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Sec- A
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classmate
Date
Page

Tutorial 1

1. int a =0, b=0
for (i=0; i< N; i++)
{
a = a + rand();
}
for (j =0; j<M; j++)
{
b = b + rand ();
}

Sol. $O(N+M)$ — TC
$O(1)$ - SC

2. int s=0, i;
for (i=0; i< N; i=i + 2)
{
sum += i;
}

Sol. i
0
2
4
6
⋮
n

$0+2+4+6+\ldots$
$a=0$
$d=2$
$S = 0 + \left(\frac{n}{2}-1\right)2$
$= O(n)$

③
```
int s = 0, i;
for (i = 0; i < n; i = i * 2)
{
    sum = s + i;
}
```

Sol.

2^0

2^1

2^2

$\vdots$

2^{k-1}

$2^{k-1} >= n$

$\log(2^{k-1}) >= \log n$

$k-1 >= \log n$

$k >= \log n + 1$

$\boxed{O(\log n)}$

④
```
int s = 0, i;
for (i = 0; i * i < n; i++)
{
    s = s + i;
}
```

Sol. $O(\sqrt{n})$

(5) int j=1; i=0;
while (i<=n)
{
i=i+j;
j++;
}

Sol. $\frac{k(k+1)}{2} >= n$

$k^2 >= n$

$k >= \sqrt{n}$

$O(\sqrt{n})$

(6) $T(n) = T(n-1) + T(n-1)+1$

$= 2T(n-1)$

$= 2.2.T(n-2)$

:

$= O(2^n)$

(7) $O(\log n)$

$T(n) = 2T(n/2)+1$

$\log_2 2 = 1$

$f(n) = 1$

$g(n) = 1$

$T.C = O(n \log(n))$

(3) $T(n) = T(n-1) + 1$

(1) $T(n-1) = T(n-2) + 1$

$T(n-2) = T(n-3) + 1$

$$T(n) = \begin{cases} 1 & n = 0 \\ T(n-1) + 1 & n > 0 \end{cases}$$

$T(n) = [T(n-2) + 1] + 1$

$T(n) = [T(n-3) + 3$

$\vdots$

$T(n) = T(n-k) + k$

let $n - k = 0$

$T(n) = T(0) + n$

$T(n) = 1 + n \longrightarrow O(n)$

(2)

$T(n) = T(n-1) + n$

$T(n-1) = T(n-2) + n-1$

$T(n-2) + T(n-3) + n - 2$

$T(n) = T(n-3) + (n-2) + (n-1) + n$

$\vdots$

$T(n) = T(n-k) + (n-(k-1)) + (n-(k-2)) \ldots (n-1) + n$

Assume $n - k = 0$

$\therefore n = k$

~~T(n) = T(n-K) + (n~~

$T(n) = T(0) + (\cancel{n} - \cancel{n} + 1) + (\cancel{n} - \cancel{n} + 2) + \ldots n$

$T(n) = 1 + n\frac{(n+1)}{2}$

$= 1 + (1)\frac{\cancel{2}}{\cancel{2}} = 2$

(iii) $T(n) = T(n/2) + 1$

$a = 1$

$b = 2$

$k = \log_2 1 = 0$

$n^0 = 1 \simeq 1$

So, $T(n) = O(1 \log n)$

$= O(\log n)$

(iv) $T(n) = 2T(n/2) + 1$

$T(n) = O(n \log n)$

(v) $O(2^n)$

(vi) $O(3^n)$

(vii) $T(n) = T(\sqrt{n}) + 1$, $n = 2^m$

$S(m) = \cancel{2}\, S\left(\frac{m}{2}\right) + 1$

$K = \log_2 1$, $k = 0$, $m^k = 1$

$T(n) = O(\log \log n)$

(8) $T(n) = T(\sqrt{n}) + n$

let $n = 2^m$

$T(2^m) = T(2^{m/2}) + 2^m$

$S(m) = T(2^m)$

$= S(m/2) + 2^m$

$K = \log_2 1 = 0$

$m^0 = 1 \neq 2^m$

$O(2^m) = O(n)$

(9)

```
int sum=0, i;
for( i=0; i<n; i++)
  sum += i;
```

$0 + 1 + 2 \ldots\ldots n$ times

$= O(n)$

(10) $N + (N-1) + (N-2) + \ldots\ldots 0$

$\Rightarrow N(N+1)/2$

$= O(N^2)$

11. for n, j runs $O(\log n)$ times
i runs $n/2$ times

$T.C = O\left(\frac{n}{2} * \log(n)\right)$

$= O(n \log n)$

12. X is asymptotically more efficient than y means x is better choice for large inputs.

[2nd correct]

13. $i = N$

$N/2$

$N/4$

$\vdots$

$\frac{N}{2^{k-1}}$

$\frac{N}{2^{k-1}} >= 1$

$N >= 2^{k-1}$

$\log N = \log_2 2^{(k-1)}$

$k-1 = \log_2 N$

$k = (\log_2 N + 1)$

$\boxed{T.C = O(\log n)}$

(14) $T(n) = 7T(n/2) + \underbrace{3n^2+2}_{f(n)}$

$a = 7, b = 2$
$K = \log_2 7 = 2.81$

$n^K = n^{2.81}$

(a) $\Theta(n^{2.81})$ ✓
(b) $O(n^3)$ ✓
(c) $O(n^{2.81})$ ✓

(15) $f2 > f4 > f3 > f1$

(16) $f(n) = \cancel{n}\ 2^{\wedge}(2n)$
$f(n) >= c * 2n)$

(b) $\Omega(2^{\wedge}n)$

17. $b = 2 \quad a = 1 \quad f(n) = n^2$
$\log_2 1 = 0 = K$
$n^0 = 1$
$n^2 > 1$

(a) $O(n^2)$

(8) $O(\log n)$

(11) $O(N^2)$

$O(N)$

2) $O(N^2+N) = O(N^2)$