LMS ADAPTIVE EQUALIZATION OF A DISPERSIVE COMMMUNICATION CHANNEL

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Abstract:

An Adaptive filter is a filter that is capable of adjusting its filter coefficients automatically to adapt the input signal with the aid of an adaptive algorithm[1]. Furthermore, an adaptive filter can also be viewed as a system with a linear filter that has a transfer function controlled by varying parameters and a means of adjusting those parameters according to an optimization algorithm. Least-mean square (LMS) algorithms which are the fundamental and widely used calculation in filtering applications are a class of adpative filters used to mimic a desired filter by finding the filter coefficients that relate to producing the least mean square of the error signal i.e the difference between the desired and actual signal. It uses the instantaneous value of the square of the error signal as an estimate of the MSE. This report concludes the effect of Eigenvalue spread, Filter order and step size on Least Mean Square Filters. It also compares standard LMS to Normalized LMS.

Introduction:

Parameters like Filter order, step size, Number of data samples, Number of iterarions and the amount of distortion each channel will introduce were given to achieve the objective of each section of this project. In each section of this project, parameters like filter order, step size, number of data samples and number of iterations will be varied and the observation of the filter for each section will be explicitly stated. The last section will be about comparing the Standard LMS to the Normalized LMS. The LMS filter that will be investigated in this project is as shown in figure 1 below:

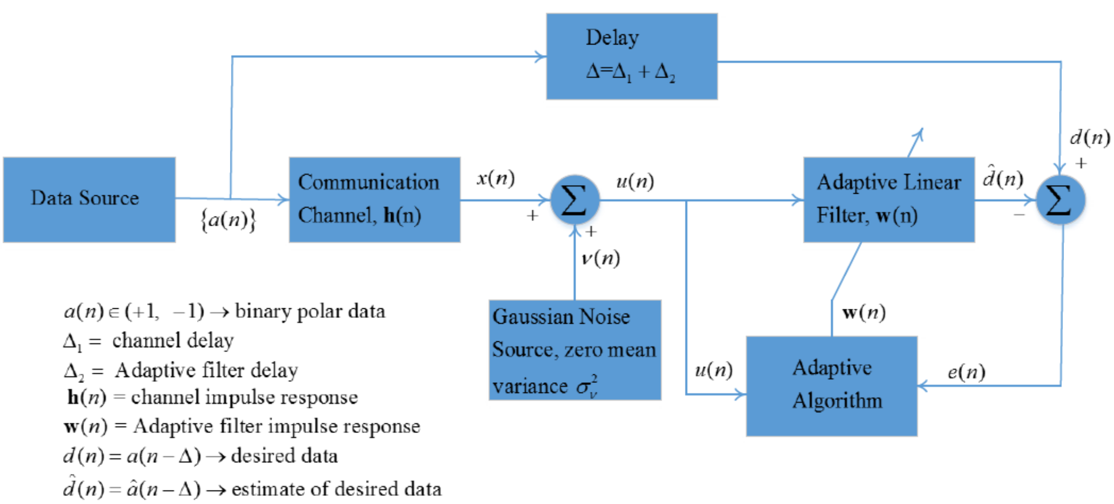
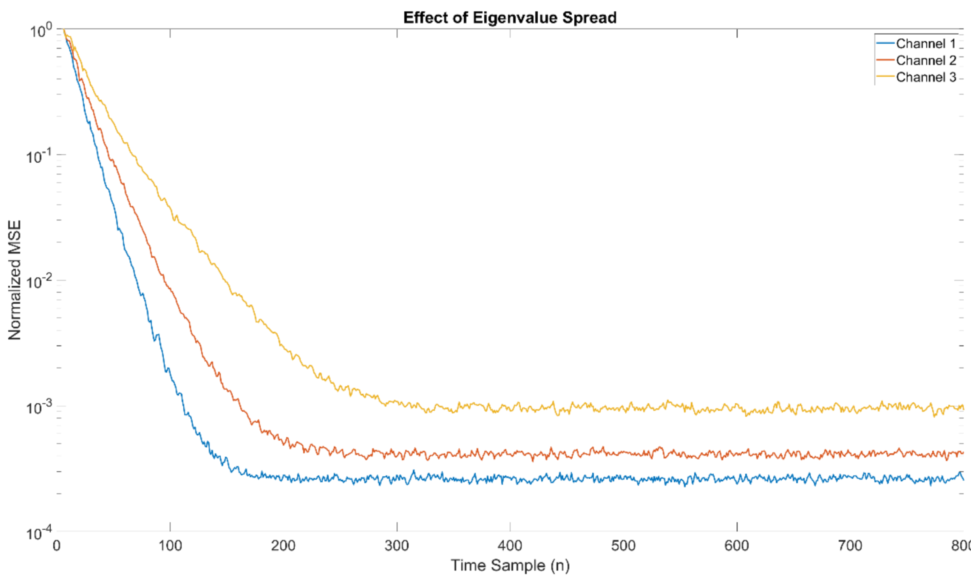


Figure 1: Diagram of LMS filter

The data source is a random sequence applied to the channel input in polar form as a(n) = ±1 which is convoluted with the communication channel h(n) to give x(n). White noise v(n) is added to give u(n) which is fed into the filter. The output of the filter $\hat{d}$(n) is substracted from the desired signal d(n) to give us the error e(n). Using the error e(n), the weight vector W(n) is adjusted so that a $\hat{d}$(n) is gotten which is very close to the deisred signal d(n) which reduces the error to a minimum.

Section 1: Eigenvalue spread changes the convergence of any adaptive algorithms and also affects the selection of step size[2]. When spectral power of input signal is non-stationary, the residual error is more in all the cases. An increase in the eigen value spread of the input correlation matrix will cause an increase in steady state error and also cause the LMS algorithm to converge more slowly.

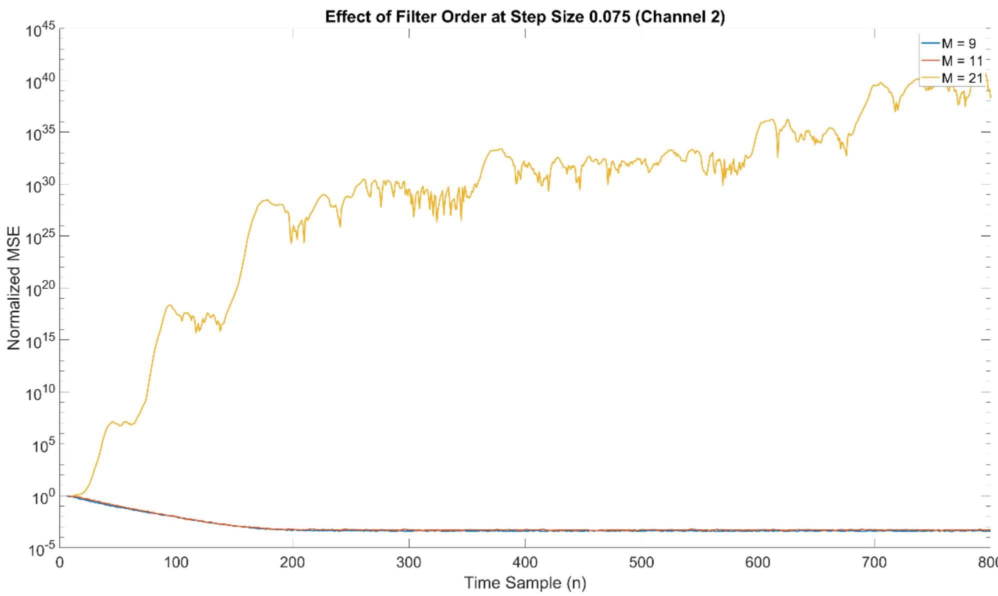
Result: From the graph below channel 1 converges faster at 170 samples, while channel 2 and 3 converge later at 250 and 350 samples respectively. Channel 3 has the most distortion, a high MSE with the highest steady state error while channel 1 has a low distortion, a low MSE with the lowest steady state error. Because of channel 1's low distortion, it converges faster at an early time sample compared to channel 2 and 3.

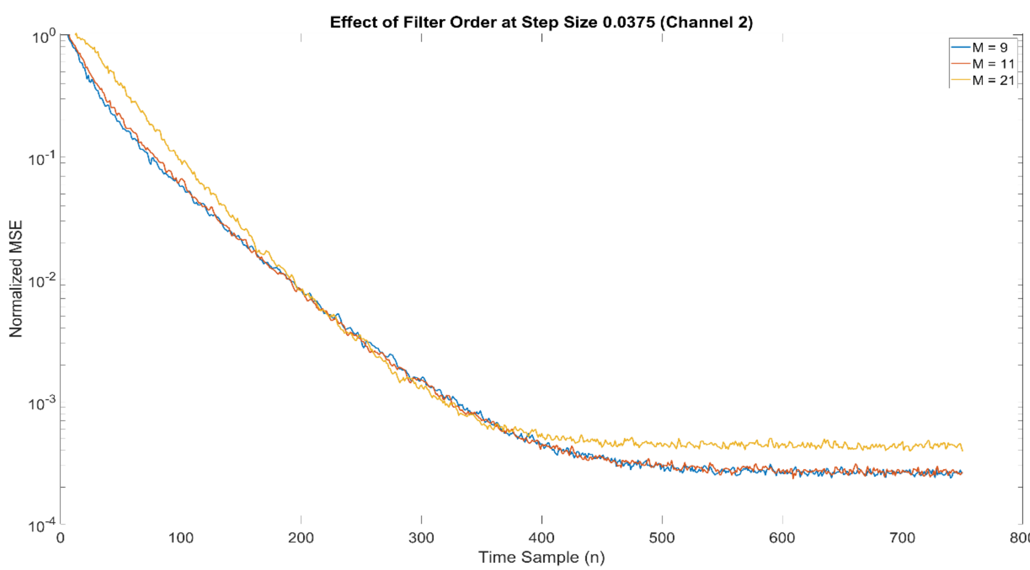


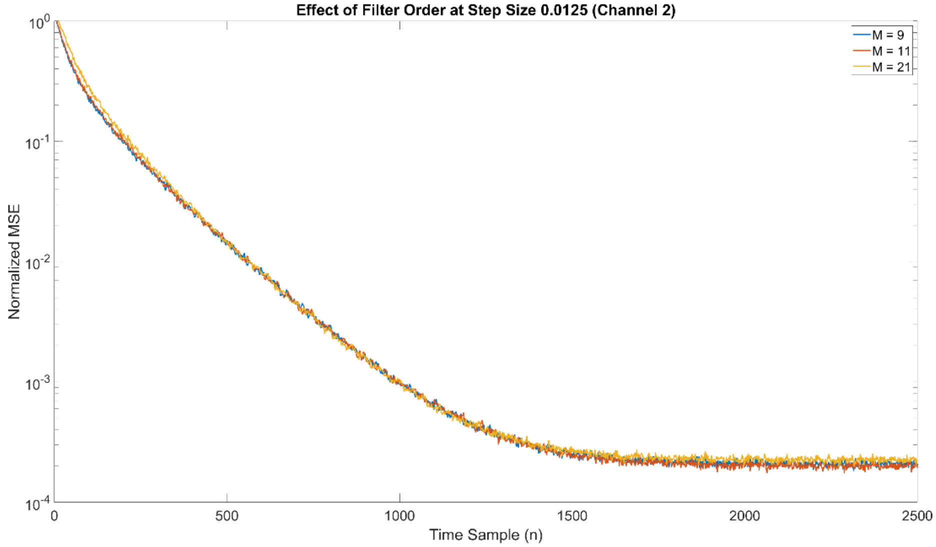
Conclusion: We can conclude that as distortion increases, the filter takes a longer time to converge making it to have a high MSE as shown in the graph above. The effect of noise also tends to cease after certain point.

Section 2: μ is the step size parameter which is varied for different step sizes. A larger step size gives faster convergence but also has a high mean square error making it have a low filtering quality. A lower step size on the other hand has a slower convergence rate which means it has a high filtering quality.

Result: From the set of graphs below, the higher the step size, the higher the MSE but the faster the convergence rate.In the first graph, filter orders 9 and 11 converge simultaneously at 160 samples and M=21 diverges with a huge increase in MSE. From the second graph; as the step size is reduced, speed of convergence is slower increasing the time of convergence to 480 samples and reducing the MSE. From the third graph, no effect of filter order is observed and the convergence happens at approximately 1700 samples.



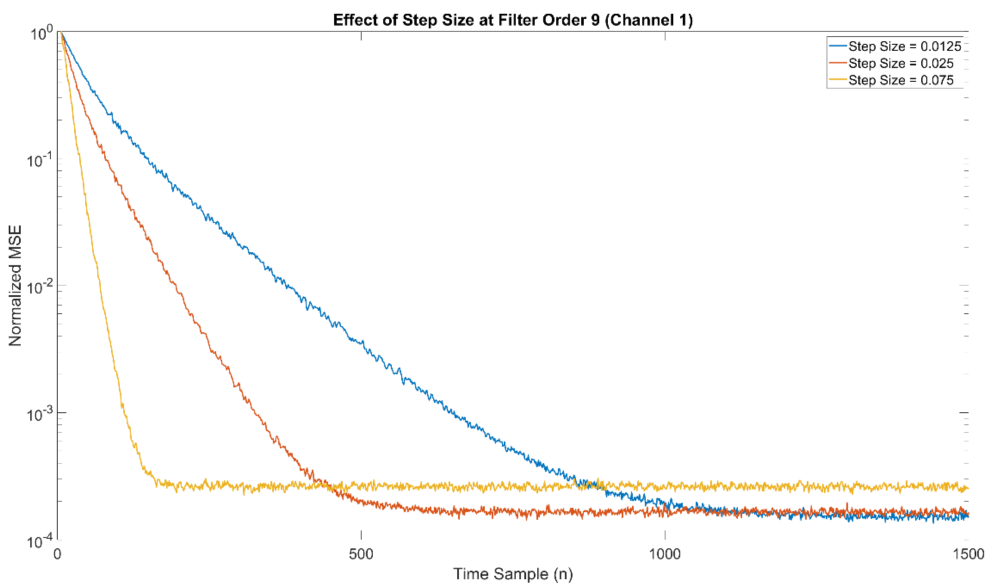




Conclusion: Based on the results gotten above from the three graphs, we can conclude that using the same step size but different filter order affects performance and Increasing the filter order doesn't necessarily improve performance.

Section 3: To study the effect of step-size parameter, the eigenvalue spread will be fixed making us focus on one channel only.

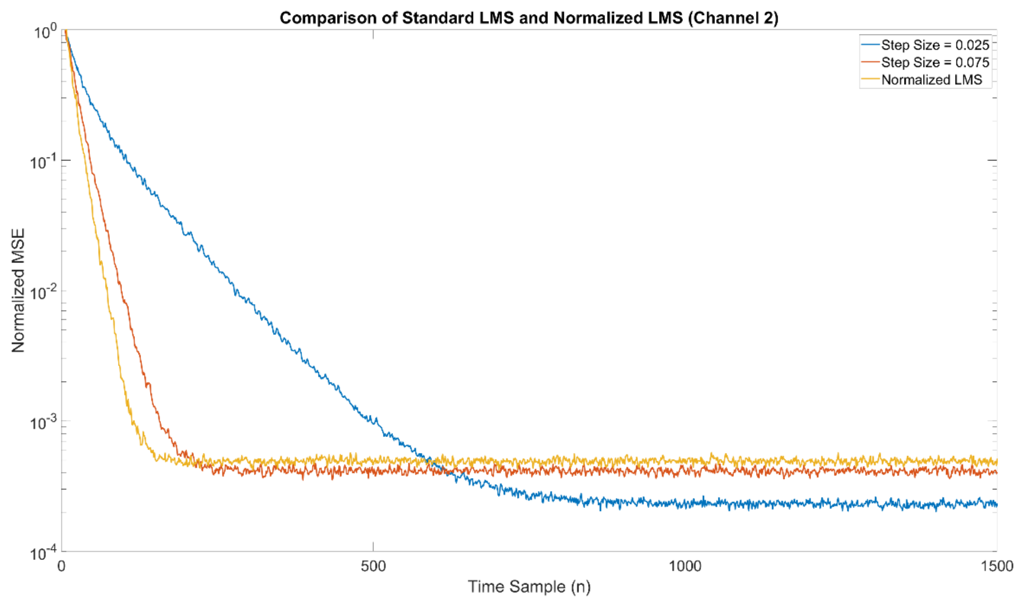
Result: From the graph below, the curve with the largest step size (μ = 0.075) converges faster but gives a high MSE as compared to step sizes (0.025 and 0.0125). 0.075 converges at 170 samples while 0.025 and 0.0125 converges at 510 and 1200 respectively.



Conclusion: Step size plays a crucial role in the rate of convergence of the algorithm. As step size increases, filter's response increase but the quality of noise filtering reduces. The step size value is chosen according to the speed of convergence and misadjustment.

Section 4: The difference between Normalized and Standard LMS is that Normalized LMS has a variable step size whereas Standard LMS has a fixed step size.

Result: From the graph below; for small step size of 0.025, there is a slow convergence but a better MSE giving a better performance. For step size of 0.075, there is faster convergence with a high MSE. The Normalized LMS converges fastest at 110 samples with the highest MSE.



Conclusion: The NLMS algorithm is an equally simple but more robust variant of the LMS algorithm[3]. It exhibits a better balance between perfomance and simplicity than the LMS algorithm. The NLMS is independent of the step size but for the Standard LMS, as step size increases the speed of convergence increases.

References:

[1] S. Haykin, Adaptive Filter Theory, Prentice Hall, Englewood Cliffs, NJ, 3rd edition, 1996.

[2] Lal C. Godara, "Application of antenna arrays to mobile communications; Part II: Beam-forming and direction-of-arrival considerations", *Proceedings of the IEEE*, vol. 85, no. 8, pp. 1195-1234, 1997.

[3] M. Rupp, "The behavior of LMS and NLMS algorithms in the presence of spherically invariant processes", *IEEE Trans. Acoust. Speech Signal Processing*, vol. 41, no. 3, pp. 1149-1160, Mar. 1993.