1 The Physics Models Used in the Collimation Routine

1.1 The Energy Loss

The energy loss is simulated using the Bethe-Bloch equation.

$$-dE/dx = \frac{Kz^2Z}{A\beta^2}[\frac{1}{2}\ln\frac{2m_ec^2\beta^2\gamma^2Tmax}{I^2} - \beta^2], \tag{1}$$

where Z is the aomic number of absorber, A is the atomic mass of absorber, m_e is the electron mass, z is the charge number of the incident particle, $K = 4\pi N_A r_e^2 m_e c^2$, r_e is the classical electron radius, N_A is the Avogadro's number, I is the mean excitation energy. β and γ are kinematic variables. T_{max} is the maximum kinetic energy which can be imparted to a free electron in a single collision.

$$T_{max} = \frac{2m_ec^2\beta^2\gamma^2}{1 + 2\gamma m_e/M + (m_e/M)^2}, \tag{2}$$

where M is the incident particle mass.

The stopping power is compared with PSTAR program of NIST in Fig. 1.

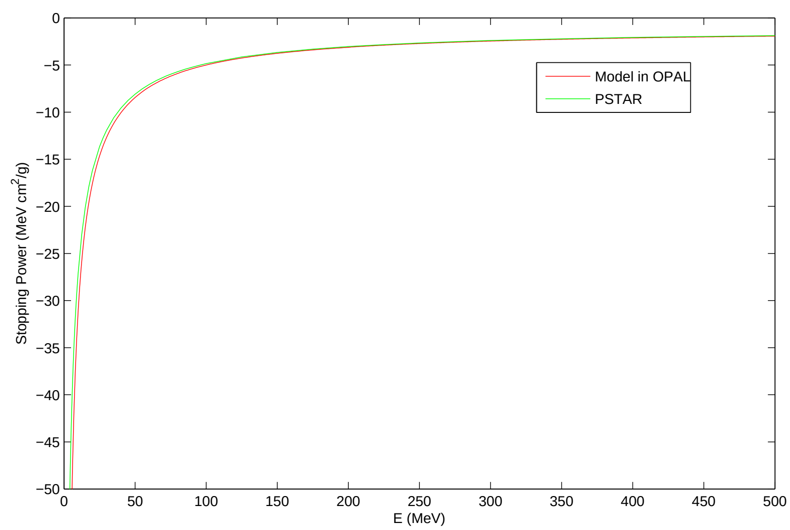

Figure 1: The comparison of stopping power with PSTAR.

Energy straggling: For relatively thick absorbers such that the number of collisions is large, the energy loss distribution is shown to be Gaussian in form.

For nonrelativistic heavy particles the spread σ_0 of the Gaussian distribution is calculated by:

$$\sigma_0^2 = 4\pi N_A r_e^2 (m_e c^2)^2 \rho \frac{Z}{A} \Delta s, \tag{3}$$

where ρ is the density, Δs is the thickness.

1.2 The Coulomb Scattering

The Coulomb scattering is treated as two independent events: the multiple Coulomb scattering and the large angle Rutherford scattering.
Using the distribution given in Classical Electrodynamics, by J. D. Jackson, the multiple- and single-scattering distributions can be written:

$$P_M(\alpha) d\alpha = \frac{1}{\sqrt{\pi}} e^{-\alpha^2} d\alpha, \tag{4}$$

$$P_S(\alpha) d\alpha = \frac{1}{8 \ln(204 Z^{-1/3})} \frac{d\alpha}{\alpha^3}, \tag{5}$$

where $\alpha = \frac{\theta}{<\Theta^2>^{1/2}} = \frac{\theta}{\sqrt{2}\theta_0}$.

the transition point is $\theta = 2.5\sqrt{2}\theta_0 \approx 3.5\theta_0$,

$$\theta_0 = \frac{13.6 MeV}{\beta c p} z \sqrt{\Delta s / X_0} [1 + 0.038 \ln(\Delta s / X_0)], \tag{6}$$

where p is the momentum, Δs is the stepsize, and X_0 is the radiation length.

1.2.1 Multiple Coulomb Scattering

Generate two independent Gaussian random variables with mean zero and variance one: z_1 and z_2. If $z_2\theta_0 > 3.5\theta_0$, start over. Otherwise,

$$x = x + \Delta s p_x + z_1 \Delta s \theta_0 / \sqrt{12} + z_2 \Delta s \theta_0 / 2, \tag{7}$$

$$p_x = p_x + z_2 \theta_0. \tag{8}$$

Generate two independent Gaussian random variables with mean zero and variance one: z_3 and z_4. If $z_4\theta_0 > 3.5\theta_0$, start over. Otherwise,

$$y = y + \Delta s p_y + z_3 \Delta s \theta_0 / \sqrt{12} + z_4 \Delta s \theta_0 / 2, \tag{9}$$

$$p_y = p_y + z_4 \theta_0. \tag{10}$$

1.2.2 Large Angle Rutherford Scattering

Generate a random number ξ_1, *if* $\xi_1 < \frac{\int_{2.5}^{\infty} P_S(\alpha)d\alpha}{\int_0^{2.5} P_M(\alpha)d\alpha + \int_{2.5}^{\infty} P_S(\alpha)d\alpha} = 0.0047$, sampling the large angle Rutherford scattering.
The cumulative distribution function of the large angle Rutherford scattering is

$$F(\alpha) = \frac{\int_{2.5}^{\alpha} P_S(\alpha)d\alpha}{\int_{2.5}^{\infty} P_S(\alpha)d\alpha} = \xi, \tag{11}$$

where ξ is a random variable. So

$$\alpha = \pm 2.5\sqrt{\frac{1}{1-\xi}} = \pm 2.5\sqrt{\frac{1}{\xi}}. \tag{12}$$

Generate a random variable P_3,
if $P_3 > 0.5$

$$\theta_{Ru} = 2.5\sqrt{\frac{1}{\xi}}\sqrt{2}\theta_0, \tag{13}$$

else

$$\theta_{Ru} = -2.5\sqrt{\frac{1}{\xi}}\sqrt{2}\theta_0. \tag{14}$$

The angle distribution after Coulomb scattering is shown in Fig. 2. The line is from Jackson's formula, and the points are simulations with Matlab. For a thickness of $\Delta s = 1e-4\ m$, $\theta = 0.5349\alpha$ (in degree).

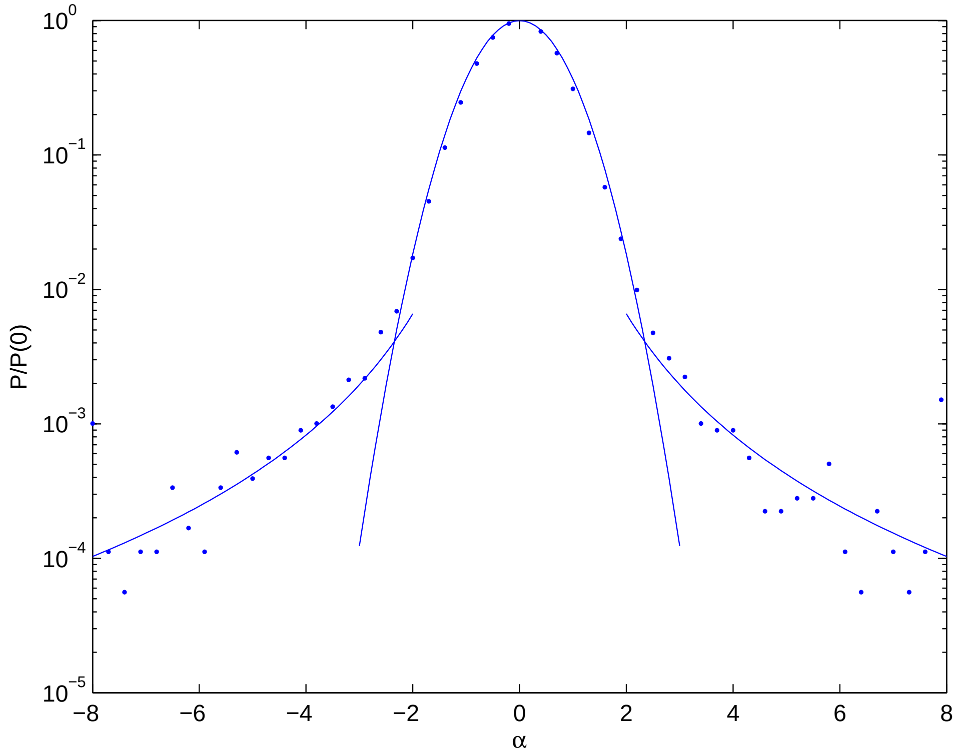

Figure 2: The comparison of Coulomb scattering with Jackson's book.

2 The diagram of CollimatorPhysics in OPAL

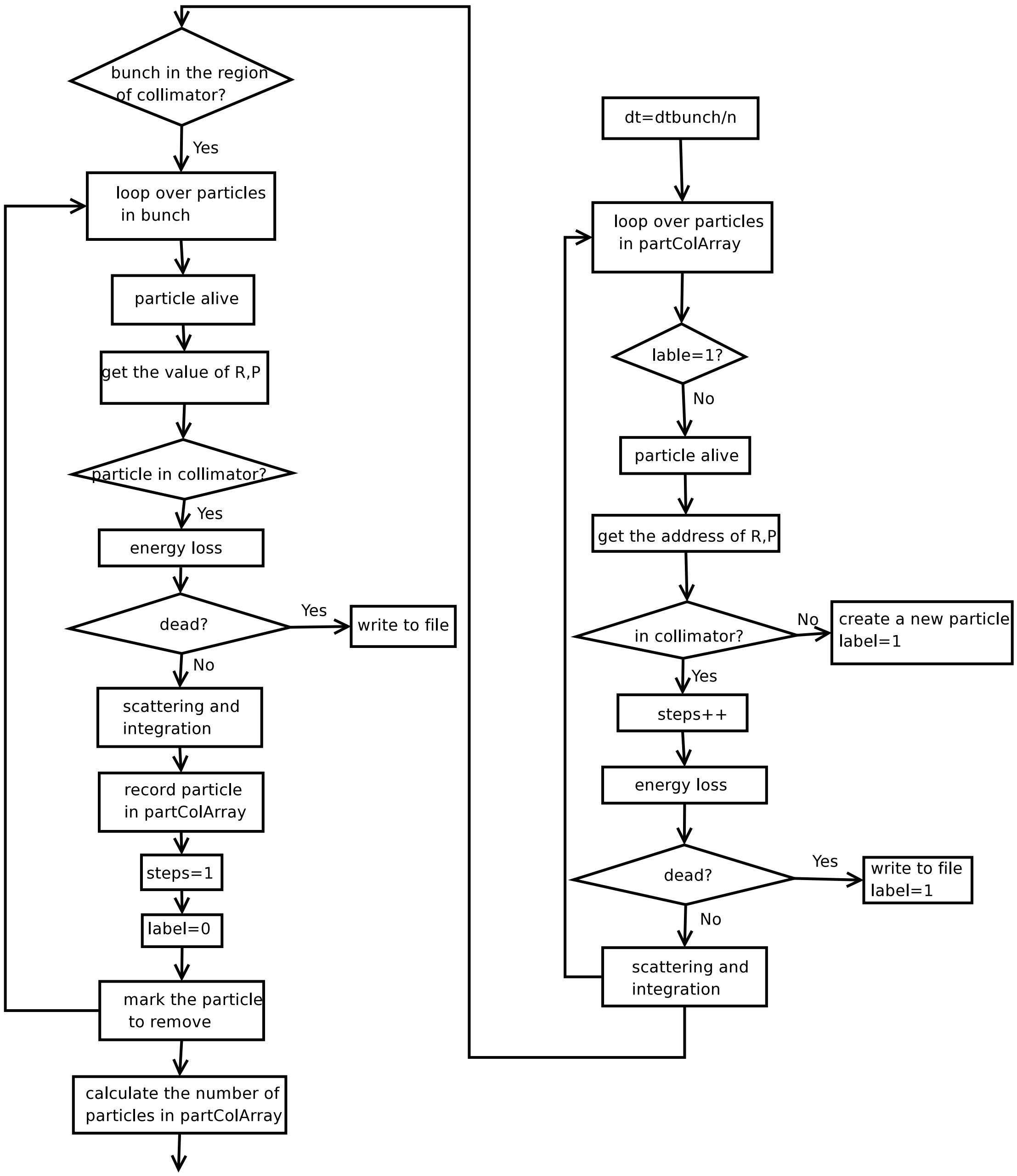

Figure 3: The diagram of CollimatorPhysics in OPAL.

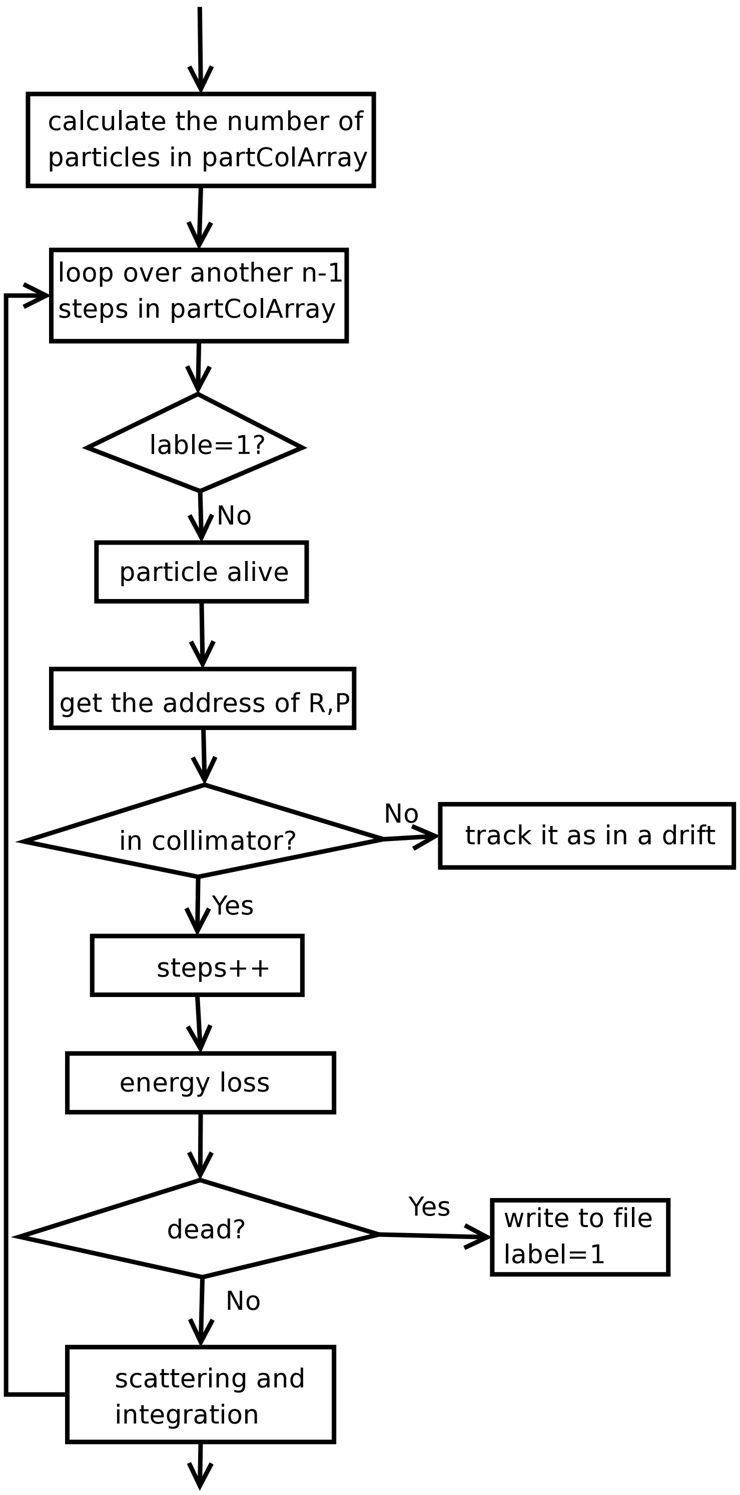

Figure 4: The diagram of CollimatorPhysics in OPAL(Continued).

3 The substep

Small step is needed in the routine of CollimatorPhysics.

If a large step is given in the main input file, in the file CollimatorPhysics.cpp, it is divided by a integer number n to make the stepsize using for the calculation of collimator physics less than 1.01e-12 s. As shown by Fig. 3 and Fig. 4 in the previous section, first we track one step for the particles already in the collimator and the newcomers, then another (n-1) steps to make sure the particles in the collimator experiencr the same time as the ones in the main bunch.

Now, if the particle leave the collimator during the (n-1) steps, we track it as in a drift and put it back to the main bunch when finishing (n-1) steps.

4 the example of Input file

KX1IPHYS: SurfacePhysics, TYPE="Collimator",MATERIAL="Cu";
KX2IPHYS: SurfacePhysics, TYPE="Collimator",MATERIAL="Graphite";
KX0I: ECollimator, L=0.09, ELEMEDGE=0.01, APERTURE={0.003,0.003},OUTFN="KX0I.h5", SURFACEPHYSICS='KX1IPHYS';
FX5: Slit, L=0.09, ELEMEDGE=0.01, APERTURE={0.005,0.003}, SURFACEPHYSICS='KX2IPHYS';
FX16: Slit, L=0.09, ELEMEDGE=0.01, APERTURE={-0.005,-0.003}, SURFACEPHYSICS='KX2IPHYS';

FX5 is a slit in x direction, the APERTURE is **POSITIVE**, the first value in APERTURE is the left part, the second value is the right part.

FX16 is a slit in y direction, the APERTURE is **NEGTIVE**, the first value in APERTURE is the down part, the second value is the up part.

5 A Simple Test

A cold Gaussian beam with $\sigma_x = \sigma_y = 5$ mm. The position of the collimator is from 0.01 m to 0.1 m, the half aperture in y direction is 3 mm. Fig. 5 shows the trajectory of particles which are either absorbed or deflected by a copper slit. As a benchmark of the collimator model in OPAL, Fig. 6 shows the energy spectrum and angle deviation at z=0.1 m after an elliptic collimator.

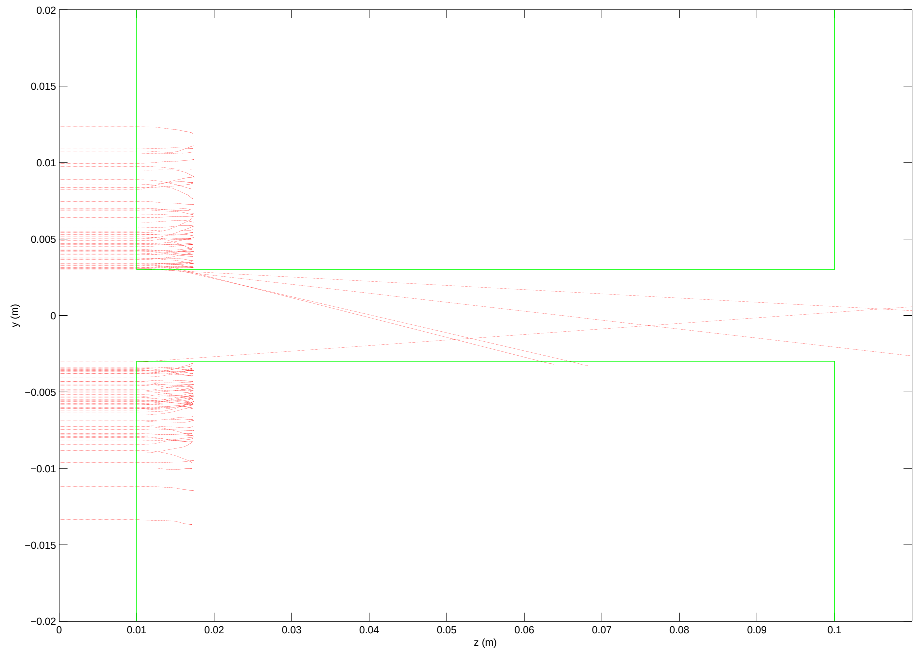

Figure 5: The passage of protons through the collimator.

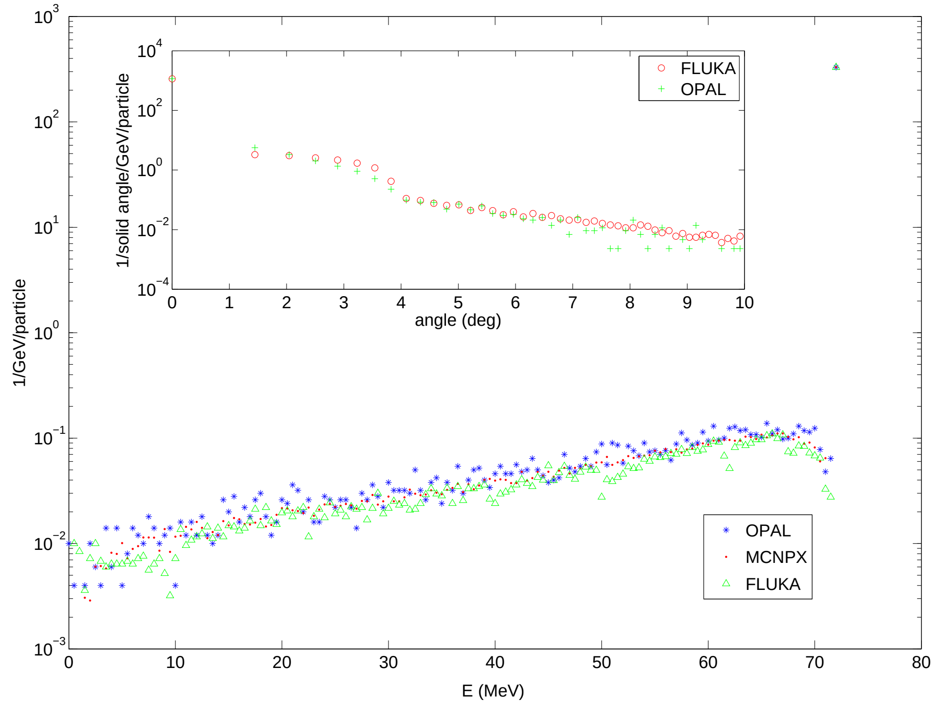

Figure 6: The energy spectrum and scattering angle at z=0.1 m