LUT Engineering Science
Data-Centric Engineering
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BM40A0902 3D Computer Vision
Exercise 5
Lines and parametric curves.

1. Line fitting with gradient method (2 points).

   The model for a line is
   $$ax + by + c = 0$$
   The parameters a, b, and c of the model can be found through minimizing
   $$\min \sum_k (ax_k + by_k + c)^2$$
   where x_k and y_k, $k = 1...n$ are the (possibly noisy) points corresponding to the line. This optimization problem can be solved with the gradient descent method defined as the iteration
   $$\theta_{i+1} = \theta_i - \eta \frac{\partial f}{\partial \theta}$$
   where θ is a vector containing the parameters of the model. For line fitting, the parameters are $\theta = [a, b, c]$. The user-definable parameter η is the step length taken in the direction $-\frac{\partial f}{\partial t}$. You can use $\theta_0 = [1, 1, 1]$. The partial derivatives with respect to θ are needed and they can be derived from the function to be minimized defined as
   $$f(\theta) = f(a, b, c) = \sum_k (ax_k + by_k + c)^2$$
   Find suitable parameters $\theta = [a, b, c]$ when the points x_k, y_k, $k = 1...512$ are given in Mat data file (xy.mat). Download the file (xy.mat), code the iteration, and check the result by plotting the original line as points and the line with the parameters θ. The result should look similar to the plot in Fig. 1.

   How many iterations are needed and what is a suitable value for the user-defined parameter η? Hint: start with a really small value for η, for example 10^{-8} and large number of iterations, at least 1000.

2. RANSAC (2 points).

   The goal of this task is to compare algebraic fitting and RANSAC for circles.

   Noisy points of a circle are given in file `circle_points.mat`, see Fig. 2. Complete the following tasks:

   (a) Load and plot the points given in `circle_points.mat`. Are there any outliers?

   (b) Now implement function for algebraic fitting of circles. The model for a circle is somewhat similar to a line, only it contains 4 parameters instead of 3:
   $$a(x^2 + y^2) + bx + cy + d = 0$$

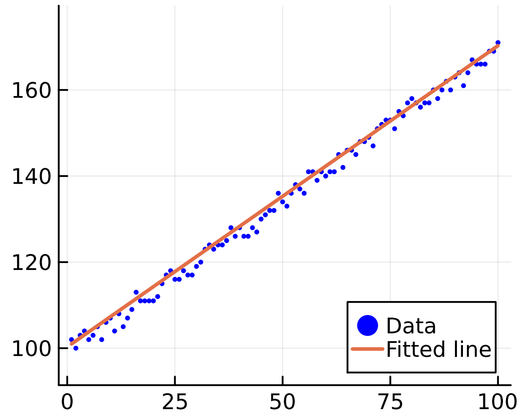

Figure 1: Line fitting. Blue dots: original points from the noisy data. Red line: a line fitted to the noisy data using gradient descent. Only a limited set of points is displayed.

As a result there will be a linear equation for each of the points, and you can use Singular Value Decomposition (SVD)[1] for solving the resulting set of linear equations (take a look at the lecture notes to see how this is done). Basically, you need first to present the setting as a minimization problem of the form $\min_{\mathbf{a}} \|X\mathbf{a}\|^2$. Then, you can use SVD to solve this. Plot the points and the fitted circle together on the same plot. Is it a good fit? Could it be improved?

(c) Implement RANSAC. General algorithm is given on slide 27 of lecture slides. The following parameters can be used:

- Minimum sampling size s can be set to 3 (the minimum number of points required to fit a circle)
- We are using algrebraic fitting, so we can use algebraic error for thresholding the points as well. We can use the following equation to find inliers

$$|a(x^2 + y^2) + bx + cy + d| < 0.1. \tag{1}$$

  If this equation is satisfied for a point, it is considered to be consistent with the model (i.e. the point is an inlier).
- The threshold t, i.e. the number of inliers that we consider to be satisfactory can be set to 360.
- Number of iterations N can be chosen according to the rules specified on slide 37. We want to have probability $p = 0.99$ of having at least one completely outlier-free sample, and we can assume that the probability of a point being an outlier is $e = 0.3$.

(d) Apply RANSAC to fit a circle to the points given in `circle_points`. Plot the points and the fitted circle together on the same plot. Is the fit using RANSAC better?

[1] See templates for instructions how to do SVD in Python and MATLAB.

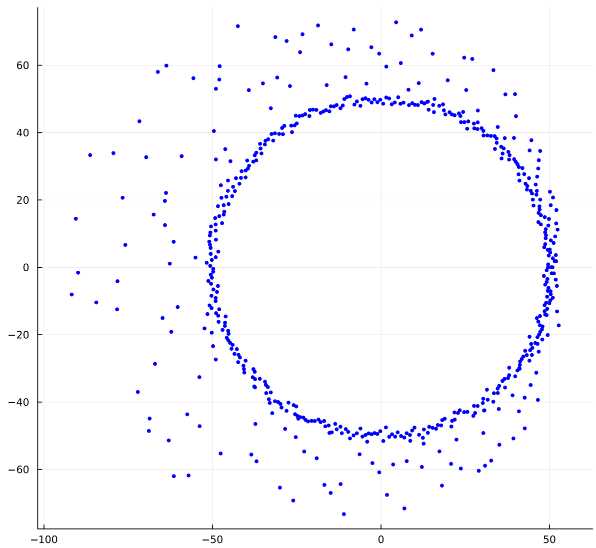

Figure 2: Noisy circle points for task 2.