ME 599/699 Robot Modeling & Control
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Drive Trains

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Permanent Magnet DC motors

We consider permanent magnet DC motors. Other kinds include *AC* motors and brushless *DC* motors.

A model for the motor torque is $\tau = K_m i_a$, if the flux in the motor is constant. The current is generated by a voltage source, and has dynamics

$$L \frac{d}{dt} i_a + R i_a = V - V_b, \tag{1}$$

where L is the motor inductance, R is the winding resistance, V_b is the back EMF and is proportional to ω_m, the motor speed.

PMDC Model

The dynamics governing θ_m are

$$
\begin{aligned}
J_m\ddot{\theta}_m + B_m\dot{\theta}_m &= \tau_m - \tau_l/r & (2)\\
&= K_m i_a - \tau_l/r & (3)
\end{aligned}
$$

We can rewrite (1) and (5) as

$$
\begin{aligned}
(Ls+R)I_a(s) &= V(s) - K_b\ s\Theta_m(s), & (4)\\
(J_m s^2 + B_m s)\Theta_m(s) &= K_m I_a(s) - \tau_l(s)/r & (5)
\end{aligned}
$$

We combine these equations to obtain

$$
s\left((Ls+R)(J_m s + B_m) + K_b K_m\right)\Theta_m(s) = K_m V(s) - \frac{(Ls+R)}{r}\tau_l(s). \tag{6}
$$

Fast Electrical Dynamics

The electrical time constant L/R is much smaller than the mechanical time constant J_m/B_m. So, we can divide by R and set L/R to zero, obtaining.

$$s\left((J_m s + B_m) + \frac{K_b K_m}{R}\right)\Theta_m(s) = \frac{K_m}{R}V(s) - \frac{1}{r}\tau_l(s).$$

Setting $u \leftarrow K_m V/R$, and $B \leftarrow B_m + K_b K_m/R$, we obtain the motor equation as

$$J\ddot{\theta}_m + B\dot{\theta}_m = u(t) - \frac{1}{r}\tau_l. \tag{7}$$

Combined Link-Actuator Model

Let's combine the motor m with the link l

$$J_m\ddot{\theta}_m + B\dot{\theta}_m = u(t) - \frac{1}{r}\tau_l \tag{8}$$

$$J_l\ddot{\theta}_l + B_l\dot{\theta}_l = \tau_l \tag{9}$$

Due to the gears, $\dot{\theta}_m = r\dot{\theta}_l$. We eliminate θ_l to obtain

$$\left(J_m r^2 + J_l\right)\ddot{\theta}_l + \left(Br^2 + B_l\right)\dot{\theta}_l = ru \tag{10}$$

Main takeaway: When gear ratio r is large, then link inertia becomes negligible compared to the motor's inertia.

Full Euler-Lagrangian Model

$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \tau + \tau_{friction} + \tau_e,$$

becomes

$$\underbrace{M(q)}_{+\text{motor inertia}} \ddot{q} + C(q,\dot{q})\dot{q} + \underbrace{B\dot{q}}_{+\text{motor friction}} + G(q) = u + \tau_e,$$

where u denotes the input due to the voltage, whereas τ was the torque acting on the link joint.

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$$u_k = r_k \frac{K_{m_k}}{R_k} V_k,$$

where $\theta_{m_k} = r_k q_k$, and $M(q) = D(q) + J$, and J is diagonal with $r_k^2 J_{m_k}$ as k^{th} diagonal element.

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- Boston Dynamics' Atlas robot uses hydraulic actuation