title

introduction

a)

We start with the given equation

$-\frac{v_{i-1}-2v_i+v_{i+1}}{h^2} = f_i$

where $i \in [1, n] \cap \mathbb{N}$ and $n \in \mathbb{N}$. We will assume that $n \geqslant 3$ and that $v_0 = v_{n+1} = 0$.

$-v_{i-1} + 2v_i - v_{i+1} = h^2 f_i \equiv \widetilde{b}_i$

$$\begin{bmatrix} -1 & 2 & -1 \end{bmatrix} \begin{bmatrix} v_{i-1} \\ v_i \\ v_{i+1} \end{bmatrix} = \widetilde{b}_i$$

Now we expand these vectors from 3 elements to n elements. Note that the ith element of the row vector should be 2 and the ith element of the column vector should be v_i after the expansion:

$$\begin{bmatrix} 0 & \cdots & -1 & 2 & -1 & \cdots & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ \vdots \\ v_{i-1} \\ v_i \\ v_{i+1} \\ \vdots \\ v_n \end{bmatrix} = \widetilde{b}_i$$

Note that in the row vector 2 can very well be the first or last element, in which case the preceding presentation can be a little misleading. We further recognize the row vector as the ith row of the $n \times n$ matrix $\mathbf{A}$ (defined such that $A_{ij} = 2\delta_{ij} - \delta_{i(j-1)} - \delta_{i(j+1)}$) and get the inner product

$\mathbf{A}_i \cdot \mathbf{v} = \widetilde{b}_i$

and remembering that $i \in [1, n]$, by the definition of matrix multiplication

$\mathbf{A}\mathbf{v} = \widetilde{\mathbf{b}}$

which is what we wanted to show.

b)

Algorithm:

- step 1
- step 2
- ???
- profit