

# Generative Models I

Jisang Han

onground@korea.ac.kr

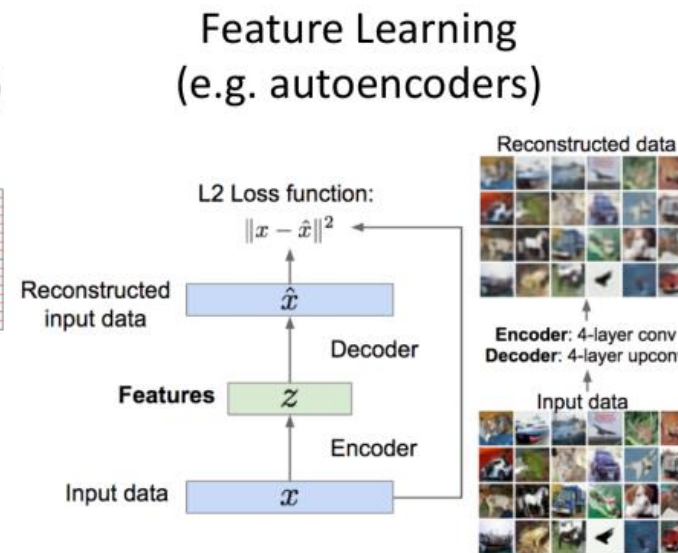
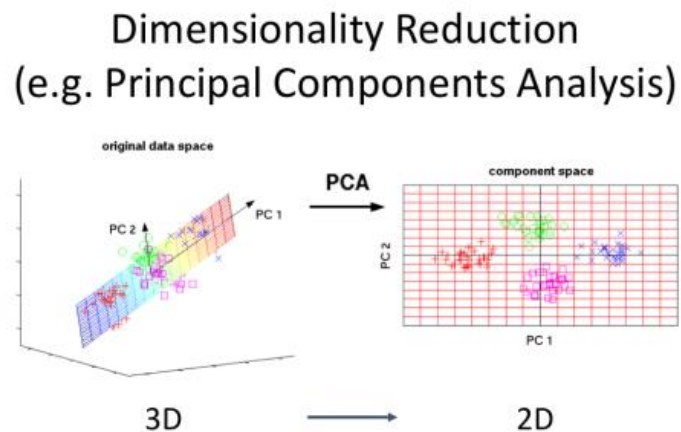
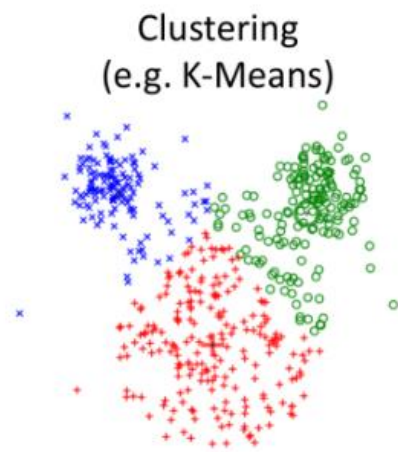
KUGODS

Department of Computer Science and Engineering, Korea University



# Supervised vs Unsupervised

	Supervised Learning	Unsupervised Learning
<b>Data</b>	$(x, y)$ : $x$ is data, $y$ is label	$x$
<b>Goal</b>	$x \rightarrow y$ 로 가는 function을 학습	Learn some underlying hidden structure of the data
<b>Exam ples</b>	Classification, regression, object detection, semantic segmentation, image captioning, etc.	Clustering, dimensionality reduction, feature learning, density estimation, etc.



# Discriminative vs Generative Models

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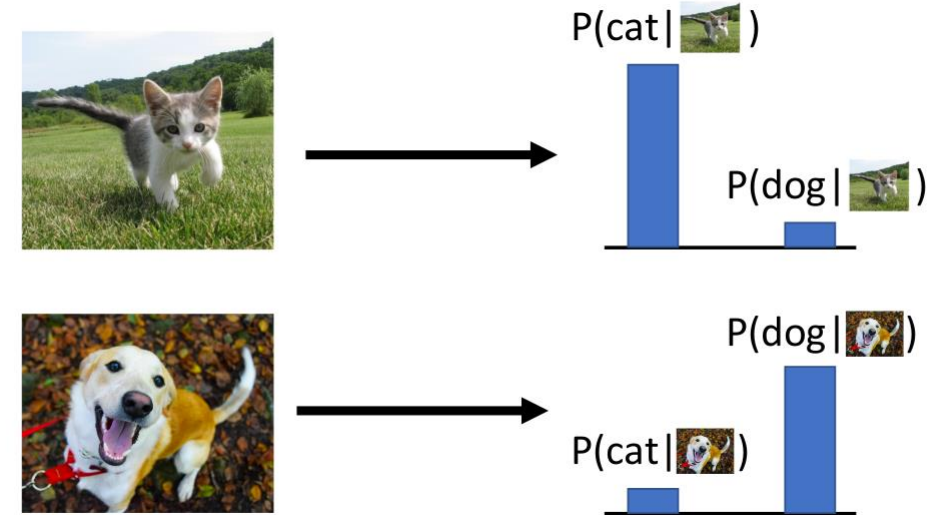
## Density Function

$$\int_X p(x) dx = 1$$

- Density Functions are normalized.
- Sum of probabilities are 1. If one is bigger then the other is smaller(competes each other).

# Discriminative vs Generative Models

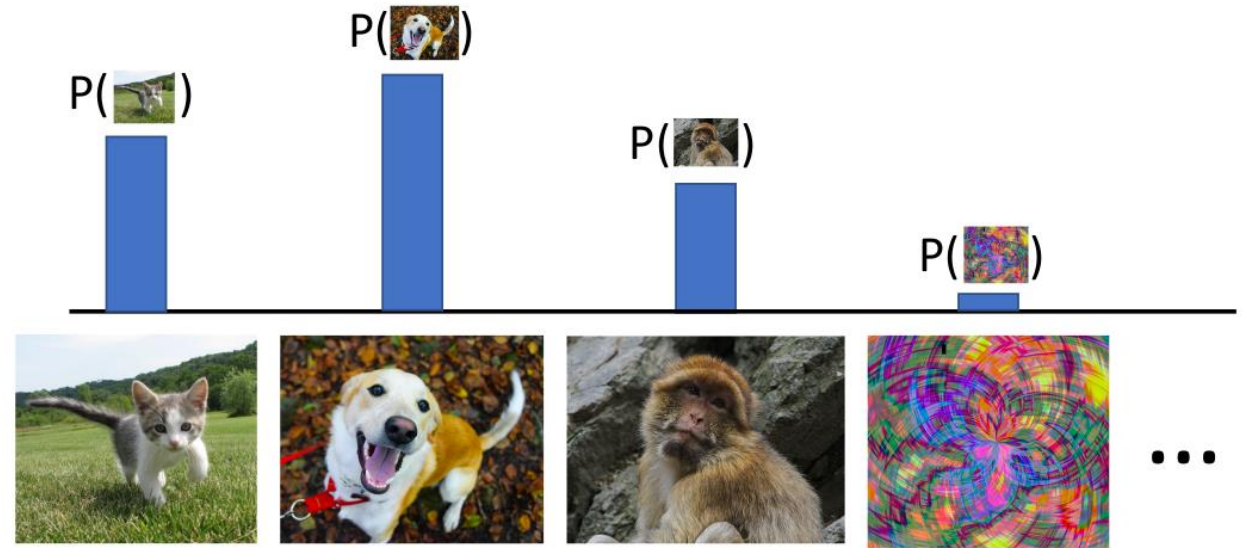
## Discriminative Model



- Learn a probability distribution  $p(y|x)$  that predicts probability of the label  $y$  conditioned on the input image  $x$
- Input is  $x$  and Output is the probability. ( $x$ 에 대한 label이 나올 확률)
- The probability is calculated even if an input different from the specified labels is given.

# Discriminative vs Generative Models

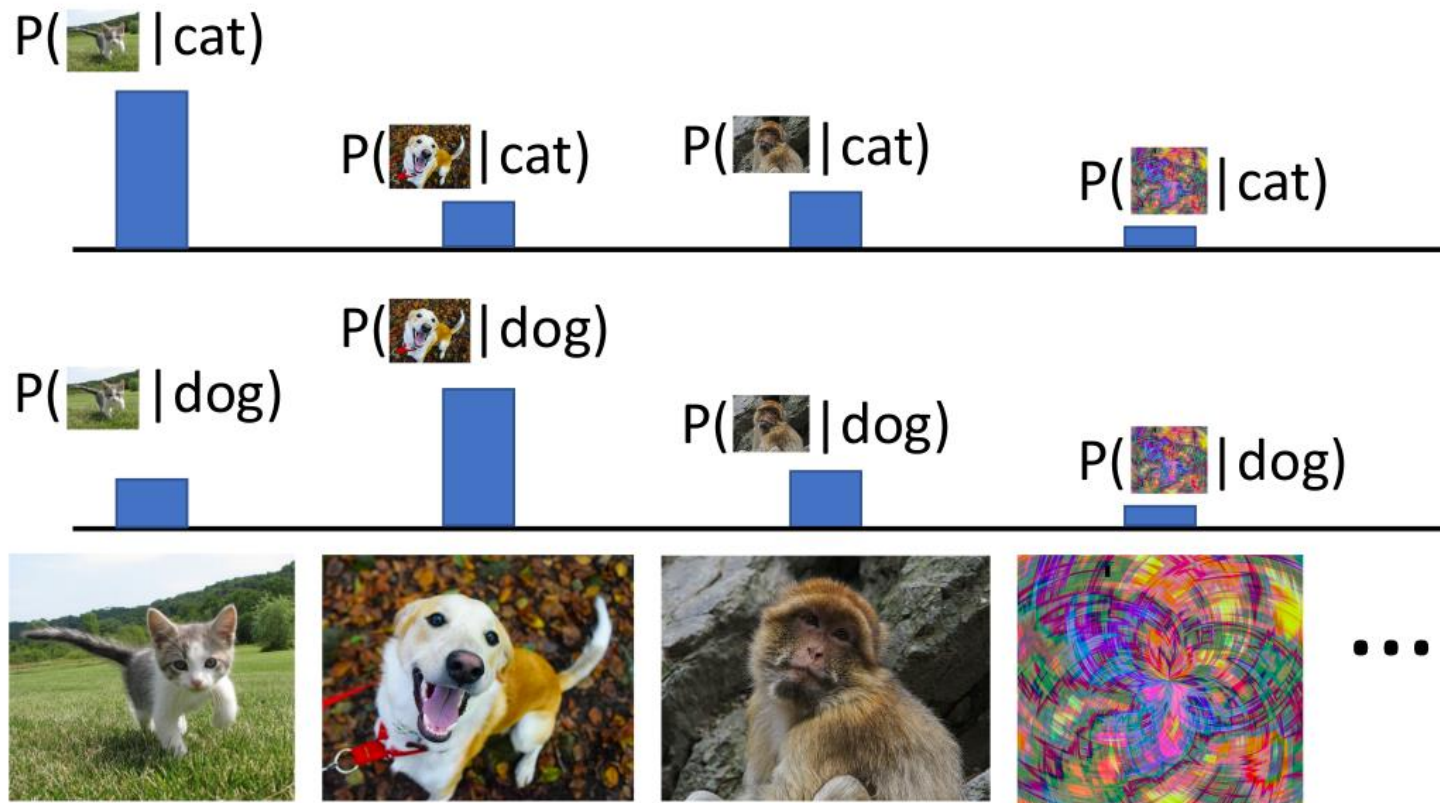
## Generative Model



- **Learn a probability distribution  $p(x)$**
- All possible images compare to the probability mass.
- A deep understanding of the image is needed. (Which is more plausible, sitting a dog or standing up?)
- The model can reject irrational inputs by giving them a very small value.

# Discriminative vs Generative Models

## Conditional Generative Model



Recall Bayes' Rule:

$$P(x | y) = \frac{P(y | x) P(x)}{P(y)}$$

Conditional Generative Model

Discriminative Model

(Unconditional) Generative Model

Prior over labels

- Learn  $p(x|y)$

# Discriminative vs Generative Models

## **Discriminative Model:**

Learn a probability distribution  $p(y|x)$



Assign labels to data  
Feature learning (supervised)

## **Generative Model:**

Learn a probability distribution  $p(x)$



Detect outliers  
Feature learning (unsupervised)  
Sample to **generate** new data

## **Conditional Generative Model:** Learn $p(x|y)$



Assign labels, while rejecting outliers!  
Generate new data conditioned on input labels



# Autoregressive Models

## Explicit Density Estimation

**Goal :** Write down an explicit function for  $p(x) = f(x, W)$  ( $x$ : data,  $W$ : learnable weight matrix)

If dataset is  $x^{(1)}, x^{(2)}, \dots, x^{(N)}$ ,

$$\begin{aligned} W^* &= \arg \max_W \prod_i p(x^{(i)}) \\ &= \arg \max_W \sum_i \log p(x^{(i)}) \\ &= \arg \max_W \sum_i \log f(x^{(i)}, W) \end{aligned}$$

Maximize the probability of training data

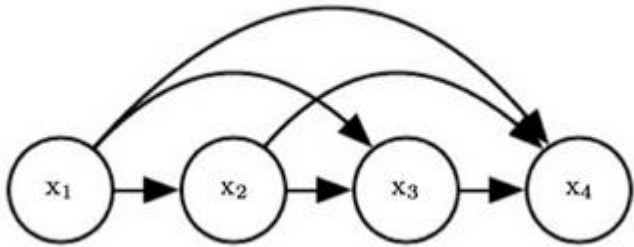
Loss Function. (Gradient Descent)



# Autoregressive Models

## Explicit Density: Autoregressive Models

- 자기 자신을 입력으로 하여 자기 자신을 예측하는 모형



- $x$ 가 여러 subparts로 이루어져있다고 가정.  $x$ 가 이미지라고 하면 subparts는 각 픽셀이다  $x = (x_1, x_2, x_3, \dots, x_T)$
- Chain rule을 사용하여 probability를 계산한다.

$$\begin{aligned} p(x) &= p(x_1, x_2, \dots, x_T) \\ &= p(x_1) p(x_2|x_1) p(x_3|x_1, x_2) \\ &= \prod_{t=1}^T p(x_t|x_1, \dots, x_{t-1}) \end{aligned}$$

# Autoregressive Models

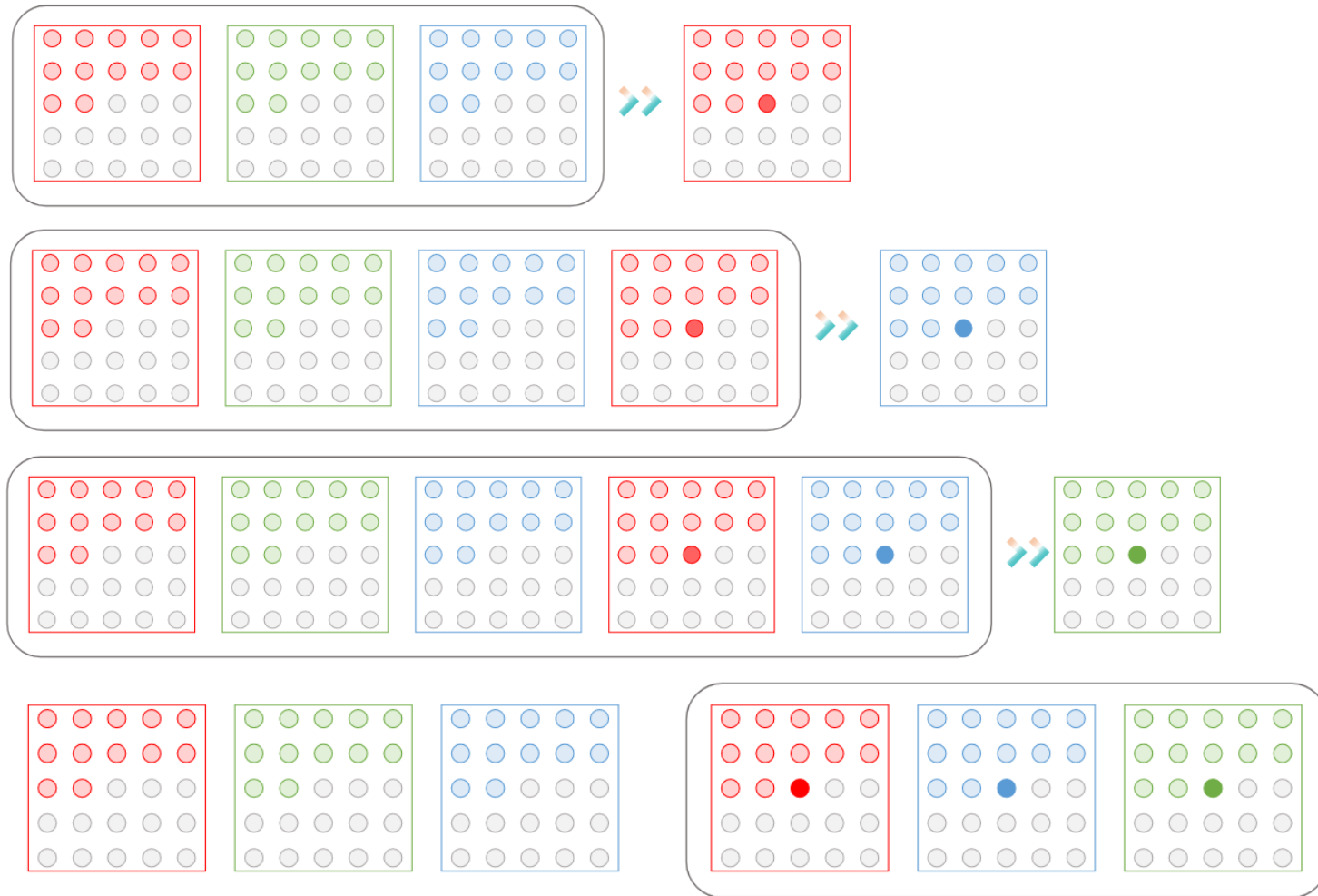
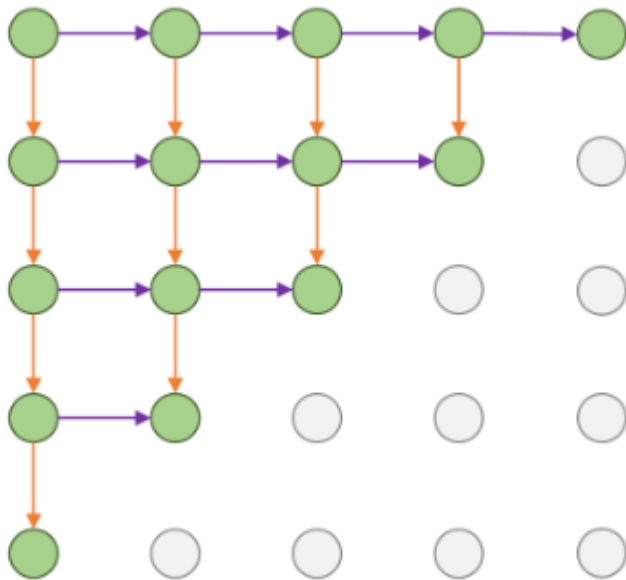
## Explicit Density: Autoregressive Models

$$\begin{aligned} p(x) &= p(x_1, x_2, \dots, x_T) \\ &= p(x_1) p(x_2|x_1) p(x_3|x_1, x_2) \\ &= \prod_{t=1}^T p(x_t|x_1, \dots, x_{t-1}) \end{aligned}$$

- 즉, sequence의 확률  $p(x)$ 는 이전 sequence가 주어졌을 때 다음 sequence가 나올 확률을 전부 곱한 것이다.
- <https://wikidocs.net/22034>

# Autoregressive Models

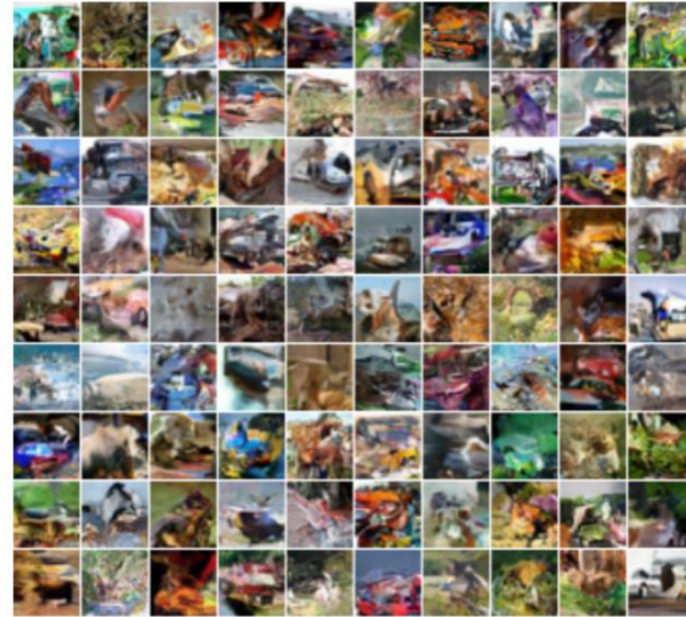
## PixelRNN



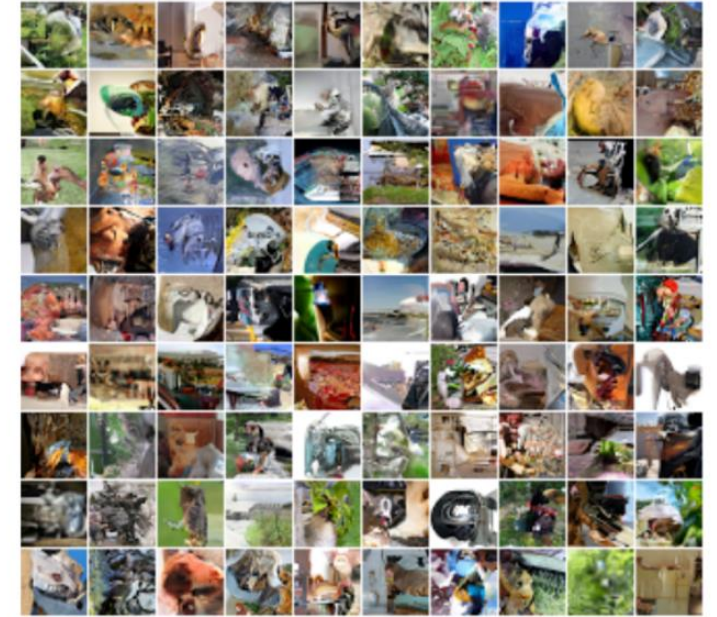
[http://dmqm.korea.ac.kr/uploads/seminar/20190705\\_Autoregressive.pdf](http://dmqm.korea.ac.kr/uploads/seminar/20190705_Autoregressive.pdf)

# Autoregressive Models

## PixelRNN



32x32 CIFAR-10



32x32 ImageNet

- 겉으로 보기에는 합리적으로 보이지만 사실 자세히 보면 거지같다..
- 그래도 edges, colors를 꽤나 그럴싸하게 생성하는 것에 의의를 가진다.
- 이는 unconditional generation이므로 test time에 내가 무엇을 생성하고 있는지 정할 수 없다

# Autoregressive Models

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## Pros and Cons

### Pros

- Likelihood  $p(x)$ 를 명시적으로 계산할 수 있다.
- 위의 장점 덕분에 좋은 evaluation metric을 얻는다. (training data와 유사한, 즉 얼마나 그럴듯한지에 대한  $p(x)$  값을 얻을 수 있기 때문이다.)
- 나름 괜찮은 결과를 얻을 수 있다. (진짜 사진같지는 않지만..)

### Cons

- Sequential Generation 이므로 느리다.

# Variational Autoencoders (VAE)

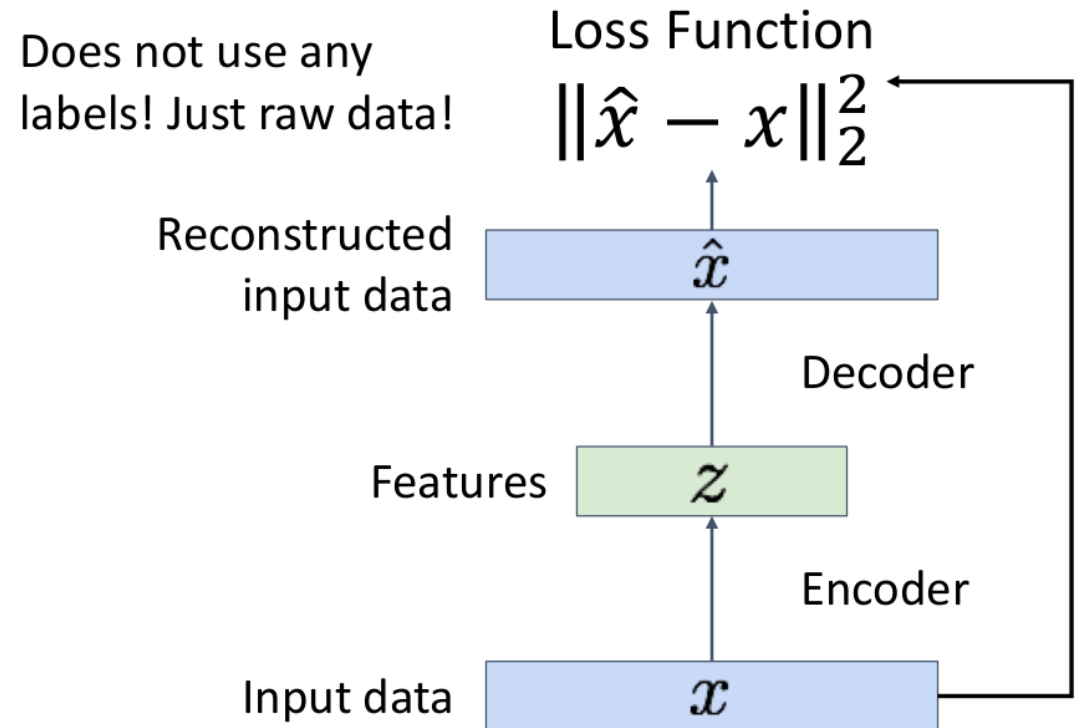
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- In PixelRNN, PixelCNN, they defined parametric density function  $p(x) = f(x, W)$  and calculate for each input. And train the model to maximize this output.
- In VAE, Instead of maximizing the actual density value, **maximize the lower bound of the density.**
- $p_{\theta}(x) = \prod_{i=1}^n p_{\theta}(x_i | x_1, \dots, x_{i-1})$

# Variational Autoencoders (VAE)

## (Regular, non-variational) Autoencoders

- Unsupervised method로, labels 없이 raw data  $x$ 로부터 feature vectors를 학습한다.  
(Unsupervised method for learning feature vectors from raw data  $x$ , without any labels)
- **Features** extracts useful information that can be used for downstream tasks.
- **Encoder** extracts features from input data,
- **Decoder** reconstruct the input data from the features.
- Encoder:
  - Originally: Linear + nonlinearity (sigmoid)
  - Later: Deep, fully-connected
  - Later: ReLU CNN (upconv)

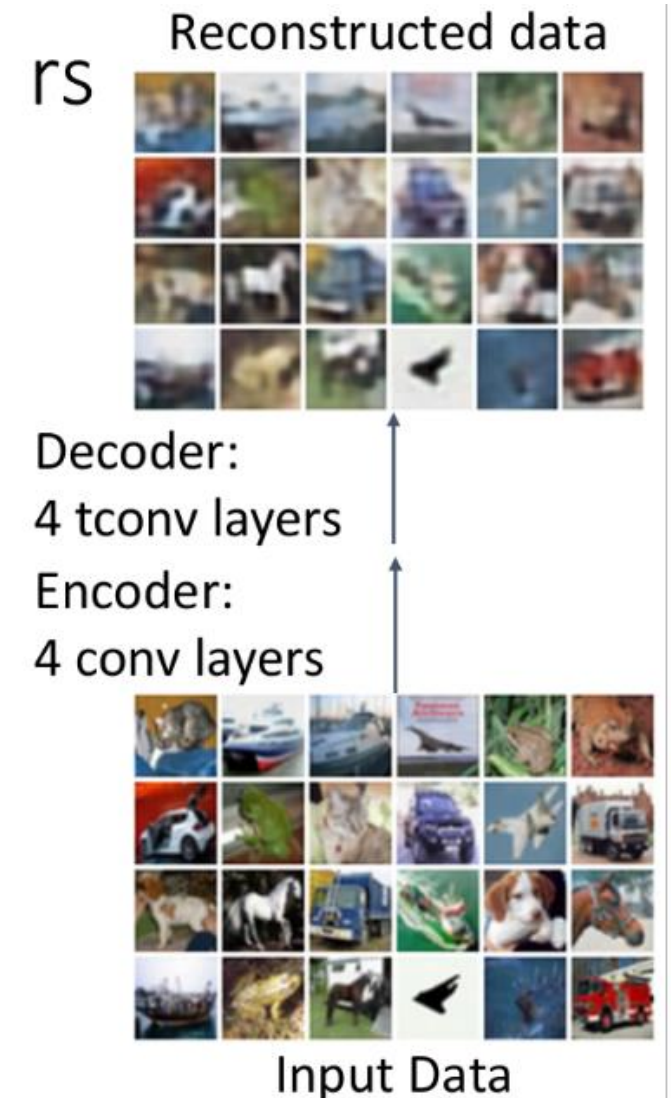




# Variational Autoencoders (VAE)

## (Regular, non-variational) Autoencoders

- We expect the effect of **compressing input data through Encoder**.
- After learning, discard the decoder and use it for the downstream task using the encoder.
- **Not probabilistic** : 학습하지 않은 new data를 sampling할 수 없다.



# Variational Autoencoders (VAE)

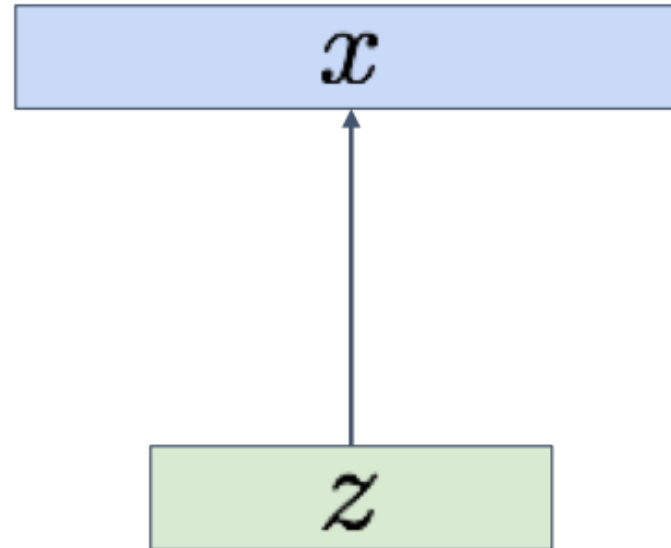
## Variational Autoencoders

Sample from  
conditional

$$p_{\theta^*}(x \mid z^{(i)})$$

Sample  $z$   
from prior

$$p_{\theta^*}(z)$$



- Autoencoder에 확률 개념을 도입하였다
- 1. raw data로부터 **latent features  $z$** 를 학습한다.
- 2. new data를 생성하기 위해 model로부터 **sampling**한다.

# Variational Autoencoders (VAE)

## Variational Autoencoders

- **Decoder:** Generating new data  $x$  from latent features  $z$  that is similar to input data but completely new.
- Sampling latent variables from prior distribution  $p_{\theta_*}(z)$ , put sampled  $z$  into the decoder to predict image  $x$
- At this time, output is not a single image but the distribution of images.
- $p_{\theta_*}(z)$  : prior distribution. PDF of  $x$ . (Gaussian distribution)
- $p_{\theta_*}(x|z^{(i)})$  : 주어진  $z$ 에서 특정  $x$ 가 나올 조건부 확률에 대한 PDF
- $\theta$  : parameter

# Variational Autoencoders (VAE)

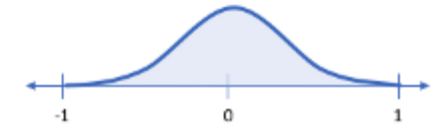
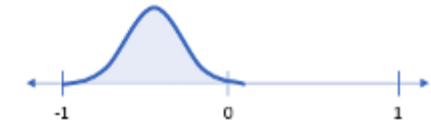
## Variational Autoencoders



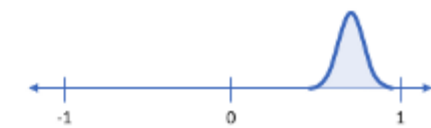
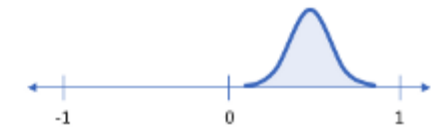
Smile (discrete value)



Smile (probability distribution)

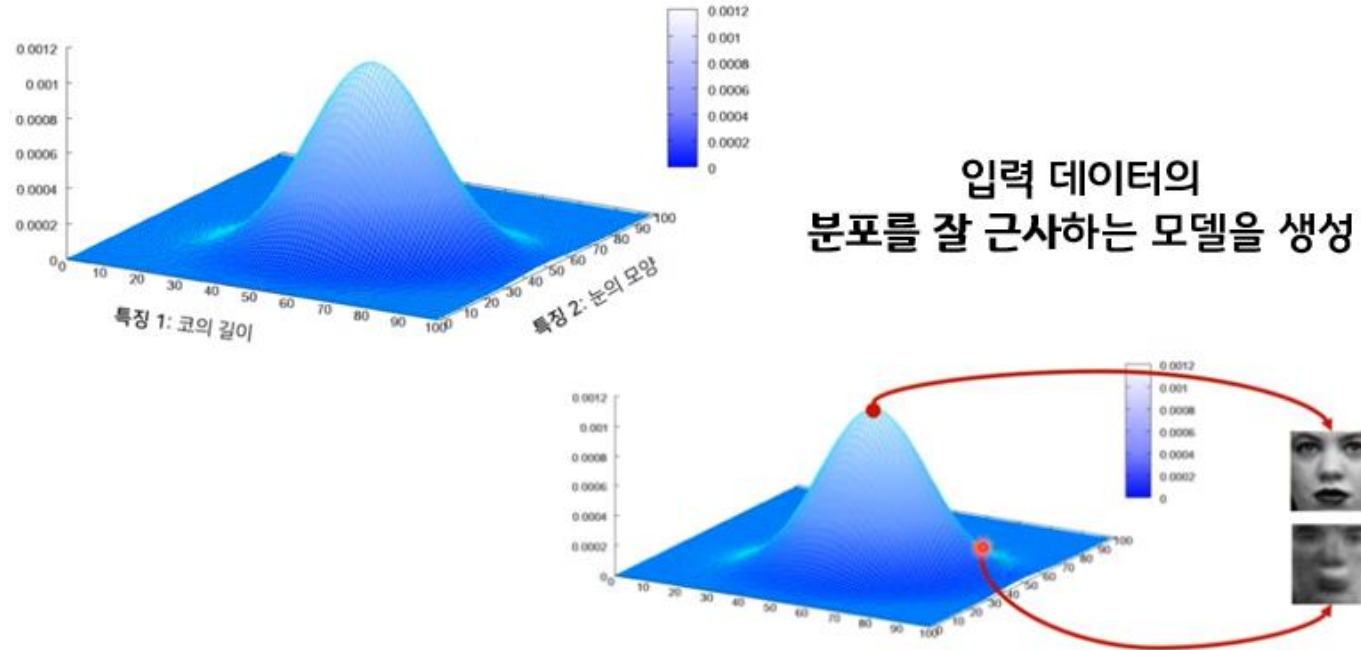


vs.



- VAE에서의  $z$ 는 AE에서의  $z$  (a value: low dimension of input data)와 다르게, 가우시안 확률분포에 기반한 확률값으로 나타낸다.
- input image가 들어오면 그 이미지의 다양한 특징들이 각각의 확률변수가 되는 확률분포를 만든다. 이 확률 분포를 잘 찾아내어 확률값이 높은 부분을 사용하면 그럴듯한 새로운 이미지를 생성할 수 있다.

# Variational Autoencoders (VAE)



- 이때 각 feature가 가우시안 분포를 따른다고 가정하고 latent  $z$ 는 각 feature의 평균과 분산값을 나타낸다.
- 예를 들어 한국인의 얼굴을 그리기 위해 눈, 코, 입 등의 feature를 Latent vector  $z$ 에 담고, 그  $z$ 를 이용해 그럴듯한 한국인의 얼굴을 그려내는 것이다. latent vector  $z$ 는 한국인 눈 모양의 평균 및 분산, 한국인 코 길이의 평균 및 분산, 한국인 머리카락 길이의 평균 및 분산 등등의 정보를 담고 있다고 생각할 수 있다.

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# Thank you!

## Q & A