N1

$\bar{c} = \bar{a} \times \bar{b}$ $\qquad \alpha \bar{a} \times \alpha b = \alpha \bar{C} = \bar{C}'$

$\bar{c}' = \bar{a}' \times \bar{b}'$ $\qquad \alpha^{-1}\bar{C}' = \underbrace{\alpha^{-1}\alpha}_{1}\bar{C} \Rightarrow \alpha^{-1}\bar{C}' = C$

$\bar{a}' = \alpha \bar{a}$

$\bar{b}' = \alpha \bar{b}$ $\qquad \alpha^{-1} = \alpha^{T}$ т.к. ортогональны

$\Rightarrow C = \alpha^{T} C'$ $\Rightarrow$ вектор

N2

1) $\{C\}_i \equiv c_i = e_{ijk} a_j b_k$, $\{C\}_k \equiv C_k = e_{ijk} a_i b_j$

$e_{ijk} a_j b_k = - e_{jik} a_j b_k = e_{jik} a_i b_k = e_{kij} a_i b_j$

$e_{kij} = -e_{ikj} = e_{ijk} \Rightarrow e_{ijk} a_j b_k = e_{kij} a_i b_j$

2) $\{\nabla f\}_i \equiv \frac{\partial f}{\partial x_i}$ $\qquad \nabla \times \nabla f = 0$

$\nabla \times \nabla f = e_{ijk} \frac{\partial f}{\partial x_i} = 0$, т.к. e_{ijk} - антисим. тензор,

∇f - вектор

3) $\{z\}_i \equiv x_i$ $\qquad \nabla \times \vec{z} = 0$?

$\nabla \times \vec{z} = e_{ijk} x_i = \vec{c}$

$c_1 = e_{123} x_1 + e_{132} x_1 = e_{123} x_1 - e_{123} x_1 = 0$

$c_2 \ldots$ аналогично

4) ∇F^2 и ∇F^α -?

$$\nabla F^2 = \frac{\partial F^2}{\partial x_i}\delta_{ij} = 2\frac{\partial F}{\partial x_i} F\,\delta_{ij}$$

$$\nabla F^\alpha = \frac{\partial F^\alpha}{\partial x_i}\delta_{ij} = \alpha F^{\alpha-1}\frac{\partial F}{\partial x_i}\delta_{ij}$$

5) $r^2 = r^2 = x_i x_i$ $\quad \nabla r^2 \quad\quad \alpha = \frac{1}{2}$

$$\{\nabla r^2\}_i = \frac{\partial x_i}{\partial x_i} x_i + x_i\frac{\partial x_i}{\partial x_i} = 2x_i$$

$$\{\nabla r^\alpha\}_i = \frac{1}{2} x_i^{-\frac{1}{2}}\frac{\partial x_i}{\partial x_i} = \frac{1}{2\sqrt{x_i}}$$

6)

№3

1) $e_{ijl}e_{ijm} = 2\delta_{jm}$

т.к. при $i=j$ или $j=l$ или $i=l$ (аналогично для ijm) $e_{ijl}=0$,

единственно при $j=m$:

$$2(\delta_{11}+\delta_{22}+\delta_{33}) = e_{123}e_{123}+e_{213}e_{213}+e_{132}e_{132}+e_{321}e_{321}+e_{312}e_{312}+e_{231}e_{231}$$

$$6 = 1\cdot1+(-1)(-1)+(-1)(-1)+1\cdot1+(-1)(-1)+1\cdot1 = 6$$

при $j \neq m$

$\delta_{jm} = 0$, $e_{ijm} = 0 \Rightarrow e_{ijl}e_{ijm} = 0$

2) $\nabla \times (a \times b) = a(\nabla \cdot b) - (\nabla \cdot a)b + (b \cdot \nabla)a - (a \cdot \nabla)b$

$$\{\nabla \times (a \times b)\}_i = e_{ijk} \frac{\partial (a \times b)_k}{\partial x_j} = e_{ijk} e_{klm} \frac{\partial (a_l b_m)}{\partial x_j} =$$

$$= \left| e_{ijk} e_{klm} = e_{ijk} e_{lmk} = \delta_{il}\delta_{jm} - \delta_{im}\delta_{jl} \right| = (\delta_{il}\delta_{jm} - \delta_{im}\delta_{jl}) \frac{\partial a_l b_m}{\partial x_j} =$$

$$= \frac{\partial a_i b_j}{\partial x_j} - \frac{\partial a_j b_i}{\partial x_j} = a_i \frac{\partial b_j}{\partial x_j} + b_j \frac{\partial a_i}{\partial x_j} - a_j \frac{\partial b_i}{\partial x_j} - b_i \frac{\partial a_j}{\partial x_j} =$$

$$= a_i(\nabla \cdot b) - b_i(\nabla \cdot a) + b_j \frac{\partial a_i}{\partial x_j} - a_j \frac{\partial b_i}{\partial x_j} = \cancel{a(\nabla \cdot b) - b(\nabla \cdot a) +}$$

$$\left| \frac{\partial}{\partial \bar{a}} = (\bar{a} \cdot \nabla), \ \frac{\partial}{\partial b} = (b \cdot \nabla) \right| = a(\nabla \cdot b) - (\nabla \cdot a)b + (b \cdot \nabla)a - (a \cdot \nabla)b$$

3) $(a \cdot \nabla)r \equiv a$

$$\{(a \cdot \nabla)r\}_i = a_i \cdot \frac{\partial}{\partial x_i} x_i = a_i \ ?$$

4) $\nabla \cdot (a \times \nabla)r = \nabla \cdot a$