Computer Graphics

The Rendering Equation

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Outline

- Raycasting
- Raytracing
- Raymarching / Sphere tracing

- Rendering equation solvers
  - Radiosity, Path tracing, Photon mapping

Quiz

- Explain recursive raytracing in simple terms.

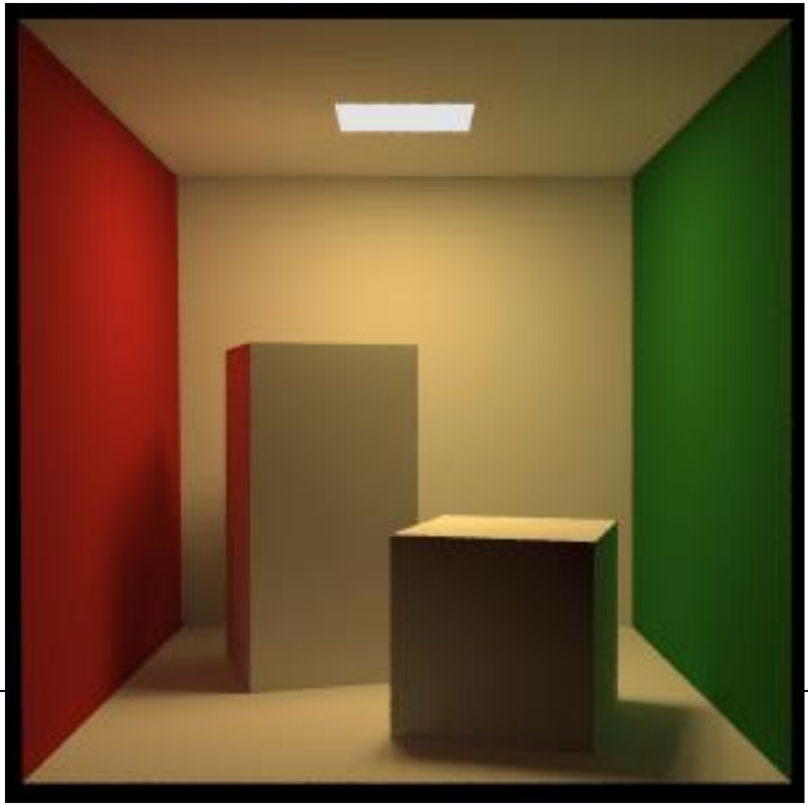
Main problem with raytracing

- Raytracing, although often producing nice results, is not a faithful model of real-world lighting.

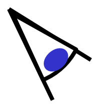
- Quiz: Why?

Main problem with raytracing

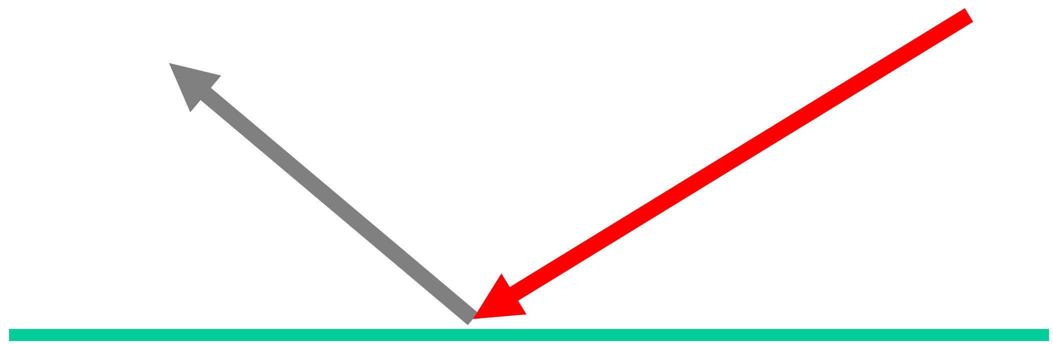
- In its pure form, it only considers “perfect” reflections and refractions, ignoring *diffuse light transfer* between surfaces.



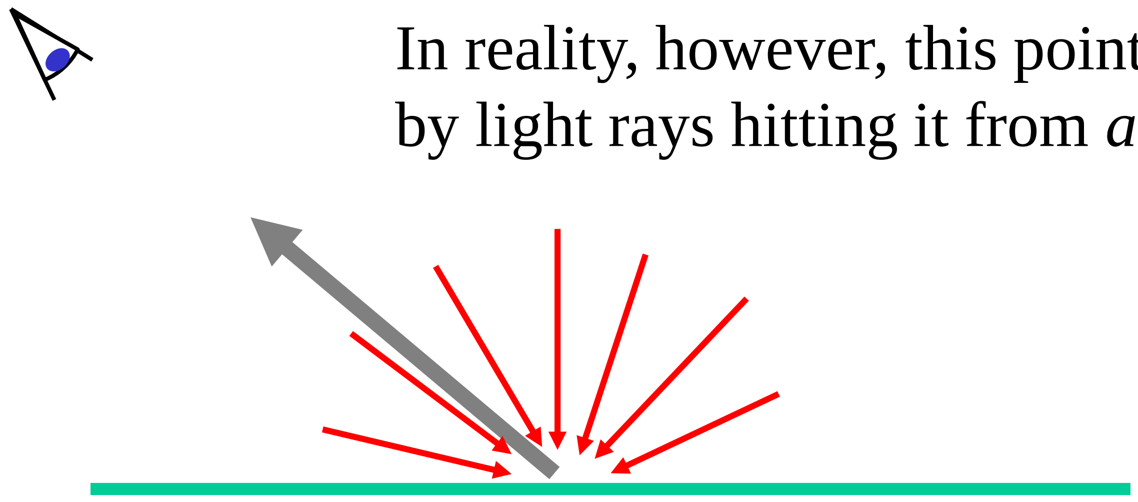
Raytracing vs Reality



Besides applying a local light model, standard raytracing only takes into account light rays affecting the point from **this** direction.

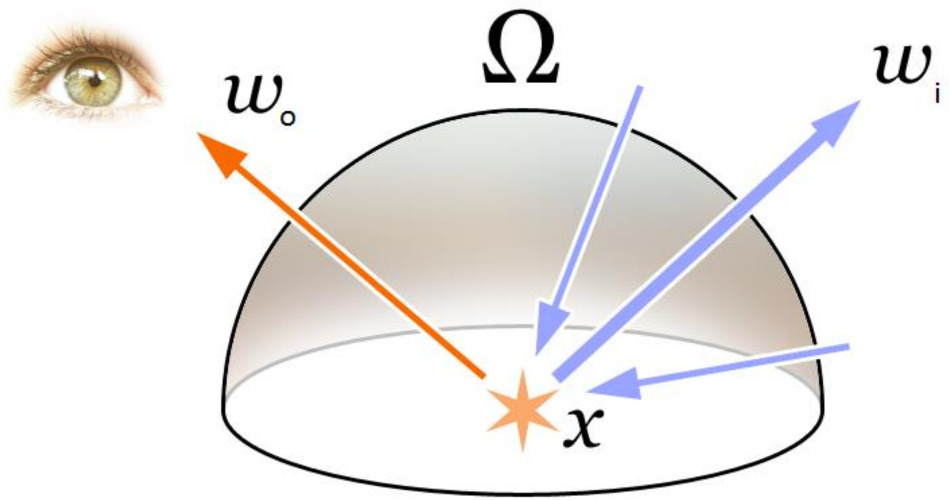


Raytracing vs Reality



In reality, however, this point is affected by light rays hitting it from *all* directions.

The Rendering Equation

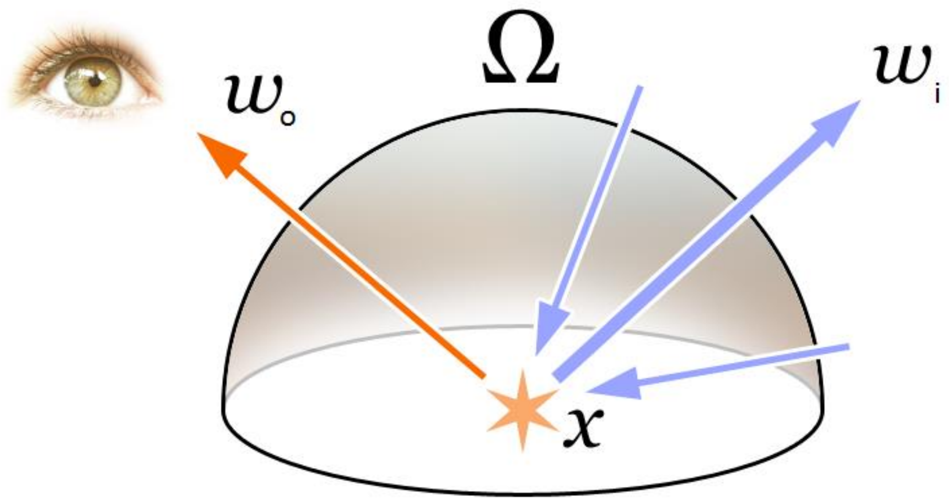


Consider light, leaving some point $\boldsymbol{x}$ in direction w_o:

$$L(\boldsymbol{w}_o, \boldsymbol{x}) =$$

http://en.wikipedia.org/wiki/File:Rendering_eq.png

The Rendering Equation

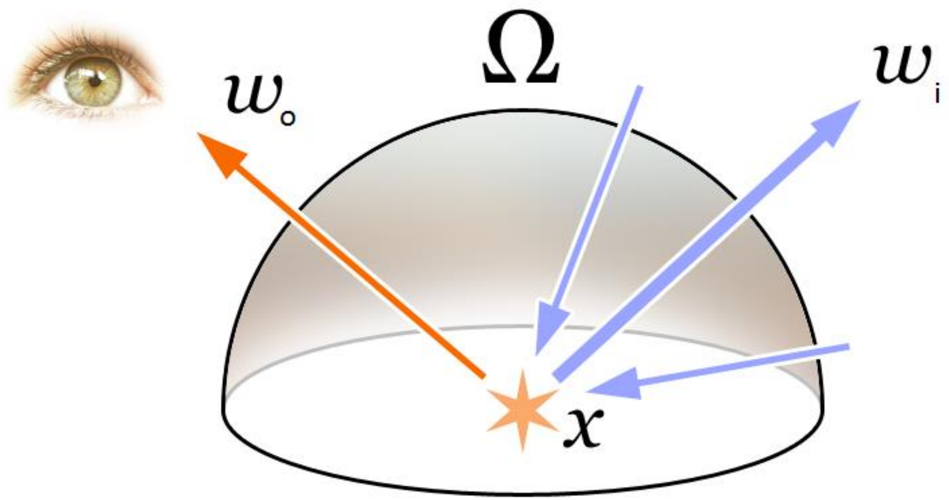


Consider light, leaving some point $\boldsymbol{x}$ in direction w_o:

$$L(\boldsymbol{w}_o, \boldsymbol{x}) = E(\boldsymbol{w}_o, \boldsymbol{x}) +$$

It consists of energy, **emitted** at that point in direction $\boldsymbol{w}_o$,…

The Rendering Equation



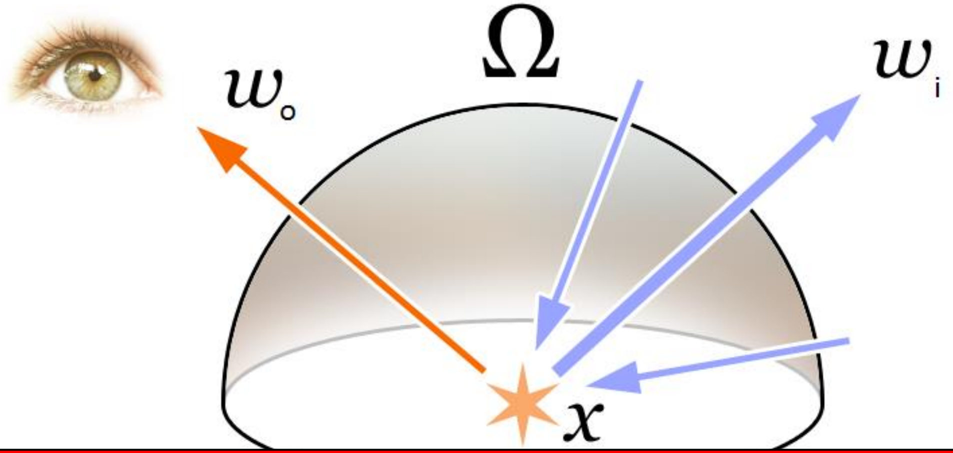
Consider light, leaving some point $\boldsymbol{x}$ in direction w_o:

…and energy, **reflected** towards $\boldsymbol{w}_o$ from direction $\boldsymbol{w}_{in}$

$$L(\boldsymbol{w}_o, \boldsymbol{x}) = E(\boldsymbol{w}_o, \boldsymbol{x}) + f(\boldsymbol{w}_{\text{in}}, \boldsymbol{w}_o) L_{\text{in}}(\boldsymbol{w}_{\text{in}}, \boldsymbol{x})$$

It consists of energy, emitted at that point in direction $\boldsymbol{w}_o$,…

The Rendering Equation

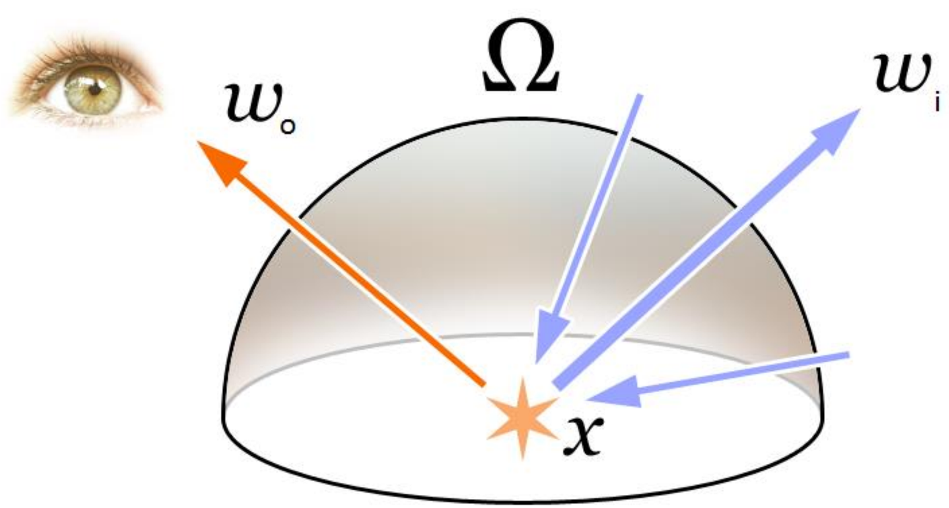


Bidirectional Reflectance Distribution Function (BRDF)
This function indicates how much energy incoming from $\boldsymbol{w}_{\text{in}}$ will be reflected towards $\boldsymbol{w}_o$. It is a property of a particular surface.

$$L(\boldsymbol{w}_o, \boldsymbol{x}) = E(\boldsymbol{w}_o, \boldsymbol{x}) + \boxed{f(\boldsymbol{w}_{\text{in}}, \boldsymbol{w}_o)} L_{\text{in}}(\boldsymbol{w}_{\text{in}}, \boldsymbol{x})$$

It consists of energy, emitted at that point in direction $\boldsymbol{w}_o$,…

The Rendering Equation



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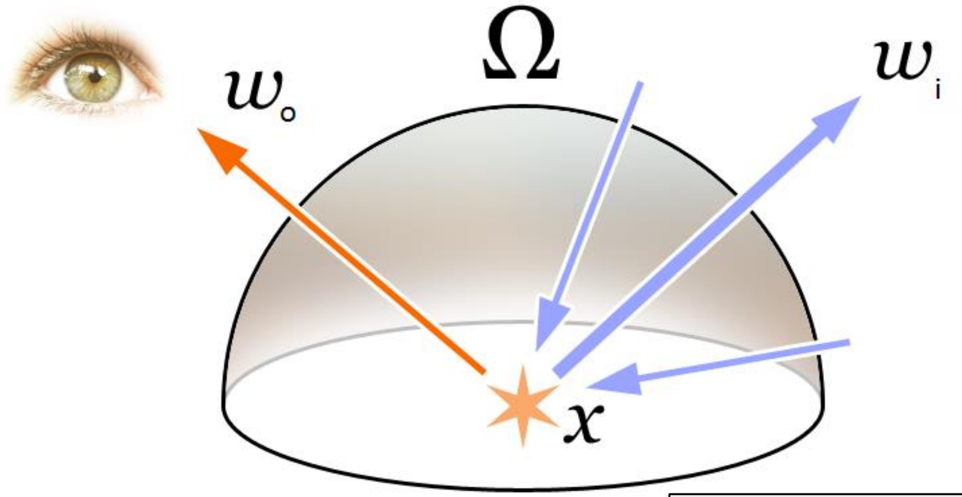


Consider light, leaving some point $\boldsymbol{x}$ in direction w_o:

…averaged over all incoming directions

$$L(\boldsymbol{w}_o, \boldsymbol{x}) = E(\boldsymbol{w}_o, \boldsymbol{x}) + \int_\Omega f(\boldsymbol{w}_{\text{in}}, \boldsymbol{w}_o) L_{\text{in}}(\boldsymbol{w}_{\text{in}}, \boldsymbol{x}) (\boldsymbol{n}^T \boldsymbol{w}_{\text{in}}) d\boldsymbol{w}_{\text{in}}$$

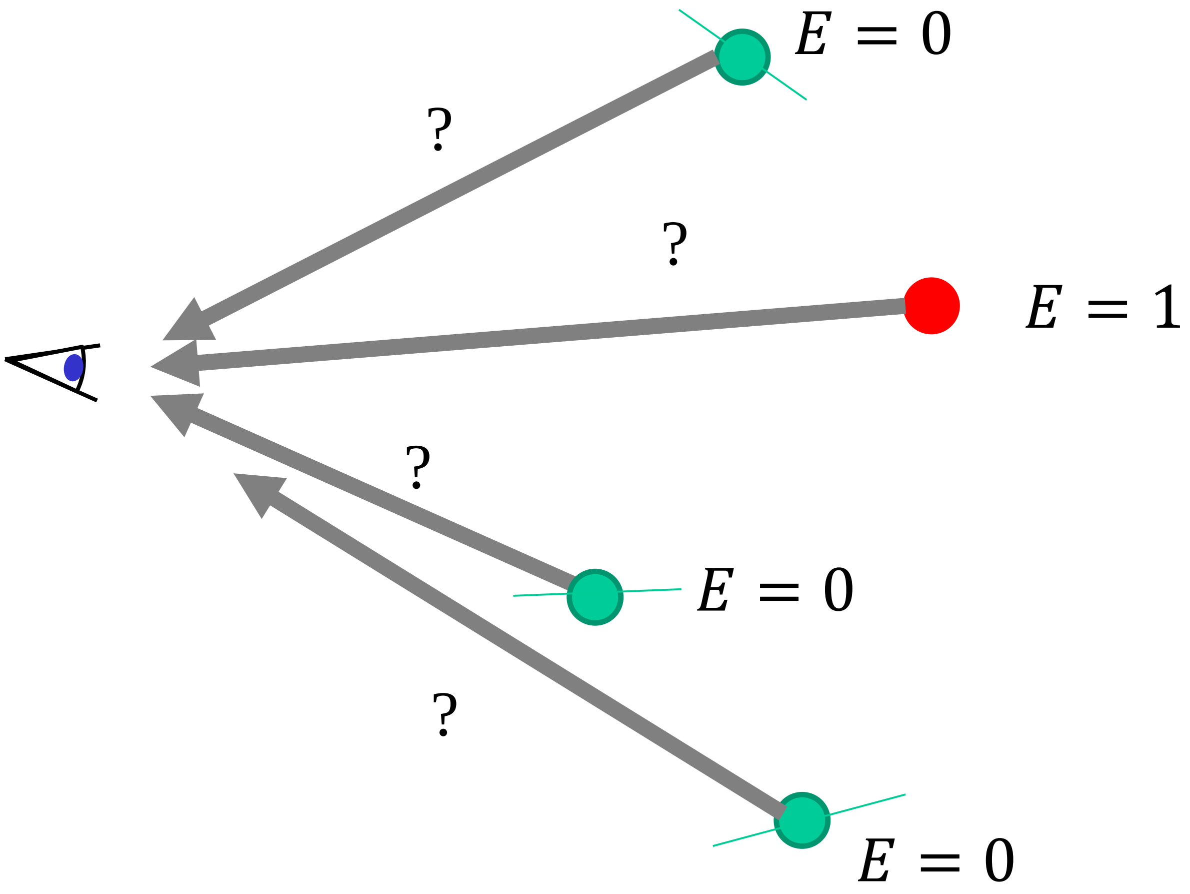
It consists of energy, emitted at that point in direction $\boldsymbol{w}_o$,…

The Rendering Equation

$$L(\boldsymbol{w}_o, \boldsymbol{x}) = E(\boldsymbol{w}_o, \boldsymbol{x}) + \int_{\Omega} f(\boldsymbol{w}_{\text{in}}, \boldsymbol{w}_o) L_{\text{in}}(\boldsymbol{w}_{\text{in}}, \boldsymbol{x})(\boldsymbol{n}^T \boldsymbol{w}_{\text{in}}) d\boldsymbol{w}_{\text{in}}$$

- The equation must hold true for all points $\boldsymbol{x}$ in the scene, for all color components (i.e. it is a system of equations, in fact).
- We initially know $E(\cdot)$ for all points (i.e. we know which points emit light)
- “Solving the rendering equation” means finding $L(\boldsymbol{w}_o, \boldsymbol{x})$ for points and light directions that reach the camera.

Rendering equation in 2D



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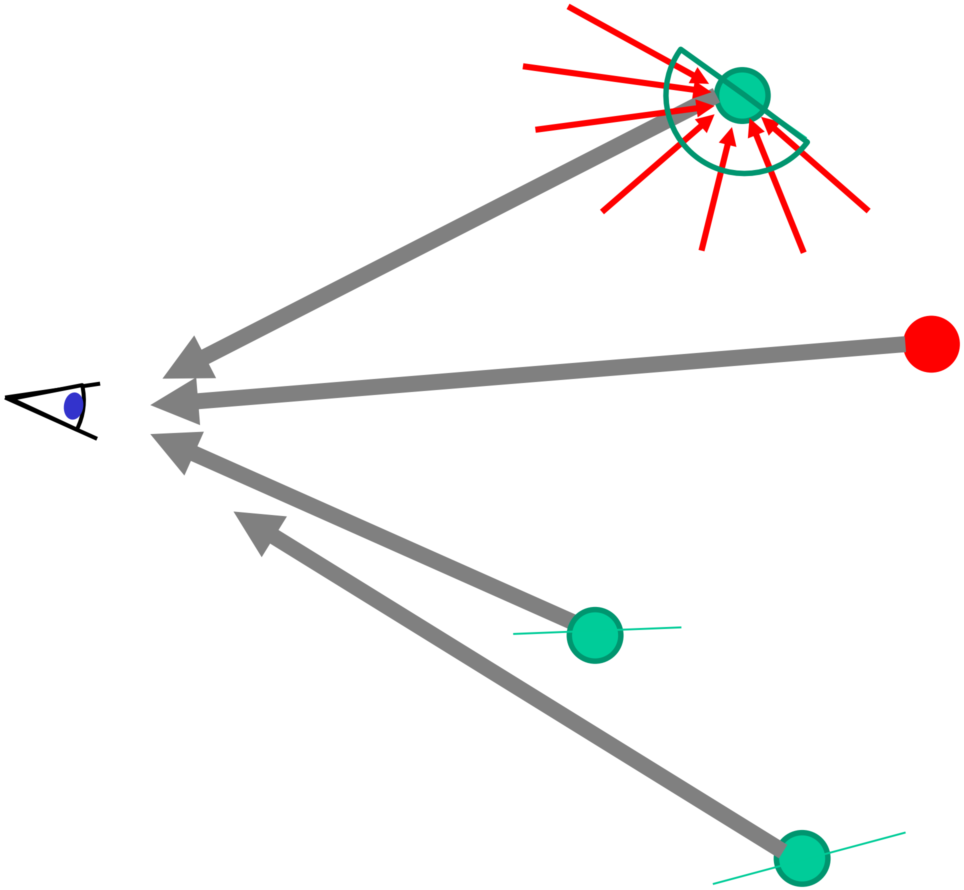

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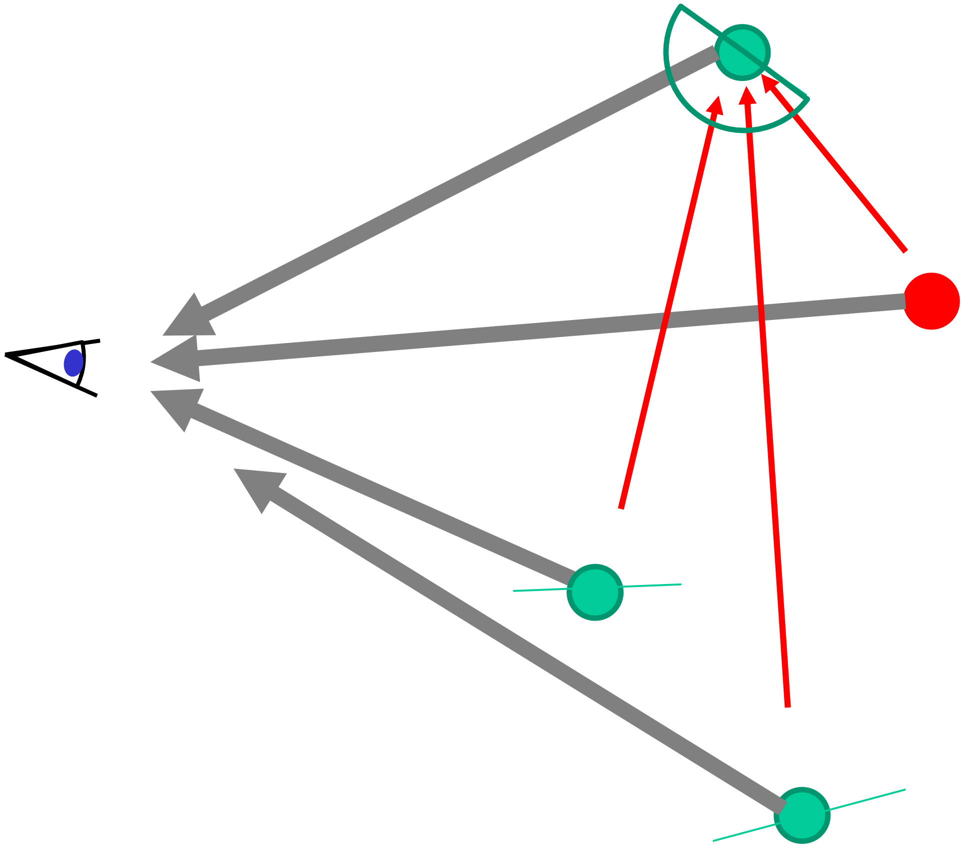

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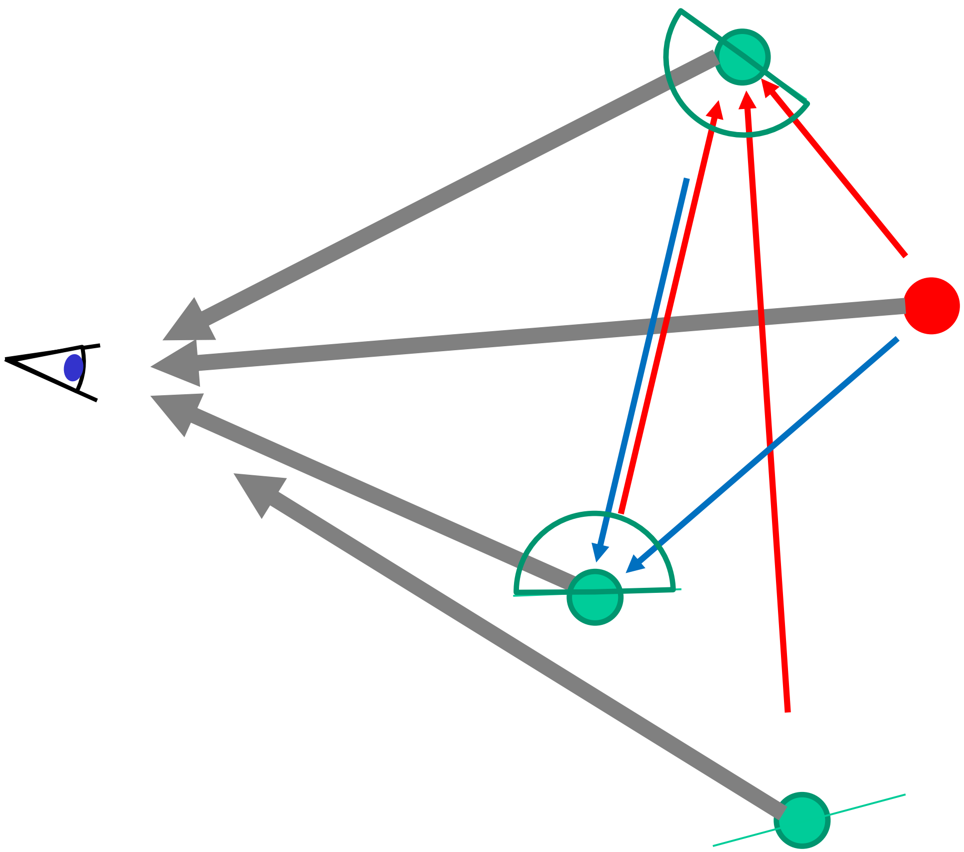

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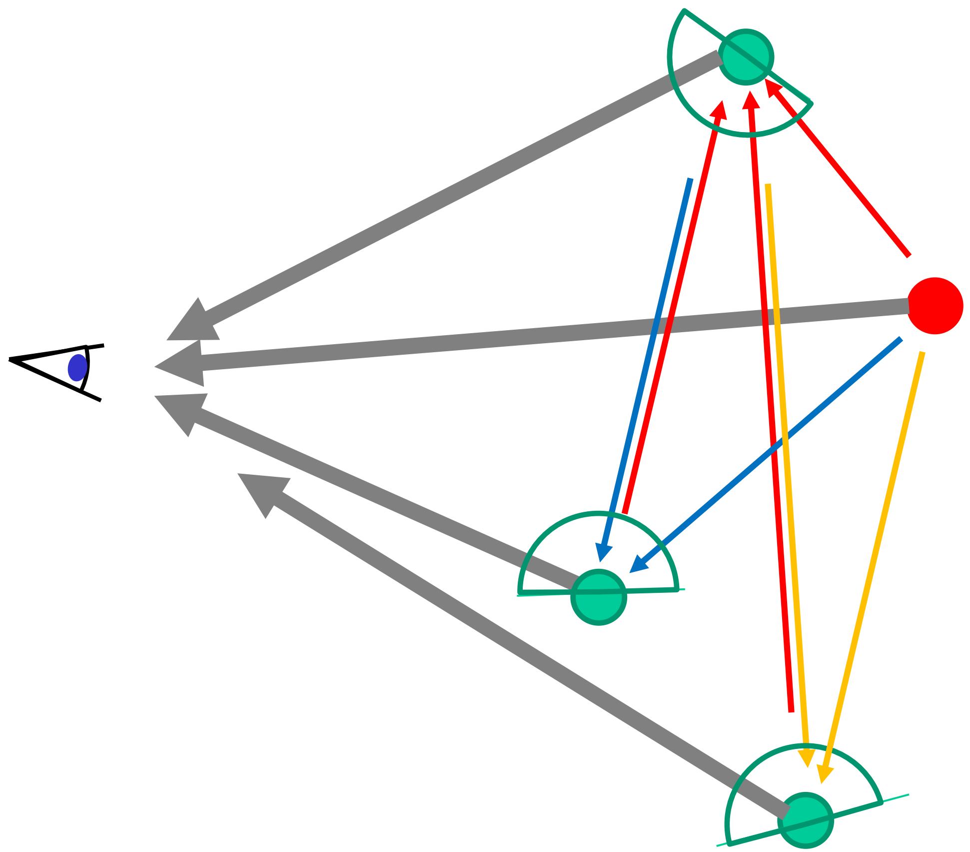

Solving the Rendering Equation

Two primary approaches

- Replace the integral with a *finite sum*:
  - **Radiosity**
- Replace the integral with a *random sample*:
  - **Path tracing**
  - **Photon mapping**

Radiosity

$$L(\boldsymbol{w}_o, \boldsymbol{x}) = E(\boldsymbol{w}_o, \boldsymbol{x}) + \int_\Omega f(\boldsymbol{w}_{\text{in}}, \boldsymbol{w}_o) L_{\text{in}}(\boldsymbol{w}_{\text{in}}, \boldsymbol{x})(\boldsymbol{n}^T \boldsymbol{w}_{\text{in}}) d\boldsymbol{w}_{\text{in}}$$

First, assume the scene consists of
a **finite** number of patches (e.g triangles)

Radiosity

$$L(\boldsymbol{w}_o, \boldsymbol{x}) = E(\boldsymbol{w}_o, \boldsymbol{x}) + \int_\Omega f(\boldsymbol{w}_{\text{in}}, \boldsymbol{w}_o) L_{\text{in}}(\boldsymbol{w}_{\text{in}}, \boldsymbol{x})(\boldsymbol{n}^T \boldsymbol{w}_{\text{in}}) d\boldsymbol{w}_{\text{in}}$$

$$L(\boldsymbol{w}_o, \boldsymbol{x}_k) = E(\boldsymbol{w}_o, \boldsymbol{x}_k) + \sum_j f(\boldsymbol{w}_{\text{in}}, \boldsymbol{w}_o) L_{\text{in}}(\boldsymbol{x}_j, \boldsymbol{x})(\boldsymbol{n}^T \boldsymbol{w}_{\text{in}})$$

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Second, assume each patch radiates equally in all directions, i.e. all lighting is *diffuse*.

Radiosity

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$$L(\boldsymbol{x}_k) = E(\boldsymbol{x}_k) + \sum_j \rho_k F_{kj} L(\boldsymbol{x}_j)$$

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$$L(\boldsymbol{x}_k) = E(\boldsymbol{x}_k) + \sum_j \rho_k F_{kj} L(\boldsymbol{x}_j)$$

$$L_k = E_k + \rho_k \sum_j F_{kj} L_j$$

Second, assume each patch radiates equally in all directions, i.e. all lighting is *diffuse*.

Radiosity

Energy created in patch k. (i.e. light source intensity)

Energy coming from another patch j.

"View factor": How much of j's energy is received by k.

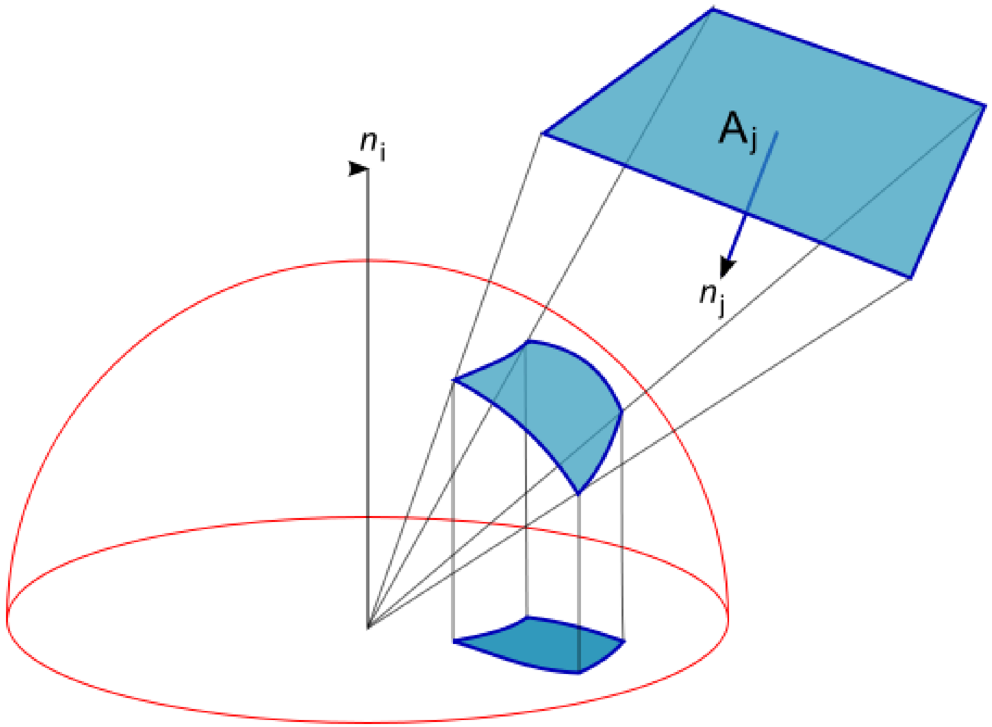
E.g. $F_{kj} = 0$ if k is occluded from j.

Energy leaving patch k.

$$\boxed{L_k} = \boxed{E_k} + \boxed{\rho_k} \sum_j \boxed{F_{kj}} \boxed{L_j}$$

How much incoming energy k re-diffuses. i.e. "patch color"

View factors



The amount of energy that patch *i* receives from patch *j* is proportional to the projected solid angle of that patch as seen from *i*

http://en.wikipedia.org/wiki/File:Nusselt_analog.svg

Radiosity

$$L_k = E_k + \rho_k \sum_j F_{kj} L_j$$

Radiosity

$$\boldsymbol{l} = \boldsymbol{e} + \boldsymbol{\rho} \cdot \boldsymbol{F}\boldsymbol{l}$$

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This is a linear equation. Analytically the solution is:

$$\boldsymbol{l} \coloneqq (\boldsymbol{I} - \boldsymbol{\rho}\boldsymbol{F})^{-1}\boldsymbol{e}$$

The number of patches is usually prohibitively large to use this approach.

Radiosity

$$\boldsymbol{l = e + \rho \cdot Fl}$$

Instead we can use the Jacobi iteration method:

$$\boldsymbol{l_0 = e}$$
$$\boldsymbol{l_1 = e + \rho \cdot Fl_0}$$
$$\boldsymbol{l_2 = e + \rho \cdot Fl_1}$$
$$\boldsymbol{l_3 = e + \rho \cdot Fl_2}$$

…

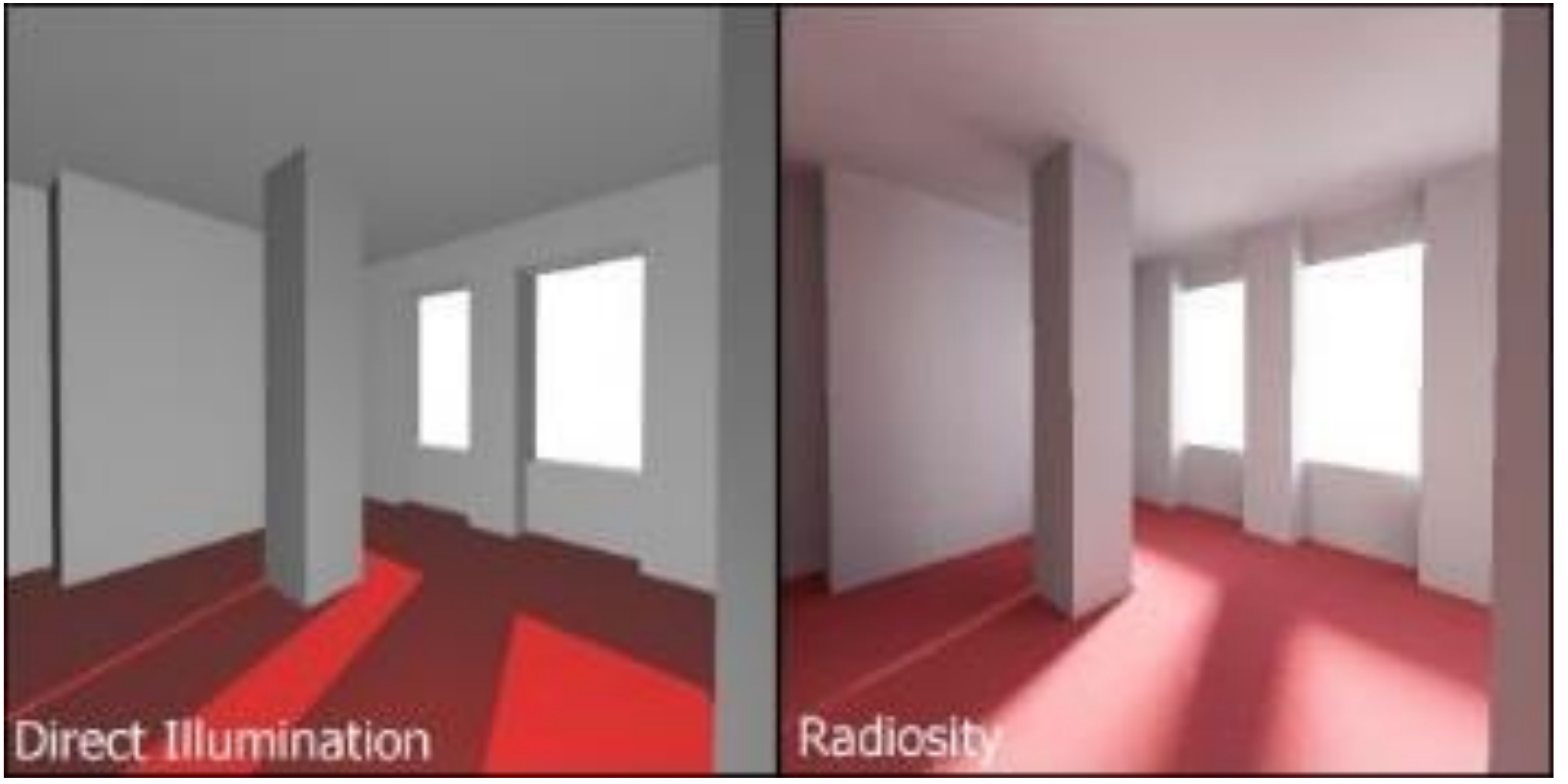
The algorithm converges in a few iterations.

Radiosity

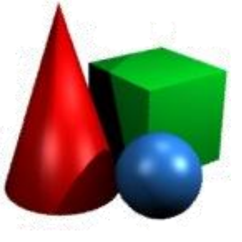
In simple terms:

- In the beginning all patches except light-emitting ones have zero *radiosity* value.
- For each patch, recompute its radiosity value as a weighted sum of received radiosity + emitted radiosity.
- Repeat several times.

Radiosity



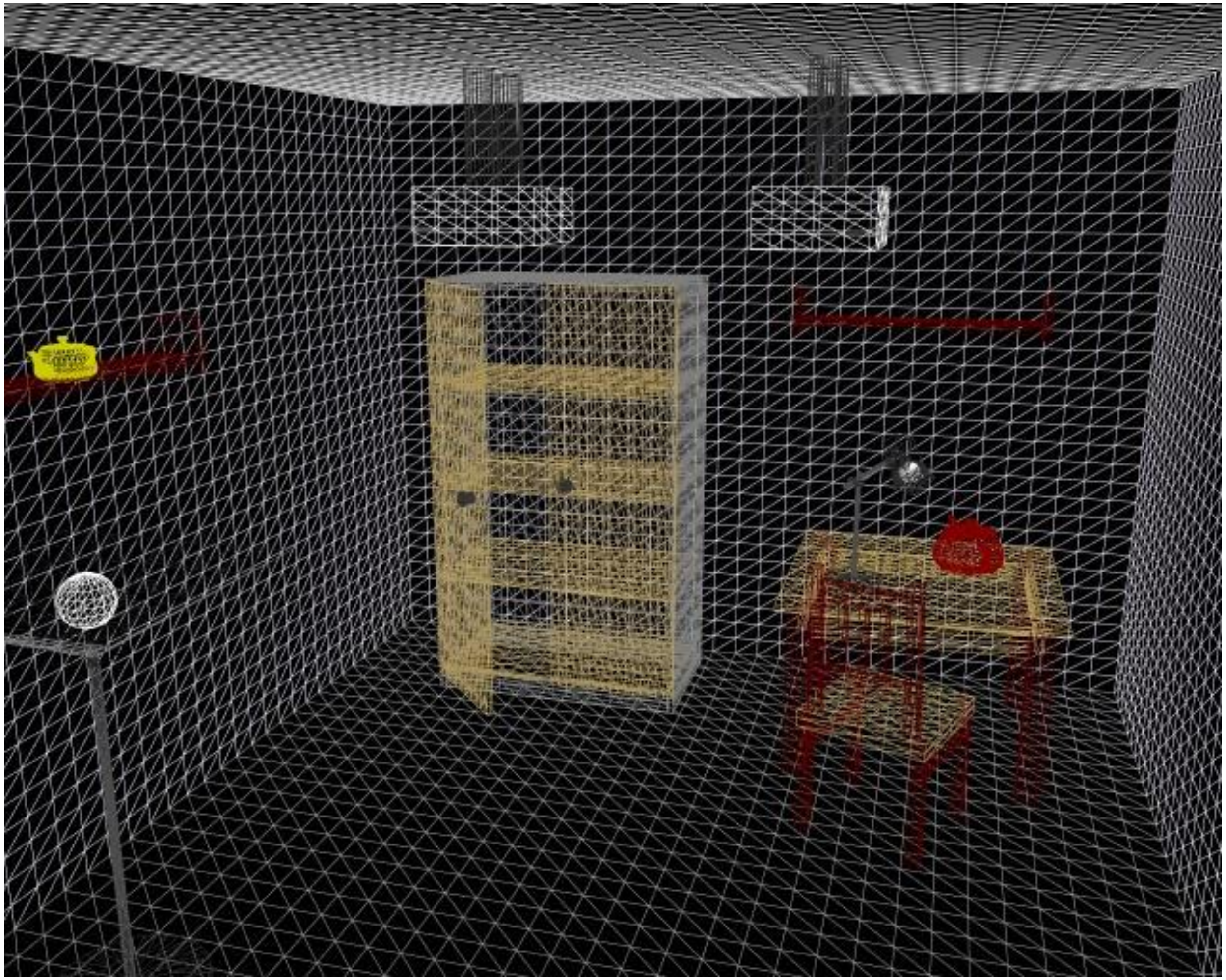
http://en.wikipedia.org/wiki/File:Radiosity_Comparison.jpg



Radiosity

http://dudka.cz/rrv/gallery

Radiosity



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Radiosity

Radiosity

Quiz

- Assuming the scene has 10000 patches:
  - How much memory is used by the iterative Radiosity algorithm?
  - How many operations are performed per iteration (i.e. what is the computational complexity)?

Quiz

- Assuming the scene has 10000 patches:
  - How much memory is used by the iterative Radiosity algorithm? ~10000 (i.e. $O(n)$)
  - How many operations are performed per iteration (i.e. what is the computational complexity)?

    ~100 000 000

    (i.e. the complexity is $O(n^2)$)

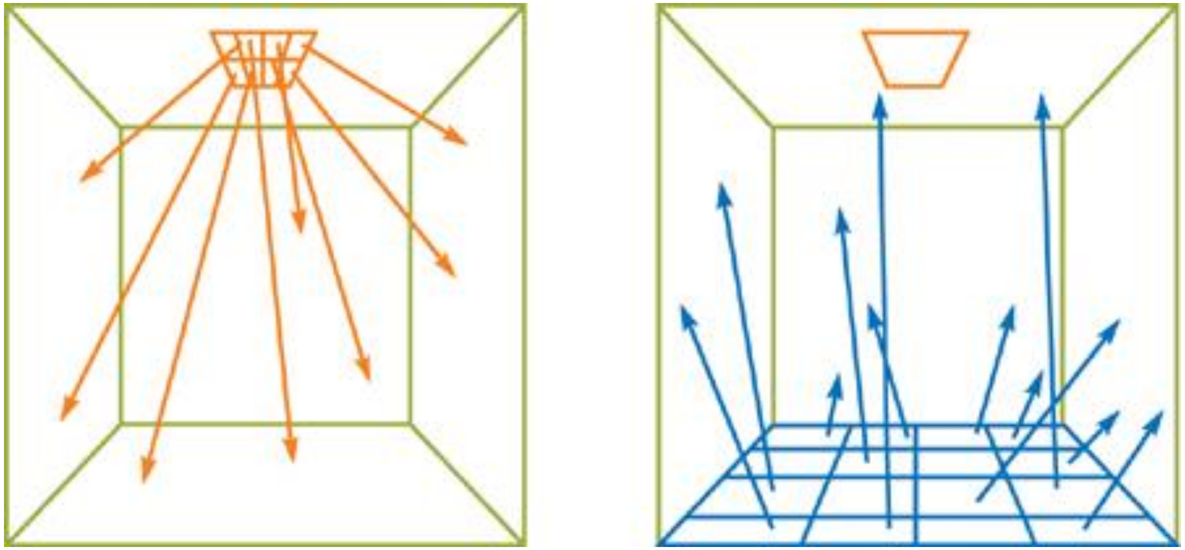
Progressive refinement radiosity

- Quadratic complexity is prohibitive for many scenes.

- A good approximation to radiosity which works in linear time per iteration (although requires more iterations) is *progressive refinement*.

Progressive refinement radiosity

- Deposit “energy chunks” on light-emitting patches only
- Pick the most energetic patch as the “shooter”, and distribute its energy to all other patches (“receivers”), according to view factors.
- Repeat



http://http.developer.nvidia.com/GPUGems2/gpugems2_chapter39.html

Quiz

- Suppose we run the radiosity algorithm for a scene.
- Then viewer position changes.
- Do we need to re-run the algorithm?

Radiosity

- Radiosity values are fixed and viewer-independent, so they can be pre-computed for the scene and stored as light maps.

- Consequently, radiosity is usually used together with other techniques (e.g. raytracing or standard pipeline).

Radiosity

- Progressive-refinement radiosity can be GPU-optimized

- The technique is pretty much always used in professional architecture / interior design CAD renderings.

Solving the Rendering Equation

Two primary approaches

- Replace the integral with a *finite sum*:
  - **Radiosity**
- Replace integral with a *random sample*:
  - **Path tracing**
  - **Photon mapping**

Monte-carlo integration

$$L(\boldsymbol{w}_o, \boldsymbol{x}) = E(\boldsymbol{w}_o, \boldsymbol{x}) + \int_\Omega f(\boldsymbol{w}_{\text{in}}, \boldsymbol{w}_o) L_{\text{in}}(\boldsymbol{w}_{\text{in}}, \boldsymbol{x})(\boldsymbol{n}^T \boldsymbol{w}_{\text{in}}) d\boldsymbol{w}_{\text{in}}$$

Rather than taking the integral, sum over a *random sample* of directions

Monte-carlo integration

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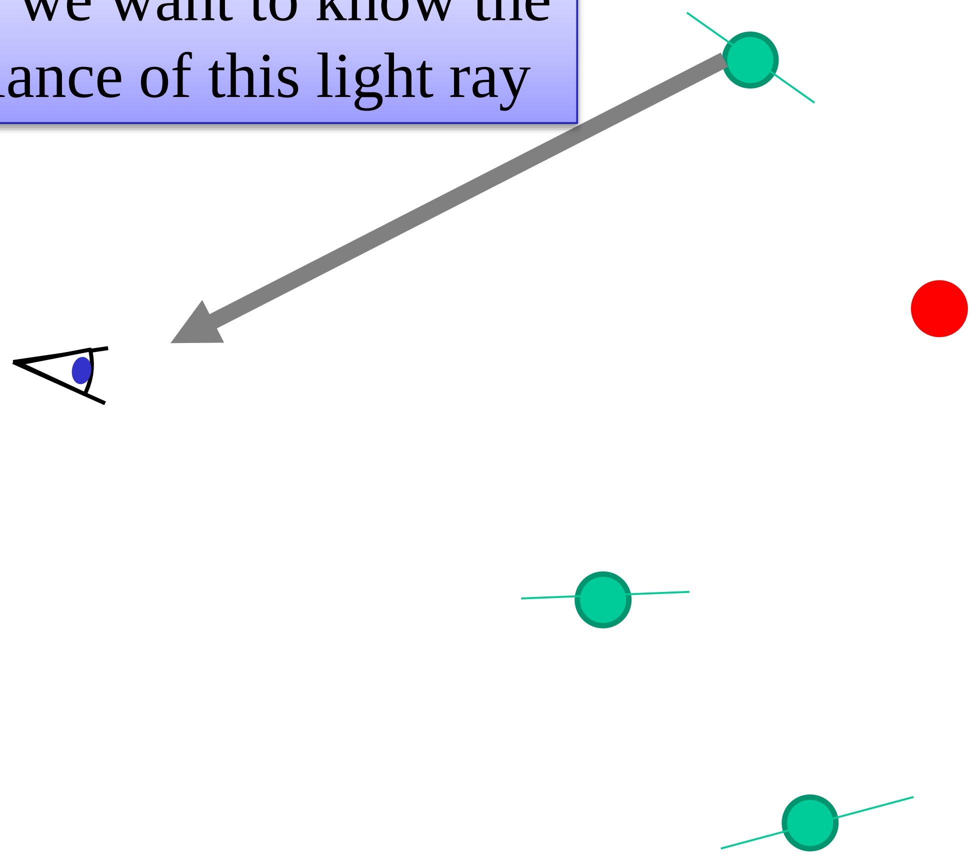
$$L(\boldsymbol{w}_o, \boldsymbol{x}) = E(\boldsymbol{w}_o, \boldsymbol{x}) + \sum_{\boldsymbol{w}_{\mathbf{in}} \in \text{Random}} f(\boldsymbol{w}_{\text{in}}, \boldsymbol{w}_o) L_{\text{in}}(\boldsymbol{w}_{\text{in}}, \boldsymbol{x}) (\boldsymbol{n}^T \boldsymbol{w}_{\text{in}})$$

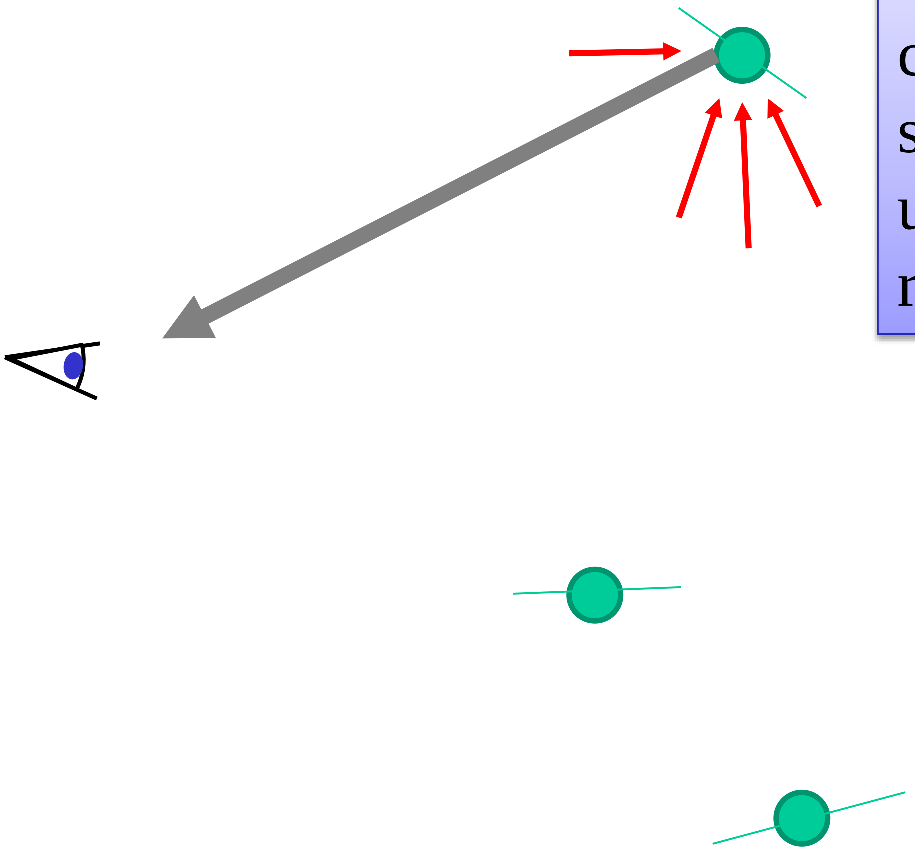
Rather than taking the integral, sum over a *random sample* of directions

Monte-carlo integration

Say we want to know the radiance of this light ray

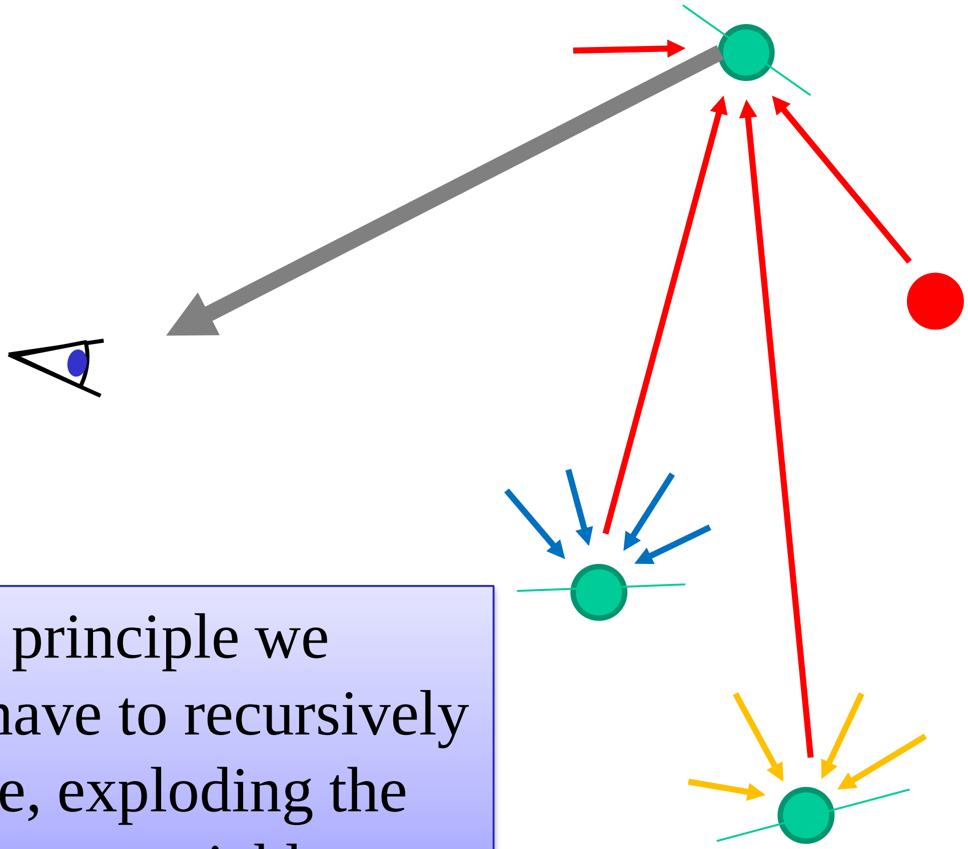


Monte-carlo integration



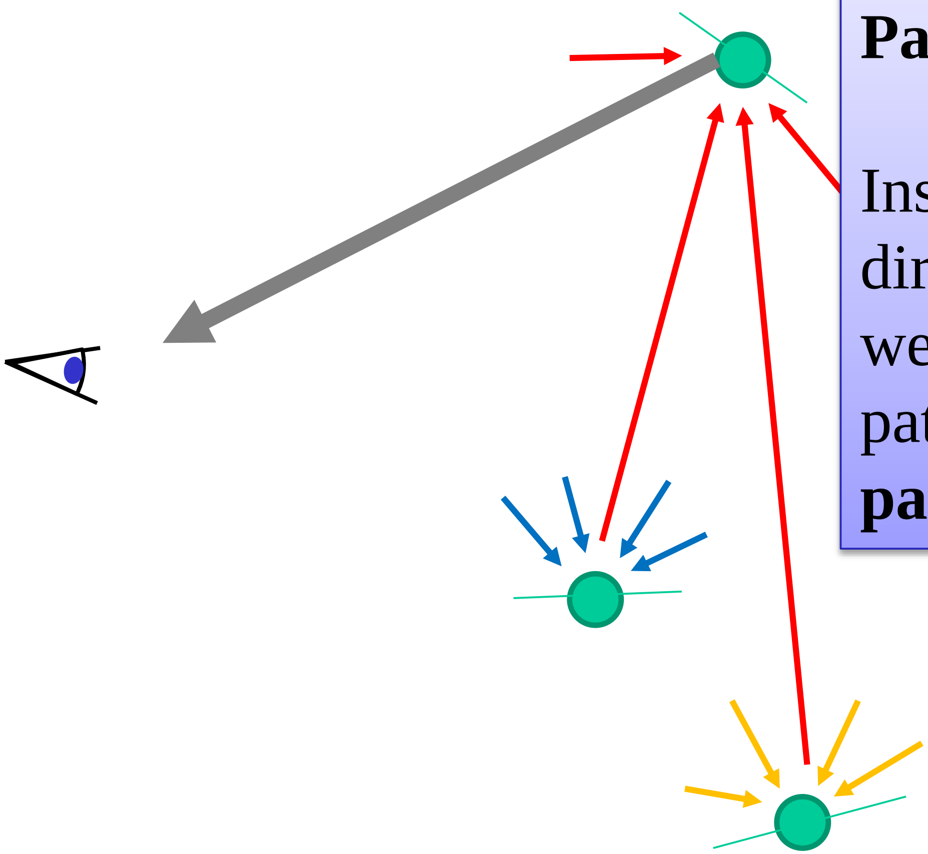
It is an integral over light coming into this point, so we can approximate it using a large enough random sample

Monte-carlo integration



Now in principle we would have to recursively continue, exploding the search space quickly

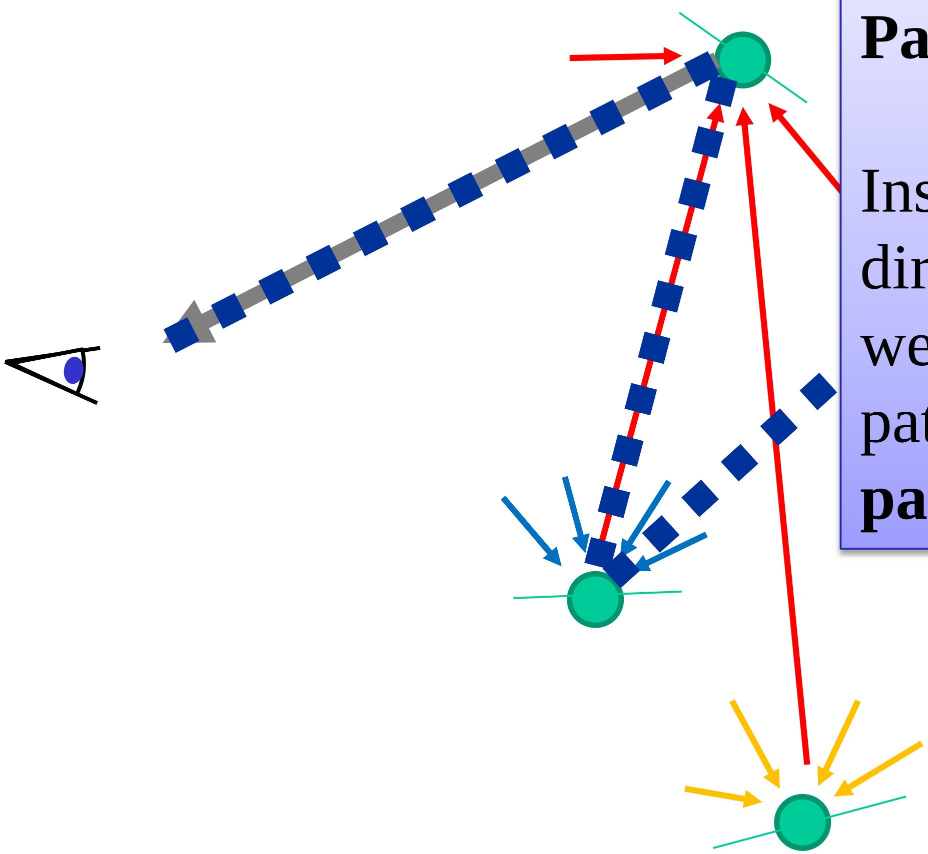
Monte-carlo integration



Path tracing

Instead of averaging directions at each step, we shoot several random paths and average **over paths**.

Monte-carlo integration



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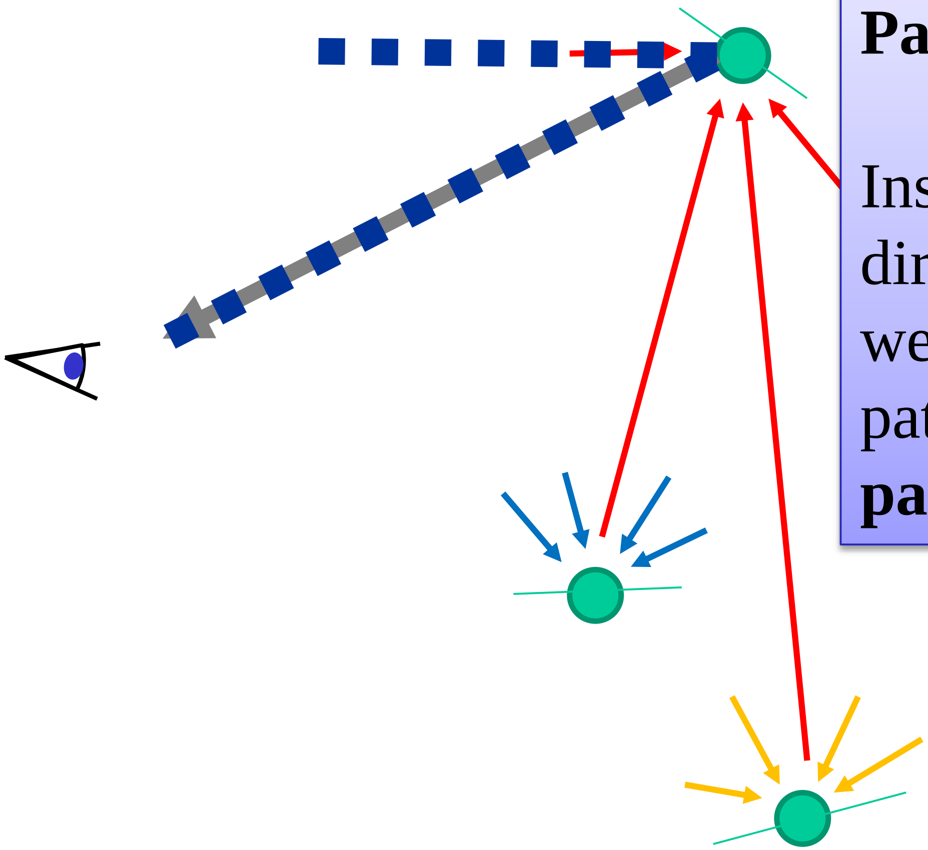

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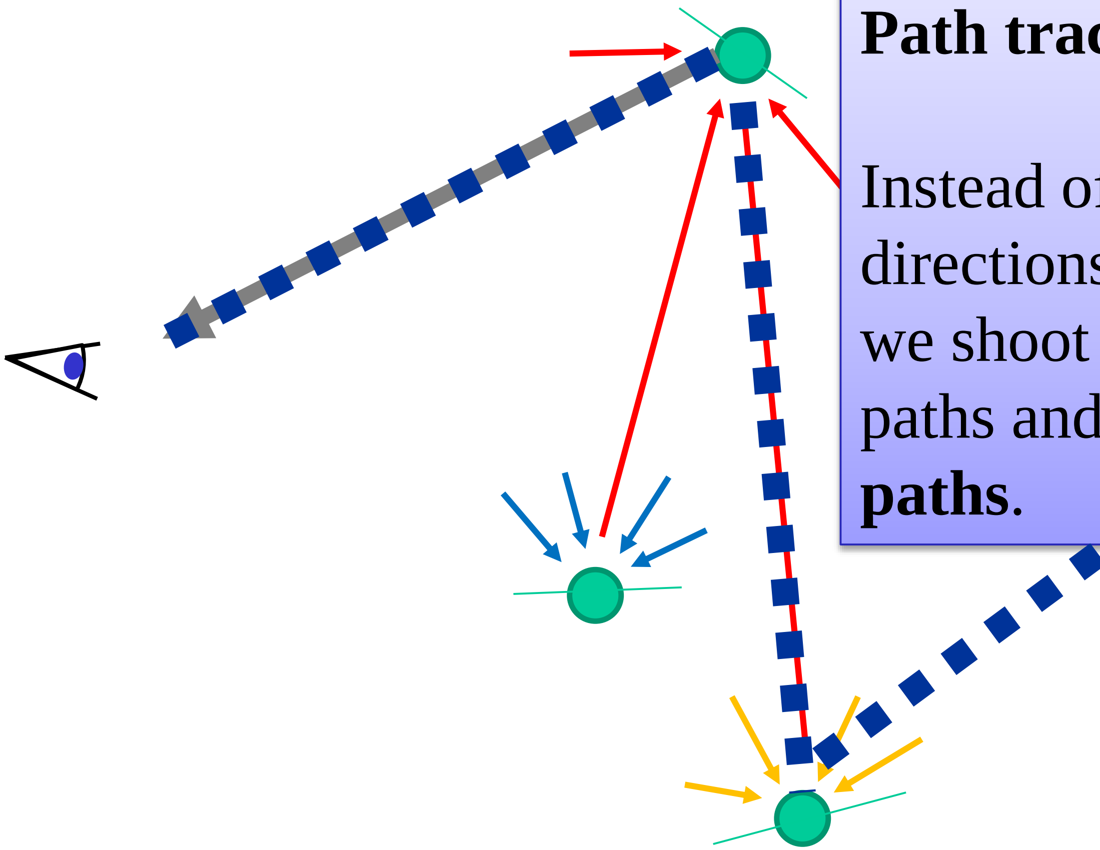


Path tracing

Instead of averaging directions at each step, we shoot several random paths and average **over paths**.

Path tracing

```
color trace_path(ray, depth) {
  hit = raycast_scene(ray);
  if (hit == None || depth is too large) return Black;
  else {
     dir = random_direction()
     new_ray = Ray(origin = hit.point,
                   direction = dir);

     incoming_color = trace_path(new_ray, depth+1)
     return hit.emittance +
             BRDF(hit.material, ray, new_ray) *
             dot(new_ray, hit.normal) *
             incoming_color
  }
```

Path tracing

- In simple terms:
  - Cast rays into the scene, each bouncing randomly around.
  - Rays that do not reach light sources contribute nothing.
  - Rays that reach light sources, contribute color, determined by the diffusion/reflectance properties of the surfaces they bounced off.

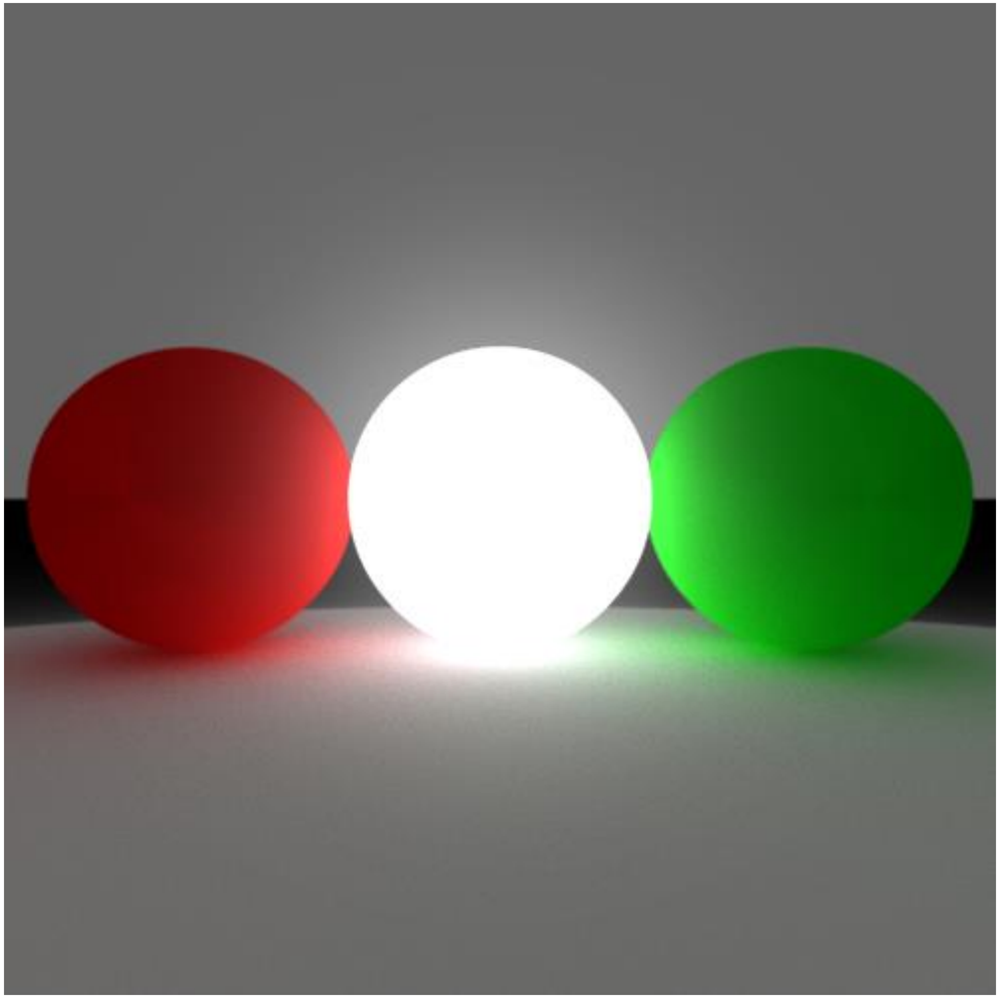
Quiz

- How is this different from conventional recursive raytracing?

Quiz

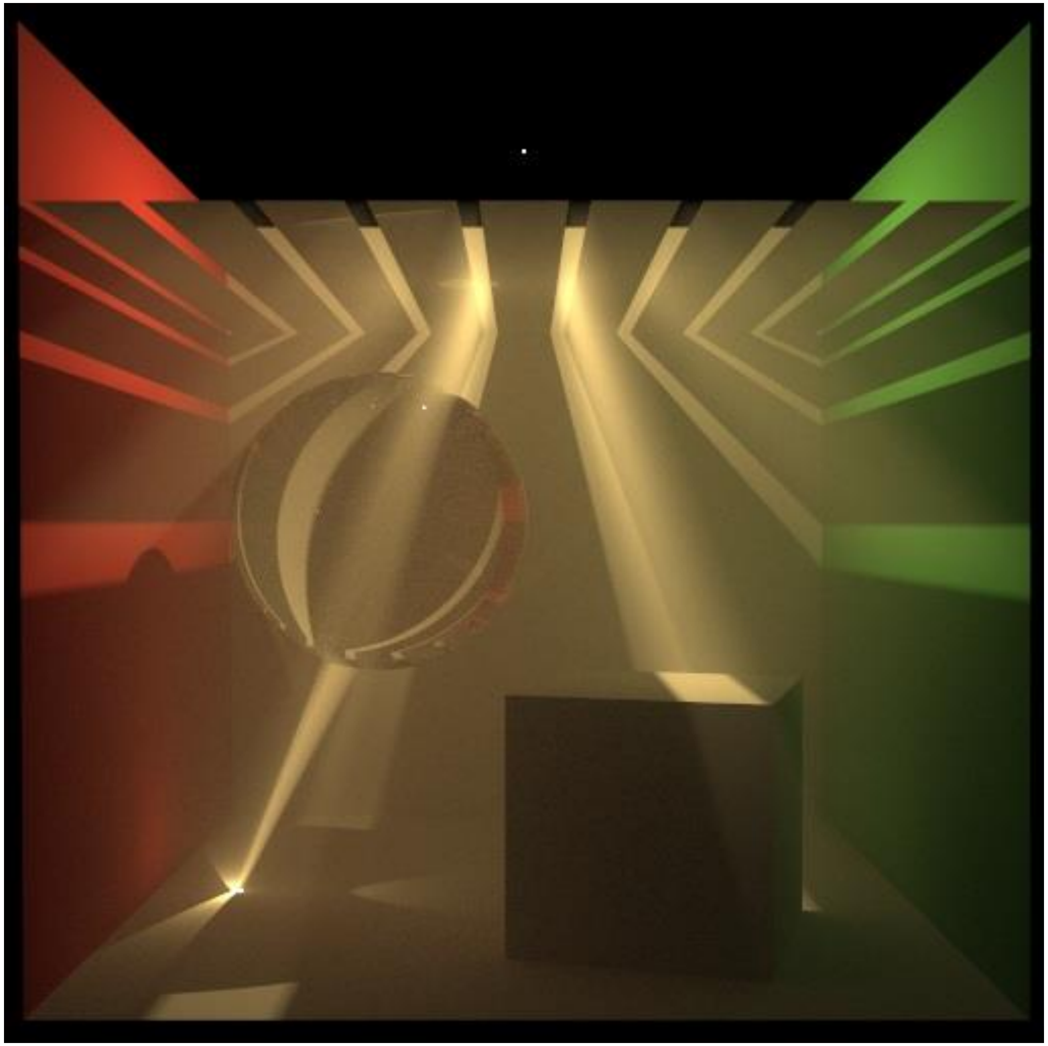
- How is this different from conventional recursive raytracing?
  - Rays bounce **randomly**, thus taking into account light diffusion, not only pure reflections or refractions.
  - **No light model computations** are performed at the bounce points. The ray **has to reach** a light source to contribute any color.

Path tracing



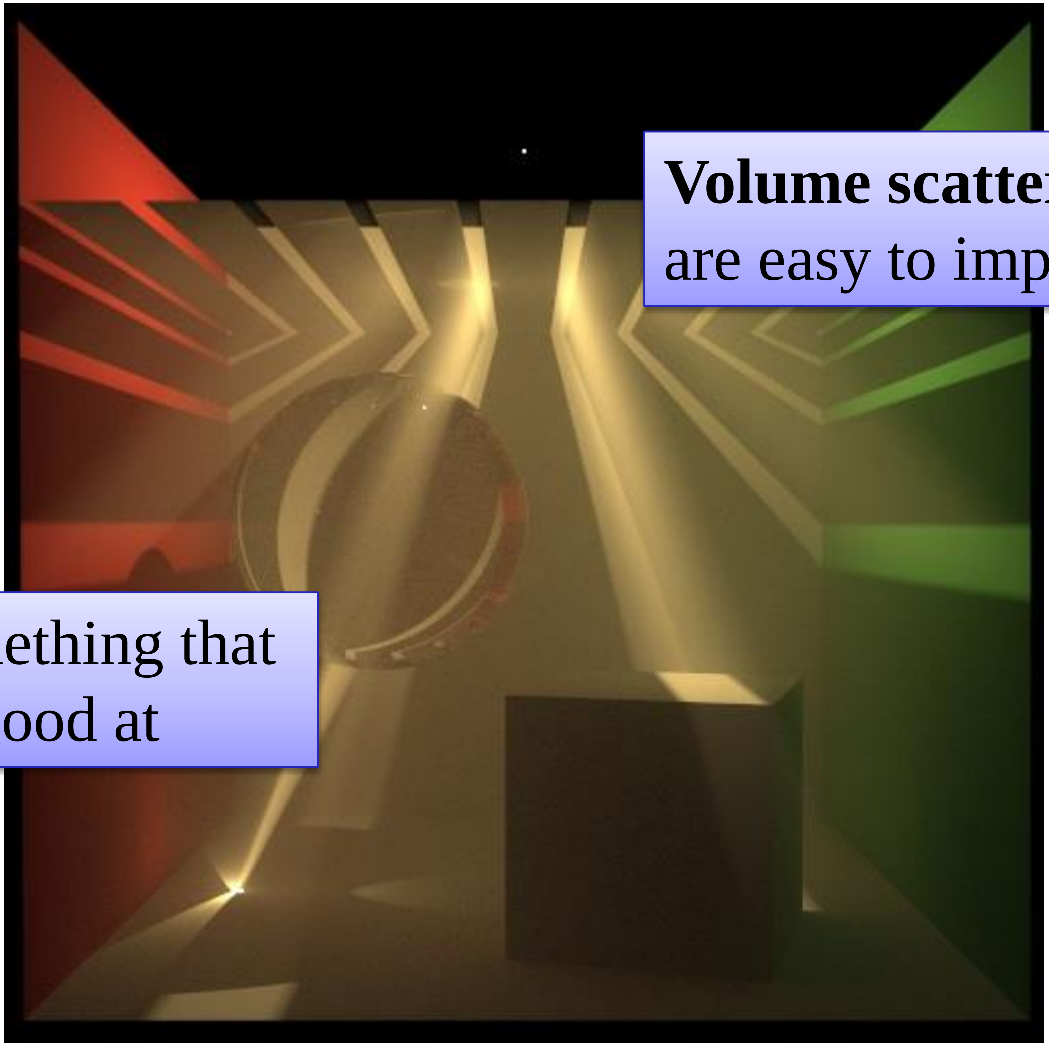
http://commons.wikimedia.org/wiki/File:Pathtrace3.png

Path tracing



http://www.sjbrown.co.uk/2012/07/15/bidirectional-path-tracing-in-participating-media/

Path tracing



Volume scattering effects are easy to implement

Caustics is something that path tracing is good at

http://www.sjbrown.co.uk/2012/07/15/bidirectional-path-tracing-in-participating-media/

Problem with Path tracing

- We have to trace the ray until it hits the light (or we are tired).

- If light sources are small (e.g. points), rays may never hit them (or hit extremely rarely).

Problem with Path tracing

- Solution?

Problem with Path tracing

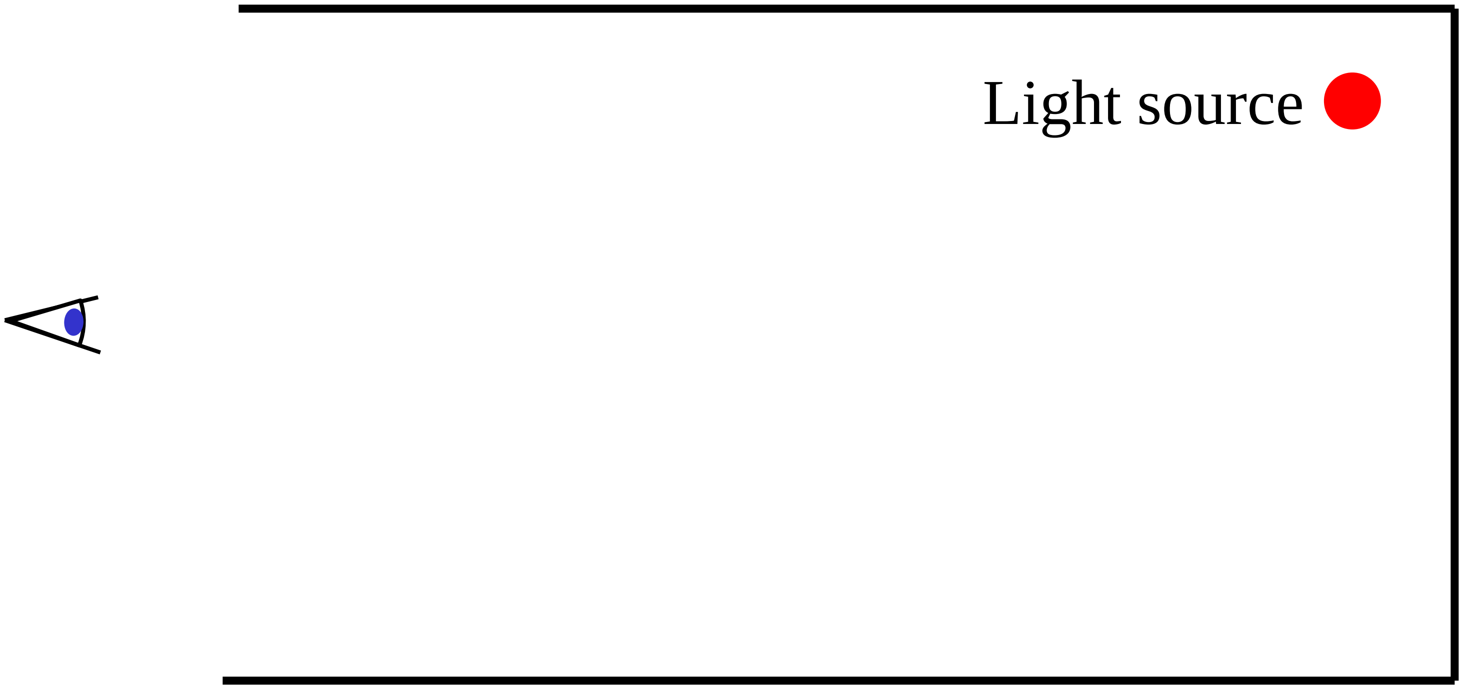
- Solution?

- We could try to trace the rays from light sources, but this leads to a similar problem: some rays wouldn't ever reach the camera.

Bidirectional path tracing

- Solution:
  - Trace a path for some steps starting from the camera.
  - Trace another path for some steps starting from the light source.
  - Connect the two parts and process the result as a single random path.

Bidirectional path tracing



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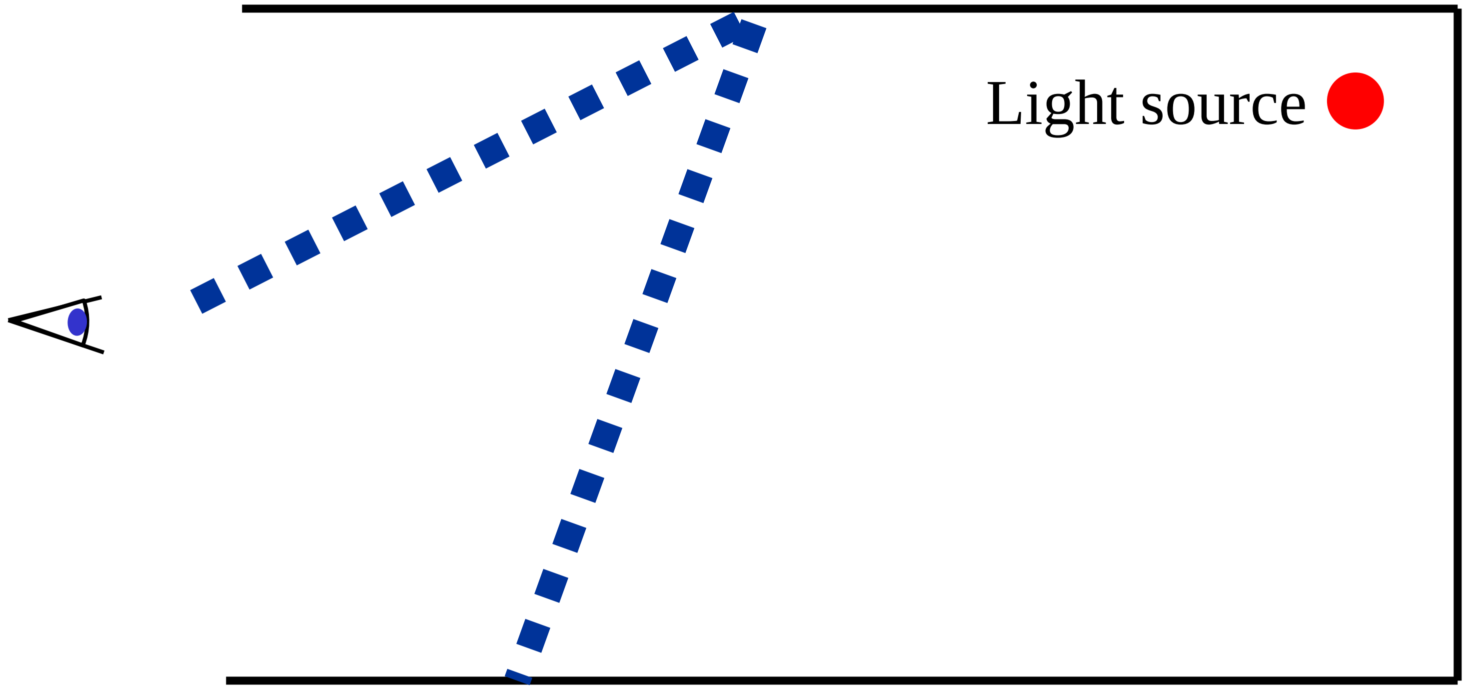

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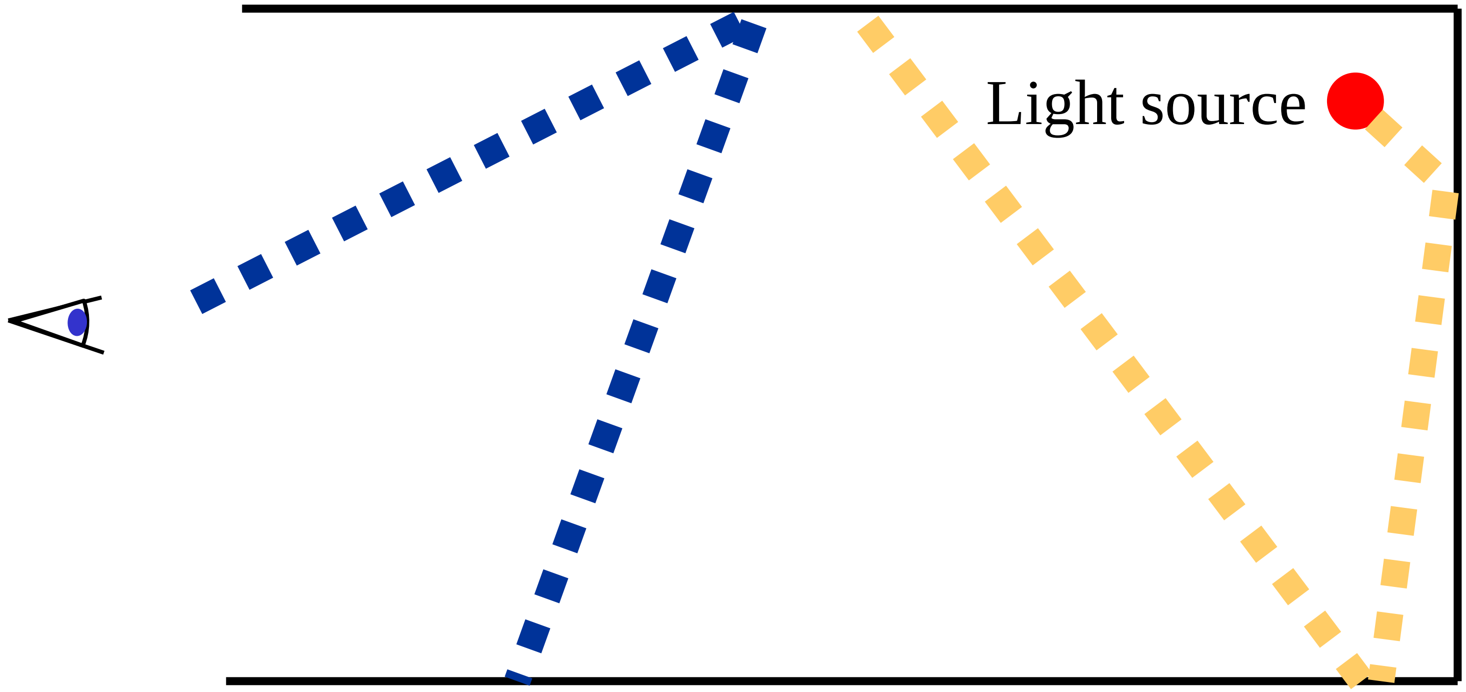
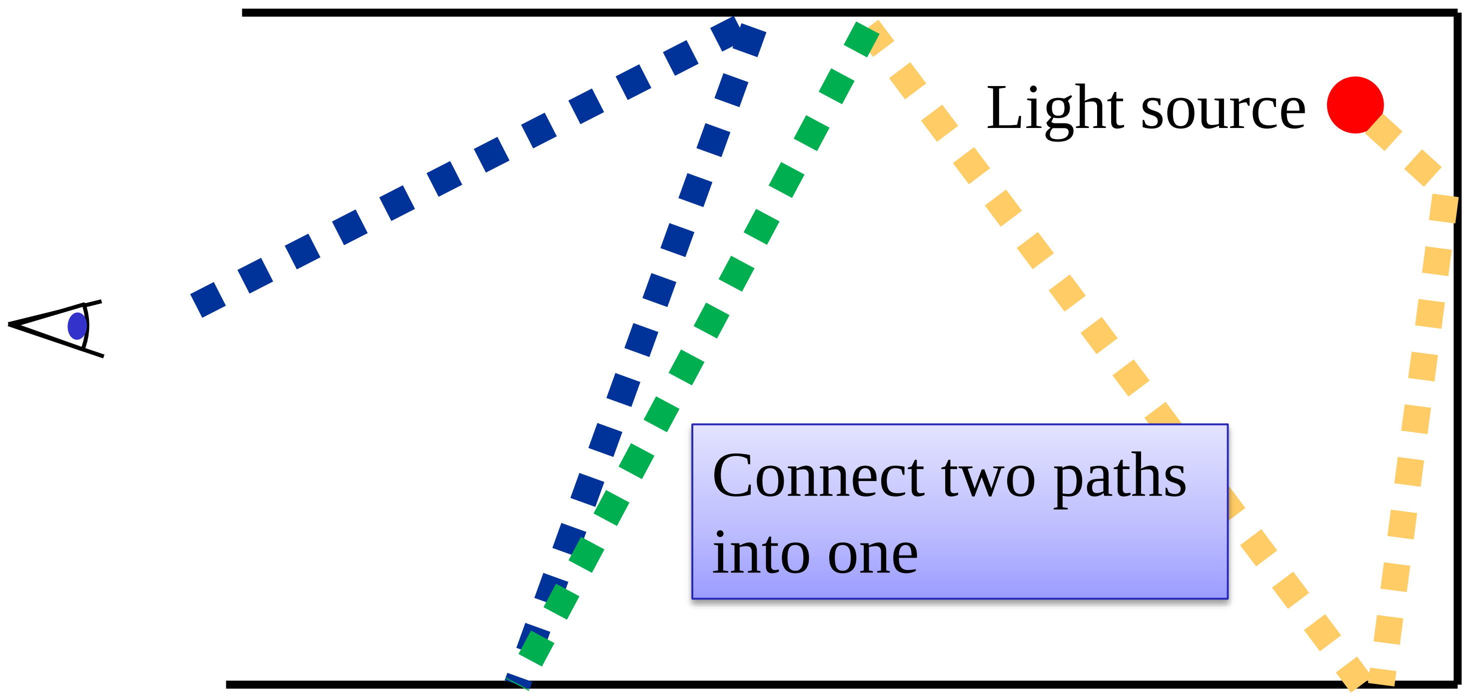


Bidirectional path tracing



Photon mapping

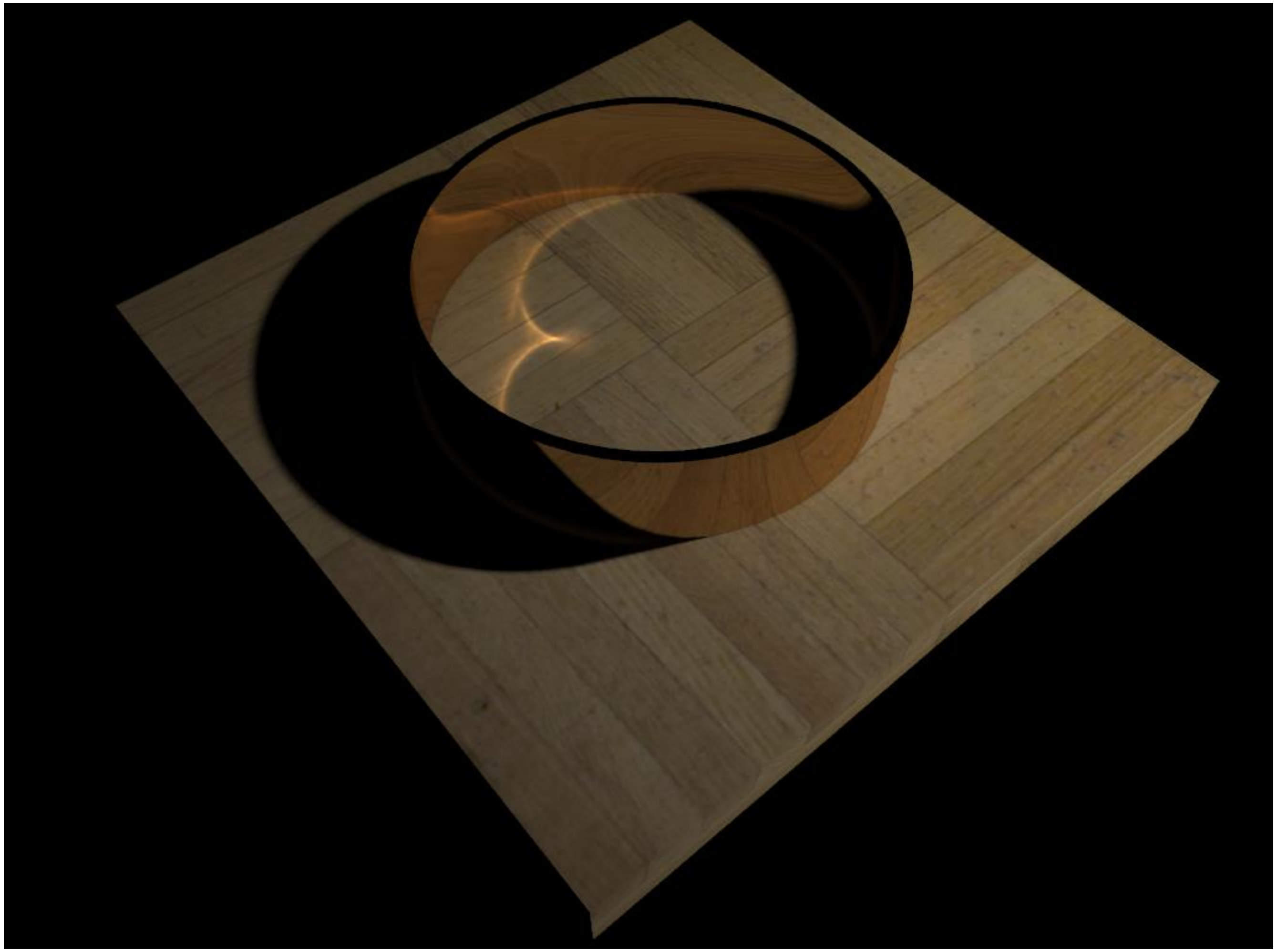
- A method “inbetween” path tracing and radiosity:
  - Simulate “photons” shooting from light sources in all directions.
  - When photon hits a surface, it is split into parts – part of its intensity “gets stuck” in the surface. Remaining intensity continues its path in some direction.
  - Eventually, many (~millions) of photons will be deposited all over the scene.

See H.W.Jensen. Global Illumination using Photon Maps

Photon mapping

- With each “stuck” photon we store the direction it came from and its color (intensity).
- We index all photons in a data structure for fast nearest-neighbor retrieval (e.g. k-d tree).
- Now for any point we can easily evaluate the rendering equation by replacing the integral with the sum over nearby photons.

Photon mapping



http://www.mpi-inf.mpg.de/departments/irg3/ws0304/lcn/projects/Michael/

Combined methods

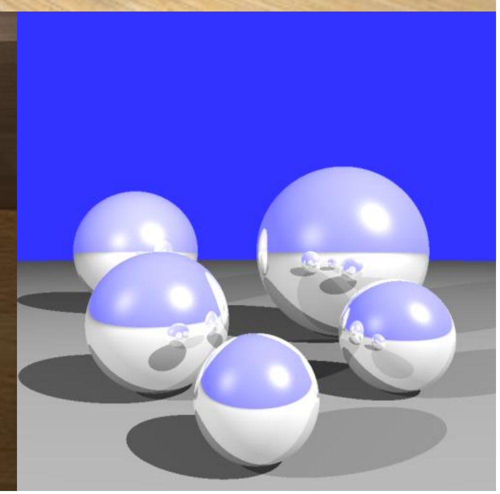
- Various combinations of methods are possible:
  - Radiosity light maps + standard pipeline
  - Radiosity / photon mapping + raytracing
  - Path tracing + Photon mapping (irradiance caching)

Summary: Core rendering methods

- **Standard graphics pipeline**
- **Direct raycasting**
  - Raytracing, Raymarching, Sphere tracing
- **Rendering equation solvers**
  - Radiosity, Path tracing, Photon mapping
- **Hybrid approaches**

Quiz





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