1 The Initial Distribution

1.1 The variables

The parameters used in the initial distribution of OPAL-t are:
sigmax the maximum half-width of the beam envelope in the x plane. [m]
sigmay the maximum half-width of the beam envelope in the y plane. [m]
sigmat one-half the longitudinal extent of the bunch of particles. [m]
pt the average momentum of the bunch. [eV]
sigmapx the maximum half-angular divergence of the beam envelope in the x plane. [eV]
sigmapy the maximum half-angular divergence of the beam envelope in the y plane. [eV]
sigmapt the half-width of the momentum interval. [eV]
corrx the correlation coefficient between x and px.
corry the correlation coefficient between y and py.
t the longitudinal position of the beam center. [m]
corrt the correlation coefficient between t and pt.
r61 the correlation coefficient between x and pt.
r62 the correlation coefficient between px and pt.
r51 the correlation coefficient between x and t.
r52 the correlation coefficient between px and t.

To generate gaussian initial distribution with dispersion, first we generate the uncorrelated gaussian inputs matrix $R = (R1, ..., R_n)$. The mean of R_i is 0 and the standard deviation squared is 1. Then we correlate R.

The correlation coefficient matrix in x and t phase space is

$$\sigma = \begin{array}{c} \begin{array}{cccc} x & px & t & pt \end{array} \\ \begin{bmatrix} 1 & corrx & r51 & r61 \\ corrx & 1 & r52 & r62 \\ r51 & r52 & 1 & corrt \\ r61 & r62 & corrt & 1 \end{bmatrix} \end{array}$$

The Cholesky decomposition of the symmetric positive-definite matrix σ is $\sigma = C^T C$, then the correlated distribution is $C^T R$.

Note: This correlation is only done for gaussian distribution now.

1.2 Unit Conversion

Convert the unit of momentum from $\beta\gamma$ to $mrad$.

$$(\beta\gamma)_z = \frac{P}{m_0 c} = \frac{Pc}{m_0 c^2}. \tag{1}$$

$$P_x[mrad] = 1000 \frac{P_x[\beta\gamma]}{(\beta\gamma)_z} \tag{2}$$

Convert the unit from eV to $\beta\gamma$.

$$\sqrt{(\frac{T}{m_0c^2}+1)^2-1}=\beta\gamma. \tag{3}$$

1.3 Example

If the initial correlation coefficient matrix

$$\sigma=\begin{bmatrix} 1 & 0.756 & 0.023 & 0.496 \\ 0.756 & 1 & 0.385 & -0.042 \\ 0.023 & 0.385 & 1 & -0.834 \\ 0.496 & -0.042 & -0.834 & 1 \end{bmatrix}$$

The corresponding input file is

Distzs3:DISTRIBUTION, DISTRIBUTION=gauss,
sigmax= 4.796e-03, sigmapx=231.0585, corrx=0.756,
sigmay= 23.821e-03, sigmapy=1.6592e+03, corry=-0.999,
t=0.466e-02, sigmat= 0.466e-02, pt=72e6,sigmapt=74.7, corrt=-0.834,
r61=0.496,r62=-0.042,r51=0.023,r52=0.385;