Hybrid resonant modes of two-sided multipactor and transition to the polyphase regime

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A plane parallel model of multipactor is studied in detail using both an analytical approach and numerical simulations. The analytical analysis is carried out within a widely exploited approximation, which assumes a fixed value for the initial velocity of secondary electrons. It is shown that the commonly accepted picture of the multipactor zones is not quite complete and should be modified by taking into account the existence of hybrid resonance modes and the important consequences of a secondary emission yield that significantly exceeds unity. Numerical simulations demonstrate that the chart of the multipactor zones is also very sensitive to a spread of the initial velocity of the electrons. In particular, the full effect of initial electron velocities cannot accurately be modeled by using a single fixed value only.
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I. INTRODUCTION

The phenomenon of electron multipactor was first discovered by Farnsworth[1] in 1934. It manifests itself as an avalanche-like increase of free electrons and is caused by secondary electron emission when electrons driven by rf fields hit the walls in vacuum devices. Since the multipactor disturbs the operation of many modern systems such as, e.g., high power microwave generators,[2] rf accelerators,[3–5] and space-borne communication,[6–8] it has been studied intensively for more than 50 years, see, e.g., the reviews in Refs. 9–12. However there still exist some fundamental questions, which have not yet been answered. These questions are mainly related to inadequate agreement between theory and experiment. On one hand, the general theoretical predictions are in good agreement with experimental results. On the other hand, the results of more detailed theoretical considerations are not confirmed by laboratory and/or numerical experiments. Moreover, so far there is no single, generally accepted opinion concerning the structure of multipactor resonant zones. Whereas some authors, following Hatch and Williams,[13] consider the zones to be overlapping,[6–8] others have a quite opposite picture involving well separated zones.[5,14–18] At the same time this point is very important, especially for modern space based communication systems where the rf field is not purely sinusoidal, in particular in situations where many carrier waves are transferred simultaneously. In this case the separation of the multipactor zones makes it possible to suppress the multipactor buildup[19–21] due to perturbations of the required resonance conditions, whereas in the picture of overlapping zones, such suppression becomes impossible. To clarify the above-mentioned uncertainty, a detailed theoretical study has been undertaken, including numerical simulations, and taking into account a spread of the initial velocities of the secondary electrons. A new kind of multimode resonant two-sided multipactor process has been discovered within the simplest plane-parallel model when the spread of initial velocities is neglected. These modes fill some part of multipactor free regions between the well-known, traditionally considered resonant zones in parameter space. The zones related to these new modes are very narrow, however the separation between the zones is also narrow. Therefore even a small spread of initial velocities may result in overlapping between these zones and can make possible an overlapping between all zones, provided the secondary emission yield is large enough. The latter prediction has been confirmed in the present work in numerical simulations with varying values of the velocity spread as well as of the secondary emission yield.

II. THEORETICAL ANALYSIS

The present analysis is restricted to the case of the simple plane-parallel model of multipactor, which has been widely exploited in the literature.[3,7–16,21] Electrons are assumed to oscillate between two parallel plates separated by a distance L. These oscillations are driven by a spatially homogeneous rf electric field directed perpendicular to the plates and varying harmonically in time $E=E_0 \sin \omega t$. The impact of an electron with the plates is accompanied by a secondary emission yield, which depends on the energy of the primary electron. Specifically, in order to release more than one secondary electron, the impact velocity, U, of the primary electron must lie within a definite interval $V_1<U<V_2$ where the characteristic velocities, V_1 and V_2 are de-

termined by the material of the plates. The equation of motion for the electrons as they move between the plates is given by

$$\frac{d^2x}{dt^2}=\omega V_\omega \sin \omega t, \tag{1}$$

where $V_\omega = eE_0/m\omega$ is the oscillatory velocity of the electrons in the rf field. We emphasize that V_ω represents a measure of both the impact and the average velocity of the electrons in the gap. A necessary condition for the multipactor process to be sustained is that the secondary electrons are accelerated by the electric field toward the second plate. Therefore the one-way transit time of the electrons should not be less than half a rf period. Taking into account the fact that the maximum value of the impact velocity equals $2V_\omega$, an estimate of the multipactor buildup conditions can be formulated as follows:

$$V_1<2V_\omega<V_2, \tag{2}$$

$$\lambda \equiv \frac{\omega L}{V_\omega}>\pi. \tag{3}$$

These qualitative conditions, which neglect the effects of initial velocity and launch phase, are consistent with all existing multipactor theories although different theories differ from each other with respect to specific details that add to the above-presented common picture.

An integration of Eq. (1) gives the following relations between impact velocity and impact time:

$$u_0=v+\cos\varphi_0-\cos\psi_0, \tag{4}$$

$$\lambda=(\psi_0-\varphi_0)(v+\cos\varphi_0)+\sin\varphi_0-\sin\psi_0. \tag{5}$$

Here it is assumed that the electron starts from the lower plate $(x=0)$ with initial velocity V_0 at time t_0 and arrives at the upper plate $(x=L)$ at time T_0 with impact velocity U_0. The following dimensionless parameters have been introduced: $u_0=U_0/V_\omega$, $v=V_0/V_\omega$, $\varphi_0=\omega t_0$ $(-\pi<\varphi_0<\pi)$, $\psi_0=\omega T_0$ (the latter parameters correspond to the phases of the field at the initial time and at the time of impact, respectively). The secondary electrons start from the upper plate at time T_0 and can be described by the same relations (4) and (5) using $\varphi_1=\psi_0-\pi k_0$, ψ_1, u_1 instead of φ_0, ψ_0, u_0 (here $k_0=2n_0-1$ is an odd number chosen so as to provide a new starting phase within the same interval $-\pi<\varphi_1<\pi$ as the first starting phase). Following the pioneering paper of Henneburg *et al.*,[22] all subsequent theories consider resonant modes of multipactor that exist when all initial velocities of electrons are fixed. The resonance condition is then expressed by the equality

$$\varphi_1=\varphi_0=\varphi_R, \tag{6}$$

where φ_R is the resonance phase, which depends on λ, v, and the order of the resonance mode n as follows:

$$\lambda=\pi(2n-1)(v+\cos\varphi_R)+2\sin\varphi_R. \tag{7}$$

There are two important requirements, which limit possible values of the resonant phase. First, the starting phase must not be too negative since in this case the electron would return to the plate of birth due to the retarding action of the rf field. Second, the resonance should be stable, i.e., provide a phase focusing effect. This means that a small deviation of φ_0 from the resonant value φ_R should result in a smaller deviation of φ_1 $(|\delta\varphi_1|\equiv|\varphi_1-\varphi_R|<|\delta\varphi_0|)$. Taking into ac-

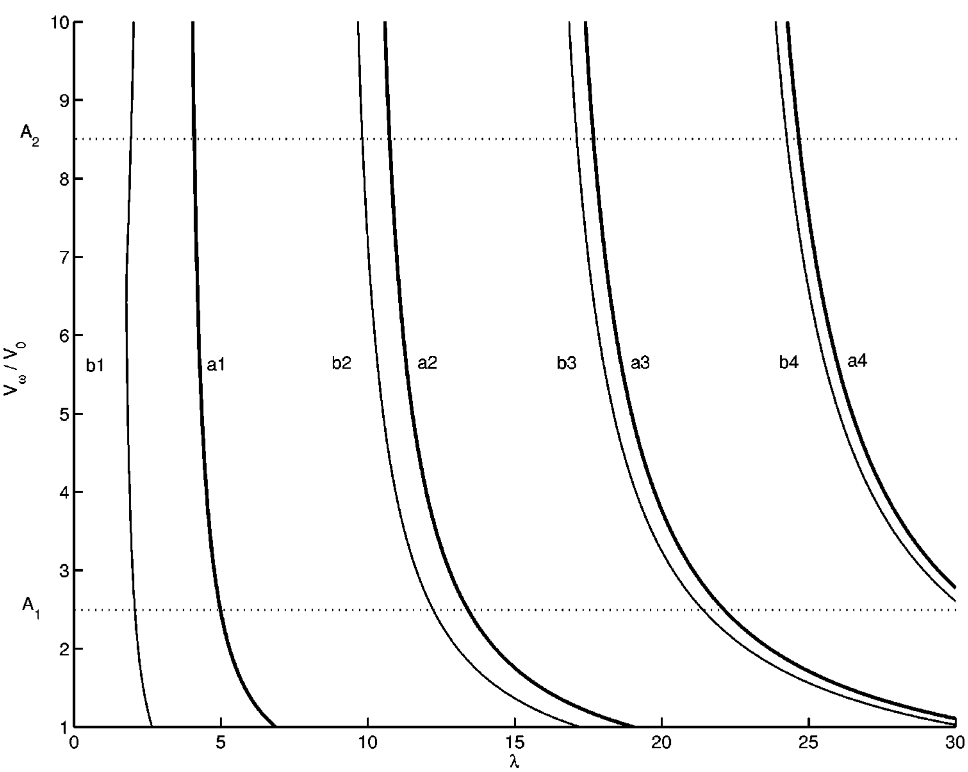


FIG. 1. The zones of ordinary resonant multipactor in the plane $V_\omega/V_0 = v^{-1}$ vs λ for the case of a single value of the initial electron velocity V_0. The thick solid lines (a1, a2, a3, and a4) indicate the upper boundaries of the multipactor zones (with $k=1$, 3, 5, and 7, respectively) as determined by the resonant conditions on the right-hand side of the inequalities (11). The thin solid lines (b1, b2, b3, and b4) indicate the lower boundaries of the same multipactor zones as determined by the left-hand side of the inequalities (11). The dotted lines indicate the boundaries of the multipactor zones as determined by the properties of the plate surfaces $(A_1=V_1/2V_0$, $A_2=V_2/2V_0)$.

count the smallness of the initial velocity ($v \ll 1$), the first condition can be expressed with high accuracy as the inequality:[21]

$$\varphi_R > -\sqrt{8v/3}. \tag{8}$$

The second condition can be expressed in explicit form:[15,16]

$$-\arctan\left(\frac{2}{\pi k}\right) - \arcsin\left(\frac{2v}{\sqrt{(\pi k)^2+4}}\right) < \varphi_R < \arctan\left(\frac{2}{\pi k}\right), \tag{9}$$

where $k=2n-1$. However for practical applications it is possible to use its approximate form:[21]

$$-\arctan\left(\frac{2}{\pi k}\right) - \frac{2v}{\sqrt{(\pi k)^2+4}} < \varphi_R < \arctan\left(\frac{2}{\pi k}\right). \tag{10}$$

These conditions determine a series of bands for the parameter λ that can be described approximately by

$$\max\left\{\begin{array}{c} \pi k - \pi k v/3 - 4\sqrt{2v/3} \\ \dfrac{(\pi k)^2-4}{(\pi k)^2+4}\sqrt{(\pi k)^2+4(1-v^2)} + \pi k v\dfrac{(\pi k)^2-4}{(\pi k)^2+4} \end{array}\right\} < \lambda < \sqrt{(\pi k)^2+4} + \pi k v, \tag{11}$$

where the first line on the left-hand side is determined by the condition (8) whereas the second line is determined by the stability condition (9). It should be noted that within the classical theory of multipactor, a resonant regime of multipactor is impossible independent of the value of the secondary emission yield, if the control parameter λ lies outside these bands. When the condition expressed by the second line on the left-hand side of Eq. (11) is not taken into account, the resonant bands inevitably overlap for higher order resonances.[13] However, the stability condition makes these bands well separated, at least when $v<1$. To complete the picture of the resonant multipactor zones, the value of the impact electron velocity must be calculated and compared with V_1 and V_2. Within the resonant bands this value is not very far from $2V_\omega$:

$$2+v > u_0 > \min\left\{\begin{array}{l} \dfrac{2\pi k}{\sqrt{(\pi k)^2+4}} + v \\ \max\left\{\begin{array}{c} 2-5v/3 \\ \dfrac{2\pi k}{(\pi k)^2+4}\sqrt{(\pi k)^2+4(1-v^2)} + v\dfrac{(\pi k)^2-4}{(\pi k)^2+4}. \end{array}\right. \end{array}\right. \tag{12}$$

Therefore the necessary condition, $V_1 < U < V_2$, for the multipactor buildup is close to that given by Eq. (2) and the zones of ordinary resonant multipactor look very simple in the plane spanned by V_ω and λ (see Fig. 1).

Generally, experimental results confirm theoretical predictions, i.e., those of Eqs. (2) and (3).[8,13,23,24] However there are certain discrepancies in the details. Specifically, Hatch and Williams[13] presented data indicating overlapping of the multipactor zones. At the same time, the results presented in Refs. 19 and 20 demonstrated unambiguously that these zones are separated. The classical experiment (Ref. 23) showed the existence of a separate first zone of multipactor. However the width of the observed zone was found to be larger than predicted by Eq. (11). It should be noted that the latter discrepancy cannot be attributed to poor knowledge of the secondary emission properties of the plates used in the experiment since the width [cf. Eq. (11)] depends neither on V_1, V_2 nor on the value of the secondary emission yield. One reason for this discrepancy may be that the spread of the normal component of the initial velocity of the secondary electrons was not taken into account. The importance of this phenomenon has been underlined many times in the literature.[9,25–28]

The first analysis of the effect of electron velocity spread was undertaken by Francis and von Engel.[9] They showed that a velocity spread leads to a spread in the transit times of electrons, thereby disturbing the resonant nature of the multipactor. This effect ultimately results in losses of electrons, losses which depend on the values of both the velocity spread and the parameter λ. Multipactor becomes possible if the secondary emission is high enough to compensate the electron losses. Therefore the bands of the control parameter λ, which are favorable for multipactor, depend crucially on the value of the secondary emission yield. In 1963 Miller and Williams presented the first results of numerical simulations that included the velocity spread effect.[26] They found some discrepancy as compared to the zones of multipactor predicted by resonant theory. However, the main emphasis in this work was given to the study of the distribution function over electron starting phases that is generated during the multipactor buildup. In the 1970s, a Russian team from the Lebedev Physics Institute paid considerable attention to the influence of velocity spread on the multipactor process[27,28] and developed the theory of the so-called polyphase regime of multipactor. In contrast to the resonant theory, which neglects the stochastic nature of the electron transit time due to the velocity spread, the polyphase theory considers the opposite limit case by assuming that the spread over transit times is much greater than the field period. This approximation is applicable for large gap widths where higher resonant modes could occur. It was predicted that in this case there are no separated resonant zones, provided that the secondary emission yield is sufficiently high (in fact it was found that it has to exceed 1.96). During the last decade the problem of velocity spread has also been discussed many times.[21,24,29,30] Nevertheless a detailed analysis of its influence on the structure of the multipactor zones has not yet been given. Following the paper by Riyopoulos *et al.*[29] one can estimate the spread, $\delta\psi_0$, over impact phases that is generated by a velocity spread, δv, assuming $\varphi_0=\varphi_R$:

$$\delta\psi_0 \approx (\pi k/2)\,\delta v. \tag{13}$$

Here $\delta v=\delta V_0/V_\omega$ is the normalized spread of the initial velocity, which is assumed to be small ($\delta v \ll 1/k$). When this phase spread is less than the characteristic width, $\Delta\varphi_R$, of the attraction region of the resonance, the phase focusing effect can compensate the effect of velocity spread almost completely. Therefore the velocity spread effect is not important in this case. However, in the opposite case ($\delta\psi_0 > \Delta\varphi_R$), the effect of a velocity spread is very important and results in considerable perturbations of the resonance between the electron oscillations and the rf field. As mentioned previously, this perturbation leads to electron losses from the resonant regime of oscillations and may finally suppress the multipactor development, if the secondary emission yield is not high enough. At the same time the perturbations of the resonance tend to give rise to an expansion of the multipactor zones with respect to the control parameter λ, if the secondary emission yield is high. An estimate of this expansion can be found from Eq. (7):

$$\delta\lambda_k \approx \pi k\,\delta v. \tag{14}$$

Note that the characteristic width of the resonant band, Δλ, is of the order of the phase width of the attraction region of the resonance, $\Delta\varphi_R$, and can be estimated as

$$\Delta\lambda_k \approx \Delta\varphi_{Rk} \approx 2/\pi k. \tag{15}$$

Therefore the effect of a velocity spread becomes important when

$$\delta v > (2/\pi k)^2. \tag{16}$$

Thus, under realistic conditions, this effect is important for all resonant modes of order $n>1$. At the same time a transition to the polyphase regime is possible when $\delta\psi_0 > 2\pi$, i.e., when

$$\delta v > 4/k. \tag{17}$$

A comparison between Eqs. (16) and (17) shows that there is a gap in parameter space where all existing theories of the multipactor process fail to be valid.

Based on the above-presented analysis one comes to the conclusion that the simple resonant theory should be quite applicable at least for the first resonant zone. Nevertheless, as mentioned earlier, the experiment (Ref. 23) demonstrated a larger width of this zone than predicted by theory. Moreover, numerical simulations[14] carried out for the case of high secondary emission yield showed a quasicontinuous region of multipactor in parameter space, without pronounced separation of the first zone. Therefore there is a definite need for more detailed analysis even of the first multipactor zone. Such an analysis is presented in Sec. III.

III. HYBRID RESONANT MODES

A complete solution of the multipactor problem taking into account a statistical spread of electron initial velocities is too complicated for analytical consideration and requires numerical simulations. However, for better understanding of numerical results, a more refined analysis of the problem is desirable. The crucial question for this analysis can be formulated as follows: Can anything be wrong with the main assumptions exploited by classical resonant theory even in the case when there is no velocity spread? The answer is yes. There are typically two specific effects which are not taken into account. The first one is known within the theory of single surface multipactor.[11,12,31,32] It is a development of the multipactor under unstable resonant phases.[31] Actually, an unstable resonant phase is associated with a phase debunching effect that is equivalent to losses of electrons. However, these losses can be compensated by secondary emission provided the latter is strong enough:

$$\sigma > \left|\frac{\delta\varphi_1}{\delta\varphi_0}\right| = \left|\frac{\pi k \sin\varphi_R + v}{2\cos\varphi_R + v}\right|, \tag{18}$$

where σ denotes the secondary emission yield. When this effect is taken into account, one can allow an expansion of the range of possible resonant phases. The resonant bands of the control parameter λ are also correspondingly expanded as described by the following equation [cf. Eq. (11)]:

$$\max\left\{\begin{array}{c}\pi k-\pi k v/3-4\sqrt{2v/3}\\ \dfrac{(\pi k)^2-4\sigma}{(\pi k)^2+4\sigma^2}\sqrt{(\pi k)^2+4\sigma^2-(\sigma+1)^2v^2}+\pi k v\,\dfrac{(\pi k)^2+4\sigma^2-2(\sigma+1)^2}{(\pi k)^2+4\sigma^2}\end{array}\right\}<\lambda<\sqrt{(\pi k)^2+4}+\pi k v. \tag{19}$$

The expansion is most pronounced for the first resonant band where it is noticeable even at moderate values of σ (see Fig. 2).

The second effect which has been missed in the majority of published papers is related to the more complicated resonant modes of the multipactor. Some of these modes were pointed out by Sombrin,[14] and later analyzed in more detail by Gilardini.[24,33] The main idea can be formulated as follows: Consider a group of electrons starting from the first plate with the same phase φ_0. As long as the initial velocity is fixed, these electrons bombard the second plate with the same impact phase ψ_0 described by Eq. (5). Consequently their secondaries will have the same starting phase $\varphi_1=\psi_0-\pi k_0$ and again generate new secondaries with the same

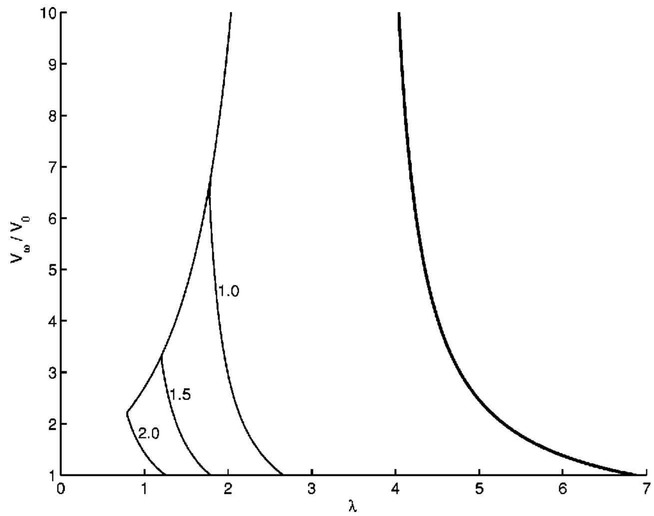


FIG. 2. The first resonant band ($k=1$) of multipactor in the plane $V_\omega/V_0 = v^{-1}$ vs λ for a case of a single value of the initial electron velocity V_0 and different fixed values of the secondary emission yield. The thick solid line indicates the upper boundary of the zone. The thin solid line indicates the lower boundary of the zone. Here the numbers at the lines correspond to the respective values of σ.

starting phase φ_2. A continuation of the process results in generation of some sequence of starting phases φ_r ($r=1, 2, \ldots$) and corresponding odd numbers k_r which can be infinite or finite (if the last starting phase is unfavorable for electrons to go away from the plate of birth). Infinite sequences correspond to the development of a multipactor and the simplest case of such a sequence is the ordinary resonance when all φ_r and k_r are the same. However, there are also other possibilities for an infinite sequence of phases to occur. For instance, any periodic sequence with some period p (i.e., with $\varphi_p = \varphi_0$ and $k_p = k_0$) can be infinite. The ordinary resonance corresponds to period $p=1$ in this sense. The modes analyzed by Gilardini correspond to period $p=2$. However, analysis and numerical simulations show that modes with a period $p>2$ are also possible.

Any mode of such type is characterized by certain finite series of odd numbers $k_0, k_1, \ldots, k_{p-1}$, which play the same role as the odd number k in the case of the ordinary resonance. To some extent, one can treat these modes as a combination of ordinary resonances of different orders and we suggest calling them hybrid resonance modes. For any possible hybrid mode, a series of starting phases $\varphi_0, \varphi_1, \ldots, \varphi_{p-1}$ is determined by the roots of a set of transcendental equations, which play the same role as Eq. (7) for the ordinary resonance

$$\lambda = (\varphi_r - \varphi_{r-1} + \pi k_{r-1})(v + \cos\varphi_{r-1}) + \sin\varphi_{r-1} + \sin\varphi_r,$$
$$r = 1,2,\ldots,p, \tag{20}$$
$$\varphi_p = \varphi_0.$$

Choosing zero index for the minimum value among φ_r, one can illustrate possible solutions of the latter equations in the plane λ vs φ_0. Generally, it is possible to state that in the plane λ vs φ_0, an infinite number of hybrid modes forms a series of separated bands as shown in Fig. 3.

The lower boundary of each region corresponds to the ordinary resonance mode

$$\lambda = \pi k(v + \cos\varphi_0) + 2\sin\varphi_0. \tag{21}$$

The upper boundary of each region can be expressed as

$$\lambda = (\pi k + \pi/2 - \varphi_0)(v + \cos\varphi_0) + 1 + \sin\varphi_0. \tag{22}$$

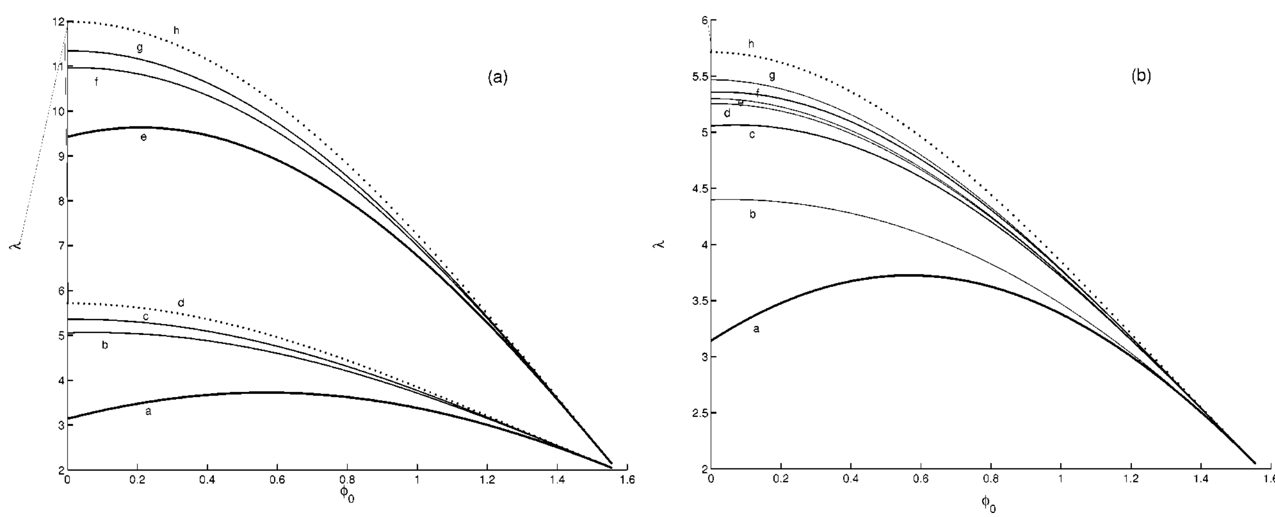


FIG. 3. The relation between λ and φ_0 that admits a solution of Eq. (20) for some sequence $k_0, k_1, \ldots, k_{p-1}$. The initial velocity is taken to be 0. In (a), the bold lines correspond to the ordinary resonances and are indicated with only one number k (curves a and e corresponding to $k=1$ and $k=3$, respectively). The thin lines correspond to hyper-resonances characterized by sequences (1,3) (curve b), (1,5) (curve c), (3,5) (curve f) and (3,7) (curve g). The dotted lines correspond to the upper boundaries of the bands filled by hybrid resonance modes of the form (1,−) (curve d) and (3,−) (curve h). (b) A blow-up of the region between the lowest order classical resonance. The bold curve corresponds to $k=1$ (curve a) and the dotted curve indicates the upper bound for hybrid modes of the order (k,−) (curve h). The second-order hybrid resonances characterized by (1,3) and (1,5) (curves c and f, respectively) are also reproduced for easy comparison. The thin lines correspond to higher order hyper-resonances of the order (1,1,3) (curve b), (1,3,3) (curve d), (1,3, 101) (curve e), (1,5,101) (curve g).

TABLE I. Lower and upper boundaries of the multipactor region within the range $5<\lambda<5.1$.

V_T/V_ω	σ_m	Lower boundary	Upper boundary
0	1	5.055	5.063
2×10^{-4}	1.2	5.026	5.082
2×10^{-4}	2	5.026	5.082
2×10^{-3}	1.2	5.045	5.088
2×10^{-3}	1.5	5.039	>5.1
2×10^{-3}	2	5.035	>5.1
2×10^{-2}	1.2	⋯	⋯
2×10^{-2}	1.5	5.067	>5.1
2×10^{-2}	2	<5	>5.1

The parameters of the particular hybrid modes with $p=2$ and $v=0$ can be expressed in explicit form as follows:

$$k_1=2k_0+1,$$

$$\lambda=\pi k_0+\pi/3+\frac{\sqrt{3}}{2}, \tag{23}$$

$$\varphi_0=0, \quad \varphi_1=\pi/3.$$

The phase stability of the hybrid modes can be analyzed in a similar manner as the phase stability of the ordinary resonance. The analysis shows that any hybrid mode is uniquely characterized by its particular band of the control parameter λ where the mode is stable. However the widths of all these bands are very small compared to the bandwidth of the stable modes of the ordinary resonance type. Specifically, when $v=0$, stable hybrid resonances are possible only for small values of φ_0 and an estimate of the stable bandwidth can be expressed as follows:

$$\Delta\lambda_{k_0,k_1,\ldots,k_{p-1}}<\frac{1}{\pi k_0}\prod_{r=1}^{p-1}\left(\frac{\cos\varphi_r+\cos\varphi_{r+1}}{\pi k_r \sin\varphi_r}\right)^2. \tag{24}$$

The hybrid mode with $p=2$, $k_0=1$, $k_1=3$ has the highest bandwidth of stability, but even this band is very narrow

$$\Delta\lambda_{1,3}\approx0.008. \tag{25}$$

It is evident that the hybrid modes are very sensitive to a velocity spread of the secondary electrons. Actually, by analogy with Eqs. (14)–(16) one can predict that a velocity spread with $\delta v>0.002$ should significantly blur any hybrid resonance. In other words, the hybrid resonance modes can be observed clearly only in numerical simulations. In fact, they were probably observed in recent simulations[5,17,18] where a velocity spread was not taken into account. Nevertheless, the existence of the hybrid modes cannot be neglected when conditions for multipactor breakdown are analyzed. As is clear from the above-given analysis, at $v=0$ there are many hybrid resonances in each band of the control parameter λ between $\lambda_l=\sqrt{(\pi k)^2+4}$ (upper boundary of the ordinary resonance band) and $\lambda_u=\pi k+\pi/2+1$ (upper boundary of the hybrid modes region). Any realistic velocity spread results in overlapping of these hybrid mode bands, provided the secondary emission yield is high enough. This phenomenon is very similar to the formation of the multipactor polyphase regime. The only difference is the threshold value of the secondary emission yield that is necessary to provide the analog with the polyphase regime. To confirm this assumption we have carried out detailed numerical simulations of the problem with different values of the velocity spread. The results of these simulations are presented in the next section.

IV. NUMERICAL SIMULATIONS

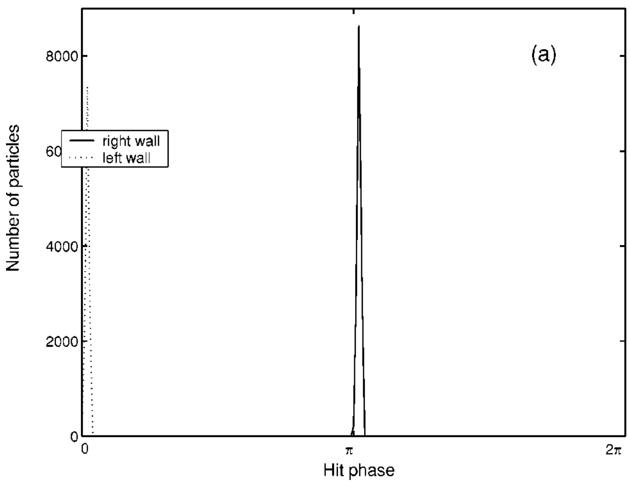
Numerical simulations have been carried out using a modified version of the one-particle particle-in-cell code originally developed and used in Ref. 30. Both plates are characterized by the same secondary emission yield. For simplicity we have assumed a step-function dependence of σ on the impact electron velocity U, i.e.,

$$\begin{aligned}\sigma&=0 \quad \text{if} \quad U<V_1,\\ \sigma&=\sigma_m \quad \text{if} \quad U>V_1,\end{aligned} \tag{26}$$

where σ_m is constant. The numerical results have been obtained for a fixed value of V_ω and different values of the velocity spread. However, the problem can be characterized by four dimension less parameters, viz. λ, V_T/V_ω, σ_m, and V_1/V_ω. Consequently the results also describe situations with different values of $V\omega$, but for a fixed ratio of V_ω/V_1. Within our series of calculations, this ratio has been set equal to the constant value 1.5. The spread of initial velocities of the secondary electrons has been approximated by a Maxwellian distribution function with varying value of the thermal velocity V_T.

TABLE II. Lower and upper boundaries of the multipactor regions within the range $2<\lambda<10$.

V_T/V_ω	v	σ_m	First classical band	Hyper-resonance	Hyper-resonance	Second classical band
0	0	1	$3.14<\lambda<3.72$	⋯	⋯	$9.42<\lambda<9.63$
0	0.02	1.2	$2.66<\lambda<3.78$	⋯	⋯	$8.90<\lambda<9.83$
0	0.02	2	$2.66<\lambda<3.78$	⋯	⋯	$8.90<\lambda<9.83$
0	0.1	1.2	$2.05<\lambda<4.04$	⋯	⋯	$9.48<\lambda<10.58$
0	0.1	2	$2.05<\lambda<4.04$	⋯	⋯	$8.67<\lambda<10.58$
0	0.3	1.2	$1.62<\lambda<4.67$	⋯	⋯	$11.16<\lambda<12.46$
0	0.3	2	$1.21<\lambda<4.67$	⋯	⋯	$10.21<\lambda<12.46$
0.02	0	1.2	$2.86<\lambda<3.96$	$5.12<\lambda<5.20$	⋯	$9.30<\lambda<9.90$
0.02	0	2	$2.70<\lambda<4.67$	$4.97<\lambda<5.62$	$5.87<\lambda<6.08$	$9.04<\lambda$
0.1	0	1.2	$2.65<\lambda<4.36$	⋯	⋯	$9.90<\lambda$
0.1	0	2	$2.25<\lambda<6.48$	⋯	⋯	$8.94<\lambda$



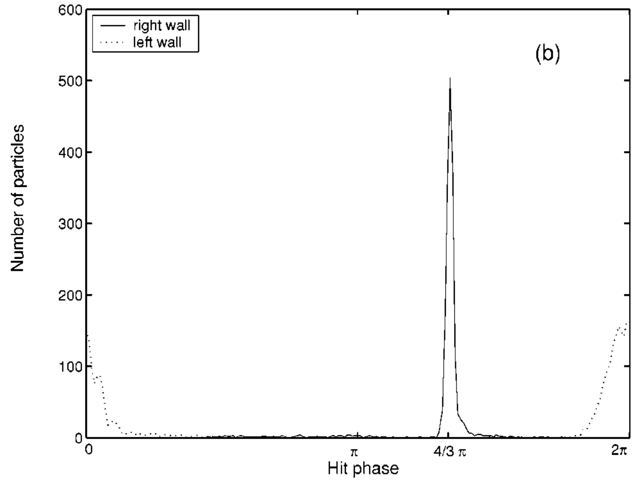


FIG. 4. Distribution of electron phases as they hit the plates relative to the rf field phases for two cases of multipactor. The case of the ordinary resonance mode is presented in (a). In this case $\lambda=3.3$, $\sigma_m=1.2$, $V_T/V_\omega=0.018$. At both plates the distributions are characterized by similar peaks around a resonance phase that is close to zero for the left plate $(x=0)$ and close to π for the right plate $(x=L)$. The case of the hybrid mode is shown in (b) ($\lambda=5.12$, $\sigma_m=1.2$, $V_T/V_\omega=0.018$). In this case the distribution is characterized by a wide peak around zero phase for the left plate whereas it has a considerably more narrow peak around $\varphi\approx 4\pi/3$ for the right plate.

$$F(V)\propto \exp\left(-\frac{V^2}{2V_T^2}\right). \tag{27}$$

The initial amount of particles was provided by setting up a small constant current between the walls. The parameter λ was varied from 2 to 10 with a step $\delta\lambda$ equal to 0.05. Some runs were done with even finer steps $\delta\lambda=0.002$ within the interval $5<\lambda<5.1$ where the simplest hybrid mode with $k_0=1,k_1=3$ should exist. The main result of the numerical simulations are presented in Tables I and II. Table I illustrates how the velocity spread and the secondary emission yield influence the multipactor breakdown region in the vicinity of the simplest hybrid mode band. The first line presents the band of the hybrid mode with $v=0$, $k_0=1,k_1=3$ given by analytical calculations. Within this band a multipactor discharge can develop for any level of the secondary emission yield that exceeds 1. When the velocity spread is very small (lines 2 and 3 in Table I), multipactor breakdown was found to occur in the simulations within a more wide λ range provided $1.2<\sigma_m<2$. In this case the λ range is not affected by the value of the secondary emission yield (as long as it exceeds unity). However, even for very small V_T (lines 4–6 of the Table I), the region of multipactor becomes sensitive to the specific value of the secondary emission yield. Finally, at $V_T\geqslant 0.01V_\omega$ (which is still unrealistically small compared to experiments), the multipactor requires a definite threshold value of $\sigma_m>1.2$ (line 7) in order to build up within the considered interval of λ. At the same time, for sufficiently high secondary emission yield, the multipactor develops within the whole interval, $5.0\leqslant\lambda\leqslant 5.1$ (line 9).

Table II illustrates the same effects as Table I, but in a wider range of the parameter λ. For comparison the first seven lines present ordinary resonance bands of the first and second order calculated using the simple theory. The bands of the hybrid modes are not shown here. These lines demonstrate how the resonant bands are modified by finite initial velocities of secondary electrons as well as by the actual value of the secondary emission yield above unity. The last four lines in Table II present results of numerical simulations for different values of V_T and σ_m. Note that at small values of V_T (lines 8 and 9), there are one or two (depending on the value of σ_m) separated regions of multipactor between the ordinary resonant bands. These regions are directly related to the hybrid modes as is clear from Fig. 4. The width of these regions increases with increasing σ_m, which indicates overlapping of several hybrid modes. It should also be noted that the boundaries of the first band are different from those predicted by simple resonant theory even at small thermal velocities. The lower boundary is found to be higher and more sensitive to the secondary emission yield than predicted by theory. This discrepancy is due to electron losses, which are stimulated by the velocity spread as discussed previously. The upper boundary is even more sensitive to the value of the secondary emission yield. At $\sigma_m=2$ this boundary lies considerably higher than the theoretical prediction. This can be explained only by an overlapping of the first resonant region with neighboring hybrid modes. Higher velocity spread (lines 10 and 11 in Table II) leads to stronger electron losses. As a result, the separated regions of the hybrid modes disappear completely for low secondary emission. At the same time, high velocity spread promotes overlapping of all hybrid modes with the first ordinary resonance. The latter effect results in a very wide region of multipactor for high values of the secondary emission.

V. CONCLUSION

A plane parallel model of multipactor breakdown has been studied in detail using both an analytical approach and numerical simulations. The analysis is based on the simple approximation which assumes some fixed value of electron initial velocity instead of a spread over these velocities. In contrast to most similar previous studies, the specific value of the secondary emission yield was taken into account when considering the conventional resonant modes of multipactor. This value is shown to contribute significantly to the expansion of the separated resonant zones. Many additional multipactor zones have been found in the parameter space between the well-known resonant zones. The new zones were associated with hybrid resonances of the multipactor. These hybrid modes are more or less complicated combinations of conventional resonances.

Numerical simulations have also been carried out taking into account the spread of initial electron velocities. The results demonstrate that even a relatively small spread significantly affects the structure of the multipactor zones. The latter become very sensitive to the value of the secondary emission yield over unity. This means that a detailed chart of the multipactor zones cannot be given based on a theory which assumes only one fixed value of the electron initial velocity. In fact, it is absolutely necessary to take into account the actual spread of the secondary electrons over initial velocities in order to obtain reliable predictions. Extensive numerical simulations will be made to analyze in more detail the influence of velocity spread and the value of the secondary emission coefficient (both difficult parameters to know for an arbitrary surface) on the multipactor zones. However, we expect that for realistic parameters this dependence will be rather smooth and that coming results will confirm the picture obtained in the present investigation.

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