1. LAWS OF COMPOSITION

DEFINITION 1. *Let E be a set. A mapping of $E\times E$ is called a law of composition on E. The value $f(x, y)$ of f for an ordered pair $(x, y) \in E \times E$ is called the composition of x and y under this law. A set with a law of composition is called a magma.*

The composition of x and y is usually denoted by writing x and y in a definite order and separating them by a characteristic symbol of the law in question (a symbol which it may be agreed to omit). Among the symbols most often used are $+$ and ., the usual convention being to omit the latter if desired; with these symbols the composition of x and y is written respectively as $x+y$, $x.y$ or xy. A law denoted by the symbol $+$ is usually called *addition* (the composition $x + y$ beind called the *sum* of x and y) and we say that it is *written additively*; a law denoted by the symbol . is usually called *multiplication* (the composition $x.y = xy$ being called the *product* for x and y) and we say that it is *written multiplicatively*.

In the general arguments of paragraphs 1 to 3 of this chapter we shall generally use the symbols $\top$ and $\perp$ to denote arbitrary laws of composition.

By an abuse of language, a mapping of a *subset* of $E\times E$ into E is sometimes called a law of composition *not everywhere defined* on E.

Examples. (1) The mappings $(X, Y) \mapsto X \cup Y$ and $(X, Y) \mapsto X \cap Y$ are laws of composition on the set of subsets of a set E.

(2) On the set $\mathbf{N}$ of natural numbers, addition, multiplication, and exponentiation are laws of composition (the compositions of $x \in \mathbf{N}$ and $y \in \mathbf{N}$ under these laws being denoted respectively by $x + y$, xy, or $x.y$ and x^y) (*Set Theory*, III, ¶3, no. 4).

(3) Let E be a set; the mapping $(X, Y) \mapsto X \circ Y$ is a law of composition on the set of subsets of $E \times E$ (*Set Theory*, II, ¶3, no. 3, Definition 6); the mapping $(f, g) \mapsto f \circ g$ is a law of composition on the set of mappings from E into E (*Set Theory*, II, ¶5, no. 2).