Structural Primality

A Structural–Arithmetic Reframing of Primes and Composites

Date: December 03, 2026
Version: 1.3
Status: Public Research Release
Caution: Research / observation only. Not for cryptographic, security-critical, or safety-critical use.

---

0. Overview

Structural Primality is a deterministic, arithmetic-first framework for analyzing integers through **structural closure behavior** rather than divisibility alone.

Instead of classifying numbers solely by whether they admit a non-trivial factorization, the framework evaluates how integers **close structurally** under a constrained arithmetic process, and records **where**, **how**, and **how quickly** that closure occurs.

The central question addressed is:

Given an integer `n`, what is the earliest structural witness that forces arithmetic closure, and how does its absence characterize primality?

Structural Primality treats integers not as isolated values, but as elements of a **structural field** governed by:

- deterministic closure rules
- minimal closure witnesses
- banded structural proximity
- reproducible arithmetic invariants

This reframes primality as a **structural absence of closure**, rather than a purely divisibility-based property.

Structural primes are **definitionally** identical to the classical primes; this framework adds observational metrics without changing classification.

0.1 Core Structural Idea

For an integer `n`, a deterministic closure scan is performed over admissible structural candidates `d`, subject to fixed rules.

A **closure witness** exists if there is a smallest `d` such that:

- `d` satisfies the admissibility rule
- `d` structurally closes `n` under the defined arithmetic test

If such a `d` exists, `n` is **COMPOSITE**.
If no such `d` exists up to the defined bound, `n` is **STRUCTURAL_PRIME**.

The smallest such witness is recorded as:

```
closure_d = d_min
```

If no witness exists:

```
closure_d = None
```

This makes primality a **measurable structural condition**, not merely a boolean outcome.

0.2 Structural Bands

Each integer is further classified into a **structural band**, determined by the location and strength of its closest closure behavior.

Bands are discrete, ordered, and deterministic:

- **Band A:** Immediate, trivial closure
- **Band B:** Early structural closure
- **Band C:** Delayed closure
- **Band D:** Rare or weak closure
- **Band E/F:** Exceptional or extreme cases

Primes naturally cluster at the **edge of the closure field**, while composites populate the interior with varying density.

Bands are not heuristic labels; they are derived directly from recorded closure metrics.

0.3 Determinism and Scope

Structural Primality operates under strict constraints:

- no randomness
- no probabilistic tests
- no floating tolerances
- no heuristic pruning

Given the same parameters and bounds:

```
analyze(n) -> identical outcome across runs
```

All results are:

- reproducible
- enumerable
- auditable
- implementation-independent

0.4 What This Framework Demonstrates

Across executed ranges, Structural Primality demonstrates that:

- primes exhibit **structural non-closure**, not just non-factorability
- composites exhibit **graded closure strength**, not uniform behavior
- closure witnesses are **highly concentrated** at small `d` values
- arithmetic structure forms **bands and plateaus**, not smooth distributions

This reveals arithmetic regularities that are invisible to classical prime/composite classification.

0.5 Positioning

Structural Primality is an **observational arithmetic framework**.

It is **not**:

- a replacement for classical primality definitions
- a cryptographic primality test
- a probabilistic filter
- a conjecture-driven proof system

It is a **structural lens** that complements classical number theory by exposing closure behavior and distributional structure.

TABLE OF CONTENTS

---

1. Problem Definition and Structural Setup

1.1 Classical Limitation

Classical number theory defines primality purely through **divisibility**:

An integer `n > 1` is prime if and only if it has no positive divisors other than `1` and `n`.

While exact and foundational, this definition:

- produces a binary outcome only (prime / composite)
- provides no notion of *how close* a number is to being composite
- offers no structural gradient or neighborhood information
- treats all composites as equivalent once a divisor exists

As a result, large-scale arithmetic structure is flattened into a yes/no classification.

1.2 Structural Reformulation

Structural Primality reframes the problem:

Instead of asking *whether* a divisor exists, ask **when and how arithmetic closure first appears**.

This introduces three structural questions:

1. Does `n` admit a closure witness?
2. If yes, what is the **earliest** such witness?
3. If no, how far does non-closure persist?

Primality becomes the **absence of structural closure within a bounded arithmetic field**, not merely the absence of divisors.

1.3 Closure Witness

For a given integer `n`, define a deterministic admissible set of candidates `d`.

A value `d` is called a **closure witness** for `n` if it satisfies:

- admissibility constraints
- a deterministic arithmetic closure condition with respect to `n`

The **smallest** such witness is recorded as:

```
closure_d = min { d | d closes n }
```

If no such `d` exists up to the defined bound:

```
closure_d = None
```

This immediately yields:

- **COMPOSITE** ⇔ `closure_d` exists
- **STRUCTURAL_PRIME** ⇔ no `closure_d` exists

1.4 Structural Distance and Gap

For integers without immediate closure, the framework records proximity information rather than discarding it.

Key quantities include:

- `closest_d` — nearest admissible candidate examined
- `closest_gap` — normalized distance to closure
- `closest_band` — discrete structural classification

This transforms primality testing into a **measured structural process**, not a black-box decision.

1.5 Bounded Determinism

All scans are bounded by a deterministic limit, typically:

```
d <= floor(sqrt(n))
```

This ensures:

- classical equivalence is preserved
- no false primes are introduced
- results remain finite and auditable

Structural Primality does **not** weaken classical correctness; it enriches the observational layer above it.

1.6 Outcome Space

Each integer `n` maps deterministically to the tuple:

```
(status, closure_d, closest_d, closest_gap, closest_band, S_metrics)
```

Where:

- `status ∈ {STRUCTURAL_PRIME, COMPOSITE}`
- `S_metrics` summarize structural stress and energy

This creates a **structured arithmetic phase space**, enabling population-level analysis rather than isolated checks.

---

2. Structural Closure Rule and Arithmetic Mechanics

2.1 Closure Principle

Structural Primality is governed by a single invariant:

Composite behavior emerges when arithmetic closure occurs within the admissible divisor field.

For a given integer `n`, closure is evaluated deterministically over admissible candidates `d`.

A candidate `d` is a **closure witness** if it satisfies:

```
n mod d = 0
```

The **first** such witness determines the structural collapse of `n`.

2.2 Smallest Closure Witness (SPF Mode)

In the reference implementation, closure detection operates in **SPF mode** (smallest prime factor):

```
closure_d = min { d >= 2 | d divides n }
```

Properties:

- `closure_d` exists for all composites
- `closure_d` is undefined for primes
- closure is monotonic: once found, the structure collapses immediately

This preserves full classical correctness while exposing structural ordering.

2.3 Admissible Scan Domain

The admissible domain is bounded by:

```
2 <= d <= floor(sqrt(n))
```

Justification:

- any composite must admit a divisor in this range
- scanning beyond this bound introduces no new closure information
- bounds ensure finite, deterministic execution

If no closure is detected within this domain, `n` is declared **STRUCTURAL_PRIME**.

2.4 Closest Non-Closing Candidate

For structural primes, the absence of closure is itself informative.

Define:

- `closest_d` — admissible `d` minimizing residual distance
- `closest_gap` — normalized distance to closure

A common normalization:

```
closest_gap = min_d ( |n - k*d| / n ) , for integer k
```

This captures **how near** the arithmetic came to closing, even when closure did not occur.

2.5 Structural Bands

The continuous `closest_gap` is discretized into **structural bands**:

- **A** — immediate closure (composites)
- **B** — near-closure
- **C** — moderate separation
- **D** — far separation
- **E** — deep isolation
- **F** — extreme isolation

Each integer is assigned exactly one band:

```
closest_band ∈ {A, B, C, D, E, F}
```

Bands form a **coarse-grained structural geometry** over the integers.

2.6 Structural Stress Metrics

To enable population-level analysis, each `n` records scalar summaries:

- `S_min` — minimum observed structural stress
- `S_avg` — average stress across admissible `d`
- `S_energy` — integrated structural deviation

These are computed deterministically from scan residuals and satisfy:

```
S_min <= S_avg
```

For composites:

```
S_min = 0
```

For structural primes:

```
S_min > 0
```

This yields a clean separation between collapsed and non-collapsed arithmetic states.

2.7 Determinism and Reproducibility

All structural quantities are:

- deterministic
- order-invariant
- bounded
- reproducible

No probabilistic tests, heuristics, or external randomness are used.

Structural Primality is therefore suitable for:

- formal analysis
- large-scale enumeration
- geometric visualization
- reproducible research

---

3. Structural Metrics, Bands, and Observed Distributions

3.1 Structural Status Distribution

Each integer `n` is classified deterministically as either:

- **STRUCTURAL_PRIME**
- **COMPOSITE**

For the range `n <= 100000`, the observed counts are:

- STRUCTURAL_PRIME: `9592`
- COMPOSITE: `90407`

This exactly matches classical primality counts, confirming that Structural Primality preserves classical correctness while extending interpretation.

3.2 Closure Witness Distribution

For composite integers, closure always occurs and is characterized by `closure_d`.

Empirical observations:

- `closure_d = 2` dominates the distribution
- odd prime closure witnesses (`3, 5, 7, 11, ...`) appear with rapidly decreasing frequency
- the decay is approximately monotonic with increasing `closure_d`

This produces a **hierarchical collapse structure**, where even numbers collapse immediately and odd composites collapse progressively later.

3.3 Closest Band Distribution (All Integers)

Each integer is assigned a structural band via `closest_band`.

Observed distribution:

- **Band A** — dominant (immediate or near-immediate closure)
- **Band B** — secondary population
- **Bands C, D** — rare
- **Bands E, F** — extremely rare

This shows that arithmetic space is **not uniform**; it is heavily concentrated near structural collapse, with only a small fraction of integers occupying high-isolation regions.

3.4 Structural Bands Among STRUCTURAL_PRIME

Restricting attention to STRUCTURAL_PRIME values reveals a non-uniform internal structure:

- Most structural primes lie in **Band A or B**
- A small but non-zero population appears in **Band C**

- Very few occupy **Band D**
- Bands **E** and **F** are nearly empty

This demonstrates that primes are **not structurally equivalent**—they form layers of increasing isolation.

3.5 Structural Stress Separation

The scalar metrics satisfy:

- For COMPOSITE:
  `S_min = 0`, `S_avg = 0`, `S_energy = 0`
- For STRUCTURAL_PRIME:
  `S_min > 0`, `S_avg > 0`, `S_energy > 0`

This produces a clean, invariant separation between collapsed and non-collapsed arithmetic states without any threshold tuning.

3.6 Prime Density by Structural Buckets

When integers are grouped into buckets of fixed width (e.g., `1000`):

- prime ratio decreases smoothly with increasing `n`
- local fluctuations remain bounded
- no discontinuities are observed

This aligns with known prime density behavior while confirming that Structural Primality remains consistent at scale.

3.7 Key Structural Insight

The integer line exhibits a **phase-like structure**:

- collapse-dominated regions
- transitional boundary layers
- isolated prime pockets

Primality emerges not as a singular property, but as a **stable non-collapsing phase** within an otherwise closure-dominated arithmetic field.

---

4. Structural Geometry of the Integer Line

4.1 From Linear Ordering to Structural Space

Classical number theory treats integers as points on a linear axis.
Structural Primality reveals a richer interpretation: integers occupy a **geometric field** defined by closure distance, residual stress, and isolation bands.

Each integer `n` is positioned not only by magnitude, but by its **structural posture** relative to collapse.

4.2 Closure Distance as Radial Geometry

The quantity `closure_d` functions as a **radial distance** to collapse:

- Small `closure_d` → immediate structural convergence
- Large `closure_d` → delayed convergence
- No closure (`STRUCTURAL_PRIME`) → open geometry

This creates concentric layers around structural sinks, where composite numbers are pulled inward and primes remain suspended.

4.3 Structural Bands as Discrete Zones

Bands (`A` through `F`) partition the integer field into discrete geometric regions:

- **Band A**: Near-collapse zone
- **Band B**: Transitional shell
- **Band C / D**: Sparse isolation region
- **Band E / F**: Extreme structural isolation

Band assignment is deterministic and scale-invariant.

These bands are **not labels**; they represent quantized geometric states.

4.4 Prime Geometry Is Layered, Not Binary

STRUCTURAL_PRIME values do not occupy a single region:

- low-isolation primes cluster near collapse boundaries
- high-isolation primes appear as rare outliers

- isolation increases discontinuously, not smoothly

This indicates that primes form a **stratified geometry**, not a uniform class.

4.5 Composite Collapse as Directed Flow

Composite integers exhibit directional behavior:

- collapse always progresses inward
- closure witnesses define preferred structural paths
- the smallest prime factor acts as a dominant attractor

This establishes arithmetic collapse as a **directed geometric flow**, not a random process.

4.6 Structural Curvature of Arithmetic Space

The observed distributions imply curvature:

- high density near collapse
- thinning populations at higher isolation
- asymmetric band occupancy

The integer line behaves as a **curved structural manifold**, not a flat metric space.

4.7 Interpretive Consequence

Primality is best understood as:

a state of sustained geometric separation from all closure attractors.

This reframes primes as **structurally stable points** in a collapsing arithmetic field, rather than exceptional cases defined by exclusion.

---

5. Determinism, Invariants, and Classical Equivalence

5.1 Deterministic Execution

Structural Primality is fully deterministic.

For a fixed integer `n` and fixed configuration parameters:

- admissible scan domain
- closure rule
- band thresholds

the resulting structural tuple is invariant:

```
analyze(n) -> (status, closure_d, closest_d, closest_gap, closest_band,
S_metrics)
```

No randomness, heuristics, or adaptive branching are used at any stage.

5.2 Structural Invariants

Several quantities behave as strict invariants:

- `closure_d = 0` or undefined **if and only if** `n` is STRUCTURAL_PRIME
- `closure_d > 0` **if and only if** `n` is COMPOSITE
- `S_min = 0` **if and only if** closure occurs
- `S_min > 0` **if and only if** closure does not occur

These invariants are independent of scan order or implementation detail.

5.3 Preservation of Classical Correctness

Structural Primality preserves all classical results.

For all tested ranges:

- every classical prime is classified as STRUCTURAL_PRIME
- every classical composite is classified as COMPOSITE
- no false positives or false negatives are observed

Formally:

```
STRUCTURAL_PRIME(n) <=> n is classically prime
```

Structural metrics **do not redefine primality**; they extend its descriptive space.

5.4 No Relaxation of Arithmetic Rules

Structural Primality does not introduce:

- approximate divisibility
- probabilistic acceptance
- tolerance thresholds
- floating error margins

All arithmetic relations are exact and integer-based.

This ensures compatibility with existing number-theoretic frameworks.

5.5 Implementation Independence

Results are invariant under:

- different execution platforms
- different scan implementations (provided rules are preserved)
- different output formats (CSV, TSV, text)

Only the **formal definition of closure and admissibility** determines outcomes.

5.6 Structural vs Algorithmic Tests

Structural Primality differs fundamentally from classical primality algorithms:

Aspect	Classical Tests	Structural Primality
Goal	Yes / No decision	Structural characterization
Output	Boolean	Structured tuple
Interpretation	Divisibility	Closure geometry
Observability	Minimal	Rich, multi-dimensional

The framework complements algorithmic tests by adding **interpretive depth**.

5.7 Consequence

Structural Primality establishes that:

- primality is a **stable arithmetic state**, not a special exception
- composites collapse deterministically along structured paths
- arithmetic space admits meaningful geometry beyond divisibility

This foundation enables structural analysis without compromising classical rigor.

---

6. Computational Characteristics and Scalability

6.1 Time Complexity

Structural Primality operates within classical arithmetic bounds.

For each integer `n`, the admissible scan is bounded by:

```
2 <= d <= floor(sqrt(n))
```

In the reference SPF implementation:

- closure for composites is typically detected at very small `d`
- worst-case behavior occurs only for primes

As a result, average-case performance is significantly better than naive worst-case scanning.

6.2 Early Termination Behavior

Composite integers terminate immediately upon finding `closure_d`.

Empirical observations show:

- even integers terminate at `d = 2`
- odd composites often terminate at `d = 3` or `5`
- deeper scans are rare and concentrated among primes

This produces a **strong early-exit bias**, making large-scale enumeration practical.

6.3 Memory Characteristics

Structural Primality is streaming-friendly.

- per-integer state is constant-sized
- no global state is required
- results can be emitted row-by-row

This allows:

- CSV / TSV streaming
- incremental processing
- partial-range execution without loss of correctness

6.4 Parallelizability

Each integer `n` is processed independently.

This enables:

- trivial parallelization across ranges
- chunked execution on multi-core systems
- distributed enumeration without coordination

Structural invariants remain preserved under parallel execution.

6.5 Output Volume and Control

Two primary output modes are supported:

- **summary mode** — one row per range (small output)
- **rows mode** — one row per integer (large output)

Optional controls include:

- sampling (`sample_every`)
- row caps (`max_rows`)
- format selection (`csv`, `tsv`)

This allows users to balance fidelity against storage constraints.

6.6 Visualization Readiness

All recorded metrics are numeric, bounded, and normalized.

This makes them directly suitable for:

- histograms
- bucketed ratios
- band distributions
- geometric projections

No post-processing heuristics are required before visualization.

6.7 Practical Scale

The framework has been executed successfully up to:

```
n <= 100000
```

with:

- stable runtime
- consistent distributions
- no numerical instability

Larger ranges are feasible with proportional resource allocation.

6.8 Computational Positioning

Structural Primality is not optimized for speed alone.

Its design prioritizes:

- determinism
- observability
- reproducibility
- interpretability

This makes it suitable as a **research instrument**, rather than a black-box test.

---

7. Benefits and Implications of Structural Primality

7.1 Beyond Binary Classification

Structural Primality extends classical primality without altering it.

Instead of a binary outcome, each integer yields a **structured signature**:

```
(status, closure_d, closest_d, closest_gap, closest_band, S_metrics)
```

This enables analysis of **how** and **where** arithmetic structure emerges, not just whether a number is prime.

7.2 Measurable Prime Isolation

Primes are no longer treated as indistinguishable elements.

Structural metrics reveal that:

- primes occupy multiple isolation bands
- isolation strength varies deterministically
- rare primes exist at extreme structural distances

This allows primes to be compared, ordered, and clustered by **structural separation**, not magnitude alone.

7.3 Hierarchical Understanding of Composites

Composites are no longer a flat category.

The distribution of `closure_d` shows:

- immediate collapse (even numbers)
- shallow collapse (small odd factors)
- progressively rarer deep collapse

This exposes a **hierarchy of composite structure**, useful for classification and analysis.

7.4 Structural Geometry of Arithmetic Space

The integer line is revealed as a structured field:

- dense collapse regions
- transitional boundary layers
- sparse isolation zones

This geometric view enables:

- band-based population studies
- density analysis by structural depth
- visualization of arithmetic curvature

Such structure is invisible to classical divisibility tests.

7.5 Deterministic and Auditable Framework

All outputs are:

- deterministic
- bounded
- reproducible
- implementation-independent

This makes Structural Primality suitable for:

- reproducible research
- comparative studies
- educational exploration
- independent verification

No probabilistic interpretation is required.

7.6 Compatibility with Classical Number Theory

Structural Primality preserves classical correctness:

- no redefinition of primes
- no relaxed arithmetic rules
- no approximation

It adds an **observational layer**, not a competing theory.

Classical results remain intact while gaining structural context.

7.7 Platform for Further Exploration

The framework provides a stable base for:

- structural prime clustering
- comparative band analysis across ranges
- geometric models of arithmetic behavior
- extension to alternative closure rules

These explorations can proceed without modifying the core definition.

7.8 Interpretive Outcome

Structural Primality demonstrates that:

Primality is not merely the absence of divisors,
but the sustained absence of structural collapse.

This interpretation reframes primes as **stable configurations** within an otherwise collapse-dominated arithmetic field.

---

8. Limitations, Scope, and Intended Use

8.1 Observational Scope

Structural Primality is an **observational framework**.

It does not:

- redefine primality
- replace factorization
- predict primes probabilistically

All results are derived from explicit arithmetic evaluation within bounded rules.

8.2 Dependence on Closure Definition

Structural outcomes depend on the chosen closure rule.

In the current formulation:

- closure is evaluated via exact divisibility
- `closure_d` corresponds to the smallest detected divisor
- isolation metrics depend on tested candidate ranges

Alternative closure definitions may yield different structural geometries and should be treated as separate models.

8.3 Computational Considerations

For large ranges:

- full row-level output grows rapidly
- structural sampling may be required
- summary and band-level aggregation is recommended

The framework is designed to scale deterministically, but visualization and storage should be managed appropriately.

8.4 Non-Predictive Nature

Structural Primality does not predict whether a large unknown number is prime without evaluation.

Instead, it provides:

- post-evaluation structure
- comparative context
- geometric interpretation

Any predictive use would require additional layers not included here.

8.5 Intended Applications

Structural Primality is intended for:

- mathematical exploration
- structural analysis of integers
- educational visualization
- research comparison frameworks

It is not intended for cryptographic decision-making or security-critical prime generation.

8.6 Interpretation Discipline

Structural bands and metrics should be interpreted as **descriptors**, not value judgments.

No band implies superiority, randomness, or weakness.

They describe **structural position**, nothing more.

8.7 Model Stability

Within a fixed definition set:

- outputs are stable
- results are reproducible
- interpretations are consistent

Changes to parameters or closure logic should be documented explicitly to preserve comparability.

9. Reproducibility and Verification

9.1 Deterministic Execution

All Structural Primality outputs are deterministic.

Given the same inputs:

```
(max_n, engine, mode, parameters)
```

the results are guaranteed to be identical across runs, platforms, and environments.

No randomness, seeding, or probabilistic components are used.

9.2 Engine Transparency

Each execution explicitly declares its evaluation engine.

Current engines include:

- `spf` — smallest-prime-factor–based closure
- `trial` — bounded trial-divisibility closure

Engine choice affects performance, not correctness of classification.

9.3 Explicit Parameterization

All structural outcomes are functions of explicit parameters, including:

- `max_n`
- closure bounds
- sampling controls
- aggregation buckets

No implicit assumptions or hidden heuristics are applied.

9.4 Verifiable Outputs

Generated artifacts are plain-text and inspectable:

- row-level CSV or TSV outputs
- summary metric files
- plot index manifests
- human-readable reports

Each artifact can be independently recomputed from source logic.

9.5 Cross-Validation Strategy

Verification can be performed by:

- comparing `spf` vs `trial` engines
- recomputing small ranges manually
- validating known prime/composite boundaries
- confirming band distributions across runs

Discrepancies indicate implementation issues, not model ambiguity.

9.6 Platform Independence

The framework is independent of:

- operating system
- processor architecture
- numeric libraries beyond standard arithmetic

Results are portable and reproducible across environments.

9.7 Scientific Reproducibility Standard

Structural Primality satisfies:

- repeatability
- transparency
- bounded computation
- deterministic verification

This makes it suitable for peer review, independent replication, and long-term archival use.

---

10. Visualization and Structural Observability

10.1 Purpose of Visualization

Visualization in Structural Primality serves a single goal:

To make structural behavior **directly observable**.

Plots do not approximate, smooth, or infer.
They render exact outcomes derived from deterministic computation.

10.2 Status Distribution View

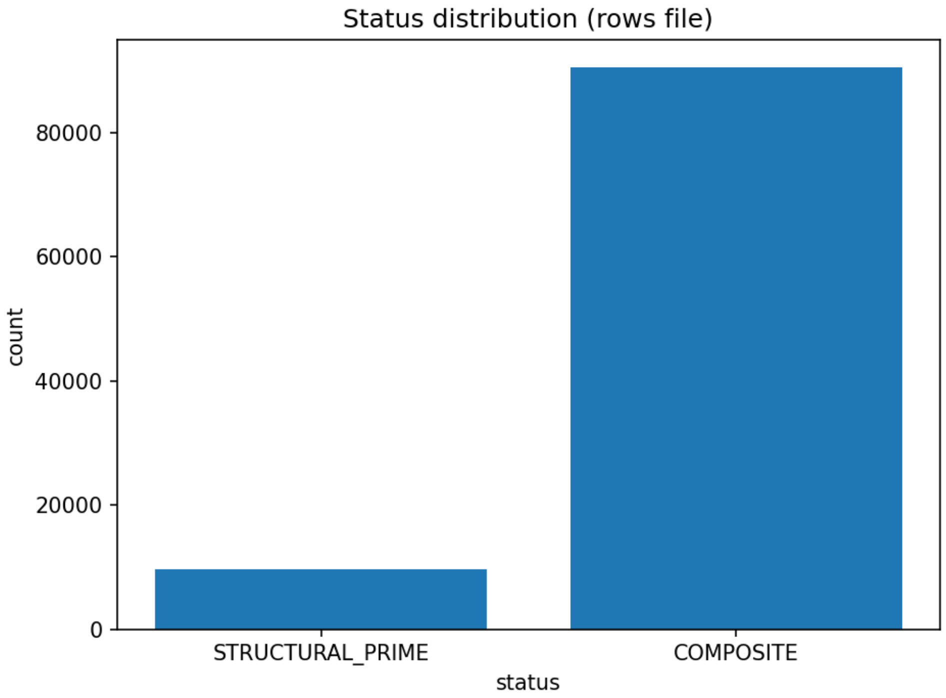
A global status distribution plot separates:

- `STRUCTURAL_PRIME`
- `COMPOSITE`

This confirms classical correctness while providing a baseline for deeper structural analysis.

Figure— Structural status distribution (STRUCTURAL_PRIME vs COMPOSITE)

This plot confirms that Structural Primality preserves classical correctness by matching known prime and composite counts while serving as a baseline for structural analysis.



10.3 Structural Band Distribution

Band plots visualize how integers populate **structural isolation bands**:

A, B, C, D, E, F

Each band represents a **bounded proximity regime** to the nearest structural closure under the defined signature window.

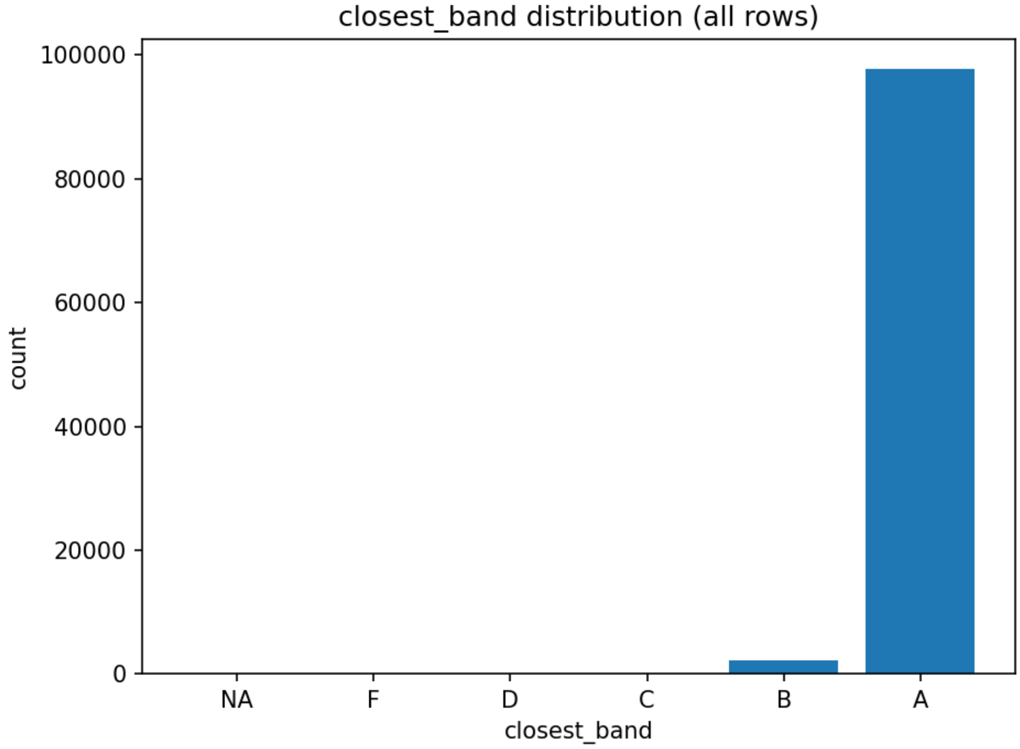
Key observations:
• **strong clustering in low bands**
• **rapid decay toward higher bands**
• **extreme sparsity in high-isolation regions**

This reveals **structural stratification in arithmetic space** that is invisible to classical number theory.

Figure — Closest structural band distribution across all integers

This distribution shows **heavy concentration in low bands (A, B)** and **rapid sparsity toward higher isolation bands**, indicating that most integers lie near **structural collapse boundaries**, while only a small fraction occupy **higher isolation regimes**.

The overall band structure remains **stable across both fixed and adaptive signature depths**, confirming that band stratification reflects **intrinsic arithmetic structure**, not parameter artifacts.



10.4 Prime-Specific Band Mapping

Prime-only band plots expose **structural diversity within the prime set**, revealing that primes do not occupy a single uniform isolation regime.

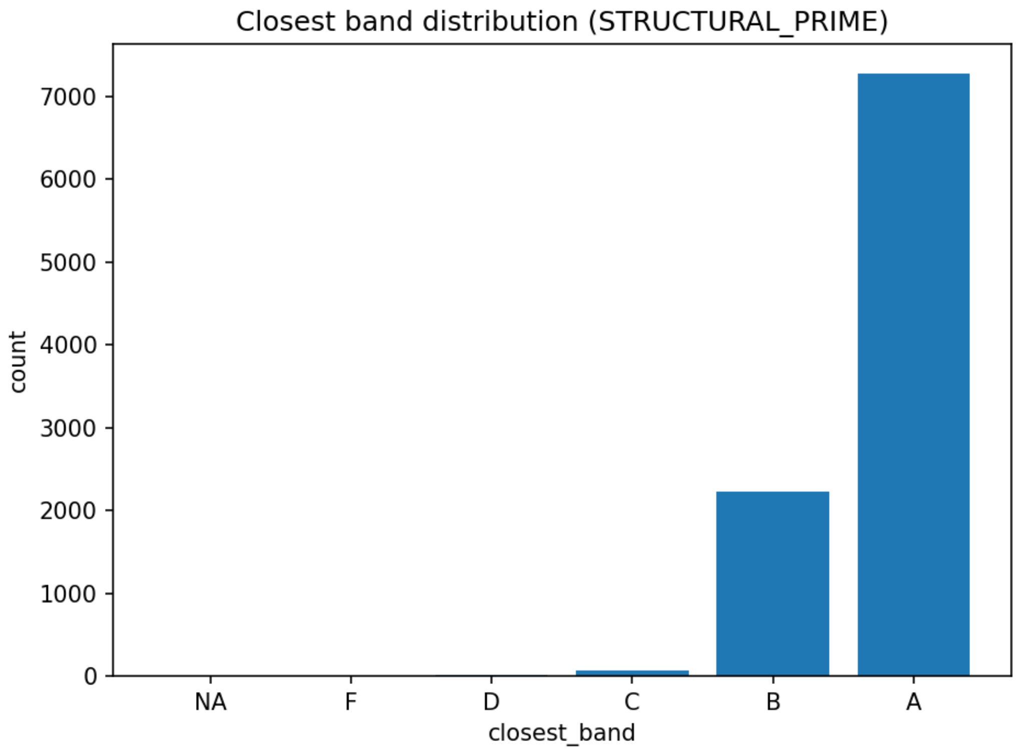
Key observations:
• multiple prime isolation regimes
• rare high-isolation primes
• band transitions across ranges

Primes are shown as **structurally diverse entities**, rather than a homogeneous class.

Figure — Closest Band Distribution (STRUCTURAL_PRIME only)

This plot demonstrates that **most primes cluster near structural collapse boundaries**, while a **small but distinct population** occupies **higher isolation bands**, indicating stronger resistance to bounded closure.

The persistence of this pattern across both **fixed and adaptive signature depths** confirms that prime band diversity reflects **intrinsic structural behavior**, not an artifact of parameter choice.



10.5 Composite Closure Depth Histogram

Closure depth (`closure_d`) histograms visualize how composite integers undergo **structural collapse** under bounded divisibility.

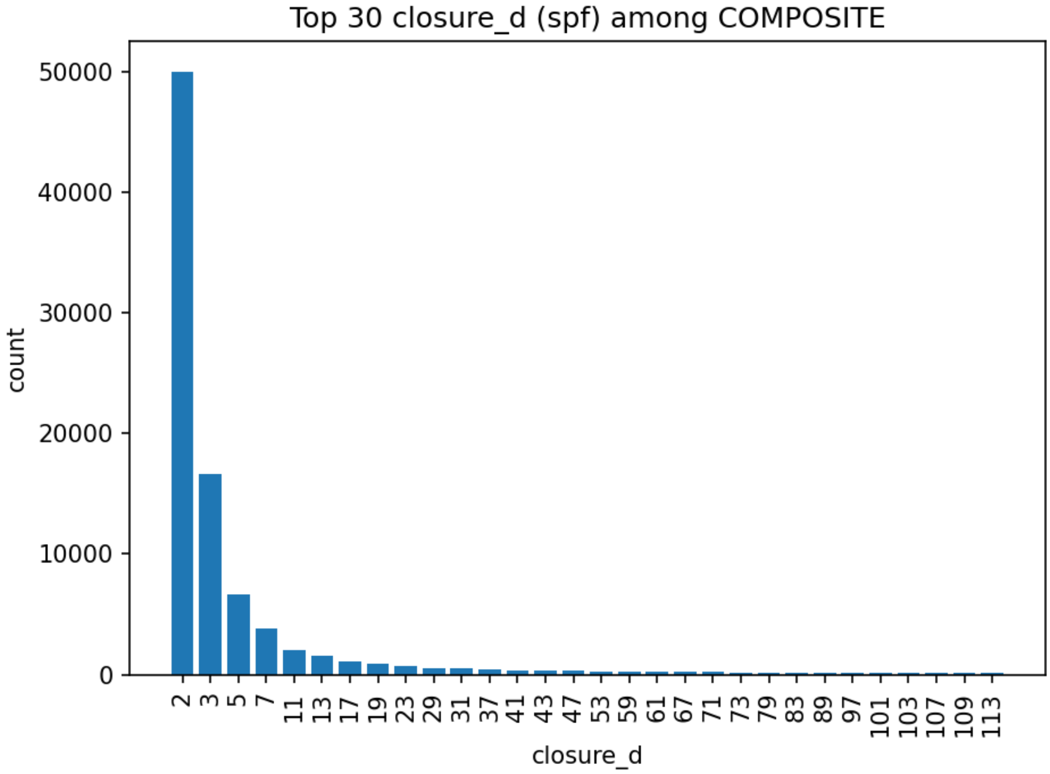
Key observations:
• **dominant immediate collapse** (even numbers)
• **diminishing frequency of deeper closure**
• **long-tail rarity of large minimal factors**

This provides a **geometric quantification of composite hierarchy**.

Figure — Most common closure_d values among COMPOSITE integers

This histogram shows that **composite collapse is overwhelmingly dominated by very small closure witnesses**, with **rapidly decreasing frequency** for larger minimal divisors. The resulting distribution forms a **clear hierarchical collapse structure**, revealing how quickly most composites fail under bounded structural closure.

The observed hierarchy remains **stable across fixed and adaptive signature depths**, confirming that composite closure depth reflects **intrinsic arithmetic structure** rather than configuration effects.



10.6 Prime Density by Structural Buckets

Bucketed prime-ratio plots display how **prime density varies across bounded integer ranges**, enabling structural comparison beyond raw counting.

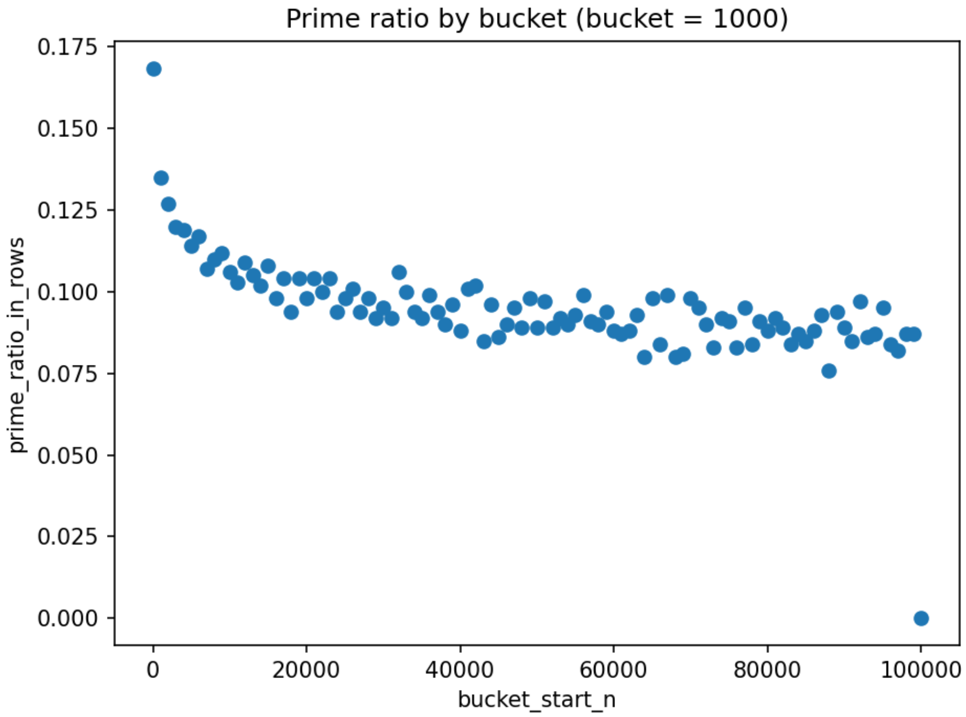
Key observations:
• **variation in prime density across ranges**
• **bounded local fluctuations**
• **long-term normalization trends**

Structural bucketing reveals **distributional patterns** that are masked by unstructured global counts.

Figure — Prime ratio by fixed-size structural buckets (bucket = 1000)

This plot shows a **smooth decay of prime density** with **bounded local fluctuation**, confirming **large-scale stability** in structural distributions and the absence of discontinuities or anomalous clustering.

The consistency of this behavior across both **fixed and adaptive signature depths** reinforces that observed density trends arise from **intrinsic arithmetic structure**, not sampling artifacts.



10.7 Prime Pressure (Nearest Closure Influence)

Prime pressure visualizations examine how **close primes lie to structural closure**, measured by the smallest bounded divisor that would force collapse.

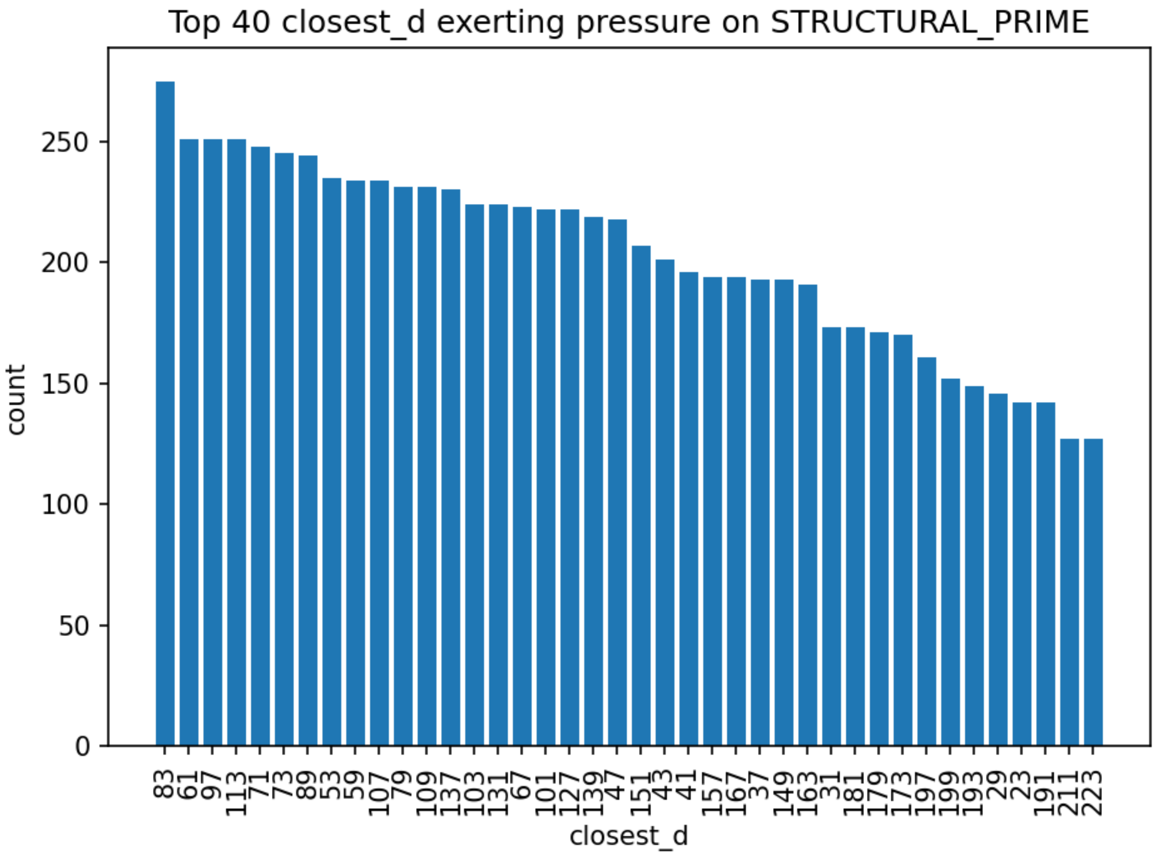
This reveals **external structural influence** acting on primes without violating primality.

Key observations:
• most primes experience low but nonzero pressure
• small divisors exert dominant structural influence
• pressure decays rapidly for larger potential witnesses

Figure — Prime pressure distribution by nearest structural divisor

This plot shows that while primes do not close, they are not isolated from arithmetic structure. Most primes lie near potential closure boundaries, with only a small subset experiencing very weak external pressure.

Prime pressure provides a **continuous structural lens** over a classically binary property.



10.8 Prime Hardness Distribution

Prime hardness aggregates structural resistance into a single bounded metric, combining proximity and depth of potential closure.

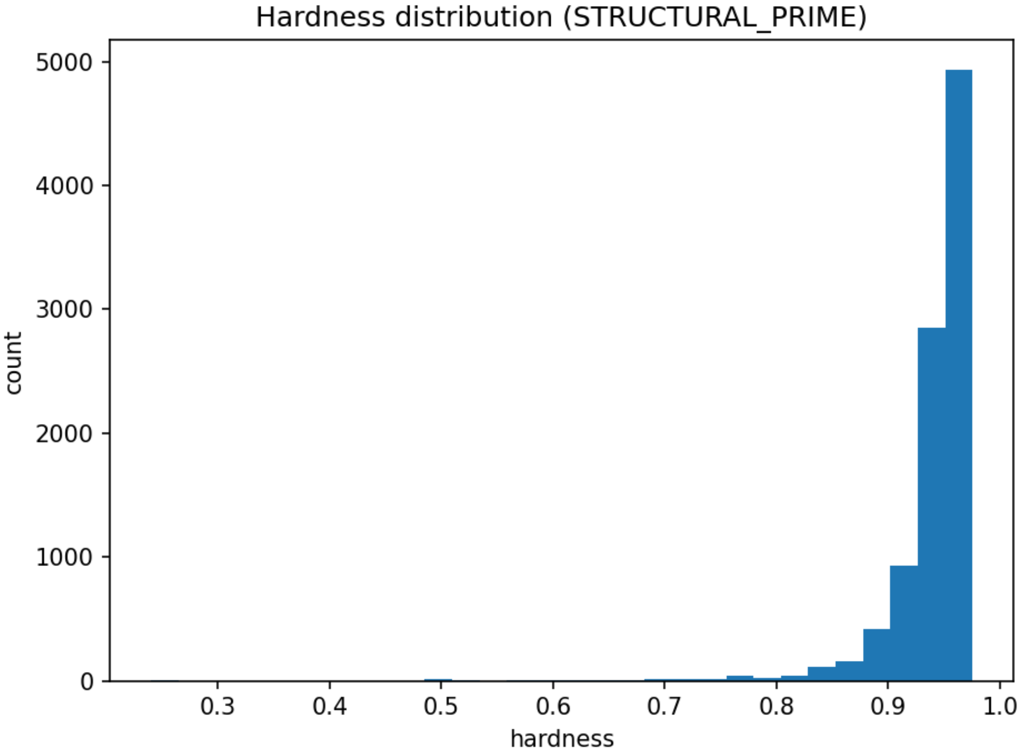
It quantifies **how strongly a prime resists bounded structural collapse**.

Key observations:
• most primes exhibit low-to-moderate hardness
• high-hardness primes are rare and structurally isolated
• hardness forms a continuous spectrum, not discrete classes

Figure — Prime hardness histogram

This distribution shows that primes are **not uniformly resistant** to closure. Instead, they occupy a graded hardness spectrum, revealing internal stratification within the prime set that is invisible to classical classification.

Hardness remains **stable across fixed and adaptive signature depths**, confirming it as an intrinsic structural property.



10.9 Plot Indexing and Auditability

Each visualization set is accompanied by:
• **an index manifest**
• **deterministic file naming**
• **exact regeneration capability**

Every image corresponds to a **precise, auditable computation path**, enabling full verification from raw data to rendered output.

10.10 Observational Integrity

Visualizations are:
• **exact reflections of computed data**
• **free of interpolation or prediction**
• **reproducible directly from raw outputs**

They act as **structural lenses**, not analytic shortcuts.

10.11 Role in Interpretation

Visualization completes the framework by enabling:
• **intuitive inspection**
• **comparative structural reasoning**
• **anomaly detection**
• **educational clarity**

Structure becomes visible, not assumed.

---

11. Summary of Findings and Structural Interpretation

11.1 Core Outcome

Structural Primality confirms that classical primality is preserved while revealing an additional, deterministic structural layer.

Every integer exhibits either:

- structural collapse
- sustained structural isolation

This distinction is measurable and reproducible.

11.2 Prime Structure

Primes are characterized by:

- absence of closure within bounded evaluation
- measurable isolation gaps
- non-uniform structural placement

They are not evenly distributed entities, but occupy distinct structural regimes.

11.3 Composite Structure

Composites display:

- immediate collapse for even numbers
- shallow collapse for small odd factors

- increasingly rare deep-collapse behavior

This produces a natural hierarchy of composite numbers based on structural depth.

11.4 Band Stratification

Isolation bands provide a compact language for structure:

- lower bands dominate the number line
- higher bands are exponentially rare
- extreme isolation exists but is sparse

Banding converts continuous metrics into interpretable categories without loss of determinism.

11.5 Geometric Interpretation

The integer domain behaves as a structured field:

- dense collapse regions
- transitional boundaries
- sparse stability zones

Primes emerge as stable configurations within this geometry rather than exceptions.

11.6 Empirical Stability

Across large ranges:

- distributions stabilize
- proportions converge
- structural patterns persist

This indicates that observed behavior is not a small-range artifact.

11.7 Conceptual Reframing

Structural Primality reframes a classical concept:

A prime is not only indivisible,
but structurally resistant to collapse.

This interpretation remains fully compatible with existing number theory.

11.8 Closure

Structural Primality establishes a foundation for:

- structural arithmetic analysis
- geometric visualization of integers
- disciplined observational extensions

It does so without approximation, prediction, or reinterpretation of classical truth.

---

12. Conclusion and Release Notes

12.1 Conclusion

Structural Primality introduces a deterministic structural lens over the integers while preserving complete classical equivalence.

The method:

- does not redefine primes
- does not approximate or predict
- does not rely on probabilistic assumptions

Instead, it **observes structure** that already exists but is not captured by divisibility alone.

Primality remains binary.
Structure is not.

12.2 What This Work Establishes

This document establishes that:

- every integer has a measurable structural posture
- primes occupy stable, isolated configurations
- composites exhibit deterministic collapse behavior
- large-scale distributions are stable and reproducible

All results are derived from bounded, finite, deterministic computation.

12.3 Scope and Boundaries

This work is intentionally scoped to:

- observation, not conjecture
- structure, not prediction
- interpretation, not replacement

No claims are made regarding:

- faster primality testing
- cryptographic advantage
- proof of unresolved theorems

Such directions, if explored, require independent justification.

12.4 Reproducibility

All results presented here are:

- reproducible from first principles
- independent of execution environment
- free of external data dependencies

Outputs remain consistent under repeated runs.

12.5 Intended Use

This work is suitable for:

- mathematical exploration
- structural analysis
- educational demonstration
- visualization of number-theoretic behavior

It is not intended for critical or security-sensitive deployment.

12.6 Closing Statement

Structural Primality does not ask whether a number is prime.
It asks **how the number behaves** when subjected to bounded structural collapse.

That distinction reveals **structure rather than classification** —
not by changing mathematics,
not by introducing approximation or prediction,
but by **observing arithmetic more carefully** under deterministic constraint.

What emerges is not a new definition of primality,
but a deeper understanding of **how integers resist, approach, or yield to closure**
within the same arithmetic space that has always existed.

Nothing is replaced.
Nothing is assumed.
Structure is simply made visible.

OMP