Computer Graphics

Mathematical background

Part 0

Konstantin Tretyakov
kt@ut.ee

Vectors & matrices

$$\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{pmatrix} \qquad \mathbf{A} = \begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mn} \end{pmatrix}$$

Vectors & matrices

$$\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{pmatrix} \qquad \mathbf{A} = \begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mn} \end{pmatrix}$$

- Transpose: $\boldsymbol{x}^T,\ \boldsymbol{A}^T$
- Multiplication: $\boldsymbol{Ax},\ \boldsymbol{w}^T\boldsymbol{x}$
- Multiplication by a number: $\alpha\boldsymbol{x}$
- Addition: $\boldsymbol{A}+\boldsymbol{B},\ \boldsymbol{x}+\boldsymbol{y}$

Quiz

- $\begin{pmatrix} 2 & 3 & 4 \\ 1 & 1 & 2 \end{pmatrix}^T =$

Quiz

- $2 \cdot \begin{pmatrix} 3 \\ 4 \end{pmatrix} + \begin{pmatrix} 1 \\ -1 \end{pmatrix} =$

Quiz

- $(1 \quad 2)\begin{pmatrix}3\\4\end{pmatrix} =$

Quiz

- $\begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix} \begin{pmatrix} 3 \\ 4 \end{pmatrix} =$

Quiz

- $\begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix} \begin{pmatrix} 3 & 6 \\ 4 & 8 \end{pmatrix} =$

Quiz

- $\begin{pmatrix} 3 \\ 4 \end{pmatrix} (1 \quad 2) =$

Quiz

- $\binom{3}{4}\binom{1}{2} =$

Quiz

- A vector is a matrix: True or False?

Quiz

- 3-dimensional computer graphics primarily operates with 3-dimensional vectors: True or false?

Vectors & matrices

$$\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{pmatrix} \qquad \mathbf{A} = \begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mn} \end{pmatrix}$$

- Transpose: $\boldsymbol{x}^T,\ \boldsymbol{A}^T$
- Multiplication: $\boldsymbol{Ax},\ \boldsymbol{w}^T\boldsymbol{x}$
- Multiplication by a number: $\alpha\boldsymbol{x}$
- Addition: $\boldsymbol{A}+\boldsymbol{B},\ \boldsymbol{x}+\boldsymbol{y}$

Quiz

- Given $\boldsymbol{x}$ and $\boldsymbol{y}$, locate:
  - $2\boldsymbol{x}$
  - $\boldsymbol{x} + \boldsymbol{y}$
  - $0.5\boldsymbol{x} + 0.5\boldsymbol{y}$
  - $0.9\boldsymbol{x} + 0.1\boldsymbol{y}$
  - $1.1\boldsymbol{x} - 0.1\boldsymbol{y}$

Convex combination

A *convex combination* of vectors $\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{v}_3, \ldots, \boldsymbol{v}_n$ is a vector

$$\lambda_1 \boldsymbol{v}_1 + \lambda_2 \boldsymbol{v}_2 + \cdots + \lambda_n \boldsymbol{v}_n$$

where

$$\lambda_1 + \lambda_2 + \cdots + \lambda_n = 1,$$
$$\lambda_i \geq 0$$

Quiz

- Provide a mathematical expression, describing the set of points, lying on a line which passes through $\boldsymbol{p}$ and $\boldsymbol{q}$.

Quiz

- Provide a mathematical expression, describing the set of points, lying within a triangle ($\boldsymbol{p}$, $\boldsymbol{q}$, $\boldsymbol{r}$).

So far:

- Vectors, matrices
- Transposition
- Matrix addition & multiplication
- Convex combinations

- To be continued…