CS3323 Fall 2006 Assignment 1
Due Friday Sept 29, by 5pm.

1. What does the following algorithm do? Analyze its worst-case running time, figure out its running time function, and express it using "Big-Oh" notation.

   Algorithm Foo (a, n):
   Input: two integers, a and n
   Output: ?

   ```
   k ← 0
   b ← 1
   while k < n do
       k ← k + 1
       b ← b * a
   return b
   ```

2. What does the following algorithm do? Analyze its worst-case running time, figure out its running time function, and express it using "Big-Oh" notation.

   Algorithm Bar (a, n):
   Input: two integers, a and n
   Output: ?

   ```
   k ← n
   b ← 1
   c ← a
   while k > 0 do
       if k mod 2 = 0 then
         k ← k/2
         c ← c * c
       else
         k ← k − 1
         b ← b * c
   return b
   ```

3. Algorithm A executes $10n \log n$ operations, while algorithm B executes n^2 operations. Determine the minimum integer value n_0 such that A executes fewer operations than B for $n \geq n_0$.

4. Prove or disprove each of the following statements:

(a) $10n^2 + 8n + 2$ is $O(n^2)$.

(b) $3(n+1)^7 + 2n \log n$ is $O(n^7)$.

(c) $3n^5 + 10n^4 \log_2 n - 10n^3 - 15n^2$ is $O(n^5)$

(d) $10n^4$ is $O(10000n^3 \log_2 n)$

5. Order the following functions by the big-O notation, starting from the smallest one.

$$3^{\log_9 n} \quad \log_8 n^3 \quad \log_{10} \log_{10} n^{100} \quad \sqrt{n} \quad n^{0.001} \quad \log_2 n \quad (\log_2 n)^2$$

6. Prove that if $f(n)$ is $O(g(n))$ and $d(n)$ is $O(h(n))$, then $f(n) + d(n)$ is $O(g(n) + h(n))$.