2	Probability Theory	1, 4, 5
3	Science	1, 4, 6, 7
4	Hindi	1, 2, 3, 5, 6, 7, 9
5	English	2, 4, 7, 8, 9
6	Biology	3, ,4, 7
7	History	3, 4, 5,6
8	Programming	5
9	Economics	4, 5
9	Sanskrit	6, 7, 9

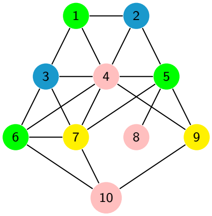
Making Schedule or Time Table

	Lecture Name	clashes with
1	Mathematics	2, 3, 4
2	Probability Theory	1, 4, 5
3	Science	1, 4, 6, 7
4	Hindi	1, 2, 3, 5, 6, 7, 9
5	English	2, 4, 7, 8, 9
6	Biology	3, ,4, 7
7	History	3, 4, 5,6
8	Programming	5
9	Economics	4, 5
9	Sanskrit	6, 7, 9



Making Schedule or Time Table

	Lecture Name	clashes with
1	Mathematics	2, 3, 4
2	Probability Theory	1, 4, 5
3	Science	1, 4, 6, 7
4	Hindi	1, 2, 3, 5, 6, 7, 9
5	English	2, 4, 7, 8, 9
6	Biology	3, ,4, 7
7	History	3, 4, 5,6
8	Programming	5
9	Economics	4, 5
9	Sanskrit	6, 7, 9



Timeslot 1	Lecture 1 (Mathematics)	Lecture 5 (English)	Lecture 6 (Biology)
Timeslot 2	Lecture 2 (Probability)	Lecture 3 (Science)	
Timeslot 3	Lecture 4 (Hindi)	Lecture 8 (Programming)	Lecture 10(Sanskrit)
Timeslot 4	Lecture 7 (History)	Lecture 9 (Economics)	

- These two problems appear to be quite different, but an appropriate choice of model shows that from the mathematical point of view they are same.

- These two problems appear to be quite different, but an appropriate choice of model shows that from the mathematical point of view they are same.
- In both cases there are some objects (regions, lectures) to which something must be assigned (colours, slots).

- These two problems appear to be quite different, but an appropriate choice of model shows that from the mathematical point of view they are same.
- In both cases there are some objects (regions, lectures) to which something must be assigned (colours, slots).
- In neither problem, do we really care what the objects are or what is being assigned to them?

(Higher) Dimension

- A notable feature of advanced mathematics is that much of it is concern with geometry in more than three dimension.

(Higher) Dimension

- A notable feature of advanced mathematics is that much of it is concern with geometry in more than three dimension.
- Lines, curves: one dimensional

(Higher) Dimension

- A notable feature of advanced mathematics is that much of it is concern with geometry in more than three dimension.
- Lines, curves: one dimensional
- Surfaces: two dimensional

(Higher) Dimension

- A notable feature of advanced mathematics is that much of it is concern with geometry in more than three dimension.
- Lines, curves: one dimensional
- Surfaces: two dimensional
- Solid objects: three dimensional

(Higher) Dimension

- A notable feature of advanced mathematics is that much of it is concern with geometry in more than three dimension.
- Lines, curves: one dimensional
- Surfaces: two dimensional
- Solid objects: three dimensional
- What is 4-dimensional or more generally n-dimensional?

(Higher) Dimension

- A notable feature of advanced mathematics is that much of it is concern with geometry in more than three dimension.
- Lines, curves: one dimensional
- Surfaces: two dimensional
- Solid objects: three dimensional
- What is 4-dimensional or more generally n-dimensional?
- What are the properties of n-dimensional objects?

(Higher) Dimension

- A notable feature of advanced mathematics is that much of it is concern with geometry in more than three dimension.
- Lines, curves: one dimensional
- Surfaces: two dimensional
- Solid objects: three dimensional
- What is 4-dimensional or more generally n-dimensional?
- What are the properties of n-dimensional objects?
- Are n-dimensional objects exit?

(Higher) Dimension

- A notable feature of advanced mathematics is that much of it is concern with geometry in more than three dimension.
- Lines, curves: one dimensional
- Surfaces: two dimensional
- Solid objects: three dimensional
- What is 4-dimensional or more generally n-dimensional?
- What are the properties of n-dimensional objects?
- Are n-dimensional objects exit?
- Why we study n-dimensional objects, $n \in \mathbb{N}$.

- How can one consider a set of properties of something without establishing first that it exists?

- How can one consider a set of properties of something without establishing first that it exists?
- One can speculate about the character of a transgender prime minister of India would be likely to have, even though there is no guarantee that there will ever be one.

- How can one consider a set of properties of something without establishing first that it exists?
- One can speculate about the character of a transgender prime minister of India would be likely to have, even though there is no guarantee that there will ever be one.

What sort of properties might we expect of 28-dimensional space?

- How can one consider a set of properties of something without establishing first that it exists?
- One can speculate about the character of a transgender prime minister of India would be likely to have, even though there is no guarantee that there will ever be one.

What sort of properties might we expect of 28-dimensional space?

- It should take 28 points to specify a point.

- How can one consider a set of properties of something without establishing first that it exists?
- One can speculate about the character of a transgender prime minister of India would be likely to have, even though there is no guarantee that there will ever be one.

What sort of properties might we expect of 28-dimensional space?

- It should take 28 points to specify a point.
- If you take 28 dimensional shape and expand it by a factor of two in every direction, then its volume should multiply by 2^{28}.

- How can one consider a set of properties of something without establishing first that it exists?
- One can speculate about the character of a transgender prime minister of India would be likely to have, even though there is no guarantee that there will ever be one.

What sort of properties might we expect of 28-dimensional space?

- It should take 28 points to specify a point.
- If you take 28 dimensional shape and expand it by a factor of two in every direction, then its volume should multiply by 2^{28}.
- If we cut an object in two pieces then the volume of the original object should be sum of the volumes of pieces.

How to define high-dimensional space

- In two dimensions you start with two directions at right angle two each other.

How to define high-dimensional space

- In two dimensions you start with two directions at right angle two each other.
- Any point in the plane is uniquely represented by an ordered pair (a, b).

How to define high-dimensional space

- In two dimensions you start with two directions at right angle two each other.
- Any point in the plane is uniquely represented by an ordered pair (a, b).
- Using Pythagoras theorem, we can prove that distance between points (a, b) and (c, d) is $\sqrt{(a-c)^2 + (b-d)^2}$.

How to define high-dimensional space

- In two dimensions you start with two directions at right angle two each other.
- Any point in the plane is uniquely represented by an ordered pair (a, b).
- Using Pythagoras theorem, we can prove that distance between points (a, b) and (c, d) is $\sqrt{(a-c)^2 + (b-d)^2}$.
- Similar representation we can use for $\mathbb{R}^3$ (3-dimensional space).

How to define high-dimensional space

- In two dimensions you start with two directions at right angle two each other.
- Any point in the plane is uniquely represented by an ordered pair (a, b).
- Using Pythagoras theorem, we can prove that distance between points (a, b) and (c, d) is $\sqrt{(a-c)^2 + (b-d)^2}$.
- Similar representation we can use for $\mathbb{R}^3$ (3-dimensional space).
- Using a slightly complicated argument we can show that distance between (a, b, c) and (d, e, f) is $\sqrt{(a-d)^2 + (b-e)^2 + (c-f)^2}$.

How to define high-dimensional space

- In two dimensions you start with two directions at right angle two each other.
- Any point in the plane is uniquely represented by an ordered pair (a, b).
- Using Pythagoras theorem, we can prove that distance between points (a, b) and (c, d) is $\sqrt{(a-c)^2 + (b-d)^2}$.
- Similar representation we can use for $\mathbb{R}^3$ (3-dimensional space).
- Using a slightly complicated argument we can show that distance between (a, b, c) and (d, e, f) is $\sqrt{(a-d)^2 + (b-e)^2 + (c-f)^2}$.
- We can generalize this and define the distance between points $(a_1, a_2, \ldots, a_n)$ and $(b_1, \ldots, b_n)$ as $\sqrt{(a_1-b_1)^2 + \ldots + (a_n-b_n)^2}$.

How to define high-dimensional space

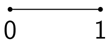
- In two dimensions you start with two directions at right angle two each other.
- Any point in the plane is uniquely represented by an ordered pair (a, b).
- Using Pythagoras theorem, we can prove that distance between points (a, b) and (c, d) is $\sqrt{(a-c)^2 + (b-d)^2}$.
- Similar representation we can use for $\mathbb{R}^3$ (3-dimensional space).
- Using a slightly complicated argument we can show that distance between (a, b, c) and (d, e, f) is $\sqrt{(a-d)^2 + (b-e)^2 + (c-f)^2}$.
- We can generalize this and define the distance between points $(a_1, a_2, \ldots, a_n)$ and $(b_1, \ldots, b_n)$ as $\sqrt{(a_1 - b_1)^2 + \ldots + (a_n - b_n)^2}$.
- Once a distance is defined we can generalize other concepts, for example n-dimensional sphere (with centre origin) $S^n = \{(x_1, \ldots, x_{n+1}) \in \mathbb{R}^{n+1} : x_1^2 + x_2^2 + \ldots + x_{n+1}^2 = 1\}$.

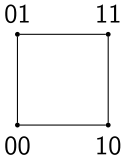
- For $n \geq 1$, let $\mathbb{R}^n = \{(x_1, \ldots, x_n) : x_i \in \mathbb{R} \text{ for each } 1 \leq i \leq n\}$.

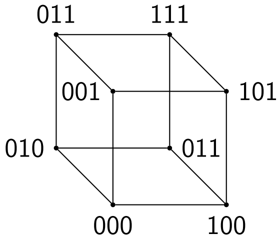
- For $n \geq 1$, let $\mathbb{R}^n = \{(x_1, \ldots, x_n) : x_i \in \mathbb{R} \text{ for each } 1 \leq i \leq n\}$.
- Once a distance is defined we can generalize other concepts, for example n-dimensional sphere (with centre origin) $S^n = \{(x_1, \ldots, x_{n+1}) \in \mathbb{R}^{n+1} : x_1^2 + x_2^2 + \ldots + x_{n+1}^2 = 1\}$.

- For $n \geq 1$, let $\mathbb{R}^n = \{(x_1, \ldots, x_n) : x_i \in \mathbb{R} \text{ for each } 1 \leq i \leq n\}$.
- Once a distance is defined we can generalize other concepts, for example n-dimensional sphere (with centre origin) $S^n = \{(x_1, \ldots, x_{n+1}) \in \mathbb{R}^{n+1} : x_1^2 + x_2^2 + \ldots + x_{n+1}^2 = 1\}$.
- Other concept we can generalize is, an interval in 1-dimension; square in 2-dimension and cube in 3-dimension.

- For $n \geq 1$, let $\mathbb{R}^n = \{(x_1, \ldots, x_n) : x_i \in \mathbb{R}$ for each $1 \leq i \leq n\}$.
- Once a distance is defined we can generalize other concepts, for example n-dimensional sphere (with centre origin) $S^n = \{(x_1, \ldots, x_{n+1}) \in \mathbb{R}^{n+1} : x_1^2 + x_2^2 + \ldots + x_{n+1}^2 = 1\}$.
- Other concept we can generalize is, an interval in 1-dimension; square in 2-dimension and cube in 3-dimension.







We can define a 4-dimensional cube, whose vertex set is given by $\{(a_1 a_2 a_3 a_4) : a_1, a_2, a_3, a_4 \in \{0, 1\}\}$ and neighbours of any vertex are vertices which differ at exactly one position.

Can four-dimensional space be visualized?

- Visualizing an object feels rather like looking at it, there are important difference between the two experiences.

Can four-dimensional space be visualized?

- Visualizing an object feels rather like looking at it, there are important difference between the two experiences.
- In a mathematical context, the important difference between being able to visualize something and not being able to is that in the former case one can somehow answer directly rather than having to stop and calculate.

Can four-dimensional space be visualized?

- Visualizing an object feels rather like looking at it, there are important difference between the two experiences.
- In a mathematical context, the important difference between being able to visualize something and not being able to is that in the former case one can somehow answer directly rather than having to stop and calculate.
- If I am asked to calculate the number of edges of a three dimensional cube, I can do it by just seeing.

Can four-dimensional space be visualized?

- Visualizing an object feels rather like looking at it, there are important difference between the two experiences.
- In a mathematical context, the important difference between being able to visualize something and not being able to is that in the former case one can somehow answer directly rather than having to stop and calculate.
- If I am asked to calculate the number of edges of a three dimensional cube, I can do it by just seeing.
- By just visualizing we can tell that number of edges in 4-dimensional cube is $12 + 12 + 8 = 32$.

Can four-dimensional space be visualized?

- Visualizing an object feels rather like looking at it, there are important difference between the two experiences.
- In a mathematical context, the important difference between being able to visualize something and not being able to is that in the former case one can somehow answer directly rather than having to stop and calculate.
- If I am asked to calculate the number of edges of a three dimensional cube, I can do it by just seeing.
- By just visualizing we can tell that number of edges in 4-dimensional cube is $12 + 12 + 8 = 32$.
- 2-dimension sphere consist of two 2-dimensional disc joining along the boundary (a circle).

Can four-dimensional space be visualized?

- Visualizing an object feels rather like looking at it, there are important difference between the two experiences.
- In a mathematical context, the important difference between being able to visualize something and not being able to is that in the former case one can somehow answer directly rather than having to stop and calculate.
- If I am asked to calculate the number of edges of a three dimensional cube, I can do it by just seeing.
- By just visualizing we can tell that number of edges in 4-dimensional cube is $12 + 12 + 8 = 32$.
- 2-dimension sphere consist of two 2-dimensional disc joining along the boundary (a circle).
- 3-dimension sphere consist of two 3-dimensional disc joining along the boundary (a 2-sphere).

Can four-dimensional space be visualized?

- Visualizing an object feels rather like looking at it, there are important difference between the two experiences.
- In a mathematical context, the important difference between being able to visualize something and not being able to is that in the former case one can somehow answer directly rather than having to stop and calculate.
- If I am asked to calculate the number of edges of a three dimensional cube, I can do it by just seeing.
- By just visualizing we can tell that number of edges in 4-dimensional cube is $12 + 12 + 8 = 32$.
- 2-dimension sphere consist of two 2-dimensional disc joining along the boundary (a circle).
- 3-dimension sphere consist of two 3-dimensional disc joining along the boundary (a 2-sphere).
- One of the pleasure of devoting one's life into mathematics research is that, as one gains in expertise, one finds that one can "just see" answers to more questions that might once have required an hours of hard thought.

What is the point of higher-dimensional geometry?

- Motion of an object by drawing an graph (considering as function of time).

What is the point of higher-dimensional geometry?

- Motion of an object by drawing an graph (considering as function of time).
- You are wondering whether it is wise to buy shares in a company, then much of the information that will help you make your decision comes in forms of numbers: the size of the workplace, the values of the various assets, the cost of raw materials, the rate of interest, and so on.

What is the point of higher-dimensional geometry?

- Motion of an object by drawing an graph (considering as function of time).
- You are wondering whether it is wise to buy shares in a company, then much of the information that will help you make your decision comes in forms of numbers: the size of the workplace, the values of the various assets, the cost of raw materials, the rate of interest, and so on.
- Suppose you want to describe a position of chair assuming it is standing up right. Then its position is completely determined by the points where two of its legs meets the floor.

Determinants

The determinant of order two is defined by the formula

$$\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \end{vmatrix} = x_1 y_2 - x_2 y_1.$$

Determinants

The determinant of order two is defined by the formula

$$\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \end{vmatrix} = x_1 y_2 - x_2 y_1.$$

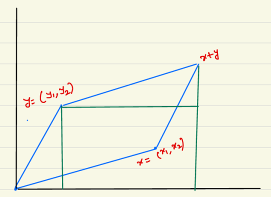
Let $x = (x_1, x_2), y = (y_1, y_2) \in \mathbb{R}^2$.

Determinants

The determinant of order two is defined by the formula

$$\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \end{vmatrix} = x_1 y_2 - x_2 y_1.$$

Let $x = (x_1, x_2), y = (y_1, y_2) \in \mathbb{R}^2$.

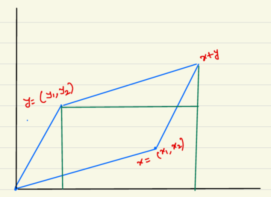


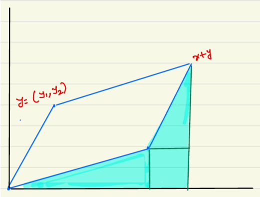
Determinants

The determinant of order two is defined by the formula

$$\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \end{vmatrix} = x_1 y_2 - x_2 y_1.$$

Let $x = (x_1, x_2), y = (y_1, y_2) \in \mathbb{R}^2$.



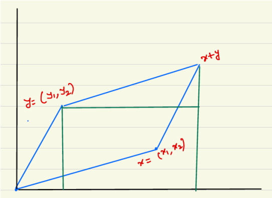


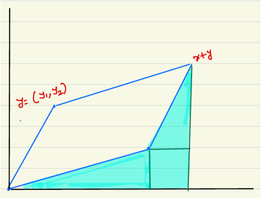
Determinants

The determinant of order two is defined by the formula

$$\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \end{vmatrix} = x_1 y_2 - x_2 y_1.$$

Let $x = (x_1, x_2), y = (y_1, y_2) \in \mathbb{R}^2$.





Area of the parallelogram is given by
$\left(\frac{1}{2}y_1y_2 + y_2x_1 + \frac{1}{2}x_1x_2\right) - \left(\frac{1}{2}x_1x_2 + x_2y_1 + \frac{1}{2}y_1y_2\right) = y_2x_1 - x_2y_1.$

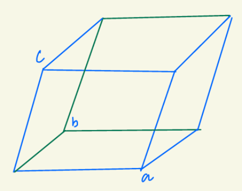
Thus we may think of the determinant of a 2×2 matrix

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

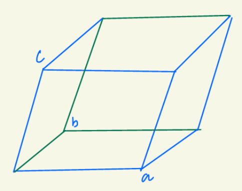
as the signed area of the parallelogram spanned by the columns $\begin{pmatrix} a \\ c \end{pmatrix}$ and $\begin{pmatrix} b \\ d \end{pmatrix}$ in $\mathbb{R}^2$.

Suppose we have given n vectors $v_1, v_2, \ldots, v_n \in \mathbb{R}^n$. We wish to define determinant as a function which attach n vectors $v_1, v_2, \ldots, v_n \in \mathbb{R}^n$ to a real number.

Suppose we have given n vectors $v_1, v_2, \ldots, v_n \in \mathbb{R}^n$. We wish to define determinant as a function which attach n vectors $v_1, v_2, \ldots, v_n \in \mathbb{R}^n$ to a real number. The number is to be thought as the signed volume of the parallelepiped spanned by $v_i' s$.



Suppose we have given n vectors $v_1, v_2, \ldots, v_n \in \mathbb{R}^n$. We wish to define determinant as a function which attach n vectors $v_1, v_2, \ldots, v_n \in \mathbb{R}^n$ to a real number. The number is to be thought as the signed volume of the parallelepiped spanned by $v_i's$.



The parallelepiped is defined to be the set $\{x \in \mathbb{R}^n : x = \sum_{i=1}^{n} \alpha_i v_i, 0 \leq \alpha_i \leq 1\}$.

Properties

- If we dilate the area of one side by, say a factor of δ, we expect that the area of the resulting parallelogram should be δ times that of the original area.

$$\det \begin{pmatrix} \delta a & b \\ \delta c & d \end{pmatrix} = \delta \det \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

Properties

- If we dilate the area of one side by, say a factor of δ, we expect that the area of the resulting parallelogram should be δ times that of the original area.

$$\det\begin{pmatrix}\delta a & b\\ \delta c & d\end{pmatrix} = \delta\det\begin{pmatrix}a & b\\ c & d\end{pmatrix}$$

- Adding one row to another causes the determinant unchanged.

$$\det\begin{pmatrix}x_1 & y_1\\ x_2 & y_2\end{pmatrix} = \det\begin{pmatrix}x_1+y_1 & y_1\\ x_2+y_2 & y_2\end{pmatrix}$$

Properties

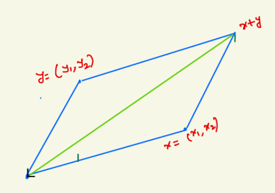
- If we dilate the area of one side by, say a factor of δ, we expect that the area of the resulting parallelogram should be δ times that of the original area.

$$\det\begin{pmatrix}\delta a & b\\ \delta c & d\end{pmatrix} = \delta \det\begin{pmatrix}a & b\\ c & d\end{pmatrix}$$

- Adding one row to another causes the determinant unchanged.

$$\det\begin{pmatrix}x_1 & y_1\\ x_2 & y_2\end{pmatrix} = \det\begin{pmatrix}x_1+y_1 & y_1\\ x_2+y_2 & y_2\end{pmatrix}$$

The volume is unaltered by cutting and rearranging the object.



- If two columns are same, then determinant is zero.
  Geometrically it means that the object does not have n-dimensional volume (but it may have lower dimensional volume).

Thank you