MTAT.03.015 Computer Graphics (Fall 2013)
Lecture V: Math exercises

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Solution for every task gives 0.5 points. Solutions are accepted on paper or via e-mail (`kt@ut.ee`) until October 16, 2013.

1. Let the horizontal field of view (*fov-X*) of some *view-frustum* be 75 degrees. Let the screen dimensions be 1280×1024. Find the corresponding vertical field of view (*fov-Y*).

2. Consider a perspective projection in two-dimensional space. We shall be projecting to the line $y = 1$ with $(0, 0)$ as the center of projection.

   - Find the projection matrix in homogeneous coordinates.
   - Explain what linear transformation does this matrix correspond to in the three-dimensional homogeneous space. Illustrations are welcome.

3. Let $ax + by + cz + d = 0$ be some plane in three-dimensional space and let $P = (p_x, p_y, p_z)$ be a point not located on this plane. Find a matrix, that performs a perspective projection from P onto this plane (in homogeneous coordinates).

4. Let $P_1 = (x_1, y_1, z_1)$, $P_2 = (x_2, y_2, z_2)$ – be points in space. Consider some attribute $\mathcal{A}$ (e.g. color) assigned to the points. Suppose that point P_1 is assigned attribute value a_1, point P_2 — value a_2 and on the line between them the attribute varies linearly.

   Let P_1^*, P_2^* — be the perspective projections of points P_1 and P_2 onto the plane $z = z_n$ with $(0, 0, 0)$ as the center of projection. Let P_t^* be a point obtained by interpolating between P_1^* and P_2^*:

   $$P_t^* = tP_1^* + (1 - t)P_2^*,$$

   and let $P_t = (x_t, y_t, z_t)$ be the point of the segment $[P_1, P_2]$ that projects into P_t^*. Show that the value a_t of the attribute at point P_t satisfies

   $$\frac{a_t}{z_t} = t\frac{a_1}{z_1} + (1 - t)\frac{a_2}{z_2}.$$

   Try to find a simple geometric proof to this fact.

It follows from this result, than when you are rasterizing a triangle, which was obtained via perspective projection, you cannot simply interpolate attribute values (e.g. colors or texture coordinates) along the screen as you did in the practice session[1].

[1] http://en.wikipedia.org/wiki/Texture_mapping#Perspective_correctness