

1) inserting n elements using

a) aggregate method

The table doubles its size when it runs out of space.

so if the original size is 1 then after insertion it doubles its size to 2 after 2 more insertions it double to size 4 etc

→ In general after k doublings the size is 2^k

⇒ pseudocode:

initialize table with capacity 1

for $i = 1$ to n :

if table is full

new table = create new table with size $2^{\text{current size}}$

then copy elements from old table to new table

table = newTable

insert element i into table

let, $k = \log(n+1) - 1$

Total cost = $O(n) * k$

= $O(n \log n)$

cost per insertion = $O(\log n)$

Runtime per insertion is $O(\log n)$

Total time is $O(n) * \log(n+1)$

b) amortizing method

charge 2 units for each insertion
when the table doubles in size from m to $2m$
credit m units.

the credit exactly pay for the copy cost of $O(m)$
total credit is $m + 2m + 4m + \dots$

$$n/2 * m = O(n)$$

→ Pseudo code

initialize table with capacity = 1

for $i = 1$ to n

if table is full

new table = create new table

with size 2^* current size.

copy elements from old
table to new table

table = new table

insert element i into table

initialize charges = 0

initialize credits = 0

for $i = 1$ to n

charges $++ 2$

if table doubled in size

from m to $2m$

credits $+= m$

$$\text{total changes} = 2^* n = O(n)$$

$$\text{Total credits} = m + 2m + \dots$$

$$n/2^* m = O(n)$$

$$\text{cost per insertion} = \text{Total} / n$$

$$= O(n) / n$$

$$= O(1)$$

Runtime per insertion

$$= O(1)$$

$$\text{Total} = O(n)$$

time