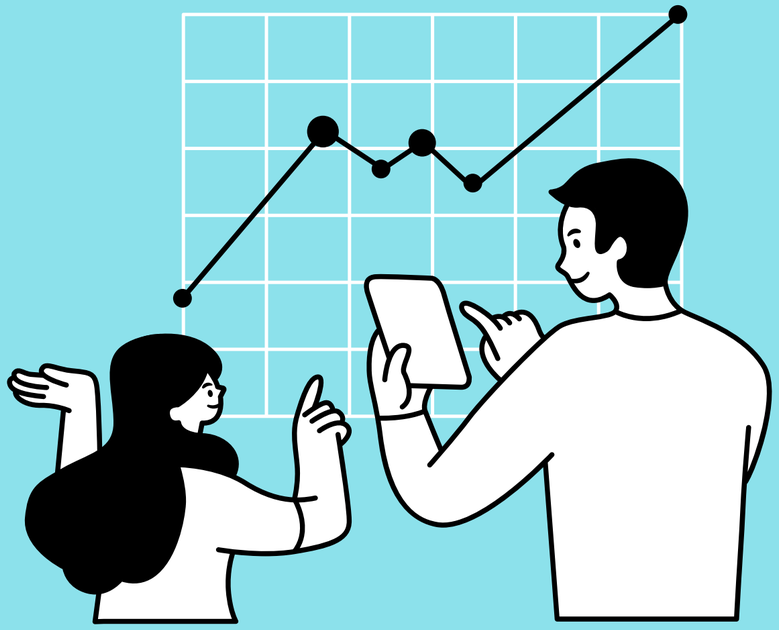
Introduction to Financial Engineering 2024

GROUP 23



Project 4: Option Pricing

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BASIC INFORMATION

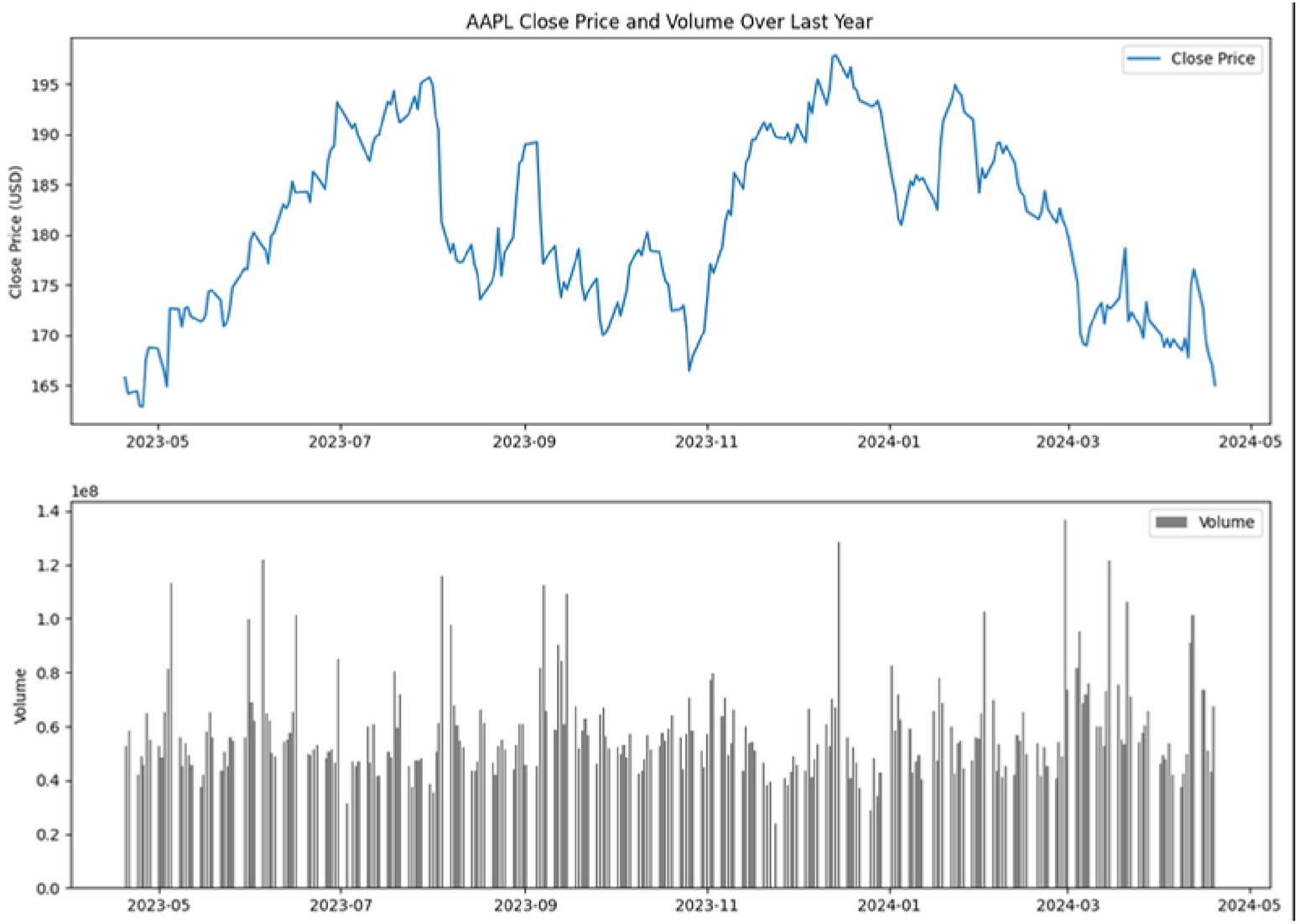
SOURCE yahoo! finance

ASSET: Apple STOCKS

U.S. Treasury bonds

Stock’s History Visualization

Apple close price and volume over last year

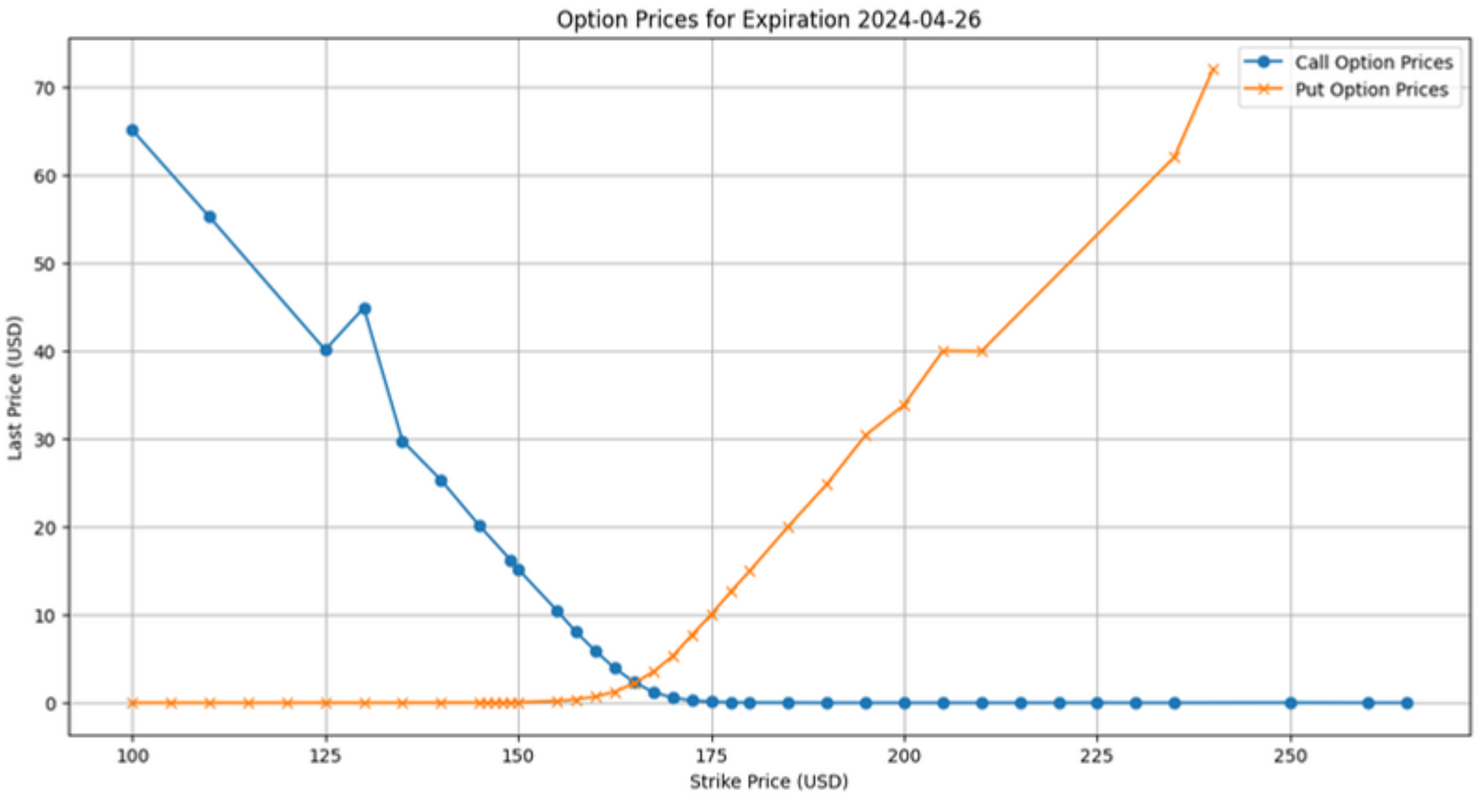


AAPL recent close price is 165 dollar

Option’s History AND Analysis

expiration on 2024-04-26

Y-axis (Last Price in USD): The vertical axis represents the last price at which the options were traded. It indicates the premium paid for the options.

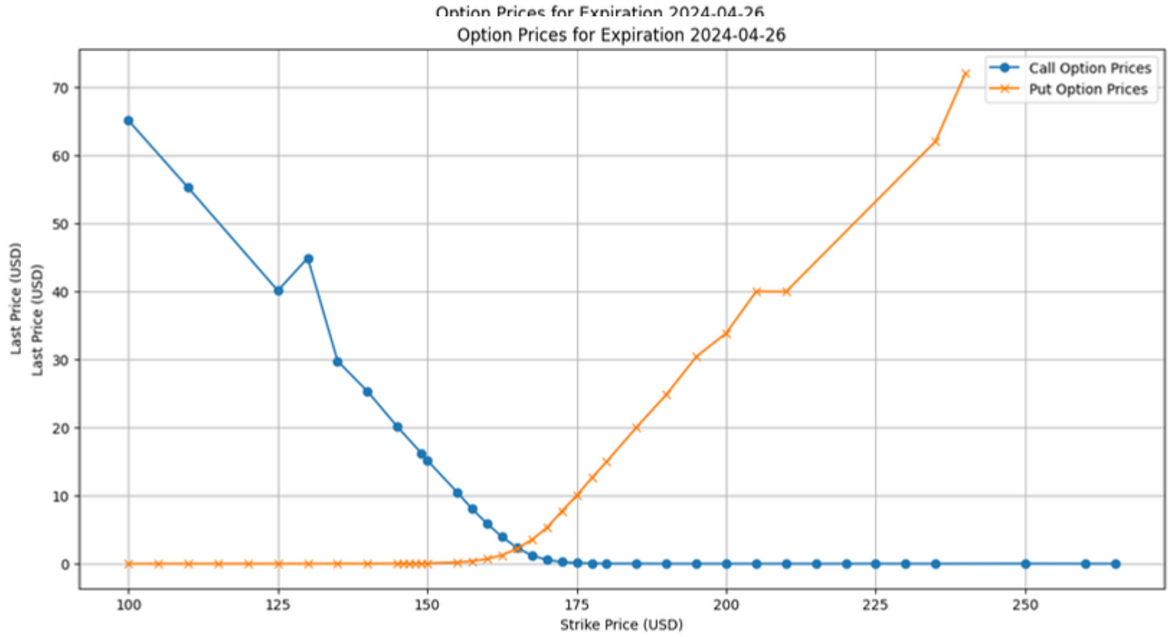



```
Maximum Call Price: $65.06, Strike: $100.0
Minimum Call Price: $0.01, Strike: $190.0
Maximum Put Price: $71.95, Strike: $240.0
Minimum Put Price: $0.01, Strike: $100.0
```

X-axis (Strike Price in USD): The horizontal axis represents the strike prices of the options. It appears to range from near 0 up to about 300 USD. The strike price is the price at which the option holder has the right to buy (for calls) or sell (for puts) the underlying asset.

Option’s History AND Analysis

expiration on 2024-04-26



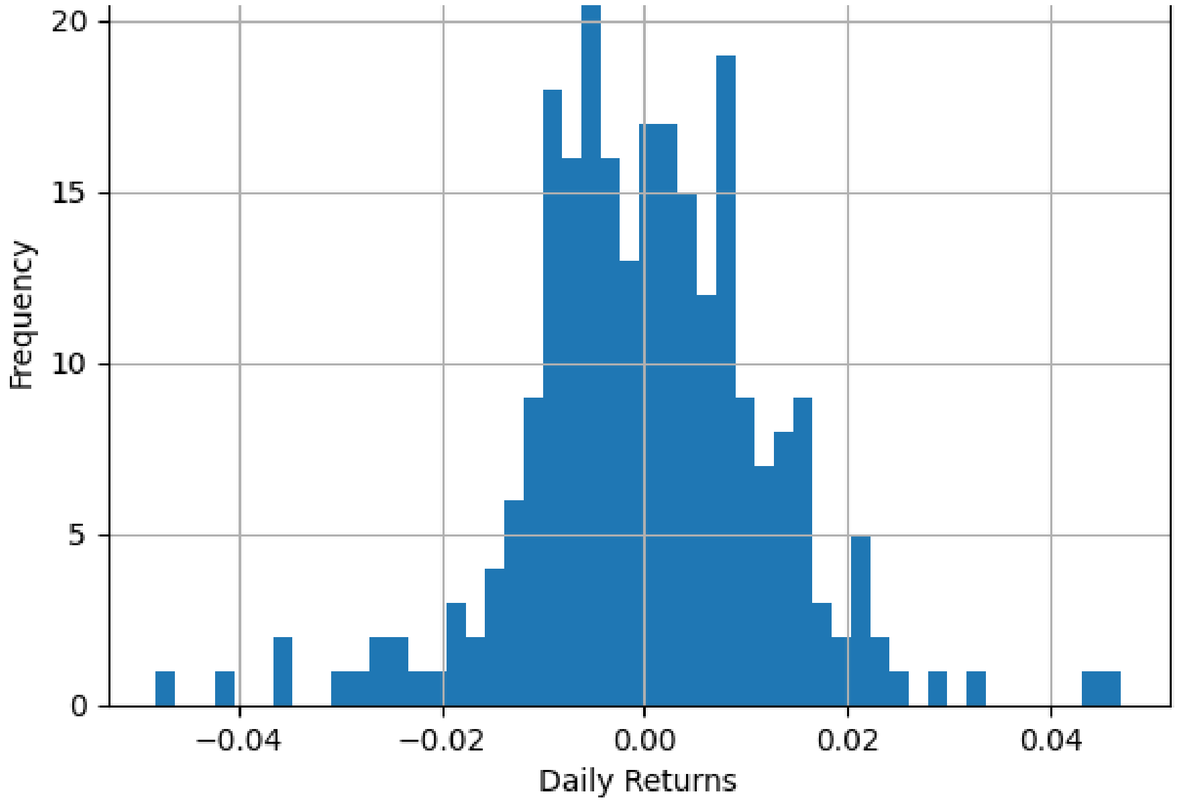
Blue Dots (Call Options): These are like tickets that let us buy Apple's stock at a future date at today's price.The graph shows that the more to the left " (lower strike prices), the more expensive these tickets are, because there's a good chance you'll get to buy Apple's stock cheaply compared to the future market price.

Orange Crosses (Put Options): they're like insurance policies that let you sell Apple's stock at a future date but at today's price. They're handy if you think the price might go down.

On the graph, these are pretty cheap if we look to the left on the ruler (lower strike prices) because it's not likely you'll need to sell Apple's stock at a low price if it goes up. But, as we move to the right (higher strike prices), they get more expensive, because there's more chance we'll use that insurance.

- If someone told you that apples are going to be super rare next year, you'd pay more for a ticket that lets you buy apples at today's cheaper prices. That's like the blue dots on the left - expensive because people are betting the price of apples will go up.
- Now, if you're an apple grower worried about prices crashing because there will be too many apples next year, you'd pay for insurance that lets you sell at today's higher prices. This insurance gets more expensive the higher the price you want to guarantee (the right side of the ruler), shown by the orange crosses.

Step 3: one-year data to estimate the annual volatility.



- **Centered Around Zero: The bulk of the bars are centered around a daily return of 0%. This suggests that on many days, the stock price didn't change drastically from the previous day's closing price.**

- **Bell Shape: The distribution of daily returns appears to have a 'bell' shape, which tells us that most of the daily returns are close to the average return, with fewer instances of extreme returns.**

- **Symmetry Around the Center: The histogram looks fairly symmetrical around the center, the x-axis shows the range of daily returns, from around -4% to +4%.**

A wider spread would indicate higher volatility. Here, most returns are within a narrow range of percentages, which suggests moderate volatility.

The output "Estimated Annual Volatility: 19.63%" refers to a measurement of how much the stock's price fluctuated over the past year.For AAPL, a volatility of 19.63% is relatively moderate

Step 4 : Used 10 years' US treasury rate as the rate of interest

Stats

Last Value	4.64%	Value from The Previous Market Day	4.59%
Latest Period	Apr 18 2024	Change from The Previous Market Day	1.09%
Last Updated	Apr 18 2024, 19:04 EDT	Value from 1 Year Ago	3.60%
Next Release	Apr 19 2024, 18:00 EDT	Change from 1 Year Ago	28.89%
Long Term Average	4.25%	Frequency	Market Daily
Average Growth Rate	3.87%	Unit	Percent
		Adjustment	N/A
		Download Source File	Upgrade

Currently : 10 Year Treasury Rate is at 4.64%, compared to 4.59% the previous market day and 3.60% last year. This is higher than the long term average of 4.25%.

Step 5: Evaluated the call/ put option price by using Binomial model on fixed different strike prices and time of maturity.

We have taken risk free rate = 0.0464 from step -4

We have calculated Call price and put price for three different strike price i.e. (130,150,170), with increase strike price the call price decreases but put price increases

```
Strike: 130, Time: 0.5 years, Call Price: 24.41, Put Price: 1.42
Strike: 130, Time: 1 years, Call Price: 29.06, Put Price: 3.17
Strike: 130, Time: 1.5 years, Call Price: 32.91, Put Price: 4.17
Strike: 150, Time: 0.5 years, Call Price: 9.09, Put Price: 5.65
Strike: 150, Time: 1 years, Call Price: 13.84, Put Price: 7.04
Strike: 150, Time: 1.5 years, Call Price: 17.85, Put Price: 7.76
Strike: 170, Time: 0.5 years, Call Price: 3.50, Put Price: 19.60
Strike: 170, Time: 1 years, Call Price: 8.07, Put Price: 20.37
Strike: 170, Time: 1.5 years, Call Price: 11.97, Put Price: 20.54
```

Step 6: Evaluate the option price by using Black Scholes Formula .

```
BS Strike: 130, Time: 0.5 years, Call Price: 24.06, Put Price: 1.08
BS Strike: 130, Time: 1 years, Call Price: 28.27, Put Price: 2.38
BS Strike: 130, Time: 1.5 years, Call Price: 32.07, Put Price: 3.33
BS Strike: 150, Time: 0.5 years, Call Price: 10.04, Put Price: 6.60
BS Strike: 150, Time: 1 years, Call Price: 15.18, Put Price: 8.38
BS Strike: 150, Time: 1.5 years, Call Price: 19.49, Put Price: 9.40
BS Strike: 170, Time: 0.5 years, Call Price: 2.95, Put Price: 19.05
BS Strike: 170, Time: 1 years, Call Price: 7.03, Put Price: 19.32
BS Strike: 170, Time: 1.5 years, Call Price: 10.87, Put Price: 19.44
```

To calculate the Black Scholes formula we assume continuous time interval compare to discrete time interval in Binomial pricing

$$C = N(d_1)S_t - N(d_2)Ke^{-rt}$$

$$\text{where } d_1 = \frac{\ln \frac{S_t}{K} + (r + \frac{\sigma^2}{2})t}{\sigma\sqrt{t}}$$

$$\text{and } d_2 = d_1 - \sigma\sqrt{t}$$

C = call option price

N = CDF of the normal distribution

S_t = spot price of an asset

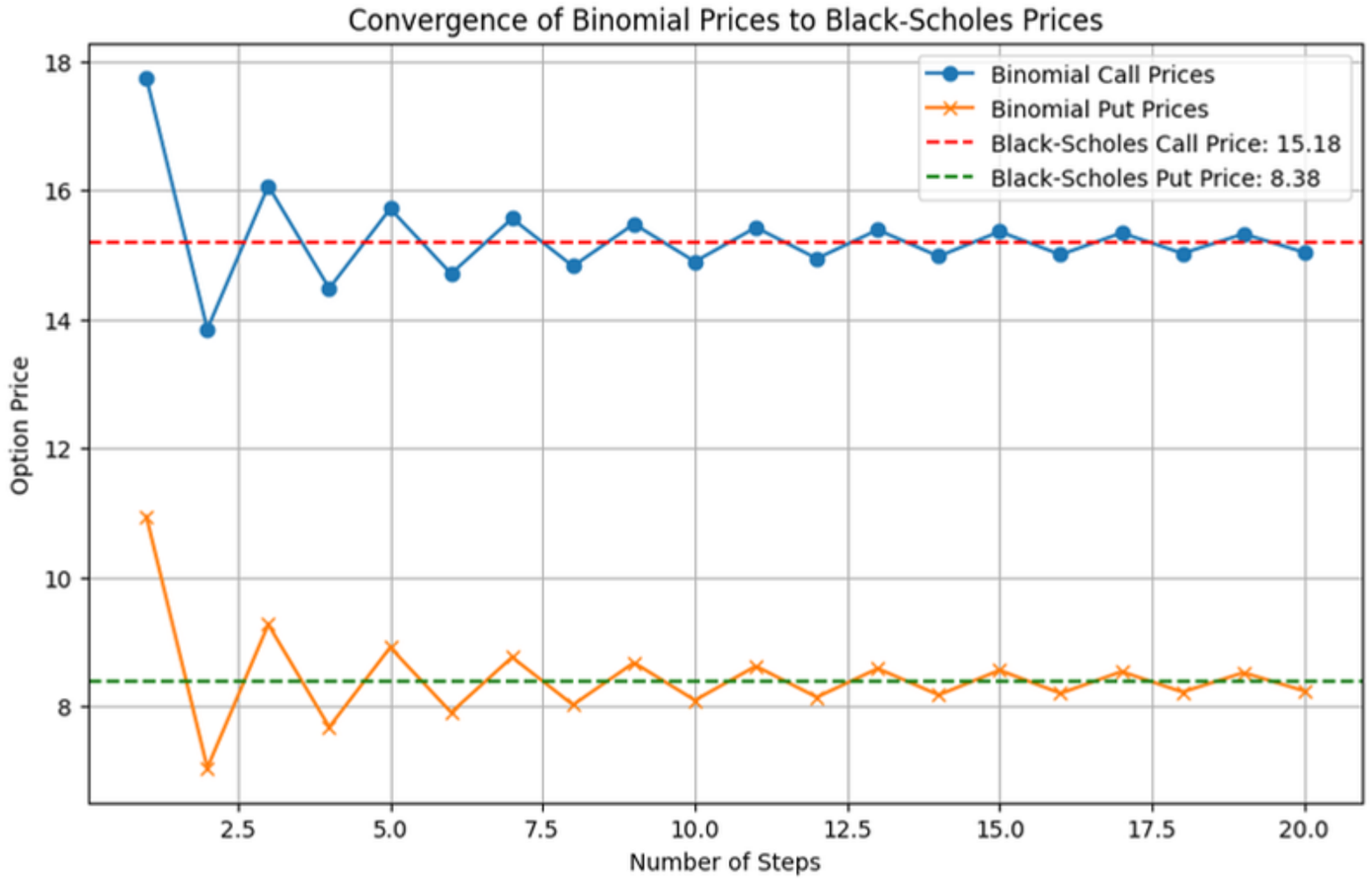
K = strike price

r = risk-free interest rate

t = time to maturity

σ = volatility of the asset

Step 7 and step 8 : Increase the number of steps in Binomial model to verify that the price converges Black Scholes Model and also plot the graph

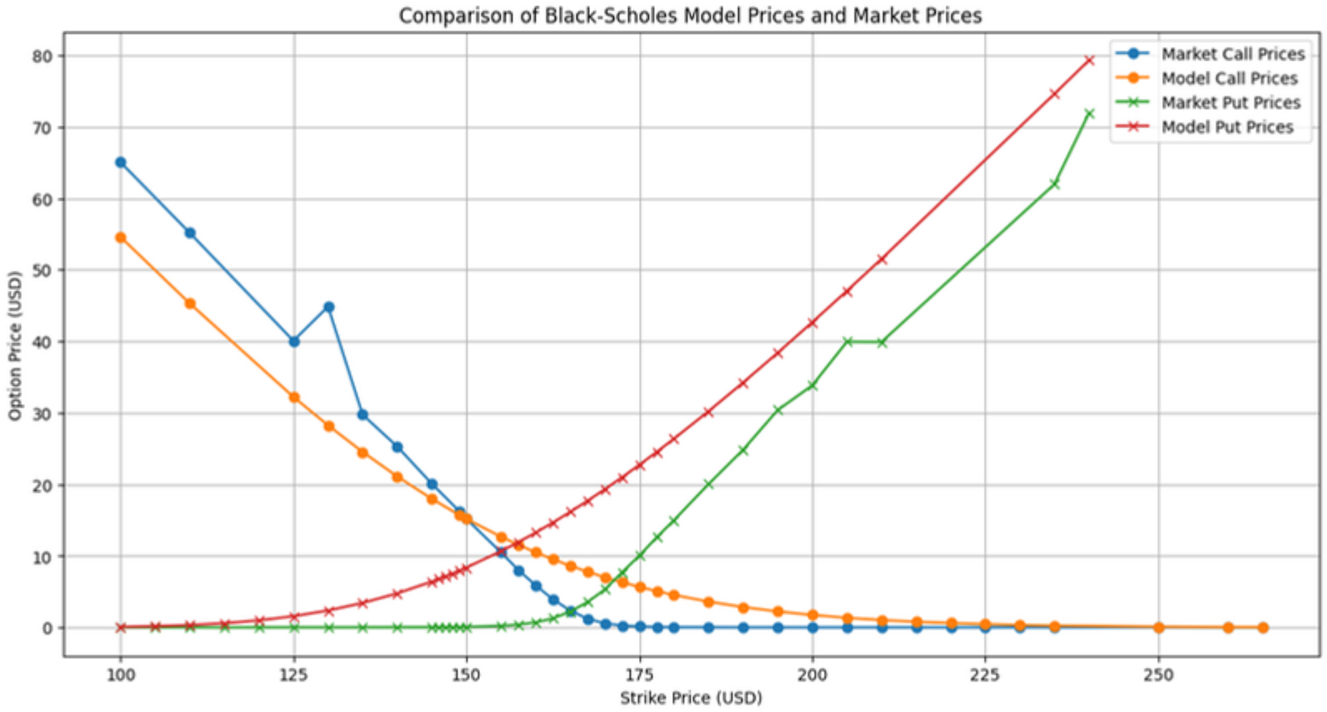


The graph shows the convergence of call and put option prices calculated by the Binomial model to the prices given by the Black-Scholes model as the number of steps in the Binomial model increases.

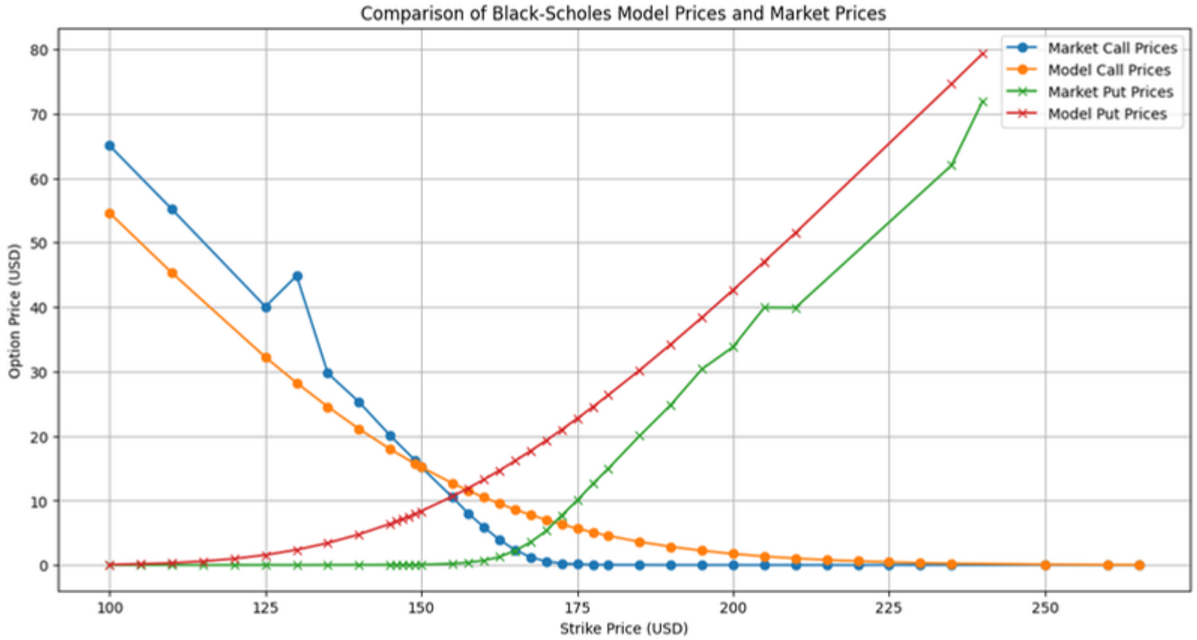
Stability: After approximately 10 steps, the prices from the Binomial model do not significantly change with additional steps, indicating that the model has reached stability and is providing consistent pricing . In summary, the graph visually confirms a key theoretical point about the Binomial option pricing model: as the number of steps increases, the Binomial model's prices converge to the continuous pricing model provided by Black-Scholes.

Step 9 : Comparision with the actual market data

Call Options: The model call prices (represented by the orange line with circles) and the market call prices (represented by the blue line with circles) start to diverge as the strike price increases. The market prices are higher than the model prices for lower strike prices and then converge around a certain point before diverging again as the strike price continues to increase.



Put Options: The market put prices (red line with crosses) and model put prices (green line with crosses) show a similar divergence pattern. For lower strike prices, the market prices are much higher than the model prices. The lines converge around the middle range of strike prices, indicating that the model prices are more in line with the market here. As the strike prices increase further, the market prices tend to be slightly higher than the model prices.



Conclusions:

1. Volatility Skew: The divergence between the model and market prices suggests a volatility skew, which is a situation where the implied volatility varies for different strike prices. This can happen due to various factors, including investor sentiment, demand and supply dynamics, or upcoming events that the market expects to affect the stock price.

2. Over- or Underpricing: Where the market prices are higher than the model prices, it could indicate that those options are overpriced by the market (or the model is underpricing them), which could be due to actual market conditions not being captured by the Black-Scholes model assumptions (e.g., constant volatility).

3. Model Limitations: The Black-Scholes model assumes constant volatility and does not take into account the actual market environment, which can lead to discrepancies when comparing with market prices that are influenced by a variety of dynamic factors.

Black Scholes vs Binomial Pricing

1. **Continuous vs. Discrete Time: The Black-Scholes model assumes continuous time, meaning that the price of the underlying asset can change infinitely over time. In contrast, the binomial model works in discrete time, breaking down the time period into smaller intervals.**
2. **Complexity: The Black-Scholes model provides a closed-form solution for option pricing, which means there's a formula that directly calculates the option price. On the other hand, the binomial model involves constructing a binomial tree to represent possible price movements over time and then calculating option prices at each node of the tree. While the Black-Scholes model may seem simpler due to its formula, implementing the binomial model might offer more flexibility in modeling complex situations.**
3. **Assumptions: Both models make certain assumptions about the behavior of the underlying asset and the market. The Black-Scholes model assumes constant volatility, continuous trading, and no dividends. The binomial model allows for more flexibility in incorporating different assumptions about volatility, interest rates, and dividend yields.**
4. **Accuracy: The Black-Scholes model is theoretically more accurate when the assumptions underlying the model hold true. However, in practice, market conditions may deviate from these assumptions, leading to discrepancies between the model's predictions and actual market prices. The binomial model, while more flexible, can also provide accurate results if the parameters are chosen appropriately.**
5. **Computational Complexity: While the Black-Scholes formula provides a straightforward calculation, the binomial model requires more computational effort, especially for complex options or when using a large number of time steps in the tree.**

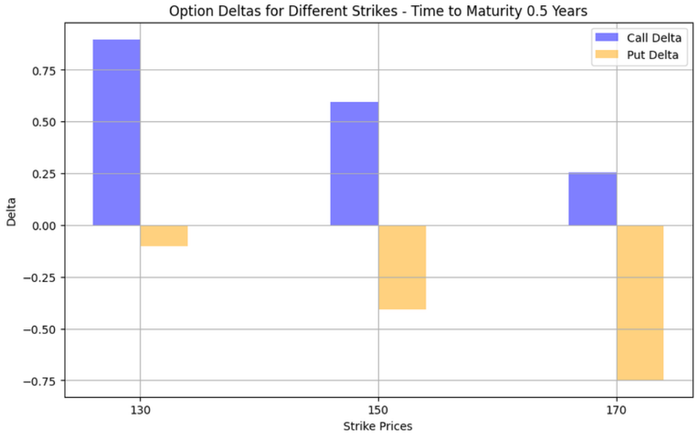
Step 10 : Delta neutral portfolio

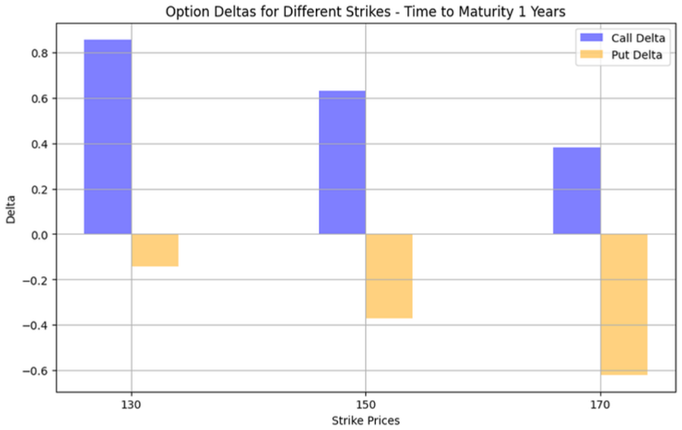
The delta of an option is a measure of how much the price of the option is expected to move per one dollar change in the price of the underlying asset

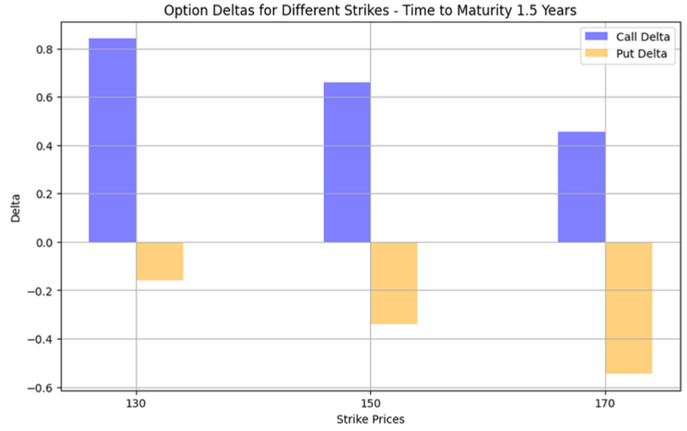
Time to Maturity and Delta Relationship:
For call options, the delta is higher for options that are further in-the-money (lower strike prices) and decreases as the strike price increases (moves out-of-the-money).
For put options, the magnitude of delta increases (becomes more negative) as the option goes further in-the-money (higher strike prices) and decreases as the strike price decreases (moves out-of-the-money).

Effect of Time on Delta:
The delta of call options for a given strike decreases slightly as time to maturity increases. This is because the probability of finishing in-the-money becomes more uncertain with more time until expiration.
The magnitude of the delta of put options increases as the time to maturity increases, particularly for in-the-money options (higher strike price).

Sign of Delta:
Call deltas are positive, indicating that the price of the call option increases as the price of the underlying asset increases.
Put deltas are negative, meaning the price of the put option increases as the price of the underlying decreases.

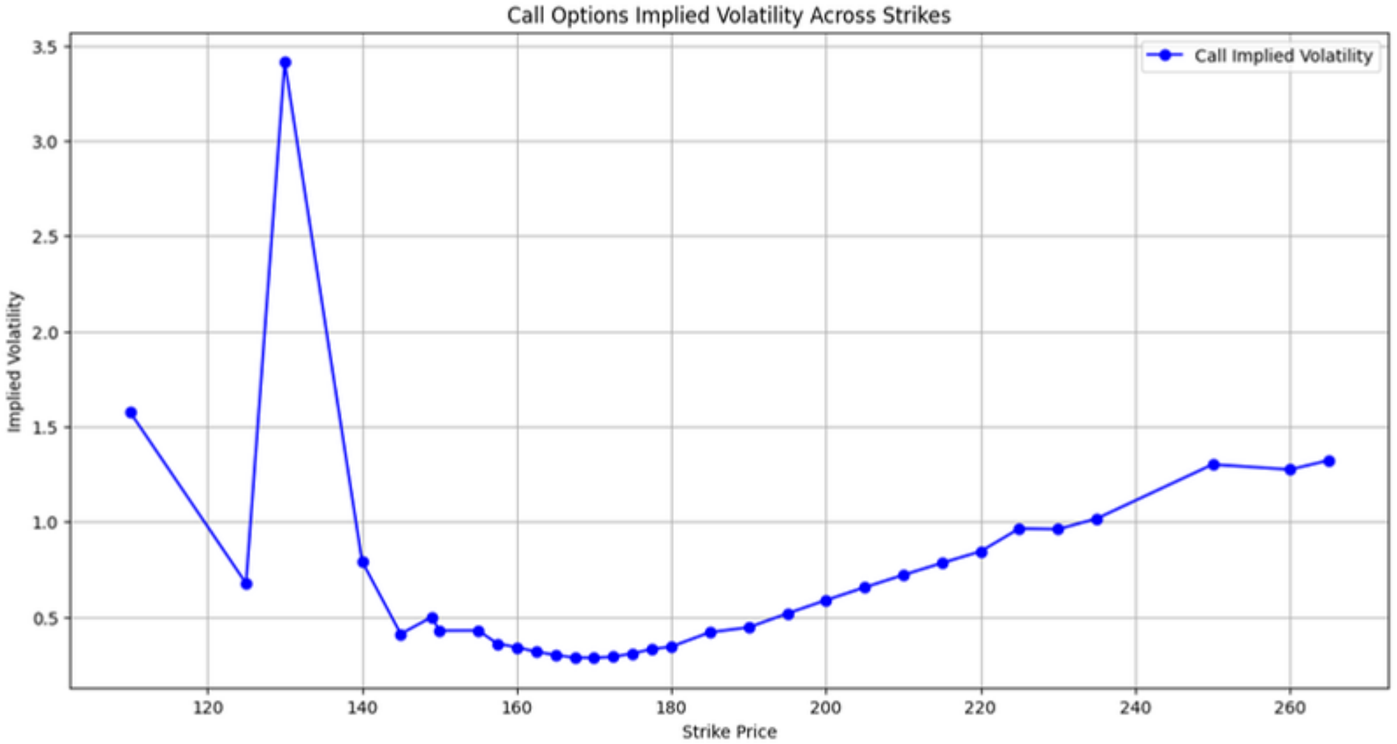






Step: 11 Implied volatility Using numerical methods "Brent method"

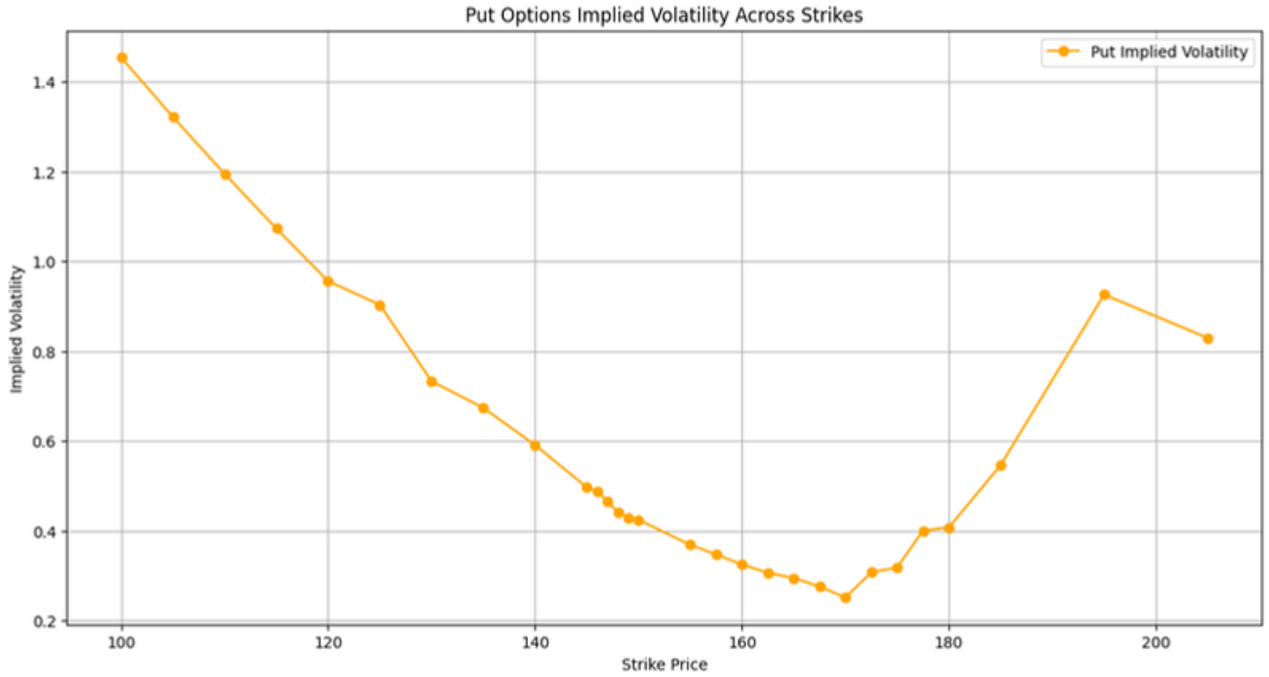
Implied volatility (IV) reflects the market's expectation of how much the price of a security (like a stock or commodity) will move over a specific period. It's expressed as an annualized percentage



Call Options Implied Volatility (First Chart):

- There is a pronounced spike in implied volatility at a lower strike price, indicating that options were priced with a much higher volatility assumption, perhaps due to a specific event or market condition expected to affect the underlying stock significantly.
- After the spike, implied volatility sharply decreases, which may suggest the options are transitioning from in-the-money to at-the-money or slightly out-of-the-money.
- As the strike price increases further, the implied volatility gradually rises, forming an upward trend. This could be indicative of a volatility skew, where the market prices in a higher likelihood of significant price movements as the option goes deeper out-of-the-money.

Step: 11 Implied volatility Using numerical methods "Brent method"



A high implied volatility means the market expects the stock to have bigger price swings by the expiration date, while a low implied volatility means the market expects the stock to be relatively stable.

Put Options Implied Volatility (Second Chart):

- The implied volatility steadily decreases as the strike price increases from deep out-of-the-money to at-the-money, which is typical in many markets due to the higher probability of in-the-money put options being exercised.
- There is a trough in the chart, after which implied volatility starts to increase. This increase for higher strike prices (moving from at-the-money to in-the-money options) suggests that the market may be pricing in some uncertainty or risk of downside in the underlying asset.

Thank You!

HOPE YOU LIKED THE PRESENTATION