Solution to Assigment(Week 1)

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1 Q1

Let A = Probability that it rains today, B = Probability that it rains tomorrow

1. P(A) = 0.6
2. P(B) = 0.5
3. P($\bar{A} \cap \bar{B}$) = 0.3

$P(A \cup B) = 1 - 0.3 = 0.7 => P(A) + P(B) - P(A \cap B) = 0.7 => P(A \cap B) = 0.4$

1. P($A \cup B$) = 0.7
2. P($A \cap B$) = 0.4
3. P($A \cap \bar{B}$) = P(A) - P($A \cap B$) = 0.6-0.4 = 0.2
4. P($A \Delta B$) = P($A \cup B$) - P($A \cap B$) = 0.7-0.4 = 0.3

2 Q2

set A = $1, 2, 3, 4, 5, 6$ set B = $1, 2, 3, 4, 5, 6$

Sample Set (S) is the cartesian product of the set A and B . S = A x B $|S|$ = 6x6 = 36
Favourable cases for X1+X2 = 8:
(2,6), (3,5), (4,4), (5,3), (6,2)
Probability of each case = $\frac{1}{6} * \frac{1}{6} = \frac{1}{36}$
Total probability = $5 * 1/36 = \frac{5}{36}$

3 Q3

We will apply Bayes Theorem.

Probability that all are girls = $\frac{1}{2^n}$ Probability of having exactly k girls = nCk * $\frac{1}{2^n}$ Probability of choosing 1 girl from k girls = $\frac{k}{n}$

By Bayes Theorem:

$$P = \frac{\frac{1}{2^n}}{\Sigma_{k=0}^{n} k/n * nCk * \frac{1}{2^n}} \tag{1}$$

$$P = \frac{1}{2^{n-1}} \tag{2}$$

4 Q4

The probability of getting head is p and of getting tail is (1-p) hence $f(x) = pf_d(x) + (1-p)f_c(x)$
Similarly $F(x) = pF_d(x) + (1-p)F_c(x)$
$E(x) = pE(x_d) + (1-p)E(x_c)$
Var(X) = $p^2Var(x_d) + (1-p)^2Var(x_c) + 2p(1-p)Cov(x_d, x_c)$

5 Q5

$Cov(Z, W) = E(ZW) - E(Z)E(W)$
$=> E((1 + X + XY^2)(1 + X)) - (E(1 + X) * (E(1 + X + XY^2))$
$=> 1 + E(2X) + E(XY^2) + E(X^2Y^2) + E(X^2) - 1 - 2E(X) - (E(X))^2 - E(X)E(XY^2) - E(XY^2)$
$=> Var(X) + E(X^2Y^2)(Since E(X) = 0)$
$=> 1 + E(X^2)E(Y^2)$
$=> 1 + 1 = 2.$

Var(X) = 1, E(X) = 0
So $E(X^2) = 1$ similarly $E(Y^2) = 1$.

6 Q6

Probability that he will get atleast one offer = 1-Probability he gets no offer
=> P = 1 - $\frac{4^4}{5^4}$
=> P = 0.59
which is approximately 60 percent so he is wrong.

7 Q7

1000 is a large sample and each event is independent so we can apply Central Limit theorem.
The Binomial distribution here will tend to Normal distribution.
Mean of this distribution(μ) = np = 1000*0.1 = 100

Variance of this distribution(σ^2) = npq = 1000*0.1*0.9 = 90

We want P($X > 120$)
=> P(X - 100/$\sqrt{90} > 20/\sqrt{90}$)
= 1 - $\phi(\frac{20}{\sqrt{90}})$ where ϕ is the CDF of normal distribution.
= 1 - 0.982 = 0.018

8 Q8

64 is quite a large sample so we can apply Central Limit theorem and approximate it as a Normal dsitribution. Let X_i be the number of sandwiches ith person gets.
E(X_i) = 0*1/4 + 1*1/2 + 2*1/4 = 1
Var(X_i) = 0*1/4 + 1*1/2 + 4*1/4 - 1 = 0.5
Total Mean = 1*64 = 64
P($L <= X <= U$) = 0.95
= P($L - 64/\sqrt{(0.5)} <= Z <= U - 64/\sqrt{(0.5)}$) = 0.95
Now lower and upper limit turn out to be 1.96
Hence our range = (64 +- 1.96 * sqrt(0.5)) = [62.61, 65.38]
So number of sandwiches should be in this range for us to be 95 percent sure.

9 Q9

1. E(X) = 0, E(Y) = 0
2. Var(X) = 1 = Var(Y)
3. Cov(X,Y) = ρ

10 Q10

This is a standard bivariate normal distribution.
E(X) = 1, E(Y) = 2, Var(X) = 4, Var(Y) = 3, Cov(X,Y) = 1
Corr(X,Y) = 1/2*root(3)
Z_1 and Z_2 are two independent normal distribution such that Z_i N(0,1)
Y = $\sqrt{3}Z_1$ + 2
X = $[\rho Z_1 + \sqrt{1-\rho^2}Z_2]2$ + 1 We can substitute Z1 from Equation 1 and then apply formula of E(X+Y) and Var(X+Y) and we will get
E($X|Y = y$) = (y+1)/3
Var($X|Y = y$) = $4 * (1 - 1/12) = 11/3$

Hence $X|Y = y \sim$ N((y+1)/3 , 11/3)

11 Q11

E(Z) = E(3X-2Y) = 3E(X) - 2E(Y) = 0
Var(Z) = Var(3X-2Y) = 9Var(X) + 4Var(Y) - 12Cov(X,Y) = 9 + 16 -12 = 13
Cov(Z,X) = Cov(3X-2Y, X) = 3Cov(X,X) - 2Cov(Y,X) = 3Var(X) - 2 = 1.
Corr(Z,X) = Cov(Z,X)/sqrt(Var(Z)*Var(X)) = 1/sqrt(13).
Z $\sim$ N(0, 13) since linear combination of bivariate normal is also normal