Mathematical Concepts

Exercise 1: Convex set

(a) A set $S = \left\{x \in \mathbb{R}^n : a^T x \leq b\right\}$is a convex set.

(b) Let $c \in \mathbb{R}^n$ and $r > 0$.Let $||\cdot||$ be an arbitrary norm defined on $\mathbb{R}^n$. Show that the open ball

$$B(c, r) = \{x \in \mathbb{R}^n : \ \ ||x - c|| < r\}$$

and closed ball

$$B[c, r] = \{x \in \mathbb{R}^n : \ \ ||x - c|| \leq r\}$$

are convex

Exercise 2:
Solve the following linear programming problem graphically.
maximize

$$z = 5x_1 + 7x_2 \tag{1}$$

subject to

$$\begin{aligned}
&3x_1 + 8x_2 \leq 12, \\
&x_1 + x_2 \leq 2, \\
&2x_1 \leq 3, \\
&x_1 \geq 0, x_2 \geq 0
\end{aligned}$$

Exercise 3:
Solve the following linear programming problem graphically.
maximize

$$z = 10x_1 + 6x_2 \tag{2}$$

subject to

$$\begin{aligned}
&5x_1 + 3x_2 \leq 30, \\
&x_1 + 2x_2 \leq 18, \\
&x_1 \geq 0, x_2 \geq 0.
\end{aligned}$$

Exercise 4:
Consider the linear programming problem:
Maximize

$$z = 2x + 3y \tag{3}$$

subject to

$$\begin{aligned}
&2x + 3y \geq 12, \\
&3x + 4y \leq 12 \\
&x \geq 0, y \geq 0.
\end{aligned}$$

Exercise 5:
Consider the linear programming problem:
Maximize

$$z = 2x_1 + 4x_2 + 3x_3$$

subject to

$$\begin{aligned} &x_1 + x_2 + x_3 \leq 12, \\ &x_1 + 3x_2 + 3x_3 \leq 24, \\ &3x_1 + 6x_2 + 4x_3 \leq 90, \\ &x_1 \geq 0, x_2 \geq 0, x_3 \geq 0. \end{aligned}$$

Exercise 6:
Solve the following linear programming problem graphically.
maximize

$$z = 5x_1 + 6x_2 \tag{4}$$

subject to

$$\begin{aligned} &x_1 + x_2 \geq 6, \\ &x_1 \leq 4, \\ &x_2 < 1, \\ &x_1 \geq 0, x_2 \geq 0. \end{aligned}$$

Exercise 7:
Consider the linear programming problem
Maximize , $z = 4x_1 + 2x_2 + 7x_3$
subject to

$$\begin{aligned} &2x_1 - x_2 + 4x_3 \leq 18, \\ &4x_1 + 2x_2 + 5x_3 \leq 10, \\ &x_1 \geq 0, x_2 \geq 0, x_3 \geq 0. \end{aligned}$$

- Write down the matrix representation of this problem.
- Transform this problem to a problem in canonical form.
- Solve the problem geometrically.

Exercise 8:
Consider the linear programming in standard form
Maximize , $z = c^T x$
subject to

$$\begin{aligned} &Ax \leq b \\ &x \geq 0. \end{aligned}$$

Show that the constraints $Ax \leq b$ may be written as

$$Ax + Ix' = b$$

where x' is a vector of slack variables.