Satyam ...

1) $A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 3 \\ 0 & -4 & -11 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & -3 & 2 \end{bmatrix}$

$R_2 \to R_2 - 2R_1$ $\quad R_4 \to R_4 - 6R_1$ $\quad R_4 \to R_4 - R_3$ $\quad R_4 \to R_4 - R_2$

$R_3 \to R_3 - 3R_1$

$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & -3 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ no. of non zero rows
rank = 3

3) $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \Rightarrow A^{-1} = \frac{1}{4-1}\begin{bmatrix} 2 & +1 \\ +1 & 2 \end{bmatrix} = \begin{bmatrix} \frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} \end{bmatrix}$

$= \begin{bmatrix} \frac{2}{3} - \lambda & \frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} - \lambda \end{bmatrix} = \left(\frac{2}{3} - \lambda\right)^2 - \left(\frac{1}{3}\right)^2 = 0$

$\boxed{a^2 - b^2 = (a-b)(a+b) = 0}$

$= \frac{1}{3} - \lambda (1 - \lambda) = 0$

$\boxed{\text{Eigen values } \lambda = 1, \frac{1}{3}}$

$\lambda = 1$

$\begin{bmatrix} \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & -\frac{1}{3} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$

$= x\frac{1}{3} + y\left(\frac{1}{3}\right) = 0$

$x = y = k$ $\quad$ Eigen vector $k\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$\lambda = \frac{1}{3}$ $\begin{bmatrix} \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$

$x = -y$ $\boxed{\text{Eigen vector} = \begin{bmatrix} k \\ -k \end{bmatrix} = k\begin{bmatrix} 1 \\ -1 \end{bmatrix}}$

$z = A + 4I$ $\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 6 & -1 \\ -1 & 6 \end{bmatrix}$

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1) $A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 3 \\ 0 & -4 & -11 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & -3 & 2 \end{bmatrix}$

$R_2 \to R_2 - 2R_1$ $\quad R_4 \to R_4 - 6R_1$ $\quad R_4 \to R_4 - R_3$ $\quad R_4 \to R_4 - R_2$

$R_3 \to R_3 - 3R_1$

$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & -3 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ no. of non zero rows
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3) $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \Rightarrow A^{-1} = \frac{1}{4-1}\begin{bmatrix} 2 & +1 \\ +1 & 2 \end{bmatrix} = \begin{bmatrix} \frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} \end{bmatrix}$

$= \begin{bmatrix} \frac{2}{3}-\lambda & \frac{1}{3} \\ \frac{1}{3} & \frac{2}{3}-\lambda \end{bmatrix} = \left(\frac{2}{3}-\lambda\right)^2 - \left(\frac{1}{3}\right)^2 = 0$

$\boxed{a^2 - b^2 = (a-b)(a+b) = 0}$

$= \frac{1}{3} - \lambda (1-\lambda) = 0$

$\boxed{\text{Eigen values } \lambda = 1, \frac{1}{3}}$

$\lambda = 1$

$\begin{bmatrix} \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & -\frac{1}{3} \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = 0$

$= x\frac{1}{3} + y\left(\frac{1}{3}\right) = 0$

$x = y = k$ $\quad$ Eigen vector $k\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$\lambda = \frac{1}{3}$ $\quad \begin{bmatrix} \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$

$x = -y$ $\boxed{\text{Eigen vector} = \begin{bmatrix} k \\ -k \end{bmatrix} = k\begin{bmatrix} 1 \\ -1 \end{bmatrix}}$

$z = A + 4I \quad \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 6 & -1 \\ -1 & 6 \end{bmatrix}$

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$$z - \lambda I = \begin{bmatrix} 6-\lambda & -1 \\ -1 & 6-\lambda \end{bmatrix}$$

$$a^2 - b = (a-b)(a+b)$$

$$(6-\lambda)^2 = (6-\lambda-1)(6-\lambda+1)$$

$$= (5-\lambda)(7-\lambda) = 0$$ $$\boxed{\text{Eigen value } \lambda = 5, 7}$$

$$\lambda = 5 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0 \quad x - y = 0$$ $$\boxed{\text{Eigen vector} = k\begin{bmatrix} 1 \\ 1 \end{bmatrix}}$$

$$x = y \qquad k = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\lambda = 7 \begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0 \quad x = -y$$ $$\boxed{\text{Eigen vector} = k\begin{bmatrix} 1 \\ -1 \end{bmatrix}}$$

$$k = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

34) $3x - 0.1y - 0.2z = 7.85$

$0.1x + 7y - 0.3z = -19.3$

$0.3x - 0.2y + 10z = 71.4$

$$x_1 = \frac{1}{3}[7.85 + 0.1y - 0.2z]$$

$$y = \frac{1}{7}[-19.3 - 0.1x + 0.3z]$$

$$z = \frac{1}{10}[71.4 - 0.3x + 0.2y]$$

<u>Iteration-1:</u>

$z = y = 0$

$$x(1) = \frac{7.85}{3} = 2.61$$

$z = 0$

$$y = \frac{1}{7}\left(-19.3 - 0.1\cdot\left(\frac{7.85}{3}\right)\right) = 2.79$$

$$z = \frac{1}{10}[71.4 - 0.3(2.61) + 0.2(2.79)]$$

$$z = \frac{1}{10}[71.4 - 0.783 + 0.558]$$

$$z(1) = 7.1175$$

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Iteration-2 :

$$x(2) = \frac{7.85 - 0.1(2.6167) - 0.2(7.1408)}{3}$$

$$= 2.9255$$

$$y(2) = \frac{19.3 - 0.1(2.9255) - 0.3(7.1408)}{7}$$

$$= 3.0123$$

$$z(2) = \frac{71.4 - 0.3(2.9255) - 0.2(3.0123)}{10}$$

$$= 7.0132$$

Iteration-3 :

$$x(3) = \frac{7.85 - 0.1(2.9255) - 0.2(7.0132)}{3} = 3.0032$$

$$y(3) = \frac{19.3 - 0.1(3.0032) - 0.3(7.0132)}{7} = 3.001$$

$$z(3) = \frac{71.4 - 0.3(3.0032) - 0.2(3.0001)}{10} = 7.00$$

$x = 3.0032$, $y = 3.0001$, $z = 7.000$

5) Define consistent & in consistent

$x + 3y + 2z = 0,\ 2x - y + 3z = 0,\ 3x - 5y + 4z = 0,\ x + 7y + 4z = 0$

Sol

$$A = \begin{bmatrix} 1 & 3 & 2 \\ 2 & -1 & 3 \\ 3 & -5 & 4 \\ 1 & 17 & 4 \end{bmatrix}$$ after performing row reduction we obtained Echelon form.

$$A = \begin{bmatrix} 1 & 3 & 2 \\ 0 & -7 & -1 \\ 0 & -14 & -2 \\ 0 & 14 & -2 \end{bmatrix}$$

$R_2 \longrightarrow R_2 - 2R_1$
$R_3 \longrightarrow R_3 - 3R_1$
$R_4 \longrightarrow R_4 - R_1$

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$$\begin{bmatrix} 1 & 3 & 2 & 0 \\ 0 & -7 & -1 & 0 \\ 0 & -14 & -2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad R_3 \to R_3 - 2R_2$$

$$\begin{bmatrix} 1 & 3 & 2 & 0 \\ 0 & -7 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

This corresponds to the system

$$x + 3y + 2z = 0$$
$$-7y - z = 0$$

Now let express the variables in terms of parameters

Let, $y = t$, $x = -3t$, $z = -7t$

So, the system has infinitely many solution given by

$x = -3t$, $y = t$, $z = -7t$

The system is consistent & dependent.

2) $T : M \to P_2$

$$T\begin{bmatrix} a & b \\ c & d \end{bmatrix} = (a-b) + (b-c)x + (c-a)x^2$$

find the rank & nullity ~~the~~ of T.

$\rightarrow$ $$T\left(\begin{bmatrix} a & b \\ b & c \end{bmatrix}\right) = (a-b) + (b-c)x + (c-a)x^2$$

Let $A = \begin{bmatrix} a & b \\ b & c \end{bmatrix}$ then

$$T(A) = (a-b) + (b-c)x + (c-a)x^2$$
$$= a - bx + c(x^2 - x + 1)$$

$\therefore$ The image of T is the set of all polynomials of degree at most 2, denoted ~~at~~ as P_2.

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Rank of T :

The rank of T is the dimension of its image since P_2 has a dimension of 3 (coefficient for x^0, x^1 and x^2) the rank of T is 3.

The ~~null~~ Null Space of symmetric matrix

$$T(A) = 0$$

this leads to the system of equations

$a - b = 0$ $\quad b - c = 0$ $\quad c - a = 0$

$\therefore a = b = c$

$\therefore$ T is the set of symmetric matrices of the form.

$\begin{bmatrix} t & t \\ t & t \end{bmatrix}$ where t is any scaler.

$\therefore$ Dimension = 1 (using only t)

$\therefore$ rank T is 3

The nullity of T is 1

6) $T : P_2 \to P_2$ is linear transformation.

$$T(a + bx + cx^2) = (a+1) + (b+1)x + (c+1)x^2$$

$\to T(a + bx + c) = (a+1) + (b+1)x + (c+1)x^2$

is a linear transformation, ~~so~~ we need to ~~ch~~ ~~st~~ check two properties.

1.) Additivity $T(u+v) = T(u) + T(v)$

2) Homoneity of degree 1.

$T(ku) = kT(u)$ for all u in the domain of T and all scalars k.

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1) $T(u+v) = T\left[(a_1 + b_1 x + c_1) + (a_2 + b_2 x + c_2)\right]$

$= T\left[(a_1 + a_2) + (b_1 + b_2)x + (c_1 + c_2)\right]$

$= (a_1 + a_2 + 1) + (b_1 + b_2 + 1)x + (c_1 + c_2 + 1)x^2$

$= (a_1 + 1) + (b_1 + 1)x + (c_1 + 1)x^2 + (a_2 + 1) + (b_2 + 1)x + (c_2 + 1)x^2$

$T(a_1 + b_1 x + c_1) + T(a_2 + b_2 x + c_2)$

So function is additive.

$\therefore$ Homogenity of Degree 1:

$T(ku) = T\left[k(a + bx + c)\right]$

$= T\left[(ka + kbx + kc) = (ka + 1) + (kb + 1)x + (kc + 1)x\right.$

$= k(a + 1) + k(b + 1)x + k(c + 1)x^2$

$= kT(a + bx + c)$

So, the function is homeneous of degree 1.

$\therefore$ It indeed linear transformation.

7) $S = \{(1, 2, 3), (3, 1, 0), (-2, 1, 3)\}$ is a basic of $V_3(R)$. In case S is not a basic, determine sub space - spanned by S.

$\rightarrow$ $S = \{(1, 2, 3), (3, 1, 0), (-2, 1, 3)\}$ can be arranged as a matrix.

$A = \begin{bmatrix} 1 & 3 & -2 \\ 2 & 1 & 1 \\ 3 & 0 & 3 \end{bmatrix}$ Now, lets perform row reduction to obtain the Echelon form.

$\begin{bmatrix} 1 & 3 & -2 \\ 2 & 1 & 1 \\ 3 & 0 & 3 \end{bmatrix}$ $R_2 \leftarrow R_2 - 2R_1$ and $R_3 \leftarrow R_3 - 3R_1$

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$$\begin{bmatrix} 1 & 3 & -2 \\ 0 & -5 & 5 \\ 0 & -9 & 9 \end{bmatrix} \quad R_3 \rightarrow R_3 + \frac{9}{5} R_1 \quad \begin{bmatrix} 1 & 3 & -2 \\ 0 & -5 & 5 \\ 0 & 0 & 0 \end{bmatrix}$$

$\therefore$ Third row of zeros indicates that the vectors in S is linearly dependent. ~~for basic of the~~

For basic of the subspace spanned by S.

$$\begin{bmatrix} 1 & 3 & -2 \\ 0 & -5 & 5 \end{bmatrix}$$ $(1, 3, 2)$ & $(0, -5, 5)$. these vectors form a basic for the subspace spanned by S.

$\therefore$ Dimension of subspace spanned by $S = 2$

$\therefore$ set S is not a basic of R^3 because the row reduced form has a row of zeros.

$\therefore$ The basic for the subspace spanned by S is $\{(1, 3, -2), (0, -5, 5)\}$

$\therefore$ The dimension of the subspace is 2.

8) Using Jacobi's method (perform 3 iterations) solve $3x - 6y + 2z = 23$, $-4x + y - z = 15$, $x - 3y + 7z = 16$, with initial values: $x_0 = 1$, $y_0 = 1$, $z = 1$

$\rightarrow$ $3x - 6y + 2z = 23$ with initial values
$-4x + y - z = -15$ $\quad x = 1, y = 1, z = 1$
$x - 3y + z(7) = 16$

Iteration-1:

$$x(1) = \frac{23 + 6y(0) - 2z(0)}{3} \simeq 9.0$$

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$$y(1) = \frac{-15 + 4(x)(1) + z(0)}{1} \simeq -9.0$$

$$z(1) = \frac{16 - x(1) - \cancel{3}\,3(y)(1)}{7} = 2.0.$$

<u>Iteration-2:</u>

$$x(2) = \frac{23 + 6y(1) - 2(z)(1)}{3} = 5.0$$

$$y(2) = \frac{-15 + 4(x)(1) + z(1)}{1} = -5.0$$

$$z(2) \Rightarrow \cancel{+16}$$
$$= \frac{16 - x(1) + 3(y)(1)}{7} \simeq 3.0$$

<u>Iteration-3:</u>

$$\cancel{x(3) = 23 + 6y(2) - 2z(2) = 26.0}$$

$$\cancel{y(3) = -15 + 4x(2) + z(2}$$

$$x(3) = \frac{23 + 6y(2) - 2z(2)}{3} = 6.0$$

$$y(3) = \frac{-15 + 4x(2) + z(2)}{1} \simeq -6.0$$

$$z(3) = \frac{16 - x(2) + 3y(2)}{7} = 2.0$$