

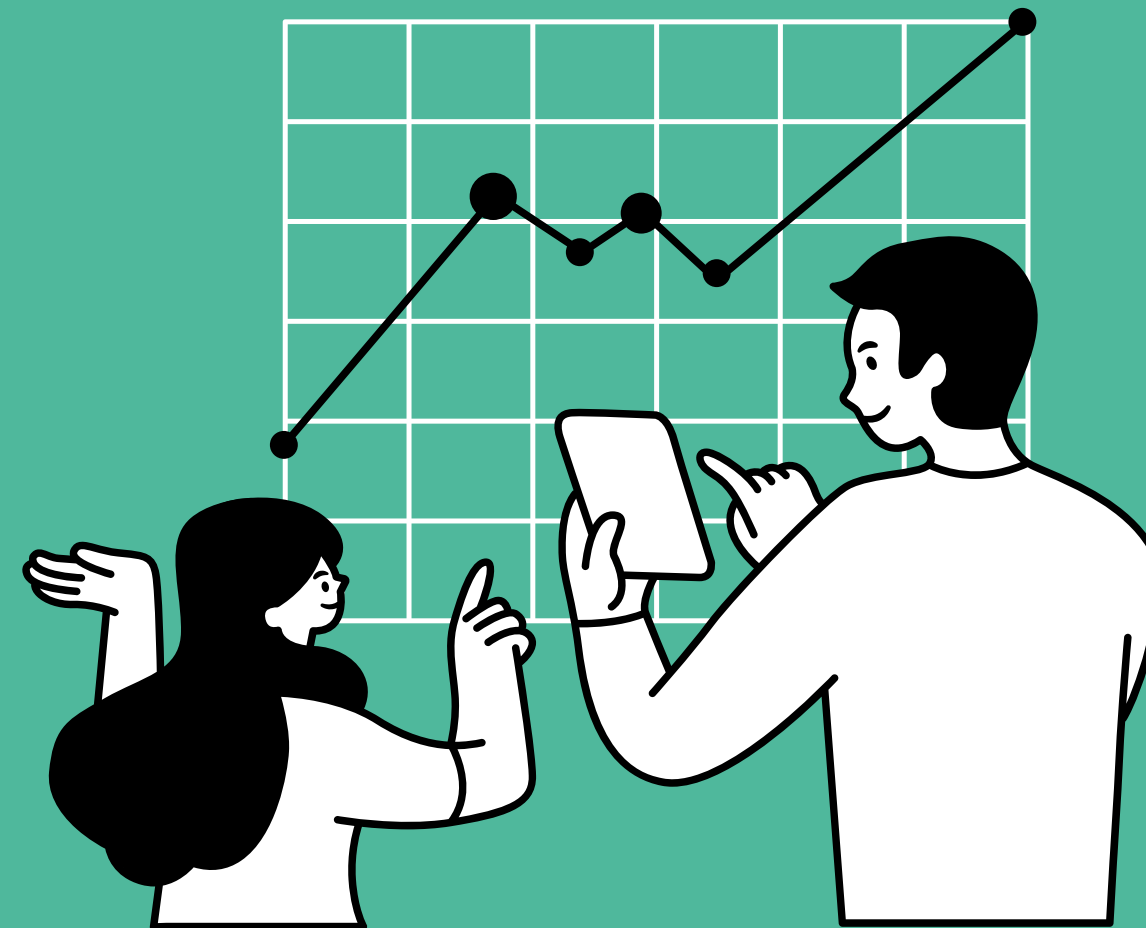


Introduction to Financial Engineering 2024

GROUP 23

# Project 2: Markowitz Portfolio Optimization

20, Feb 2023



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# BASIC INFORMATION

## SOURCE



## ASSET

# 10 chosen assets

tickers = ["AAPL", "GOOGL",  
"MSFT", "AMZN", "TSLA", "NFLX",  
"JPM", "GS", "BND", "VOO"]

## DURATION AND DATA

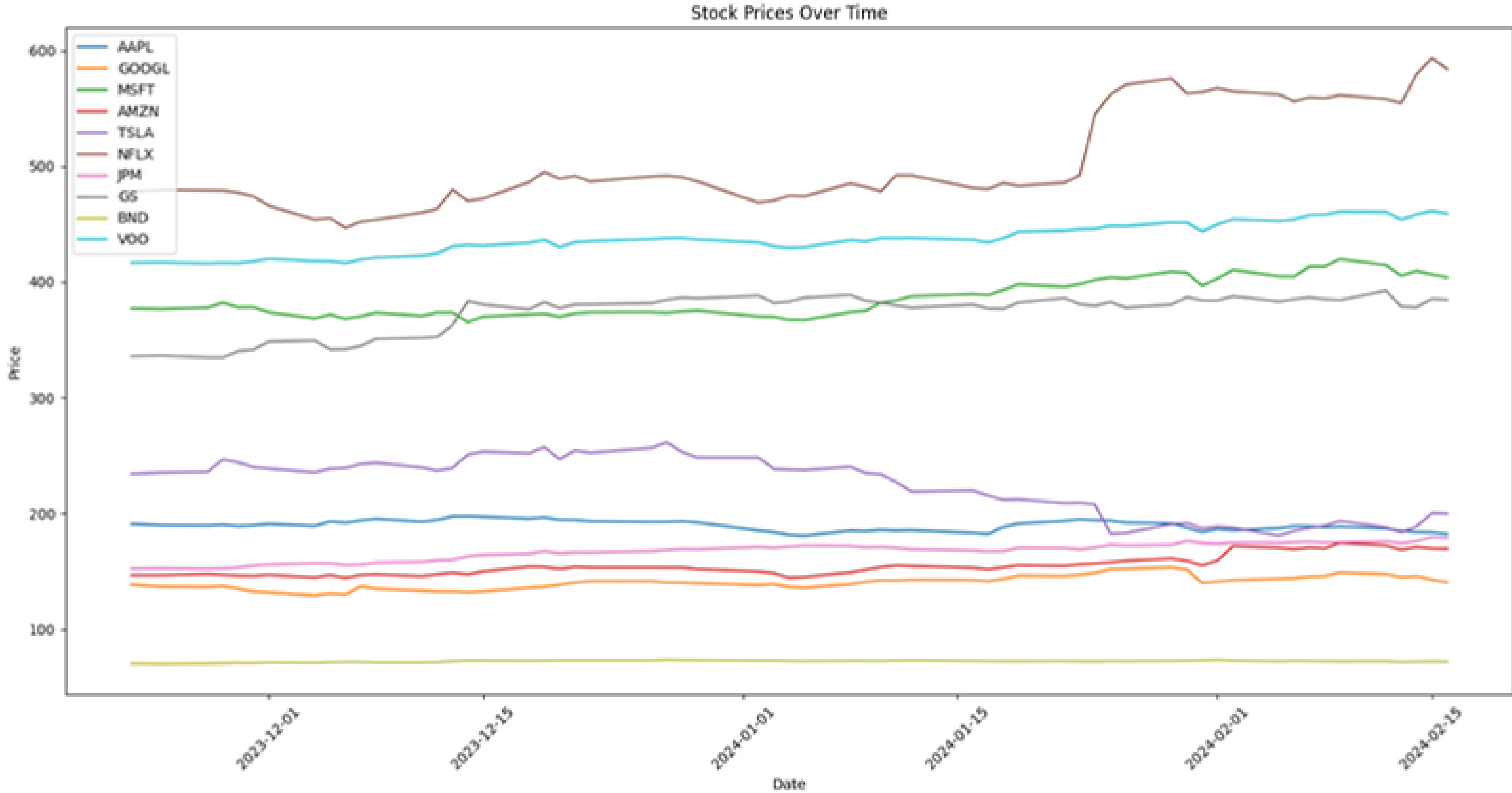
The stock prices for assets over  
the last 3 months

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Date	AAPL	AMZN	BND	GOOGL
2023-11-22 00:00:00	191.06619262695312	146.7100067138672	70.29431915283203	138.49000549316406
2023-11-24 00:00:00	189.7279052734375	146.74000549316406	69.95722198486328	136.69000244140625
2023-11-27 00:00:00	189.54812622070312	147.72999572753906	70.383544921875	136.41000366210938
2023-11-28 00:00:00	190.1573486328125	147.02999877929688	70.68098449707031	137.1999969482422
2023-11-29 00:00:00	189.128662109375	146.32000732421875	71.04783630371094	134.99000549316406
2023-11-30 00:00:00	189.70791625976562	146.08999633789062	70.81979370117188	132.52999877929688
2023-12-01 00:00:00	190.9962921142578	147.02999877929688	71.39745330810547	131.86000061035156
2023-12-04 00:00:00	189.18858337402344	144.83999633789062	71.10911560058594	129.27000427246094
2023-12-05 00:00:00	193.1735076904297	146.8800048828125	71.56648254394531	130.99000549316406
2023-12-06 00:00:00	192.07492065429688	144.52000427246094	71.77527618408203	130.02000427246094
2023-12-07 00:00:00	194.02243041992188	146.8800048828125	71.75538635253906	136.92999267578125
2023-12-08 00:00:00	195.46058654785156	147.4199981689453	71.40740203857422	134.99000549316406
2023-12-11 00:00:00	192.93380737304688	145.88999938964844	71.41734313964844	133.2899932861328

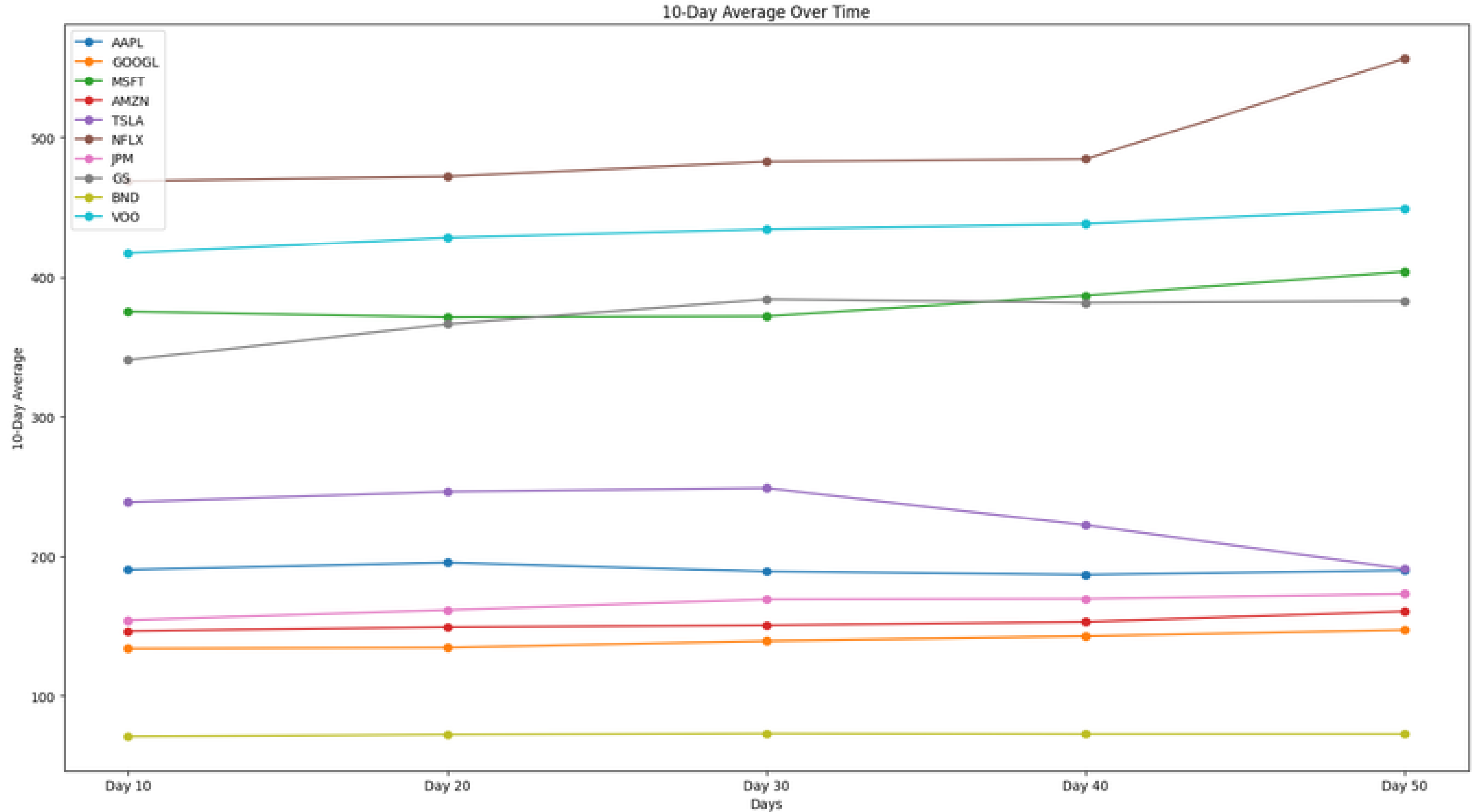
GS	JPM	MSFT	NFLX	TSLA	VOO
335.8819274902344	152.3903045654297	377.1525573730469	478.0	234.2100067138672	416.4214172363281
336.38775634765625	152.59901428222656	376.7333068847656	479.55999755859375	235.4499969482422	416.65045166015625
334.9595031738281	152.2511749267578	377.9111328125	479.1700134277344	236.0800018310547	415.9234619140625
334.8999938964844	152.59901428222656	381.9936218261719	479.0	246.72000122070312	416.3317565917969
340.260009765625	153.37425231933594	378.15069580078125	477.19000244140625	244.13999938964844	416.0628967285156
341.5400085449219	155.1234588623047	378.2106018066406	473.9700012207031	240.0800018310547	417.67620849609375
348.42999267578125	155.87879943847656	373.8187255859375	465.739990234375	238.8300018310547	420.1260986328125
349.3900146484375	157.02175903320312	368.4586486816406	453.8999938964844	235.5800018310547	418.01483154296875
341.75	157.0018768310547	371.8323669433594	455.1499938964844	238.72000122070312	417.8853454589844
341.9700012207031	155.35205078125	368.1192321777344	446.7300109863281	239.3699951171875	416.1425476074219
344.6199951171875	155.8291015625	370.2652893066406	452.0	242.63999938964844	419.39910888671875
350.8299865722656	157.5485076904297	373.53924560546875	453.760009765625	243.83999633789062	421.1817626953125

# VISUALIZATION STOCK Price OVER 3 months





# VISUALIZATION STOCK Price @10 Days Average





## **TSLA (Tesla Inc.)**

Tesla is an electric vehicle and clean energy company recognized for its innovation in automotive technology.



## **AMZN (Amazon.com Inc.)**

Amazon is a major e-commerce company providing products and services, including cloud computing.



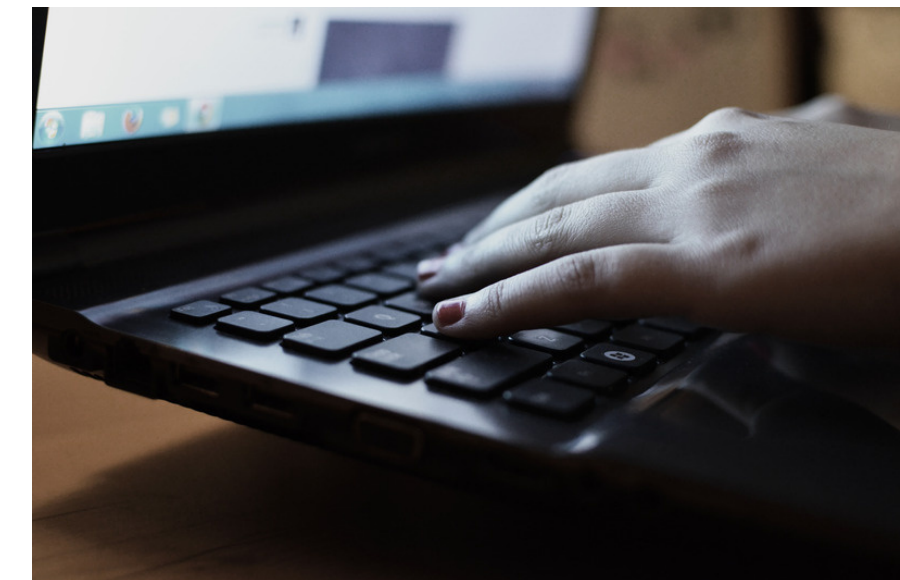
## **AAPL (Apple Inc.)**

Known for its popular consumer electronics like iPhone and Mac.



## **GOOGL (Alphabet Inc. - Class A)**

Alphabet is Google's parent company, operating in online advertising, cloud computing, and software development.



## **MSFT (Microsoft Corporation)**

Microsoft is a top tech-firm offering software products, hardware devices, and cloud services.





## **NFLX (Netflix Inc.)**

Netflix is a popular streaming service offering a vast library of TV shows, movies, and original content.



## **GS (The Goldman Sachs Group Inc.)**

Goldman Sachs is a leading investment bank and financial services company.



## **BND**

(Vanguard Total Bond Market ETF): BND is an ETF tracking the performance of the U.S. Aggregate Bond Index, offering exposure to investment-grade bonds.



## **JPM (JPMorgan Chase & Co.)**

JPMorgan Chase is a global financial services firm providing banking and investment services.



## **V00 (Vanguard S&P 500 ETF)**

V00 is an ETF mirroring the S&P 500 Index's performance, providing exposure to the largest U.S. publicly traded companies.

**>> These assets cover various sectors such as technology, finance, e-commerce, automotive, entertainment, and the broader market through bond and S&P 500 index exposure. The selection of assets should align with investment goals, risk tolerance, and market outlook.**

# SIMPLE RETURNS

Simple returns are a common way to measure the percentage change in the value of an asset over a period of time. It provide a straightforward measure of the percentage change in the asset's value from one period to the next, making them easy to interpret and compare across different assets and time periods.

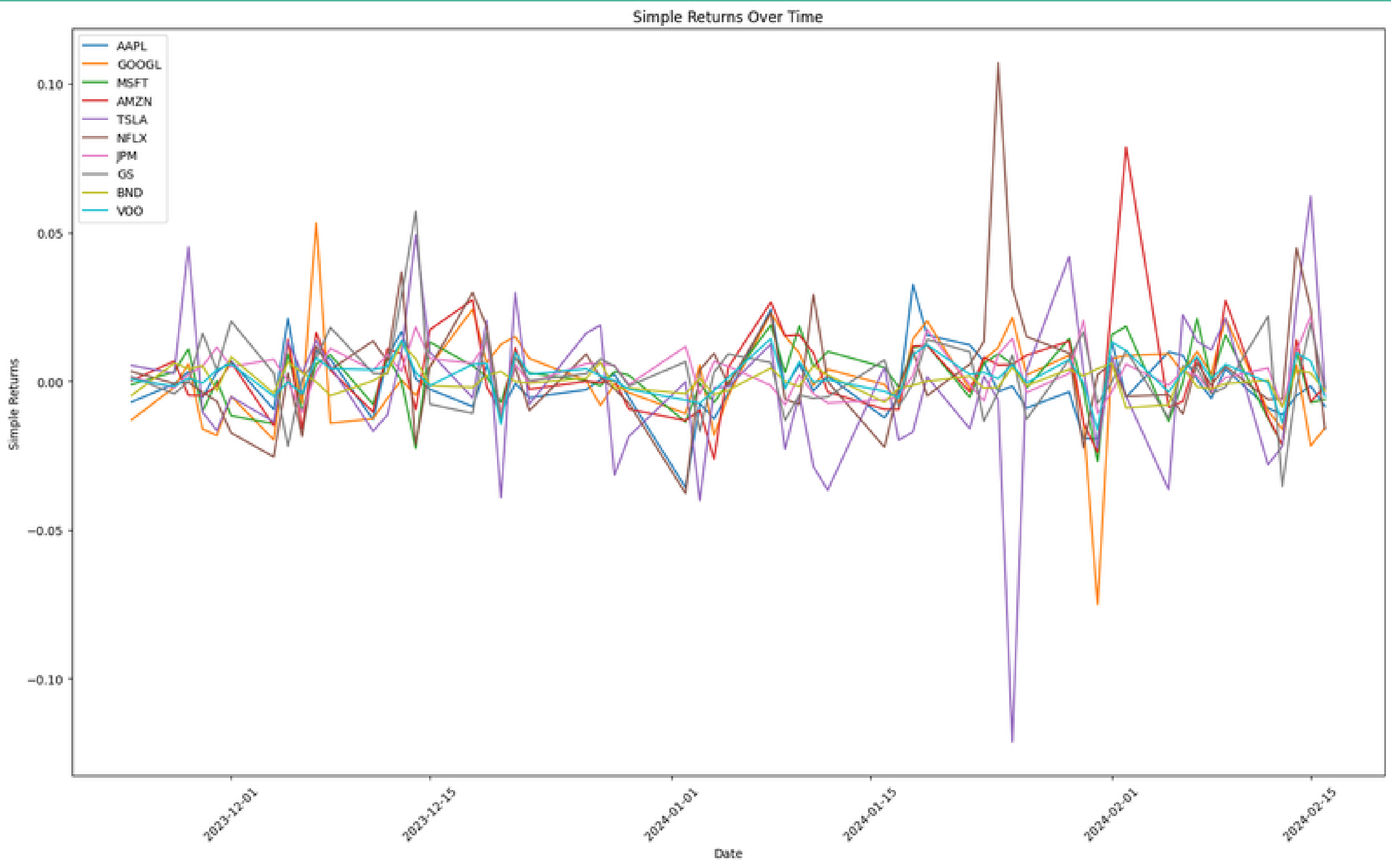
The formula for calculating simple returns  $R_t$  for a given time period  $t$  is:

$$R_t = \frac{P_t - P_{t-1}}{P_{t-1}} \times 100\%$$

- $P_t$  is the price of the asset at time  $t$ .
- $P_{t-1}$  is the price of the asset at the previous time period  $t - 1$ .

	AAPL	AMZN	BND	GOOGL	GS	JPM	MSFT	NFLX	TSLA	VOO
Date										
2023-11-22	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
2023-11-24	-0.007004	0.000204	-0.004796	-0.012997	0.001506	0.001370	-0.001112	0.003264	0.005294	0.000550
2023-11-27	-0.000948	0.006747	0.006094	-0.002048	-0.004246	-0.002279	0.003126	-0.000813	0.002676	-0.001745
2023-11-28	0.003214	-0.004738	0.004226	0.005791	-0.000178	0.002285	0.010803	-0.000355	0.045069	0.000982
2023-11-29	-0.005410	-0.004829	0.005190	-0.016108	0.016005	0.005080	-0.010060	-0.003779	-0.010457	-0.000646

# VISUALIZATION SIMPLE RETURN OVER 3 months

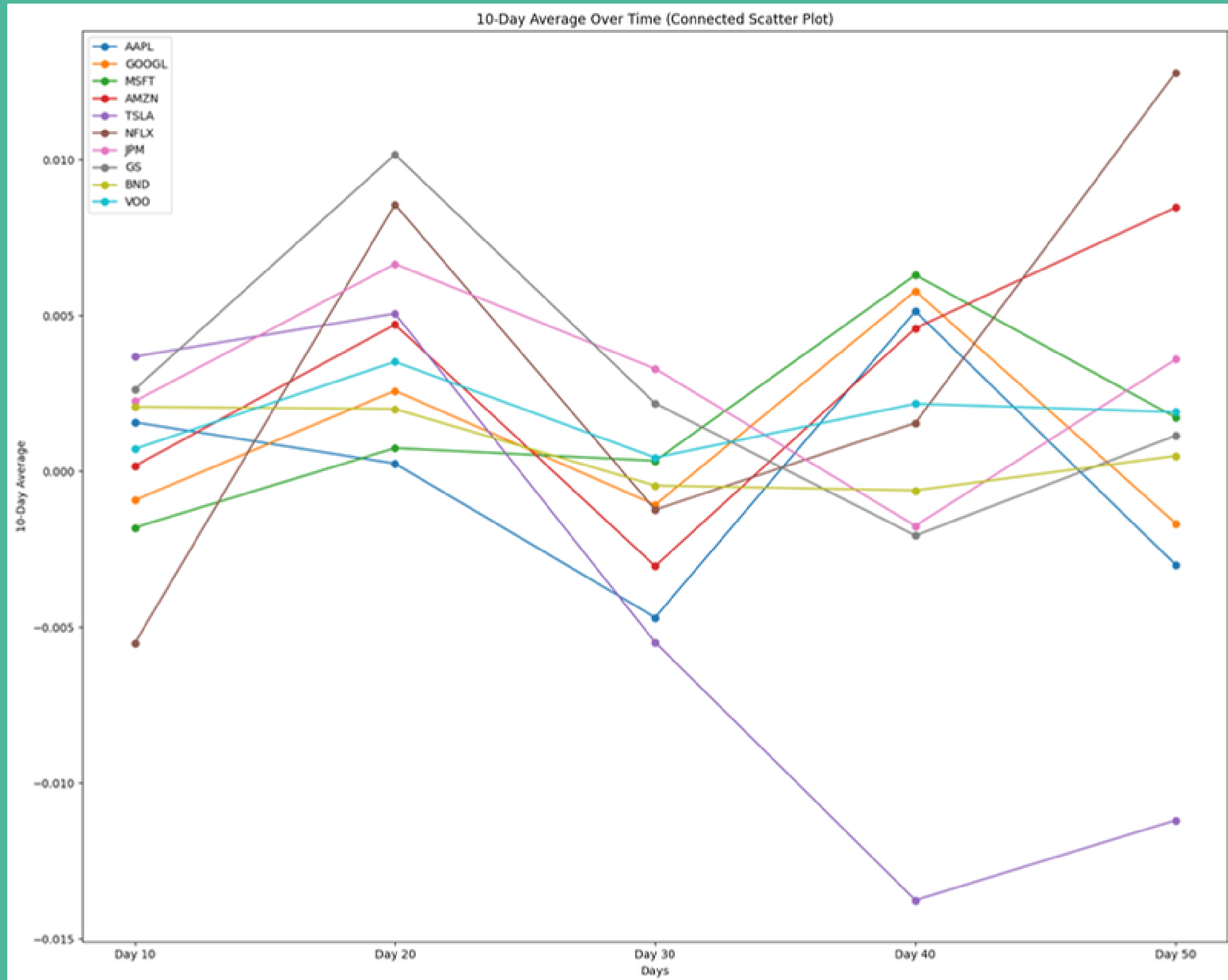




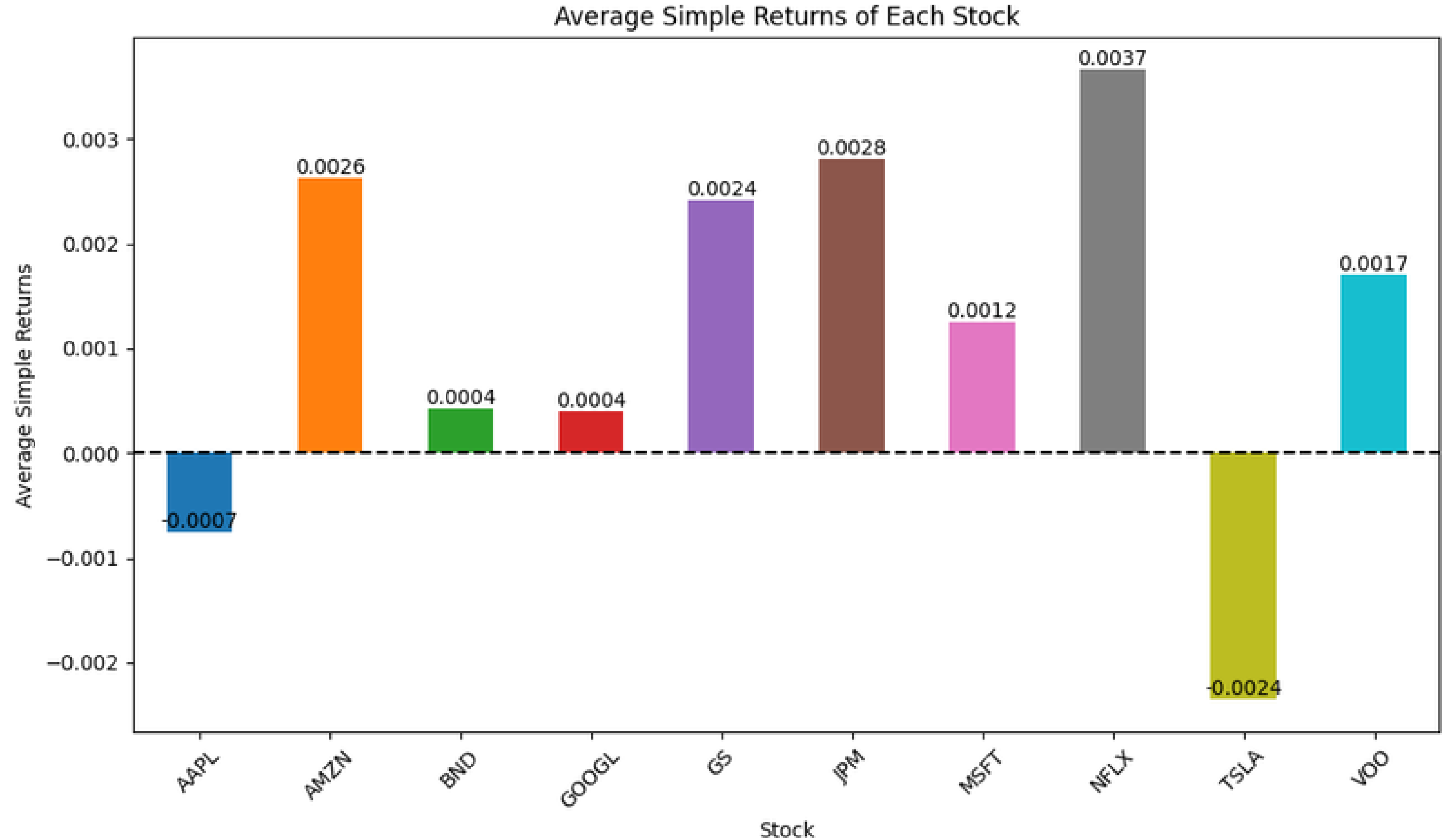
**Y AXIS : AVERAGES of  
period 10 DATA ROWS**

**X AXIS : LAST DAY of SET**

This provides with an average  
'SIMPLE RETURN' value for  
each set, smoothing out short-  
term fluctuations and helping  
us to observe longer-term  
trends in the data.



# VISUALIZATION avg. of SIMPLE RETURN OVER 3 months



# LOG RETURNS

Log returns, also known as logarithmic returns or continuously compounded returns, are an alternative way to measure the percentage change in the value of an asset over time.

The formula for calculating log returns  $r_t$  for a given time period  $t$  is:

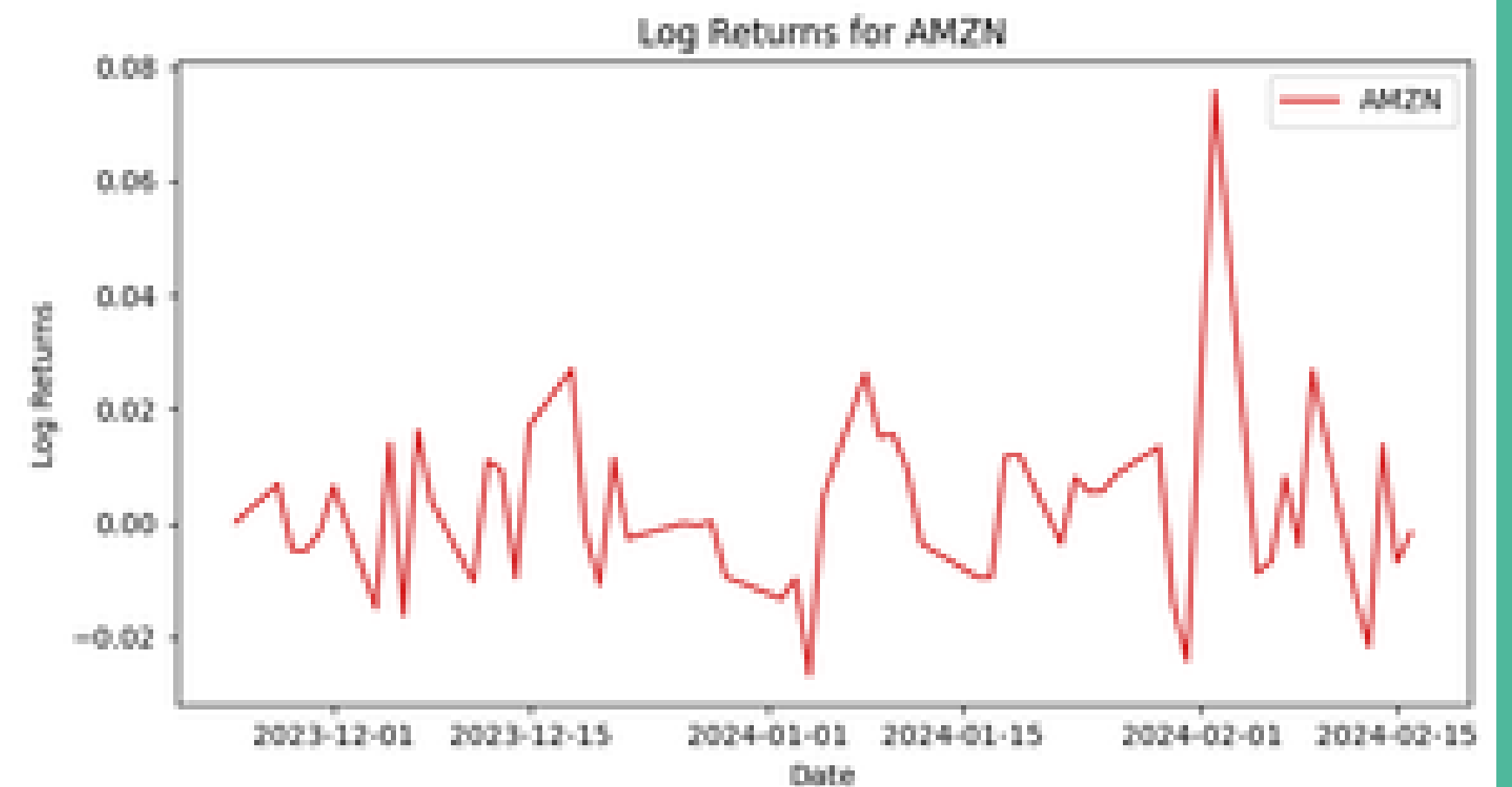
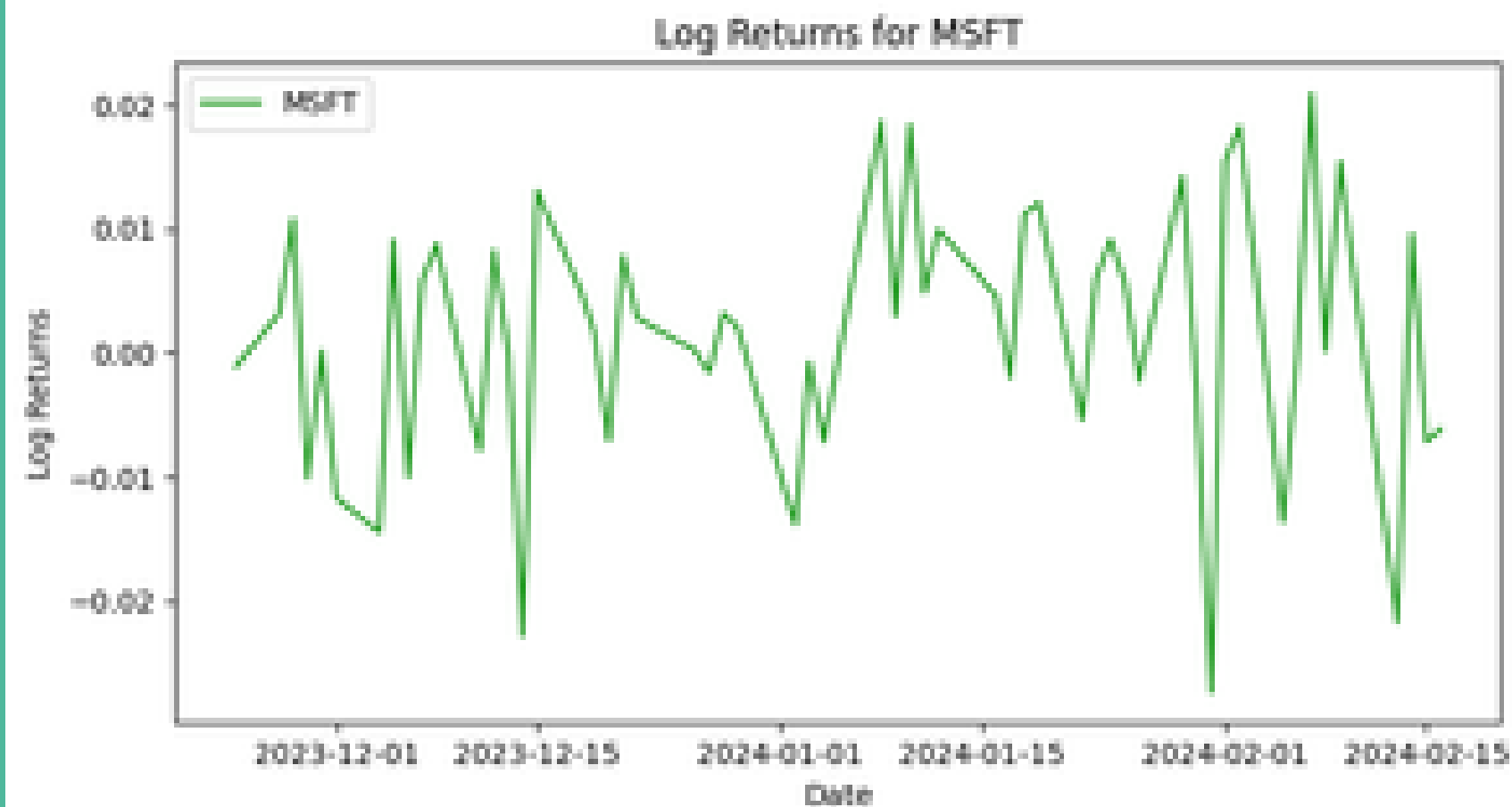
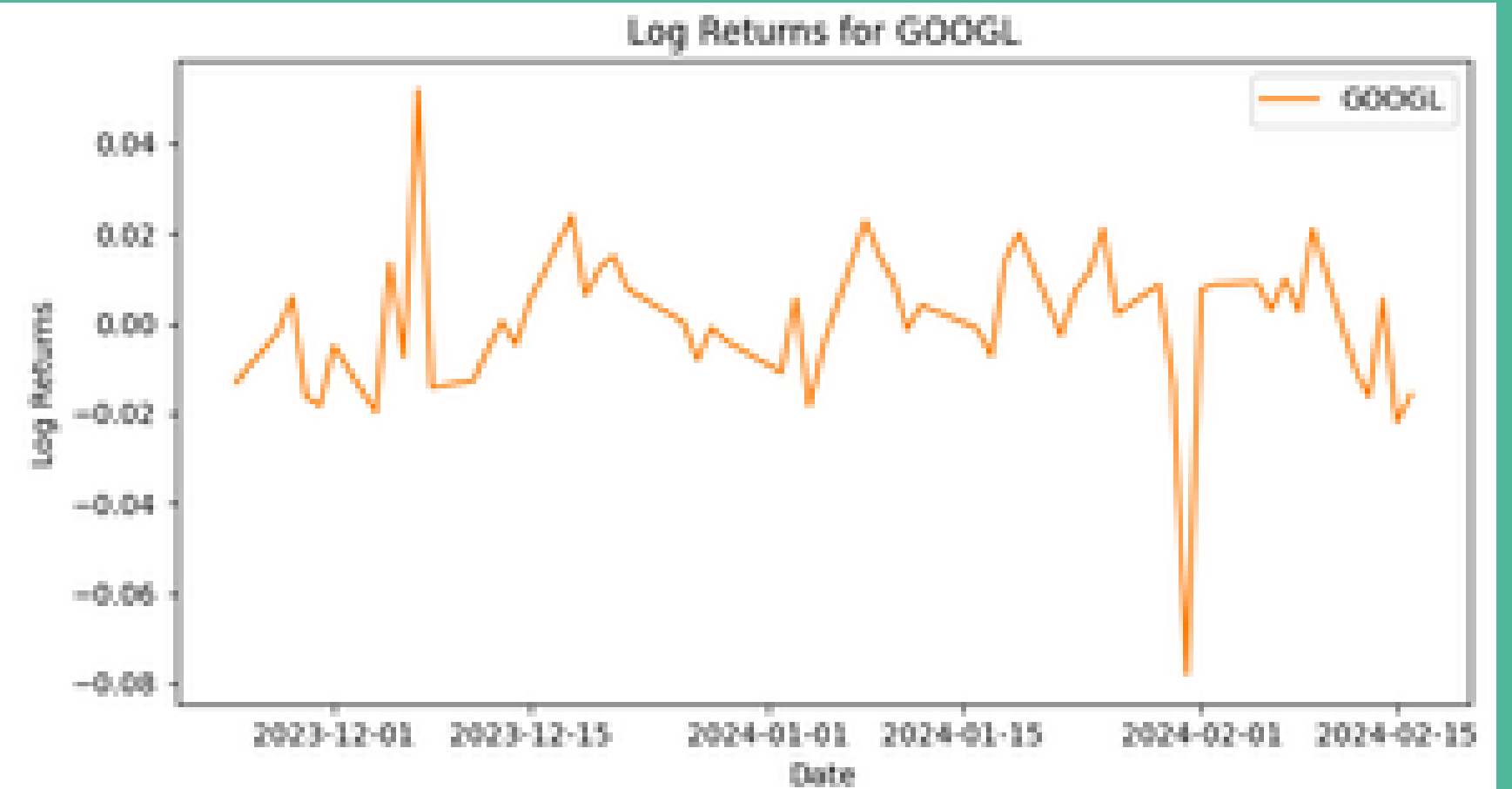
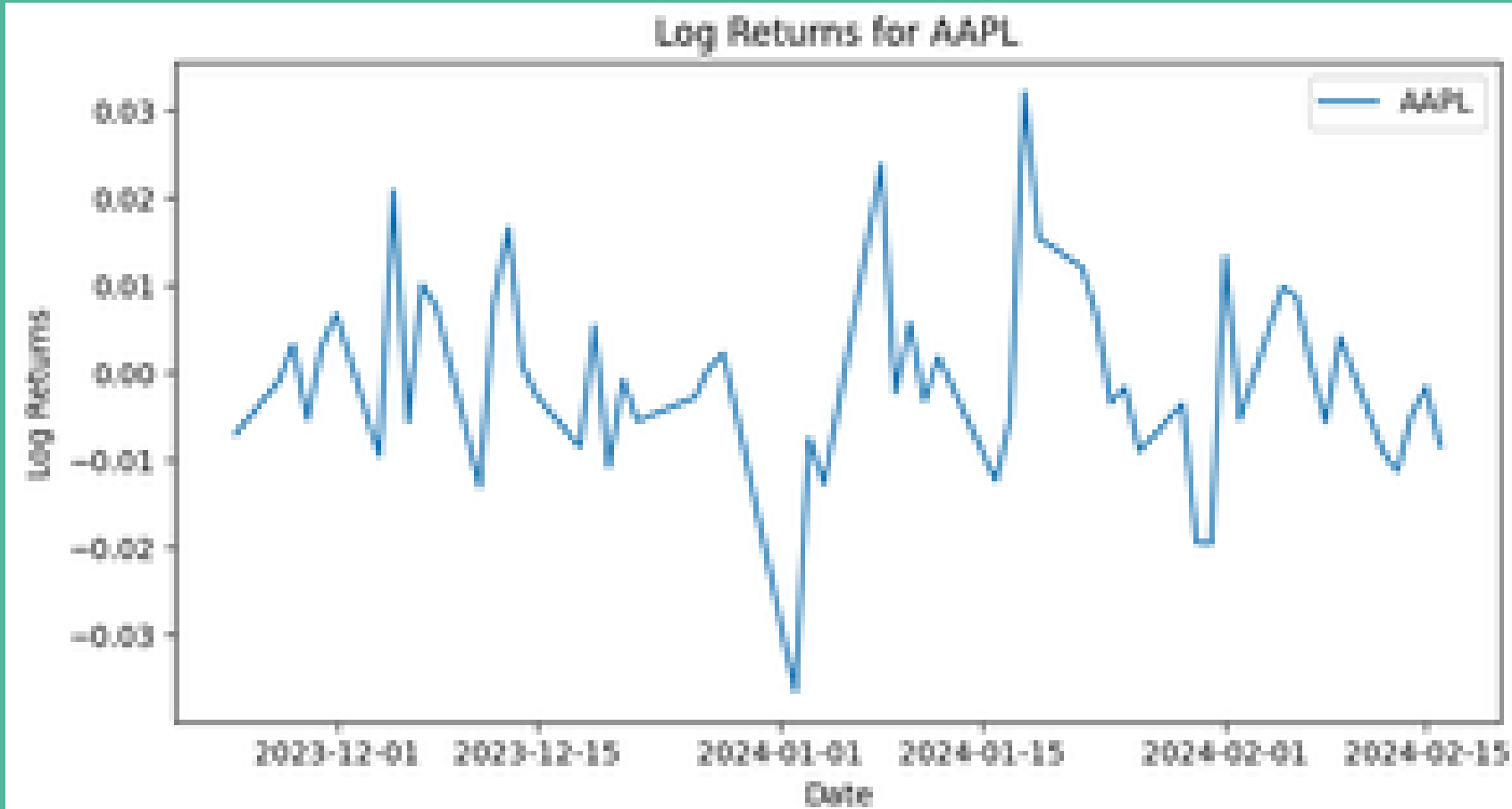
$$r_t = \ln \left( \frac{P_t}{P_{t-1}} \right)$$

- $P_t$  is the price of the asset at time  $t$ .
- $P_{t-1}$  is the price of the asset at the previous time period  $t - 1$ .

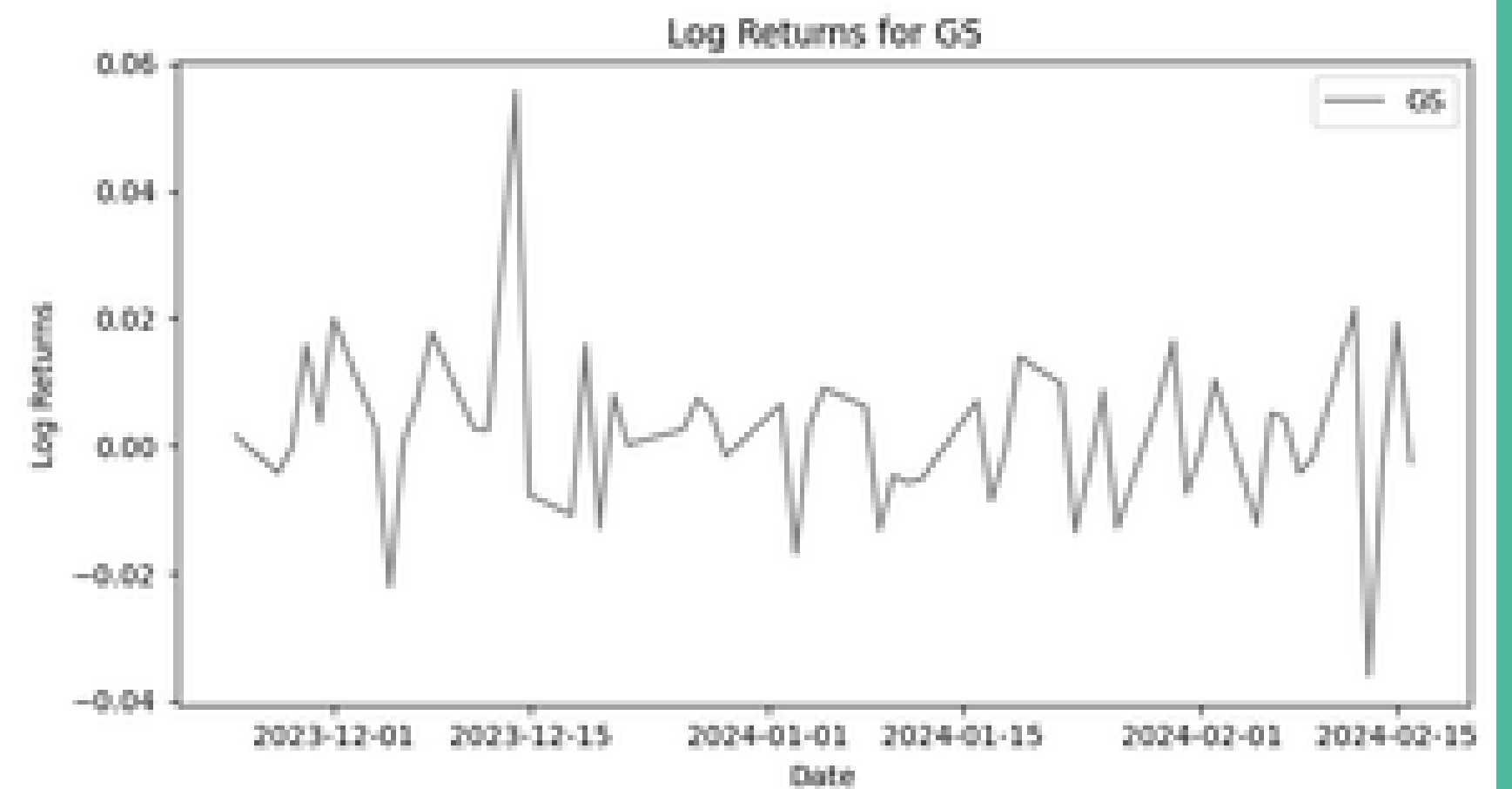
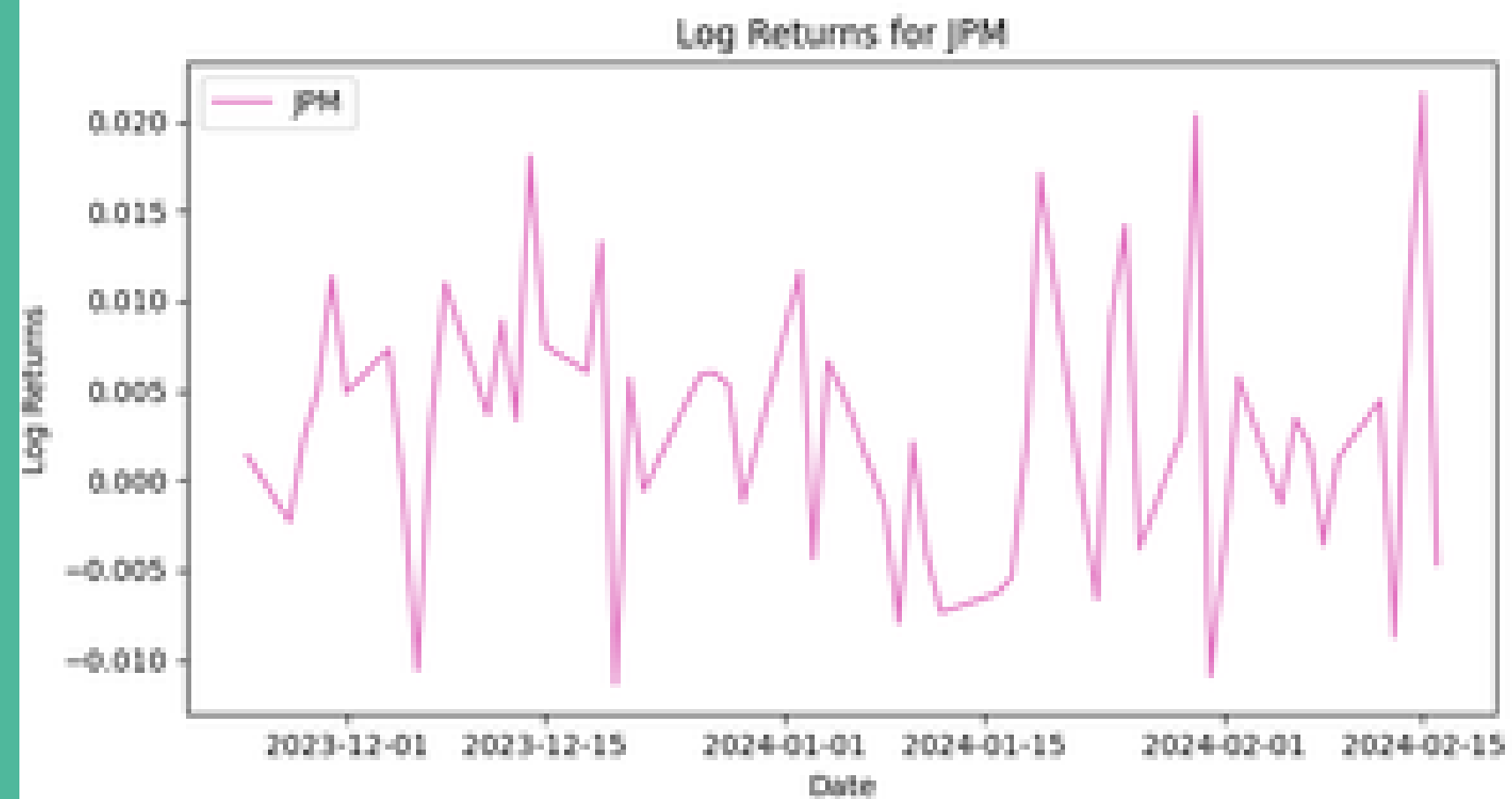
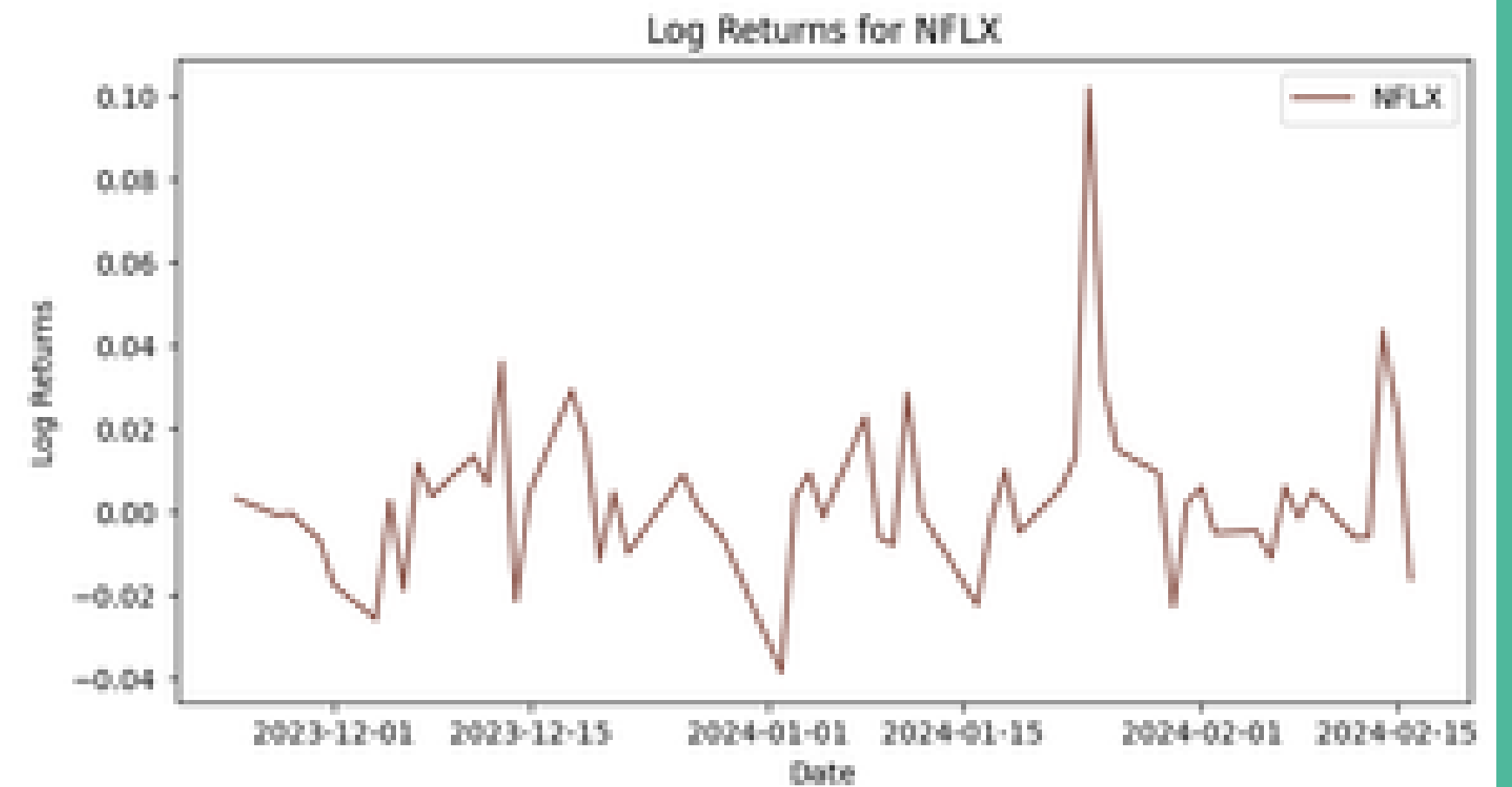
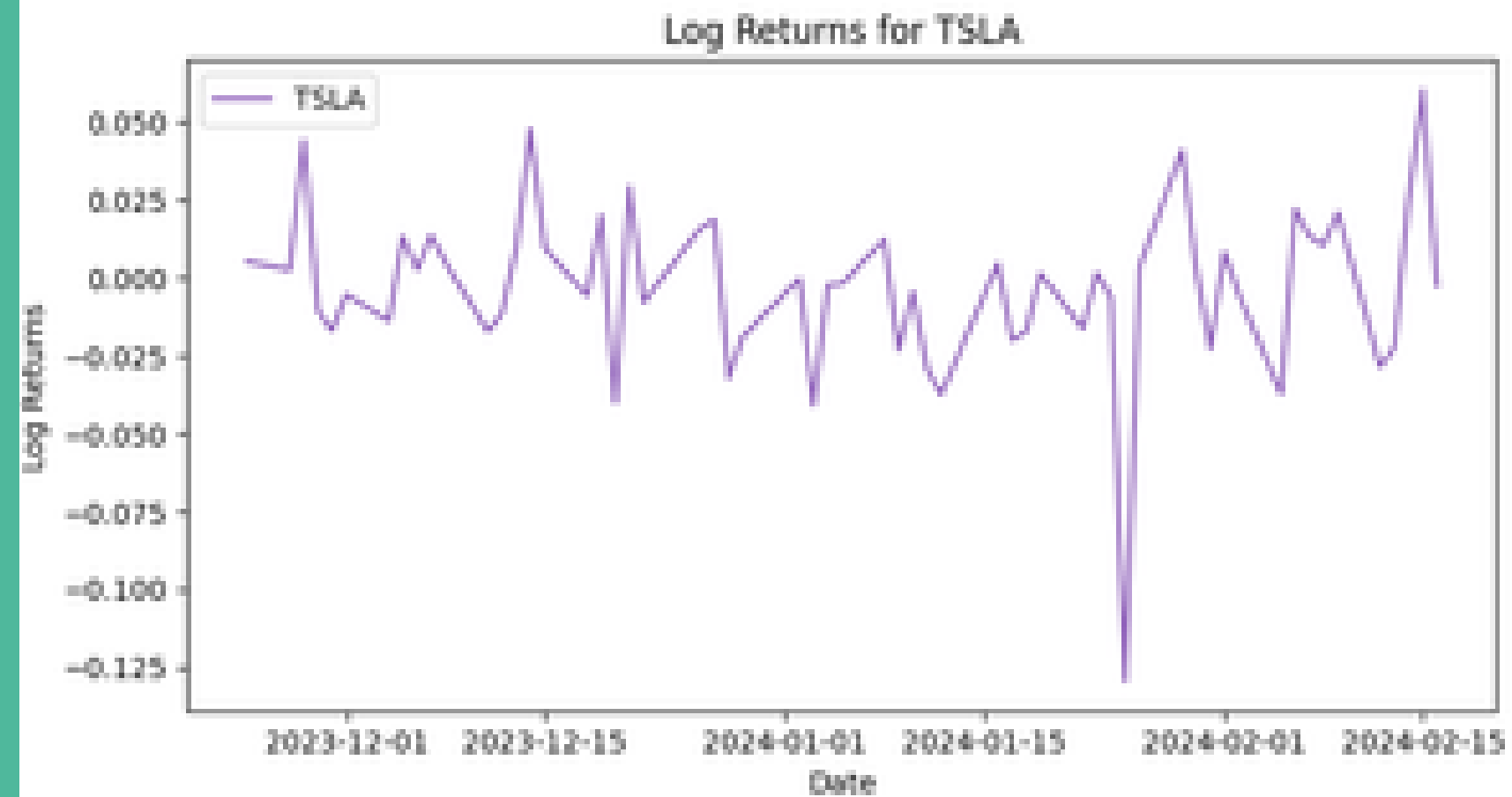
	AAPL	AMZN	BND	GOOGL	GS	JPM	MSFT	NFLX	TSLA	VOO
Date										
2023-11-24	-0.007029	0.000204	-0.004807	-0.013083	0.001505	0.001369	-0.001112	0.003258	0.005280	0.000550
2023-11-27	-0.000948	0.006724	0.006076	-0.002051	-0.004255	-0.002282	0.003122	-0.000814	0.002672	-0.001746
2023-11-28	0.003209	-0.004750	0.004217	0.005775	-0.000178	0.002282	0.010745	-0.000355	0.044083	0.000981
2023-11-29	-0.005424	-0.004841	0.005177	-0.016239	0.015878	0.005067	-0.010111	-0.003786	-0.010512	-0.000646
2023-11-30	0.003058	-0.001573	-0.003215	-0.018392	0.003755	0.011340	0.000158	-0.006771	-0.016770	0.003870



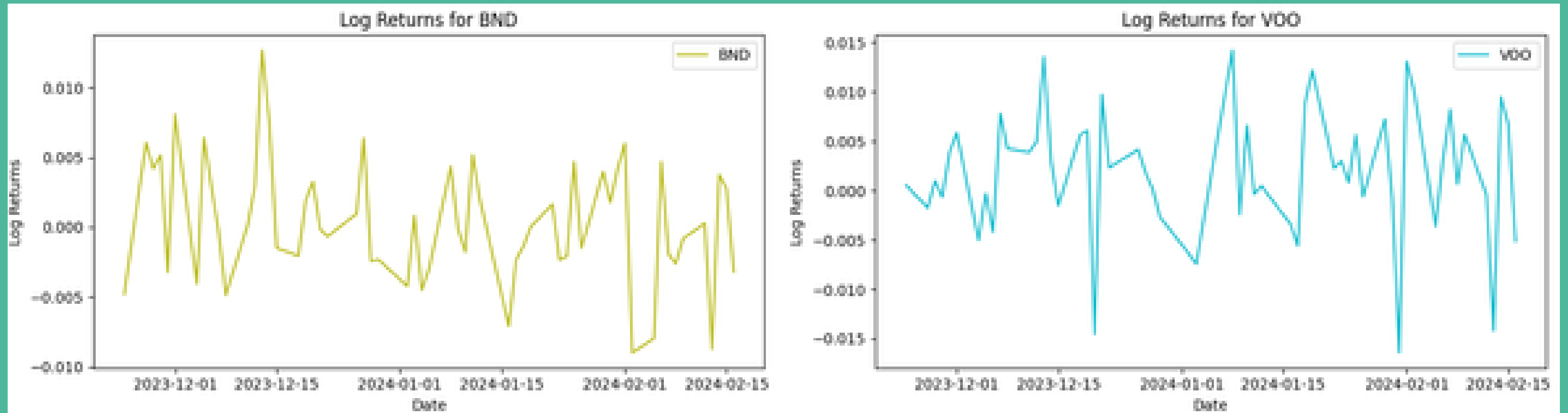
# VISUALIZATION LOG RETURN OVER 3 months



# VISUALIZATION LOG RETURN OVER 3 months



# VISUALIZATION LOG RETURN OVER 3 months

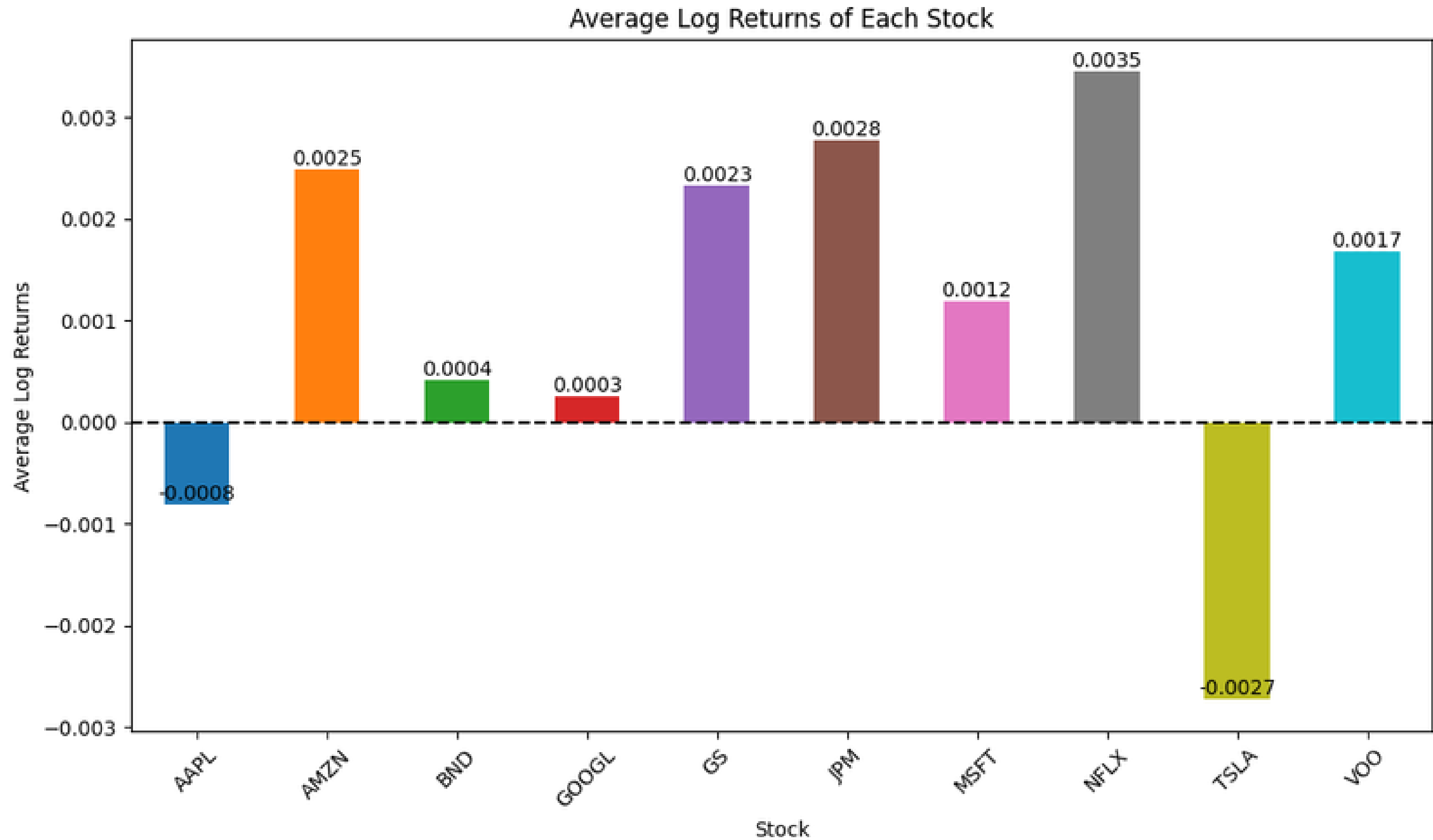


## Log returns have several advantages over simple returns:

- Additivity: Log returns are additive over time periods, making them suitable for continuous compounding.
- Statistical properties: Log returns have more desirable statistical properties for many financial applications, such as being approximately normally distributed.
- Interpretability: Log returns represent the continuously compounded growth rate of an investment, making them easier to interpret and compare across different assets and time periods.



# VISUALIZATION avg. of LOG RETURN OVER 3 months



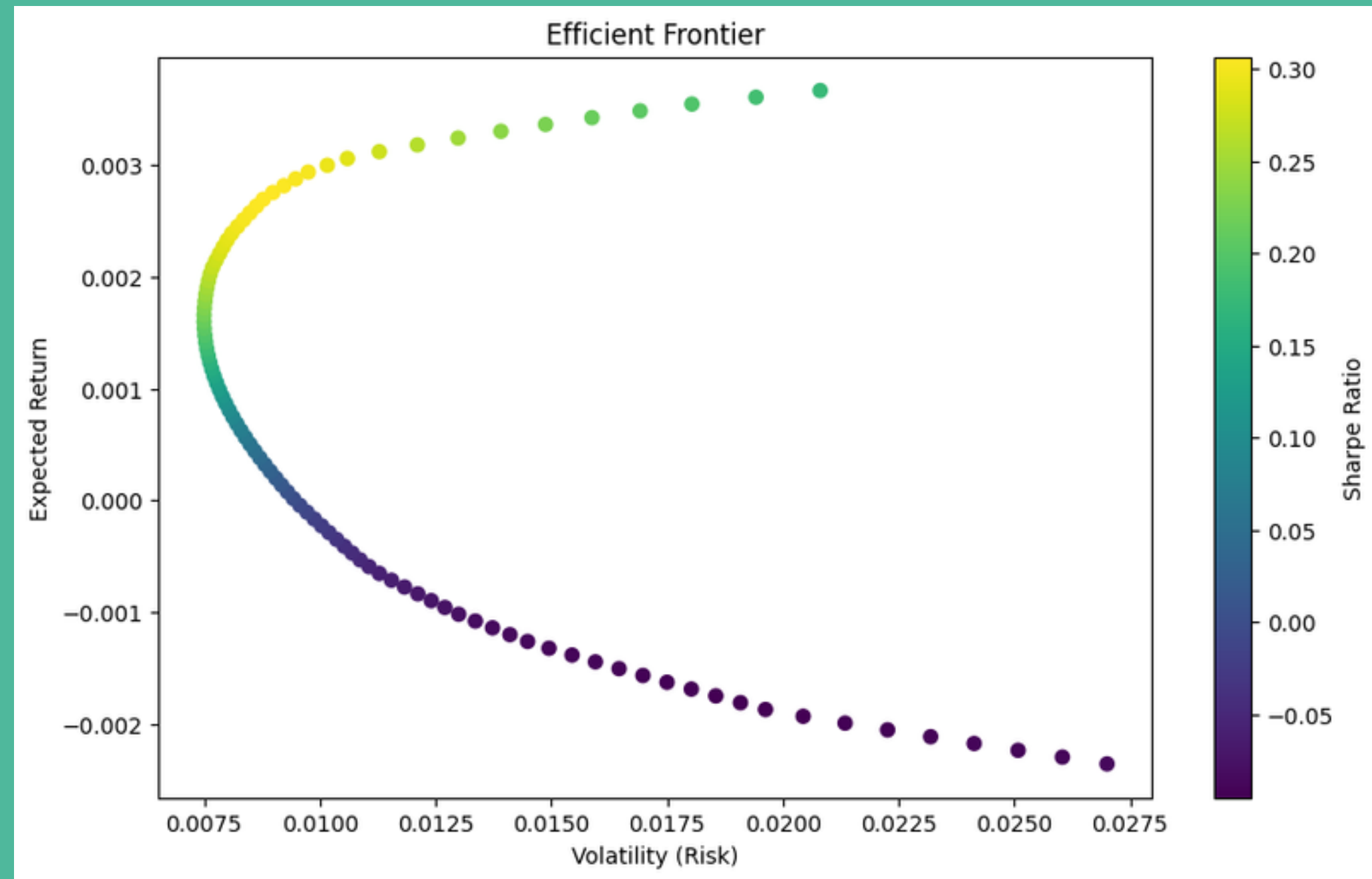
## Markowitz's mean-variance optimization using the following steps:

- Mean returns and covariance matrix are calculated.
- Initial weights and an objective function are defined to maximize returns.
- Constraints and bounds are set to ensure proper weight allocation.
- Efficient frontier points are iteratively determined, optimizing portfolio weights for different expected return levels.
- Efficient returns and volatilities are stored for visualization.
- The optimization process is conducted using the minimize function from **SciPy**.

SciPy solve the mean-variance optimization problem by finding the best combination of investments in a portfolio. It does this by iteratively adjusting the allocation of assets to maximize returns while minimizing risk, based on the historical performance of each asset and their relationships with each other.

**The color of each point represents the **Sharpe Ratio**, which is a measure of risk-adjusted return.**

- Higher Sharpe Ratio values indicate better risk-adjusted returns.
- Points with warmer colors (e.g., red) indicate portfolios with higher returns relative to their risk,)
- While points with cooler colors (e.g., blue) represent portfolios with lower returns relative to their risk.



**The x-axis represents the volatility or risk of each portfolio, while the y-axis represents the expected return.**

**It shows the relationship between expected return and volatility (risk) for different portfolios. Each point on the graph represents a portfolio with a specific combination of assets.**



# RISK MEASURES USED ARE BETA, STANDARD DEVIATION, AND THE SHARPE RATIO.

## Beta:

Beta measures the sensitivity of an asset's returns to changes in the market. A beta of 1 indicates that the asset's returns move in line with the market, while a beta greater than 1 implies higher volatility than the market, and a beta less than 1 suggests lower volatility.

## Standard Deviation:

Standard deviation measures the dispersion or volatility of an asset's returns around its mean. A higher standard deviation indicates higher volatility and therefore higher risk, while a lower standard deviation implies lower risk.

## Sharpe Ratio:

The Sharpe ratio is a measure of risk-adjusted return and assesses the excess return of an investment per unit of risk. A higher Sharpe ratio indicates better risk-adjusted performance, as the asset generates more return per unit of risk.

### 1. Beta ( $\beta$ ):

$$\beta = \frac{\text{Covariance}(r_a, r_m)}{\text{Variance}(r_m)}$$

where:

- $r_a$  is the asset's returns.
- $r_m$  is the market's returns.

### 2. Standard Deviation ( $\sigma$ ):

$$\sigma = \sqrt{\frac{\sum_{i=1}^n (r_i - \bar{r})^2}{n}}$$

where:

- $r_i$  is the individual return.
- $\bar{r}$  is the average return.
- $n$  is the number of observations.

### 3. Sharpe Ratio:

$$\text{Sharpe Ratio} = \frac{r_a - r_f}{\sigma_a}$$

where:

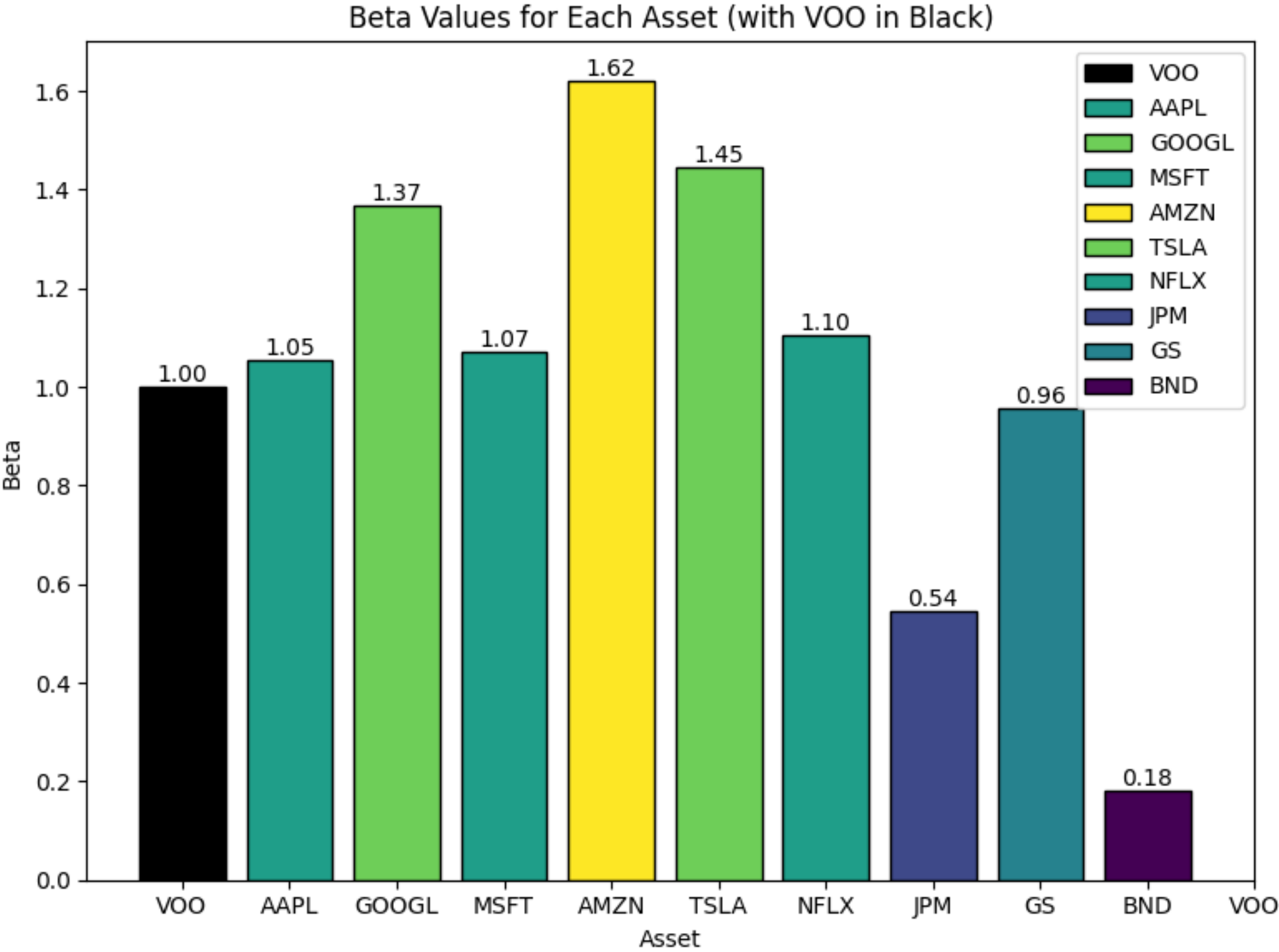
- $r_a$  is the asset's average return.
- $r_f$  is the risk-free rate.
- $\sigma_a$  is the standard deviation of the asset's returns

V00 (Vanguard S&P 500 ETF)

market\_index = "V00"

Risk Measures for Each Asset:

	Standard Deviation	Beta	Sharpe Ratio
AAPL	0.011123	1.052907	-0.067222
GOOGL	0.0169	1.366232	0.023251
MSFT	0.01069	1.072315	0.116477
AMZN	0.016249	1.619543	0.161292
TSLA	0.027005	1.446093	-0.087229
NFLX	0.020824	1.104932	0.175964
JPM	0.007716	0.544794	0.364263
GS	0.013566	0.956229	0.178417
BND	0.004428	0.182143	0.095536
V00	0.006505	1.0	0.261624



**Point 1 (Low Risk Tolerance):**

AAPL, MSFT, AMZN, JPM, and BND have significant weights, indicating higher importance in the diversified portfolio.

GOOGL, TSLA, and NFLX have zero weight, implying they are not included in this conservative portfolio.

Weights for Point 1 (Risk Tolerance Level 1):

AAPL: 0.264583  
GOOGL: 0.000000  
MSFT: 0.124374  
AMZN: 0.127981  
TSLA: 0.000000  
NFLX: 0.000000  
JPM: 0.025930  
GS: 0.000000  
BND: 0.457131  
VOO: 0.000000

**Point 2 (High Risk Tolerance):**

GOOGL, TSLA, NFLX, and GS have higher weights, suggesting a more concentrated portfolio.

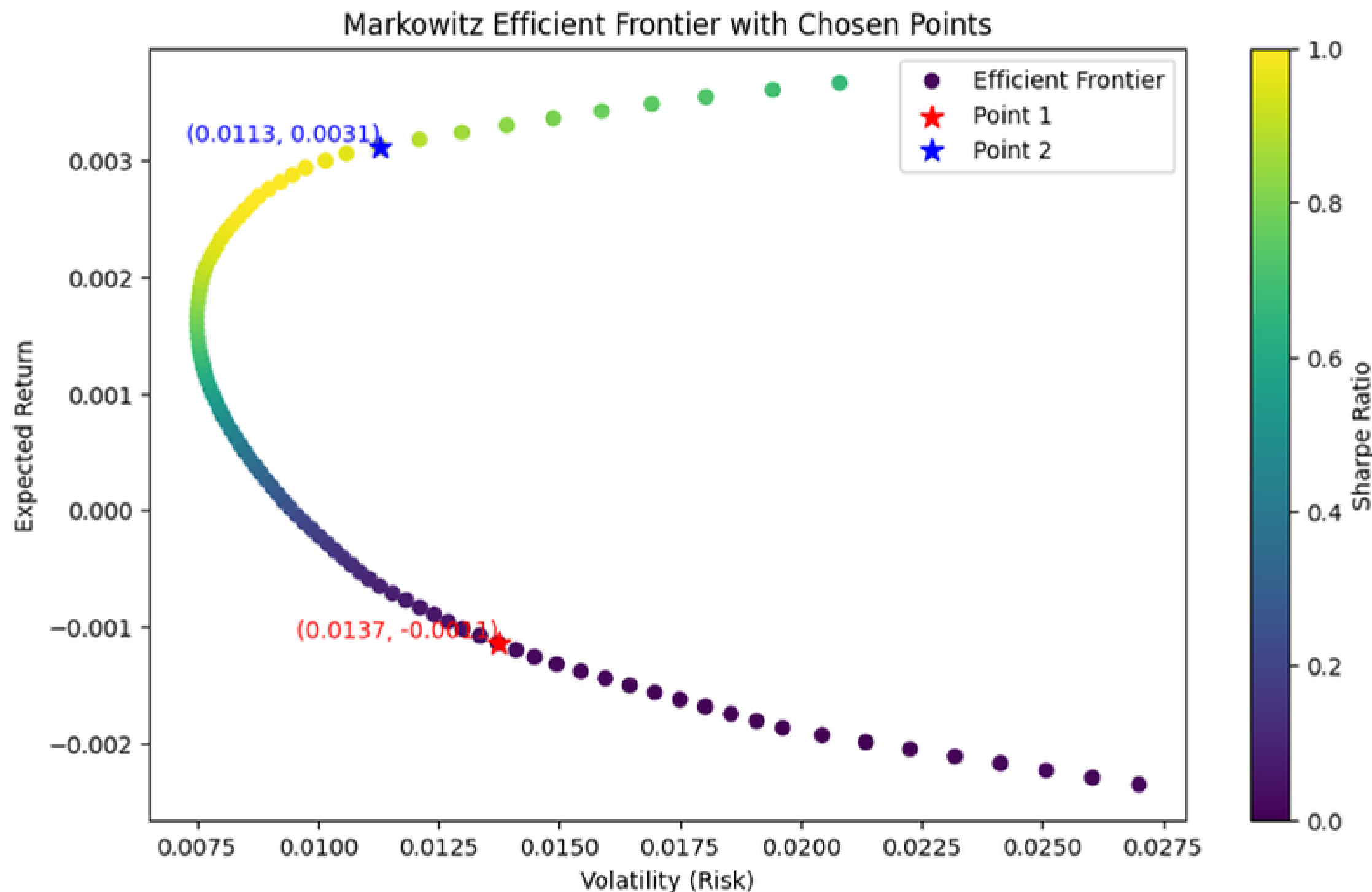
AAPL, MSFT, AMZN, JPM, BND, and VOO have zero weight, indicating they are less relevant in this riskier strategy.

Weights for Point 2 (Risk Tolerance Level 2):

AAPL: 0.000000  
GOOGL: 0.181599  
MSFT: 0.000000  
AMZN: 0.000000  
TSLA: 0.128495  
NFLX: 0.231894  
JPM: 0.000000  
GS: 0.458012  
BND: 0.000000  
VOO: 0.000000



**CALCULATING WEIGHTS BY CHOOSING TWO POINTS ON THE EFFICIENT FRONTIER  
REPRESENTING TWO DIFFERENT RISK TOLERANCE LEVELS.**



**Higher volatility indicates that the price of the asset can change dramatically over a short period, implying higher risk. Conversely, lower volatility suggests that the price is more stable and less prone to large fluctuations, indicating lower risk.**

**The portfolio allocation changes at different risk tolerance levels along the efficient frontier.**

## A brief discussion of the trade-off between risk and return in our portfolio choices.

- **High-Risk, High-Return Assets:**

Assets like "TSLA" (Tesla) and "NFLX" (Netflix) are often considered high-risk, high-return investments. These companies operate in dynamic industries and are subject to rapid technological changes and market trends. While these assets have the potential for high returns, they also come with higher volatility and risk. Factors such as regulatory changes, competition, and market sentiment can significantly impact their stock prices.

- **Moderate-Risk, Moderate-Return Assets:**

"AAPL" (Apple) and "MSFT" (Microsoft) are examples of well-established technology companies with a history of stable performance. They are considered moderate-risk, moderate-return assets. These companies often have diversified revenue streams, strong market positions, and established customer bases. While they may not offer the same level of potential returns as high-risk assets, they tend to exhibit more stability.

- **Low-Risk, Lower-Return Assets:**

"BND" (Vanguard Total Bond Market ETF) is an example of a low-risk, lower-return asset. Bonds are generally considered safer investments compared to stocks, and they provide regular interest payments. The trade-off here is that the potential returns from bonds are typically lower than those from stocks. Investors often include bonds in their portfolios for capital preservation and income stability.

- **Market Exposure:**

"VOO" (Vanguard S&P 500 ETF) represents exposure to the broader market. The S&P 500 includes a diversified mix of large-cap stocks from various sectors. While the S&P 500 is subject to market fluctuations, it is considered a benchmark for the overall performance of the U.S. stock market. It provides a way for investors to participate in the long-term growth of the economy.

## Limitations of Markowitz optimization and its real-world applications.

**Normality Assumption:** Markowitz assumes asset returns follow a normal distribution, but real markets can have extreme events. ForEx : the 2008 financial crisis saw much larger stock market declines than predicted by normal distributions.

**Estimation Error:** Relying on historical data for future predictions can lead to errors. For instance, if historical data suggests low volatility in a market, but sudden geopolitical events increase uncertainty, the model may not accurately account for this.

**Sensitivity to Inputs:** Small changes in expected returns or risk measures can drastically alter portfolio recommendations. For instance, slightly adjusting expected returns upward might heavily favor riskier assets, potentially exposing the portfolio to greater risk.

**Single-Period Focus:** Markowitz optimization considers a single period, overlooking changing market conditions. For instance, a portfolio optimized for a bull market may underperform in a bear market.

**Ignoring Transaction Costs:** The model doesn't factor in transaction costs, leading to unrealistic recommendations. For example, high-frequency trading strategies may generate significant transaction costs, affecting portfolio performance.

**Assumptions of Stability:** Markowitz assumes stable market parameters, but real-world volatility and interest rates fluctuate. For instance, unexpected changes in central bank policies can lead to rapid shifts in interest rates, affecting bond prices.

**Complexity of Short Selling:** While short selling can enhance portfolio flexibility, it introduces complexity and risk. For example, shorting a stock that unexpectedly surges can lead to significant losses if the position is not properly managed.



# Thank You!

