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## 一 数列基础

1. 等比数列

通项公式 

前n项和 

推导如下（错项相减，超级适用于等比数列）：



2 平方和公式

https://gss1.bdstatic.com/-vo3dSag_xI4khGkpoWK1HF6hhy/baike/s%3D442/sign=8cbc53b969061d957946363c49f50a5d/6a600c338744ebf83ff14ebedef9d72a6159a7aa.jpg

**证法一　（归纳猜想法）：**

1、

https://gss1.bdstatic.com/9vo3dSag_xI4khGkpoWK1HF6hhy/baike/s%3D35/sign=ffaaeb4b9252982201333fc6d6caec7a/bd315c6034a85edf343829af4b540923dc547582.jpg

时，

https://gss1.bdstatic.com/9vo3dSag_xI4khGkpoWK1HF6hhy/baike/s%3D183/sign=bfe86f07fc1f4134e4370176161d95c1/0824ab18972bd4072c0c21197a899e510eb3090e.jpg

2、

https://gss3.bdstatic.com/-Po3dSag_xI4khGkpoWK1HF6hhy/baike/s%3D37/sign=45376f0dc91349547a1eee63544e0ada/eac4b74543a98226d414fcab8882b9014b90eb61.jpg

时，

https://gss3.bdstatic.com/7Po3dSag_xI4khGkpoWK1HF6hhy/baike/s%3D209/sign=cd1e927ba50f4bfb88d099543a4e788f/86d6277f9e2f07086724d7e4e824b899a901f235.jpg

3、设

https://gss3.bdstatic.com/7Po3dSag_xI4khGkpoWK1HF6hhy/baike/s%3D36/sign=cf9bb1fff9f2b211e02e8348cb80c617/3ac79f3df8dcd10065022fad708b4710b8122fa4.jpg

https://gss2.bdstatic.com/9fo3dSag_xI4khGkpoWK1HF6hhy/baike/s%3D89/sign=7af8458c219759ee4e506dc2b3fb97e9/91ef76c6a7efce1bef14e94ba851f3deb48f6577.jpg

时，公式成立，即

https://gss3.bdstatic.com/7Po3dSag_xI4khGkpoWK1HF6hhy/baike/s%3D254/sign=cca12a5ff536afc30a0c3860871beb85/1c950a7b02087bf4c53f3076f0d3572c10dfcf5a.jpg

则当

https://gss1.bdstatic.com/9vo3dSag_xI4khGkpoWK1HF6hhy/baike/s%3D59/sign=e3080bec372ac65c6305667afaf2ca00/a686c9177f3e670910b2927339c79f3df9dc5588.jpg

时，

https://gss0.bdstatic.com/-4o3dSag_xI4khGkpoWK1HF6hhy/baike/s%3D190/sign=a45c76b097cad1c8d4bbf82e4f3f67c4/0eb30f2442a7d9335c8782ecaf4bd11373f00105.jpg

https://gss3.bdstatic.com/-Po3dSag_xI4khGkpoWK1HF6hhy/baike/s%3D196/sign=e21fc8ff4dc2d562f608d4e4d11090f3/bd3eb13533fa828bfd39fb7cff1f4134970a5a12.jpg

https://gss3.bdstatic.com/-Po3dSag_xI4khGkpoWK1HF6hhy/baike/s%3D191/sign=abbca499adc3793179688220dac5b784/63d9f2d3572c11df7aa0d9bc612762d0f703c238.jpg

https://gss3.bdstatic.com/-Po3dSag_xI4khGkpoWK1HF6hhy/baike/s%3D156/sign=6097afac9045d688a702b6a192c37dab/024f78f0f736afc3da5b59fab119ebc4b64512dd.jpg

https://gss3.bdstatic.com/7Po3dSag_xI4khGkpoWK1HF6hhy/baike/s%3D159/sign=017e135f0e33874498c52b79680ed937/5882b2b7d0a20cf4c6f7b3d074094b36acaf995e.jpg

https://gss0.bdstatic.com/94o3dSag_xI4khGkpoWK1HF6hhy/baike/s%3D230/sign=0fb6ac3837fae6cd08b4ac623fb20f9e/4bed2e738bd4b31c1f9cd9b485d6277f9e2ff819.jpg

也满足公式。

根据[数学归纳法](https://baike.baidu.com/item/%E6%95%B0%E5%AD%A6%E5%BD%92%E7%BA%B3%E6%B3%95" \t "_blank)，对一切自然数n有

https://gss2.bdstatic.com/9fo3dSag_xI4khGkpoWK1HF6hhy/baike/s%3D276/sign=9022ac0f562c11dfdad1b82455256255/d8f9d72a6059252d26d176e8359b033b5ab5b91c.jpg

成立。

**证法二 （利用恒等式）：**

https://gss2.bdstatic.com/-fo3dSag_xI4khGkpoWK1HF6hhy/baike/s%3D187/sign=b06992e2adaf2eddd0f14de1ba110102/5fdf8db1cb134954cf430480574e9258d0094af1.jpg

https://gss0.bdstatic.com/-4o3dSag_xI4khGkpoWK1HF6hhy/baike/s%3D187/sign=683e4e3af11f3a295ec8d1c6ae24bce3/7e3e6709c93d70cfdfad45e9f9dcd100bba12bfb.jpg

https://gss3.bdstatic.com/-Po3dSag_xI4khGkpoWK1HF6hhy/baike/s%3D263/sign=2cf4cb0ba8ec8a13101a50e6c4019157/2934349b033b5bb5b232cf1a37d3d539b700bc69.jpg

…………

https://gss0.bdstatic.com/-4o3dSag_xI4khGkpoWK1HF6hhy/baike/s%3D180/sign=dde0ac1b3ac79f3d8be1e0388aa3cdbc/72f082025aafa40f5f262739aa64034f79f01975.jpg

https://gss2.bdstatic.com/9fo3dSag_xI4khGkpoWK1HF6hhy/baike/s%3D180/sign=edbcfaab9f2f07085b052e08d925b865/bf096b63f6246b6066595a90eaf81a4c500fa2fb.jpg

求和得：

https://gss3.bdstatic.com/-Po3dSag_xI4khGkpoWK1HF6hhy/baike/s%3D446/sign=6acc6dcfb151f819f525024eecb64a76/a1ec08fa513d26970a30e3b954fbb2fb4216d877.jpg

,

由于

https://gss0.bdstatic.com/-4o3dSag_xI4khGkpoWK1HF6hhy/baike/s%3D192/sign=ed7f729fd01373f0f13f6b96960e4b8b/29381f30e924b899953d93dc6f061d950b7bf6d5.jpg

（可由倒序求和得到）,

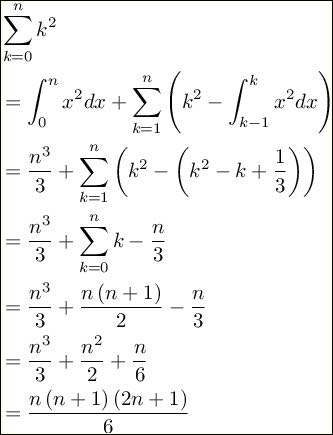
代入上式得：

https://gss2.bdstatic.com/9fo3dSag_xI4khGkpoWK1HF6hhy/baike/s%3D394/sign=17f38656b3b7d0a27fc90294ffed760d/2cf5e0fe9925bc31e6a7eb7c5fdf8db1ca137058.jpg

整理后得：

https://gss3.bdstatic.com/7Po3dSag_xI4khGkpoWK1HF6hhy/baike/s%3D281/sign=b0414f1778310a55c024d9fc86444387/63d9f2d3572c11dfee9b6ac7622762d0f603c288.jpg

**证法三 （无敌积分法）**



1. 立方和公式



证明从略