Project 5, FYS 3150 / 4150, fall 2013

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All our source code can be found at our GitHub repository for this project: https://github.com/OPSand/Project5/

1 Introduction

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2 Theory

2.1 Derivation of τ_{crunch} (optional)

(OPS)

2.2 Lack of a singularity in our model

While our "universe" has the same mass distribution as the closed universe discussed above, it is not closed. If it were, we would be able to go in a straight line and after travelling a distance $2R$, we would end up right where we started due to curvature. This is not the case in our model; our geometry is flat in space. Furthermore, we use Newtonian gravity and not General Relativity (i.e. we take all distances at face value without factoring in the scale factor of the universe). While the mass will certainly clump together and reach equilibrium, space itself will not shrink under the gravitational attraction, and the only blueshifts (or redshifts) observed inside this model should come from the peculiar motions of the celestial bodies. τ_{crunch} has no direct physical interpretation in our universe in itself - however, it is easy to calculate and scales similarly to the relaxation time τ_{relax}, which is the time needed for the system to reach equilibrium.

(TODO: Can we show this? Perhaps! Maybe even use it instead? :p)

2.3 G in units of ly, $M_{\sun}$ and τ_{crunch}

We can rewrite τ_{crunch} given in years

$$\tau_{crunch} = \sqrt{\frac{3\pi}{32 G_{yr} \rho_0}}$$

as

$$G_{yr} = \frac{3\pi}{32 \tau_{crunch}^2 \rho_0}.$$

But this is completely independent of the time unit... unless of course c = 1 somewhere, relating the time and distance units. In that case we have ly −> y (probably a safe bet =p).

Switching time units to τ_{crunch}, we get that $\tau_{crunch} = 1$ in these units, so

$$G = \frac{3\pi}{32\rho_0} = \frac{\pi^2 R_0^3}{8\mu N}$$

where for the latter equality we have used the definitions of average mass $\mu = \frac{M}{N}$ and initial mass density for a sphere $\rho_0 = \frac{M}{V_0} = \frac{\mu N}{\frac{4}{3}\pi R_0^3} = \frac{3\mu N}{4\pi R_0^3}$.

This means our gravitational constant and our time unit both depend on N, R_o and μ.

2.4 Gravitational potential with modified gravity

Starting with the magnitude of the force

$$F = \frac{GMm}{r^2 + \epsilon^2} = \frac{GMm}{\epsilon^2} \frac{1}{(\frac{r}{\epsilon})^2 + 1} = \frac{GMm}{\epsilon^2} \frac{1}{u^2 + 1}$$

where we have used the subsitution $u = \frac{r}{\epsilon}$ which gives $\frac{du}{dr} = \frac{1}{\epsilon}$, hence $dr = \epsilon \cdot du$, and so

$$E_p = m\Phi = \int F dr = \frac{GMm}{\epsilon^2} \int \frac{1}{u^2 + 1} \epsilon du = \frac{GMm}{\epsilon} \arctan(u) + C$$

We want $E_p \to 0$ as $r \to \infty$, and since $\arctan(u) \to \frac{\pi}{2}$ as $u \to \infty$, we achieve this by choosing $C = -\frac{\pi}{2}$:

$$E_p = \frac{GMm}{\epsilon} (\arctan(\frac{r}{\epsilon}) - \frac{\pi}{2}) \qquad (\epsilon > 0)$$

3 Results and analysis

3.1 ?

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4 Conclusion

What we learned:

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4.1 Critique

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