Computer Graphics

Raycasting. Raytracing. Raymarching.

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Next

- Raycasting
- Raytracing
- Raymarching / Sphere tracing

- Rendering equation solvers
  - Radiosity, Photon mapping, Path tracing

The “Emission theory”

- Ancient greeks (up to Aristotle and even later) believed that you see by using beams emanating from your eyes.

- “Winer et al. (2002) have found recent evidence that as many as 50% of American college students believe in emission theory.[3]”

http://en.wikipedia.org/wiki/Emission_theory_(vision)

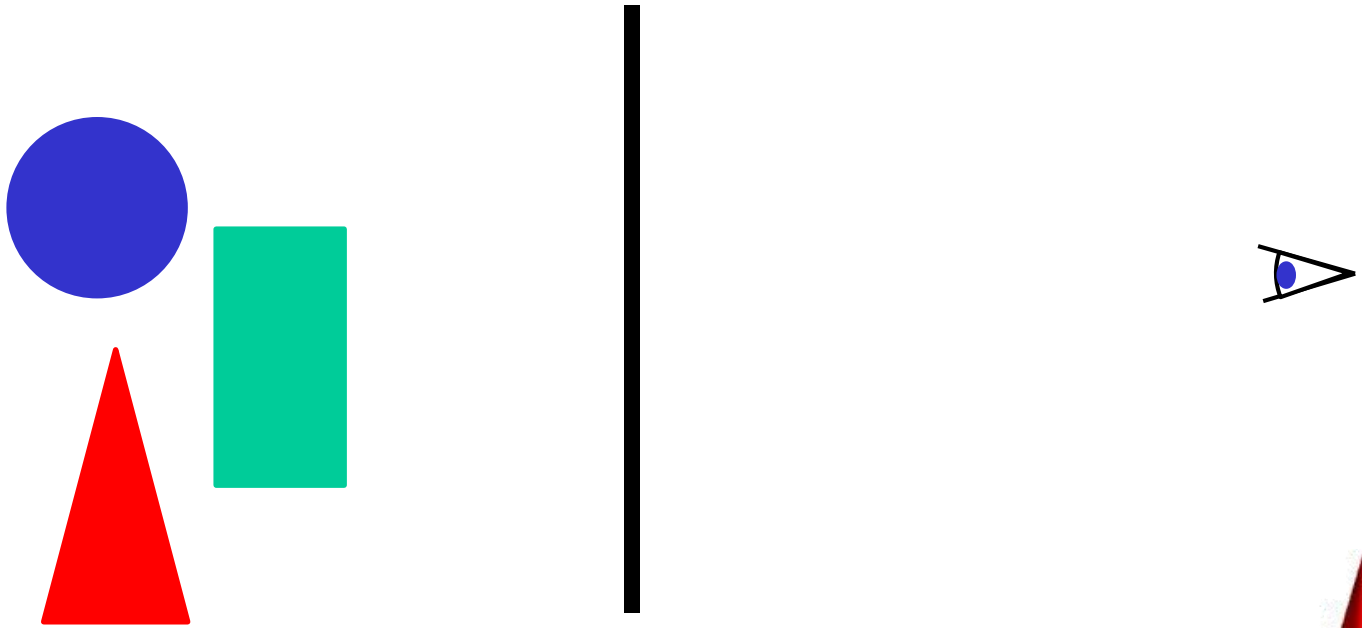
The “Emission theory”

- Mathematically, the two theories are equivalent in many respects, because light propagation rules are mostly direction-independent.

- Consequently, we can use the “Emission” approach in rendering. It is called raycasting.

Graphics Pipeline vs Raycasting

- **Graphics pipeline:**
  - For each object, find its location on screen (and then color each pixel appropriately)



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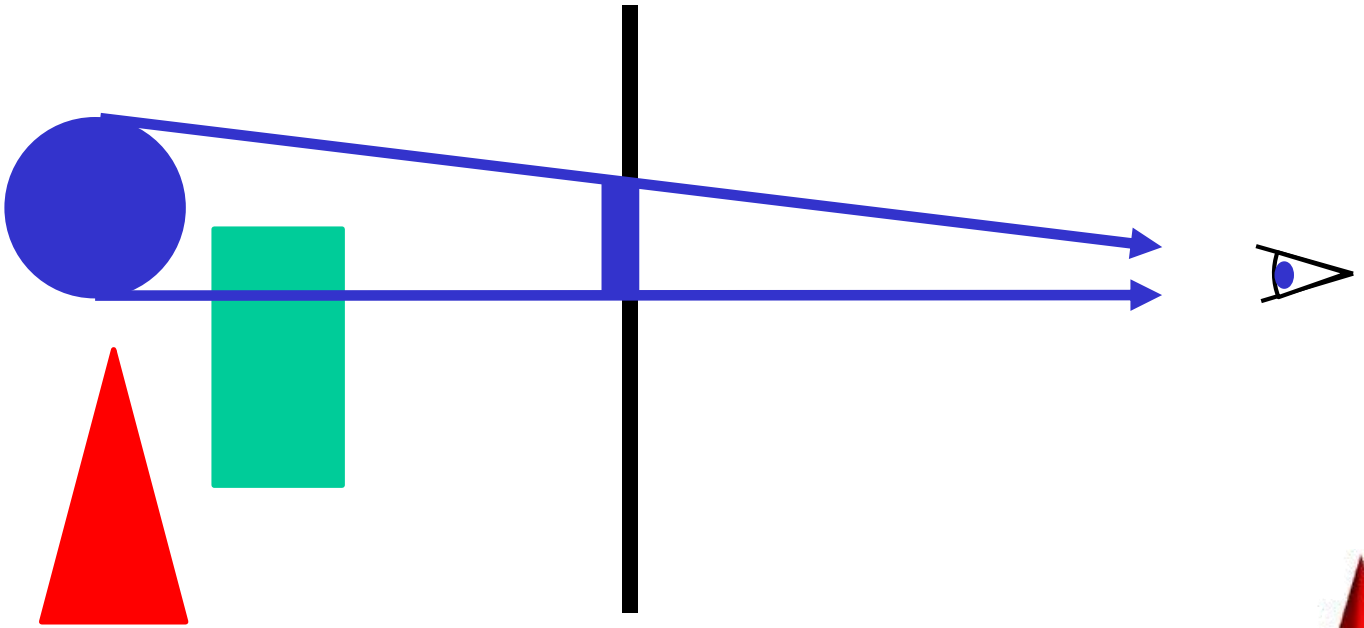

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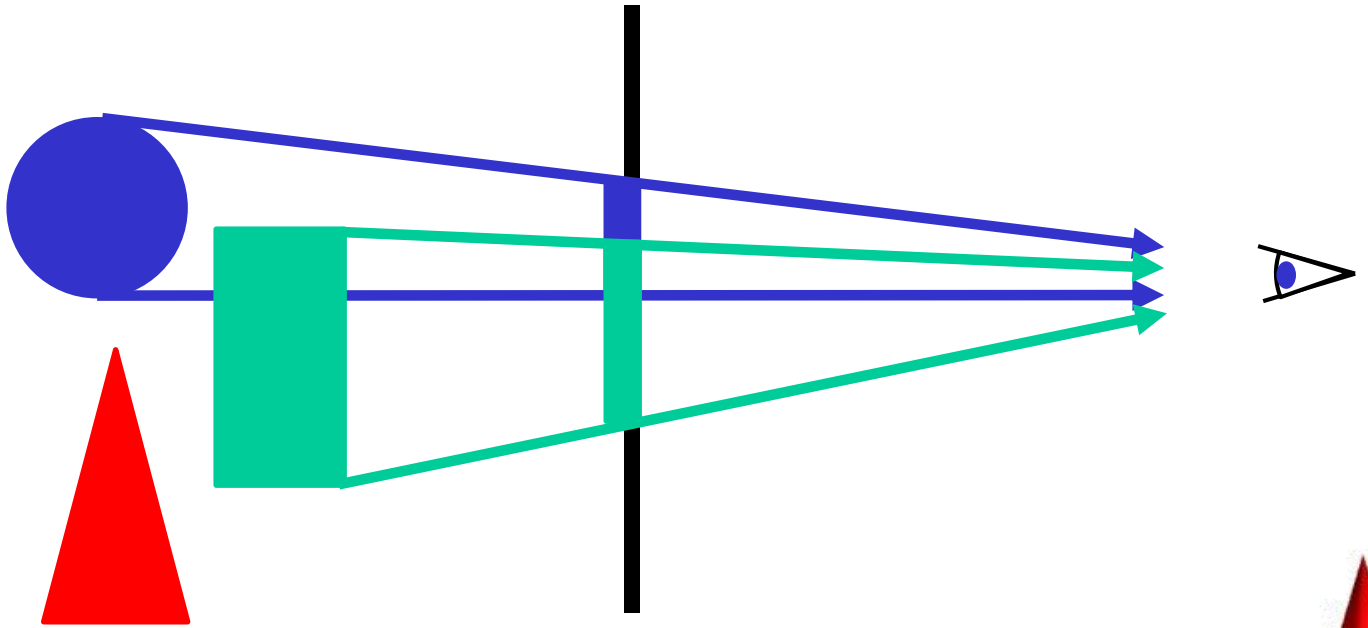

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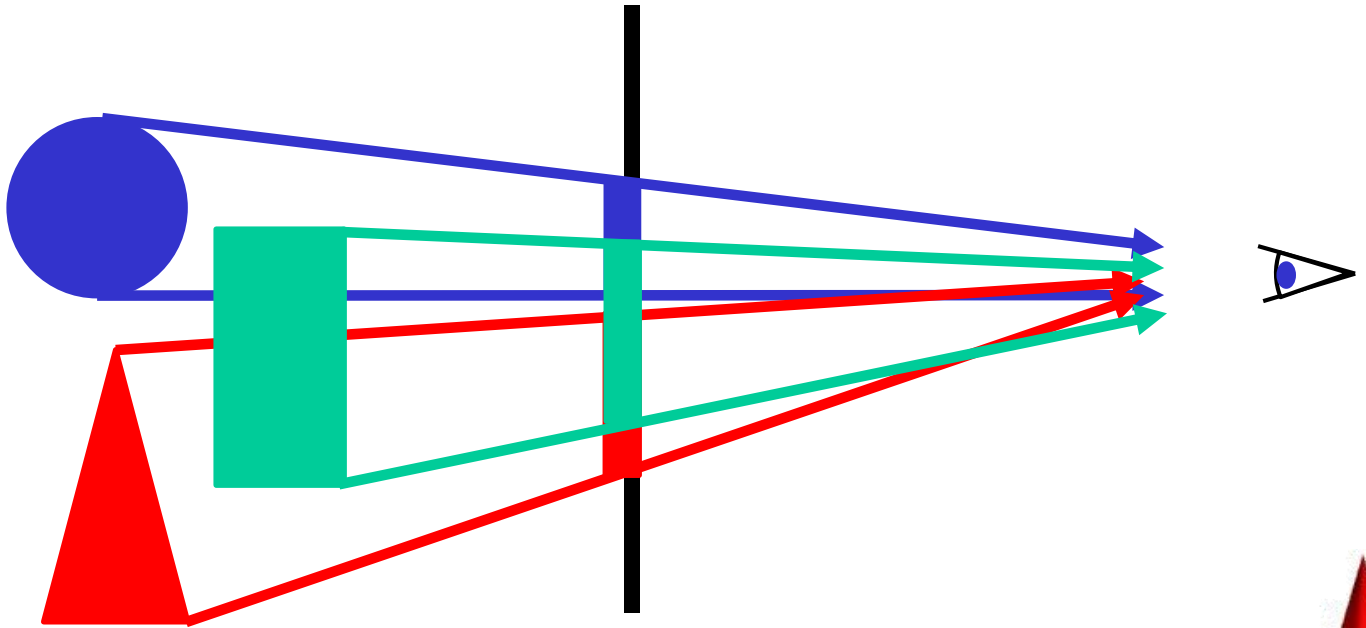

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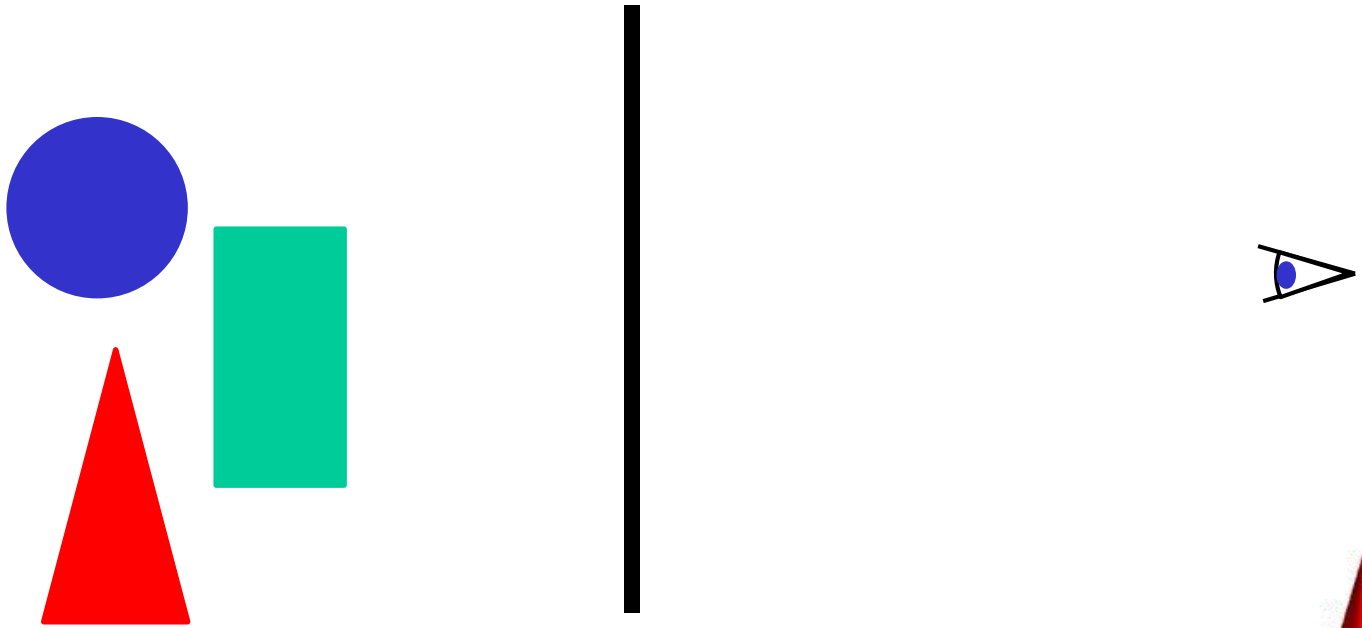

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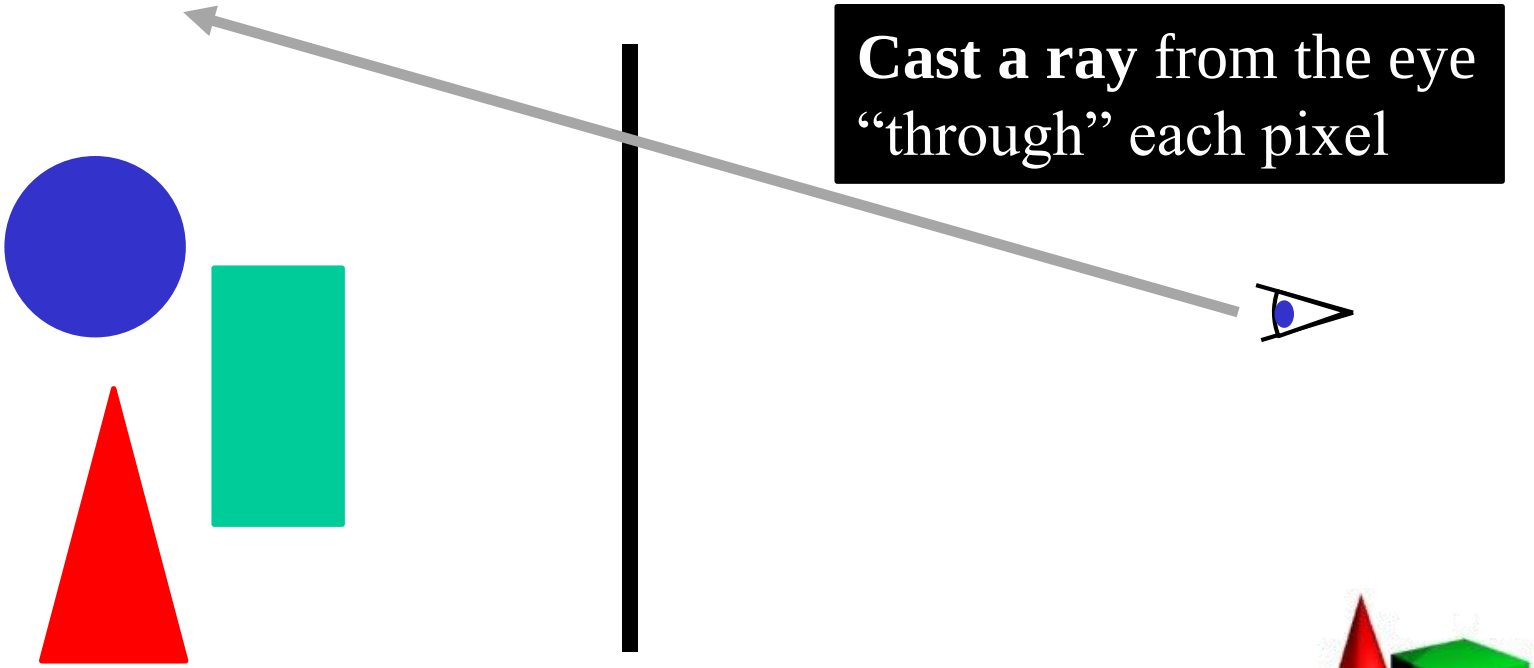

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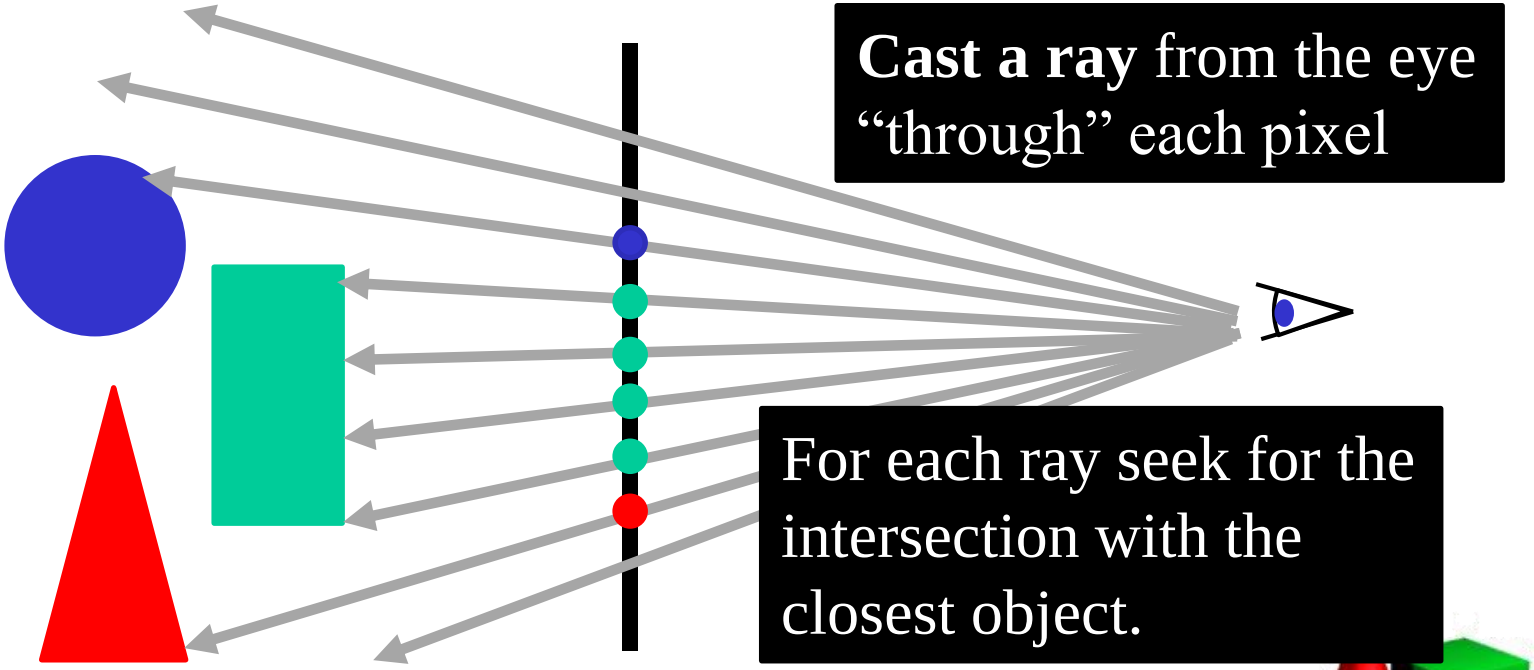
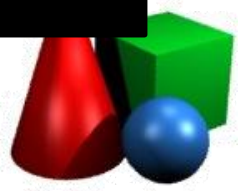
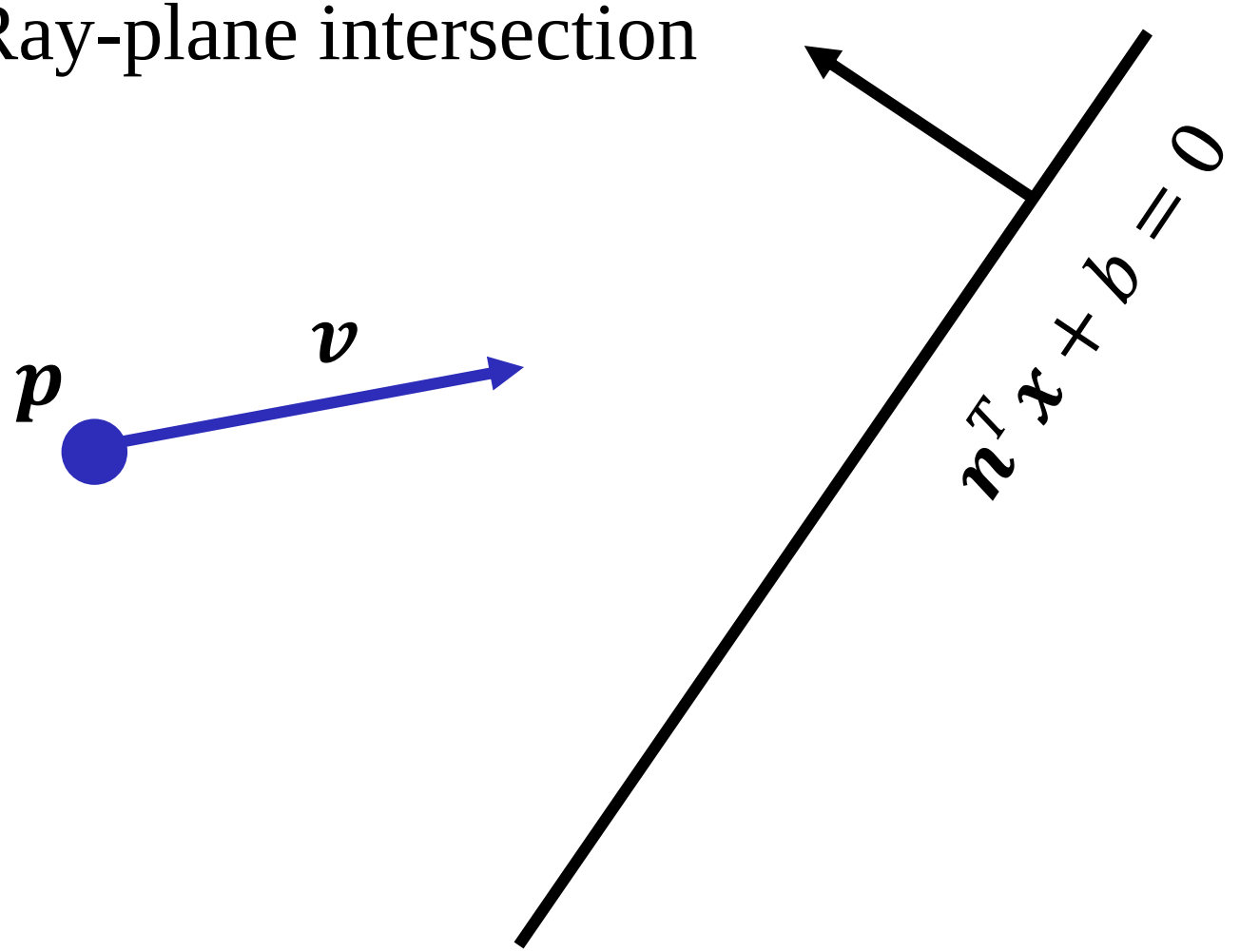




Example: Plane

Ray-plane intersection



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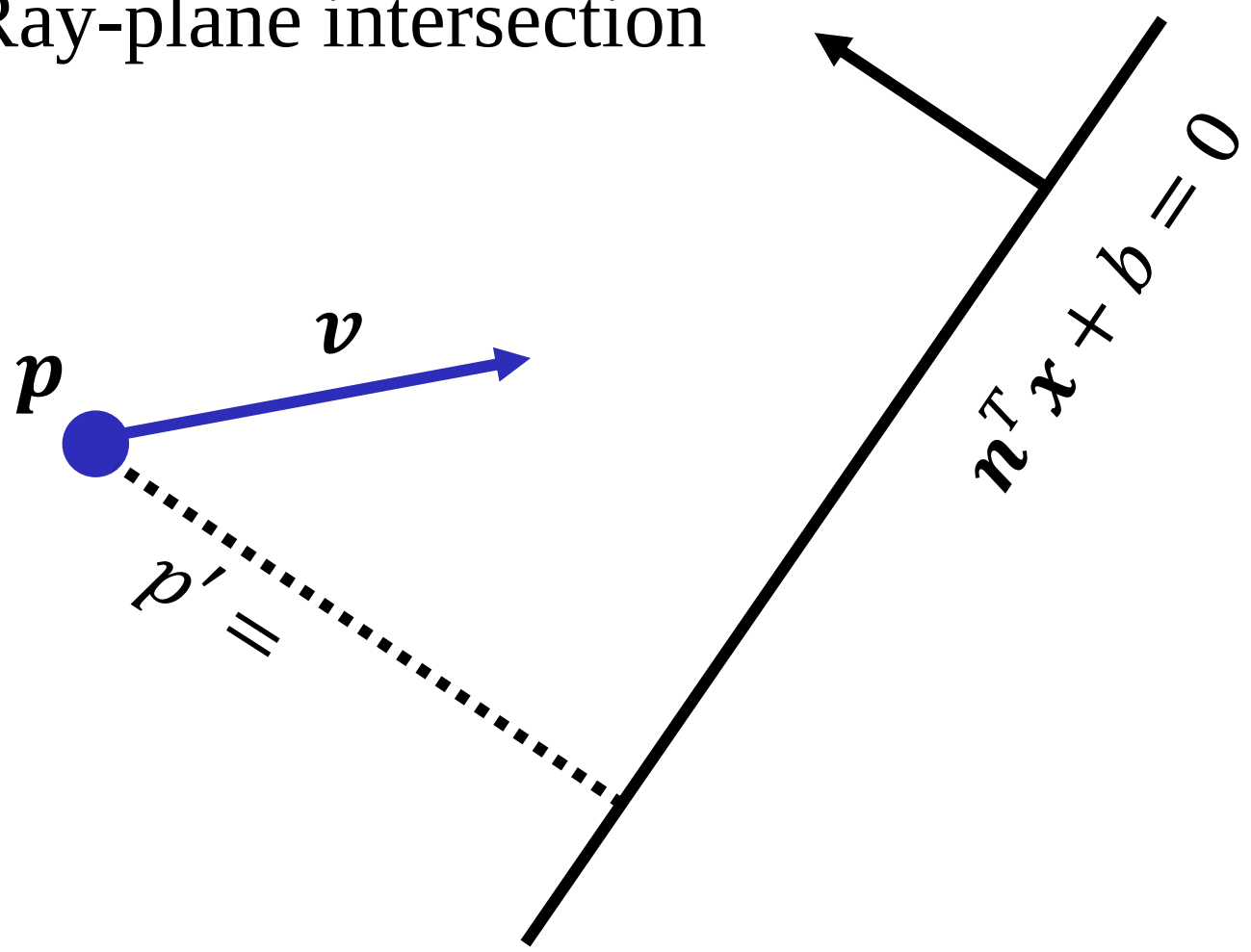

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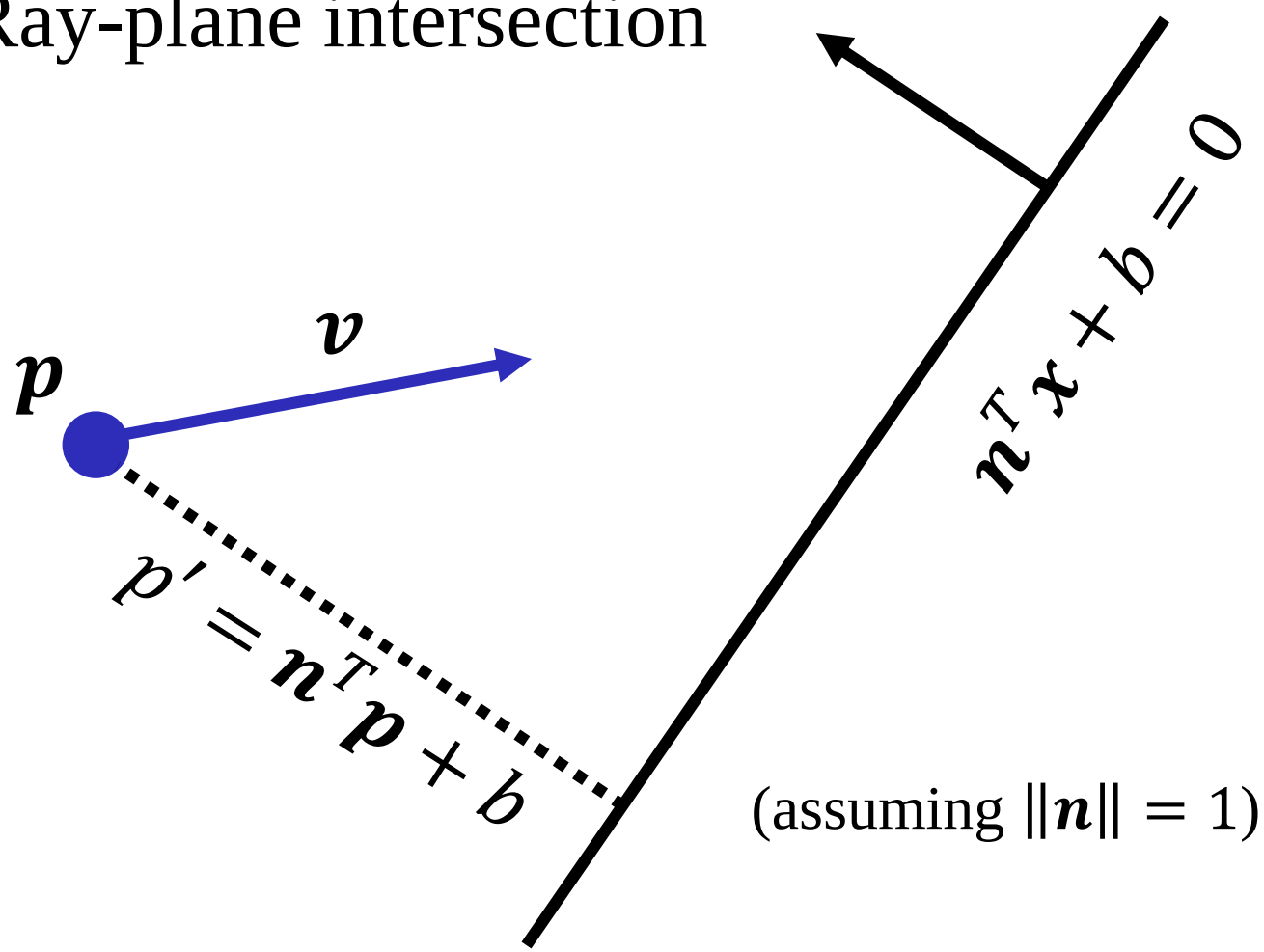

(assuming $\|\boldsymbol{n}\| = 1$)

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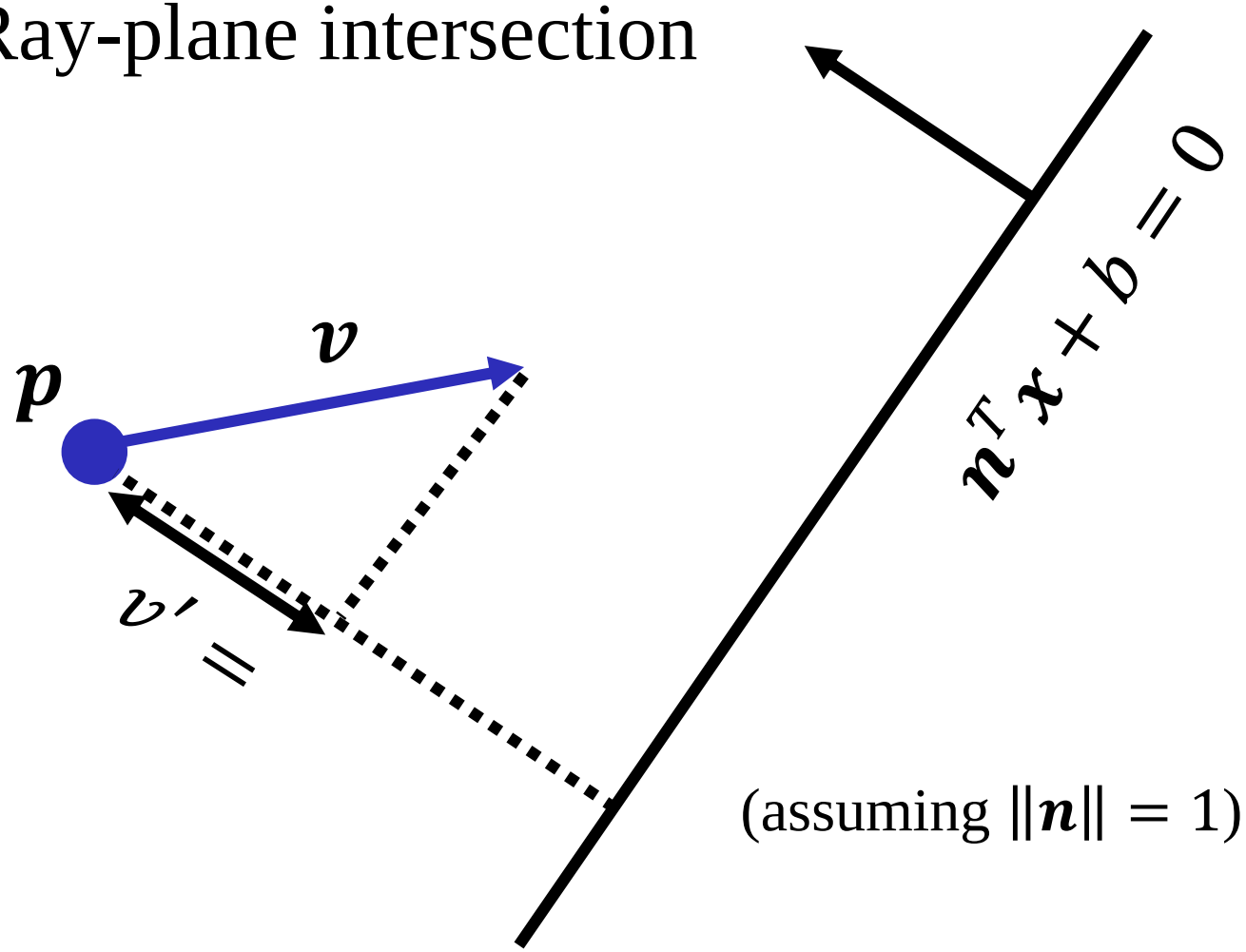

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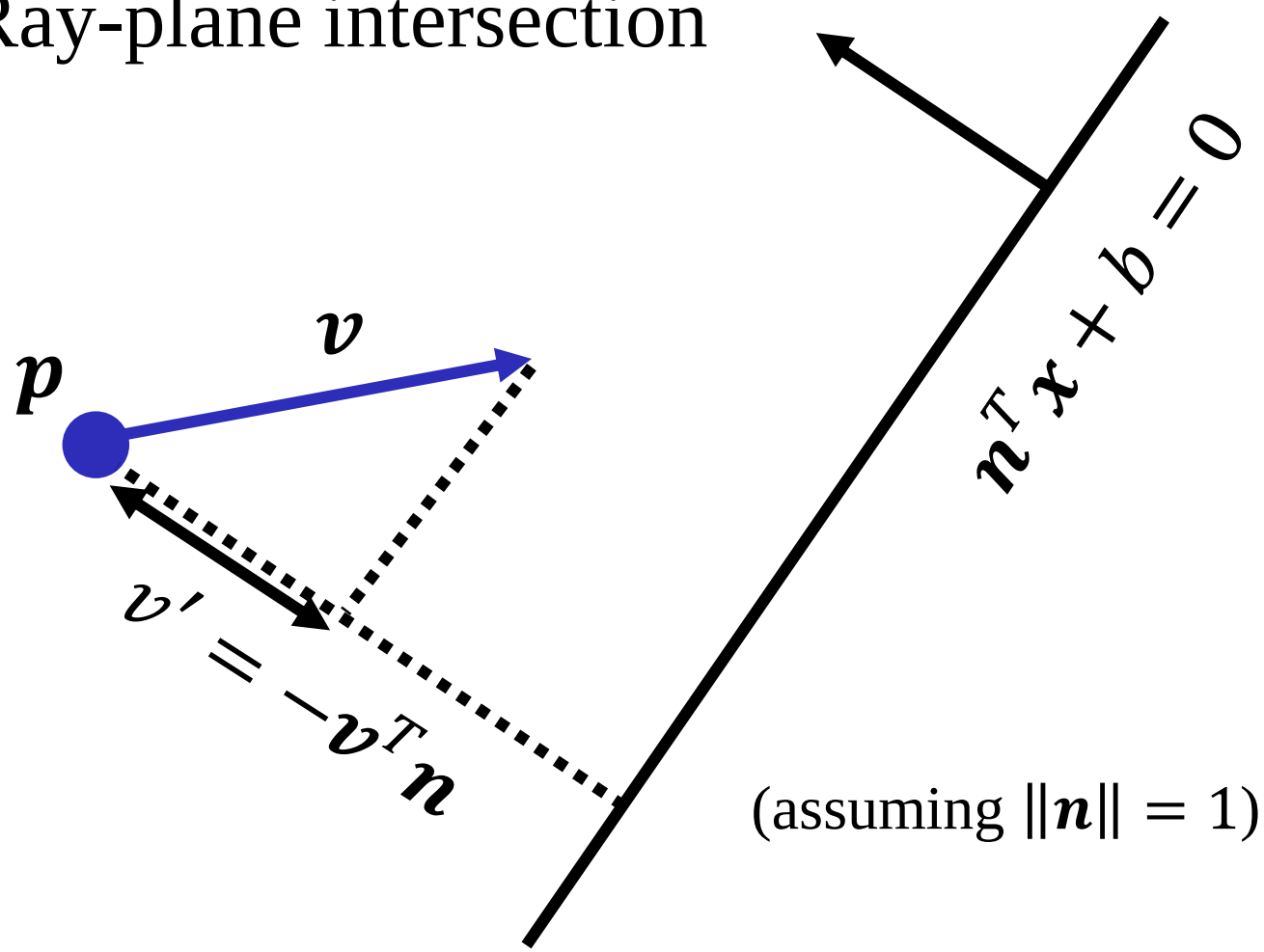

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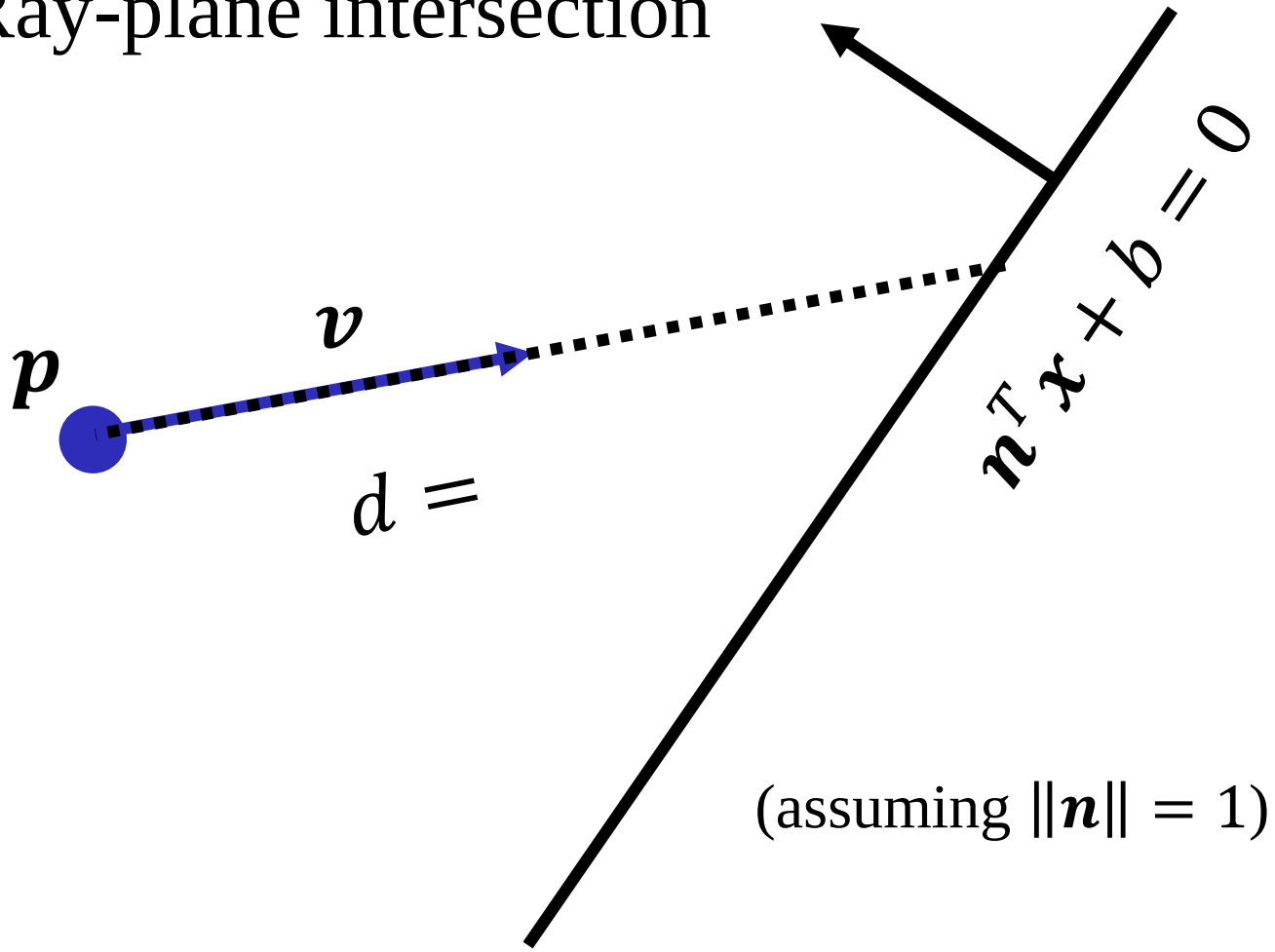

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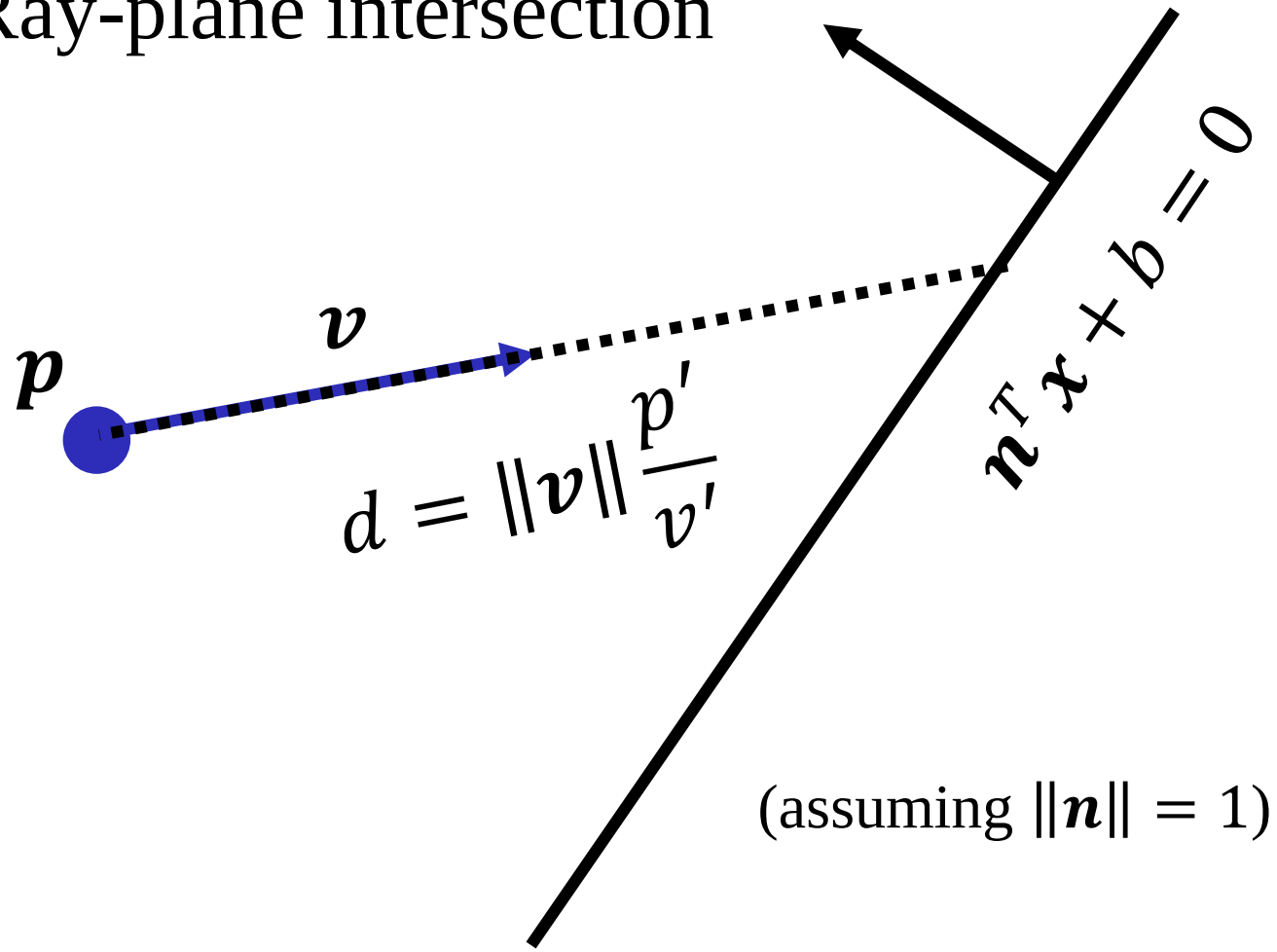

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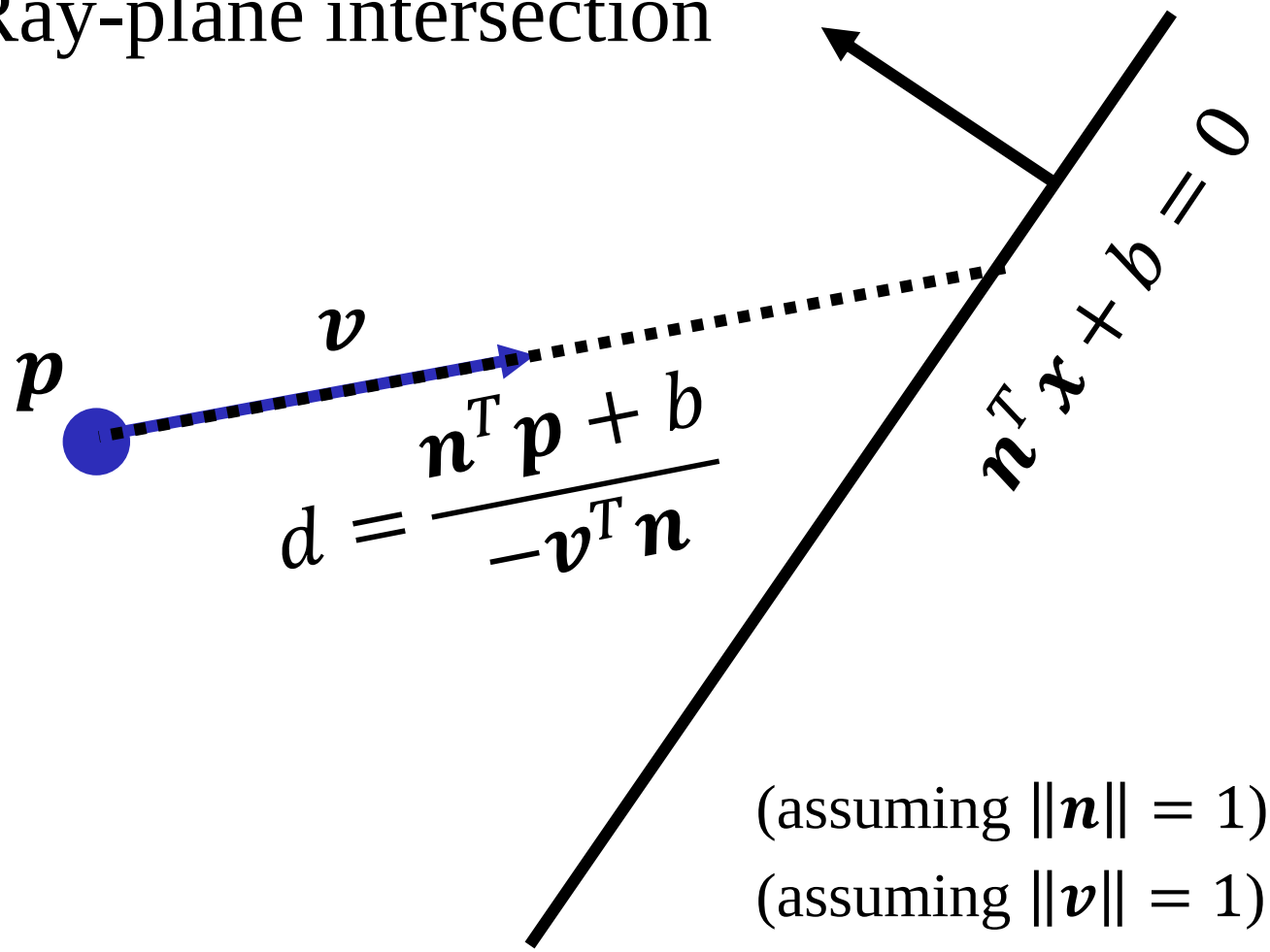

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Example: Plane

```
float raycast_plane(vec3 from, vec3 dir,
                    vec3 n,    float b) {
   if (dot(n, dir) == 0)
      return -1;
   else
      return -(dot(n, from) + b)/dot(n, dir);
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Separate ray-surface intersection routine can be derived for every type of surface.

Quiz

- Suppose we have n objects, each with screen area ~1000 pixels. Let the total screen area be 800x600 = 480 000 pixels.

- Complexity of (naïve) graphics pipeline:
  - ?
- Complexity of (naïve) raycasting:
  - ?

Quiz

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- Complexity of (naïve) raycasting:
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Quiz

- Suppose we have n objects, each with screen area ~1000 pixels. Let the total
  - $1000\,n$
- Complexity of (naïve) raycasting:
  - $480\,000\,n$

In the worst case (each object occupies the whole screen) the complexities are equal.

Graphics Pipeline vs Raycasting

- Theoretically, the complexity of naïve raycasting is not worse than that of the graphics pipeline.

Graphics Pipeline vs Raycasting

- Theoretically, the complexity of naïve raycasting is not worse than that of the graphics pipeline.
- In fact, in the old days, properly-done (1D) raycasting was more efficient.

Graphics Pipeline vs Raycasting

- However,
  - Raycasting parallelizes over **outputs** (pixels) and uses **full scene data for each pixel**.
  - GP parallelizes over **inputs** (splits input into triangles) which makes memory access of each subtask *local*.
- For large scenes the latter approach fits the architecture of modern hardware much better (shared memory access is slow).

Benefits of Raycasting

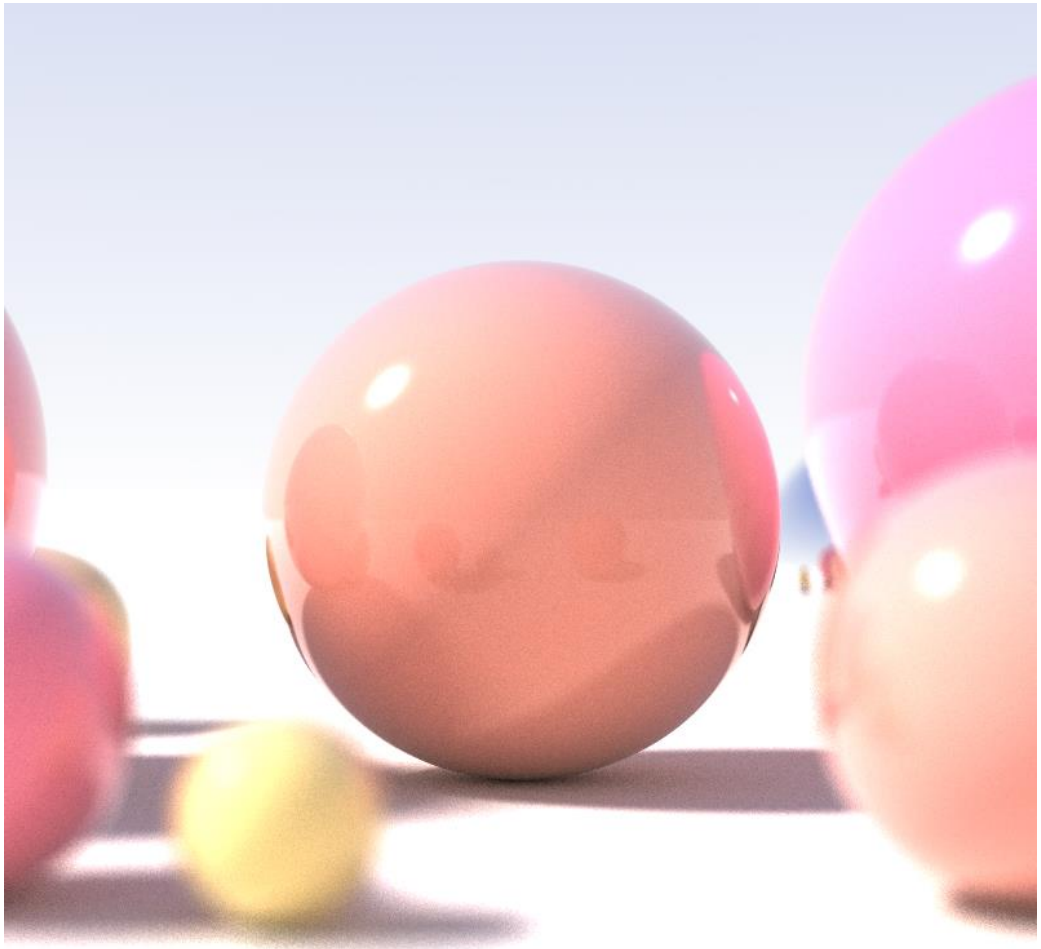
- On the other hand, having full access to the scene while rendering lets you implement **global illumination** effects much more naturally than in the standard graphics pipeline.

Global vs Local illumination

- **Local illumination model**:
  - Use only the information about light sources in lighting computations. Other objects in the scene do not matter.

- **Global illumination model**:
  - Take into account light scattering and occlusion due to **all** objects in the scene:
    - Shadows, reflection, refraction, ambient scattering, ambient occlusion

Global vs Local illumination



http://en.wikipedia.org/wiki/File:Recursive_raytrace_of_a_sphere.png

Raytracing

- **Raytracing** :=
  - Basic raycasting
  - + Shadows
  - + Reflections
  - + Refractions
  - + Recursion
  - + Backwards raytracing
  - + Stochastic raytracing

Shadows via Raycasting

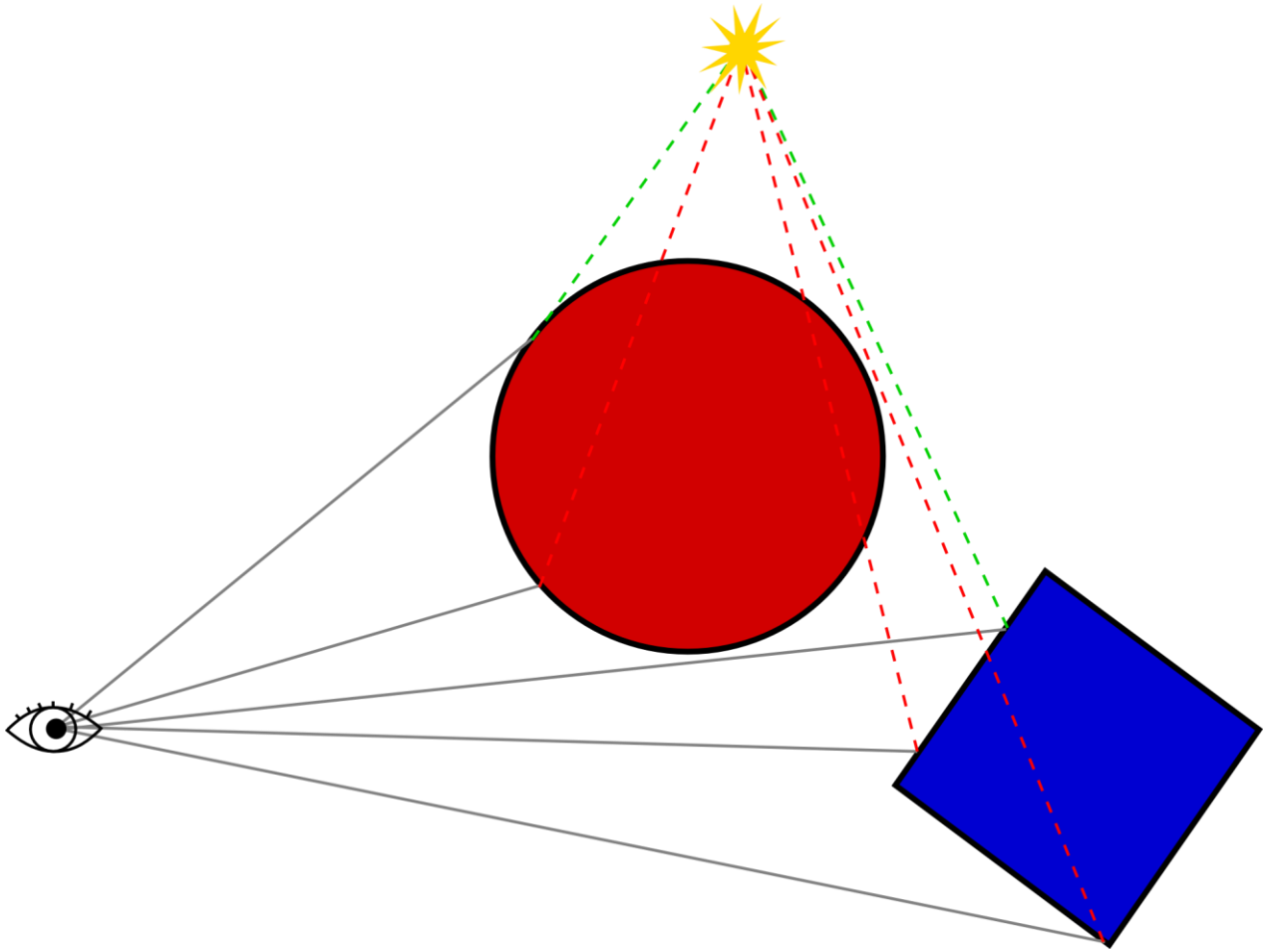
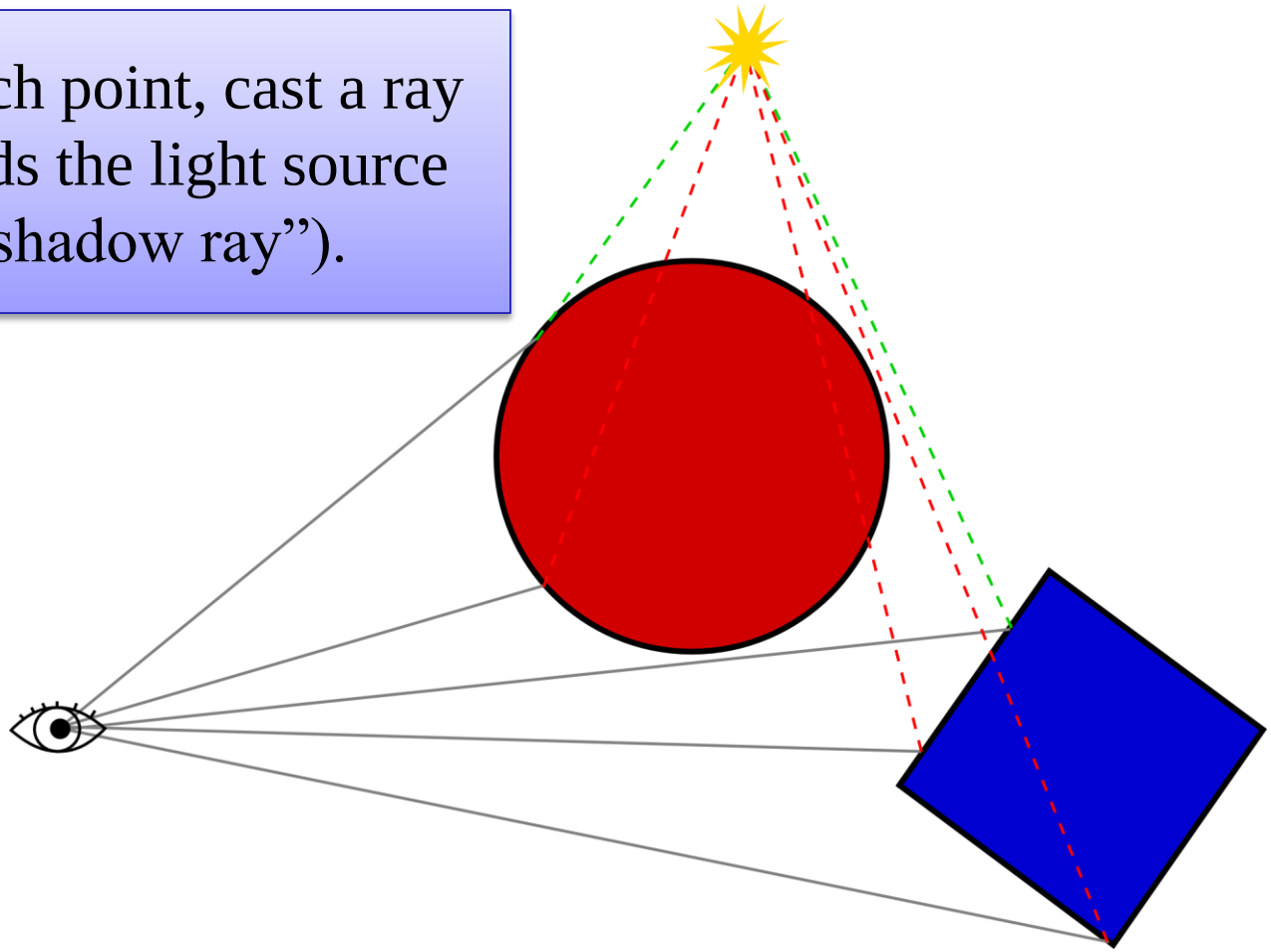


Illustration © Mark Fishel

Shadows via Raycasting

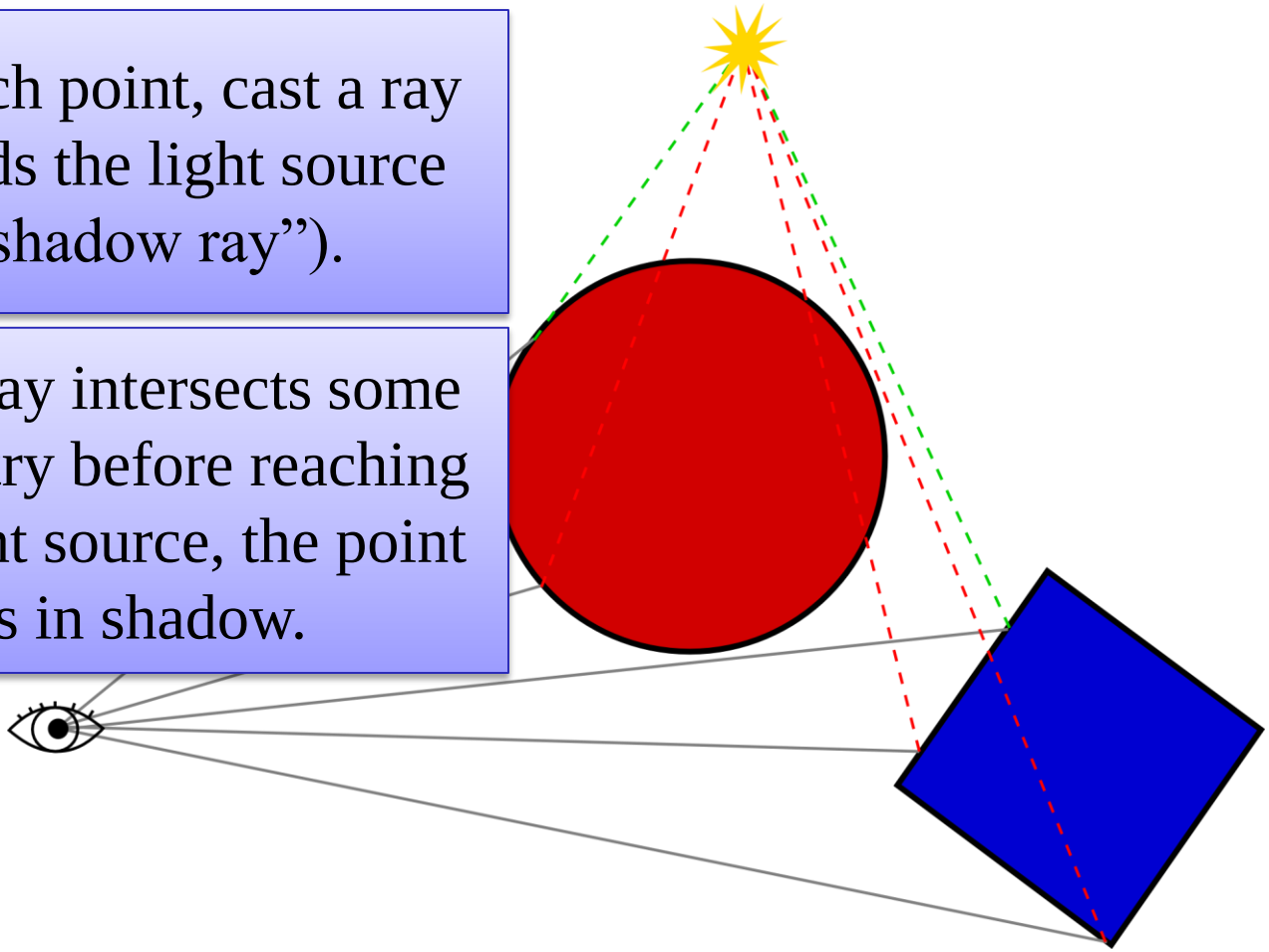
For each point, cast a ray towards the light source ("shadow ray").



Shadows via Raycasting

For each point, cast a ray towards the light source (“shadow ray”).

If the ray intersects some geometry before reaching the light source, the point is in shadow.



Reflections via Raycasting

- For each point, continue the ray in a **reflected** direction, and combine the local lighting of the point with whatever is hit by the reflected ray.

$$C_{local\ model} + k_{refl} C_{refl}$$

(where k_{refl} is the "reflectivity coefficient")

Refractions via Raycasting

- For each point, continue the ray in a **refracted** direction, and combine the local lighting of the point with whatever is hit by the reflected ray + refracted ray

$$C_{local\ model} + k_{refl} C_{refl} + k_{refr} C_{refr}$$

Recursive Raytracing

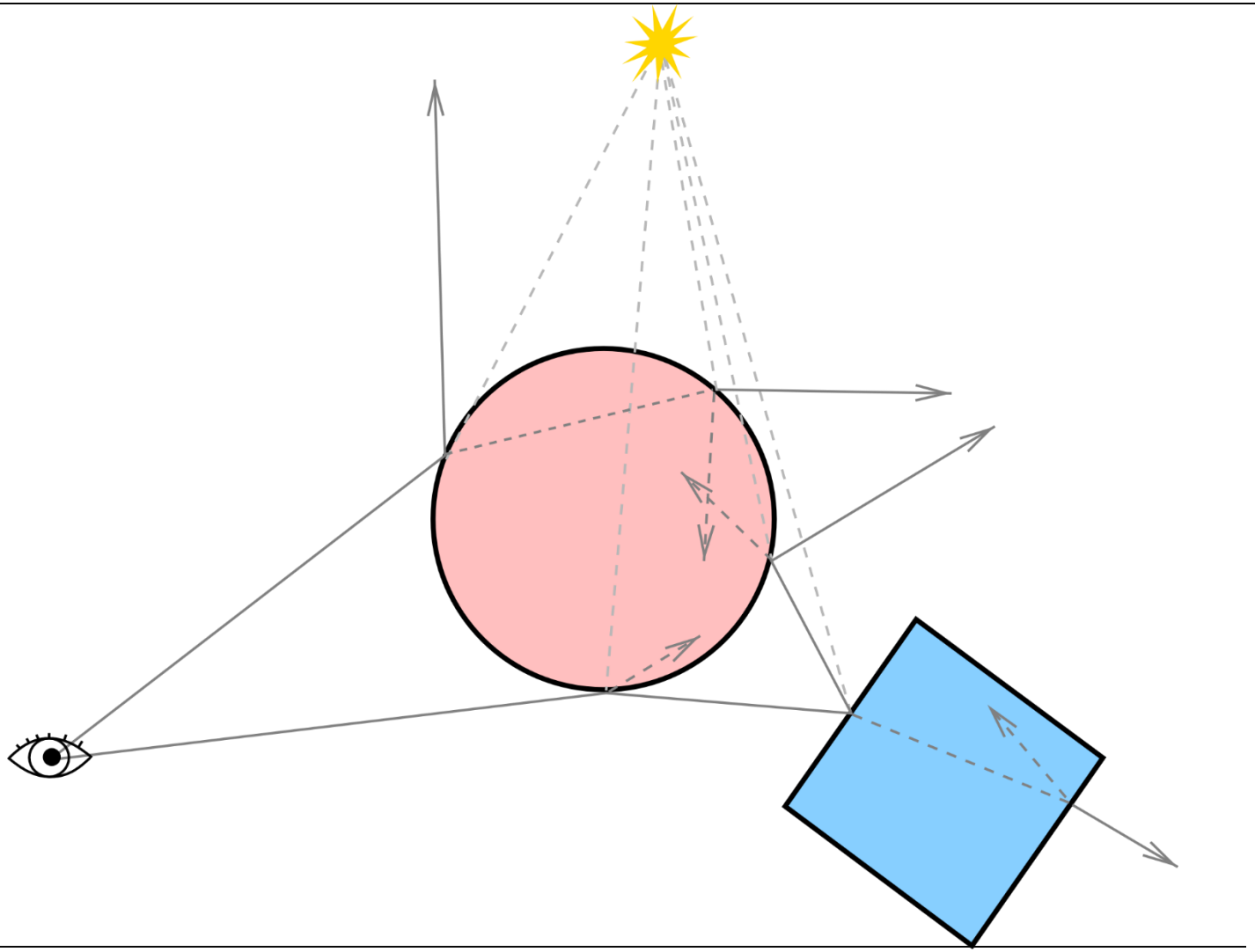


Illustration © Mark Fishel

Faster Raytracing

- **BVHs & BSPs**:
  - Quiz: What were those?

Faster Raytracing

- **BVHs & BSPs**:
  - Bounding Volume Hierarchies allow to quickly cull away irrelevant portions of the scene. Binary Space Partitioning trees let you seek for ray intersections in a front-to-back order.

Faster Raytracing

- "**Object z-buffer**":
  - Pre-render the scene using the Graphics pipeline, storing for each pixel the id of the topmost object rendered at that pixel.
  - This effectively replaces the first raycasting step with a graphics pipeline, making it faster.

Faster Raytracing

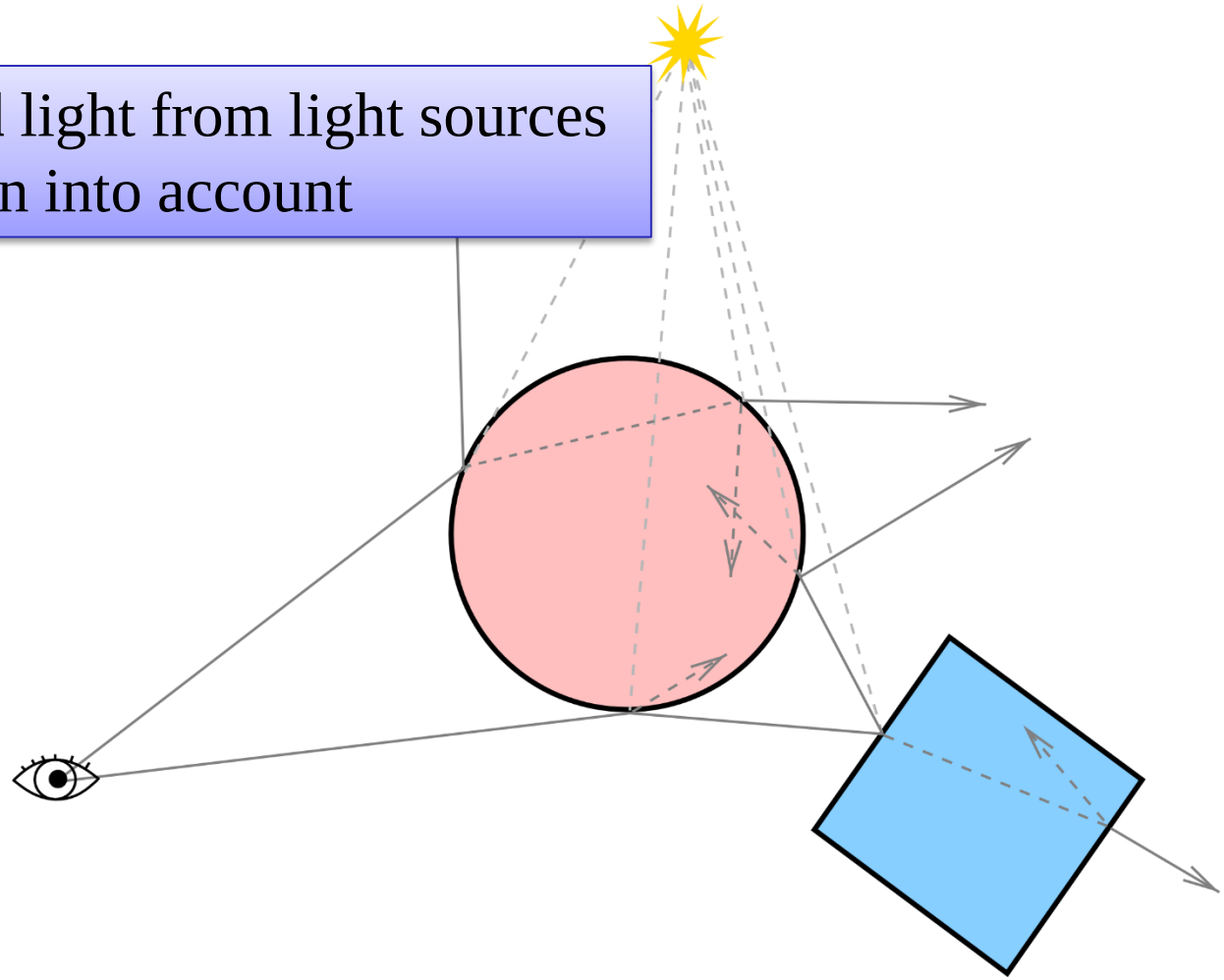
- **Adaptive recursion depth**:
  - Keep track, for each ray, what will be its final contribution to the pixel it originated from.
  - E.g. if after hitting a surface we reflect with reflection coefficient 0.2, then refract with coefficient 0.1, we may stop as further computations won’t contribute more than 0.02.

Faster Raytracing

- **Shadow mapping**:
  - Instead of casting each “shadow ray”, use a precomputed depth buffer like in shadow mapping.

Refracted lighting problem

Refracted light from light sources is not taken into account



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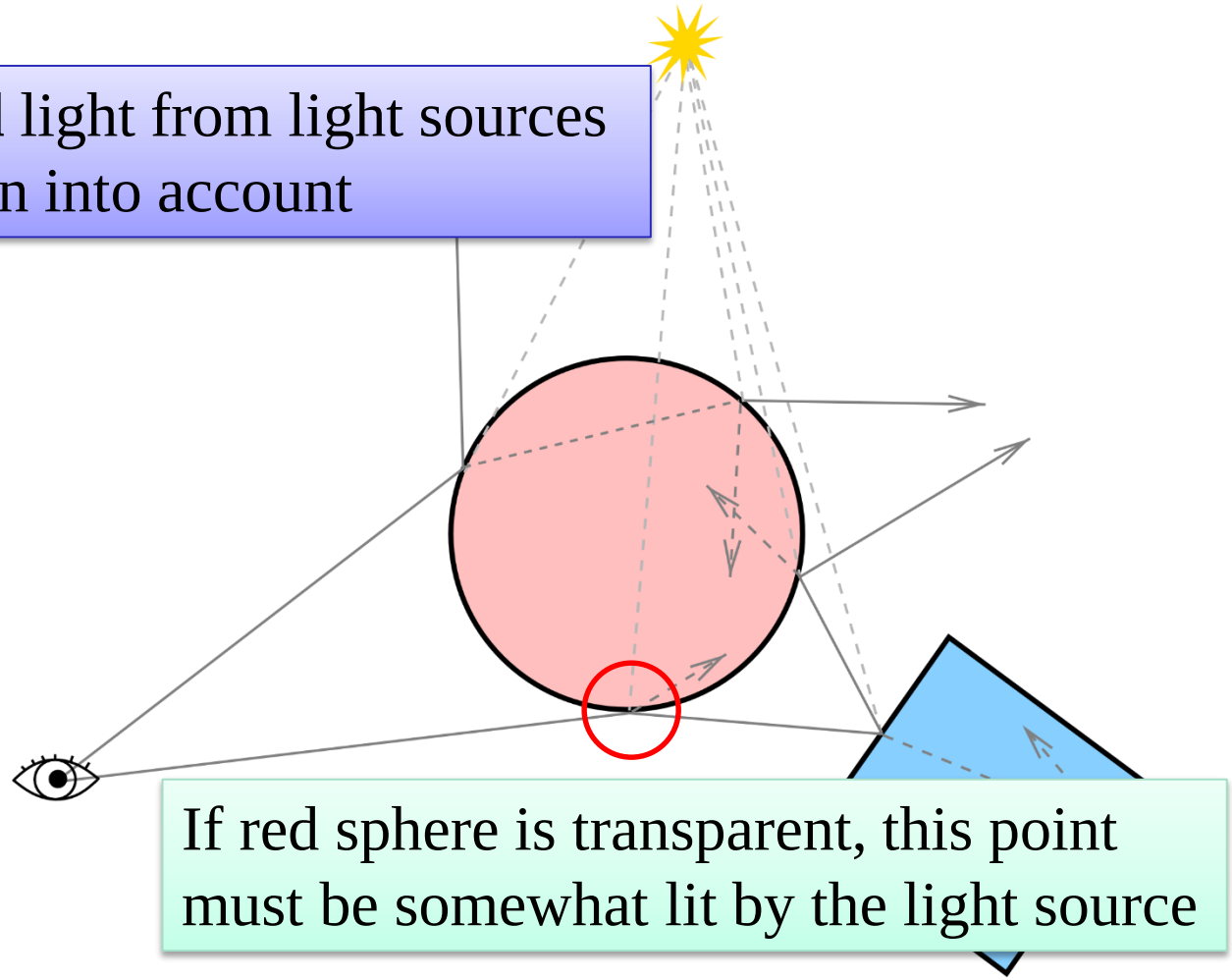


If red sphere is transparent, this point must be somewhat lit by the light source

Backwards Raytracing

- **Solution**:
  - Split geometry into patches.
  - Make a raytracing pass with rays originating from the light sources
  - For each patch store its lighting data.
  - Use this data in the normal forward pass.

Aliasing

- Aliasing can be a considerable problem for raytracing implementations.

- Solutions:
  - Cone tracing, Beam tracing, Pencil tracing
  - **Stochastic raytracing**

Stochastic raytracing

- Recall that a common way of dealing with aliasing artifacts is by using a convolution with a filter.

- In layman's terms: send multiple rays for each pixel, sampling slightly different directions, and average the results.

Basic antialiasing

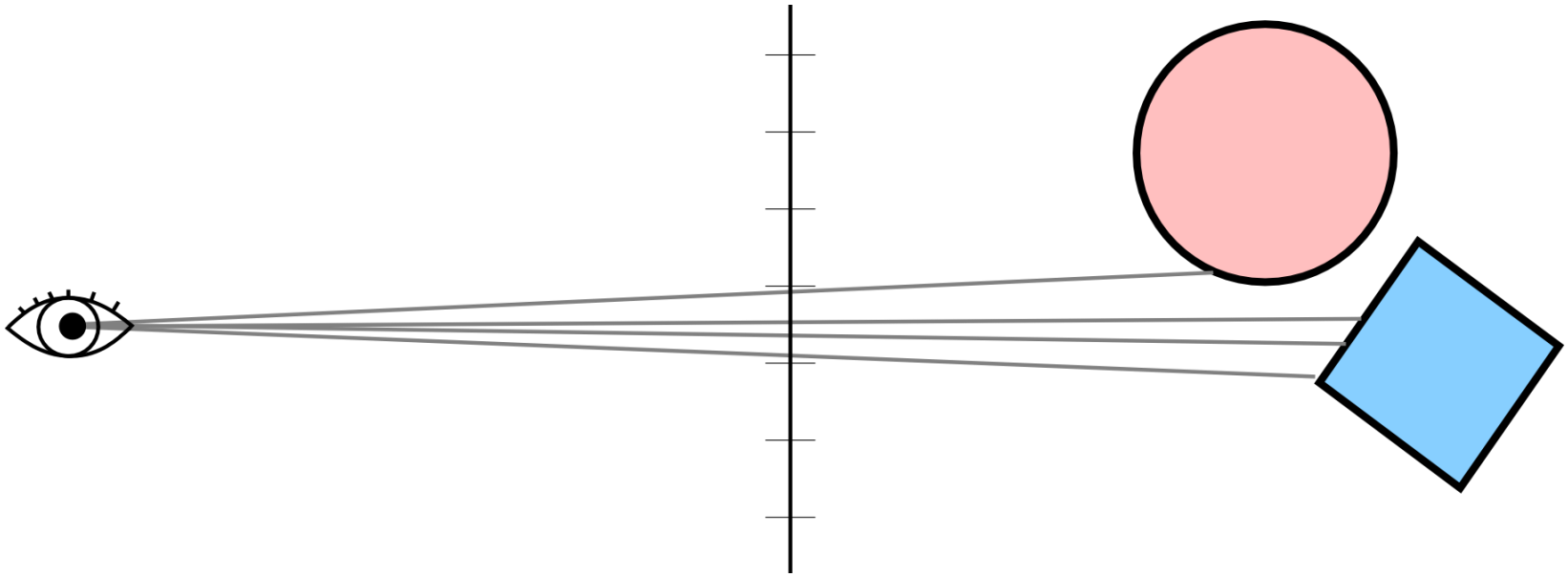
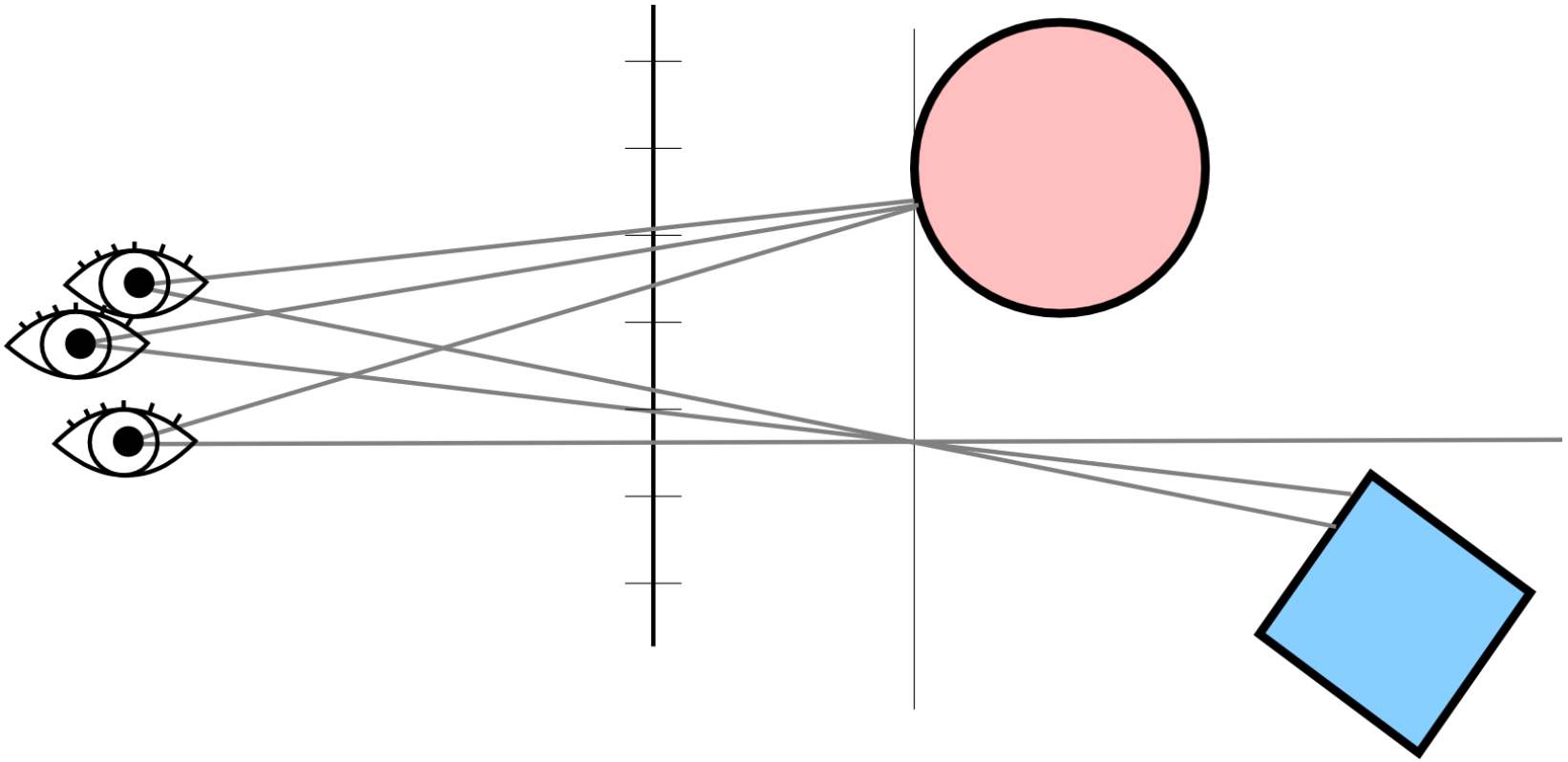


Illustration © Mark Fishel

Multiple rays per pixel

- For each ray we can vary many parameters:
  - Ray direction (basic antialiasing)
  - Ray origin (depth of field)
  - Object positions (motion blur)
  - Light source positions (soft shadows)
  - Ray light frequency (true refractions)

Depth of Field



True refraction

One ray per light frequency

Combined stochastic rays

- Suppose we want to do
  - 4x4-grid jittered antialiasing
  - + 4x4-grid jittered DOF.
  - + 10-frame motion blur

- How many rays per pixel should we send?

Combined stochastic rays

- Suppose we want to do
  - 4x4-grid jittered antialiasing
  - + 4x4-grid jittered DOF.
  - + 10-frame motion blur

- We do not need to send 16x16x10 = 2560 rays for all possible combinations!
- Choosing, say, 4x4 random parameter combinations can suffice.

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- Rendering equation solvers
  - Radiosity, Photon mapping, Path tracing

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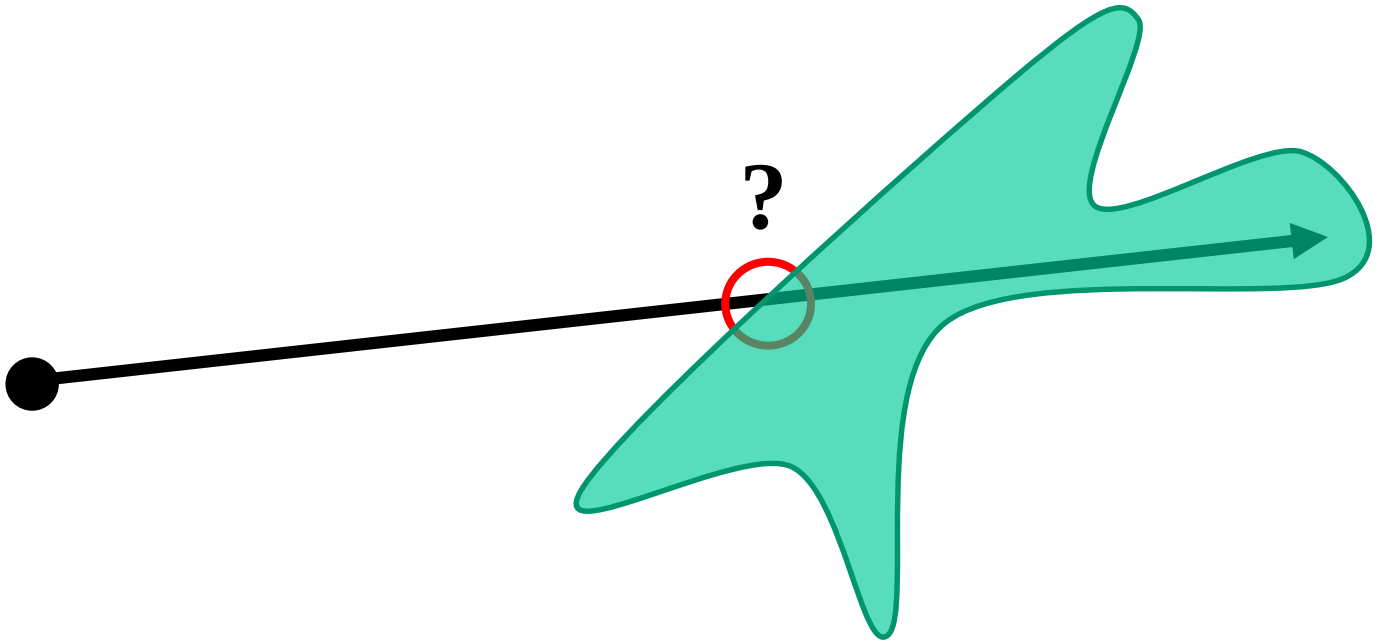
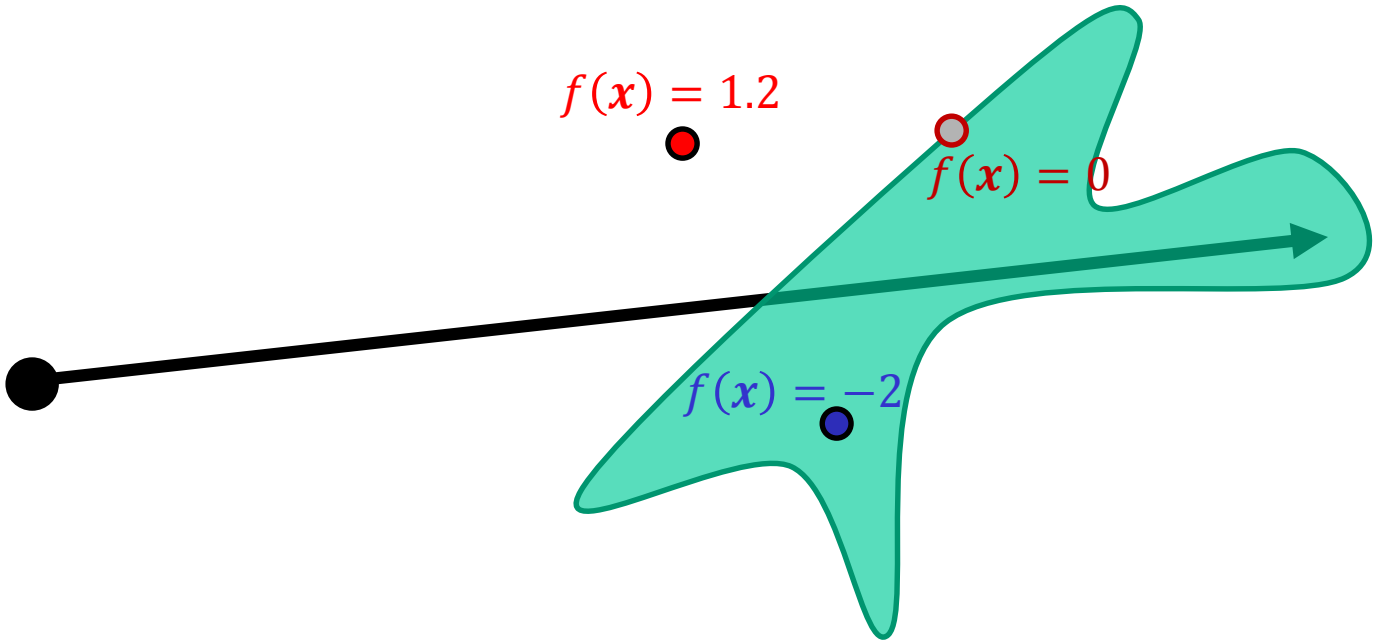
Problems with Raytracing

- Finding ray intersections with complex shapes can be hard, and not always analytically possible.



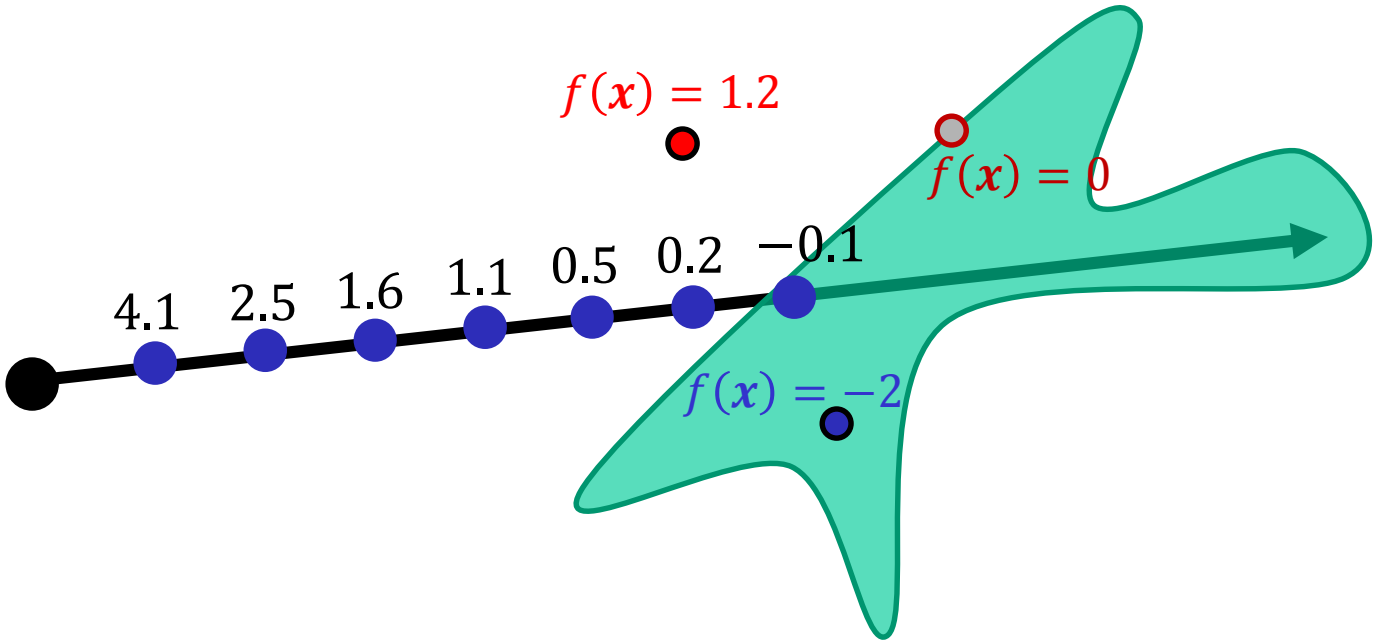
Raymarching

- Idea: use implicit surface representation:
  - $f(\boldsymbol{x}) = 0$, for points $\boldsymbol{x}$ on the surface.
  - $f(\boldsymbol{x}) > 0$, for points outside the shape.



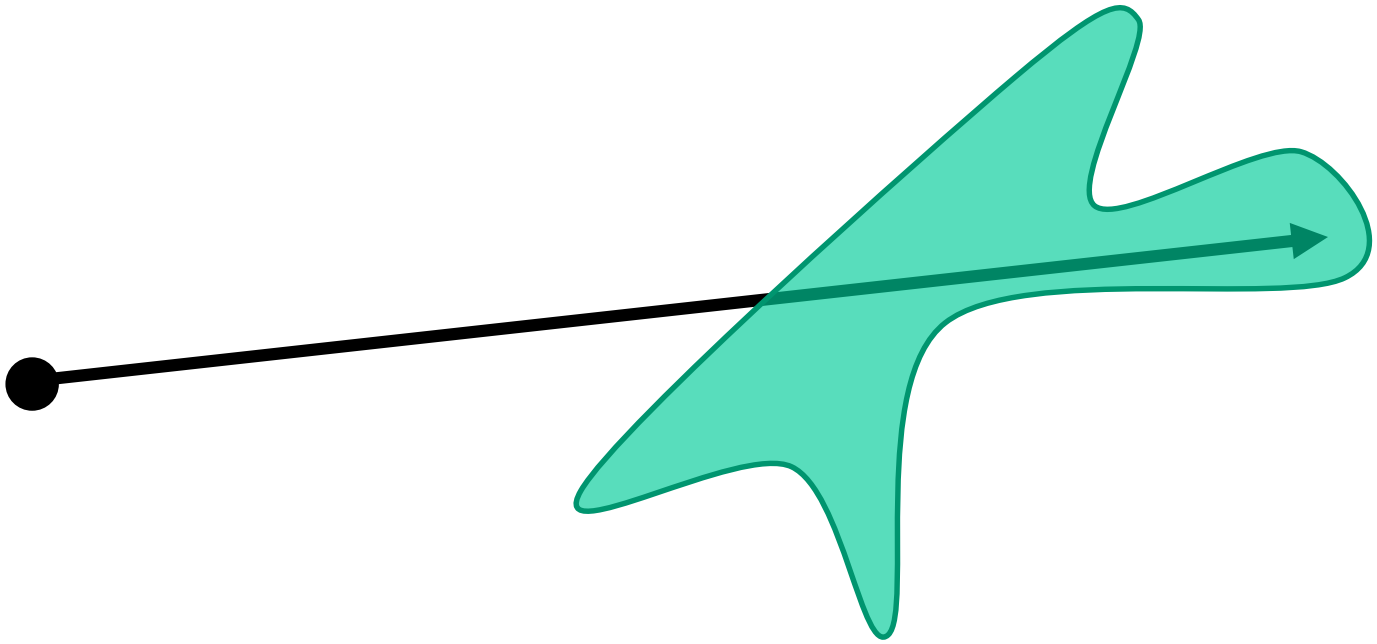
Raymarching

- Now we can “walk” along the ray in small steps, seeking for the point where $f(\boldsymbol{x}) = 0$



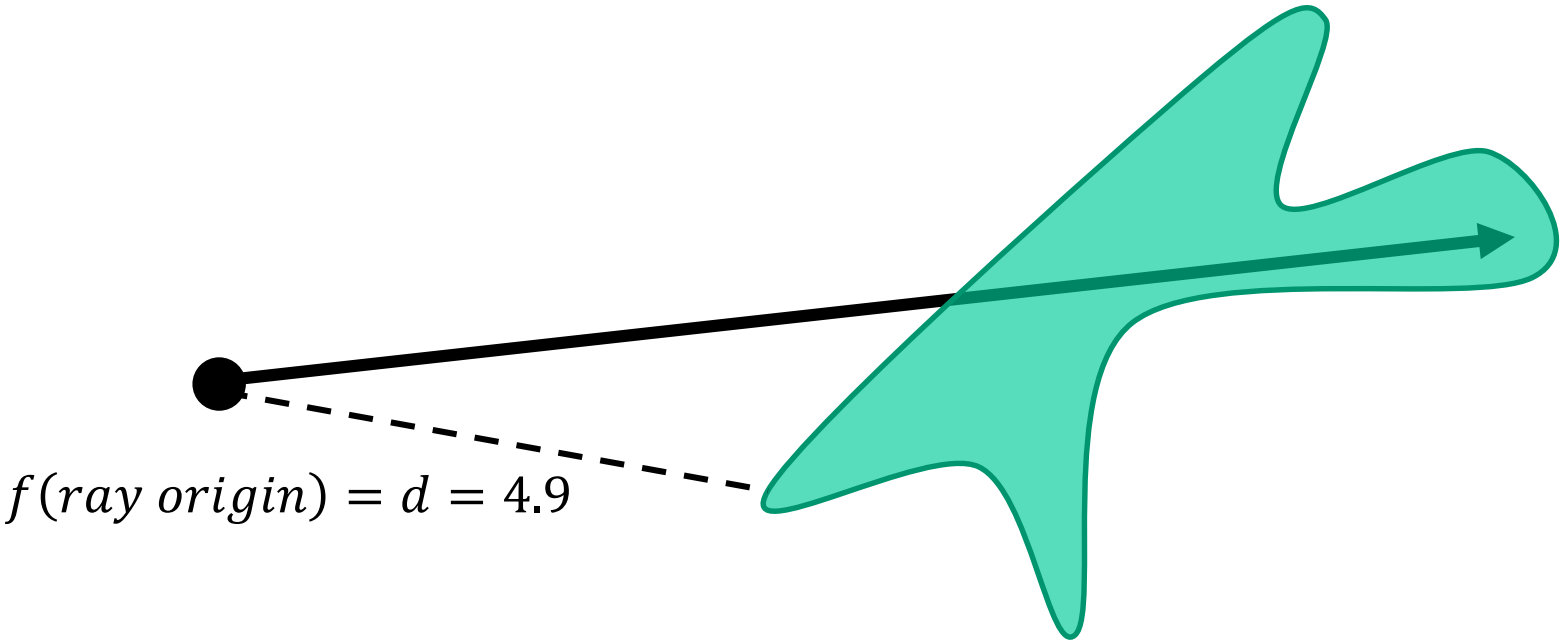
Sphere tracing

- Even better if $f(\boldsymbol{x})$ is a **lower bound on the distance** to any shape in the scene. Then we can safely use step size $f(\boldsymbol{x})$ along the ray.



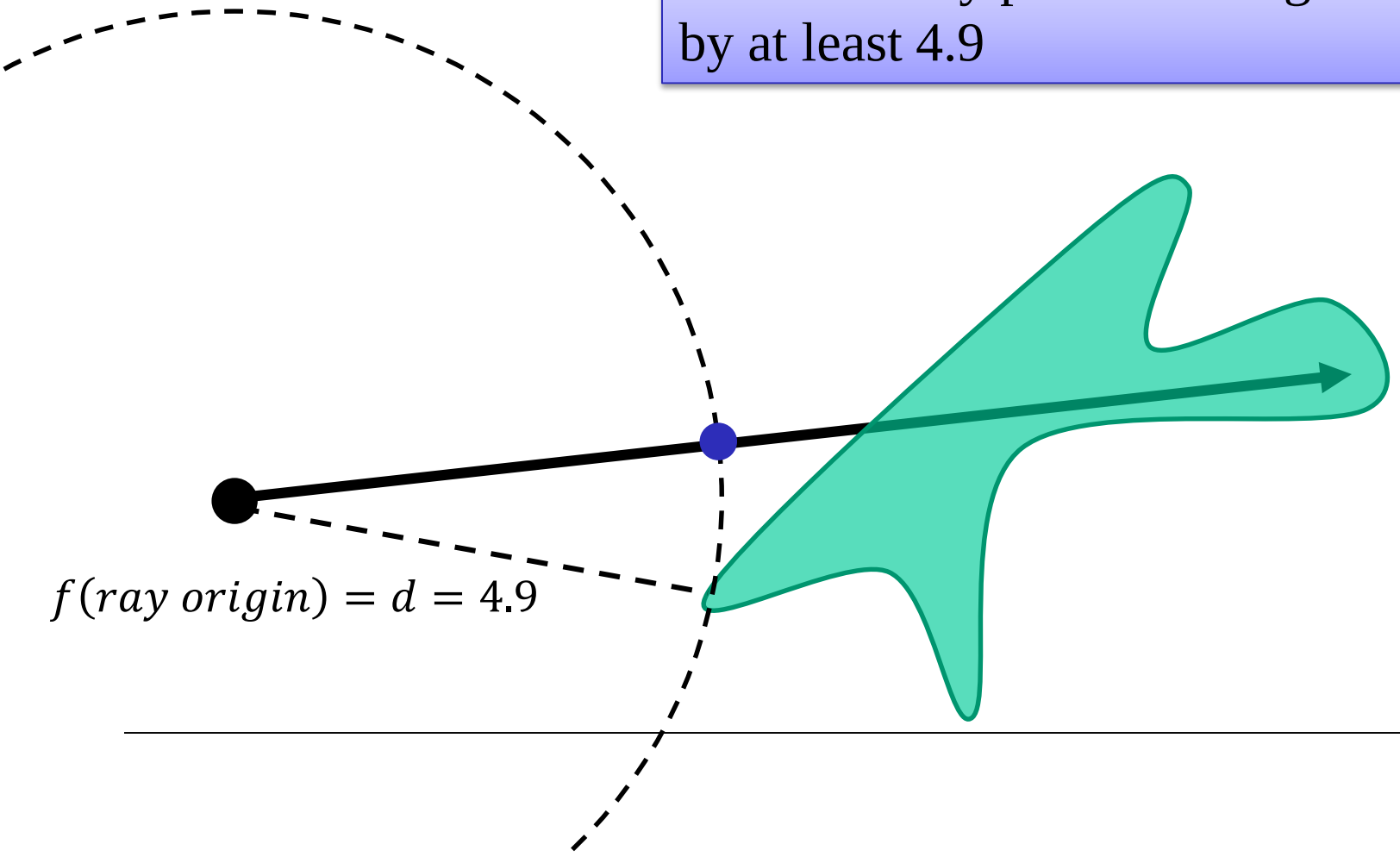
John C Hart. Sphere tracing: a geometric method for the antialiased ray tracing of implicit surfaces
http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.87.339&rep=rep1&type=pdf

Sphere tracing



Sphere tracing

We can safely proceed along the ray by at least 4.9



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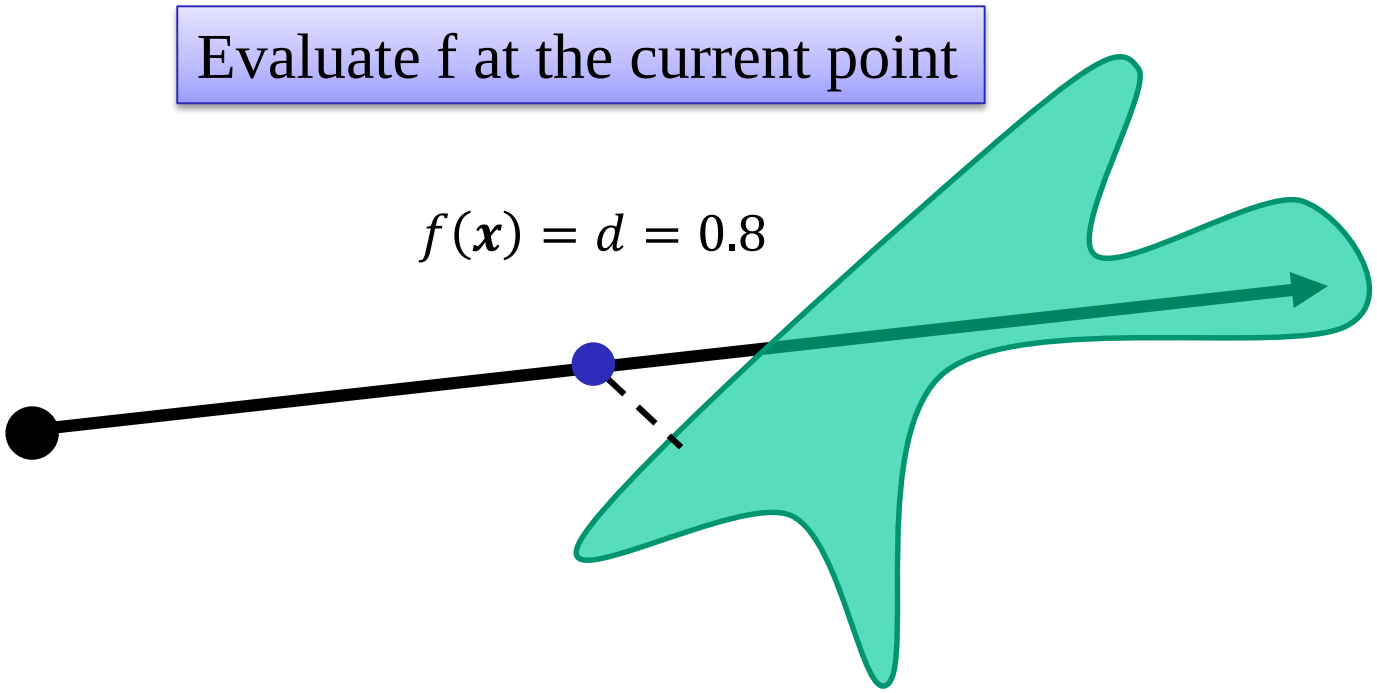
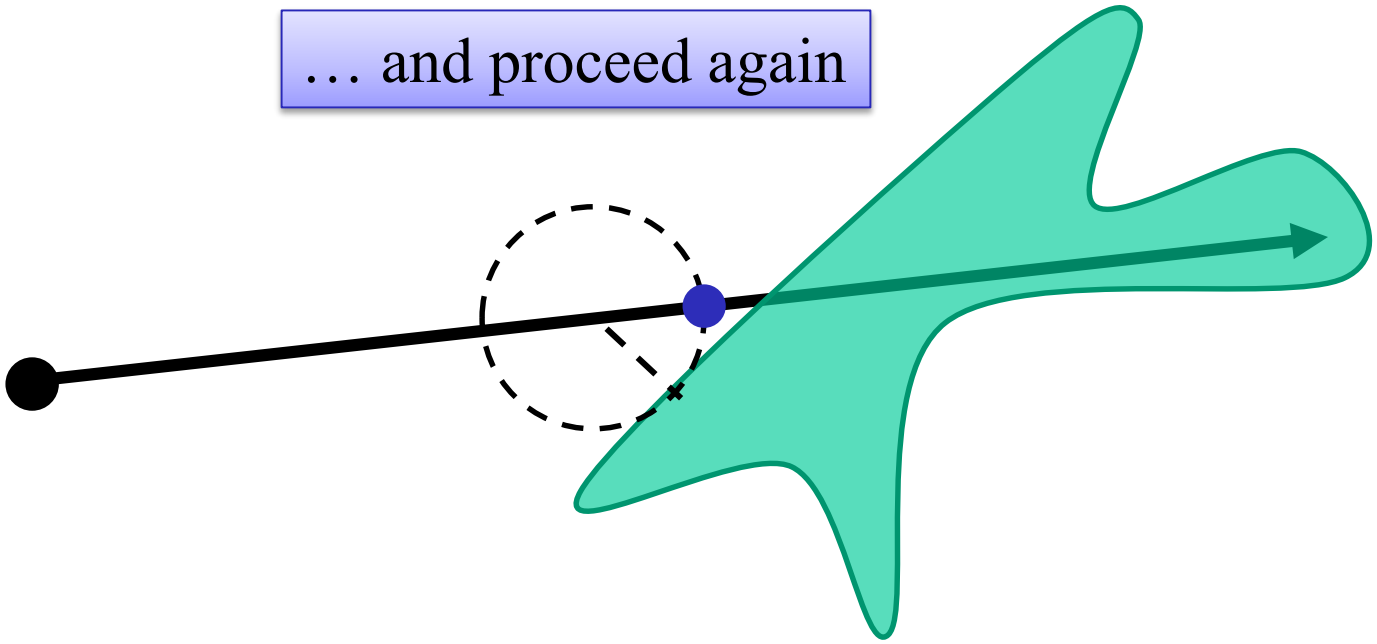
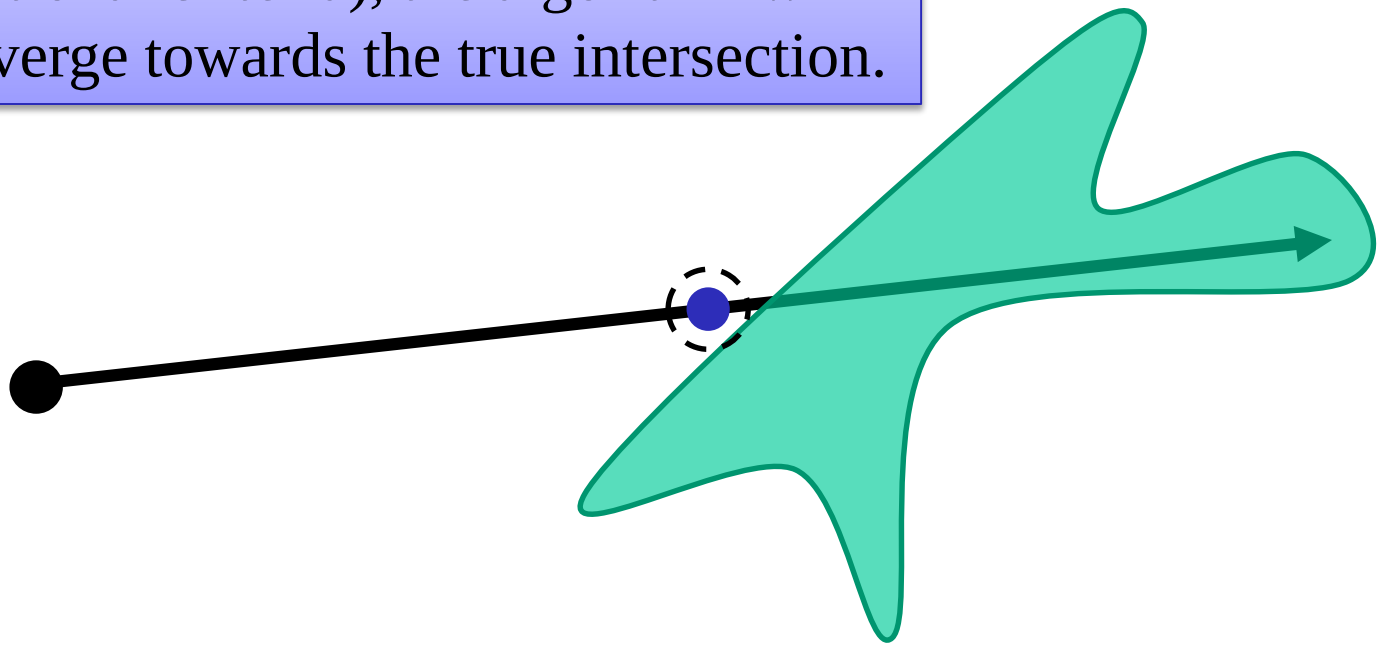


Sphere tracing



Sphere tracing

If $f(x)$ corresponds to an actual distance (or a lower bound with some additional criteria), the algorithm will converge towards the true intersection.



Sphere tracing / Raymarching

```
float raymarch(vec3 from, vec3 dir, function f)
{
   vec3 x = from;
   int steps = MAX_STEPS;
   while (f(x) > 0.0001 && steps-- > 0)
     x += f(x)*dir;
   return length(x - from);
}
```

Distance fields

- To do raymarching we have to model the whole scene in the form of a single *distance field function* $f(\boldsymbol{x})$.

- It turns out that distance fields let you model quite complicated shapes rather naturally.

Quiz

- Devise a distance function for a sphere of radius 1 centered at (0, 0, 0).

Quiz

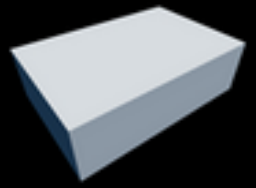
- Devise a distance function for a sphere of radius 1 centered at (0, 0, 0).

```
float sphere(vec3 x) {
    return length(x) - 1;
}
```

Sphere - signed

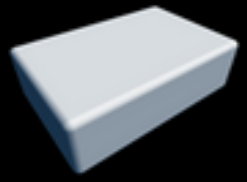
```
float sdSphere( vec3 p, float s )
{
  return length(p)-s;
}
```

Box - unsigned



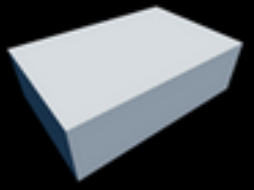
```
float udBox( vec3 p, vec3 b )
{
  return length(max(abs(p)-b,0.0));
}
```

Round Box - unsigned



```
float udRoundBox( vec3 p, vec3 b, float r )
{
  return length(max(abs(p)-b,0.0))-r;
}
```

Box - signed

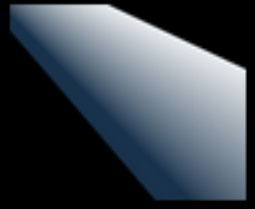


```
float sdBox( vec3 p, vec3 b )
{
  vec3 d = abs(p) - b;
  return min(max(d.x,max(d.y,d.z)),0.0) +
         length(max(d,0.0));
}
```

Torus - signed

```
float sdTorus( vec3 p, vec2 t )
{
  vec2 q = vec2(length(p.xz)-t.x,p.y);
  return length(q)-t.y;
}
```

Cylinder - signed



```
float sdCylinder( vec3 p, vec3 c )
{
  return length(p.xz-c.xy)-c.z;
}
```

Cone - signed

```
float sdCone( vec3 p, vec2 c )
{
    // c must be normalised
    float q = length(p.xy);
    return dot(c,vec2(q,p.z));
}
```

Plane - signed

```
float sdPlane( vec3 p, vec4 n )
{
  // n must be normalised
  return dot(p,n.xyz) + n.w;
}
```

 http://iquilezles.org/www/articles/distfunctions/distfunctions.htm

See also: http://www.viz.tamu.edu/faculty/ergun/research/implicitmodeling/papers/sm99.pdf

Quiz

- Given two distance functions corresponding to some shapes, devise a distance function that corresponds to the union of these shapes:

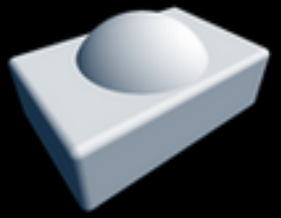
Quiz

- Given two distance functions corresponding to some shapes, devise a distance function that corresponds to the union of these shapes:

```
float union(vec3 x) {
      return min(f1(x), f2(x));
}
```

Constructive Solid Geometry

Union

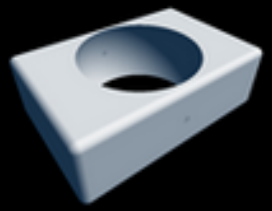


```
float opU( float d1, float d2 )
{
    return min(d1,d2);
}
```

Intersection

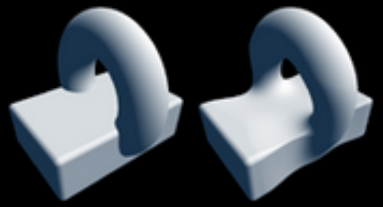
```
float opI( float d1, float d2 )
{
    return max(d1,d2);
}
```

Subtraction



```
float opS( float d1, float d2 )
{
    return max(-d1,d2);
}
```

Blend



```
float opBlend( vec3 p )
{
    float d1 = primitiveA(p);
    float d2 = primitiveB(p);
    return smin( d1, d2 );
}
```

http://iquilezles.org/www/articles/distfunctions/distfunctions.htm

Quiz

- Given a distance function describing a shape centered at (0, 0, 0). Devise a distance function for the same shape, centered at (1, 1, 1):

Quiz

- Given a distance function describing a shape centered at (0, 0, 0). Devise a distance function for the same shape, centered at (1, 1, 1):

```
float translate(vec3 x) {
    return f(x - vec3(1, 1, 1))
}
```

Arbitrary transformations

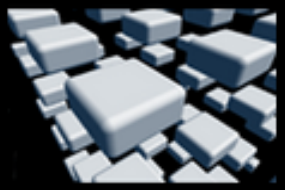
Rotation/Translation

```
vec3 opTx( vec3 p, mat4 m )
{
    vec3 q = invert(m)*p;
    return primitive(q);
}
```

Scale

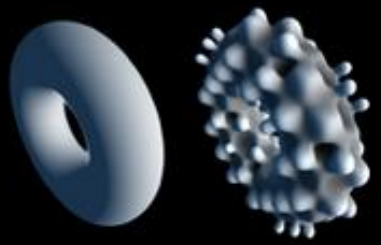
```
float opScale( vec3 p, float s )
{
    return primitive(p/s)*s;
}
```

Repetition



```
float opRep( vec3 p, vec3 c )
{
    vec3 q = mod(p,c)-0.5*c;
    return primitve( q );
}
```

Displacement

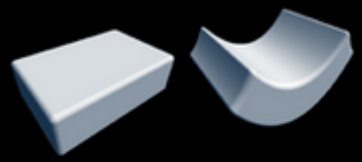


```
float opDisplace( vec3 p )
{
    float d1 = primitive(p);
    float d2 = displacement(p);
    return d1+d2;
}
```

Twist

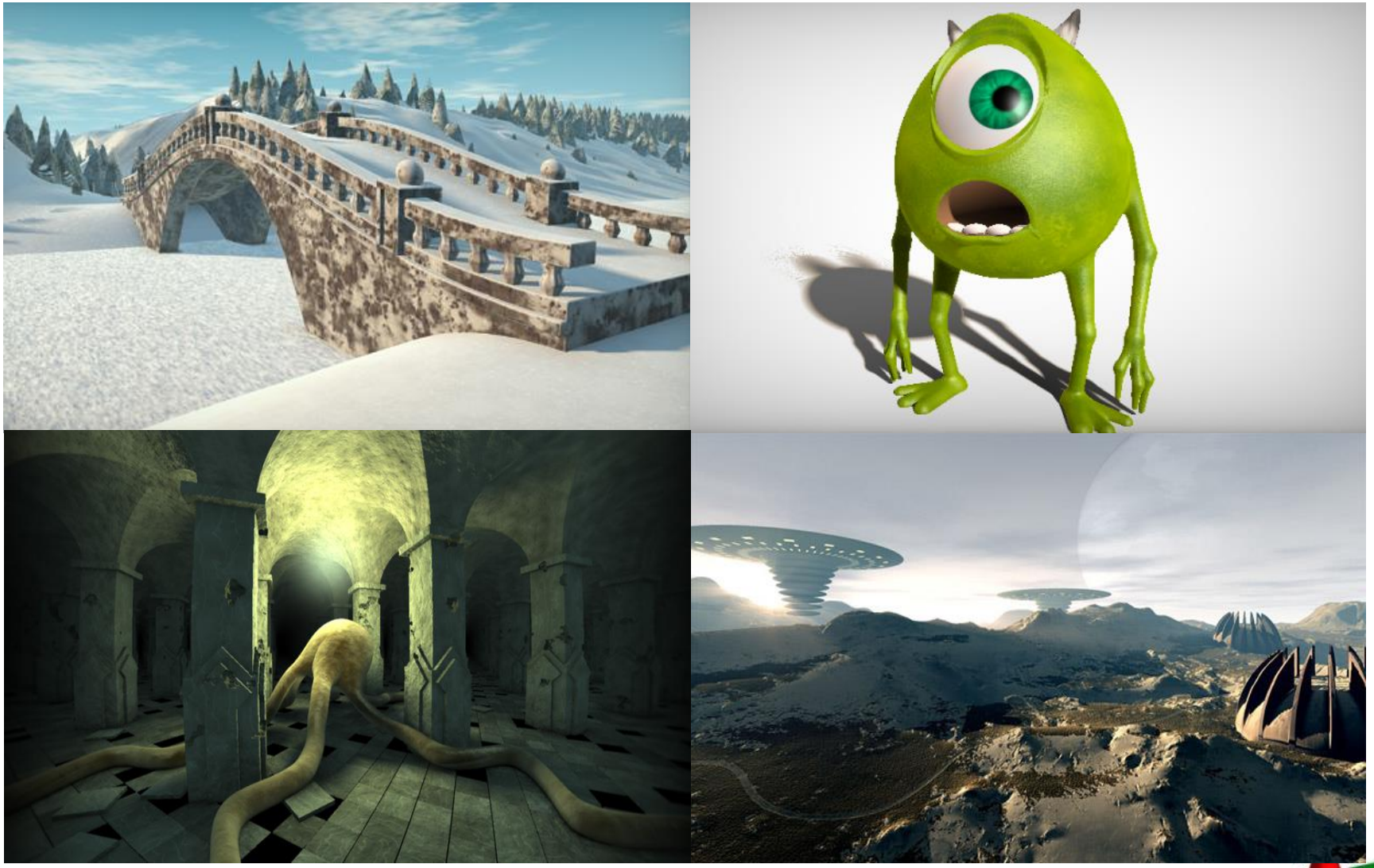
```
float opTwist( vec3 p )
{
    float c = cos(20.0*p.y);
    float s = sin(20.0*p.y);
    mat2   m = mat2(c,-s,s,c);
    vec3   q = vec3(m*p.xz,p.y);
    return primitive(q);
}
```

Cheap Bend



```
float opCheapBend( vec3 p )
{
    float c = cos(20.0*p.y);
    float s = sin(20.0*p.y);
    mat2   m = mat2(c,-s,s,c);
    vec3   q = vec3(m*p.xy,p.z);
    return primitive(q);
}
```

http://iquilezles.org/www/articles/distfunctions/distfunctions.htm



Next time

- Raycasting
- Raytracing
- Raymarching / Sphere tracing

- Rendering equation solvers
  - Radiosity, Photon mapping, Path tracing