

Assignment-I

Ans 1:- Asymptotic notations are languages that allow us to analyze an algorithm running time by identifying its behaviour as the input size of algorithm.

Types:-

(a) Big O: It is commonly used for worst case, and gives upper bound for the growth rate of runtime of algorithm.

Ex:- Big O notation for linear search is $O(n)$

(b) Big Omega: It is notation used for best case complexity, it provides as with an asymptotic lower bound.

Ex:- Big Omega of linear search is $\Omega(1)$

(c) Theta: It is used for tight bound on the growth rate of runtime of algo.

Ex:- Theta of linear search is $\Theta(n)$.

(d) Small Omega: To denote lower bound (that is not asymptotic tight).

Ans 2:- for $(i=1 \text{ to } n)$
 $\{ \text{if } i \text{ is even; } \}$

$\Rightarrow O(\log n)$

Ans 3:- $T(n) = 3T(n-1)$
 $T(1) = 1$
 $T(2) = 3T(1) = 3$
 $T(3) = 3T(2) = 9$
 $T(4) = 3T(3) = 27$
 \vdots
 $T(n) = (n-1)^3$

Time complexity $\rightarrow O(3^n)$

Ans 4:- $T(n) = 2(T(n-1) - 1)$
 $T(n-1) = 2T(n-2) - 1$
 $T(n) = 4T(n-2) - 2 - 1$
 $T(n-2) = 2T(n-3) - 1$
 $T(n) = 8T(n-3) - 4 - 2 - 1$
 $T(n-3) = 2T(n-4) - 1$
 $T(n) = 16T(n-4) - 8 - 4 - 2 - 1$
 $T(n) = 2^k - 2^3 - 2^2 - 2^1 - 2^0$
 $= O(1)$

Ans 5:-

S	J	
1	1	
3	2	$O(\sqrt{n})$
6	3	
10	4	

Ans 6:- $i * i = n$
 $i^2 = n$
 $i = \sqrt{n}$
 $O(\sqrt{n})$

Ans 7:- $O(n \log^2 n)$

Ans 9:- Total $T = O(n \log n)$

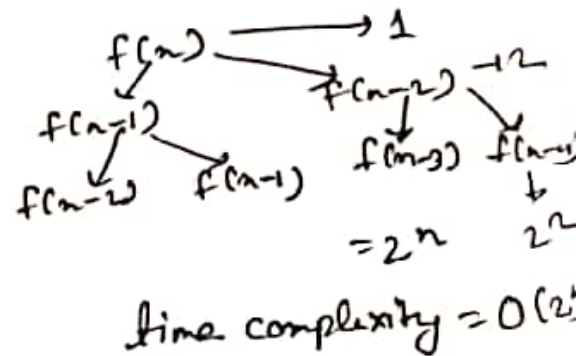
Ans 10:- n^k is $O(c^k)$ as for example
 of :: when we take $n=2, k=2, c=2$
 Then $2^2 \leq 2^2$ so c^k is upper limit of n^k .

Ans 11:-

j=1	i=0
1	1
2	3
3	6
4	10

The series is nearly dependent on i as 2^i
 so $O(2^n)$

Ans 12: Space complexity
 $= O(n)$ as clear call of $(n-1)$



Ans 13:- $n \log n$

```
for (i=0; i < n; i++)
  for (j=0; j < n; j++)
    c++;
```

```
n3
for (i=0; i < n; i++)
  for (j=0; j < n; j++)
    for (k=0; k < n; k++)
      c++;
```

$\log(\log n)$

```
int func(int n) {
  if (n == 1)
    return n;
  else
    return func(sqrt(n)) + func(sqrt(n));
}
```

Ans 14:- $T(n) = T\left(\frac{n}{2}\right) + T\left(\frac{n}{4}\right) + cn^2$

Using master

$a=2, b=2$

$c=1$

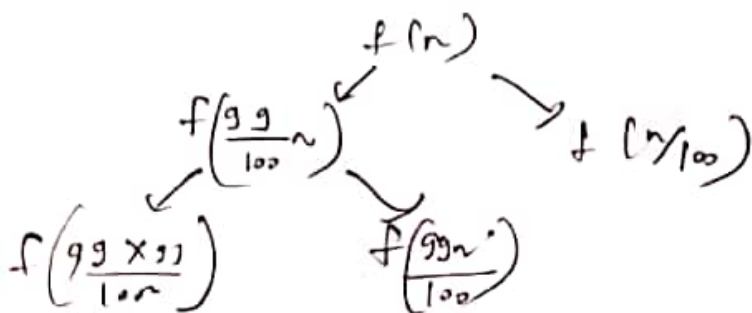
$f(n) > n^c \text{ if } n^2 > 1$

$O(n^2)$

Ans 15: $O(n\sqrt{n})$

Ans 16:- $O(\log \log n)$

Ans 17: $T(n) = T\left(\frac{99}{100}n\right) + T\left(\frac{n}{100}\right)$



$= O(\log n)$

Ans 18:- a) $100 < \log \log n < \log n < \sqrt{n} < n \log(1) < n \log n < n^2 < 2^n < 2^{2n} < 4^n < n!$

b) $1 < \log \log n < \sqrt{\log n} < \log^2 n < \log n < 2 \log n < n < 2n < 4n < n^2 < n! < 2(n)^n < n!$

c) $96 < \log_2 n < \log_3 n < \log_4 n < \log n! < n \log n < n \log_2 n < 8n^2 < 8n^3 < 8n < n!$

Ans 19: linear (arr, key) {
 for (int i=0; i<n; i++)
 if (arr[i] == key)
 return i;
 return -1;

Ans 20: Insert (arr, n) {
 if (n <= 1) return;
 recursively for n-1 element
 Insert sorted

3 Pick last element
 arr [i] &
 Insert into sorted

Iterations:-

```
Insert(arr, n) {  
    for (i = 1; i < n; i++)  
    {  
        Pick arr[i] & insert into arr [0, ..., i-1]  
    }  
}
```

	Stable	Inplace	Online
Bubble Sort		✓	✗
Selection "	✓	✓	✗
Insertion "	✗	✓	✓

Ans 22:-

	Best	Avg	Worst
Bubble	$O(n^2)$	$O(n^2)$	$O(n^2)$
Selection	$O(n^2)$	$O(n^2)$	$O(n^2)$
Insertion	$O(n)$	$O(n)$	$O(n^2)$

Ans 23:- Recursive:-

```
Binary(arr, l, r, key) {  
    if (l < r) {  
        mid = l + (r - l) / 2;  
        if (arr[mid] == key) return 1;  
        if (key < arr[mid])  
            Binary(l, mid - 1, key);  
        else  
            Binary(mid + 1, r, key);  
    }  
}
```

Ans 24:-
 $T(n) \neq T(n/2) + 1$

Iterative:-

```
while (l < r)  
{  
    mid = l + (r - l) / 2;  
    if (arr[mid] == key) return 1;  
    if (key < arr[mid])  
        r = mid - 1;  
    else  
        l = mid + 1;  
}
```