# Generative Models I

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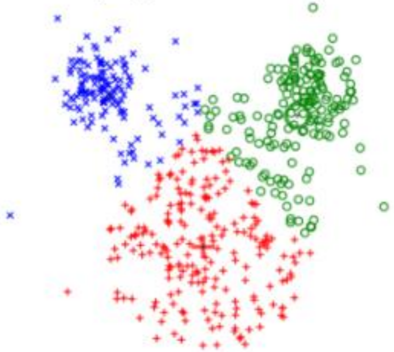
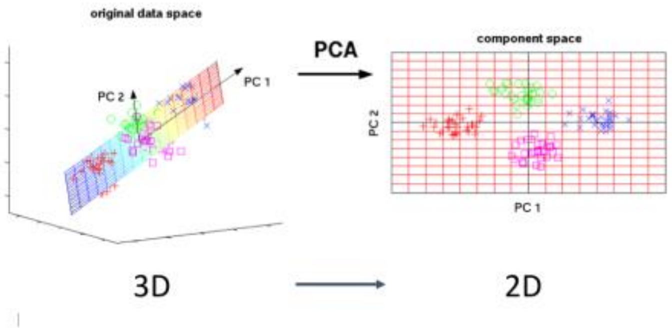
# Supervised vs Unsupervised

| | Supervised Learning | Unsupervised Learning |
|---|---|---|
| **Data** | $(x, y)$ : $x$ is data, $y$ is label | $x$ |
| **Goal** | $x \to y$ 로 가는 function을 학습 | Learn some underlying hidden structure of the data |
| **Examples** | Classification, regression, object detection, semantic segmentation, image captioning, etc. | Clustering, dimensionality reduction, feature learning, density estimation, etc. |

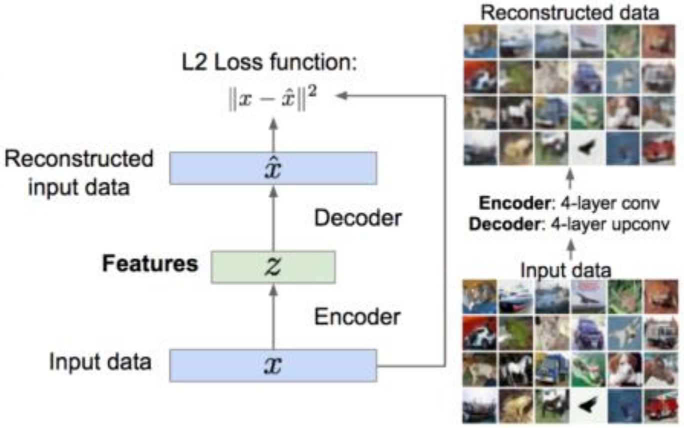
Clustering
(e.g. K-Means)



Dimensionality Reduction
(e.g. Principal Components Analysis)




Feature Learning
(e.g. autoencoders)



# **Discriminative** vs **Generative** Models

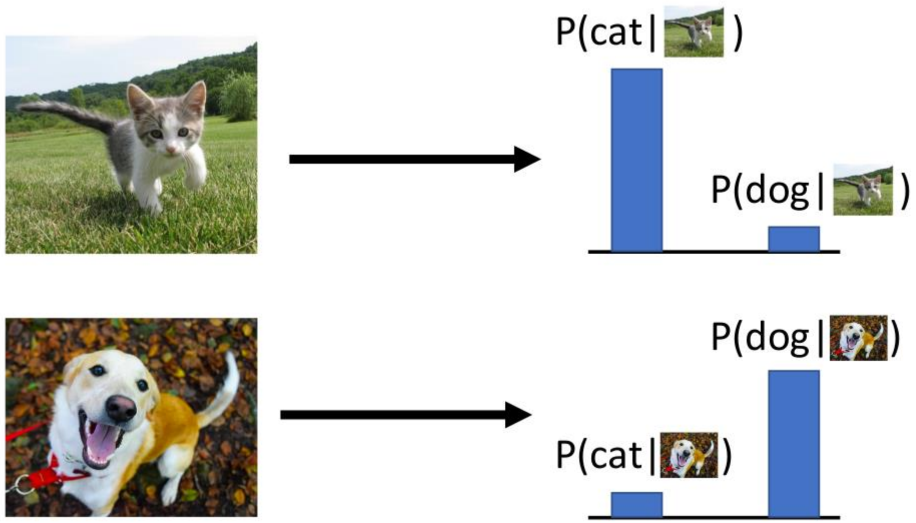
**Density Function**

$$\int_X p(x)\,dx = 1$$

- Density Functions are normalized.
- Sum of probabilities are 1. If one is bigger then the other is smaller(compete each other).

# **Discriminative** vs **Generative** Models

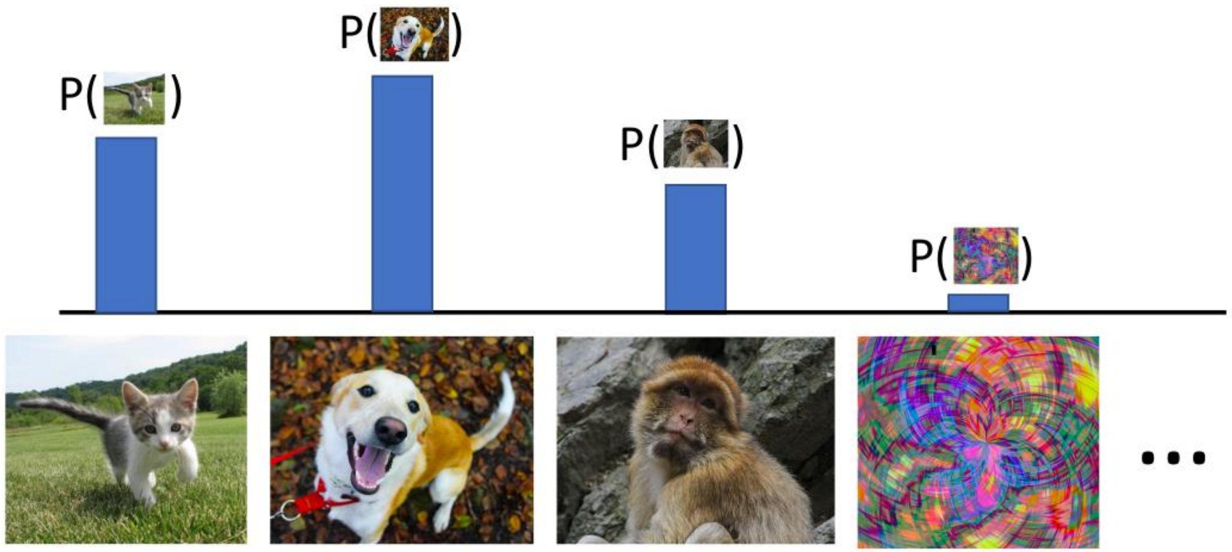
## Discriminative Model




- **Learn a probability distribution p(y|x) that predicts probability of the label y conditioned on the input image x**
- Input is $x$ and Output is the probability. (x에 대한 label이 나올 확률)
- The probability is calculated even if an input different from the specified labels is given.

# **Discriminative** vs **Generative** Models

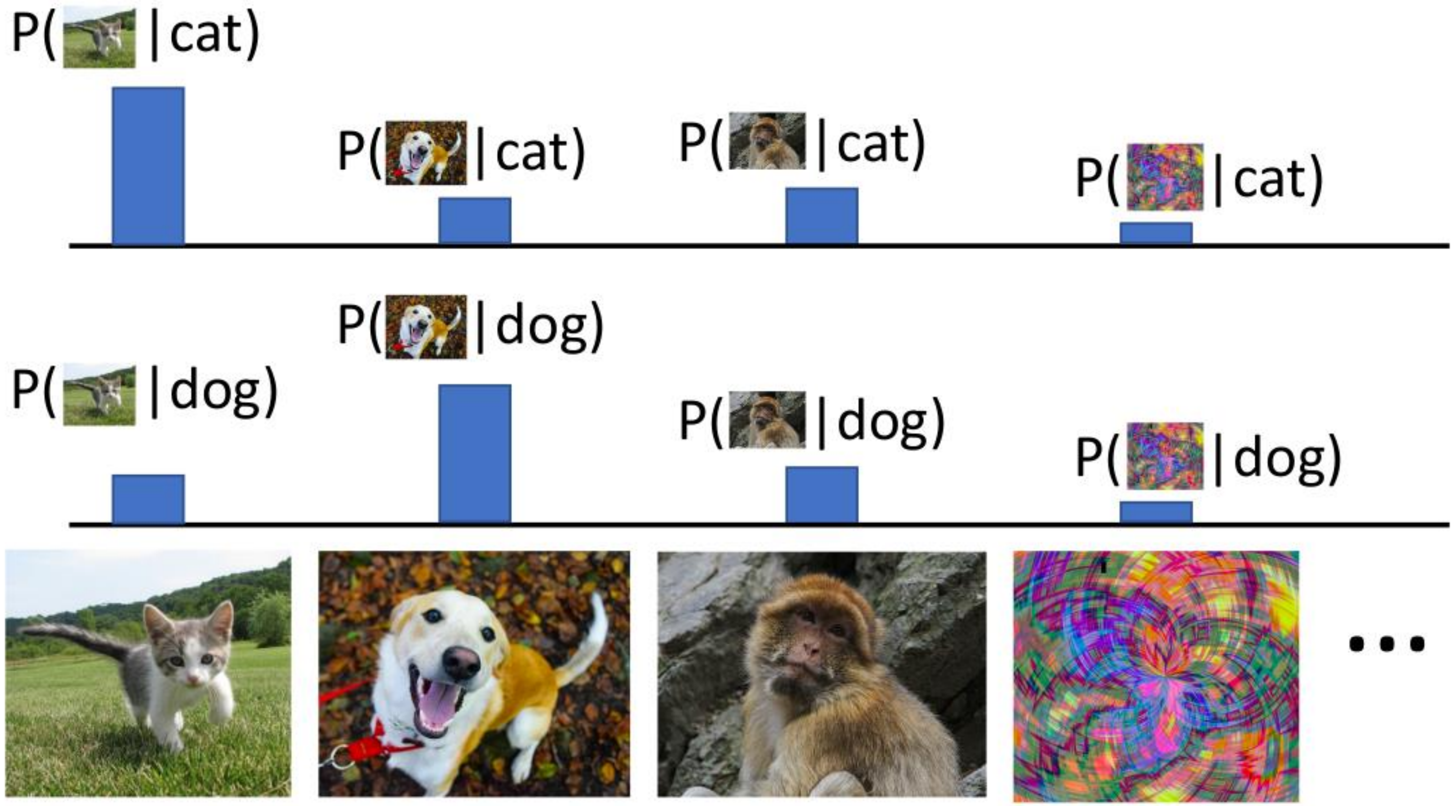
## Generative Model



- **Learn a probability distribution p(x)**
- All possible images compete to the probability mass.
- A deep understanding of the image is needed. (Which is more plausible, sitting a dog or standing up?)
- The model can reject irrational inputs by giving them a very small value.

# **Discriminative** vs **Generative** Models

## Conditional Generative Model




Recall **Bayes' Rule:**

Discriminative Model

(Unconditional) Generative Model

$$\boxed{P(x \mid y)} = \frac{\boxed{P(y \mid x)}}{\boxed{P(y)}}\boxed{P(x)}$$

Conditional Generative Model

Prior over labels

- **Learn p(x|y)**

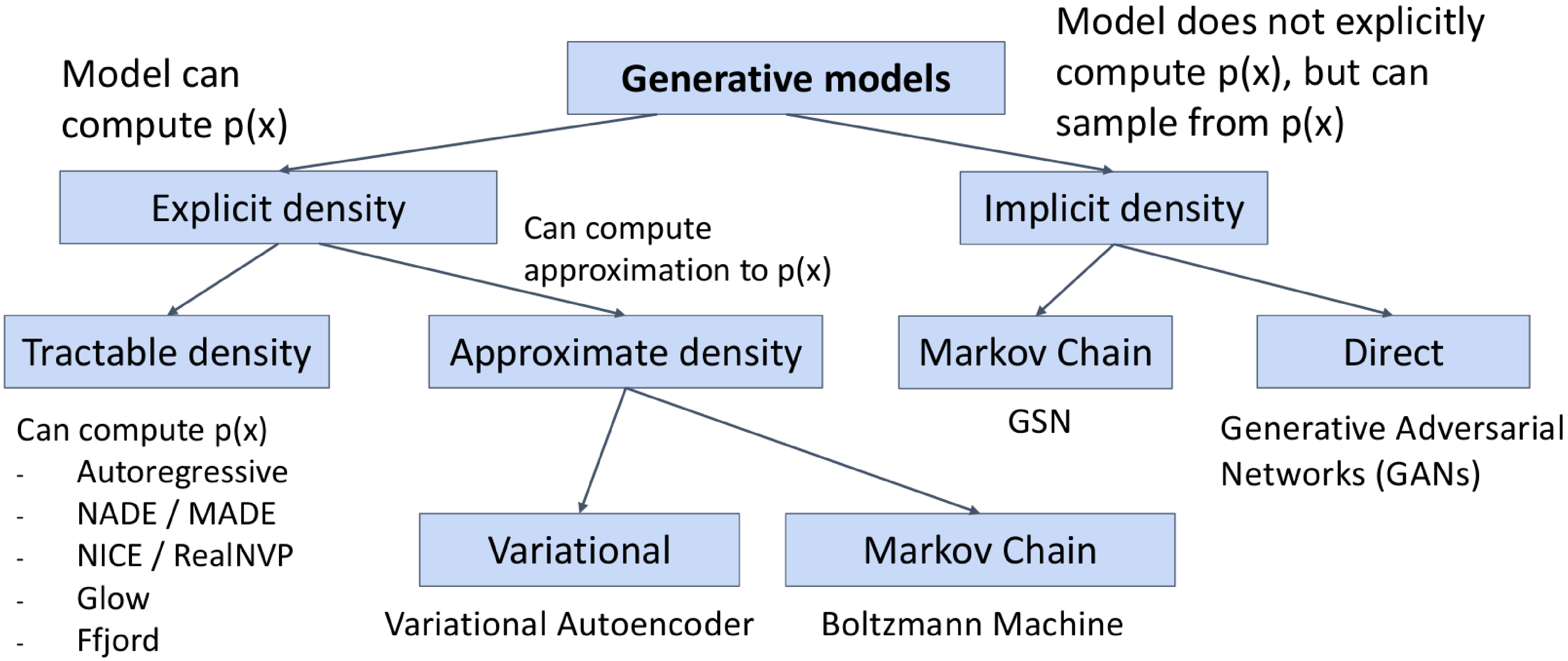
# **Discriminative** vs **Generative** Models

**Discriminative Model:**
Learn a probability distribution p(y|x) → Assign labels to data
Feature learning (supervised)

**Generative Model**:
Learn a probability distribution p(x) → Detect outliers
Feature learning (unsupervised)
Sample to **generate** new data

**Conditional Generative Model:** Learn p(x|y) → Assign labels, while rejecting outliers!
Generate new data conditioned on input labels

# Taxonomy of Generative Models



# Autoregressive Models

**Explicit Density Estimation**

**Goal :** Write down an explicit function for $p(x) = f(x, W)$ ($x$: data, $W$: learnable weight matrix)

If dataset is $x^{(1)}, x^{(2)}, \dots, x^{(N)}$,

$$W^* = \arg\max_W \prod_i p(x^{(i)})$$

Maximize the probability of training data

$$= \arg\max_W \sum_i \log p(x^{(i)})$$

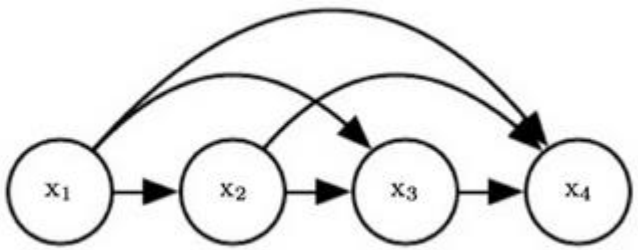
$$= \arg\max_W \sum_i \log f(x^{(i)}, W)$$

Loss Function. (Gradient Descent)

# Autoregressive Models

## Explicit Density: Autoregressive Models

- 자기 자신을 입력으로 하여 자기 자신을 예측하는 모형




- $x$가 여러 subparts로 이루어져있다고 가정. $x$가 이미지라고 하면 subparts는 각 픽셀이다 $x = (x_1, x_2, x_3, ..., x_T)$
- Chain rule을 사용하여 probability를 계산한다.

$$\begin{aligned} p(x) &= p(x_1, x_2, ..., x_T) \\ &= p(x_1)\, p(x_2|x_1)\, p(x_3|x_1, x_2) \\ &= \prod_{t=1}^{T} p(x_t|x_1, ..., x_{t-1}) \end{aligned}$$

# Autoregressive Models

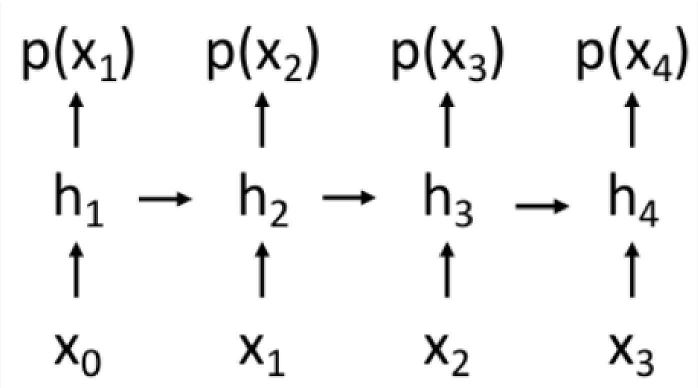
## Explicit Density: Autoregressive Models

$$
\begin{aligned}
p(x) &= p(x_1, x_2, \ldots, x_T) \\
&= p(x_1)\, p(x_2|x_1)\, p(x_3|x_1, x_2) \\
&= \prod_{t=1}^{T} p(x_t|x_1, \ldots, x_{t-1})
\end{aligned}
$$

- 즉, sequence의 확률 $p(x)$는 이전 sequence가 주어졌을 때 다음 sequence가 나올 확률을 전부 곱한 것이다.
- https://wikidocs.net/22034

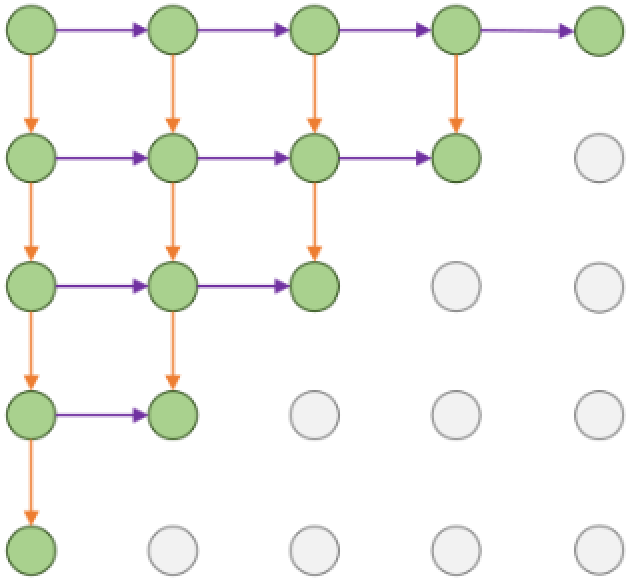
# Autoregressive Models

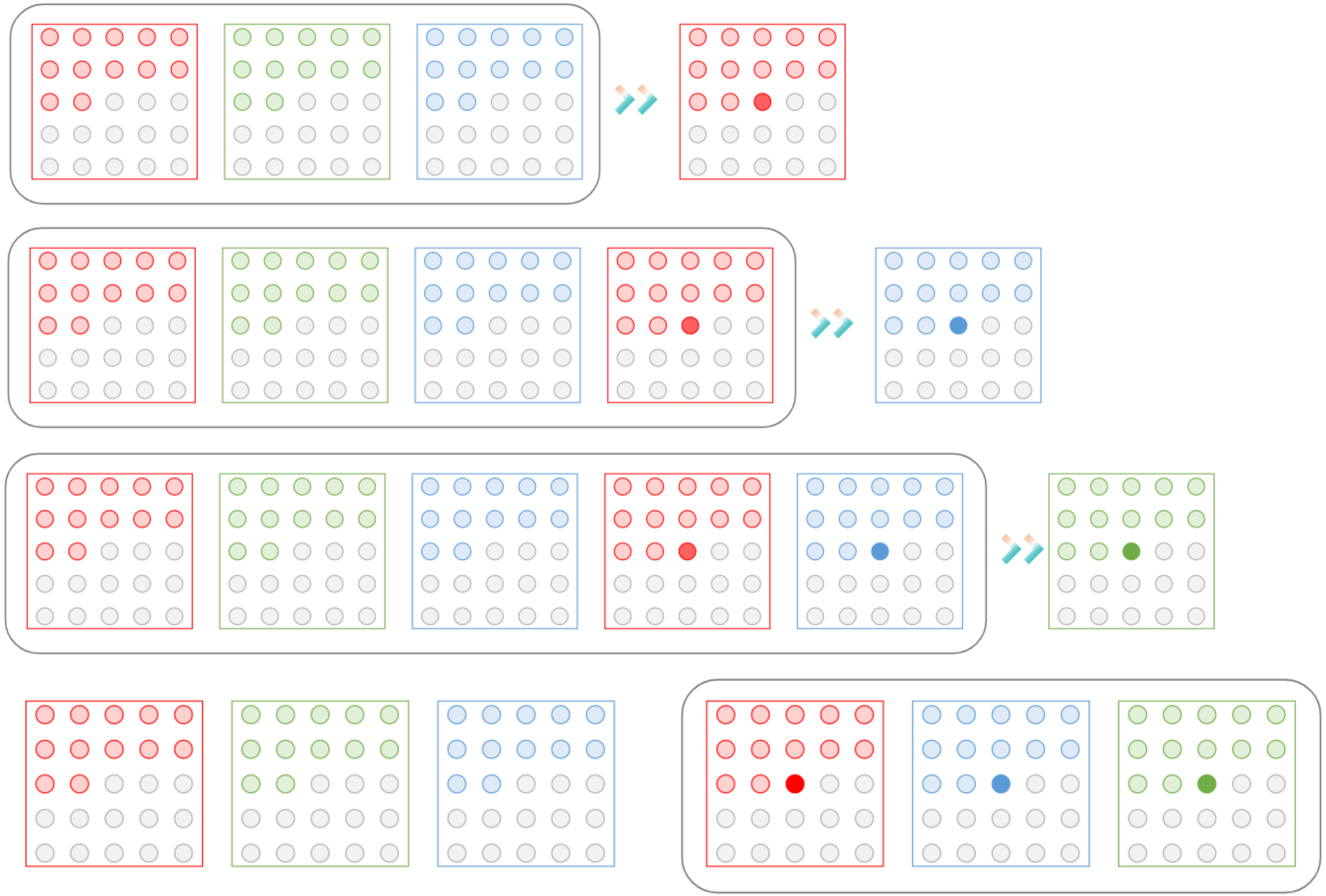
**Explicit Density: Autoregressive Models**



# Autoregressive Models

## PixelRNN

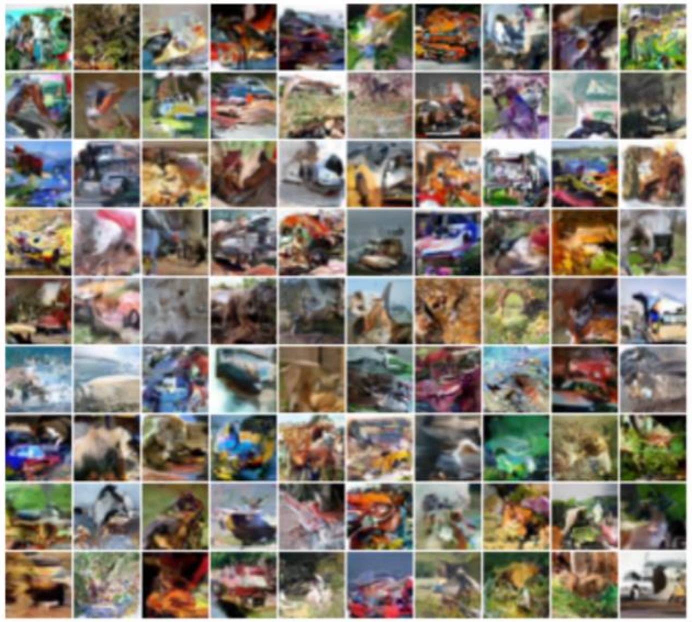




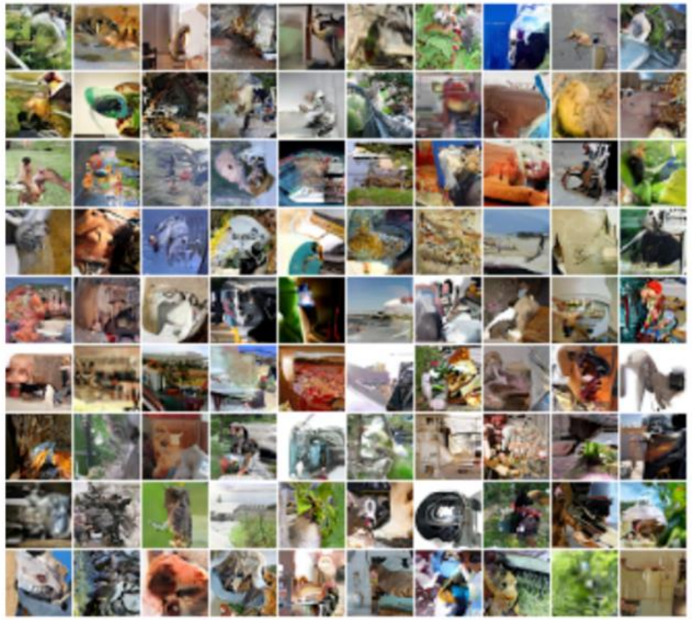
http://dmqm.korea.ac.kr/uploads/seminar/20190705_Autoregressive.pdf

# Autoregressive Models

## PixelRNN



32x32 CIFAR-10



32x32 ImageNet

- 겉으로 보기에는 합리적으로 보이지만 사실 자세히 보면 거지같다..
- 그래도 edges, colors를 꽤나 그럴싸하게 생성하는 것에 의의를 가진다.
- 이는 unconditional generation이므로 test time에 내가 무엇을 생성하고 있는지 정할 수 없다

# Autoregressive Models

## Pros and Cons

### Pros

- Likelihood p(x)를 명시적으로 계산할 수 있다.
- 위의 장점 덕분에 좋은 evaluation metric을 얻는다. (training data와 유사한, 즉 얼마나 그럴듯한지에 대한 p(x) 값을 얻을 수 있기 때문이다.)
- 나름 괜찮은 결과를 얻을 수 있다. (진짜 사진같지는 않지만..)

### Cons

- Sequential Generation 이므로 느리다.

# Variational Autoencoders (VAE)

- In PixelRNN, PixelCNN, they defined parametric density function $p(x) = f(x, W)$ and calculate for each input. And train the model to maximize this output.
- In VAE, Instead of maximizing the actual density value, **maximize the lower bound of the density.**
- $p_\theta(x) = \prod_{i=1}^{n} p_\theta(x_i | x_1, \dots, x_{i-1})$

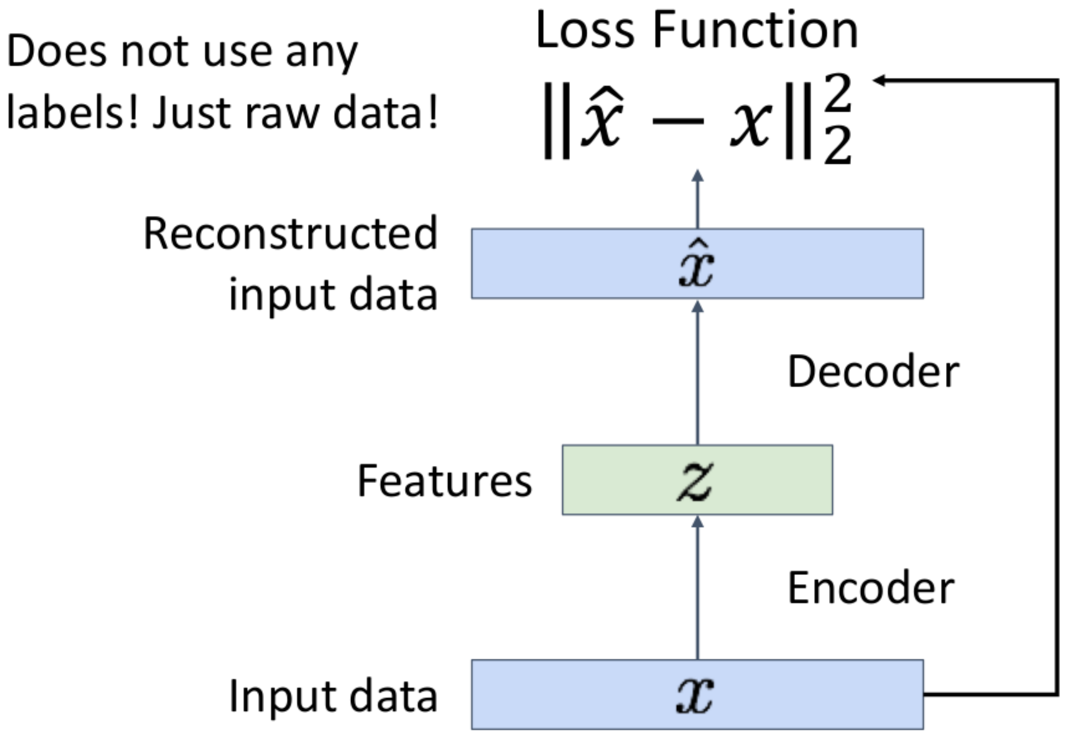
# Variational Autoencoders (VAE)

## (Regular, non-variational) Autoencoders

- Unsupervised method로, labels 없이 raw data x로부터 feature vectors를 학습한다.

(Unsupervised method for learning feature vectors from raw data x, without any labels)

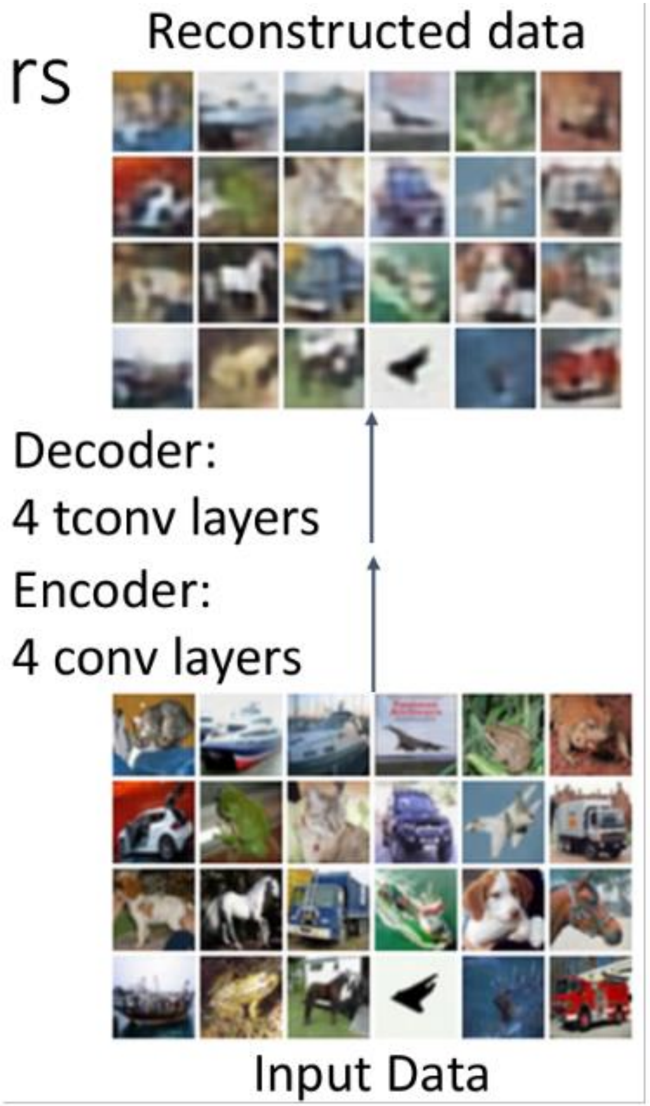
- **Features** extracts useful information that can be used for downstream tasks.
- **Encoder** extracts features from input data,
- **Decoder** reconstruct the input data from the features.
- Encoder:
  - Originally: Linear + nonlinearity (sigmoid)
  - Later: Deep, fully-connected
  - Later: ReLU CNN (upconv)



# Variational Autoencoders (VAE)

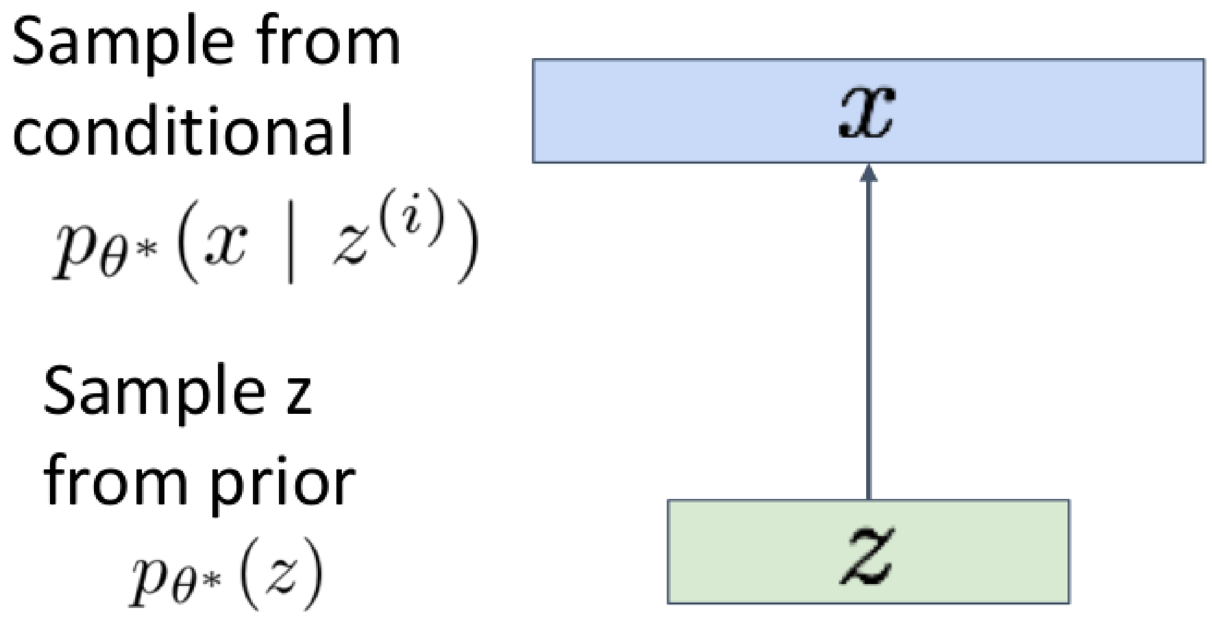
## (Regular, non-variational) Autoencoders

- We expect the effect of **compressing input data through Encoder.**
- After learning, discard the decoder and use it for the downstream task using the encoder.
- **Not probabilistic** : 학습하지 않은 new data를 sampling할 수 없다.



# Variational Autoencoders (VAE)

## Variational Autoencoders




- Autoencoder에 확률 개념을 도입하였다
- 1. raw data로부터 **latent features z**를 학습한다.
- 2. new data를 생성하기 위해 model로부터 **sampling**한다.

# Variational Autoencoders (VAE)

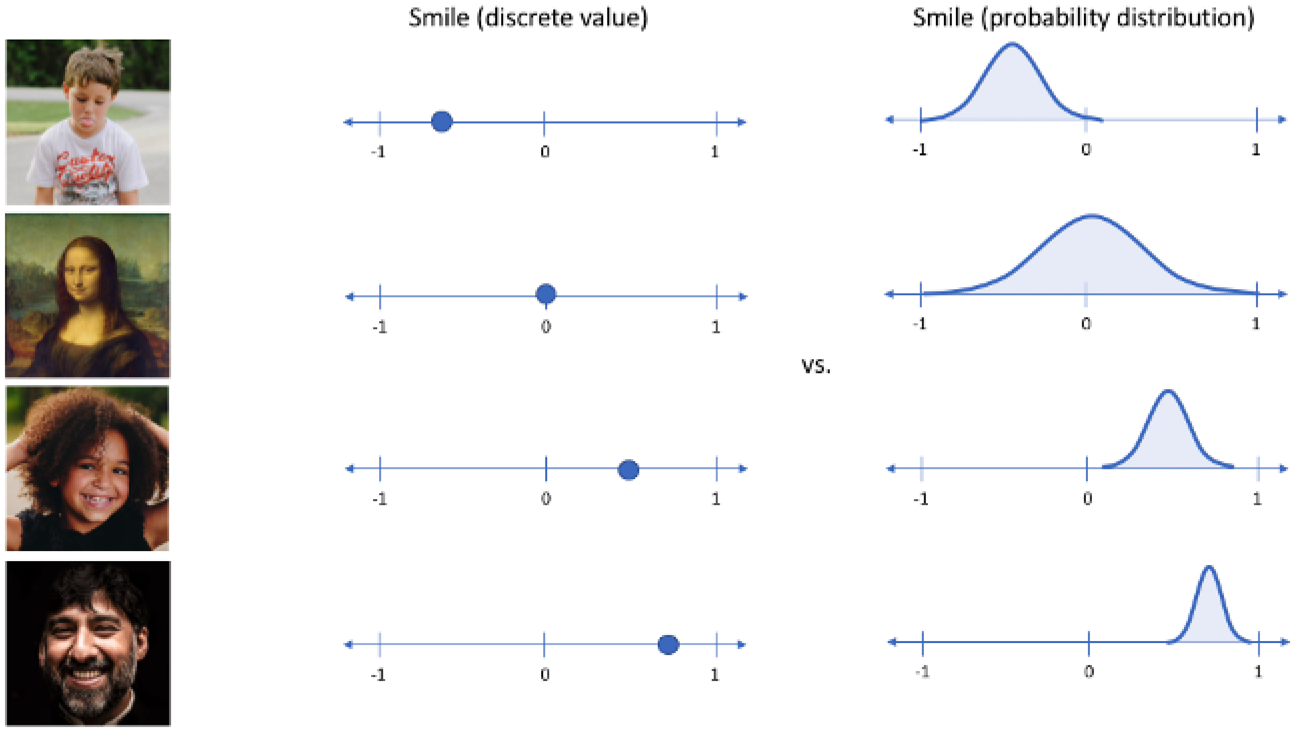
## Variational Autoencoders

- **Decoder**: Generating new data $x$ from latent features $z$ that is similar to input data but completely new.
- Sampling latent variables from prior distribution $p_{\theta_*}(z)$, put sampled $z$ into the decoder to predict image $x$
- At this time, output is not a single image but the distribution of images.

- $p_{\theta_*}(z)$ : prior distribution. PDF of $x$. (Gaussian distribution)
- $p_{\theta_*}(x|z^{(i)})$ : 주어진 z에서 특정 $x$가 나올 조건부 확률에 대한 PDF
- $\theta$ : parameter

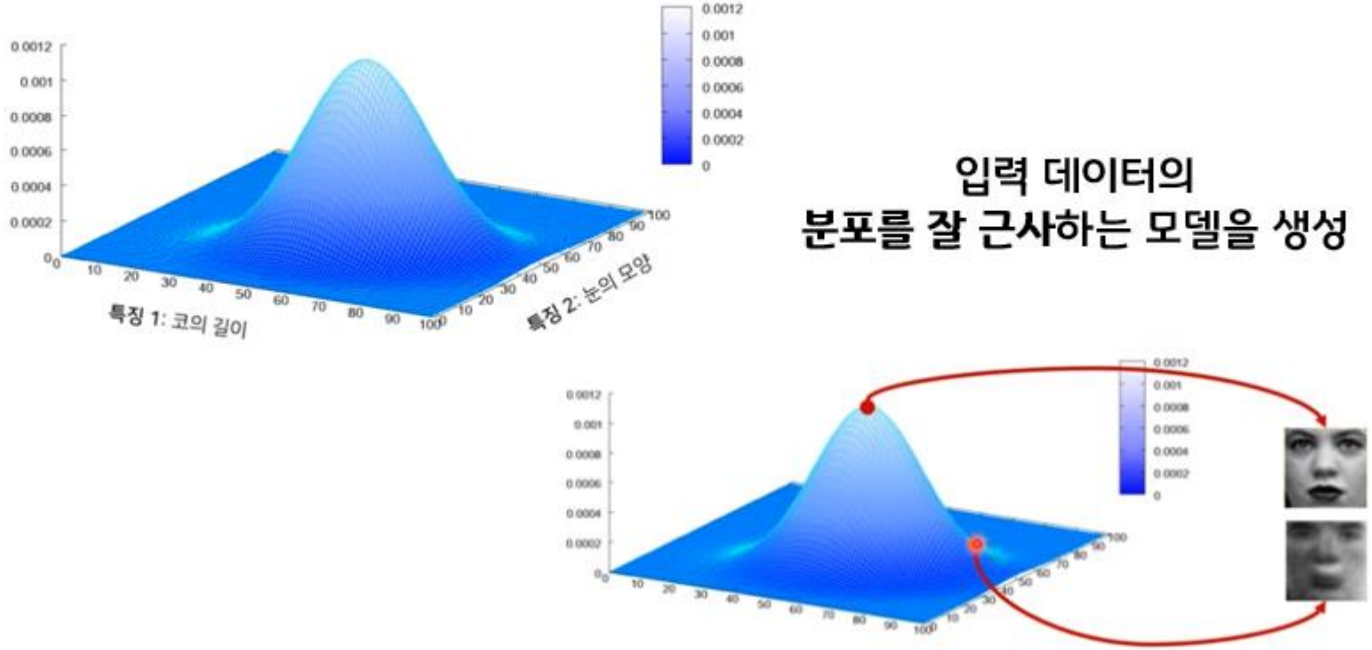
# Variational Autoencoders (VAE)

**Variational Autoencoders**




- VAE에서의 z는 AE에서의 z(a value: low dimension of input data)와 다르게, 가우시안 확률분포에 기반한 확률값으로 나타낸다.
- input image가 들어오면 그 이미지의 다양한 특징들이 각각의 확률변수가 되는 확률분포를 만든다. 이 확률 분포를 잘 찾아내어 확률값이 높은 부분을 사용하면 그럴듯한 새로운 이미지를 생성할 수 있다.

# Variational Autoencoders (VAE)




- 이때 각 feature가 가우시안 분포를 따른다고 가정하고 latent z는 각 feature의 평균과 분산값을 나타낸다.
- 예를 들어 한국인의 얼굴을 그리기 위해 눈, 코, 입 등의 feature를 Latent vector z에 담고, 그 z를 이용해 그럴듯한 한국인의 얼굴을 그려내는 것이다. latent vector z는 한국인 눈 모양의 평균 및 분산, 한국인 코 길이의 평균 및 분산, 한국인 머리카락 길이의 평균 및 분산 등등의 정보를 담고 있다고 생각할 수 있다.

# Generative Models II

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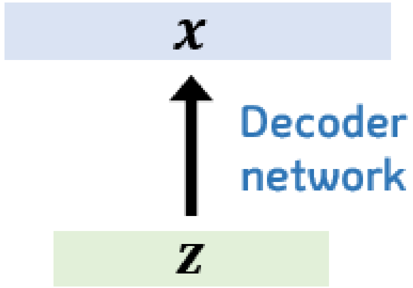
# Mathematical meaning

- If a model parameter $\theta$ is given, the higher $p_{\theta_*}(x)$ (The probability that the answer we want is $x$) the better model.
- Train parameters to maximize $p_{\theta_*}(x)$

**Decoder**




z latent variable의 확률분포

$p_\theta(z)$

z가 given일 때 x의 확률분포

$p_\theta(x|z^{(i)})$

**어떻게 학습?**

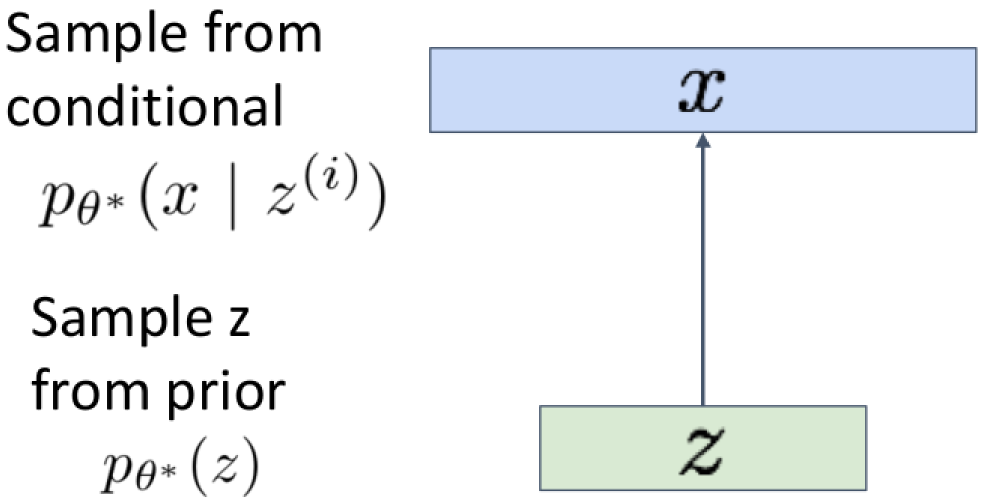
네트워크의 출력값이 있을 때
우리가 원하는 정답 x가 나올 확률이 높길바람

= x의 likelihood를 최대화하는 확률분포 찾자

**Maximize**

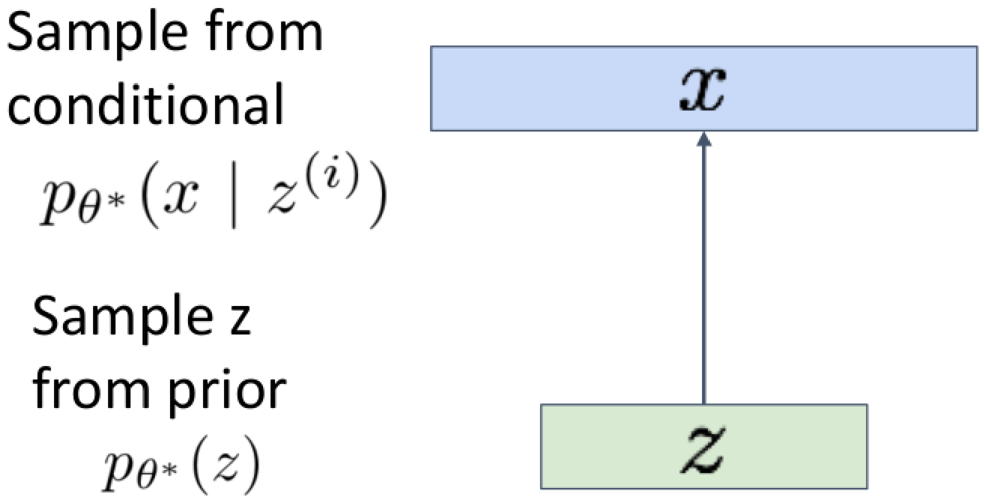
$$p_\theta(x) = \int p_\theta(z)p_\theta(x|z)dz$$

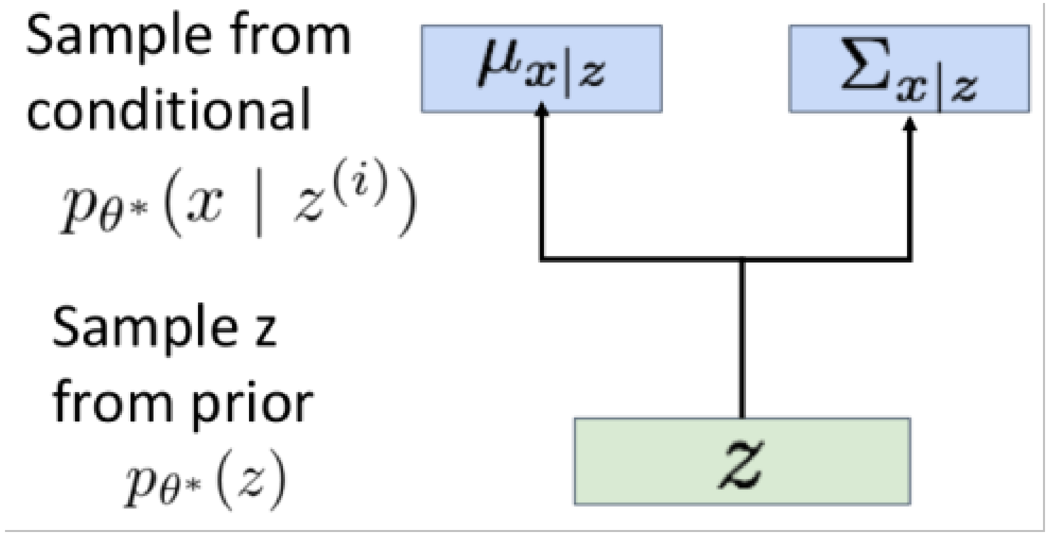
# Variational Autoencoders




- When image comes in, $p(z)$ is the appropriate Gaussian distribution for each pixel of image.
- That is, each pixel has a Gaussian distribution with $\mu, \sigma$
- But if it's a **high resolution** image, it's going to have a lot of values. Therefore, the **diagonal gaussian distribution** is used instead of the general Gaussian distribution. That is, when $z$ is given, there is no covariance between pixels of the generated image. **Pixels are independent.**

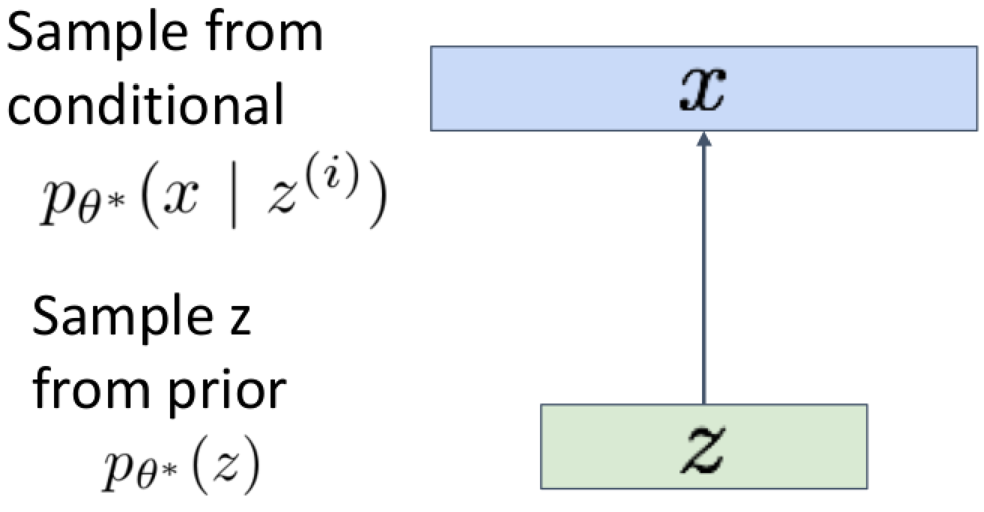
# Variational Autoencoders

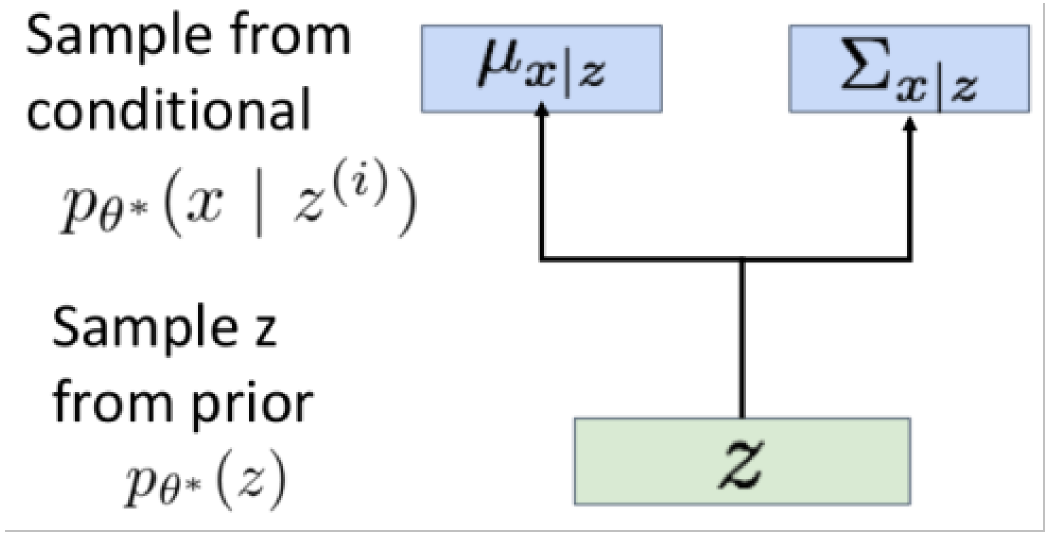



- **Decoder** outputs mean $\mu_{x|z}$ and diagonal covariance $\sum_{x|z}$ for the input $z$
- Then sample $x$ from the above Gaussian distribution

# Variational Autoencoders - Train



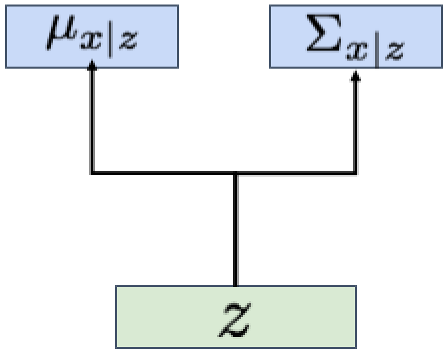

- **Maximize likelihood of data $p_{\theta_*}(x)$**

$$p_\theta(x) = \frac{p_\theta(x|z)p_\theta(z)}{p_\theta(z|x)} \qquad q_\phi(z|x) \approx p_\theta(z|x) \qquad p_\theta(x) = \frac{p_\theta(x|z)p_\theta(z)}{p_\theta(z|x)} \approx \frac{p_\theta(x|z)p_\theta(z)}{q_\phi(z|x)}$$

# Variational Autoencoders - Train

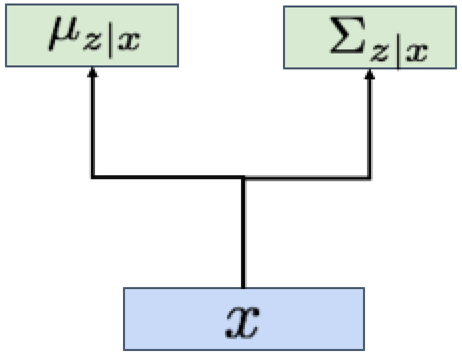
**Decoder network** inputs latent code z, gives distribution over data x

$$p_\theta(x \mid z) = N(\mu_{x|z}, \Sigma_{x|z})$$




**Encoder network** inputs data x, gives distribution over latent codes z

$$q_\phi(z \mid x) = N(\mu_{z|x}, \Sigma_{z|x})$$




$$\log p_\theta(x) = \log \frac{p_\theta(x|z)p(z)}{p_\theta(z|x)}$$

$$= \log \frac{p_\theta(x|z)p(z)q_\phi(z|x)}{p_\theta(z|x)q_\phi(z|x)}$$

$$= \log p_\theta(x|z) - \log \frac{q\phi(z|x)}{p(z)} + \log \frac{q_\phi(z|x)}{p_\theta(z|x)}$$

wrap in an expectation since it doesn't depend on $z$

$$\log p_\theta(x) = E_{z\sim q\phi(z|x)}[\log p_\theta(x)]$$

$$= E_z[\log p_\theta(x|z)] - E_z\left[\log \frac{q_\phi(z|x)}{p(z)}\right] + E_z\left[\log \frac{q_\phi(z|x)}{p_\theta(z|x)}\right]$$

$$= E_{z\sim q_\phi(z|x)}[\log p_\theta(x|z)] - D_{KL}(q_\phi(z|x), p(z)) + D_{KL}(q_\phi(z|x), p_\theta(z|x))$$

# Variational Autoencoders - Train

$$\log p_\theta(x) = E_{z\sim q\phi(z|x)}[\log p_\theta(x)]$$

$$= E_z[\log p_\theta(x|z)] - E_z\left[\log \frac{q_\phi(z|x)}{p(z)}\right] + E_z\left[\log \frac{q_\phi(z|x)}{p_\theta(z|x)}\right]$$

$$= E_{z\sim q_\phi(z|x)}[\log p_\theta(x|z)] - D_{KL}(q_\phi(z|x), p(z)) + D_{KL}(q_\phi(z|x), p_\theta(z|x))$$

$$\boxed{\log p_\theta(x) \geq E_{z\sim q_\phi(z|x)}[\log p_\theta(x|z)] - D_{KL}(q_\phi(z|x), p(z))}$$

Lower bound of likelihood

Through this, encoders and decoders are learned jointly to maximize the variable lower bound of data-like hood.

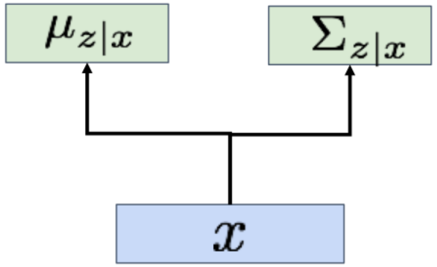
# Variational Autoencoders

Jointly train **encoder** q and **decoder** p to maximize
the **variational lower bound** on the data likelihood

$$\log p_\theta(x) \geq E_{z \sim q_\phi(z|x)}[\log p_\theta(x|z)] - D_{KL}\left(q_\phi(z|x), p(z)\right)$$

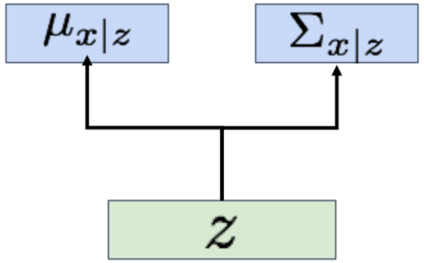
**Encoder Network**

$$q_\phi(z \mid x) = N(\mu_{z|x}, \Sigma_{z|x})$$




**Decoder Network**

$$p_\theta(x \mid z) = N(\mu_{x|z}, \Sigma_{x|z})$$



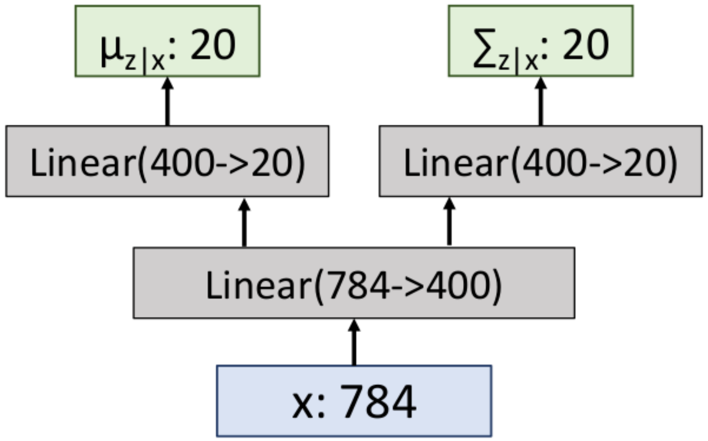
# Example : Fully connected VAE

MNIST Dataset

- $x = 28 \times 28$ image, flattened to 784-dim vector
- $z = 20$-dim vector (hyper-parameter)

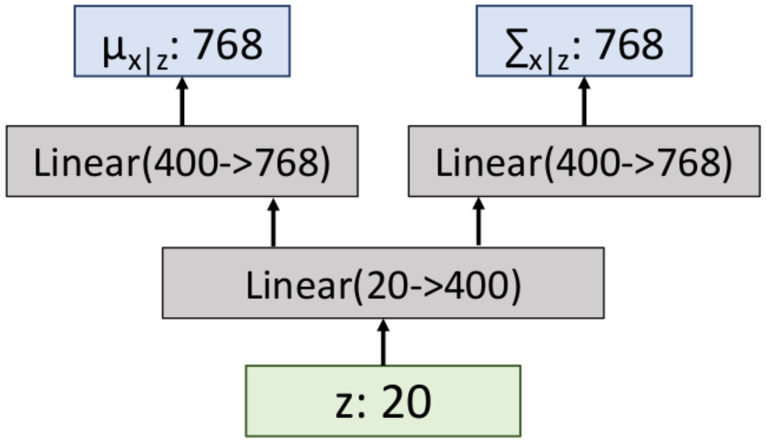
**Encoder Network**

$$q_\phi(z \mid x) = N(\mu_{z|x}, \Sigma_{z|x})$$




**Decoder Network**

$$p_\theta(x \mid z) = N(\mu_{x|z}, \Sigma_{x|z})$$



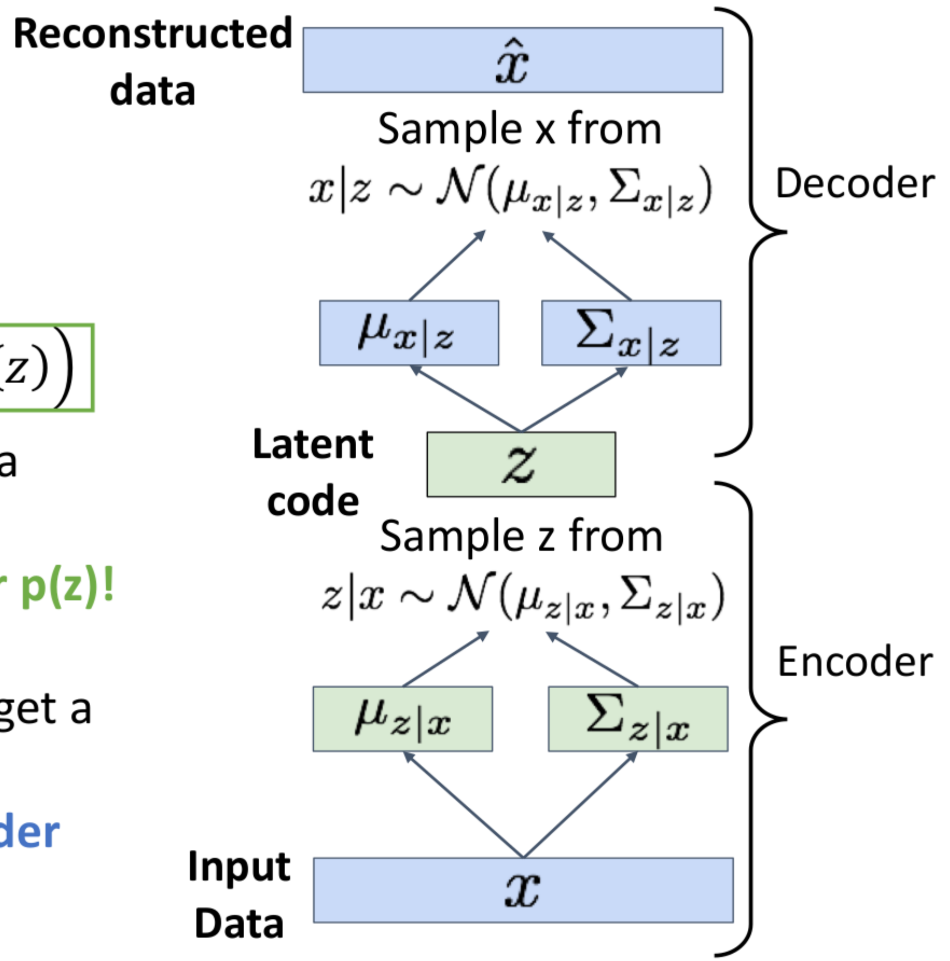
# Example : Fully connected VAE

## Variational Autoencoders

Train by maximizing the **variational lower bound**

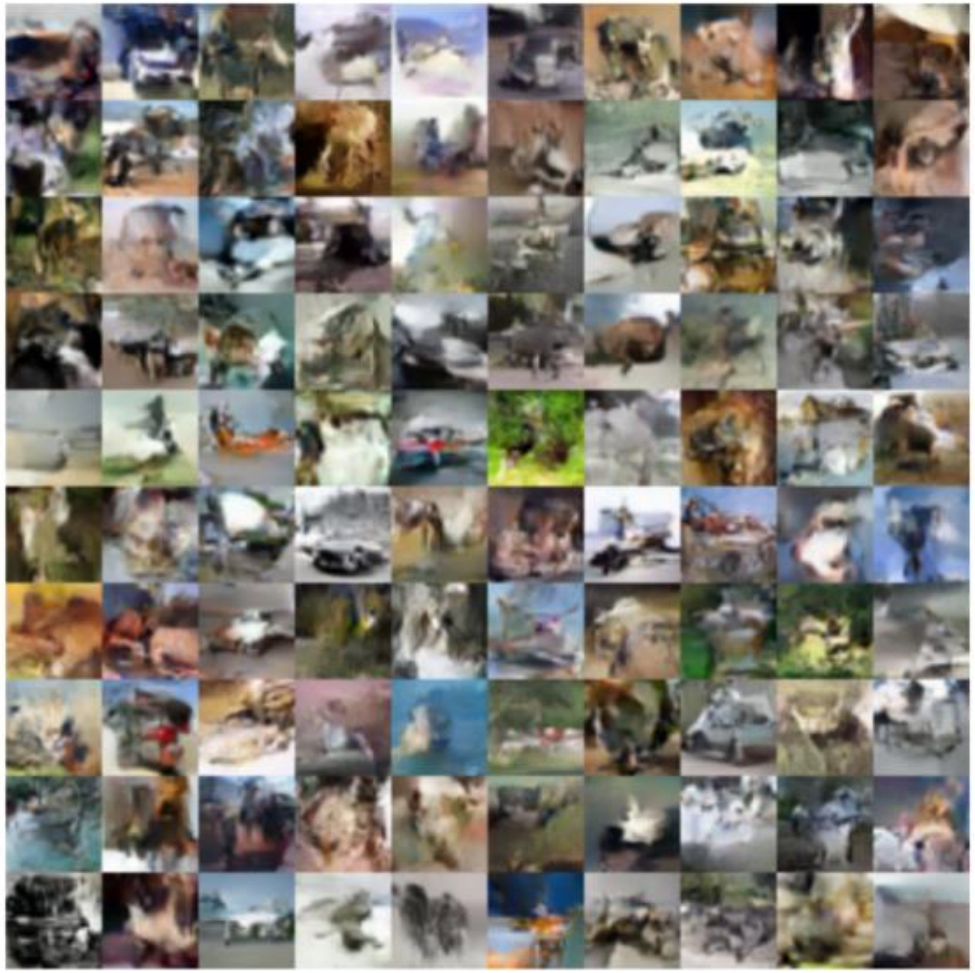
$$\boxed{E_{z\sim q_\phi(z|x)}[\log p_\theta(x|z)]} - \boxed{D_{KL}\left(q_\phi(z|x), p(z)\right)}$$

1. Run input data through **encoder** to get a distribution over latent codes
2. **Encoder output should match the prior p(z)!**
3. Sample code z from encoder output
4. Run sampled code through **decoder** to get a distribution over data samples
5. **Original input data should be likely under the distribution output from (4)!**
6. Can sample a reconstruction from (4)

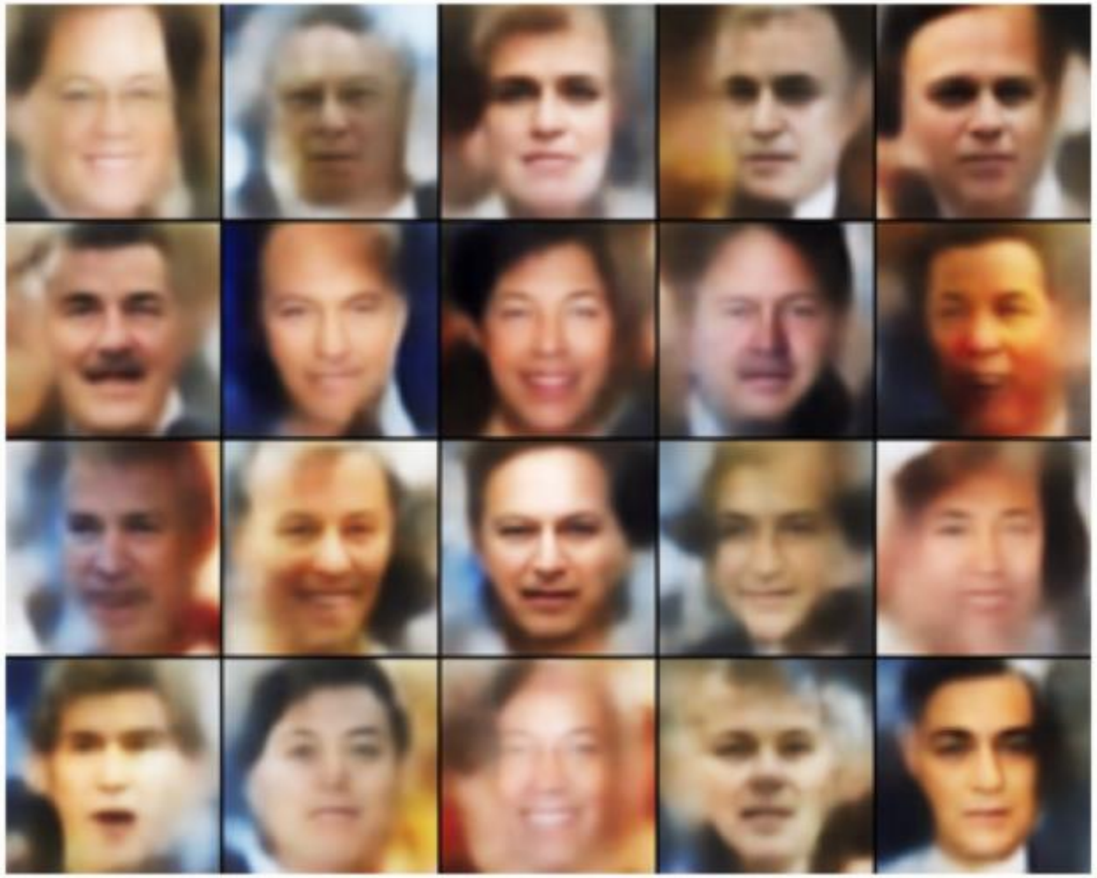


# Variational Autoencoders

32x32 CIFAR-10



Labeled Faces in the Wild



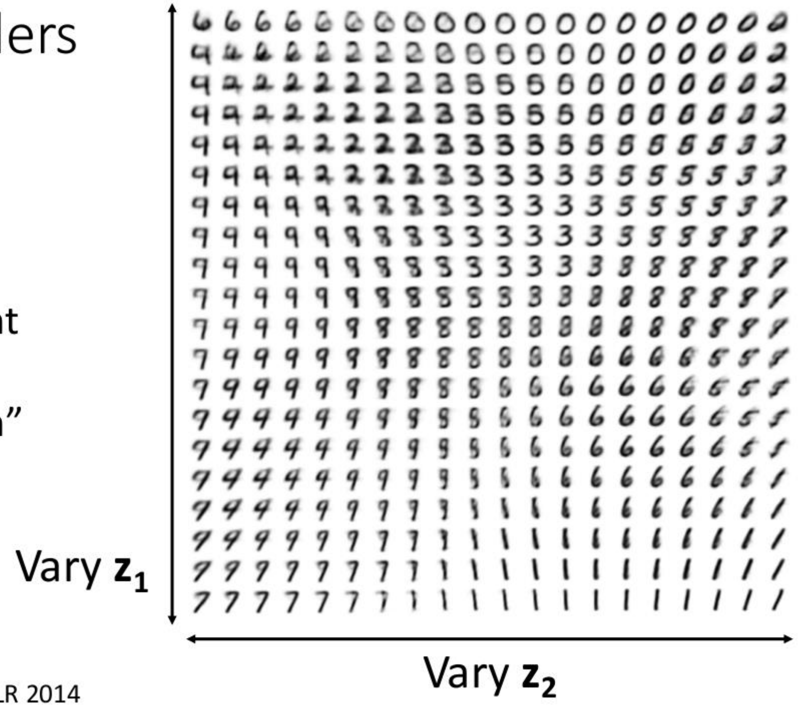
# Variational Autoencoders

- **VAE : Editing with z**

Variational Autoencoders

The diagonal prior on p(z) causes dimensions of z to be independent

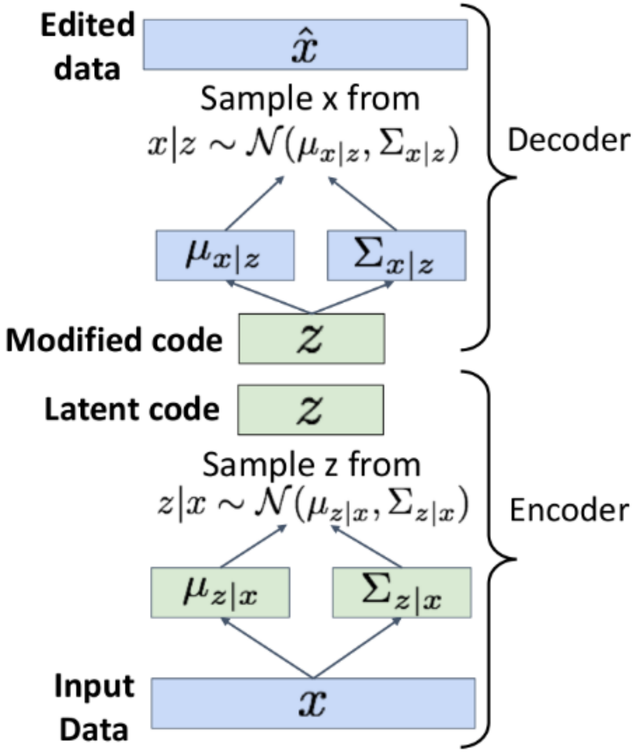
"Disentangling factors of variation"




Kingma and Welling, Auto-Encoding Variational Beyes, ICLR 2014

# Variational Autoencoders

- **VAE : Editing with z**



# Variational Autoencoders

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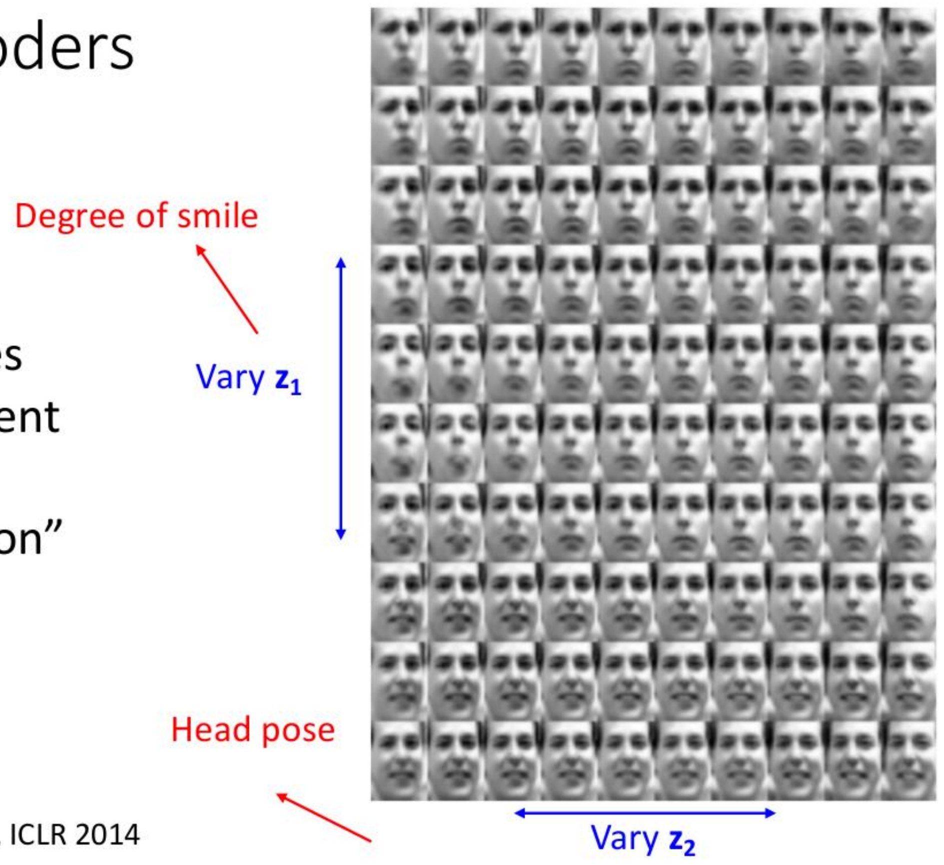
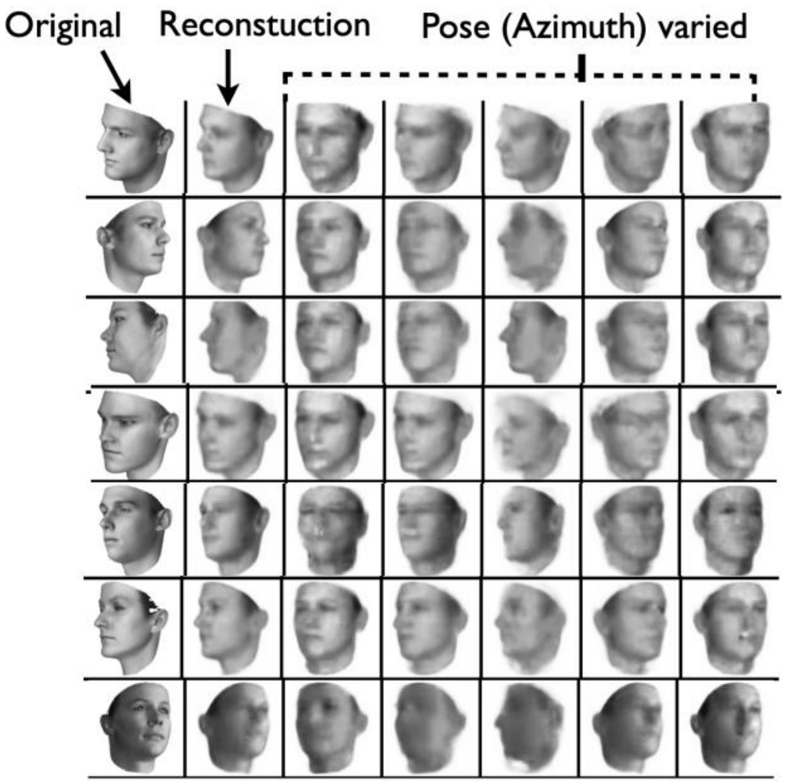
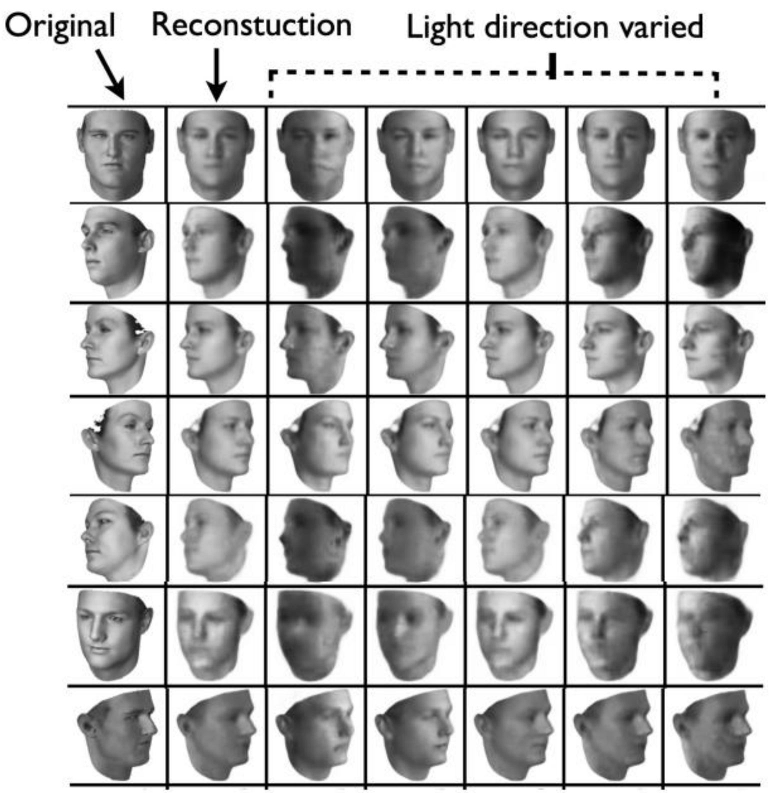



Kingma and Welling, Auto-Encoding Variational Beyes, ICLR 2014

# Variational Autoencoders

- **VAE : Editing with z**



# Variational Autoencoders

- **VAE : Summary**

Probabilistic spin to traditional autoencoders => allows generating data
Defines an intractable density => derive and optimize a (variational) lower bound

**Pros:**

- Principled approach to generative models
- Allows inference of q(z|x), can be useful feature representation for other tasks

**Cons:**

- Maximizes lower bound of likelihood: okay, but not as good evaluation as PixelRNN/PixelCNN
- Samples blurrier and lower quality compared to state-of-the-art (GANs)

**Active areas of research:**

- More flexible approximations, e.g. richer approximate posterior instead of diagonal Gaussian, e.g., Gaussian Mixture Models (GMMs)
- Incorporating structure in latent variables, e.g., Categorical Distributions

**Autoregressive models**

- Directly maximize p(data)
- High-quality generated images
- Slow to generate images
- No explicit latent codes

**Variational models**

- Maximize lower-bound on p(data)
- Generated images often blurry
- Very fast to generate images
- Learn rich latent codes

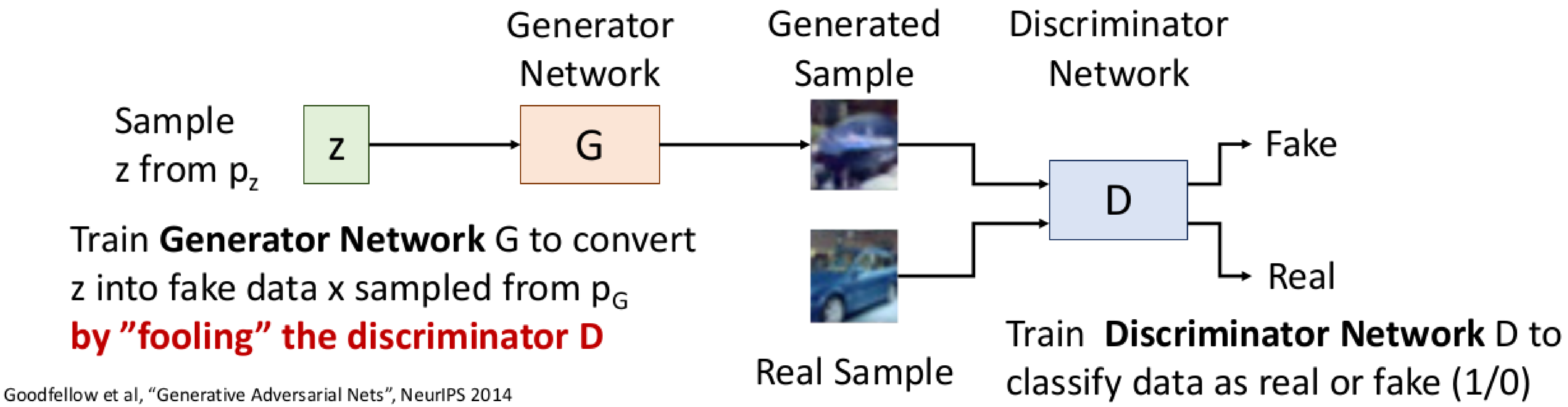
# Generative Adversarial Networks

| Models | 설명 |
| --- | --- |
| **Autoregressive models** | training data의 likelihood를 직접(directly) maximize한다. $p_\theta(x) = \prod_{i=1}^{N} p_\theta(x_i \mid x_1, ..., x_{i-1})$ |
| **VAE** | latent $z$를 추가하였고, likelihood의 lower bound를 maximize한다. $p_\theta(x) = \int_Z p_\theta(x \mid z) p(z)\, dz \geq E_{z \sim q_\phi(z \mid x)}[\log p_\theta(x \mid z)] - D_{KL}(q_\phi(z \mid x), p(z))$ |
| **GANs** | $p(x)$를 모델링하는 것을 포기한다. 그러나 $p(x)$로부터 샘플링할 수 있도록 한다. |

# Generative Adversarial Networks

- **Setup**
  - $p_{\text{data}}(x)$ : real data distribution
  - $x_i$ : our train data from $p_{\text{data}}(x)$
- **Idea**
  - Suppose a latent variable $z$ with $p(z)$ which is a simple prior (diagonal Gaussian, unformed distribution, etc.).
  - Sample $z$ from $p(z)$ and pass through **Generative Network $\boldsymbol{G}$.**
  - $\boldsymbol{x} = \boldsymbol{G}(\boldsymbol{z})$
  - Then the $x$ is from Generative distribution $p_G$.
  - Therefore we want $p_G = p_{\text{data}}$ (Our generative distribution to be real data distribution)

# Generative Adversarial Networks




- **Generater**
  - By sampling $x$ from $p_G$, train the model to generate an image that the Discriminator get fooled to think that the image was from $p_{\text{data}}$.
- **Discriminator**
  - Train to discriminate the generated sample and a real sample (real/fake(1/0)).
- Jointly train the two networks. Then $p_G$ will converge to $p_{\text{data}}$.

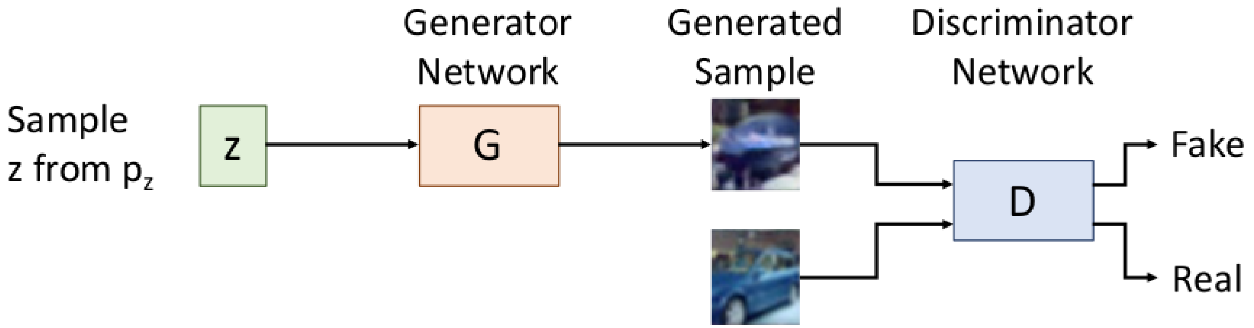
# GANs : Training Objective

- Train **Generator G** and **Discriminator D** jointly by **minmax game**.

$$\min_G \max_D (E_{x \sim p_{data}}[\log D(x)] + E_{z \sim p(z)}[\log (1 - D(G(z)))])$$

# GANs : Training Objective

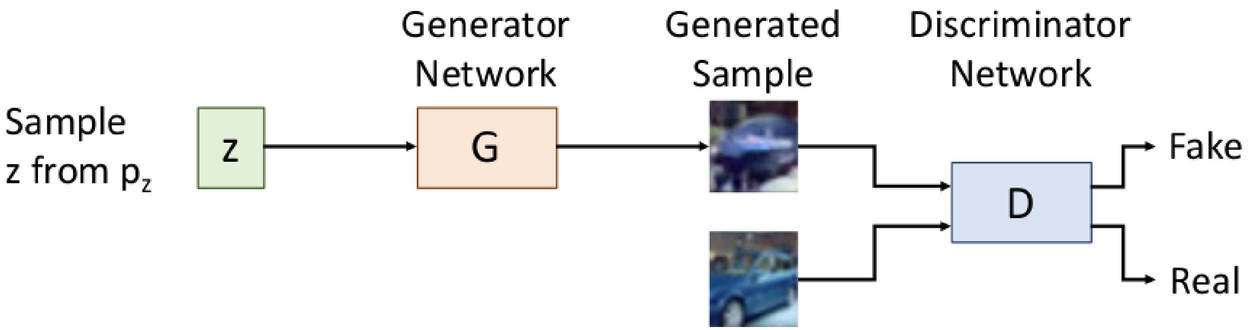
$$\min_{G} \max_{D} \left( \overbrace{E_{x\sim p_{data}}[\log D(x)]}^{\text{Discriminator wants } D(x) = 1 \text{ for real data}} + E_{z\sim p(z)}\left[\overbrace{\underbrace{\log\left(1 - D(G(z))\right)}_{\text{Generator wants } D(x) = 1 \text{ for fake data}}}^{\text{Discriminator wants } D(x) = 0 \text{ for fake data}}\right]\right)$$




- $x$ sampled from $p_{data}$, which is the real data to be REAL
- If $D(x) < 1$, it passes log term and becomes very small negative value. So we train $D(x) = 1$ that the whole term can be maximized by $D$.

# GANs : Training Objective

$$\min_{G} \max_{D} \left( \overbrace{E_{x\sim p_{data}}[\log D(x)]}^{\substack{\text{Discriminator wants} \\ D(x)\,=\,1 \text{ for real data}}} + E_{z\sim p(z)} \left[ \overbrace{\underbrace{\log\left(1 - D(G(z))\right)}_{\substack{\text{Generator wants} \\ D(x)\,=\,1 \text{ for fake data}}}}^{\substack{\text{Discriminator wants} \\ D(x)\,=\,0 \text{ for fake data}}} \right] \right)$$




- $z$ sampled from $p(z)$, pass it to Generator G, and G outputs generated sample $G(z)$. Train Discriminator to discriminate the generated sample $G(z)$ is fake. (fake to be FAKE)
- Generator G trains Discriminator D to discriminate $G(z)$ is REAL. (fake to be REAL)

# GANs

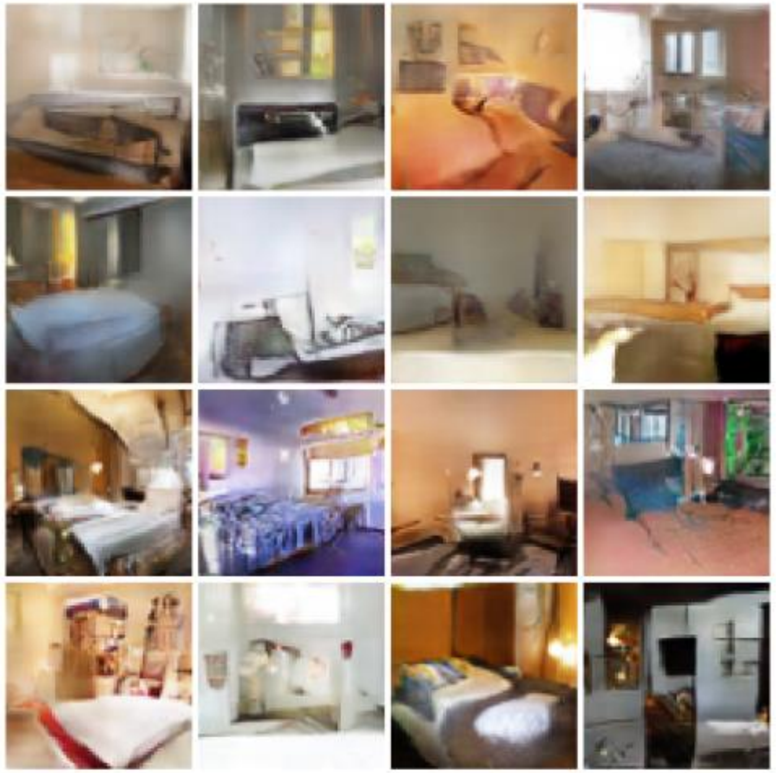
Generated samples

Nearest neighbor from training set

Goodfellow et al, "Generative Adversarial Nets", NeurIPS 2014

# GANs

## Wasserstein GAN (WGAN)



Arjovsky, Chintala, and Bouttou, “Wasserstein GAN”, 2017

## WGAN with Gradient Penalty (WGAN-GP)

Gulrajani et al, “Improved Training of Wasserstein GANs”, NeurIPS 2017

# GANs

256 x 256 bedrooms

1024 x 1024 faces

Karras et al, "Progressive Growing of GANs for Improved Quality, Stability, and Variation", ICLR 2018

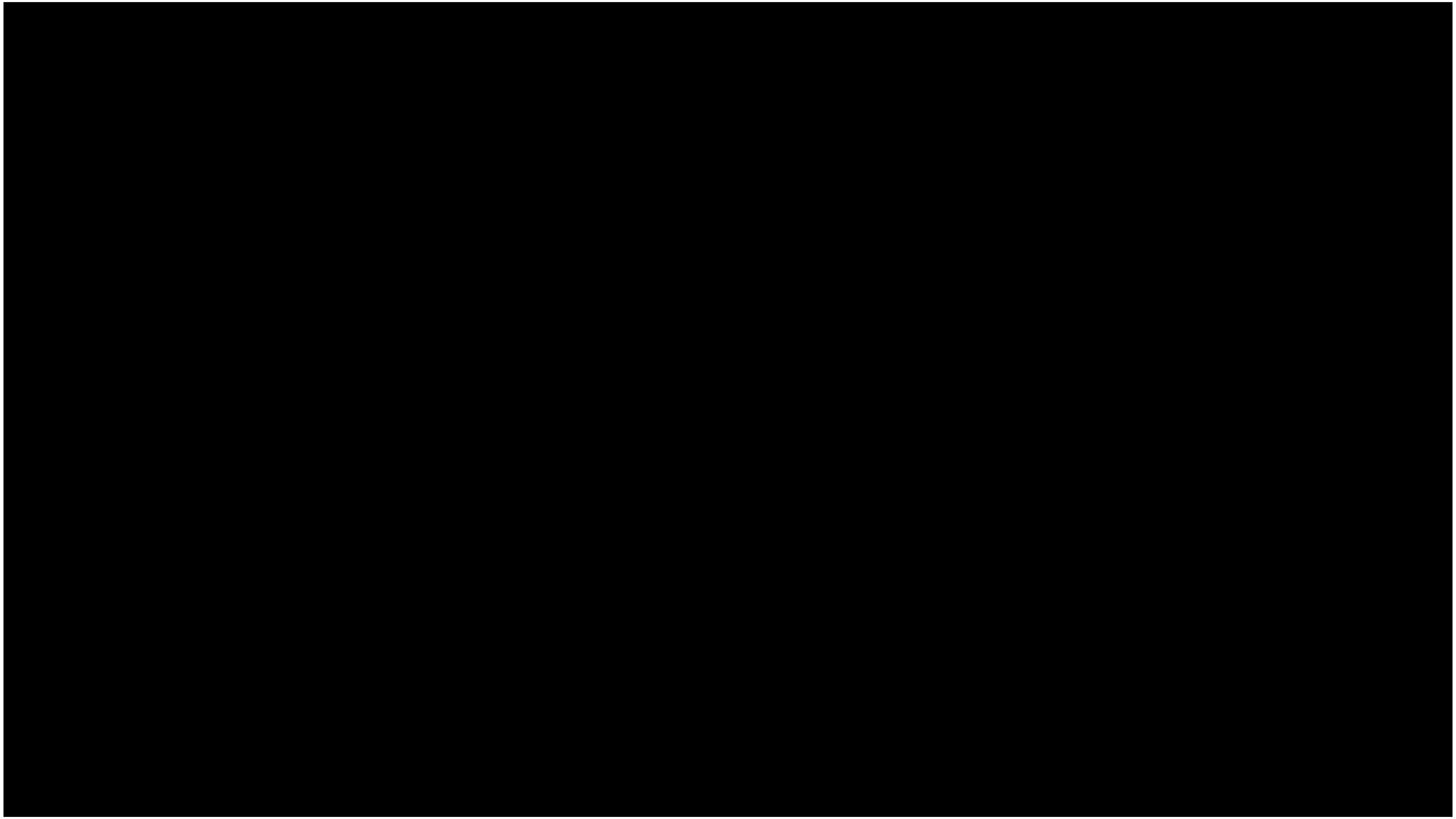
# GANs

512 x 384 cars

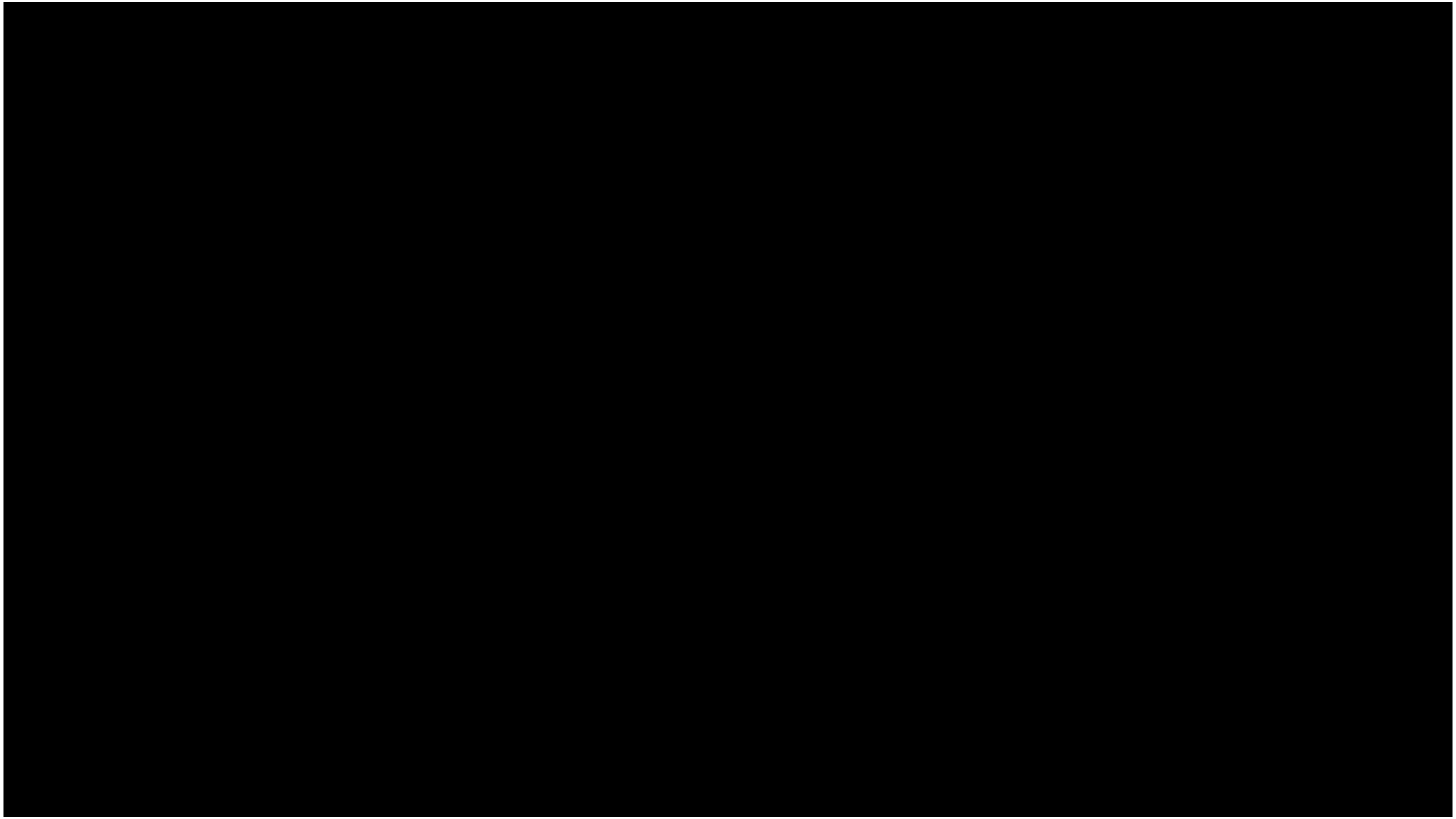
1024 x 1024 faces

Karras et al, "A Style-Based Generator Architecture for Generative Adversarial Networks", CVPR 2019

# GANs



# GANs



# GANs

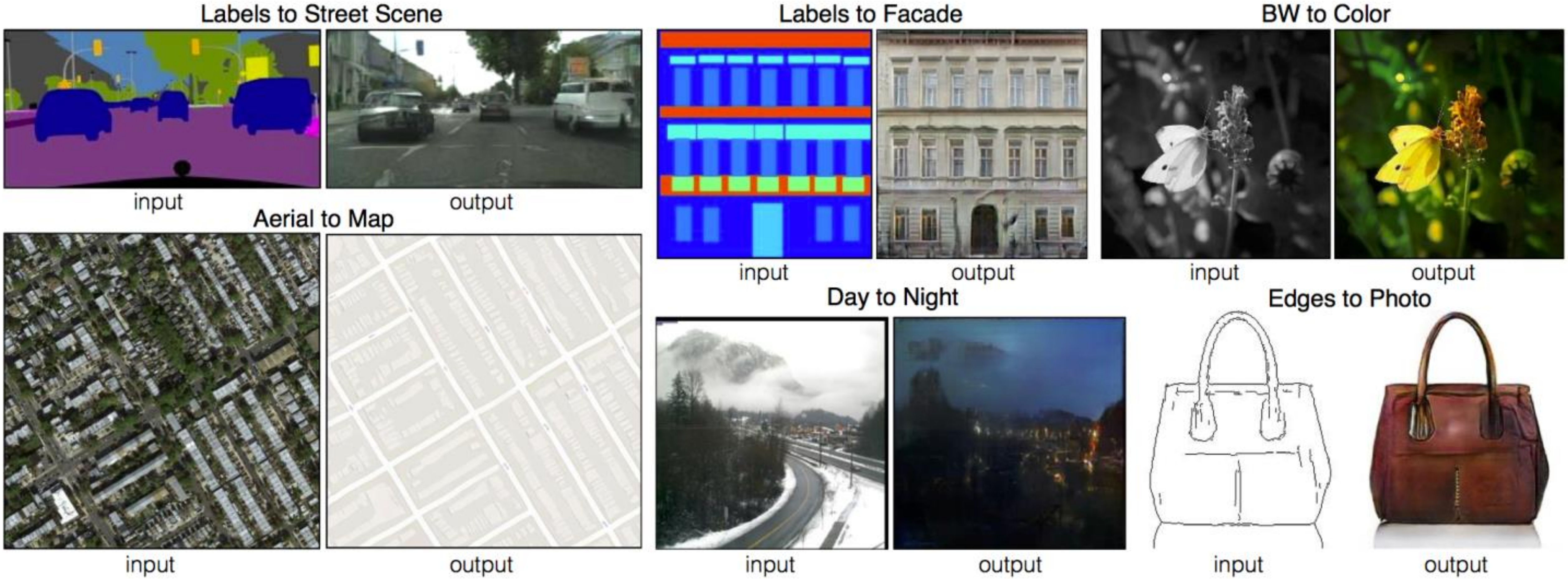

Zhang et al, "StackGAN++: Realistic Image Synthesis with Stacked Generative Adversarial Networks.", TPAMI 2018
Zhang et al, "StackGAN: Text to Photo-realistic Image Synthesis with Stacked Generative Adversarial Networks.", ICCV 2017
Reed et al, "Generative Adversarial Text-to-Image Synthesis", ICML 2016

# GANs

bicubic (21.59dB/0.6423) | SRResNet (23.53dB/0.7832) | SRGAN (21.15dB/0.6868) | original

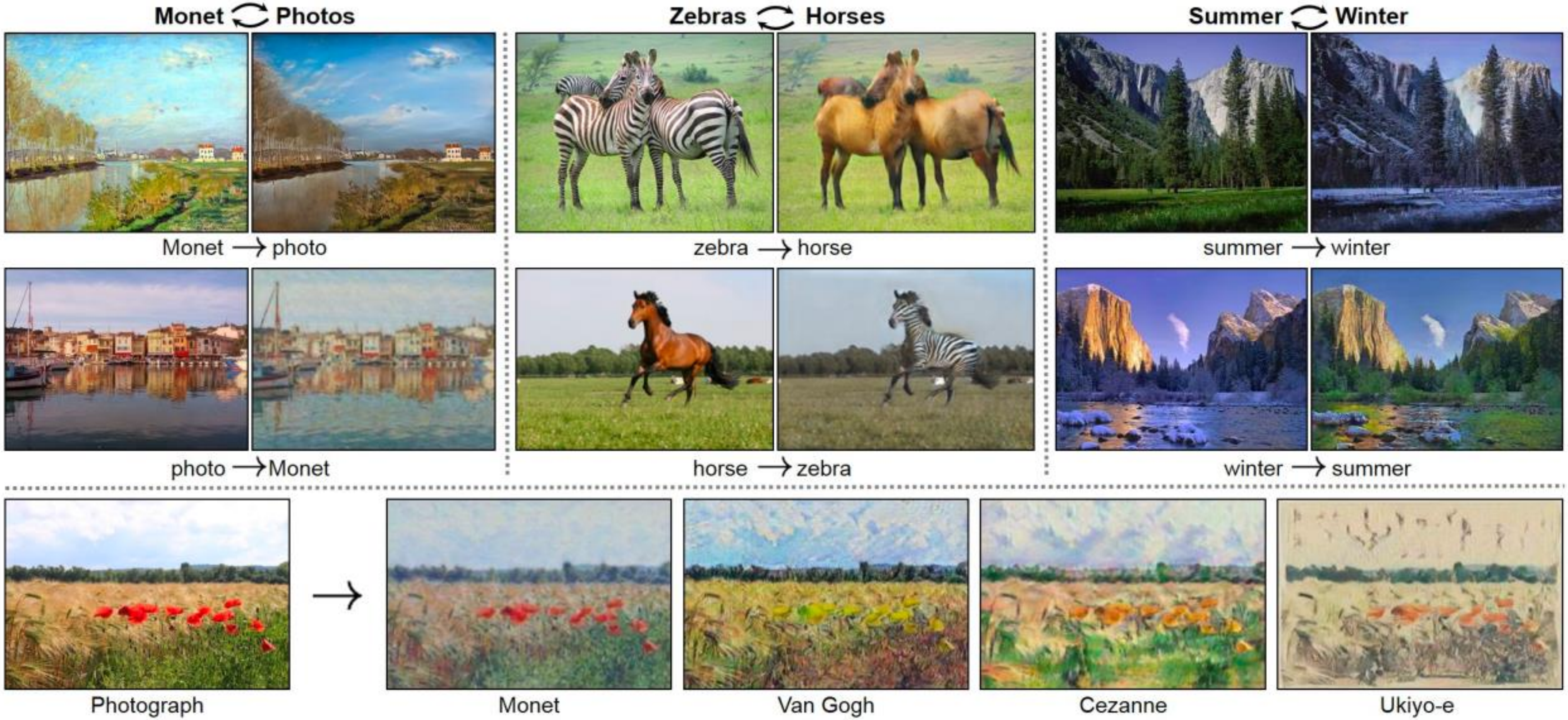
Ledig et al, “Photo-Realistic Single Image Super-Resolution Using a Generative Adversarial Network”, CVPR 2017

# GANs




Isola et al, "Image-to-Image Translation with Conditional Adversarial Nets", CVPR 2017

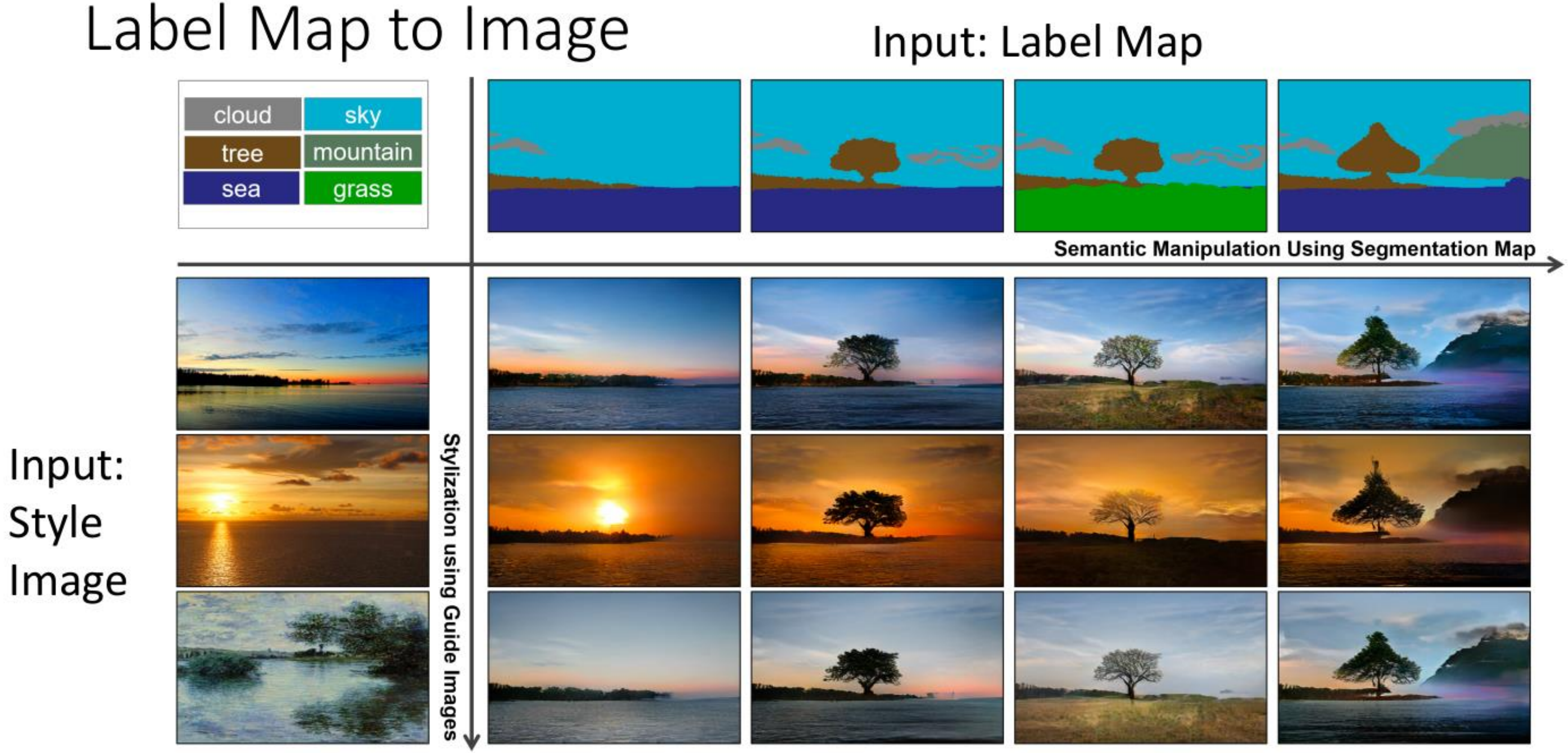
# GANs




Zhu et al, "Unpaired Image-to-Image Translation using Cycle-Consistent Adversarial Networks", ICCV 2017

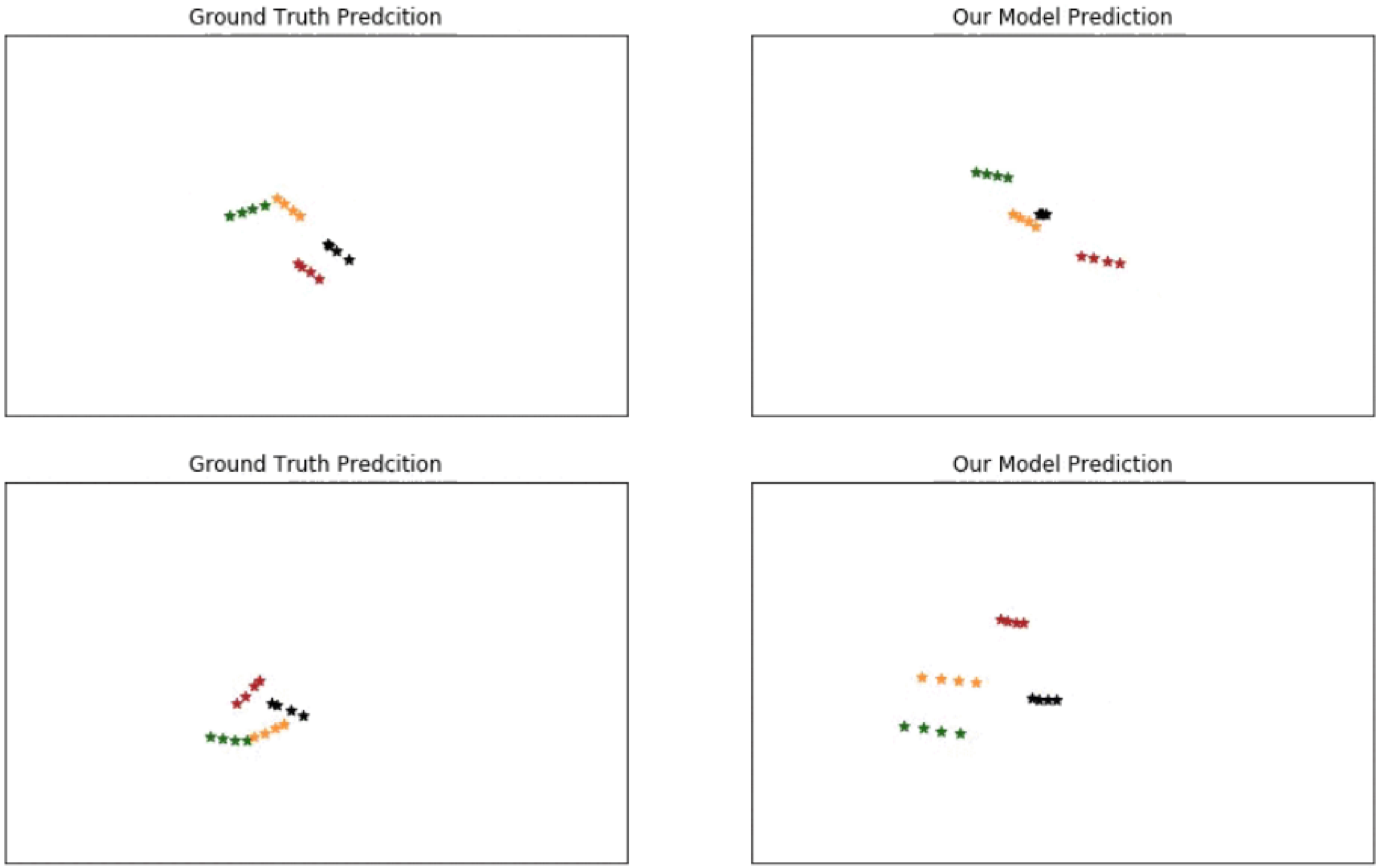
# GANs

## Label Map to Image




Park et al, “Semantic Image Synthesis with Spatially-Adaptive Normalization”, CVPR 2019

# GANs




Gupta, **Johnson**, Li, Savarese, Alahi, “Social GAN: Socially Acceptable Trajectories with Generative Adversarial Networks”, CVPR 2018

# Thank you!
# Q & A