Homework 3 - Fuzzy Control

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Due : 20.06.2021

1 Introduction

For this assignment membership functions of fuzzy sets are selected as trapezoids. The notation for a trapezoid membership function is given in Equation 1.

$$\Lambda_{(x)}^{\{x_1,x_2,x_3,x_4\}} = \begin{cases} 0 & ; x < x_1 \\ \frac{x-x_1}{x_2-x_1} & ; x_1 \leq x < x_2 \\ 1 & ; x_2 \leq x < x_3 \\ \frac{x_4-x}{x_4-x_3} & ; x_3 \leq x < x_4 \\ 0 & ; x > x_4 \end{cases} \tag{1}$$

Center of mass is used for defuzzyfication. The relations are formed from the unions of Cartesian products of fuzzy sets. Also the set operations $\wedge$ and $\vee$ correspond to $min(.)$ and $max(.)$ operations respectively.

See the Appendix for the Python codes.

2 Vaccination v1

You can only measure the current percentage of the vaccinated people π. In this manner, design a fuzzy controller which takes π as the input and provides δ as the output to vaccinate 60% of the people and maintain that percentage.Partition the sets where the measurement and the output lie into 3 fuzzy sets.

- Inputs: Vaccinated Percentage in Population $\pi \in [0, 1]$
- Controller Output: Percentage to Vaccinate $\delta \in [-0.25, +0.25]$

2.1 Solution and Implementation

The sets should be arranged such that when the ratio of vaccinated is less than 60% the output control should increase the number of vaccinated, and similarly when the ratio of vaccinated is more than 60% the output control should decrease the number of vaccinated people. Additionally if the number of vaccinated is around 60% the output should not change the number of vaccinated too much.

Furthermore it should be noted that the whole input space should be covered by the fuzzy sets. If there is a region excluded in the input space there would not be any output controls applied for that region.

The fuzzy sets and their relations are decided with linguistic definitions as follows;

Inputs Fuzzy Sets:

- **LVR:** Low Vaccinated Ratio
- **SVR:** Satisfactory Vaccinated Ratio
- **HVR:** High Vaccinated Ratio

Output Fuzzy Sets:

- **NC:** Decrease the number of vaccinated
- **ZC:** Do not change the number of vaccinated
- **PC:** Increase the number of vaccinated

Relations:

- **LVR** $\rightarrow$ **PC** : if (Low Vaccinated Ratio) then (Increase the number of vaccinated)
- **SVR** $\rightarrow$ **ZC** : if (Satisfactory Vaccinated Ratio) then (Do not change the number of vaccinated)
- **HVR** $\rightarrow$ **NC** : if (High Vaccinated Ratio) then (Decrease the number of vaccinated)

2.1.1 Fuzzy Partition 1:

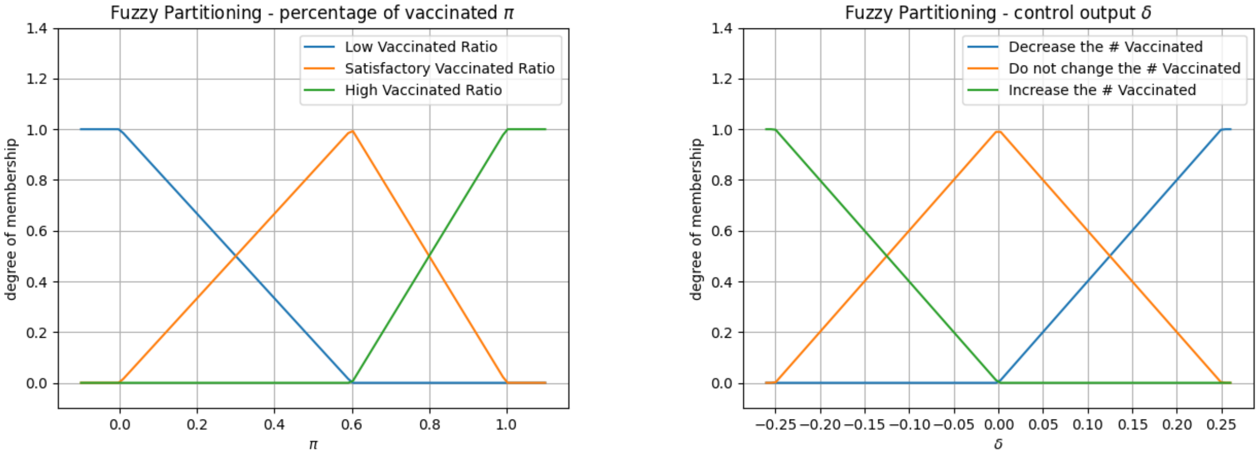
For the starting trial, fuzzy sets are arranged with boundary of their cores at the most representative values of vaccinated ratios, that are 0, 0.6 and 1 for Low, Satisfactory and High Vaccinated Ratios and -0.25, 0, 0.25 for Decreasing, Keeping Constant and Increasing the number of vaccinated people. The supports of the sets reach up to the cores of adjacent fuzzy sets. Membership functions for Input and Output Fuzzy set are shown in Fig. **??** and given below,

Membership Functions of Inputs Fuzzy Sets:

- $\mu_{(\pi)}^{LVR} = \Lambda_{(\pi)}^{\{0,0,0,0.6\}}$
  LVR: Low Vaccinated Ratio
- $\mu_{(\pi)}^{SVR} = \Lambda_{(\pi)}^{\{0,0.6,0.6,1\}}$
  SVR: Satisfactory Vaccinated Ratio
- $\mu_{(\pi)}^{HVR} = \Lambda_{(\pi)}^{\{0.6,1,1,1\}}$
  HVR: High Vaccinated Ratio

Membership Functions of Output Fuzzy Sets:

- $\mu_{(\delta)}^{NC} = \Lambda_{(\delta)}^{\{-0.25,-0.25,-0.25,0\}}$
  NC: Decrease the number of vaccinated
- $\mu_{(\delta)}^{ZC} = \Lambda_{(\delta)}^{\{-0.25,0,0,0.25\}}$
  ZC: Do not change the number of vaccinated
- $\mu_{(\delta)}^{PC} = \Lambda_{(\delta)}^{\{0,0.25,0.25,0.25\}}$
  PC: Increase the number of vaccinated



(a) Membership Functions of Input Fuzzy Sets (b) Membership Functions of Output Fuzzy Sets

Figure 1: Fuzzy Partitioning - Trial 1

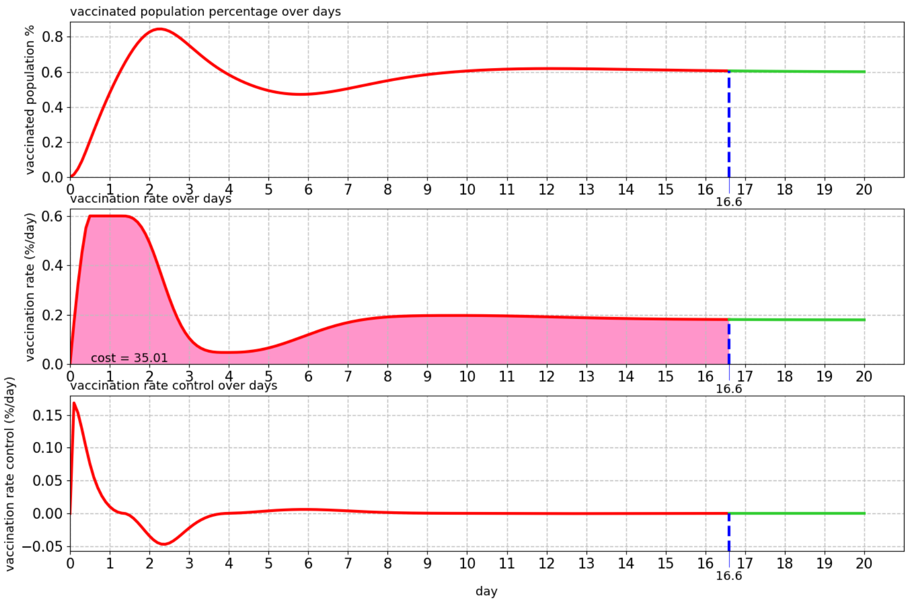


Figure 2: Vaccinated Percentage Curve, Vaccination Cost and Applied Control - Trial 1

From the vaccination curves in Fig. **??**, we can see that the partitioning with the specified fuzzy sets result in a;

- convergence within 16.6 days.

- vaccination cost of 35.01 units.

From the vaccinated population percentage curve it can be noted that implemented controller is not responsive enough to quickly compensate over-shoots and under-shoots. If a more aggressive control output is applied a faster convergence and less cost can be obtained.

2.1.2 Fuzzy Partition 2:

In order to increase the speed of convergence centre of mass of controller outputs are taken further apart from the origin. Also note that the ZC set acts as a regularization term, if its membership is increased the center of mass at the output will be brought closer to 0. Therefore, to decrease this effect, the width of ZC's support is decreased.

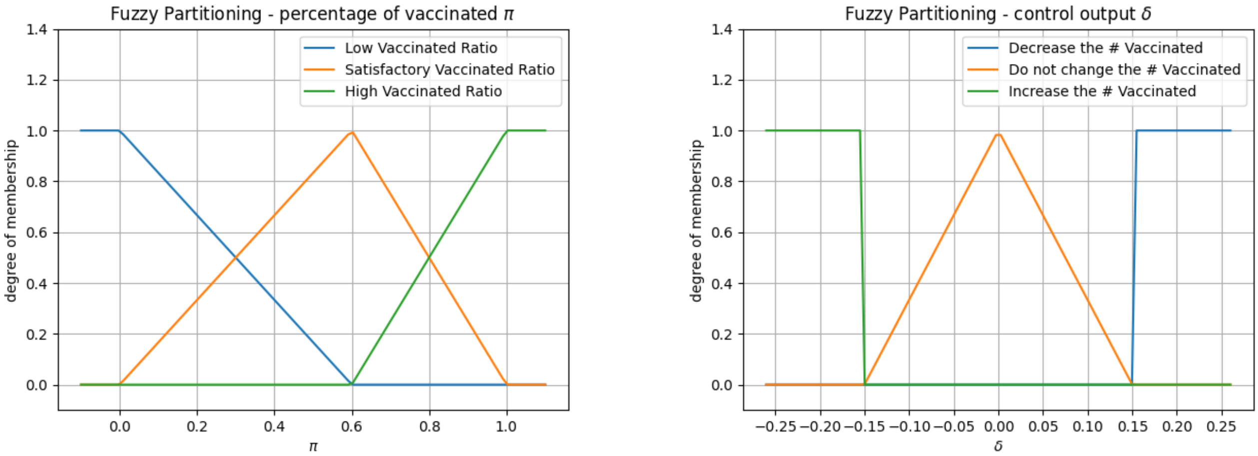
Membership functions for Input and Output Fuzzy set are shown in Fig. **??** and given below,

Membership Functions of Inputs Fuzzy Sets:

- $\mu_{(\pi)}^{LVR} = \Lambda_{(\pi)}^{\{0,0,0,0.6\}}$
  LVR: Low Vaccinated Ratio
- $\mu_{(\pi)}^{SVR} = \Lambda_{(\pi)}^{\{0,0.6,0.6,1\}}$
  SVR: Satisfactory Vaccinated Ratio
- $\mu_{(\pi)}^{HVR} = \Lambda_{(\pi)}^{\{0.6,1,1,1\}}$
  HVR: High Vaccinated Ratio

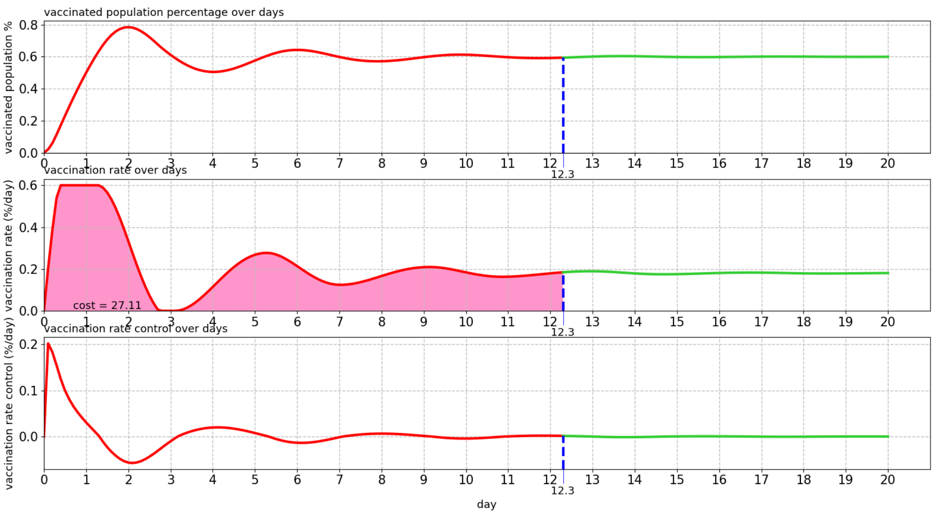
Membership Functions of Output Fuzzy Sets:

- $\mu_{(\delta)}^{NC} = \Lambda_{(\delta)}^{\{-0.25,-0.25,-0.15,-0.15\}}$
  NC: Decrease the number of vaccinated
- $\mu_{(\delta)}^{ZC} = \Lambda_{(\delta)}^{\{-0.15,0,0,0.15\}}$
  ZC: Do not change the number of vaccinated
- $\mu_{(\delta)}^{PC} = \Lambda_{(\delta)}^{\{0.15,0.15,0.25,0.25\}}$
  PC: Increase the number of vaccinated



(a) Membership Functions of Input Fuzzy Sets (b) Membership Functions of Output Fuzzy Sets

Figure 3: Fuzzy Partitioning - Trial 1



From the vaccination curves in Fig. **??**, we can see that the partitioning with the specified fuzzy sets result in a;

- convergence within 12.3 days.
- vaccination cost of 27.11 units.

From the vaccinated population percentage curve it can be seen that convergence is faster and therefore vaccination cost is less. However, by decreasing the width of ZC's support the system can be made faster.

Figure 4: Vaccinated Percentage Curve, Vaccination Cost and Applied Control - Trial 2

2.1.3 Fuzzy Partition 3:

In order to increase the speed of convergence and decrease the effect of regularization the widths of SVR and ZC's supports are decreased. With a decreased width of support for SVR, the regularization effect will only be present for input values closer to the set value of 60%. Also boundaries of the other fuzzy output sets are tuned to give a better performance.

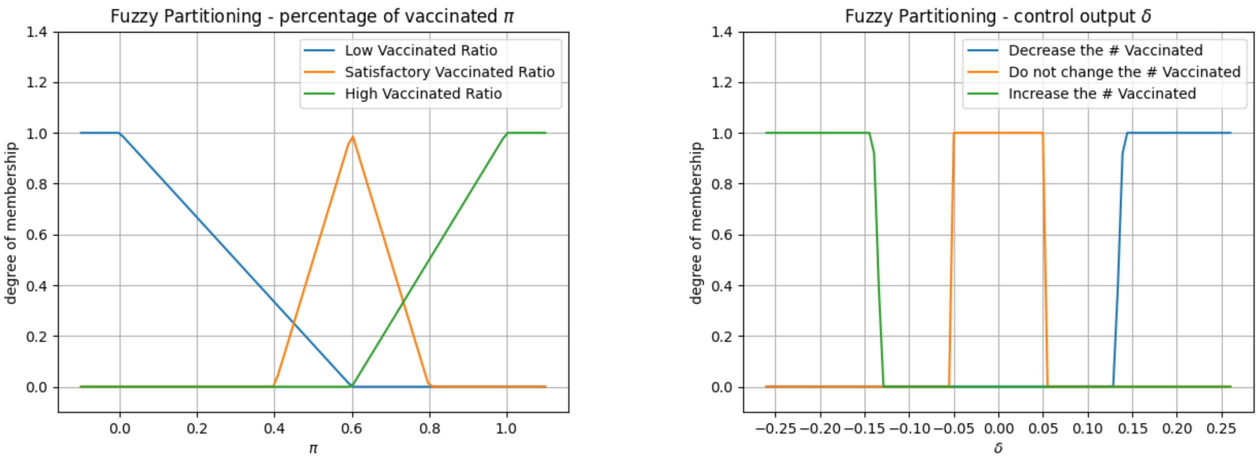
Membership functions for Input and Output Fuzzy set are shown in Fig. **??** and given below,

Membership Functions of Inputs Fuzzy Sets:

- $\mu_{(\pi)}^{LVR} = \Lambda_{(\pi)}^{\{0,0,0,0.6\}}$ **LVR:** Low Vaccinated Ratio
- $\mu_{(\pi)}^{SVR} = \Lambda_{(\pi)}^{\{0,0.6,0.6,1\}}$ **SVR:** Satisfactory Vaccinated Ratio
- $\mu_{(\pi)}^{HVR} = \Lambda_{(\pi)}^{\{0.6,1,1,1\}}$ **HVR:** High Vaccinated Ratio

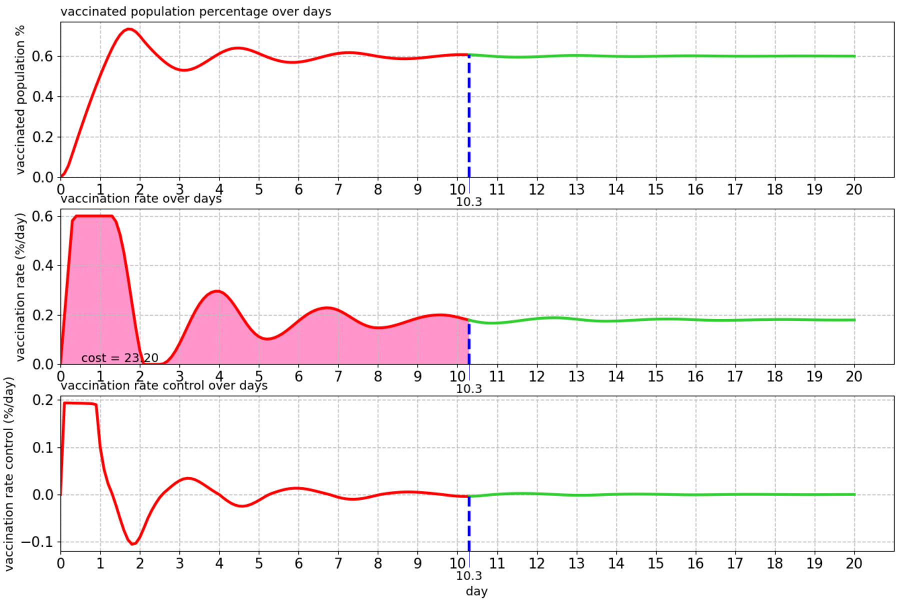
Membership Functions of Output Fuzzy Sets:

- $\mu_{(\delta)}^{NC} = \Lambda_{(\delta)}^{\{-0.25,-0.25,-0.15,-0.15\}}$ **NC:** Decrease the number of vaccinated
- $\mu_{(\delta)}^{ZC} = \Lambda_{(\delta)}^{\{-0.15,0,0,0.15\}}$ **ZC:** Do not change the number of vaccinated
- $\mu_{(\delta)}^{PC} = \Lambda_{(\delta)}^{\{0.15,0.15,0.25,0.25\}}$ **PC:** Increase the number of vaccinated



(a) Membership Functions of Input Fuzzy Sets (b) Membership Functions of Output Fuzzy Sets

Figure 5: Fuzzy Partitioning - Trial 3



From the vaccination curves in Fig. **??**, we can see that the partitioning with the specified fuzzy sets result in a;

- convergence within 10.3 days.
- vaccination cost of 23.20 units.

It should be noted that even though three steps of optimization are shown in the report, there were many trials done during the tuning of fuzzy set boundaries. With 4 parameters for each fuzzy set the hyper-parameter space has 12 dimensions. The parameter search can be more easily implemented using meta-heuristics.

Figure 6: Vaccinated Percentage Curve, Vaccination Cost and Applied Control - Trial 3

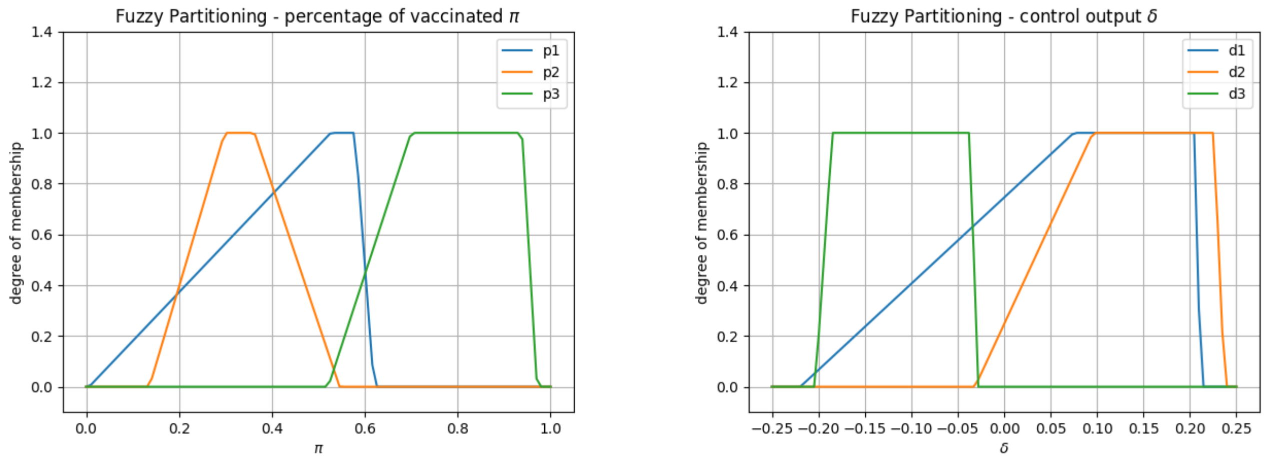
2.1.4 Fuzzy Partition with Hill Climbing Algorithm:

Hill-climbing algorithm is implemented for hyper-parameter search. Three random fuzzy sets are created both for input and output. The sets are arbitrary and therefore they do not have linguistic definitions. However after the parameter search is completed their definitions can be found by interpreting the partitions.

The input sets p_1, p_2 and p_3 are respectively matched with the output sets d_1, d_2 and d_3. Their fuzzy relations are set as;

- $p_1 \rightarrow d_1$: if (the vaccinated percentage is a member of p1) then (apply output from d1)
- $p_2 \rightarrow d_2$: if (the vaccinated percentage is a member of p2) then (apply output from d2)
- $p_3 \rightarrow d_3$: if (the vaccinated percentage is a member of p3) then (apply output from d3)

Hill-climbing algorithm is run for 3000 generations for each seed. The best partitioning and the resulting Vaccination Curves are shown in Fig. **??** and in Fig. **??**.



(a) Membership Functions of Input Fuzzy Sets

(b) Membership Functions of Output Fuzzy Sets

Figure 7: Fuzzy Partitioning - Trial HC456

The fuzzy sets in Fig. **??** can be described as follows;

Inputs Fuzzy Sets:

- p_2: Certainly Below the Satisfactory Vaccinated Ratio
- p_1: Below the Satisfactory Vaccinated Ratio
- p_3: Above the Satisfactory Vaccinated Ratio

Output Fuzzy Sets:

- d_3: Certainly Decrease the Vaccinated Ratio
- d_1: Sustain the Increase in the Vaccinated Ratio
- d_2: Increase the Vaccinated Ratio

With the found fuzzy partitions:

- Vaccinated ratio is increased certainly, if it is below satisfactory, therefore the output quickly rises to 60%.
- Also the system sustains increase in vaccinated ratio, even if the vaccinated ratio is just below 60%. This compensates the negative feedback due to increased vaccinated population and also under-shoots.
- At last if the vaccinated ratio is above 60% the system quickly decreases the ratio of vaccinated, which decreases over-shoots and speeds up the convergence.

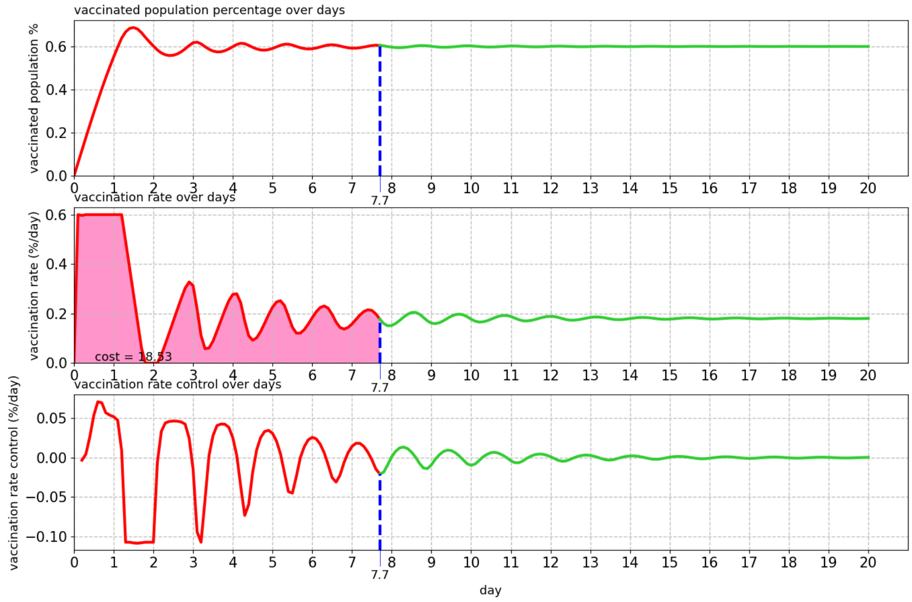


Figure 8: Vaccinated Percentage Curve, Vaccination Cost and Applied Control - Trial HC456

From the vaccination curves in Fig. **??**, we can see that the partitioning with the found fuzzy sets result in a;

- convergence within 7.7 days.

- vaccination cost of 18.53 units.

The found results are better than that of Trial 3 as it was expected, since the hyper-parameters are found by an optimization algorithm. Also the found partition is not trivial and gives a perspective of other possible considerations that can be taken into account when designing a fuzzy controller.

3 Vaccination v2

You can measure the current percentage of the vaccinated people π and the effective vaccination rate $\dot{\pi}$. In this manner, design a fuzzy controller which takes π and $\dot{\pi}$ as the input and provides δ as the output to vaccinate 60% of the people and maintain that percentage. Partition the each input into 3, output into 5 fuzzy sets. Decrease oscillations, overshoot and convergence time.

- Inputs: Vaccinated Percentage in Population $\pi \in [0, 1]$ and Effective Vaccination Rate $\dot{\pi} \in [-1, 1]$
- Controller Output: Percentage to Vaccinate $\delta \in [-0.25, +0.25]$

3.1 Solution and Implementation

The sets should be arranged such that when the ratio of vaccinated is less than 60% the output control should increase the number of vaccinated, and similarly when the ratio of vaccinated is more than 60% the output control should decrease the number of vaccinated people. Additionally if the number of vaccinated is around 60% the output should not change the number of vaccinated too much and should reduce the over and under-shoots.

The fuzzy sets and their relations are set with linguistic definitions as follows;

Inputs Fuzzy Sets:

- **LVR:** Low Vaccinated Ratio
- **SVR:** Satisfactory Vaccinated Ratio
- **HVR:** High Vaccinated Ratio
- **LDVR:** Low Effective Vaccination
- **SDVR:** Nearly Zero Effective Vaccination
- **HDVR:** High Effective Vaccination

Output Fuzzy Sets:

- **NNC:** Highly Decrease the number of vaccinated
- **NC:** Softly Decrease the number of vaccinated
- **ZC:** Do not change the number of vaccinated
- **PC:** Softly Increase the number of vaccinated
- **PPC:** Highly Increase the number of vaccinated

Relations:

		$\dot{\pi}$		
		LDVR	SDVR	HDVR
π	LVR	PPC	PC	ZC
	SVR	PC	ZC	NC
	HVR	ZC	NC	NNC

- **LVR $\wedge$ LDVR $\rightarrow$ PPC** : if (Low Vaccinated Ratio and Low Effective Vaccination) then (Highly Increase the number of vaccinated)
- **(SVR $\wedge$ LDVR) $\vee$ (SDVR $\wedge$ LVR) $\rightarrow$ PC** : if (Satisfactory Vaccinated Ratio and Low Effective Vaccination) or (Nearly Zero Effective Vaccination and Low Vaccinated Ratio) then (Softly Increase the number of vaccinated)
- **(HVR $\wedge$ LDVR) $\vee$ (SVR $\wedge$ SDVR) $\vee$ (LVR $\wedge$ HDVR) $\rightarrow$ ZC** : if ((High Vaccinated Ratio and Low Effective Vaccination) or (Satisfactory Vaccinated Ratio and Nearly Zero Effective Vaccination) or (Low Vaccinated Ratio and High Effective Vaccination)) then (Do not change the number of vaccinated)
- **(HVR $\wedge$ SDVR) $\vee$ (SVR $\wedge$ HDVR) $\rightarrow$ NC** : if ((High Vaccinated Ratio and Nearly Zero Effective Vaccination) or (Satisfactory Vaccinated Ratio and High Effective Vaccination)) then (Softly Decrease the number of vaccinated)
- **HVR $\wedge$ HDVR $\rightarrow$ NNC** : if (High Vaccinated Ratio and High Effective Vaccination) then (Highly Decrease the number of vaccinated)

3.1.1 Fuzzy Partition 1:

For the starting trial, fuzzy sets are arranged with boundary of their cores at the most representative values of vaccinated ratios, that are 0, 0.6 and 1 for Low, Satisfactory and High Vaccinated Ratios; -1, 0 and 1 for Low, Nearly Zero and High Effective Vaccination; and -0.25, 0, 0.25 for Decreasing, Keeping Constant and Increasing the number of vaccinated people. The supports of the sets reach up to the cores of adjacent sets. Membership functions for Input and Output Fuzzy set are shown in Fig. **??** and given below,

Membership Functions of Inputs Fuzzy Sets:

- $\mu_{(\pi)}^{LVR} = \Lambda_{(\pi)}^{\{0,0,0,0.6\}}$

  LVR: Low Vaccinated Ratio
- $\mu_{(\pi)}^{SVR} = \Lambda_{(\pi)}^{\{0,0.6,0.6,1\}}$

  SVR: Satisfactory Vaccinated Ratio
- $\mu_{(\pi)}^{HVR} = \Lambda_{(\pi)}^{\{0.6,1,1,1\}}$

  HVR: High Vaccinated Ratio
- $\mu_{(\dot{\pi})}^{LDVR} = \Lambda_{(\dot{\pi})}^{\{1,-1,-1,0\}}$

  LDVR: Low Effective Vaccination
- $\mu_{(\dot{\pi})}^{SDVR} = \Lambda_{(\dot{\pi})}^{\{-1,0,0,1\}}$

  SDVR: Nearly Zero Effective Vaccination
- $\mu_{(\dot{\pi})}^{HDVR} = \Lambda_{(\dot{\pi})}^{\{0,1,1,1\}}$

  HDVR: High Effective Vaccination

Membership Functions of Output Fuzzy Sets:

- $\mu_{(\delta)}^{NNC} = \Lambda_{(\delta)}^{\{-0.25,-0.25,-0.25,-0.125\}}$

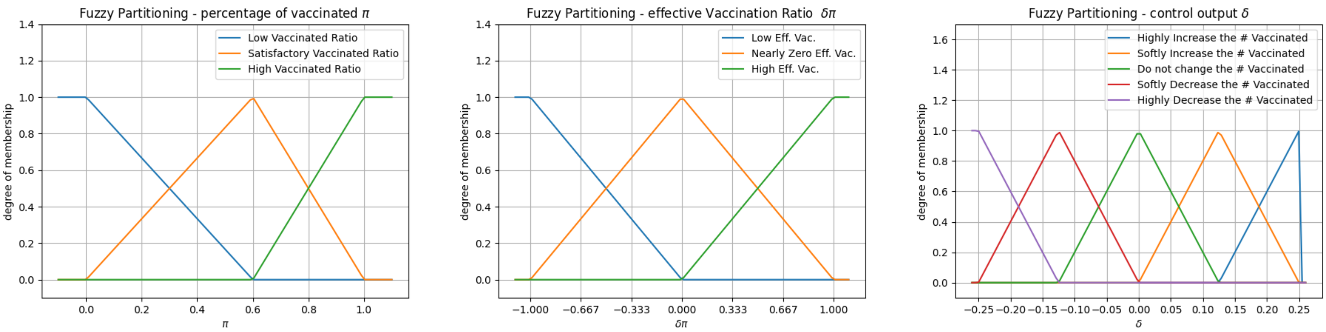
  NNC: Highly Decrease the number of vaccinated
- $\mu_{(\delta)}^{NC} = \Lambda_{(\delta)}^{\{-0.25,-0.125,-0.125,0\}}$

  NC: Softly Decrease the number of vaccinated
- $\mu_{(\delta)}^{ZC} = \Lambda_{(\delta)}^{\{-0.125,0,0,0.125\}}$

  ZC: Do not change the number of vaccinated
- $\mu_{(\delta)}^{PC} = \Lambda_{(\delta)}^{\{0,0.125,0.125,0.25\}}$

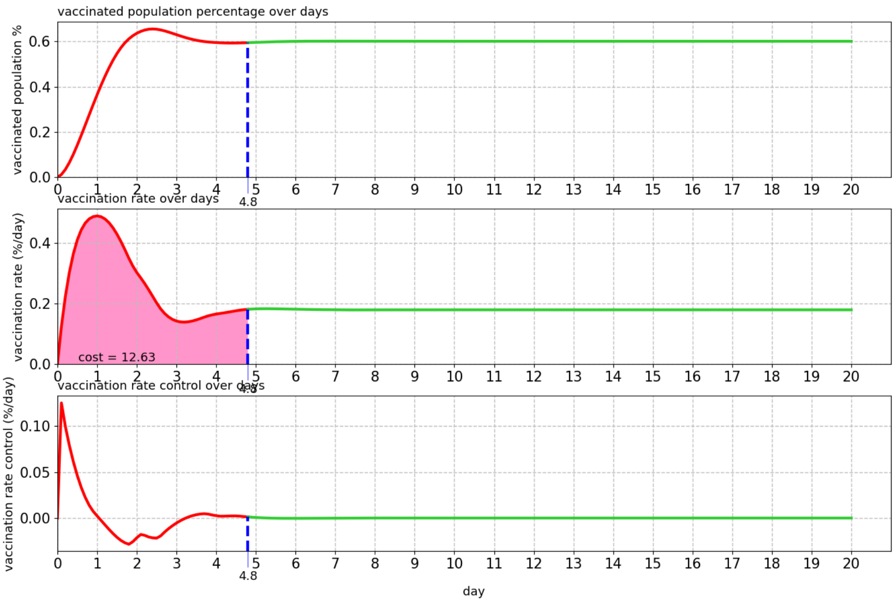
  PC: Softly Increase the number of vaccinated
- $\mu_{(\delta)}^{PPC} = \Lambda_{(\delta)}^{\{0.125,0.25,0.25,0.25\}}$

  PPC: Highly Increase the number of vaccinated



(a) Membership Functions of π (b) Membership Functions of $\dot{\pi}$ (c) Membership Functions of δ

Figure 9: Fuzzy Partitioning - Trial 1



From the vaccination curves in Fig. **??**, we can see that the partitioning with the specified fuzzy sets result in a;

- convergence within 4.8 days.
- vaccination cost of 12.63 units.

With the new controller both the convergence time and the vaccination cost decreased further. With the extra knowledge on the derivative of vaccination rate, high variations in the vaccination derivative are punished by the intermediate partitions of the output. Therefore, overshoots have decreased and system response became more damped.

Figure 10: Vaccinated Percentage Curve, Vaccination Cost and Applied Control - Trial 1

3.1.2 Fuzzy Partition 2:

On the previous trial it was observed that convergence is faster when there are less over-shoots . Therefore in this trial the system response was tried to be set to over-damped without considering the convergence of vaccinated population ratio. To do so supports of the sets LVR, SVR and HVR are made more selective by decreasing their widths.

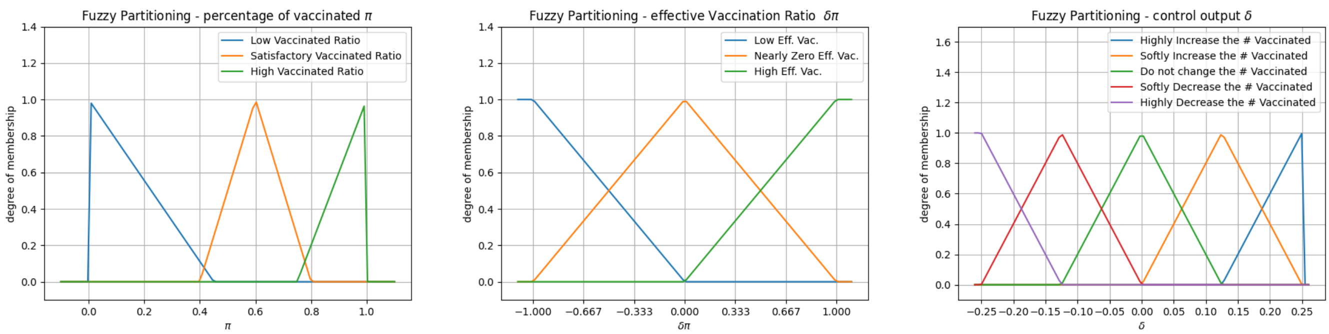
Membership functions for Input and Output Fuzzy set are shown in Fig. **??** and given below,

Membership Functions of Inputs Fuzzy Sets:

- $\mu_{(\pi)}^{LVR} = \Lambda_{(\pi)}^{\{0,0,0,0.45\}}$
  LVR: Low Vaccinated Ratio
- $\mu_{(\pi)}^{SVR} = \Lambda_{(\pi)}^{\{0.4,0.6,0.6,0.8\}}$
  SVR: Satisfactory Vaccinated Ratio
- $\mu_{(\pi)}^{HVR} = \Lambda_{(\pi)}^{\{0.75,1,1,1\}}$
  HVR: High Vaccinated Ratio
- $\mu_{(\dot{\pi})}^{LDVR} = \Lambda_{(\dot{\pi})}^{\{1,-1,-1,0\}}$
  LDVR: Low Effective Vaccination
- $\mu_{(\dot{\pi})}^{SDVR} = \Lambda_{(\dot{\pi})}^{\{-1,0,0,1\}}$
  SDVR: Nearly Zero Effective Vaccination
- $\mu_{(\dot{\pi})}^{HDVR} = \Lambda_{(\dot{\pi})}^{\{0,1,1,1\}}$
  HDVR: High Effective Vaccination

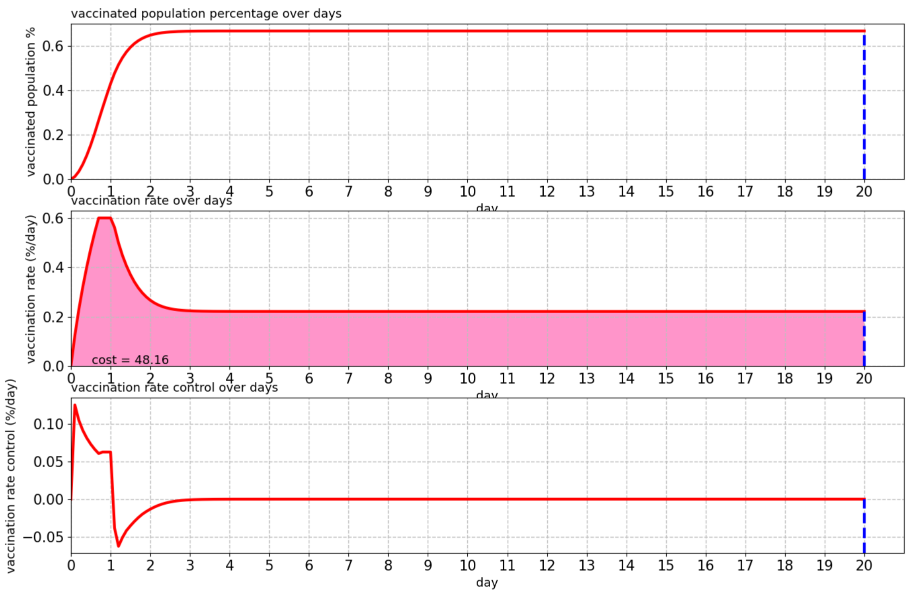
Membership Functions of Output Fuzzy Sets:

- $\mu_{(\delta)}^{NNC} = \Lambda_{(\delta)}^{\{-0.25,-0.25,-0.25,-0.125\}}$
  NNC: Highly Decrease the number of vaccinated
- $\mu_{(\delta)}^{NC} = \Lambda_{(\delta)}^{\{-0.25,-0.125,-0.125,0\}}$
  NC: Softly Decrease the number of vaccinated
- $\mu_{(\delta)}^{ZC} = \Lambda_{(\delta)}^{\{-0.125,0,0,0.125\}}$
  ZC: Do not change the number of vaccinated
- $\mu_{(\delta)}^{PC} = \Lambda_{(\delta)}^{\{0,0.125,0.125,0.25\}}$
  PC: Softly Increase the number of vaccinated
- $\mu_{(\delta)}^{PPC} = \Lambda_{(\delta)}^{\{0.125,0.25,0.25,0.25\}}$
  PPC: Highly Increase the number of vaccinated



(a) Membership Functions of π (b) Membership Functions of $\dot{\pi}$ (c) Membership Functions of δ

Figure 11: Fuzzy Partitioning - Trial 2



If the vaccinated percentage is neither too large nor too low (which corresponds to the magnitude of over-shoots), the selective fuzzy sets on vaccination ratio give the control of the output to the fuzzy sets of the effective vaccinated ratio. Therefore over-shoots are reduced with the punishment of the error term. From the vaccination curves in Fig. **??**, we can see that there are not any over-shoots but there is a positive steady state error.

Figure 12: Vaccinated Percentage Curve, Vaccination Cost and Applied Control - Trial 2

3.1.3 Fuzzy Partition 3:

The steady state error can be set to zero by changing the position of the core of EVR. This is equivalent to shifting the equilibrium point by a desired amount.

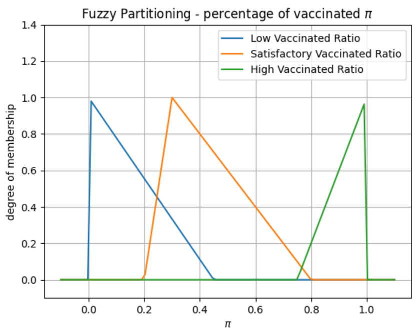
Membership functions for Input and Output Fuzzy set are shown in Fig. **??** and given below,

Membership Functions of Inputs Fuzzy Sets:

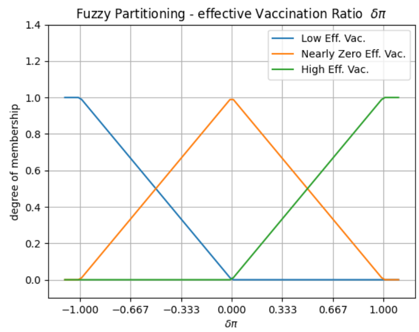
- $\mu_{(\pi)}^{LVR} = \Lambda_{(\pi)}^{\{0,0,0,0.45\}}$
  LVR: Low Vaccinated Ratio
- $\mu_{(\pi)}^{SVR} = \Lambda_{(\pi)}^{\{0.4,0.3,0.3,0.8\}}$
  SVR: Satisfactory Vaccinated Ratio
- $\mu_{(\pi)}^{HVR} = \Lambda_{(\pi)}^{\{0.75,1,1,1\}}$
  HVR: High Vaccinated Ratio
- $\mu_{(\dot{\pi})}^{LDVR} = \Lambda_{(\dot{\pi})}^{\{1,-1,-1,0\}}$
  LDVR: Low Effective Vaccination
- $\mu_{(\dot{\pi})}^{SDVR} = \Lambda_{(\dot{\pi})}^{\{-1,0,0,1\}}$
  SDVR: Nearly Zero Effective Vaccination
- $\mu_{(\dot{\pi})}^{HDVR} = \Lambda_{(\dot{\pi})}^{\{0,1,1,1\}}$
  HDVR: High Effective Vaccination

Membership Functions of Output Fuzzy Sets:

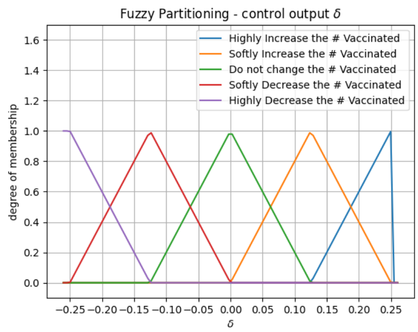
- $\mu_{(\delta)}^{NNC} = \Lambda_{(\delta)}^{\{-0.25,-0.25,-0.25,-0.125\}}$
  NNC: Highly Decrease the number of vaccinated
- $\mu_{(\delta)}^{NC} = \Lambda_{(\delta)}^{\{-0.25,-0.125,-0.125,0\}}$
  NC: Softly Decrease the number of vaccinated
- $\mu_{(\delta)}^{ZC} = \Lambda_{(\delta)}^{\{-0.125,0,0,0.125\}}$
  ZC: Do not change the number of vaccinated
- $\mu_{(\delta)}^{PC} = \Lambda_{(\delta)}^{\{0,0.125,0.125,0.25\}}$
  PC: Softly Increase the number of vaccinated
- $\mu_{(\delta)}^{PPC} = \Lambda_{(\delta)}^{\{0.125,0.25,0.25,0.25\}}$
  PPC: Highly Increase the number of vaccinated



(a) Membership Functions of π

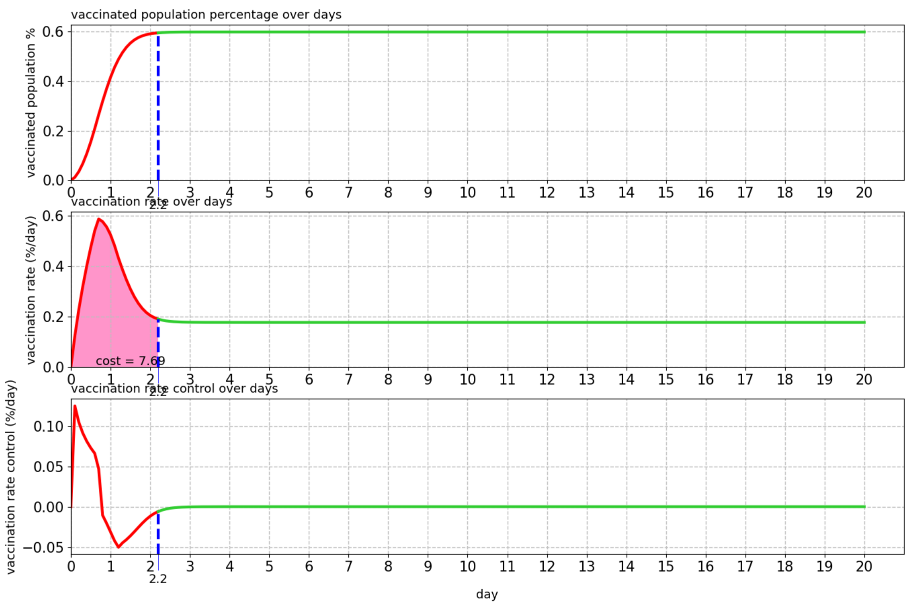


(b) Membership Functions of $\dot{\pi}$



(c) Membership Functions of δ

Figure 13: Fuzzy Partitioning - Trial 3



From the vaccination curves in Fig. **??**, we can see that the partitioning with the specified fuzzy sets result in a;

- convergence within 2.2 days.
- vaccination cost of 7.69 units.

Now that there are not any overshoots or non-zero steady state errors, the controller outperforms all other configurations.

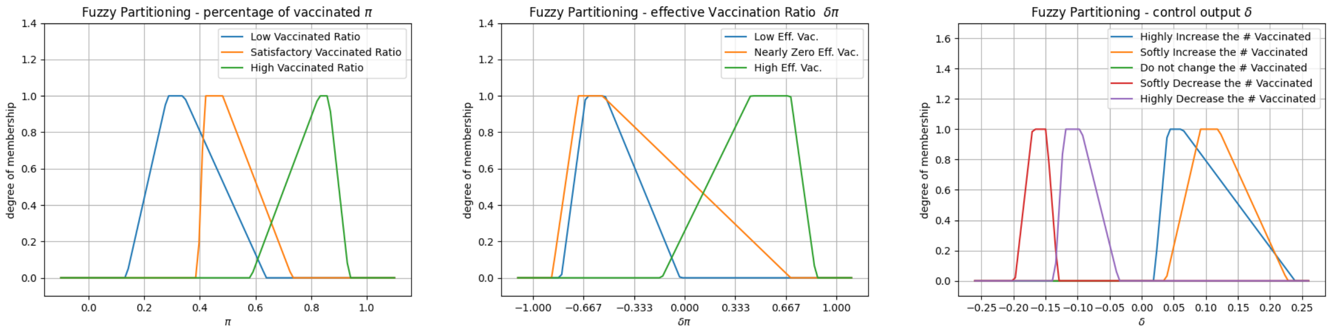
It should be noted that with 4 parameters for each fuzzy set the hyper-parameter space has 44 dimensions. The parameter search can be done better by using meta-heuristics.

Figure 14: Vaccinated Percentage Curve, Vaccination Cost and Applied Control - Trial 3

3.1.4 Fuzzy Partition with Hill Climbing Algorithm:

Hill-climbing algorithm is implemented for hyper-parameter search. Instead of starting with random fuzzy sets and relations, the previously defined relation is used. Also the fuzzy sets are created and mutated randomly around their most representative crisp values (which are stated in Fuzzy Partition 1).

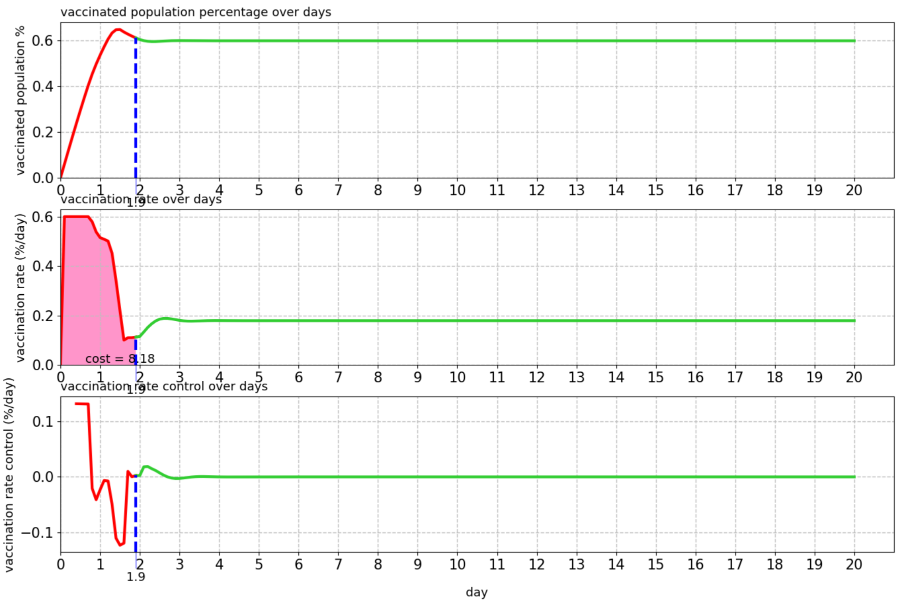
Hill-climbing algorithm is run for 1000 generations for each seed. The best partitioning and the resulting Vaccination Curves are shown in Fig. **??** and Fig. **??**.



(a) Membership Functions of π (b) Membership Functions of $\dot{\pi}$ (c) Membership Functions of δ

Figure 15: Fuzzy Partitioning - Trial HC3

Linguistic definitions are preserved for π, with a skewed EVR similar to that of Fuzzy Partition 3. Also, for $\dot{\pi}$ Fuzzy Sets of LDVR and EDVR are merged together. It is interesting to see that, for δ, the fuzzy sets NNC and PPC changed definitions with NC and PC respectively. Presumably after the solution shows over-damped response the improvements start to require faster initial response. Since the memberships values of NC and PC are more probable to be greater than the membership values NNC and PPC, they switched position in favor of more drastic outputs. Also the fuzzy set ZC seems to be eliminated. Merging of fuzzy sets LDVR and EDVR along with the disappearance of ZC show that the problem can be solved with fewer number of fuzzy sets.



From the vaccination curves in Fig. **??**, we can see that the partitioning with the specified fuzzy sets result in a;

- convergence within 1.9 days.
- vaccination cost of 8.18 units.

More drastic control effort has brought a small over-shoot and slightly increased the cost of vaccination. However convergence time has decreased.

Figure 16: Vaccinated Percentage Curve, Vaccination Cost and Applied Control - Trial HC3

4 Conclusion

The controllers for v1 and v2 correspond to proportional (P) and proportional derivative controllers (PD) in classical control theory. With the latter, rapid fluctuations of the system output can be decreased.

In accordance with the classical control, introduction of the effective vaccination ratio measurements along with the dense partitioning of the output space has decreased both the required time for convergence and the vaccination cost. Therefore, v2-controllers were more successful. Furthermore, for the v2-controllers a trade-off between control effort (cost) and convergence time is observed.

It is also seen that the trivially set linguistic definitions may not always give the best solution. From the hyper-parameter search with hill climbing algorithm, non-trivial partitions are observed, analyzed and their effects are discussed.