July 17 OPLSS 2018

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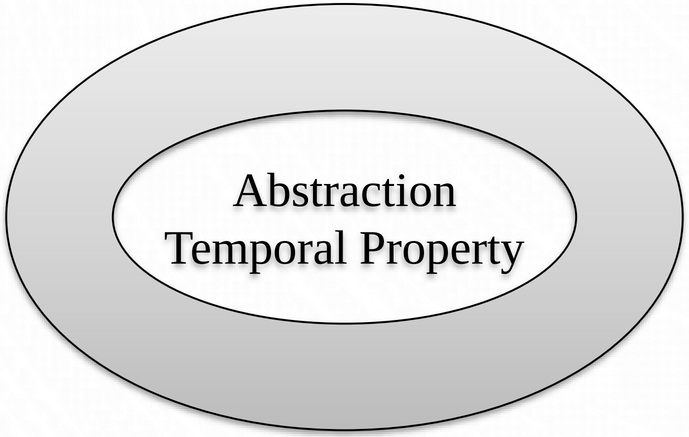
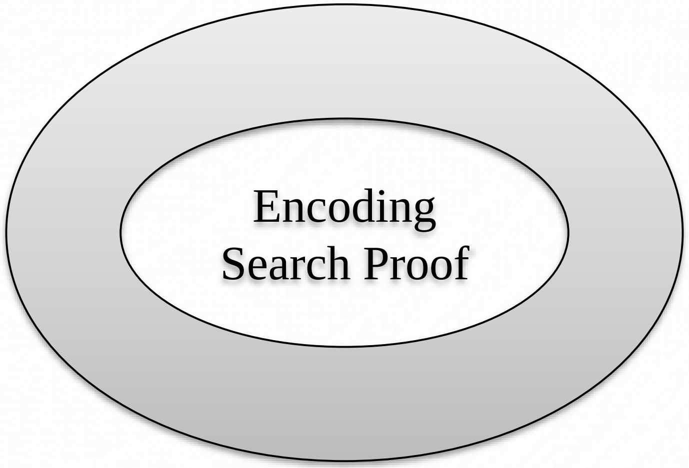
TEMPORAL LOGIC AND PROGRAM ANALYSIS

- From Temporal Logic to Program Analysis





➢ Temporal Logic

Atomic Propositions:

- Printer is busy
- He is a lawyer

Modal Logic:

- It is raining
- It will rain tomorrow
- It might rain tomorrow

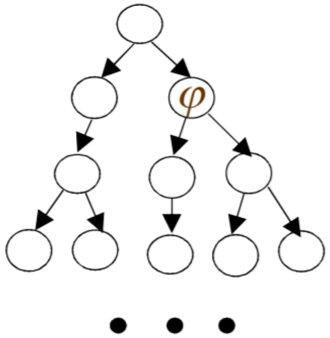
Temporal modalities

↓

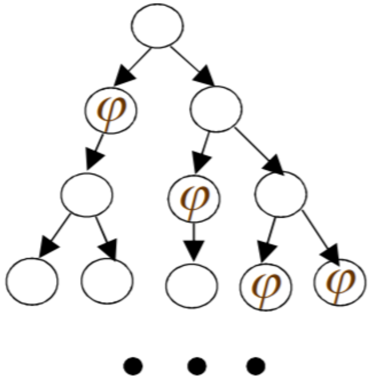
Temporal Logic

LTL, CTL

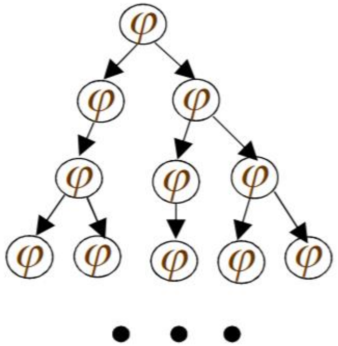
- Quantifiers over paths: A, E
- Path specific quantifiers: X, G, F, U, W



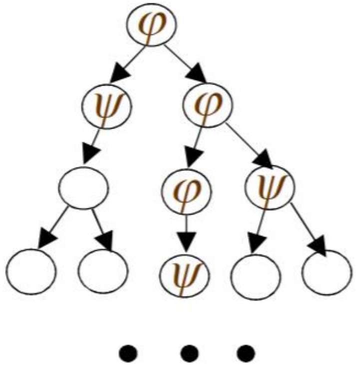
EX φ (exists next)



AF φ (all future)



AG φ (all global)



A[φ U ψ] (all until)

➢ Semantics of $\forall CTL$

$$\frac{\alpha(s)}{R, s \vDash \alpha} \quad \frac{R, s \vDash \varphi_1 \quad R, s \vDash \varphi_2}{R, s \vDash \varphi_1 \wedge \varphi_2} \quad \frac{R, s \vDash \varphi_1 \ \vee \ R, s \vDash \varphi_2}{R, s \vDash \varphi_1 \vee \varphi_2}$$

$$\frac{\forall (s_0, s_1, ...).\ s_0 = s \Rightarrow \exists i \geq 0.\ R, s_i \vDash \varphi}{R, s \vDash \mathsf{AF}\varphi}$$

$$\frac{\forall (s_0, s_1, ...).\ s_0 = s \Rightarrow \forall i \geq 0.\ R, s_i \vDash \varphi}{R, s \vDash \mathsf{AG}\varphi}$$

$$\frac{\begin{array}{c}\forall (s_0, s_1, ...).\ s = s_0 \Rightarrow (\forall i \geq 0.\ R, s_i \vDash \varphi_1) \vee \\ (\exists j \geq 0.\ R, s_j \vDash \varphi_2 \wedge \forall i \in [0, j).\ R, s_i \vDash \varphi_1)\end{array}}{R, s \vDash \mathsf{A}[\varphi_1 \mathsf{W} \varphi_2]}$$

A universal CTL formula:

- it uses only universal temporal connectives (AX, AF, AU, AG) with negation applied to the level of atomic propositions.

➢ Temporal Properties

Safety
Something "bad" will never happen

- AG ¬bad
- e.g., mutual exclusion: no two processes are in their critical section at once
- Safety = if false then there is a finite counter example

Liveness
Something "Good" will always happen

- AG AF good
- e.g., every request is eventually serviced
- Liveness = if false then there is an infinite counterexample

Every universal temporal logic formula can be decomposed into a conjunction of safety and liveness.

Program Analysis

- Derive properties of a program for all possible input values, in order to characterize the set of all possible output values or to find problems in its internal structure

$$\mathrm{M} = (\mathrm{S}, \mathrm{R}, \mathrm{I}) \mid R \in S \times S$$

$$\pi = (s_0, s_1, s_2, \ldots s_t), s_0 \in I \xrightarrow{\mathrm{P}}$$

$$\sigma : \mathrm{var} \rightarrow \mathrm{var}$$

$$\pi = \sigma_0, \sigma_1, \sigma_2, \ldots \mathrm{s.t.} \sigma_0 \in I$$

$$\mathit{Infinit} : \forall \sigma \in \Sigma, \exists \sigma', (\sigma, \sigma') \in \mathrm{R}$$

$l_0:$ x := 3;
$l_1:$ if (y > 0)
$l_2:$ C1
else
$l_3:$ C2

$$\pi = \begin{array}{r} x: \\ y: \\ pc: \end{array} \begin{bmatrix} 0 \\ 1 \\ l_0 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \\ l_1 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \\ l_2 \end{bmatrix}$$

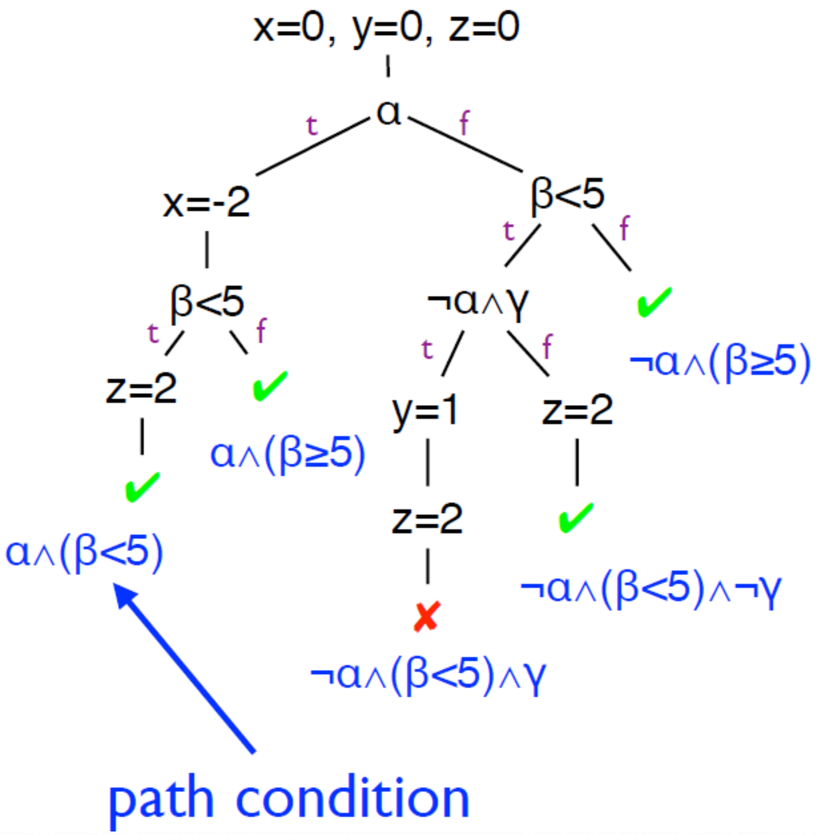
$$\pi' = \begin{array}{r} x: \\ y: \\ pc: \end{array} \begin{bmatrix} 0 \\ -2 \\ l_0 \end{bmatrix} \begin{bmatrix} 3 \\ -2 \\ l_1 \end{bmatrix} \begin{bmatrix} 3 \\ -2 \\ l_3 \end{bmatrix}$$

$$\mathit{Rank\ \ Functions}$$

$$M = \{\mathrm{f}_i \mid \forall (\mathrm{s}, \mathrm{s}') \in \mathrm{R}, \mathrm{f}_i(\mathrm{s}') \prec \mathrm{f}_i(\mathrm{s})\}$$

➢ Symbolic Execution

```
int a = α, b = β, c = γ;
                 // symbolic
int x = 0, y = 0, z = 0;
if (a) {
  x = -2;
}
if (b < 5) {
  if (!a && c)  { y = 1; }
  z = 2;
}
assert(x+y+z!=3)
```




```
; Variable declarations
(declare-fun x () Int)
(declare-fun y () Int)
(declare-fun z () Int)

; Constraints
(assert (= x -2))
(assert (= y 0))
(assert (= z 2))

; Solve
(assert(not (= (+ (+ x y) z)
3)))
(check-sat)
```

➢ Encoding Temporal Property as Program Analysis

Cook B., Koskinen E., Vardi M. (2011) *Temporal Property Verification as a Program Analysis Task*. CAV 2011.

$INV_1 : \forall s, \psi, \mathcal{M}, R.\ \text{if } R, s \not\models \varphi \text{ then } \mathcal{E}(\langle s, \varphi\rangle, \mathcal{M}, R) \text{ can return } \mathsf{false}$

$INV_2 : \forall s, \psi, \mathcal{M}, R.\ \mathcal{E}(\langle s, \psi\rangle, \mathcal{M}, R) \text{ can return } \mathsf{true}$

$$\exists\mathcal{M}.\ \forall s \in I.\ \mathcal{E}_R^{\mathcal{M}}(s, \varphi) \text{ cannot return } \mathsf{false} \quad \Rightarrow \quad P \models \varphi$$

```
let rec E(⟨s, ψ⟩, M, R) : bool =
  match(ψ) with
  | α → return α(s)
  | ψ′∧ψ″ →
      if (∗) return E(⟨s, ψ′⟩, M, R)
      else return E(⟨s, ψ″⟩, M, R);
  | ψ′∨ψ″ →
      if (E(⟨s, ψ′⟩, M, R)) return true;
      else return E(⟨s, ψ″⟩, M, R);
  | AGψ′ →
      while (true) {
        if (¬ E(⟨s, ψ′⟩, M, R))
          return false;
        if (∗) return true;
        s := choose({s′ | R(s, s′)});
      }
  | AFψ′ → local dup := false; local ′s ;
      while (true) {
        if (E(⟨s, ψ′⟩, M, R)) return true;
        if (dup ∧ ¬(∃f ∈ M. f(s) ≺ f(′s)))
          return false;
        if (¬ dup ∧ ∗) { dup := true; ′s := s; }
        if (∗) return true;
        s := choose({s′ | R(s, s′)});
      }
  | A[ψ′Wψ″] →
     while(true) {
       if (¬E(⟨s, ψ′⟩, M, R))
         return E(⟨s, ψ″⟩, M, R);
       if (∗) return true;
       s := choose({s′ | R(s, s′)});
     }
```

➢ Quick Take Away

Encoding Search Proof Properties Hold

Rank Functions

Program

Transition Systems

Safety

Liveness

Temporal Logic

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Q&A

Thanks!