

$$i) 2x - 3y + 7z = 5, \quad 3x + y - 3z = 13, \quad 2x + 9y - 4z = 32$$

$$A = \begin{bmatrix} 2 & -3 & 7 \\ 3 & 1 & -3 \\ 2 & 19 & -47 \end{bmatrix}, \quad x = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad B = \begin{bmatrix} 5 \\ 13 \\ 32 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -3 & 7 \\ 3 & 1 & -3 \\ 0 & 22 & -54 \end{bmatrix}, \quad B = \begin{bmatrix} 5 \\ 13 \\ 27 \end{bmatrix}$$

$$R_2 \rightarrow R_2 - \frac{3}{2} R_1$$

$$\begin{bmatrix} 2 & -3 & 7 \\ 0 & 15 & -\frac{27}{2} \\ 0 & 22 & -54 \end{bmatrix}, \quad B = \begin{bmatrix} 5 \\ 15 \\ 27 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -3 & 7 \\ 0 & 15 & -\frac{27}{2} \\ 0 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 5 \\ 15 \\ 0 \end{bmatrix}$$

Rank of A = 2  
Rank of A:B = 3

$\rho(A) \neq \rho(A:B)$  Inconsistency.

$$ii) 2x - y + 3z = 8, \quad -x + 2y + z = 4, \quad 3x + y - 4z = 0$$

$$A = \begin{bmatrix} 2 & -1 & 3 \\ -1 & 2 & 1 \\ 3 & 1 & -4 \end{bmatrix}, \quad B = \begin{bmatrix} 8 \\ 4 \\ 0 \end{bmatrix}, \quad x = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 2 & 1 \\ 2 & -1 & 3 \\ 3 & 1 & -4 \end{bmatrix}, \quad B = \begin{bmatrix} 4 \\ 8 \\ 0 \end{bmatrix}$$

$$A = \begin{bmatrix} -1 & 2 & 1 \\ 0 & 3 & 5 \\ 0 & 7 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} 4 \\ 16 \\ 12 \end{bmatrix}$$

B.Karthikaya

$$A = \begin{bmatrix} -1 & 2 & 1 \\ 0 & 3 & 5 \\ 0 & 7 & -1 \end{bmatrix} \quad B = \begin{bmatrix} 4 \\ 16 \\ 12 \end{bmatrix}$$

$$A = \begin{bmatrix} -1 & 2 & 1 \\ 0 & 3 & 5 \\ 0 & 0 & \frac{-38}{5} \end{bmatrix} \quad B = \begin{bmatrix} 4 \\ 16 \\ 12 \end{bmatrix}$$

$$R_3 \rightarrow R_3 - \frac{7}{3}R_2$$

$$= \begin{bmatrix} -1 & 2 & 1 \\ 0 & 3 & 5 \\ 0 & -1 & \frac{35}{3} \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 2 & 1 \\ 0 & 3 & 5 \\ 0 & -\frac{38}{3} & 0 \end{bmatrix}$$

$\delta(A) = \delta(A:B)$  consistency.

x(iii)

$$\text{Q) } 4x - y = 12, \quad -x + 5y - 2z = 0, \quad -2x + 4z = -8$$

$$A = \begin{bmatrix} 4 & -1 & 0 \\ -1 & 5 & -2 \\ -2 & 0 & 4 \end{bmatrix} \quad x = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad B = \begin{bmatrix} 12 \\ 0 \\ -8 \end{bmatrix}$$

$$A = \begin{bmatrix} 4 & -1 & 0 \\ -1 & 5 & -2 \\ -4 & 0 & 8 \end{bmatrix} \quad B = \begin{bmatrix} 12 \\ 0 \\ -16 \end{bmatrix}$$

$$A = \begin{bmatrix} 4 & -1 & 0 \\ -1 & 5 & -2 \\ 0 & -1 & 8 \end{bmatrix} \quad B = \begin{bmatrix} 12 \\ 0 \\ -4 \end{bmatrix}$$

$$A = \begin{bmatrix} 4 & -1 & 0 \\ -4 & 20 & -8 \\ 0 & -1 & 8 \end{bmatrix} \quad B = \begin{bmatrix} 12 \\ 0 \\ -4 \end{bmatrix}$$

$$A = \begin{bmatrix} 4 & -1 & 0 \\ 0 & 19 & -8 \\ 0 & -1 & 8 \end{bmatrix} \quad B = \begin{bmatrix} 12 \\ 0 \\ -4 \end{bmatrix}$$

$$A = \begin{bmatrix} 4 & -1 & 0 \\ 0 & 19 & -8 \\ 0 & 18 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 12 \\ 0 \\ 8 \end{bmatrix}$$

$\delta(A) = \delta(A:B)$  consistency.

B. Kavithi Kanya.

b) For what values of  $\lambda$  and  $\mu$ .

$$x+y+z=6, x+2y+3z=10, x+2y+\lambda z=\mu$$

i) no solution, ii) a unique solution (iii) infinite no. of solutions.

$$Ax = B$$

Sol:

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 2 & \lambda \end{bmatrix}, \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 10 \\ \mu \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & \lambda-3 \end{bmatrix}$$

$$A:B = \begin{bmatrix} 1 & 1 & 1 & 6 \\ 1 & 2 & 3 & 10 \\ 1 & 2 & \lambda & \mu \end{bmatrix}$$

i) No solution ( $S(A) \neq S(A:B)$ )

$$\lambda=3, \mu \neq 10$$

(ii) a unique solution.  $S(A)=S(A:B)$

$$\lambda \neq 3, \mu \text{ any value}$$

iii) Infinite solution  $S(A)=S(A:B) < 3n$

$$\lambda=3, \mu=10$$

$n \rightarrow$  no. of unknowns  
 $x, y, z$

B. Keerthi Kya

c) Find for what value of  $\lambda$ . B. Karthikaya  
Have a solution. &  
Find  $\lambda$  to solve for each value of  $\lambda$ .

$$x+y+z=1$$

$$x+2y+4z=\lambda$$

$$x+4y+10z=\lambda^2$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 4 & 10 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ \lambda \\ \lambda^2 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix}$$

$$A:B = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & \lambda-1 \\ 0 & 0 & 0 & \lambda^2-1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & \lambda-1 \\ 0 & 0 & 0 & \lambda^2-1 \end{bmatrix}$$

$$f(A) = 2$$

$$f(A:B) = 2$$

$$\lambda^2 - 3\lambda + 2 = 0$$

$$\lambda^2 - 2\lambda - \lambda + 2 = 0$$

$$\lambda(\lambda-1) - \lambda(\lambda-1) = 0$$

$$(\lambda-1)(\lambda-2) = 0$$

$$\lambda = 1, 2$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & \lambda-1 \\ 0 & 0 & 0 & \lambda^2-1-3\lambda+3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & \lambda-1 \\ 0 & 0 & 0 & \lambda^2-3\lambda+2 \end{bmatrix}$$

$$f(A) = f(A:B) \neq n-2$$

3 unknowns

Case-1  $\lambda = 1$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$x+y+z=1$$

$$y+3z=0$$

$$z=k$$

$$y=-3k$$

$$x = 1 - y - z$$

$$= 1 + 3k - k = 1 + 2k$$

Case-2  $\lambda = 2$

3 minors are zero

not col of h.c.f.

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{aligned} x+y+z &= 1 \\ y+3z &= 1 \\ y &= 1-3z \\ z &= k \end{aligned}$$

$$x = 1 - 1 + 3k - k = 2k$$

d) Find the sol<sup>n</sup> of the system of equations.

$$x+3y-2z=0, \quad 2x-y+4z=0, \quad x-11y+14z=0.$$

B.Karthikeya

$$A = \begin{bmatrix} 1 & 3 & -2 \\ 2 & -1 & 4 \\ 1 & -11 & 14 \end{bmatrix}, \quad x = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\overset{R_2-R_1}{\sim} \begin{bmatrix} 1 & 3 & -2 \\ 0 & -7 & 8 \\ 0 & -14 & 16 \end{bmatrix} \overset{R_3-2R_2}{\sim} \begin{bmatrix} 1 & 3 & -2 \\ 0 & -7 & 8 \\ 0 & 0 & 0 \end{bmatrix} \therefore$$

e) Find for what values of  $\lambda$  the given equations  $3x+y-\lambda z=0$ ,  $4x-2y-3z=0$ ,  $2\lambda x+4y+\lambda z=0$ , may possess non-trivial solution and solve them completely in each case

$$\begin{aligned} 12x &= 0 & \text{S}(A) = \text{S}(A \cup E) \\ 2 &\neq 0 & = 2 \neq n \\ -7y &= 0 & \text{Infinite solution} \\ y &= 0 \\ x &\neq 0. \end{aligned}$$

$$A = \begin{bmatrix} 3 & +1 & -\lambda \\ 4 & -2 & -3 \\ 2\lambda & 4 & \lambda \end{bmatrix} \sim \begin{bmatrix} 3 & +1 & -\lambda \\ 4 & -2 & -3 \\ 2x+3 & 4+1 & 0 \end{bmatrix} \text{ Infinite}$$

$R_3 = R_3 + R_1$

$$\begin{bmatrix} 12 & 4 & -4\lambda \\ 12 & -6 & -9 \\ 2x+3 & 5 & 0 \end{bmatrix} \sim \begin{bmatrix} 12 & 4 & -4\lambda \\ 0 & -10 & -9+4\lambda \\ 2x+3 & 5 & 0 \end{bmatrix}$$

$R_1 = R_1/4, R_2 = R_2/3$

$$\begin{bmatrix} 12 & 4 & -4\lambda \\ 0 & -10 & -9+4\lambda \\ 2x+3 & 5 & 0 \end{bmatrix} \sim \begin{bmatrix} 12 & 4 & -9 \\ 0 & -10 & -9+4\lambda \\ 2x+3 & 5 & 0 \end{bmatrix}$$

$R_2 = R_2 - R_1$

$$\begin{bmatrix} 12 & 4 & -9 \\ 0 & -10 & -9+4\lambda \\ 2x+3 & 5 & 0 \end{bmatrix} \sim \begin{bmatrix} 12 & 4 & -9 \\ 0 & -10 & -9+4\lambda \\ 2x+3 & 5 & 0 \end{bmatrix}$$

$R_1 = R_1 + R_2$

$$\begin{aligned} 12x+6y-9z &= 0 \\ -10y+(-9+4\lambda)z &= 0 \\ 2x+3 &= 0 \quad \frac{-9+4\lambda}{2} = 0 \\ x &= \frac{-3}{2} \quad \lambda = \frac{9}{4} \\ 10y &= (-9+4\lambda)z \\ z &= 10y/(-9+4\lambda) \end{aligned}$$

$$\begin{aligned} 12x+6y-9z &= 0 \\ -10y+(-9+4\lambda)z &= 0 \\ (2x+3)x+5y &= 0 \\ -(2x+3)\frac{x}{2}y &= \frac{9+4\lambda}{5} \end{aligned}$$

B. Kafthikeya

$$12x - 6y - 9z = 0$$

$$12x + \frac{6(2\lambda+3)x}{5} - 9\left(\frac{10y}{(-9+4\lambda)}\right) = 0$$

$$12x + \frac{6(2\lambda+3)}{5}x + 9\left(\frac{10^2}{(-9+4\lambda)}\right)\left(\frac{2\lambda+3}{5}\right)x = 0$$

$$12 + \frac{6(2\lambda+3)}{5} + \frac{18(2\lambda+3)}{(2(4\lambda-9)} = 0$$

$$12(4\lambda-9)5 + 6(2\lambda+3)(4\lambda-9) + 18(5)(2\lambda+3) = 0$$

$$240\lambda - 540 + 180\lambda + 270 + 48\lambda^2 - 108\lambda + 72\lambda - 162 = 0$$

$$48\lambda^2 + 384\lambda - 432 = 0$$

$$\lambda^2 + 8\lambda - 9 = 0$$

$$\lambda^2 + 9\lambda - \lambda - 9 = 0$$

$$\lambda(\lambda+1) + 9(\lambda+1) = 0$$

$$(\lambda+9)(\lambda+1) = 0 \Rightarrow \lambda = -1, \lambda = -9$$

$$\lambda = 1$$

$$-x = y, z = -2y$$

$$12x - 6y - 9z = 0$$

$$12(-y) - 6y - 9(-2y) = 0$$

$$18y - 18y = 0$$

$y \neq 0$  non-trivial solution

$$\lambda = -9$$

$$y = \left(-\frac{2x+3}{5}\right)x$$

$$z = \frac{10y}{(-9+4\lambda)}$$

$$z = \frac{(-18+3)}{5}x = +\frac{3}{5}x$$

$$= \frac{-10y}{-45} = \frac{2y}{-9}$$

$$12x - 6y - 9z = 0$$

$$12\left(\frac{y}{3}\right) - 6y - 9\left(\frac{2y}{-9}\right) = 0$$

$$4y - 6y + 2y = 0$$

$y = 0$  trivial solution

It has no non-trivial solution

## Assignment - 2

(Q)

$$1) [1, 0, 0], [1, 1, 0], [1, 1, 1]$$

$$\begin{bmatrix} c_1 + c_2 + c_3 & c_2 + c_3 & c_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}$$

$$c_3 = 0, c_2 = 0, c_1 = 0$$

unique solution  
linearly independent.

$$2) [6, 0, 3, 1, 4, 2], [0, -1, 2, 7, 0, 5], [1, 2, 3, 0, -1, 9, 8, -11]$$

$$\begin{bmatrix} 6c_1 + 12c_3 & -1c_2 + 3c_3 & 3c_1 + 2c_2 & c_1 + 7c_2 - 19c_3 \\ 4c_1 + 8c_3 & 5c_2 - 11c_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$c_1 + 2c_3 = 0$$

$$c_1 + 2c_3 = 0$$

$$3c_1 + 2c_2 = 0$$

$$-c_2 + 3c_3 = 0$$

$$c_1 + 7c_2 - 19c_3 = 0$$

$$5c_2 - 11c_3 = 0$$

$$c_2 = 3c_3$$

$$c_1 = 0$$

$$15c_3 - 11c_3 = 0$$

$$4c_3 = 0$$

$$c_3 = 0$$

unique solution. linearly independent

B.Karthikeya.

$$2) [7 \ -3 \ 11 \ -6], [-56 \ 24 \ -88 \ 48]$$

B-Karthika

$$\begin{bmatrix} 7c_1 - 56c_2 & -3c_1 + 24c_2 & 11c_1 - 88c_2 & -6c_1 + 48c_2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 8k \\ k \end{bmatrix} = k \begin{bmatrix} 8 \\ 1 \end{bmatrix}$$

$$7c_1 = 56c_2$$

$$c_1 = 8c_2$$

$$c_1 = 8k$$

linearly dependent

$$3) [-1 \ 5 \ 0], [16 \ 8 \ -3], [-64 \ 56 \ 9]$$

$$\begin{bmatrix} -1c_1 + 16c_2 - 64c_3 & 5c_1 + 8c_2 + 56c_3 & -3c_2 + 9c_3 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}$$

$$-3c_2 + 9c_3 = 0$$

$$3. 3c_3 = 3c_2$$

$$c_2 = \frac{3c_3}{3k}$$

$$-c_1 + 16c_2 - 64c_3 = 0$$

$$16c_2 = 64c_3 + c_1$$

$$16(3c_3) = 64c_3 + c_1$$

$$\Rightarrow c_1 = 0$$

$$\text{vector} = k \begin{bmatrix} 0 \\ 3k \\ k \end{bmatrix} = k \begin{bmatrix} 0 \\ 3 \\ 1 \end{bmatrix}$$

linearly dependent

$$4) [1 \ -1 \ 1]; [1 \ 1 \ -1]; [-1 \ 1 \ 1] \cdot [0 \ 1 \ 0]$$

$$\begin{bmatrix} c_1 + c_2 - c_3 & -c_1 + c_2 + c_3 + c_4 & c_1 - c_2 + c_3 \end{bmatrix}$$

$$c_1 + c_2 - c_3 = 0$$

$$-c_1 + c_2 + c_3 + c_4 = 0$$

$$c_1 + c_2 = c_3$$

$$c_1 = c_2 + c_3 + c_4$$

$$c_1 + c_2 + c_3 = c_3$$

$$c_4 = -2k$$

$$2c_1 = 0 \Rightarrow c_1 = 0.$$

$$c_1 - c_2 + c_3 = 0$$

$$c_2 = c_1 + c_3.$$

$$c_2 = c_3 = k.$$

linearly dependent

5)  $\begin{bmatrix} 2 & -4 \end{bmatrix}, \begin{bmatrix} 1 & 9 \end{bmatrix}, \begin{bmatrix} 3 & 5 \end{bmatrix}$

$$\begin{bmatrix} 2C_1 + C_2 + 3C_3 \\ 2C_1 + C_2 + 3C_3 \\ 4C_1 + 2C_2 + 6C_3 \end{bmatrix} - [4C_1 + 9C_2 + 5C_3] = \begin{bmatrix} 0 & 0 \end{bmatrix}$$

$$2C_1 + C_2 + 3C_3 = 0$$

$$4C_1 + 2C_2 + 6C_3 = 0.$$

$$9C_2 + 5C_3 + 2C_2 + 6C_3 = 0$$

$$11C_2 + 11C_3 = 0$$

$$C_2 = -C_3 = -K.$$

$$4C_1 = 9CK - 5K$$

$$C_1 = 4K$$

$$C_1 = K.$$

vector =  $\begin{bmatrix} K \\ 4K \\ -K \end{bmatrix} = K \begin{bmatrix} 1 \\ 4 \\ -1 \end{bmatrix}$ , linearly dependent

6)  $\begin{bmatrix} 3 & -2 & 0 & 4 \end{bmatrix}, \begin{bmatrix} 5 & 0 & 0 & 1 \end{bmatrix}; \begin{bmatrix} -6 & 1 & 0 & 1 \end{bmatrix}$

$$\begin{bmatrix} 2 & 0 & 0 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 3C_1 + 5C_2 - 6C_3 + 2C_4 \\ 2C_1 + C_2 + C_3 + 3C_4 \\ -2C_1 + C_3 \\ 2C_1 = C_3 \end{bmatrix}$$

$$\begin{bmatrix} -2C_1 + C_3 \\ 4C_1 + C_2 + C_3 + 3C_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}$$

$$-2C_1 + C_3$$

$$4C_1 + C_2 + C_3 + 3C_4 = 0$$

$$2C_1 + C_2 + C_3 + 3C_4 = 0$$

$$3C_1 + 5C_2 - 6C_3 + 2C_4 = 0$$

$$C_1 = K$$

$$\underline{\underline{17C_1 + 11C_3 - 13C_4 = 0}}$$

$$C_3 = 2C_1$$

$$17K + 11(2K) = 13C_4$$

$$4C_1 + C_2 + C_3 + 3C_4$$

$$\cancel{17K + 22K} = 13C_4$$

$$4(K) + C_2 + 2K + 3(2K) = 0$$

$$\frac{32K}{13} = C_4 = 3K$$

$$C_2 + 15K = 0$$

$$C_2 = -15K.$$

$$\text{vector} = \begin{bmatrix} K \\ -15K \\ 2K \\ 3K \end{bmatrix} = K \begin{bmatrix} 1 \\ -15 \\ 2 \\ 3 \end{bmatrix}$$

linearly dependent

Bharthayya.

$\Rightarrow [3 \ 4 \ 7], [2 \ 0 \ 3], [8 \ 2 \ 3], [5 \ 5 \ 6]$

$$\begin{bmatrix} 3C_1 + 2C_2 + 8C_3 + 5C_4 \\ 4C_1 + 2C_3 + 5C_4 \\ = [0 \ 0 \ 0] \end{bmatrix} \quad 4C_1 + 2C_3 + 5C_4 = 7C_1 + 3C_2 + 3C_3 + 5C_4$$

Eigen Values

$$A = \begin{bmatrix} -2 & 2 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}$$

$$3C_1 + 2C_2 + 8C_3 + 5C_4 = 0$$

$$\underline{4C_1 + 2C_3 + 5C_4 = 0}$$

$$\underline{-C_1 + 2C_2 + 6C_3 = 0}$$

$$C_1 = 2C_2 + 6C_3$$

$$5C_1 = 10C_2 + 30C_3$$

$$7C_1 + 3C_2 + 3C_3 + 6C_4 = 0$$

$$\underline{4C_1 + 6C_2 + 6C_3 + 12C_4 = 0}$$

$$\underline{-9C_1 + 6C_2 + 24C_3 + 15C_4 = 0}$$

$$\underline{5C_1 - 18C_3 - 3C_4 = 0}$$

$$5C_1 = 18C_3 + 3C_4$$

$$10C_2 + 30C_3 = 18C_3 + 3C_4$$

$$12C_3 = 3C_4 - 10C_2$$

$$4C_1 + 2C_3 + 5C_4 = 0$$

$$36C_1 + 18C_3 + 45C_4 = 0$$

$$\underline{5C_1 - 18C_3 - 3C_4 = 0}$$

$$41C_1 + 42C_4 = 0$$

$$41C_1 = -42C_4$$

$$C_1 = -\frac{42}{41}C_4 = -\frac{42}{41}K.$$

$$= \begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \end{bmatrix} = K \begin{bmatrix} -\frac{42}{41} \\ \frac{10543}{492} \\ 1 \\ \frac{1506}{216} \end{bmatrix}$$

$$18C_3 = 5C_1 - 3C_4$$

linearly dependent

$$C_3 = 5\left(-\frac{42}{41}\right)K - 3(K)\left(\frac{1}{6}\right)$$

$$= \frac{(210(6) - 6(41))}{(41)(6)} K = \frac{(1260 + 216)K}{216}$$

$$\frac{42}{41}K = 2C_2 + 8\left(\frac{251}{38}\right)K \Rightarrow \frac{1506K}{216}$$

$$2C_2 = -\frac{42K}{41} - \frac{251}{6}K$$

$$C_2 = -\left(\frac{252 + 10291}{2 \times 246}\right)K = \frac{10543K}{492}$$

B-Kar thi kya.

$$\frac{6 \times 36 + 35}{36}$$

$$1) \quad A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix} \quad | \cdot A \rightarrow I = 0$$

$$= \begin{bmatrix} -2-\lambda & 2 & -3 \\ 2 & 1-\lambda & -6 \\ -1 & -2 & -\lambda \end{bmatrix} = (-2-\lambda)(-\lambda(1-\lambda)+6(-2))$$

$$= -2(-\lambda^2 + 2\lambda + 6(-1))$$

$$= -3(-4 + 1 - \lambda)$$

$$= (-2-\lambda)(-\lambda + \lambda^2 - 12) + 2(+\lambda + 6) + 3(+3 + \lambda)$$

$$= +2\lambda - 2\lambda^2 + 24 + \lambda^2 - \lambda^3 + 12\lambda + 2\lambda + 12 + 9 + 3\lambda$$

$$\textcircled{Q} 19\lambda - \lambda^2 - \lambda^3 + 45$$

$$\lambda^3 + \lambda^2 - 19\lambda - 45 = 0$$

$$2) \quad A = \begin{bmatrix} 1 & 1 & 1 \\ -1 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix} \quad | \cdot A - \lambda I = 0$$

$$= \begin{bmatrix} 4-\lambda & 0 & 1 \\ -2 & 1-\lambda & 0 \\ -2 & 0 & 1-\lambda \end{bmatrix} = (4-\lambda)(1-\lambda)(1-\lambda) + 1(+2(1-\lambda))$$

$$= (1-\lambda)((4-\lambda)(1-\lambda) + 2)$$

$$= (4-\lambda - 4\lambda + \lambda^2 + 2)$$

$$\lambda = -1, 2, 3.$$

Eigenvalues.

$$x = -1$$

$$\begin{bmatrix} 5 & 0 & 1 \\ -2 & 2 & 0 \\ -2 & 0 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$5x + z = 0$$

$$-2x + 2y = 0$$

$$-2x + 2z = 0$$

$$\underline{10x + 2z = 0}$$

$$-12x = 0$$

$$x = 0$$

$$(5-1)x + (0-1)z - 2y = 0 \rightarrow z = 0$$

$$(0-1) + (2-1)y = 0 \rightarrow y = 0$$

$$(5-1+0-1) = 0$$

$$\text{Eigen vector} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$(5+0)x + (0+0)z + (0+0)y = 0$$

$$\lambda = \frac{2k+1}{24} + \frac{2k+1}{24} + \frac{2k+1}{24} + k = \frac{6k+3}{24} + k = \frac{6k+3+24k}{24} = \frac{30k+3}{24} = \frac{10k+1}{8}$$

$$\begin{bmatrix} 2 & 0 & 1 \\ -2 & -1 & 0 \\ -2 & 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x+k \\ 2x+z = 0 \\ -2x-y = 0 \\ -2x-z = 0 \end{bmatrix}$$

$$x = -k$$

$$z = 2k$$

$$y = -2k$$

$$\text{Eigen vector} = k \begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix}$$

$$x = 3$$

$$\begin{bmatrix} 1 & 0 & 1 \\ -2 & 0 & 0 \\ -2 & 0 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$x + z = 0$$

$$-2x - 2y = 0$$

$$-2x - 2z = 0$$

$$x = -z = -k$$

$$z = k$$

$$y = -x$$

$$y = k$$

$$\text{vector}_2 = k \begin{bmatrix} -k \\ k \\ k \end{bmatrix} = +k \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$$

$$3) = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 0 & 0 \\ -1 & 0 & 3 \end{bmatrix}$$

$$A - \lambda I = \begin{bmatrix} 5-\lambda & 0 & 0 \\ 0 & -\lambda & 0 \\ -1 & 0 & 3-\lambda \end{bmatrix}$$

$$(5-\lambda)(-\lambda)(3-\lambda) = 0$$

$$\lambda = 0, 3, 5.$$

$$\lambda = 0$$

$$\begin{bmatrix} 5 & 0 & 0 \\ 0 & 0 & 0 \\ -1 & 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$5x=0 \Rightarrow x=0$   
 $-x+3z=0 \Rightarrow z=0$   
 vector<sub>1</sub> =  $k \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

$$\lambda = 3$$

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & -3 & 0 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$2x=0$   
 $-3y=0$   
 $-x=0$

$$\text{vector}_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\lambda = 5$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & -5 & 0 \\ -1 & 0 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$-5y=0$   
 $-x-2z=0$   
 $x=2z = -2k$

$$\text{vector}_3 = \begin{bmatrix} -2k \\ 0 \\ k \end{bmatrix} = k \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}$$

$$4) \begin{bmatrix} 0 & 0 & 0 \\ 0 & 3 & 4 \\ 0 & 0 & -2 \end{bmatrix} = \begin{bmatrix} 0-\lambda & 0 & 0 \\ 0 & 3-\lambda & 4 \\ 0 & 0 & -2-\lambda \end{bmatrix}$$

$$\rightarrow ((3-\lambda)(-2+\lambda)) = 0 \Rightarrow (\lambda-3)(\lambda+2) = 0$$

$$\lambda = 0, 3, -2$$

$$\lambda = 0$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 3 & 4 \\ 0 & 0 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 3y + 4z = 0 \\ -2z = 0 \end{bmatrix}$$

$$z = 0, y = 0$$

$$\text{vector}_2 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\lambda = 3$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 4 \\ 0 & 0 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 4z = 0 \\ -5z = 0 \end{bmatrix}$$

$$\text{vector}_2 \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

$$\lambda = -2$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 5 & 4 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 5y + 4z = 0 \\ 5y - 4z = -4k \end{bmatrix}$$

$$y = \frac{-4}{5} k$$

$$k \begin{bmatrix} 0 \\ -\frac{4}{5} \\ 1 \end{bmatrix}$$