

Deep Learning with Neural Networks

**Advanced topics in
Deep Probabilistic Modelling**

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Multimodal generative models

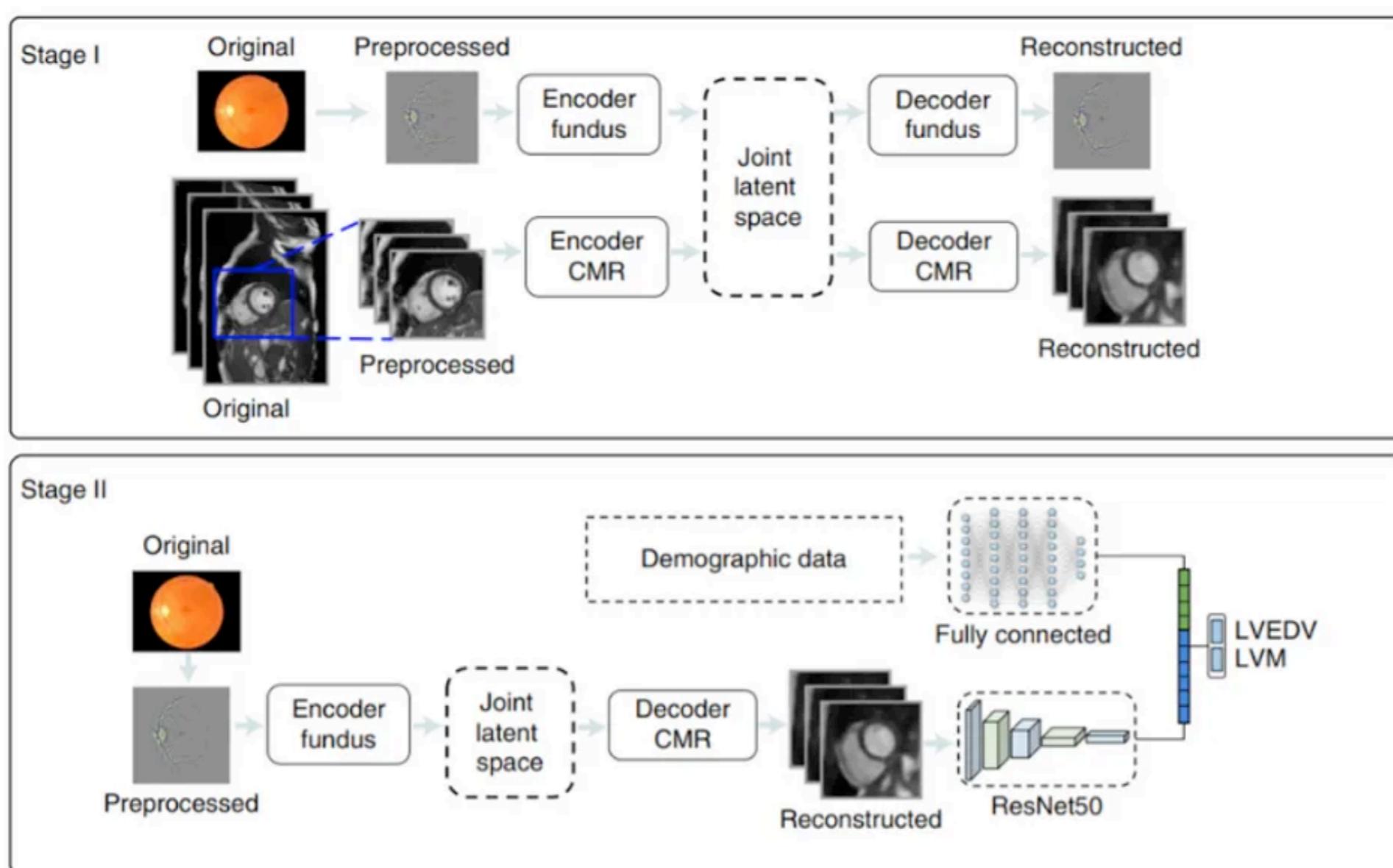
Prediction of cardiovascular risk factors from retinal fundus photographs via deep learning

Ryan Poplin, Avinash V. Varadarajan, Katy Blumer, Yun Liu, Michael V. McConnell, Greg S. Corrado, Lily

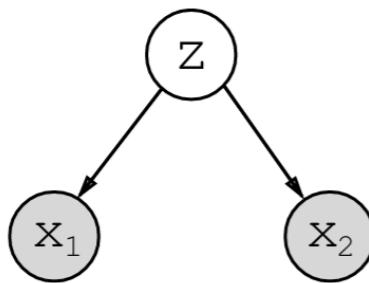
Peng & Dale R. Webster

Nature Biomedical Engineering 2, 158–164 (2018) | Cite this article

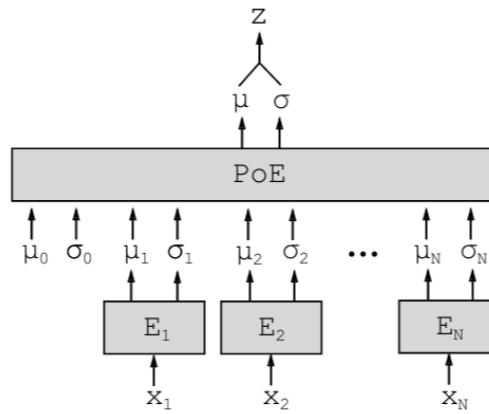
25k Accesses | 808 Citations | 2368 Altmetric | Metrics



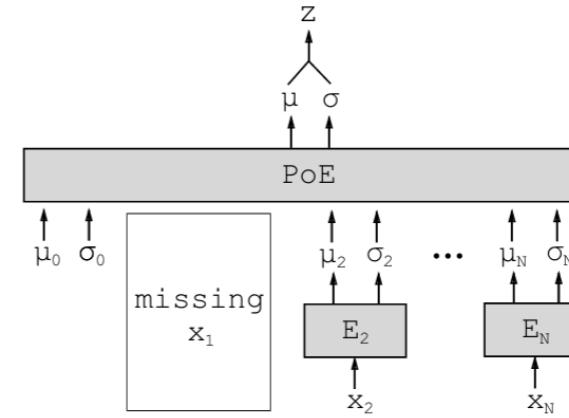
Multimodal Generative Models for Scalable Weakly-Supervised Learning



(a)



(b)



(c)

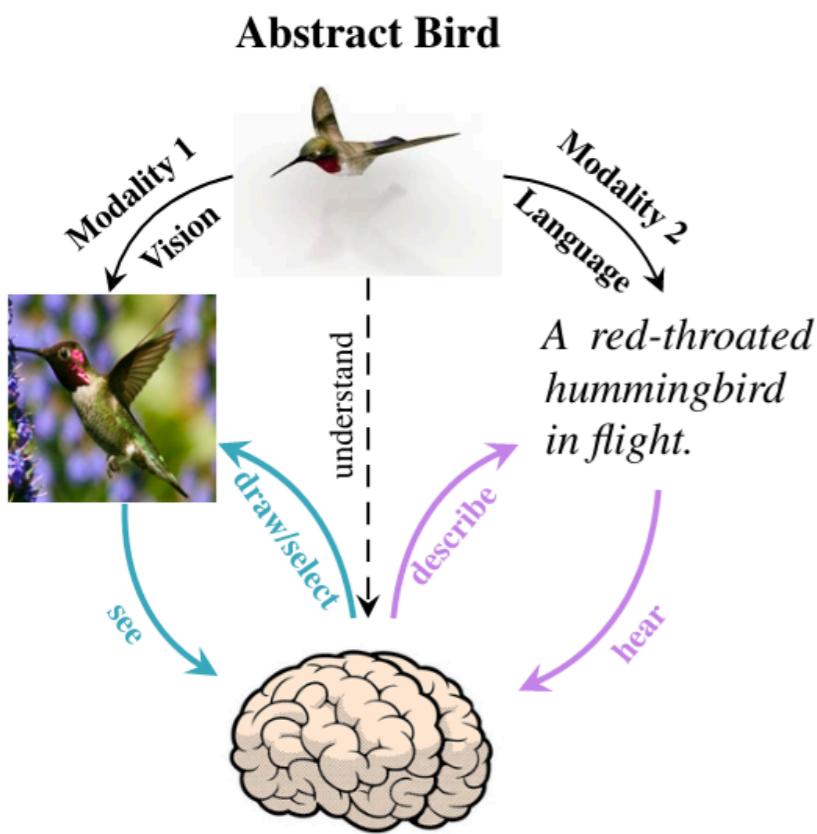
$$\text{ELBO}(X) \triangleq \mathbb{E}_{q_\phi(z|X)} \left[\sum_{x_i \in X} \lambda_i \log p_\theta(x_i|z) \right] - \beta \text{KL}[q_\phi(z|X), p(z)].$$

$$p(z|x_1, \dots, x_N) \propto p(z) \prod_{i=1}^N \tilde{q}(z|x_i)$$

Product-of-experts (MoE)

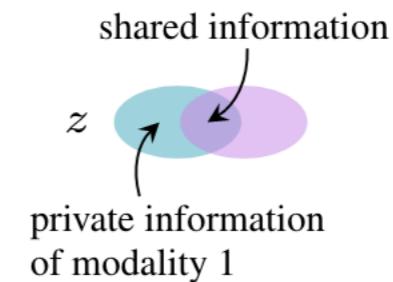
Variational Mixture-of-Experts Autoencoders for Multi-Modal Deep Generative Models

$$\mathcal{L}_{\text{ELBO}}(\mathbf{x}_{1:M}) = \mathbb{E}_{\mathbf{z} \sim q_{\Phi}(\mathbf{z} | \mathbf{x}_{1:M})} \left[\log \frac{p_{\Theta}(\mathbf{z}, \mathbf{x}_{1:M})}{q_{\Phi}(\mathbf{z} | \mathbf{x}_{1:M})} \right]$$

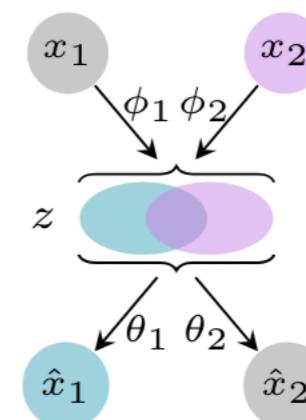


$$q_{\Phi}(\mathbf{z} | \mathbf{x}_{1:M}) = \sum_m \alpha_m \cdot q_{\phi_m}(\mathbf{z} | \mathbf{x}_m)$$

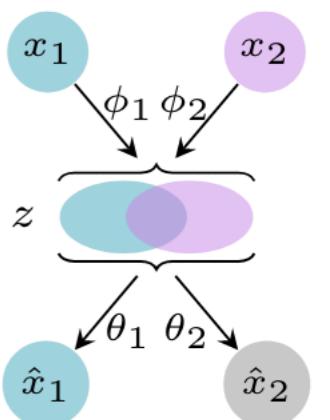
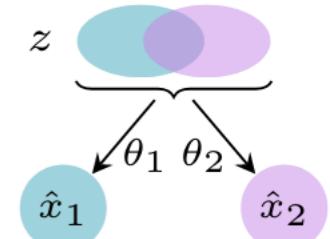
Mixture-of-experts (MoE)



(a) Latent Factorisation (b) Joint Generation



(c) Cross Generation (d) Synergy



Generative classifiers as multimodal VAEs?

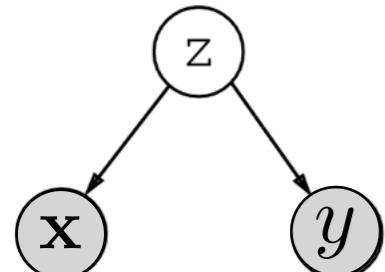
Deep Generative classifiers

$$(\mathbf{x}, y), \quad y \in \{0, 1\} \quad \mathbf{x} \in \mathbb{R}^D, D \gg 1$$

Learn $p(\mathbf{x}, y)$ instead $p(y|\mathbf{x})$

- Semi-supervised learning
- Robust against outliers and adversarial attacks

Naive Approach:

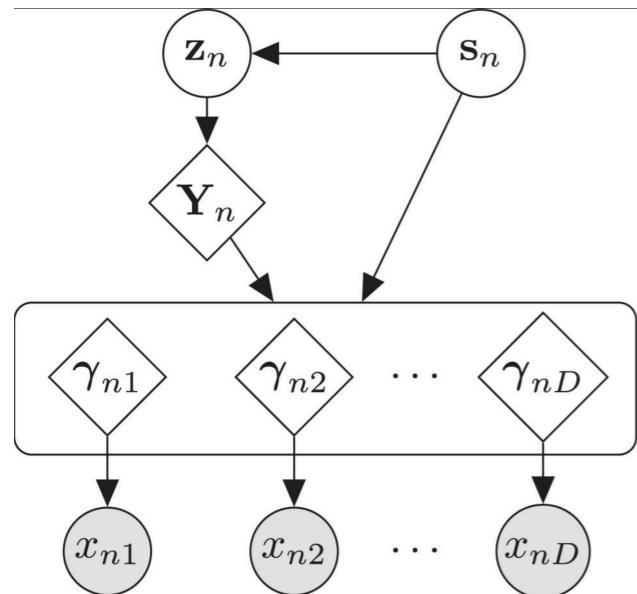


$$\text{ELBO}(X) \triangleq \mathbb{E}_{q_\phi(z|X)} \left[\sum_{x_i \in X} \lambda_i \log p_\theta(x_i|z) \right] - \beta \text{KL}[q_\phi(z|X), p(z)].$$

The reconstruction term for \mathbf{x} dominates the ELBO!

- Normalization to balance reconstruction terms
- Change generative model!

Deep Generative classifiers



Pattern Recognition

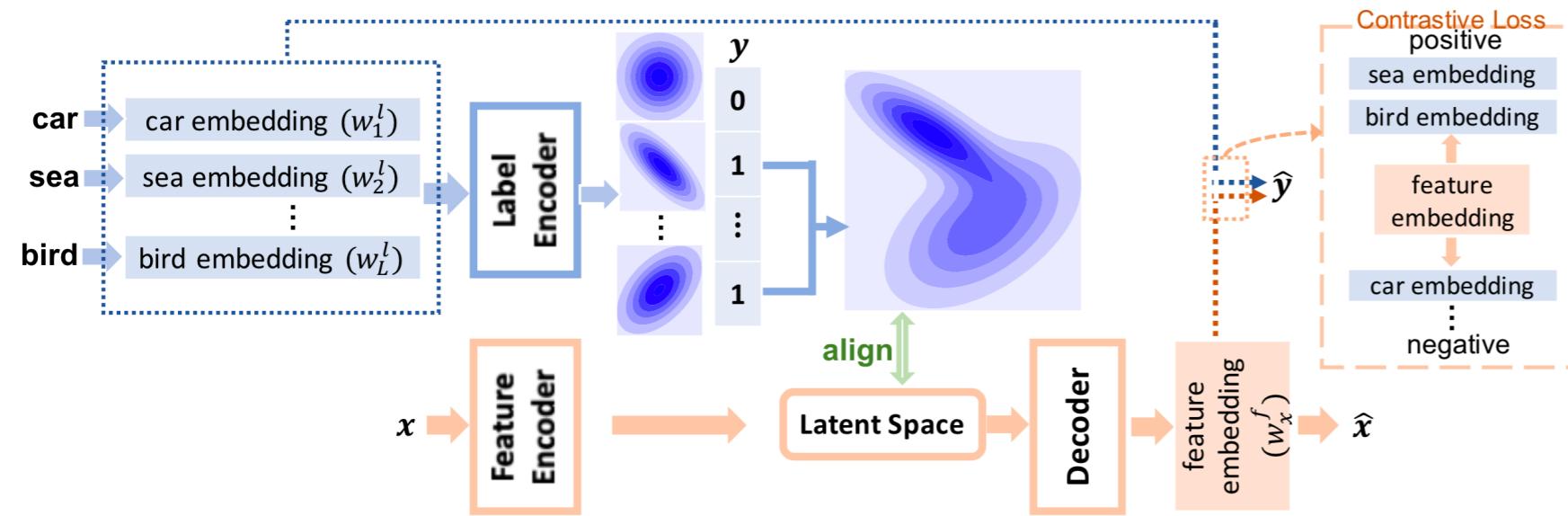
Volume 107, November 2020, 107501



Handling incomplete heterogeneous data using
VAEs

Gaussian Mixture Variational Autoencoder with Contrastive Learning
for Multi-Label Classification

Junwen Bai¹ Shufeng Kong¹ Carla Gomes¹

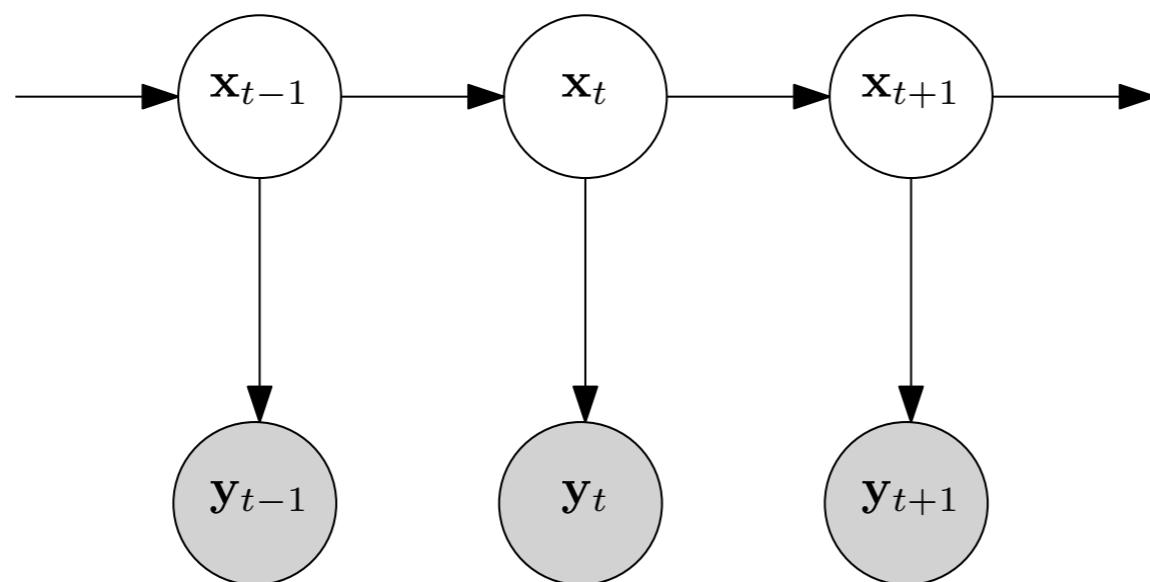


Unsupervised models for time series

Hidden Markov Processes

If the observed sequence $\mathbf{y}_{1:T}$ is a noisy version of the (first order) Markov process $\mathbf{x}_{1:T}$

$$p(\mathbf{y}_{1:T}, \mathbf{x}_{1:T}) = p(\mathbf{y}_1|\mathbf{x}_1)p(\mathbf{x}_1) \dots p(\mathbf{y}_t|\mathbf{x}_t)p(\mathbf{x}_t|\mathbf{x}_{t-1}) \dots \\ \dots p(\mathbf{y}_T|\mathbf{x}_T)p(\mathbf{x}_T|\mathbf{x}_{T-1})$$



- ▶ Discrete x_t (s_t): Hidden Markov Model (HMM)
- ▶ Continuous \mathbf{x}_t : State Space Model (SSM)

Beyond the Kalman Filter: temporal variational autoencoders

$$p(\mathbf{x}_{\leq T}, \mathbf{z}_{\leq T}) = \prod_{t=1}^T p(\mathbf{x}_t \mid \mathbf{z}_{\leq t}, \mathbf{x}_{<t}) p(\mathbf{z}_t \mid \mathbf{x}_{<t}, \mathbf{z}_{<t}).$$

$\mathbf{z}_t \sim \mathcal{N}(\boldsymbol{\mu}_{0,t}, \text{diag}(\boldsymbol{\sigma}_{0,t}^2))$, where $[\boldsymbol{\mu}_{0,t}, \boldsymbol{\sigma}_{0,t}] = \varphi_\tau^{\text{prior}}(\mathbf{h}_{t-1})$,

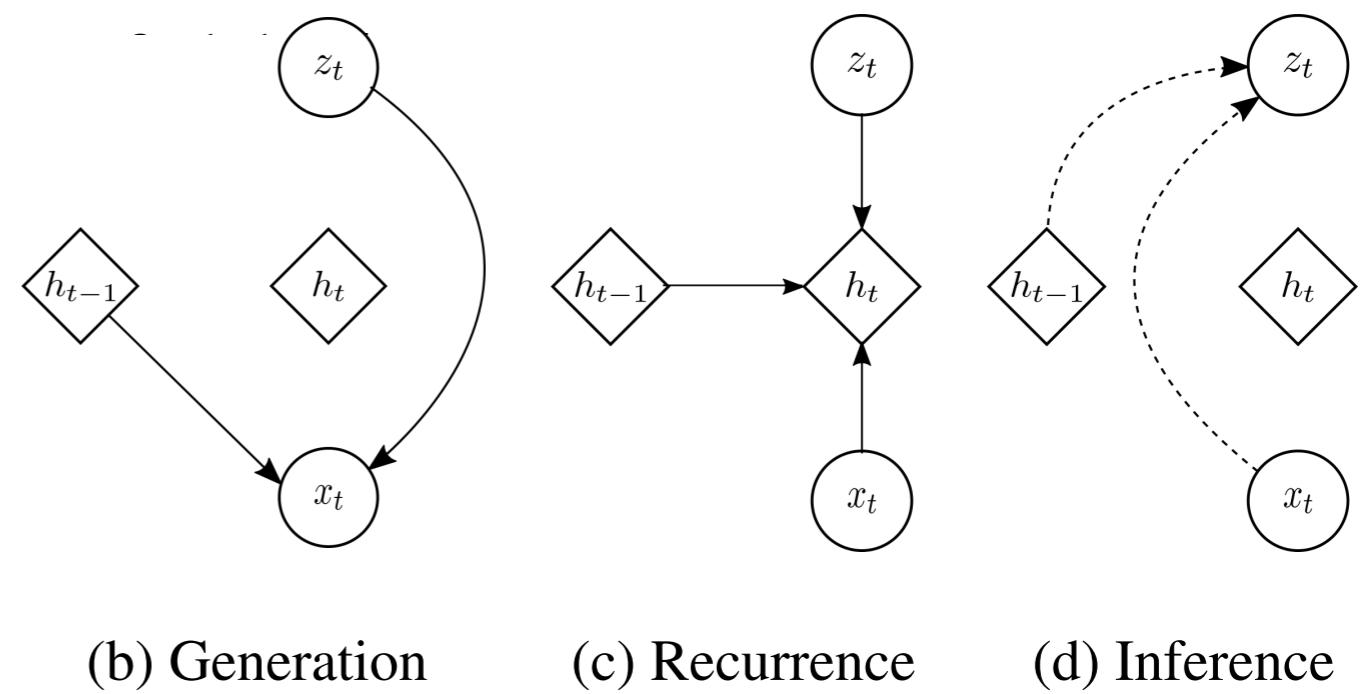
$\mathbf{x}_t \mid \mathbf{z}_t \sim \mathcal{N}(\boldsymbol{\mu}_{x,t}, \text{diag}(\boldsymbol{\sigma}_{x,t}^2))$, where $[\boldsymbol{\mu}_{x,t}, \boldsymbol{\sigma}_{x,t}] = \varphi_\tau^{\text{dec}}(\varphi_\tau^{\mathbf{z}}(\mathbf{z}_t), \mathbf{h}_{t-1})$,

The RNN updates its hidden state using the recurrence equation:

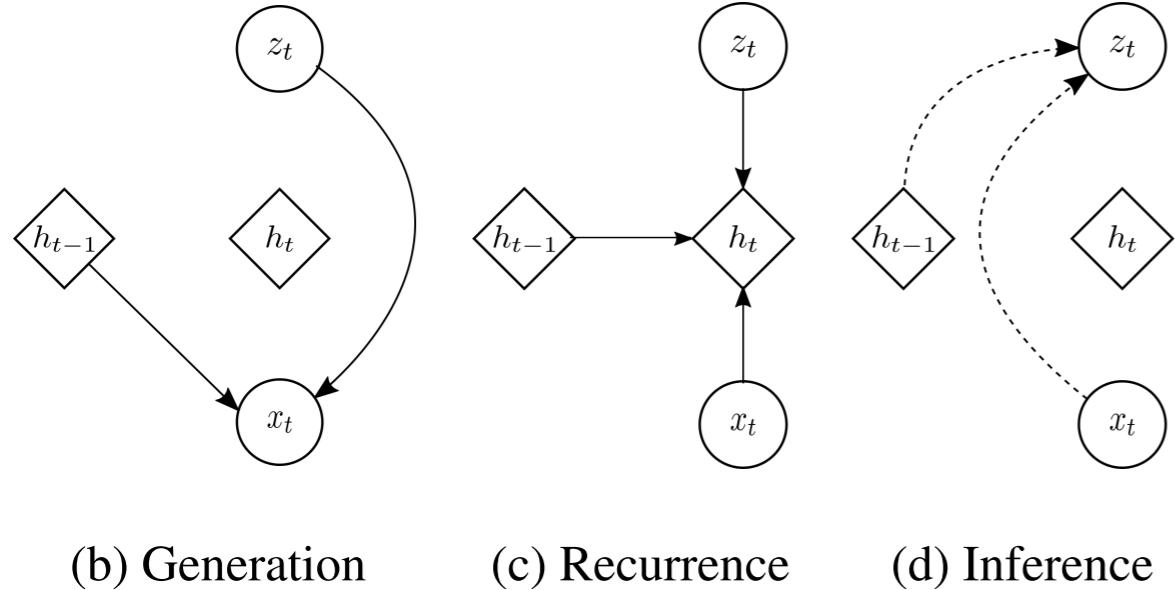
$$\mathbf{h}_t = f_\theta (\varphi_\tau^{\mathbf{x}}(\mathbf{x}_t), \varphi_\tau^{\mathbf{z}}(\mathbf{z}_t), \mathbf{h}_{t-1}),$$

A Recurrent Latent Variable Model for Sequential Data

Junyoung Chung, Kyle Kastner, Laurent Dinh, Kratarth Goel,
Aaron Courville, Yoshua Bengio*
Department of Computer Science and Operations Research
Université de Montréal
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Beyond the Kalman Filter: temporal variational autoencoders



$\mathbf{z}_t \mid \mathbf{x}_t \sim \mathcal{N}(\boldsymbol{\mu}_{z,t}, \text{diag}(\boldsymbol{\sigma}_{z,t}^2))$, where $[\boldsymbol{\mu}_{z,t}, \boldsymbol{\sigma}_{z,t}] = \varphi_\tau^{\text{enc}}(\varphi_\tau^{\mathbf{x}}(\mathbf{x}_t), \mathbf{h}_{t-1})$

$$q(\mathbf{z}_{\leq T} \mid \mathbf{x}_{\leq T}) = \prod_{t=1}^T q(\mathbf{z}_t \mid \mathbf{x}_{\leq t}, \mathbf{z}_{<t}).$$

Learning The objective function becomes a timestep-wise variational lower bound using Eq. (8) and Eq. (10):

$$\mathbb{E}_{q(\mathbf{z}_{\leq T} \mid \mathbf{x}_{\leq T})} \left[\sum_{t=1}^T (-\text{KL}(q(\mathbf{z}_t \mid \mathbf{x}_{\leq t}, \mathbf{z}_{<t}) \| p(\mathbf{z}_t \mid \mathbf{x}_{<t}, \mathbf{z}_{<t})) + \log p(\mathbf{x}_t \mid \mathbf{z}_{\leq t}, \mathbf{x}_{<t})) \right]. \quad (11)$$

A Recurrent Latent Variable Model for Sequential Data

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Beyond the Kalman Filter: temporal variational autoencoders

Sequential Neural Models with Stochastic Layers

Marco Fraccaro[†]

Søren Kaae Sønderby[‡]

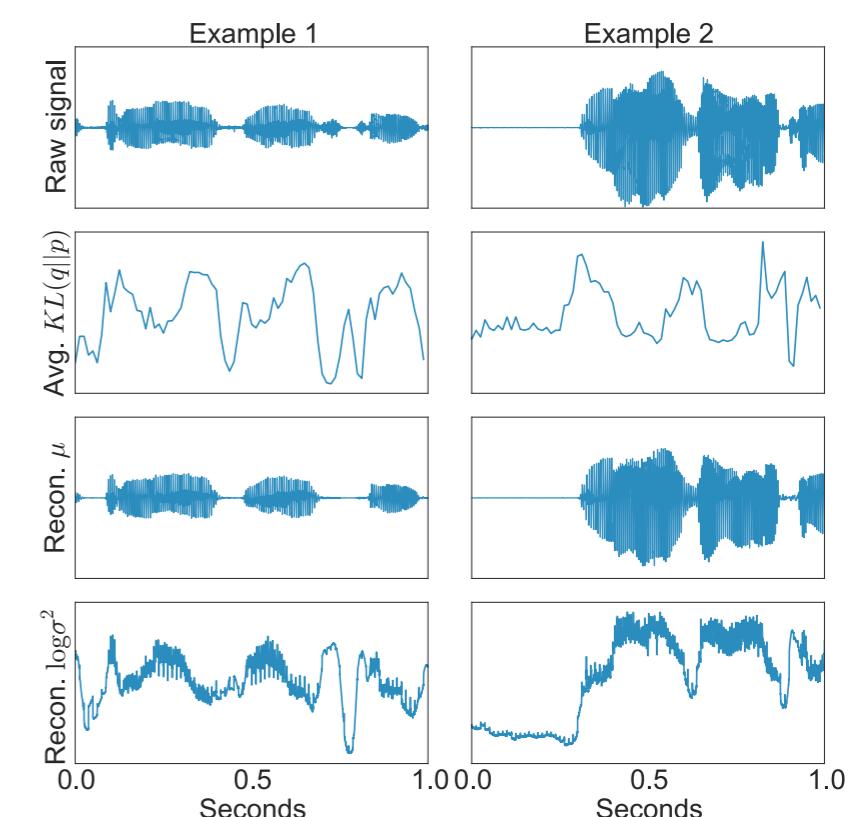
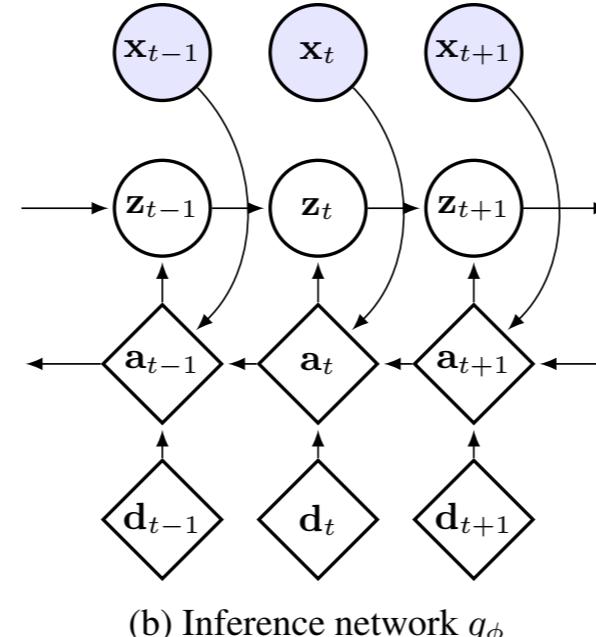
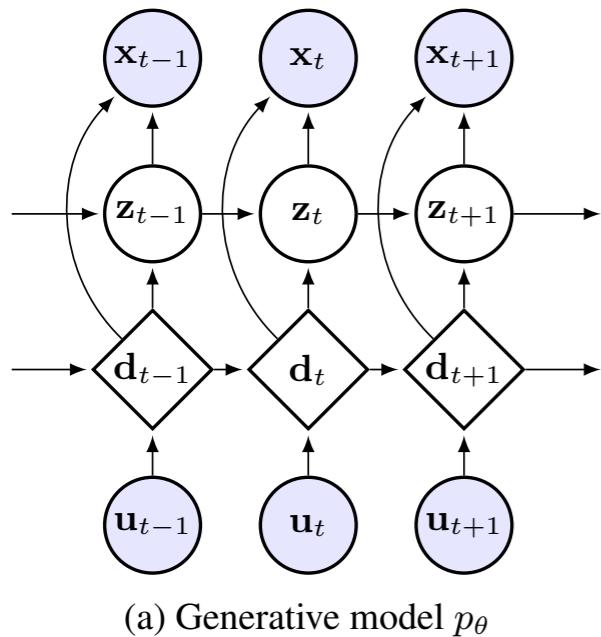
Ulrich Paquet^{*}

Ole Winther^{†‡}

[†] Technical University of Denmark

[‡] University of Copenhagen

^{*} Google DeepMind



$$\mathcal{F}_i(\theta, \phi) = \mathbb{E}_{q_\phi} \left[\log p_\theta(\mathbf{x}_{1:T} | \mathbf{z}_{1:T}, \tilde{\mathbf{d}}_{1:T}) \right] - \text{KL} \left(q_\phi(\mathbf{z}_{1:T} | \tilde{\mathbf{d}}_{1:T}, \mathbf{x}_{1:T}, \mathbf{z}_0) \parallel p_\theta(\mathbf{z}_{1:T} | \tilde{\mathbf{d}}_{1:T}, \mathbf{z}_0) \right)$$

Beyond the Kalman Filter: temporal variational autoencoders

Sequential Neural Models with Stochastic Layers

Marco Fraccaro[†]

Søren Kaae Sønderby[‡]

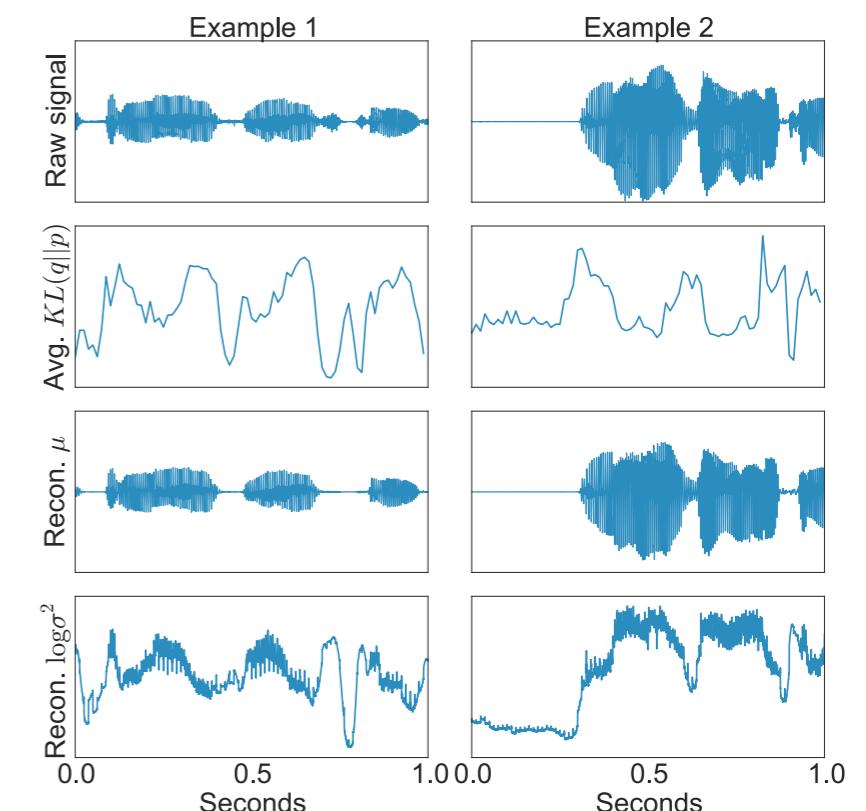
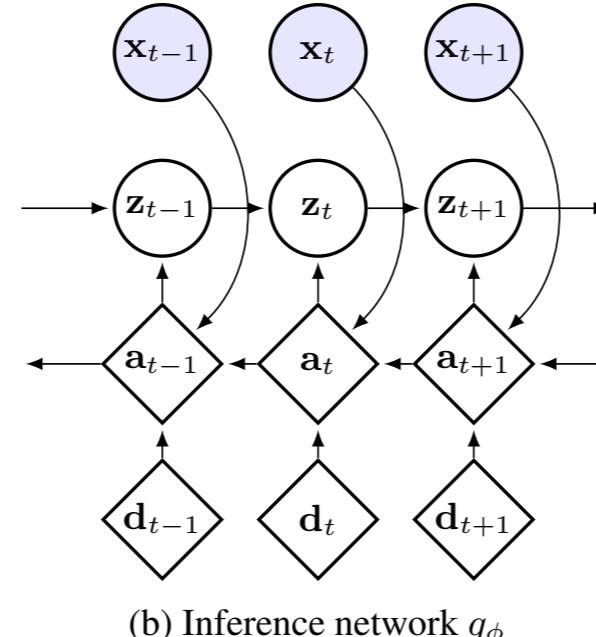
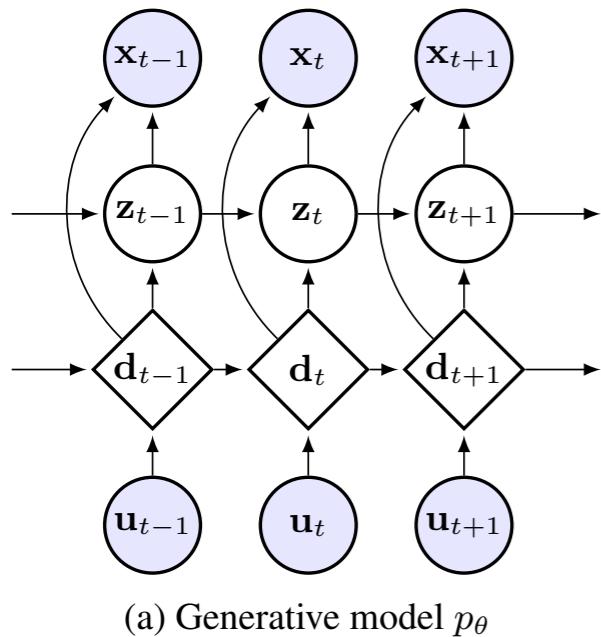
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$$\mathcal{F}_i(\theta, \phi) = \mathbb{E}_{q_\phi} \left[\log p_\theta(\mathbf{x}_{1:T} | \mathbf{z}_{1:T}, \tilde{\mathbf{d}}_{1:T}) \right] - \text{KL} \left(q_\phi(\mathbf{z}_{1:T} | \tilde{\mathbf{d}}_{1:T}, \mathbf{x}_{1:T}, \mathbf{z}_0) \parallel p_\theta(\mathbf{z}_{1:T} | \tilde{\mathbf{d}}_{1:T}, \mathbf{z}_0) \right)$$

Beyond the Kalman Filter: temporal variational autoencoders

Deep Kalman Filters

Rahul G. Krishnan Uri Shalit David Sontag
 Courant Institute of Mathematical Sciences
 New York University

$$z_t = G_t z_{t-1} + B_t u_{t-1} + \epsilon_t \text{ (action-transition)}, \quad x_t = F_t z_t + \eta_t \text{ (observation)},$$



$$z_1 \sim \mathcal{N}(\mu_0; \Sigma_0)$$

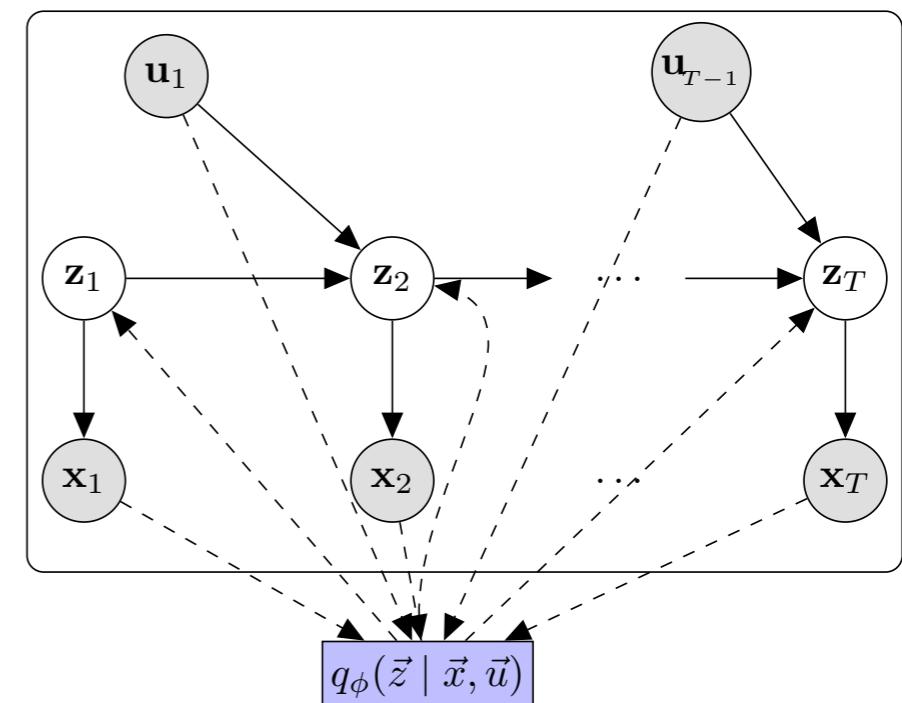
$$z_t \sim \mathcal{N}(G_\alpha(z_{t-1}, u_{t-1}, \Delta_t), S_\beta(z_{t-1}, u_{t-1}, \Delta_t))$$

$$x_t \sim \Pi(F_\kappa(z_t)).$$

$$\log p_\theta(\vec{x} | \vec{u}) \geq \mathcal{L}(x; (\theta, \phi)) =$$

$$\sum_{t=1}^T \mathbb{E}_{q_\phi(z_t | \vec{x}, \vec{u})} [\log p_\theta(x_t | z_t)] - \text{KL}(q_\phi(z_1 | \vec{x}, \vec{u}) || p_0(z_1))$$

$$- \sum_{t=2}^T \mathbb{E}_{q_\phi(z_{t-1} | \vec{x}, \vec{u})} [\text{KL}(q_\phi(z_t | z_{t-1}, \vec{x}, \vec{u}) || p_0(z_t | z_{t-1}, u_{t-1}))].$$



Hybrid autoregressive and latent variable models

A Hierarchical Latent Variable Encoder-Decoder Model for Generating Dialogues

Iulian V. Serban^{*}, Alessandro Sordoni[†], Ryan Lowe[◊], Laurent Charlin[◊], Joelle Pineau[◊], Aaron Courville^{*} and Yoshua Bengio^{*}

$$P_{\theta}(w_1, \dots, w_M) = \prod_{m=2}^M P_{\theta}(w_m \mid w_1, \dots, w_{m-1}) P(w_1).$$

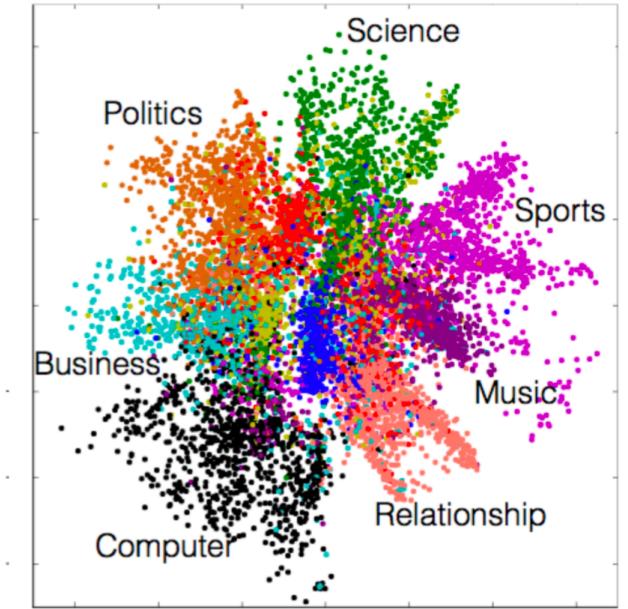


$$P_{\theta}(\mathbf{z}_n \mid \mathbf{w}_1, \dots, \mathbf{w}_{n-1}) = \mathcal{N}(\boldsymbol{\mu}_{\text{prior}}(\mathbf{w}_1, \dots, \mathbf{w}_{n-1}), \Sigma_{\text{prior}}(\mathbf{w}_1, \dots, \mathbf{w}_{n-1})),$$

$$P_{\theta}(\mathbf{w}_n \mid \mathbf{z}_n, \mathbf{w}_1, \dots, \mathbf{w}_{n-1}) = \prod_{m=1}^{M_n} P_{\theta}(w_{n,m} \mid \mathbf{z}_n, \mathbf{w}_1, \dots, \mathbf{w}_{n-1}, w_{n,1}, \dots, w_{n,m-1}),$$

$$\begin{aligned} Q_{\psi}(\mathbf{z}_n \mid \mathbf{w}_1, \dots, \mathbf{w}_N) &= Q_{\psi}(\mathbf{z}_n \mid \mathbf{w}_1, \dots, \mathbf{w}_n) = \mathcal{N}(\boldsymbol{\mu}_{\text{posterior}}(\mathbf{w}_1, \dots, \mathbf{w}_n), \Sigma_{\text{posterior}}(\mathbf{w}_1, \dots, \mathbf{w}_n)) \\ &\approx P_{\psi}(\mathbf{z}_n \mid \mathbf{w}_1, \dots, \mathbf{w}_N), \end{aligned} \tag{5}$$

$$\begin{aligned} \log P_{\theta}(\mathbf{w}_1, \dots, \mathbf{w}_N) &\geq \sum_{n=1}^N -\text{KL}[Q_{\psi}(\mathbf{z}_n \mid \mathbf{w}_1, \dots, \mathbf{w}_n) \parallel P_{\theta}(\mathbf{z}_n \mid \mathbf{w}_1, \dots, \mathbf{w}_{n-1})] \\ &\quad + \mathbb{E}_{Q_{\psi}(\mathbf{z}_n \mid \mathbf{w}_1, \dots, \mathbf{w}_n)} [\log P_{\theta}(\mathbf{w}_n \mid \mathbf{z}_n, \mathbf{w}_1, \dots, \mathbf{w}_{n-1})], \end{aligned}$$



Hybrid autoregressive and latent variable models

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$$P_\theta(w_1, \dots, w_M) = \prod_{m=2}^M P_\theta(w_m \mid w_1, \dots, w_{m-1}) P(w_1).$$

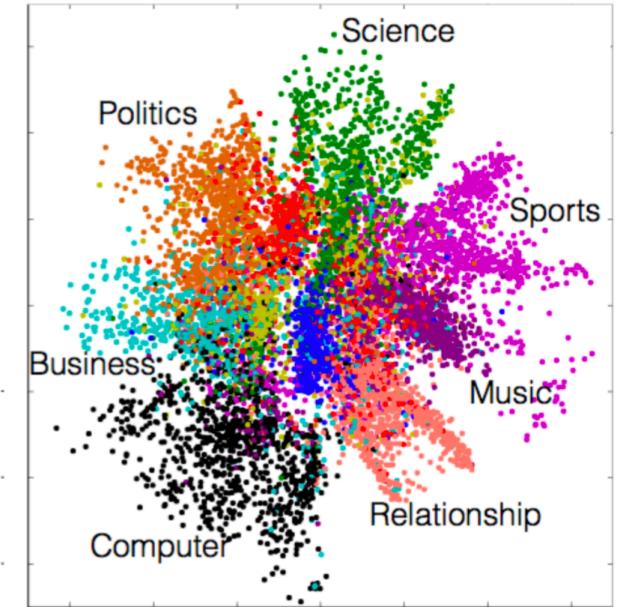


$$P_\theta(\mathbf{z}_n \mid \mathbf{w}_1, \dots, \mathbf{w}_{n-1}) = \mathcal{N}(\boldsymbol{\mu}_{\text{prior}}(\mathbf{w}_1, \dots, \mathbf{w}_{n-1}), \boldsymbol{\Sigma}_{\text{prior}}(\mathbf{w}_1, \dots, \mathbf{w}_{n-1})),$$

$$P_\theta(\mathbf{w}_n \mid \mathbf{z}_n, \mathbf{w}_1, \dots, \mathbf{w}_{n-1}) = \prod_{m=1}^{M_n} P_\theta(w_{n,m} \mid \mathbf{z}_n, \mathbf{w}_1, \dots, \mathbf{w}_{n-1}, w_{n,1}, \dots, w_{n,m-1}),$$

$$\begin{aligned} Q_\psi(\mathbf{z}_n \mid \mathbf{w}_1, \dots, \mathbf{w}_N) &= Q_\psi(\mathbf{z}_n \mid \mathbf{w}_1, \dots, \mathbf{w}_n) = \mathcal{N}(\boldsymbol{\mu}_{\text{posterior}}(\mathbf{w}_1, \dots, \mathbf{w}_n), \boldsymbol{\Sigma}_{\text{posterior}}(\mathbf{w}_1, \dots, \mathbf{w}_n)) \\ &\approx P_\psi(\mathbf{z}_n \mid \mathbf{w}_1, \dots, \mathbf{w}_N), \end{aligned} \quad (5)$$

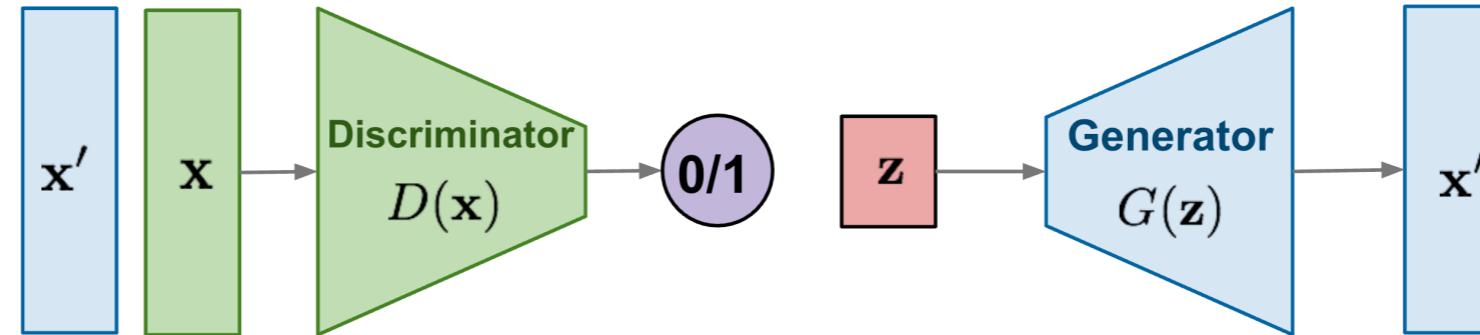
$$\begin{aligned} \log P_\theta(\mathbf{w}_1, \dots, \mathbf{w}_N) &\geq \sum_{n=1}^N -\text{KL}[Q_\psi(\mathbf{z}_n \mid \mathbf{w}_1, \dots, \mathbf{w}_n) \parallel P_\theta(\mathbf{z}_n \mid \mathbf{w}_1, \dots, \mathbf{w}_{n-1})] \\ &\quad + \mathbb{E}_{Q_\psi(\mathbf{z}_n \mid \mathbf{w}_1, \dots, \mathbf{w}_n)} [\log P_\theta(\mathbf{w}_n \mid \mathbf{z}_n, \mathbf{w}_1, \dots, \mathbf{w}_{n-1})], \end{aligned}$$



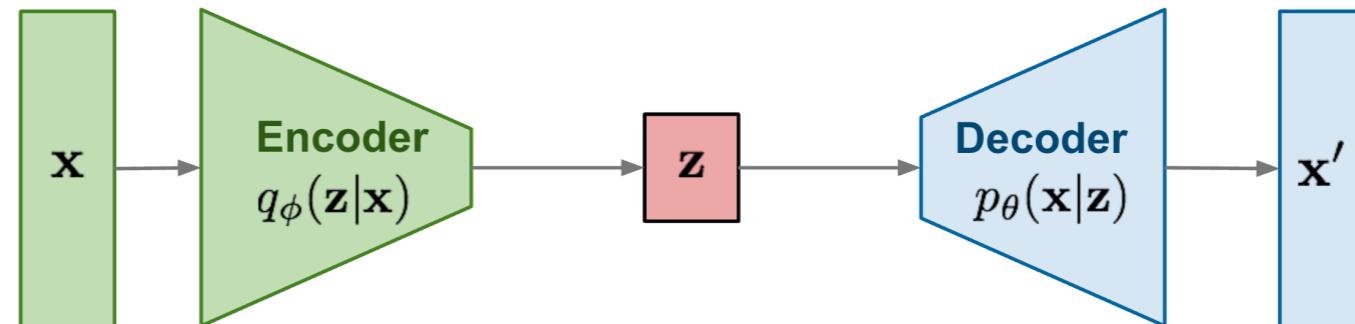
**Latent variable adds topic coherence
and improved long-term memory**

Normalizing Flows

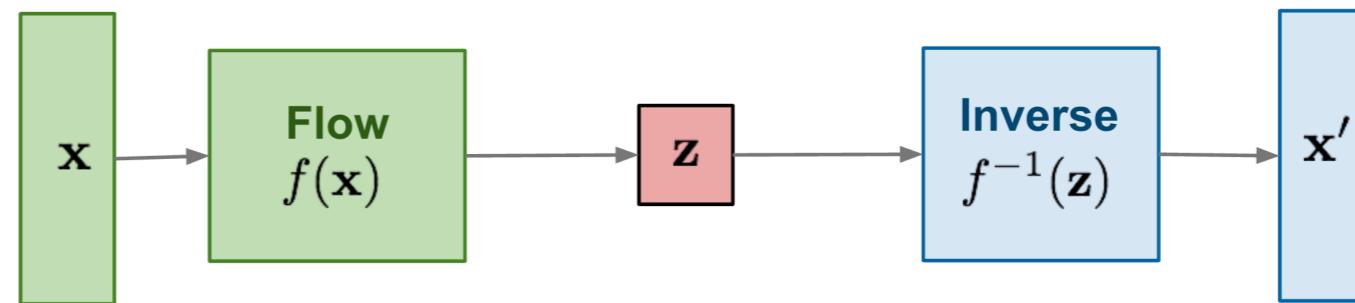
GAN: Adversarial training



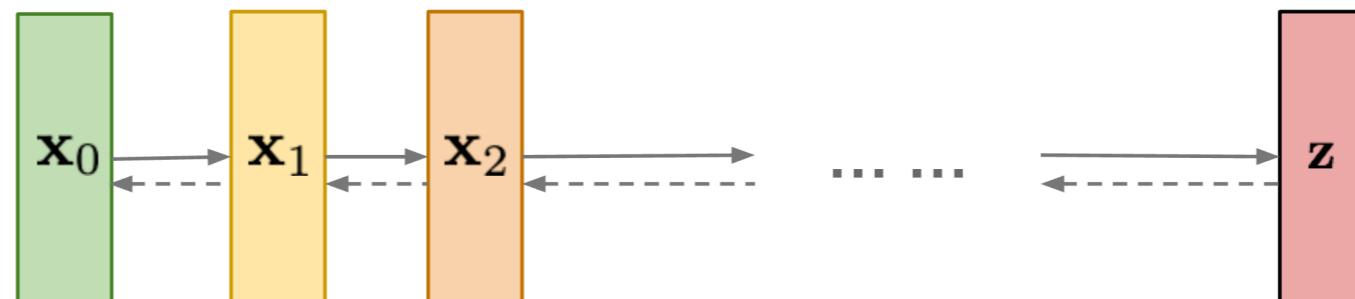
VAE: maximize variational lower bound



Flow-based models:
Invertible transform of distributions



Diffusion models:
Gradually add Gaussian noise and then reverse



Normalizing Flows Models

- Invertible Functions with Neural Networks.
- Same input-output dimension!
- No dimensionality reduction

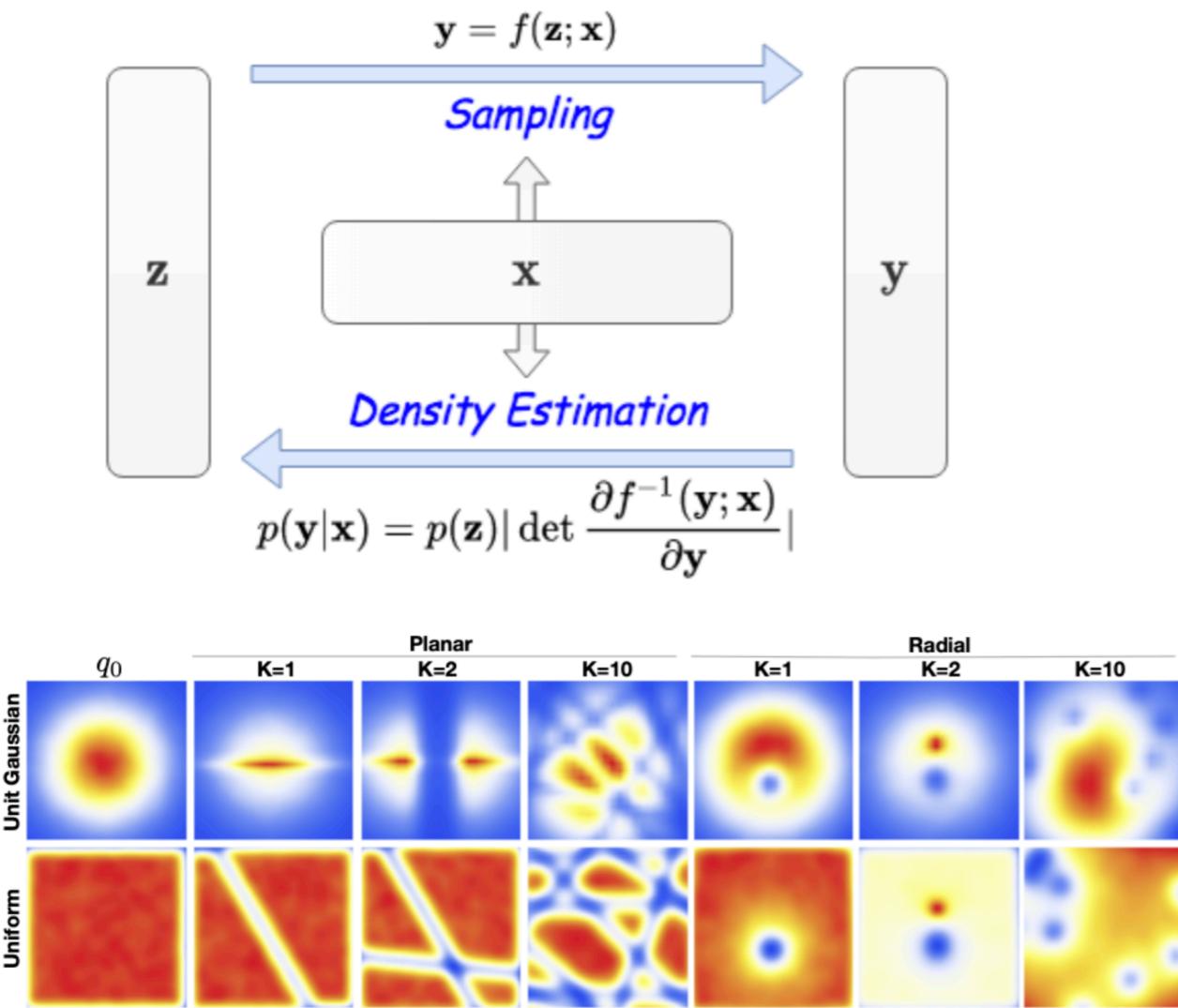


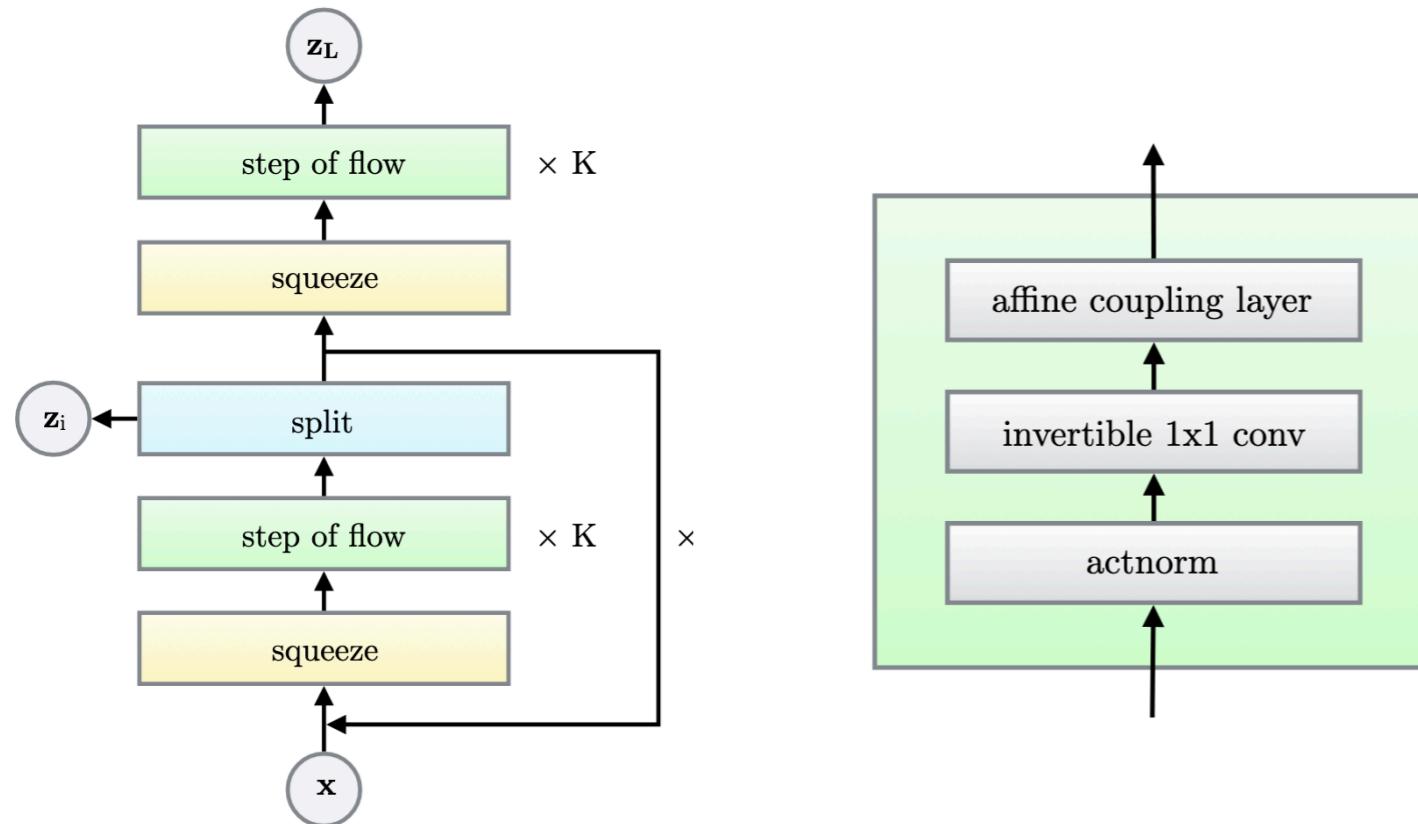
Figure 1. Effect of normalizing flow on two distributions.

Glow: Generative Flow with Invertible 1×1 Convolutions



Figure 1: Synthetic celebrities sampled from our model; see Section 3 for architecture and method, and Section 5 for more results.

Normalizing Flows Models

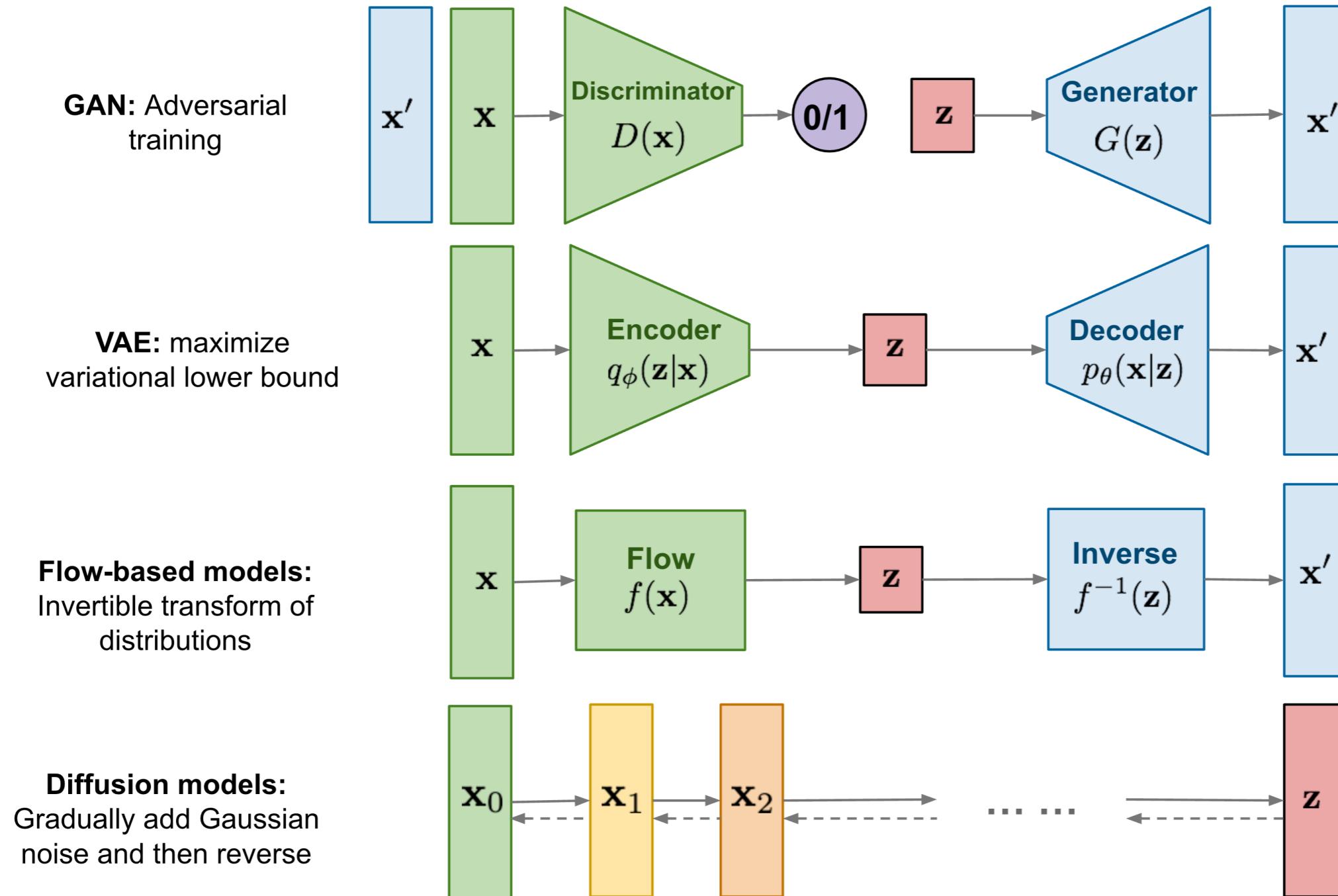


$$\mathbf{x} \xleftarrow{\mathbf{f}_1} \mathbf{h}_1 \xleftarrow{\mathbf{f}_2} \mathbf{h}_2 \cdots \xleftarrow{\mathbf{f}_K} \mathbf{z}$$

$$\begin{aligned} \log p_{\theta}(\mathbf{x}) &= \log p_{\theta}(\mathbf{z}) + \log |\det(d\mathbf{z}/d\mathbf{x})| \\ &= \log p_{\theta}(\mathbf{z}) + \sum_{i=1}^K \log |\det(d\mathbf{h}_i/d\mathbf{h}_{i-1})| \end{aligned}$$

Description	Function	Reverse Function	Log-determinant
Actnorm. See Section 3.1.	$\forall i, j : \mathbf{y}_{i,j} = \mathbf{s} \odot \mathbf{x}_{i,j} + \mathbf{b}$	$\forall i, j : \mathbf{x}_{i,j} = (\mathbf{y}_{i,j} - \mathbf{b})/\mathbf{s}$	$h \cdot w \cdot \text{sum}(\log \mathbf{s})$
Invertible 1×1 convolution. $\mathbf{W} : [c \times c]$. See Section 3.2.	$\forall i, j : \mathbf{y}_{i,j} = \mathbf{W}\mathbf{x}_{i,j}$	$\forall i, j : \mathbf{x}_{i,j} = \mathbf{W}^{-1}\mathbf{y}_{i,j}$	$h \cdot w \cdot \log \det(\mathbf{W}) $ or $h \cdot w \cdot \text{sum}(\log \mathbf{s})$ (see eq. (10))
Affine coupling layer. See Section 3.3 and (Dinh et al., 2014)	$\mathbf{x}_a, \mathbf{x}_b = \text{split}(\mathbf{x})$ $(\log \mathbf{s}, \mathbf{t}) = \text{NN}(\mathbf{x}_b)$ $\mathbf{s} = \exp(\log \mathbf{s})$ $\mathbf{y}_a = \mathbf{s} \odot \mathbf{x}_a + \mathbf{t}$ $\mathbf{y}_b = \mathbf{x}_b$ $\mathbf{y} = \text{concat}(\mathbf{y}_a, \mathbf{y}_b)$	$\mathbf{y}_a, \mathbf{y}_b = \text{split}(\mathbf{y})$ $(\log \mathbf{s}, \mathbf{t}) = \text{NN}(\mathbf{y}_b)$ $\mathbf{s} = \exp(\log \mathbf{s})$ $\mathbf{x}_a = (\mathbf{y}_a - \mathbf{t})/\mathbf{s}$ $\mathbf{x}_b = \mathbf{y}_b$ $\mathbf{x} = \text{concat}(\mathbf{x}_a, \mathbf{x}_b)$	$\text{sum}(\log(\mathbf{s}))$

Diffusion Models





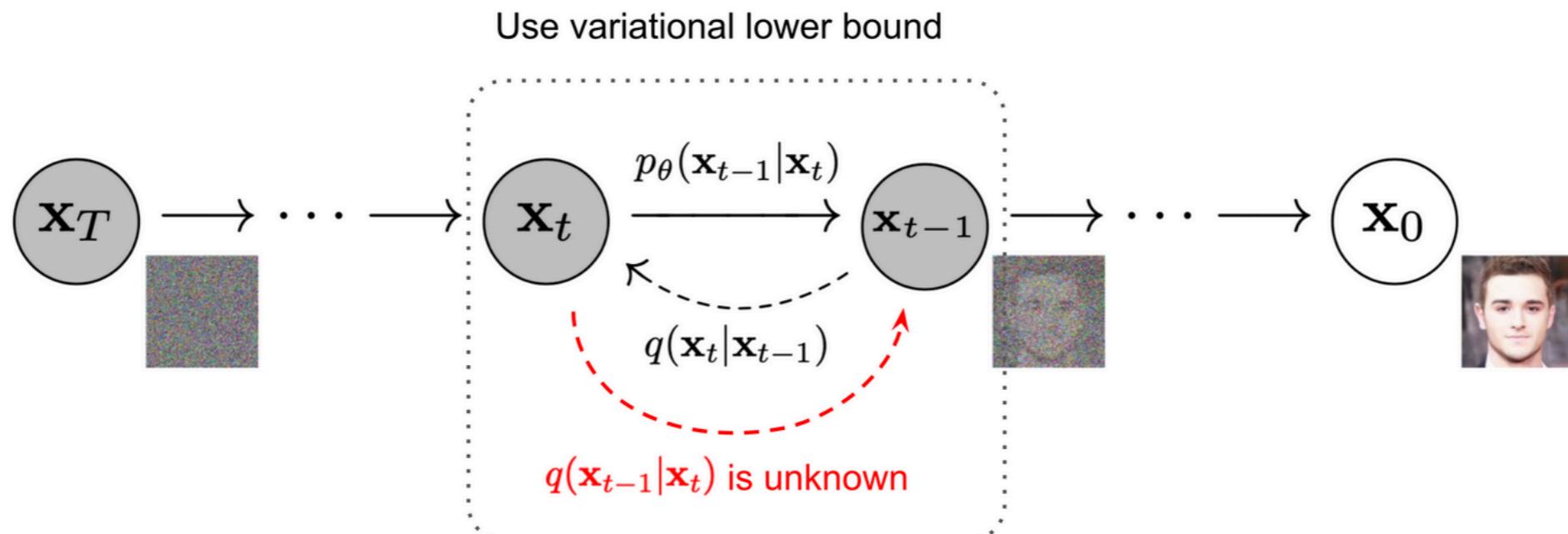
Denoising Diffusion Probabilistic Models

Jonathan Ho
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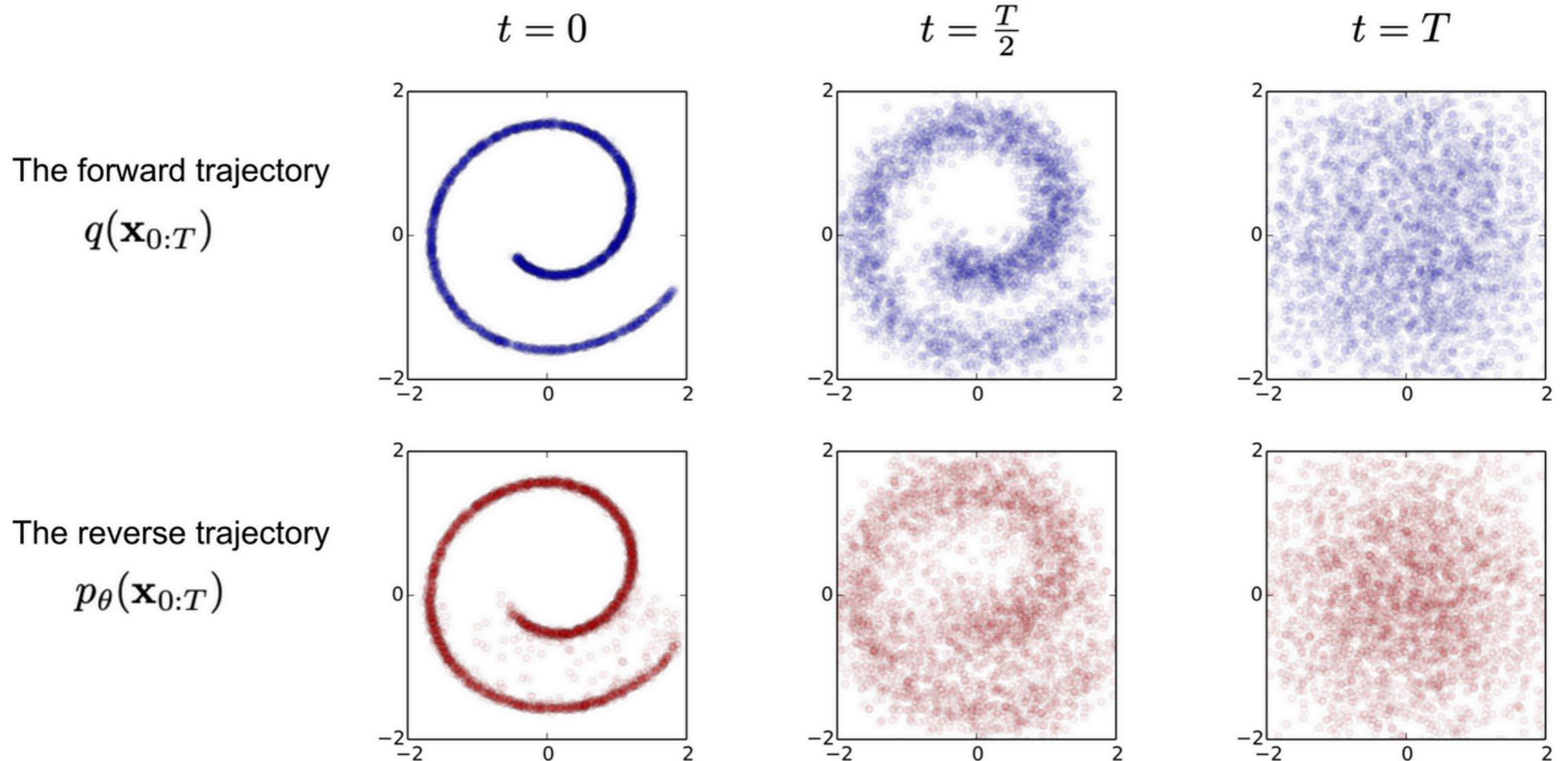
Ajay Jain
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$$q(\mathbf{x}_t | \mathbf{x}_{t-1}) = \mathcal{N}(\mathbf{x}_t; \sqrt{1 - \beta_t} \mathbf{x}_{t-1}, \beta_t \mathbf{I}) \quad q(\mathbf{x}_{1:T} | \mathbf{x}_0) = \prod_{t=1}^T q(\mathbf{x}_t | \mathbf{x}_{t-1})$$

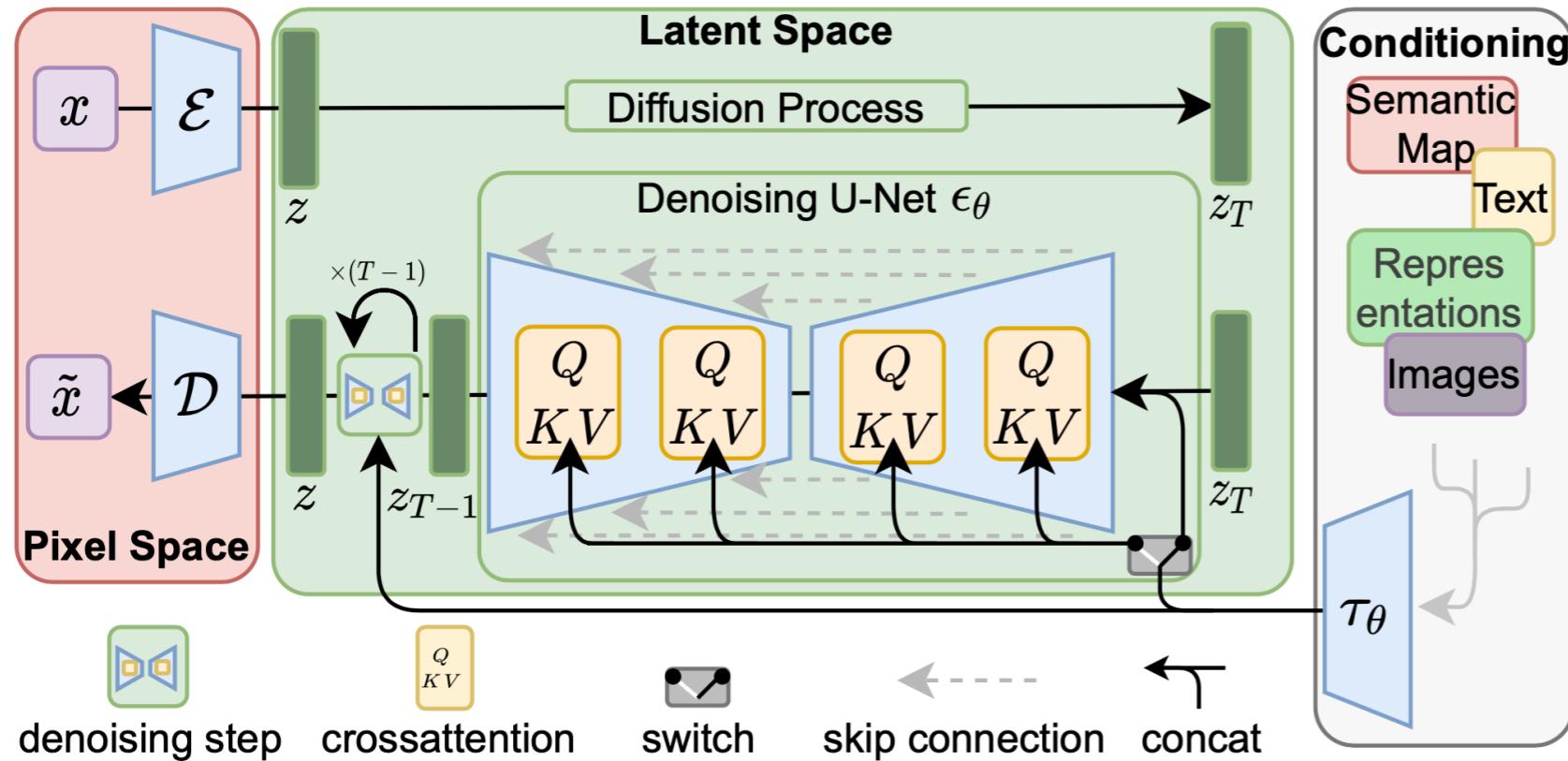


$$p_{\theta}(\mathbf{x}_{0:T}) = p(\mathbf{x}_T) \prod_{t=1}^T p_{\theta}(\mathbf{x}_{t-1}|\mathbf{x}_t) \quad p_{\theta}(\mathbf{x}_{t-1}|\mathbf{x}_t) = \mathcal{N}(\mathbf{x}_{t-1}; \boldsymbol{\mu}_{\theta}(\mathbf{x}_t, t), \boldsymbol{\Sigma}_{\theta}(\mathbf{x}_t, t))$$



$$-\log p_{\theta}(\mathbf{x}_0) \leq -\log p_{\theta}(\mathbf{x}_0) + D_{\text{KL}}(q(\mathbf{x}_{1:T}|\mathbf{x}_0) \| p_{\theta}(\mathbf{x}_{1:T}|\mathbf{x}_0))$$

Stable Diffusion!



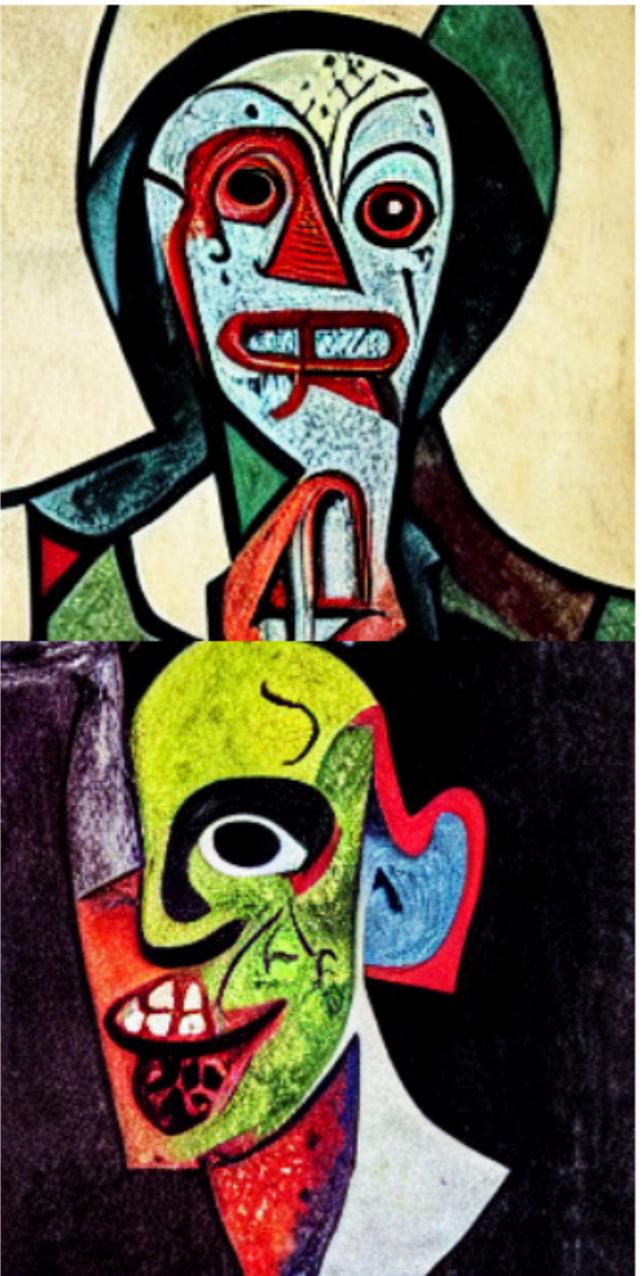
$$\text{Attention}(\mathbf{Q}, \mathbf{K}, \mathbf{V}) = \text{softmax}\left(\frac{\mathbf{Q}\mathbf{K}^\top}{\sqrt{d}}\right) \cdot \mathbf{V}$$

where $\mathbf{Q} = \mathbf{W}_Q^{(i)} \cdot \varphi_i(\mathbf{z}_i)$, $\mathbf{K} = \mathbf{W}_K^{(i)} \cdot \tau_\theta(y)$, $\mathbf{V} = \mathbf{W}_V^{(i)} \cdot \tau_\theta(y)$

and $\mathbf{W}_Q^{(i)} \in \mathbb{R}^{d \times d_\epsilon^i}$, $\mathbf{W}_K^{(i)}, \mathbf{W}_V^{(i)} \in \mathbb{R}^{d \times d_\tau}$, $\varphi_i(\mathbf{z}_i) \in \mathbb{R}^{N \times d_\epsilon^i}$, $\tau_\theta(y) \in \mathbb{R}^{M \times d_\tau}$

Stable Diffusion!

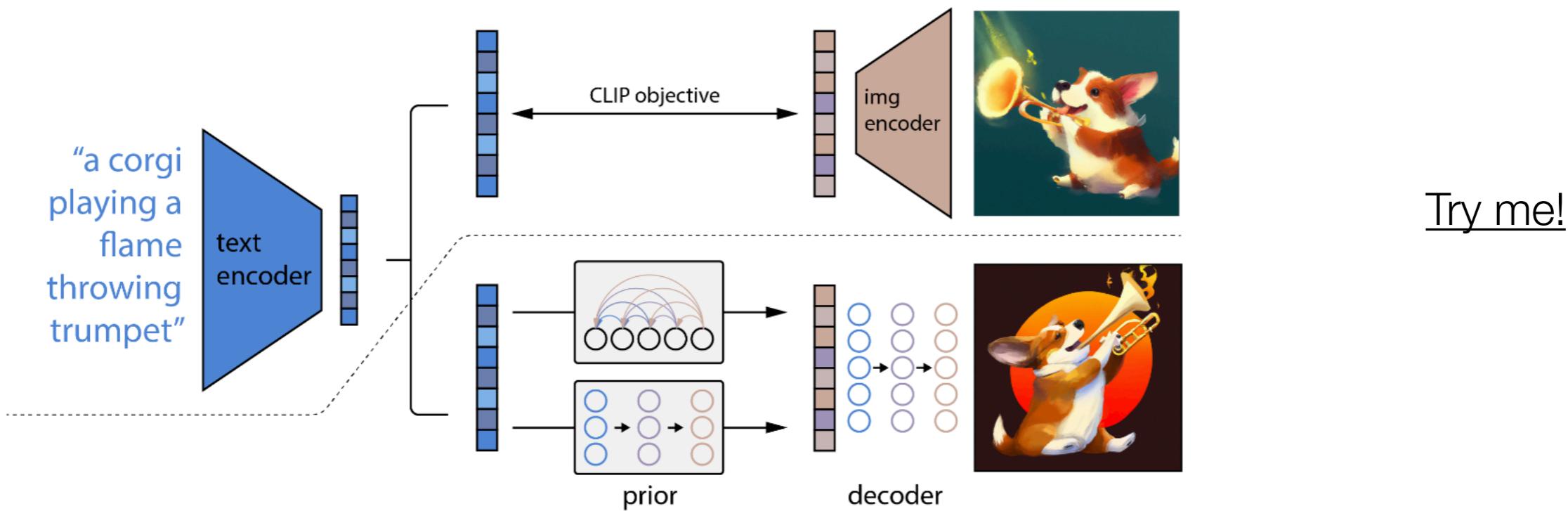
'A zombie in the style of Picasso'



'An image of an animal half mouse half octopus'



Try me!



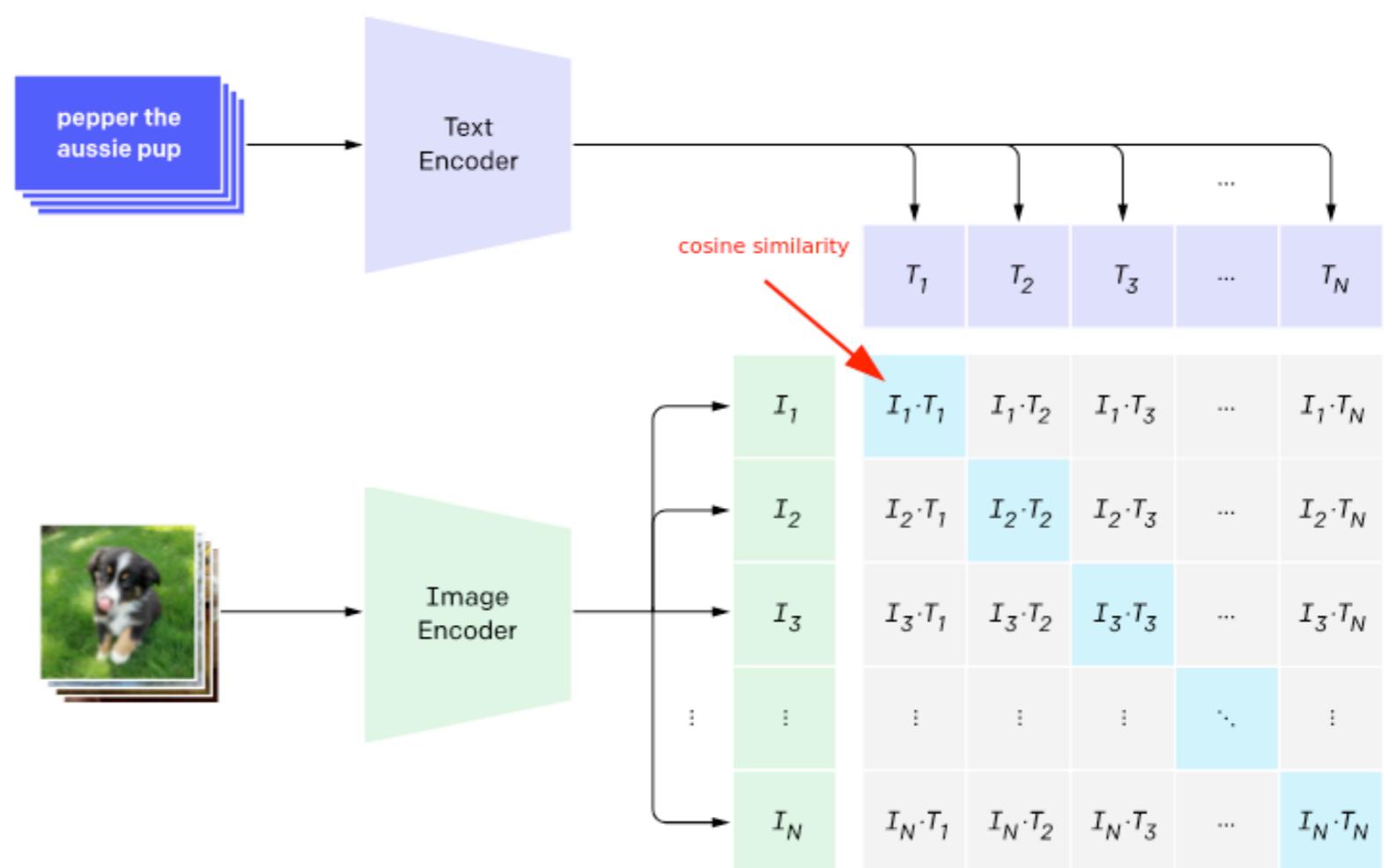
Two kinds of prior distributions are proposed with roughly similar performance:

- An autoregressive prior over a discrete latent space (VQ-VAE)
- **Diffusion prior** just like in Stable Diffusion

CLIP pretraining to jointly encode images/text

- Encodes images and text to similar embeddings
- Text and image have separate transformer encoders
- Visual encoder is **ViT (vision transformer)**
- Text encoder is **GPT-2 transformer**
- Trained on 256 GPUs for 2 weeks

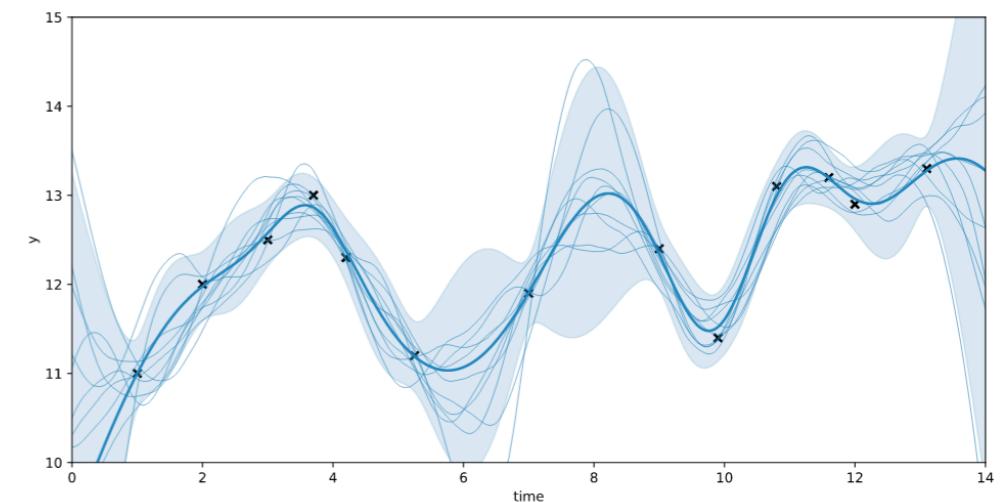
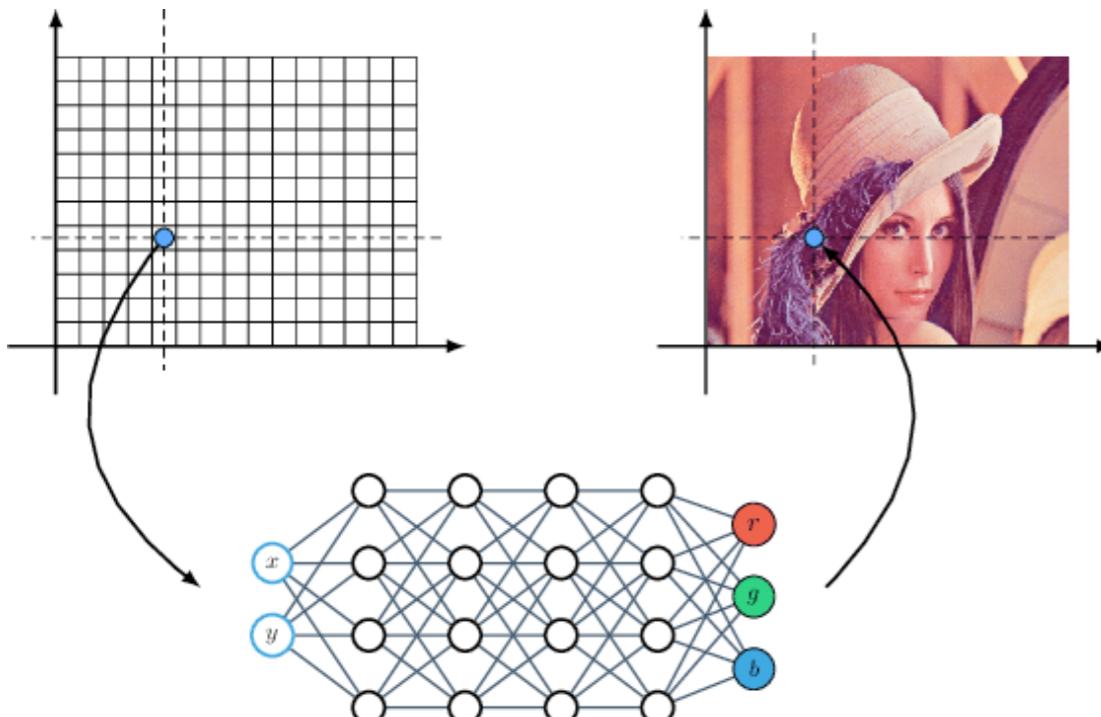
1. Contrastive pre-training



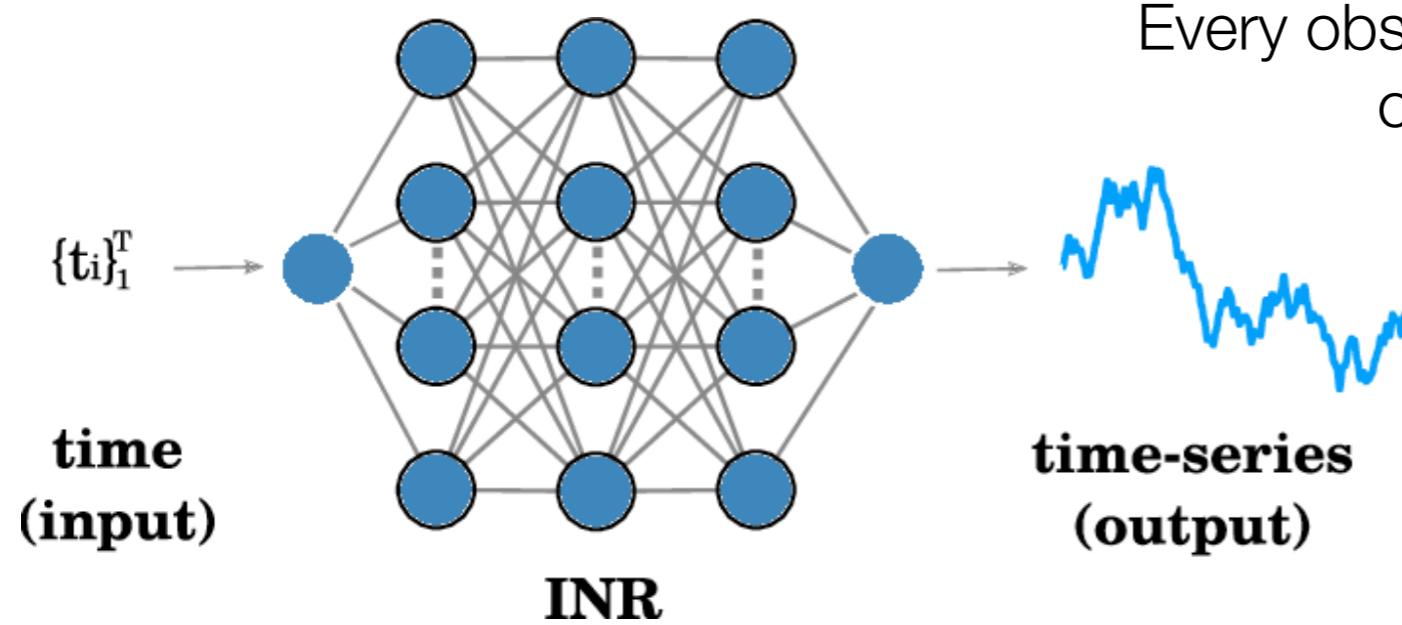
Neural Implicit Representations

Basic idea

- Implicit neural representations is about **parameterizing a continuous differentiable signal** with a neural network

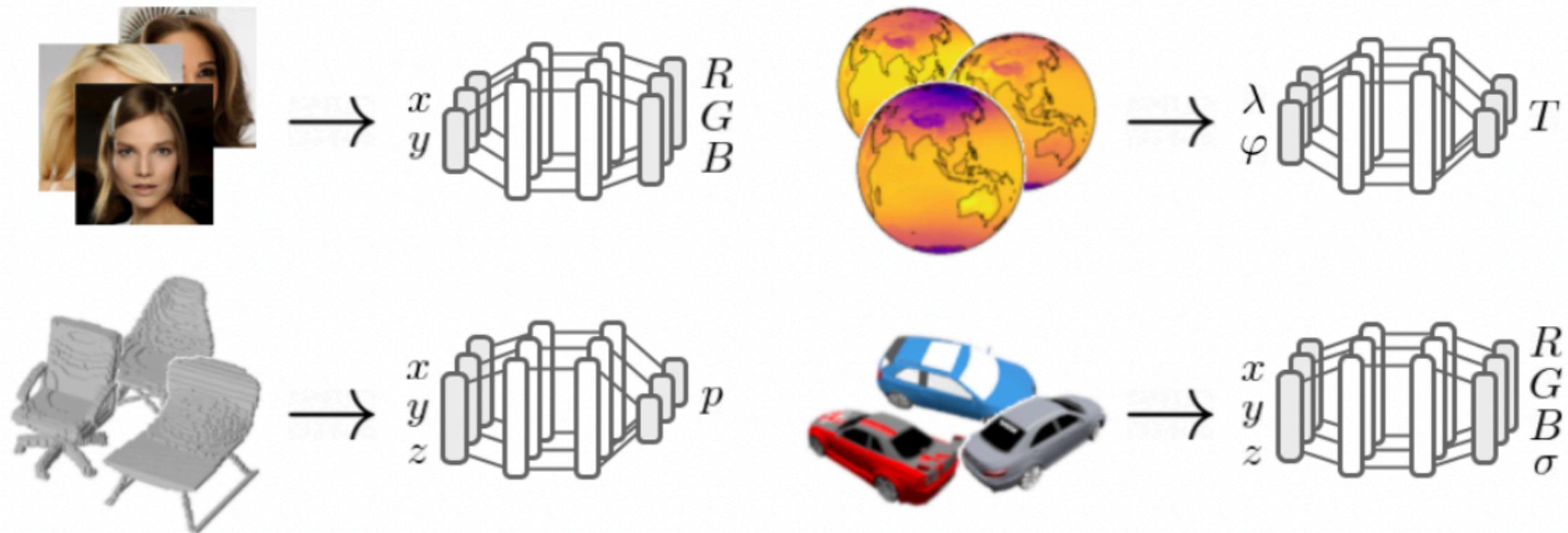


We learn parametric models for functions!
Every observation (pixel in an image) is an evaluation
of such function at a given location



Basic idea

- No matter what kind of data you have, the NN nature is not changed!!
- It scales much better with resolution than array representations!



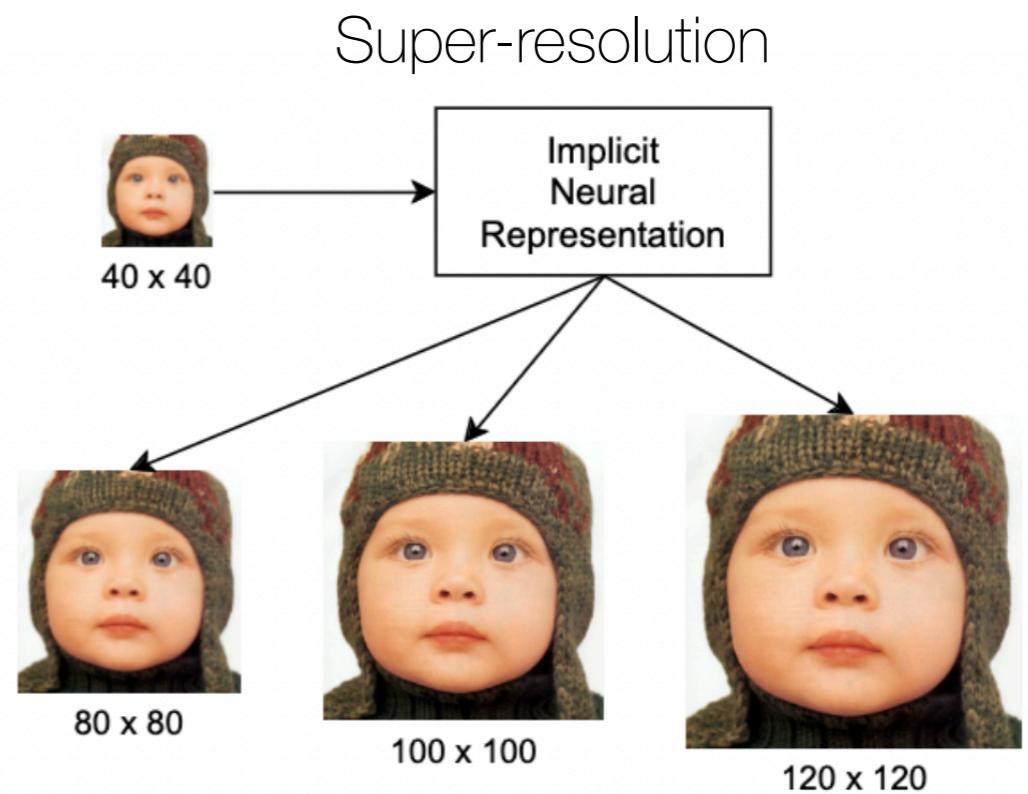
Implicit Neural Representations with Periodic Activation Functions (SIREN)

$$\Phi(\mathbf{x}) = \mathbf{W}_n (\phi_{n-1} \circ \phi_{n-2} \circ \dots \circ \phi_0)(\mathbf{x}) + \mathbf{b}_n, \quad \mathbf{x}_i \mapsto \phi_i(\mathbf{x}_i) = \sin(\mathbf{W}_i \mathbf{x}_i + \mathbf{b}_i)$$

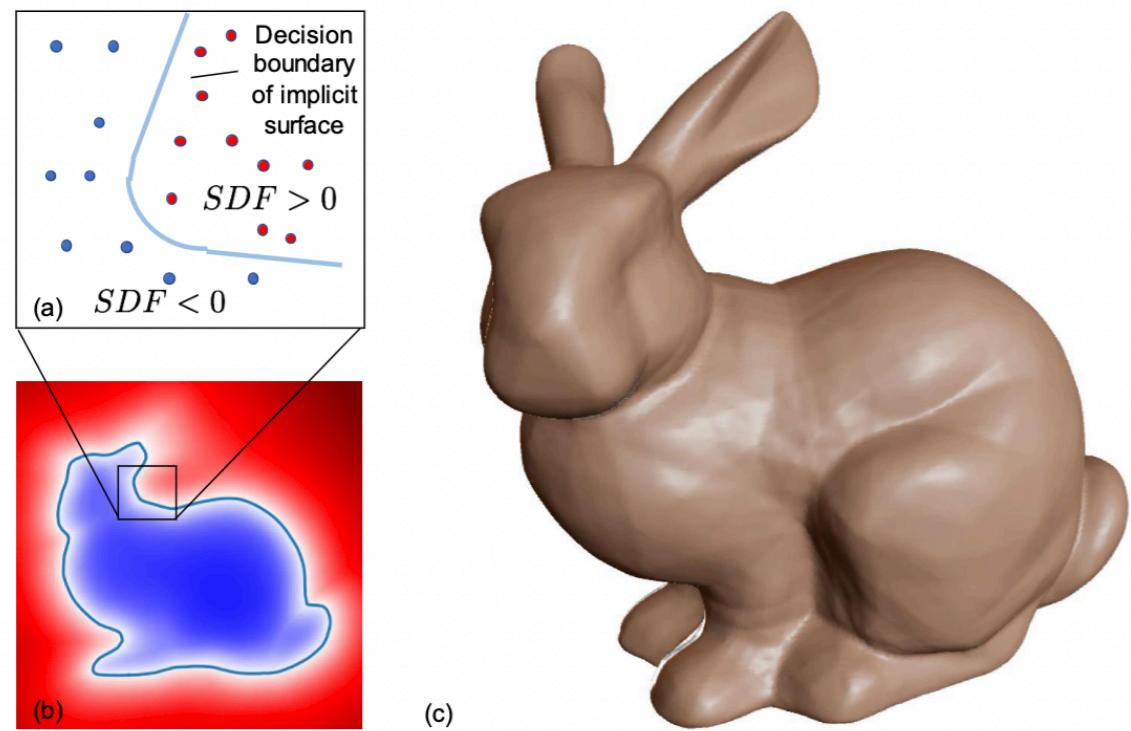
$$\tilde{\mathcal{L}} = \sum_i \|\Phi(\mathbf{x}_i) - f(\mathbf{x}_i)\|^2$$



Reconstruction



DeepSDF: Learning Continuous Signed Distance Functions for Shape Representation



$$SDF(\mathbf{x}) = s : \mathbf{x} \in \mathbb{R}^3, s \in \mathbb{R}.$$

$$SDF(\cdot) = 0 \quad \text{Surface}$$

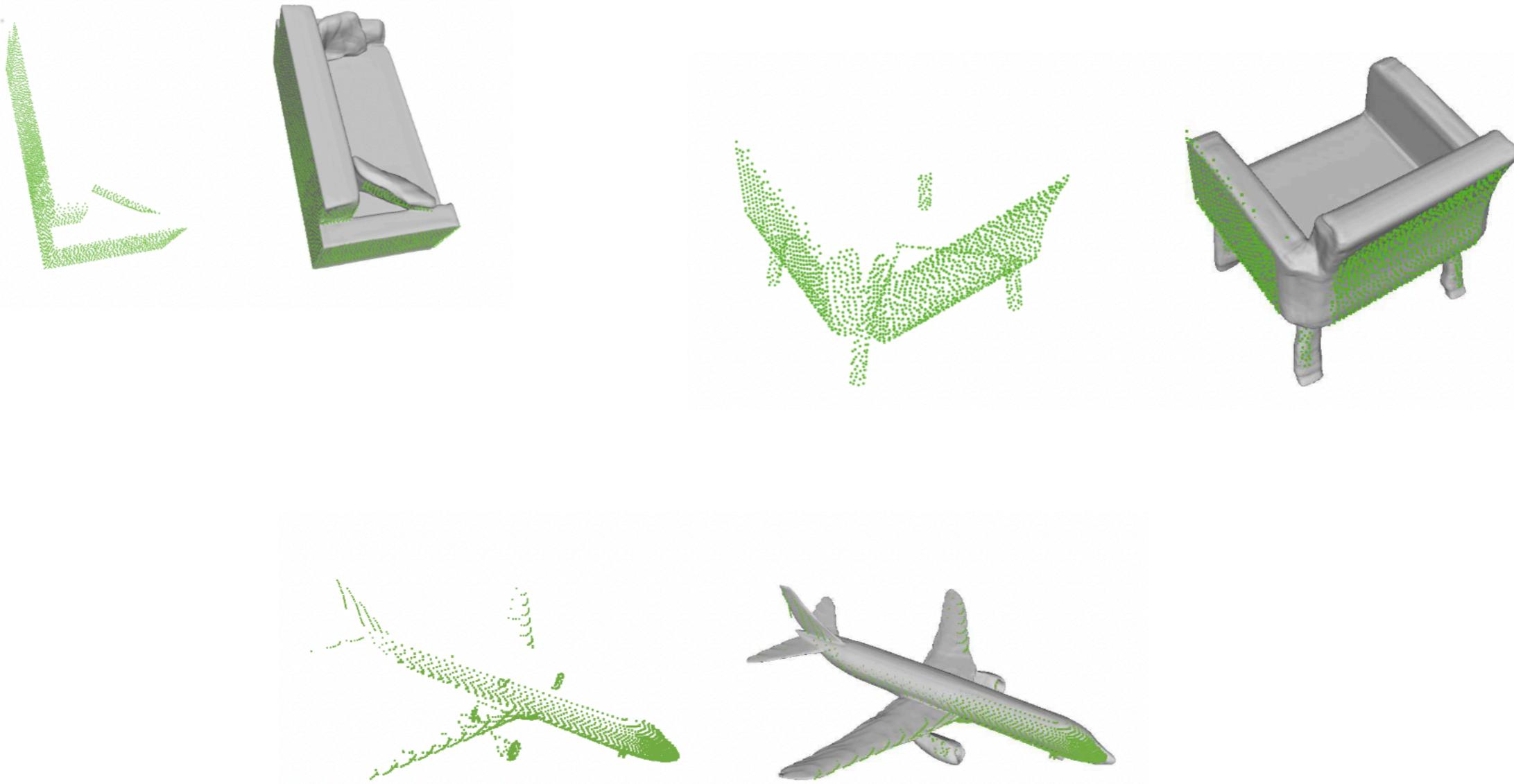
$$f_{\theta}(\mathbf{x}) \approx SDF(\mathbf{x}), \forall \mathbf{x} \in \Omega.$$

$$X := \{(\mathbf{x}, s) : SDF(\mathbf{x}) = s\}.$$

Figure 2: Our DeepSDF representation applied to the Stanford Bunny: (a) depiction of the underlying implicit surface $SDF = 0$ trained on sampled points inside $SDF < 0$ and outside $SDF > 0$ the surface, (b) 2D cross-section of the signed distance field, (c) rendered 3D surface recovered from $SDF = 0$. Note that (b) and (c) are recovered via DeepSDF.

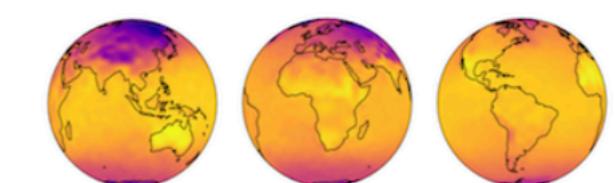
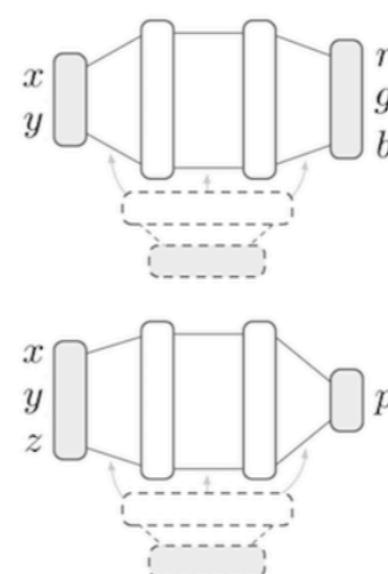
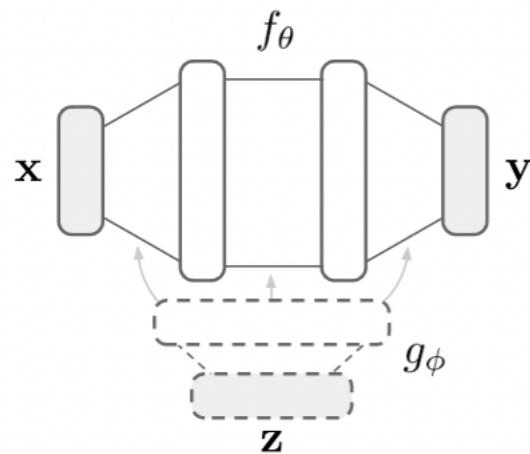
$$\mathcal{L}(f_{\theta}(\mathbf{x}), s) = |\text{clamp}(f_{\theta}(\mathbf{x}), \delta) - \text{clamp}(s, \delta)|.$$

DeepSDF: Learning Continuous Signed Distance Functions for Shape Representation



Probabilistic INR models

Generative Models as Distributions of Functions



Function generator use a **hypernetwork!**

A **GAN-style loss** function is used to train the generator



Figure 5: Training procedure for GASP: 1. Sample a function and evaluate it at a set of coordinate locations to generate fake point cloud. 2. Convert real data sample to point cloud. 3. Discriminate between real and fake point clouds.

Generative Models as Distributions of Functions

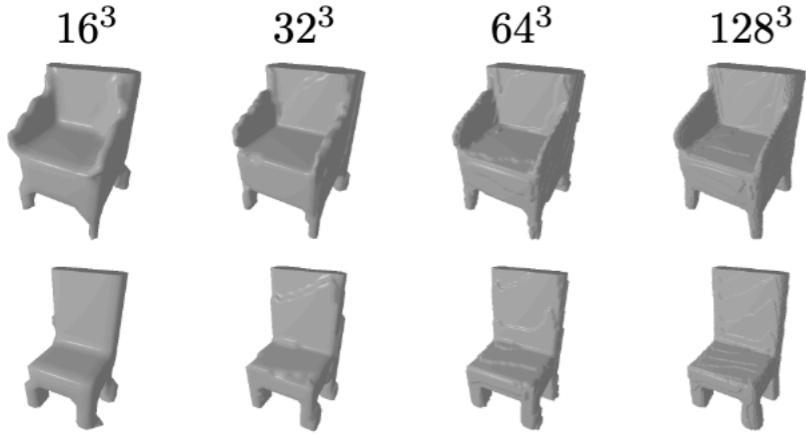


Figure 10: Evaluating the same function at different resolutions. As samples from our model can be probed at arbitrary coordinates, we can increase the resolution to render smoother meshes.

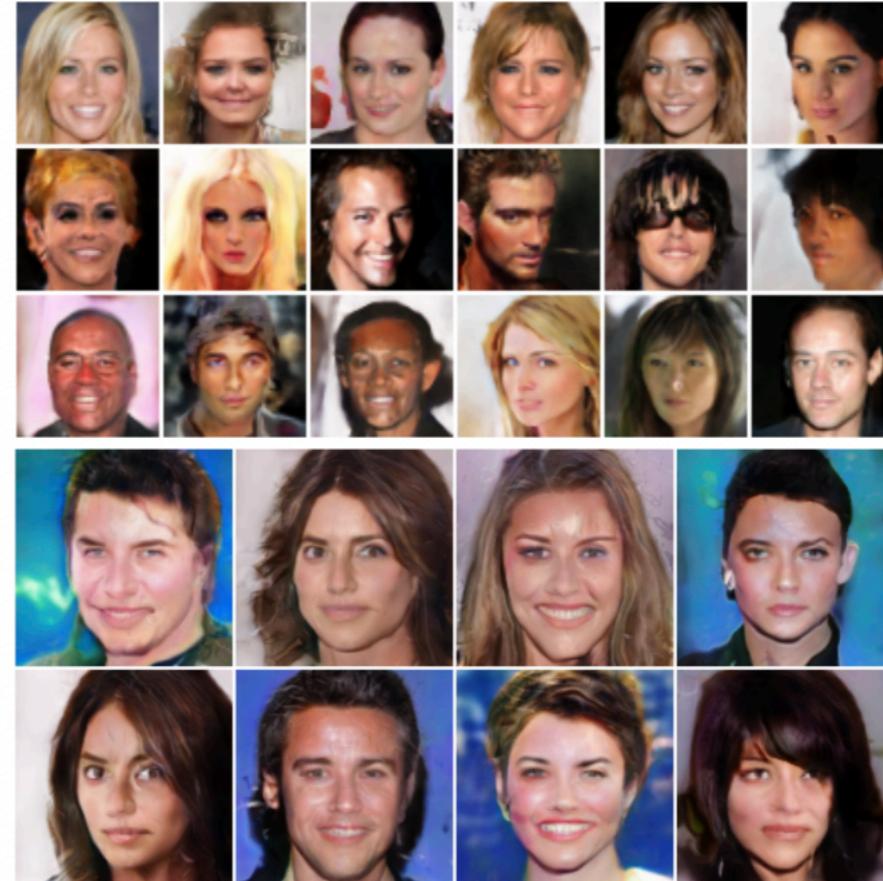


Figure 6: Samples from our model trained on CelebAHQ 64×64 (top) and 128×128 (bottom). Each image corresponds to a function which was sampled from our model and then evaluated on the grid. To produce this figure we sampled 5 batches and chose the best batch by visual inspection.

Generative Models as Distributions of Functions

Still not many things in SOTA. We have an ICML :
VAMoH!

VARIATIONAL MIXTURE OF HYPERGENERATORS FOR LEARNING DISTRIBUTIONS OVER FUNCTIONS

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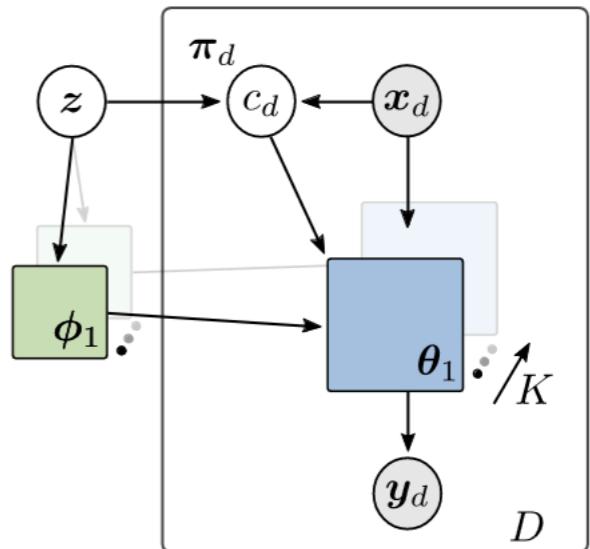
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Madrid, Spain

Isabel Valera
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Saarbrücken, Germany



■ HyperNetwork ■ Data generator



(a) Generative model

