ME 599/699 Robot Modeling & Control
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Feedforward and Feedback Control

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The task of motion control is to achieve that change

Ideal Behavior

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Ideally, $q_d(t) = q(t)$ at all times

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If we can apply large enough forces, we may assume that we can practically instantaneously set velocities. Eg: low inertia wheels.

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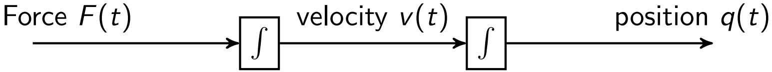

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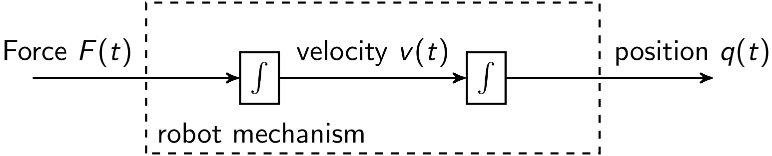

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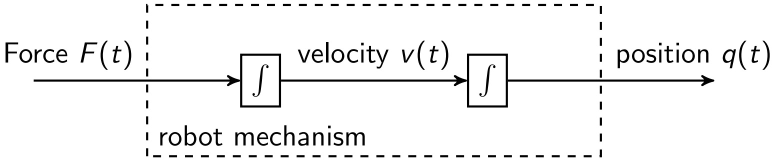

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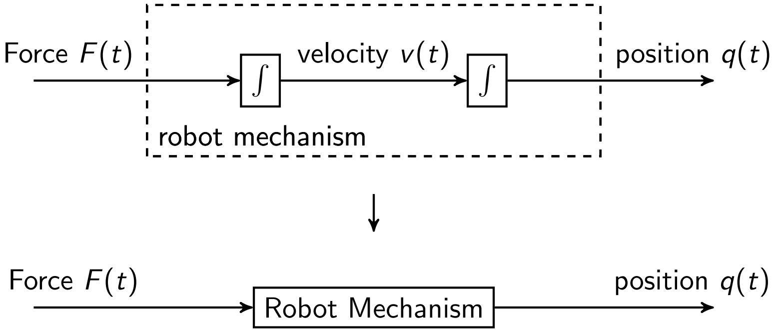

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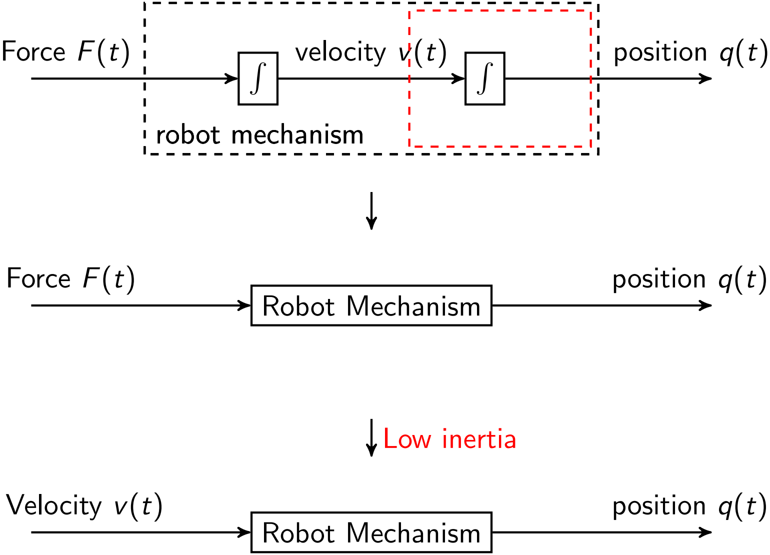

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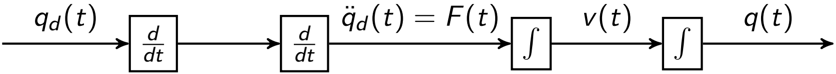

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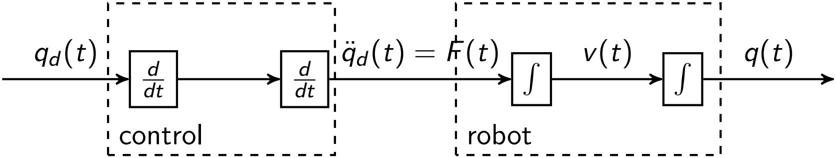

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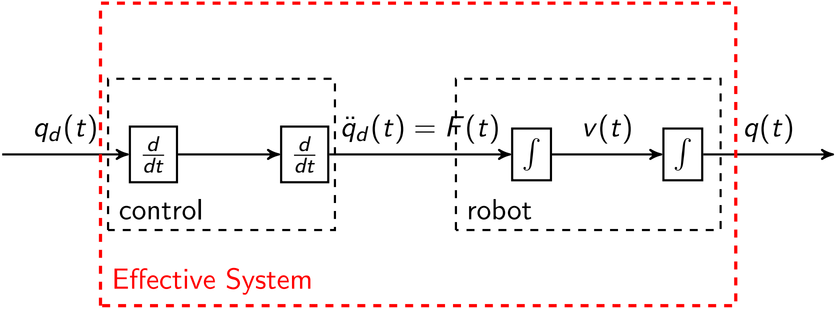
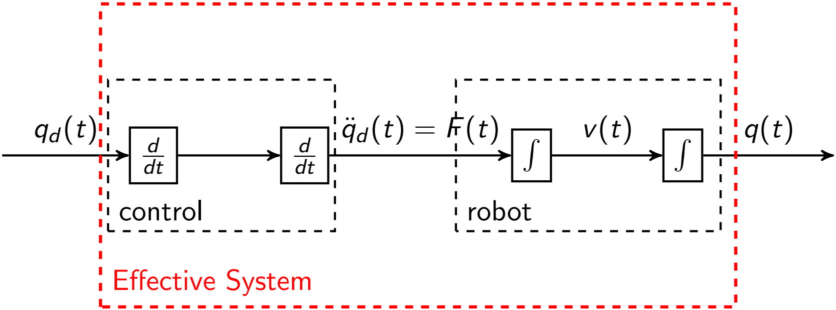


Model Inversion: Open-Loop



We've managed to make our robot react to desired position instantly, by

- Setting the initial condition
- Knowing the 'input' $q_d(t)$ perfectly

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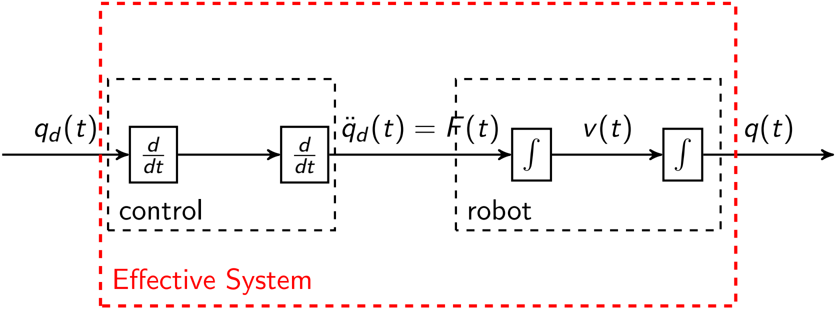


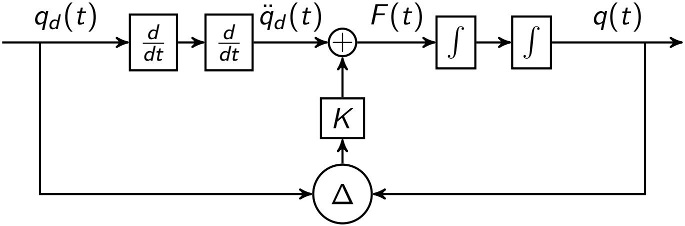
Issues:

- What if desired position is not known ahead of time?
- What if a disturbance force $f_d(t)$ acts, so that input is $F(t) + f_d(t) \neq \ddot{q}_d(t)$?

Closed-Loop Control

One way to account for these issues is to also react to errors in position:

$$F(t) = \ddot{q}_d(t) + K\left(q_d(t) - q(t)\right)$$



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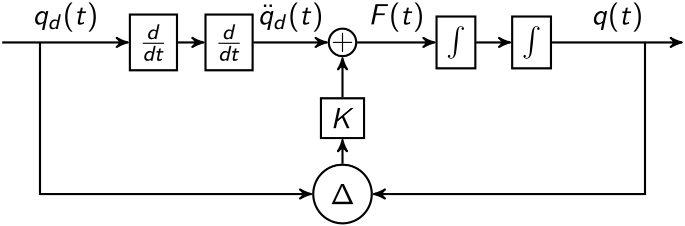

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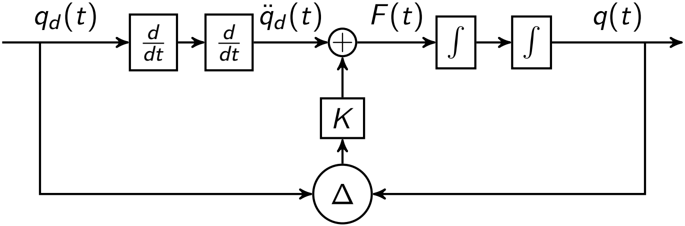

Issue: How do we choose K? Why will a good choice be possible?
Another issue: How do you know what $q(t)$ is?

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We then related force to change in $q(t)$ using an ordinary differential equation:

$$\frac{d^2}{dt^2}q(t) = F(t).$$

This ODE is a forward dynamics model: how the state and input affect the change in state.

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For point-mass:

$$\dot{x} = \begin{bmatrix} \dot{q} \\ F(t) \end{bmatrix} = f(x(t), F(t)).$$