A Behavioral Heterogeneous Agent New Keynesian Model

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Abstract

We develop a New Keynesian model with household heterogeneity and bounded rationality in the form of cognitive discounting. The behavioral heterogeneous agent New Keynesian model is consistent with recent empirical facts about the effectiveness and the transmission mechanisms of monetary and fiscal policy: monetary policy is amplified through indirect general equilibrium effects, fiscal multipliers on consumption are positive and the model delivers empirically-realistic intertemporal marginal propensities to consume. Simultaneously, and consistent with the data, the model resolves the forward guidance puzzle and remains stable at the effective lower bound as the model features equilibrium determinacy even under an interest-rate peg. The model is analytically tractable and nests a wide range of existing models as special cases, none of which can produce all the listed features within one model. We further show how the main insights from the tractable model extend to a quantitative version of the model, how the model-implied household expectations can be aligned with recent findings from survey data, and how to derive an equivalence result between heterogeneous household models with bounded rationality and those featuring incomplete information and learning.

Keywords: Behavioral Macroeconomics, Heterogeneous Households, Monetary Policy, Forward Guidance, Fiscal Policy, New Keynesian Puzzles, Determinacy, Lower Bound

JEL Codes: E21, E52, E62, E71

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1 Introduction

Recent empirical evidence on the transmission mechanisms and effectiveness of monetary and fiscal policies challenges the basic model of monetary policy, the New Keynesian model, along many dimensions: Monetary policy is transmitted to household consumption to a large extent through indirect, general equilibrium effects. Government spending increases private consumption substantially. Households' marginal propensities to consume (MPCs) out of transitory income changes are high on average in the year of the received income windfall and also remain high even in the year after. Announcements of future monetary policy changes, on the other hand, have relatively weak effects on current economic activity. Despite these weak effects of forward guidance, advanced economies have not faced large instabilities during long spells at the binding effective lower bound.[1]

In this paper, we propose a new framework which accounts for all these empirical facts *simultaneously*. To this end, we construct a New Keynesian model featuring household heterogeneity and bounded rationality in the form of cognitive discounting. The resulting behavioral heterogeneous agent New Keynesian model—or *behavioral HANK*—is analytically tractable which enables a clear understanding of the two frictions and how they interact. We show that it is indeed the *interaction* of bounded rationality and household heterogeneity that allows our model to be reconciled with the empirical evidence. Moreover, the model nests a broad spectrum of existing models—including the standard New Keynesian model, rational HANK models, and representative agent models which depart from the full-information rational expectations hypothesis (FIRE). None of these other models, however, can account for the listed empirical facts simultaneously.

To arrive at our framework, we extend the textbook New Keynesian model by household heterogeneity and bounded rationality in ways that preserve the tractability of the model. We assume that there are two groups of households, namely savers and hand-to-mouth households, and households face an uninsurable, idiosyncratic risk of switching their type. This generates heterogeneity in income, MPCs, and a precautionary-savings motive for the households. We introduce bounded rationality by the means of cognitive discounting. Households anchor their expectations about future macroeconomic variables to the steady state but are myopic or inattentive to future deviations from it. As a result, average ex-

[1] See, e.g., Ampudia et al. (2018), Slacalek et al. (2020) Samarina and Nguyen (2019) and Holm et al. (2021) for the empirical relevance of indirect channels in the transmission of monetary policy, Galí et al. (2007), Perotti (2007) or Dupor et al. (2021) for empirical evidence on the positive consumption response to fiscal spending, Auclert et al. (2018), Fagereng et al. (2021), Jappelli and Pistaferri (2020), Auclert (2019) and Patterson (2019) document empirical patterns of MPCs and see, for example, Del Negro et al. (2015), D'Acunto et al. (2020), Miescu (2022) and Roth et al. (2021) for empirical evidence on the (in-)effectiveness of forward guidance and Debortoli et al. (2020) and Cochrane (2018) on the stability at the lower bound.

pectations underreact to news, as empirically documented in Coibion and Gorodnichenko (2015), Bordalo et al. (2020) and Angeletos et al. (2021).[2]

In the behavioral HANK model, indirect general equilibrium effects account for large parts of how monetary policy is transmitted to consumption. Consistent with the data, households that exhibit high MPCs are more exposed to the business cycle in the behavioral HANK model. Thus, after an expansionary monetary policy shock (and likewise after a fiscal spending shock), high MPC households disproportionately benefit from the increase in output. This leads to an amplification of contemporaneous monetary policy through general equilibrium via a Keynesian-type multiplier. In addition, in the useful benchmark of a constant real interest rate, these general equilibrium effects also generate positive fiscal multipliers on consumption.

Even though the behavioral HANK model generates amplification of contemporaneous shocks through these indirect effects, the model does not suffer from the forward guidance puzzle: announced changes in the interest rate in the future have weaker effects on today's output than a current change in the interest rate and the effectiveness on today's output decreases with the horizon of the announcement. There are two competing forces shaping the effectiveness of a forward guidance shock on today's output. First, the general equilibrium amplification channel that is at work in response to contemporaneous monetary policy shocks is, ceteris paribus, compounded over time. The reason is that when savers expect higher consumption in the future, they decrease their precautionary savings today as they would disproportionately benefit from the increase in output in the hand-to-mouth state. Yet, the behavioral agents cognitively discount both this indirect general equilibrium effect as well as the direct effects of the future interest rate changes. This dampens the effects of forward guidance. With every reasonable calibration, the second channel dominates, leading to a dampening of the effects of forward guidance. Additionally, the behavioral HANK model remains stable during prolonged periods at the effective lower bound (ELB), as the model features equilibrium determinacy under an interest-rate peg for a large area of the parameter space.

The intertemporal MPCs (iMPCs) have been shown to be key statistics for monetary and fiscal policy analyses (Auclert et al. (2018), Wolf (2021), Kaplan and Violante (2020)). We derive the iMPCs in the behavioral HANK model analytically. To the best of our knowledge, we are the first ones to do so in a HANK model featuring a departure from FIRE. The behavioral HANK model quantitatively matches the empirical iMPCs. Thanks to the

[2] We show in Appendix A.7 how we can microfound our behavioral setup by the means of a noisy-signal extraction problem of otherwise rational agents. Angeletos and Lian (2017) show how other forms of bounded rationality or lack of common knowledge lead to observationally-equivalent outcomes but abstract from household heterogeneity.

closed-form solution, the model also sheds light on how the iMPCs depend on household heterogeneity frictions and bounded rationality. Boundedly-rational households tend to save more than rational households out of the income windfall as they cognitively discount the decrease in their future marginal utility which lowers the current MPC. As time progresses, however, bounded rationality increases the aggregate MPC as the behavioral savers start to consume their (higher) savings. These dynamic effects are particularly pronounced when idiosyncratic risk is relatively high.

We demonstrate that the behavioral HANK model can have *qualitatively different* policy implications than its rational counterpart by applying our framework to study the most effective timing of monetary policy. Consider an overheating economy which the monetary authority wants to tame by hiking interest-rates by a cumulative $x\%$. This rate hike can be implemented immediately or by raising the rate $\frac{x}{k}\%$ over k consecutive periods. A well-known feature of the RANK model is that monetary policy becomes more effective the more it is back-loaded. While this is also the case in the rational HANK model, the opposite is true in the behavioral HANK model: monetary policy is more effective when it is completely *front-loaded*, i.e., when $k = 1$. The increased effectiveness is driven by the fact that the hand-to-mouth agents' incomes contract more strongly leading to a strong decrease in aggregate demand. Thus, the increased effectiveness of front-loading the policy comes at the cost of an increase in inequality.

We show that the main insights of our tractable behavioral HANK model carry over to more quantitative models. To this end, we construct a quantitative behavioral HANK model which, in its rational expectation limit, collapses to a standard one-asset HANK model. The same general equilibrium forces as in the tractable model lead to an amplification of contemporaneous monetary policy shocks. Yet, also in the quantitative behavioral HANK model, there is no forward guidance puzzle, but the effectiveness of a change in the interest rate declines in the horizon. This is in contrast to the rational counterpart, in which the forward guidance puzzle is aggravated.

We extend our tractable baseline framework in several dimensions. First, we allow for sticky wages and show how the interplay of sticky wages, household heterogeneity and bounded rationality leads to hump-shaped responses of macroeconomic variables in response to aggregate shocks, as documented empirically (see, e.g., Auclert et al. (2020)). When forming their expectations, the behavioral households do not fully incorporate the implications of wage stickiness on future consumption in different states and, thus, on their idiosyncratic risk. As a consequence, the economy grows stronger than expected during the first quarters after the shock which generates a hump-shaped response. We also show that the interaction of bounded rationality, sticky wages and household heterogeneity generates an initial underreaction of households' expectations about future output, followed by a delayed overshooting, which is consistent with recent findings from survey expectations data (see Angeletos et al. (2021) and Adam et al. (2020)). This is the case although in our setup expectations are purely forward looking.

Second, we show how to extend our framework to derive an equivalence result between models with bounded rationality and models of incomplete information and learning. To this end, we assume that behavioral agents anchor their beliefs to *past observations* of the respective variable instead of the respective steady state values. This extended behavioral HANK model is observationally equivalent to models featuring incomplete information and learning (see Angeletos and Huo (2021) and Gallegos (2021)) that induce myopia and anchoring in the aggregate IS equation.[3]

Related Literature The literature so far treats the empirical facts laid out in the Introduction mostly independently from each other. The HANK and TANK literature – both with quantitative and analytical models – have highlighted the transmission of monetary policy through indirect, general equilibrium effects (Kaplan et al. (2018), Auclert (2019), Auclert et al. (2020), Bilbiie (2020)), positive fiscal multipliers on consumption (Auclert et al. (2018), Galí et al. (2007)), and the role of iMPCs (Auclert et al. (2018), Cantore and Freund (2021), Kaplan and Violante (2020)). On the other side, HANK models have also been used to solve the forward guidance puzzle (McKay et al. (2016), McKay et al. (2017), Hagedorn et al. (2019)).

Werning (2015) and Bilbiie (2021) combine the themes of policy amplification and forward guidance puzzle in HANK. While these two papers focus on slightly different explanation mechanisms, both establish a trade-off inherent in models with household heterogeneity: if HANK models amplify contemporaneous monetary policy (and fiscal policy) through redistributing towards high MPC households, HANK models also dampen precautionary savings desires after a forward guidance shock which further aggravates the forward guidance puzzle.[4] One of our contributions is that our behavioral HANK model overcomes this so-called *Catch-22* (Bilbiie (2021)).[5]

[3] Angeletos and Huo (2021) derive an equivalence result between models with incomplete information and learning with models which include behavioral myopia and an additional friction such as habit persistence or adjustment costs. We now complement their equivalence result with a behavioral model that solely relies on one behavioral friction.

[4] Acharya and Dogra (2018) construct a pseudo-RANK model, in which they isolate and highlight the role of precautionary savings dynamics in explaining the solution/aggravation of the forward guidance puzzle.

[5] Bilbiie (2021) provides two theoretical possibilities of how to sidestep the Catch-22. The first possibility is a pure risk channel which can, in theory, break the comovement of income risk and inequality. Yet to do so, it requires a calibration which seems highly at odd with the data. A second possibility is to drastically narrow down the policy space: in a world in which monetary policy is described by Wicksellian price level

A mostly-detached strand of the literature suggests to relax the assumption of full-information rational expectations (FIRE) to weaken the effectiveness of future monetary policies, thereby resolving the forward guidance puzzle (Wiederholt (2015), Angeletos and Lian (2018), Andrade et al. (2019), Gabaix (2020), Pfäuti (2021) and Roth et al. (2021)). We complement these papers by introducing household heterogeneity in terms of iMPCs, asset-market participation status, and exposure to the business cycle. This way, our model cannot only resolve the forward guidance puzzle (and other NK puzzles) but also simultaneously deliver amplification of contemporaneous monetary and fiscal policy through indirect channels, as well as match empirical estimates of iMPCs.

We share the combination of household heterogeneity and some deviation from FIRE with Farhi and Werning (2019), Auclert et al. (2020), Broer et al. (2021), Angeletos and Huo (2021), Laibson et al. (2021), Gallegos (2021), and Bonciani and Oh (2022). In contrast to all these papers, we offer analytical insights into how the two frictions matter for policy analysis, and how the interaction of the two frictions is key to reconcile the model with recent empirical facts as outlined above.

Outline. The rest of the paper is structured as follows. We present our behavioral HANK model in Section 2 and our main analytical results in Section 3. In Section 4, we present our model extensions, and Section 5 concludes.

2 A Behavioral HANK Model

In this section, we present our tractable NK model featuring household heterogeneity and bounded rationality (BR).

2.1 Structure of the Model

Households. The economy is populated by a unit mass of households, indexed by $j \in [0, 1]$. Households obtain utility from (non-durable) consumption, C_t^j, and dis-utility from working N_t^j. Households discount future utility at rate $\beta \in [0, 1]$. We assume a standard CRRA

targeting or fiscal policy follows a nominal bond rule, there would be no Catch-22. Hagedorn et al. (2019) use a similar description of fiscal policy to solve the forward guidance puzzle in a quantitative HANK model, in which contemporaneous monetary policy is amplified. Similarly, Kaplan et al. (2016) show that in their quantitative HANK model in Kaplan et al. (2018), there is no Forward Guidance puzzle, conditional on specific fiscal policy responses to a monetary policy shock. In contrast, in our model, there is no Catch-22 *independently* of the exact specification of monetary and fiscal policy.

utility function

$$U(C_t^j, N_t^j) \equiv \frac{(C_t^j)^{1-\gamma}}{1-\gamma} - \frac{(N_t^j)^{1+\varphi}}{1+\varphi}, \tag{1}$$

where φ denotes the inverse Frisch elasticity and γ denotes the relative risk aversion. For most of the paper, we focus on $\gamma = 1$, that is, log-utility $log(C_t^j)$.

Households can save or borrow in government bonds, paying nominal interest i_t, and acquire shares of intermediate monopolistic firms. We allow for the possibility that households participate in financial markets infrequently. When they do participate, they can freely buy or sell bonds and shares and receive the intermediate firm profits, D_t. Otherwise, they simply receive the payoff from their previously acquired bonds. We denote households participating in financial markets by S as they will be Savers in equilibrium, and the non-participants by H as they will be $H$and-to-mouth. A saver remains a saver with probability s and becomes hand-to-mouth with probability $1-s$. Hand-to-mouth households remain hand-to-mouth with probability h and switch with probability $1-h$. In what follows, we focus on stationary equilibria where $\lambda \equiv \frac{1-s}{2-s-h}$ denotes the constant share of hand-to-mouths.

We use the same simplyfing assumptions as in Bilbiie (2021) which allow for a tractable solution. In particular, we assume that households belong to a family whose utilitarian intertemporal welfare is maximized by its family head. The head can only provide insurance within types but not across types, i.e., the head pools all the resources within types. When households switch from the saver to the hand-to-mouth type, they only keep their government bonds. Stocks cannot be used to self-insure. Using the in- and outflows between both groups and the stationary distribution, we get the following relationships between real, per-capita, beginning-of-period-t+1 bonds, B_{t+1}^j and end-of-period-t per-capita real values (before moving across types), Z_{t+1}^j:

$$\begin{aligned}(1-\lambda)B_{t+1}^S &= s(1-\lambda)Z_{t+1}^S + (1-h)\lambda Z_{t+1}^H \\ \lambda B_{t+1}^H &= (1-s)(1-\lambda)Z_{t+1}^S + h\lambda Z_{t+1}^H,\end{aligned}$$

which, after using the definition of λ, can be re-written as

$$\begin{aligned}B_{t+1}^S &= sZ_{t+1}^S + (1-s)Z_{t+1}^H \\ B_{t+1}^H &= (1-h)Z_{t+1}^S + hZ_{t+1}^H.\end{aligned} \tag{2}$$

We allow for the possibility that the family head is boundedly rational (BR) in the way we describe in detail in Section 2.3.[6] The program of the family head is

$$W\left(B_t^S, B_t^H, \iota_t\right) = \max_{\{C_t^S, C_t^H, Z_{t+1}^S, Z_{t+1}^H, N_t^S, N_t^H, \iota_{t+1}\}} \Big[(1-\lambda)U\left(C_t^S, N_t^S\right) + \lambda U\left(C_t^H, N_t^H\right)\Big] + \beta \mathbb{E}_t^{BR} W\left(B_{t+1}^S, B_{t+1}^H, \iota_{t+1}\right)$$

subject to the flow budget constraints of the savers

$$C_t^S + Z_{t+1}^S + v_t \iota_{t+1} = W_t N_t^S + \iota_t (v_t + D_t) + \frac{1+i_{t-1}}{1+\pi_t} B_t^S + T_t^S, \tag{3}$$

and the hand-to-mouth households

$$C_t^H + Z_{t+1}^H = W_t N_t^H + T_t^H + \frac{1+i_{t-1}}{1+\pi_t} B_t^H, \tag{4}$$

as well as the non-negativity constraints

$$Z_{t+1}^H, Z_{t+1}^S \geq 0,$$

where W_t is the real wage, ι_t are the shares of stocks traded at price v_t, B_t denotes the liquid asset holdings (government bonds), and T_t^j are transfers to type-j households. As we will detail below, we assume that these transfers are financed by a proportional tax on profits, τ^D, such that they entail a redistribution from S households (who own the firms) to H households.

The optimality conditions are given by the savers' Euler equation

$$U'\left(C_t^S\right) \geq \beta \mathbb{E}_t^{BR} \Big[R_t \left(sU'\left(C_{t+1}^S\right) + (1-s)U'\left(C_{t+1}^H\right)\right)\Big] \tag{5}$$

$$\text{and } 0 = Z_{t+1}^S \Big[U'\left(C_t^S\right) - \beta \mathbb{E}_t^{BR} \Big[R_t \left(sU'\left(C_{t+1}^S\right) + (1-s)U'\left(C_{t+1}^H\right)\right)\Big]\Big],$$

the Euler equation of the hand-to-mouth households

$$U'\left(C_t^H\right) \geq \beta \mathbb{E}_t^{BR} \Big[R_t \left((1-h)U'\left(C_{t+1}^S\right) + hU'\left(C_{t+1}^H\right)\right)\Big] \tag{6}$$

$$\text{and } 0 = Z_{t+1}^H \Big[U'\left(C_t^H\right) - \beta \mathbb{E}_t^{BR} \Big[R_t \left((1-h)U'\left(C_{t+1}^S\right) + hU'\left(C_{t+1}^H\right)\right)\Big]\Big],$$

[6] We show in Appendix A.7 how the family head's expectation can be understood as an average expectation over all households' expectations within family where each household receives a noisy signal about the future state.

and the demand for shares

$$U'\left(C_t^S\right) \geq \beta \mathbb{E}_t^{BR}\left[\frac{v_{t+1}+D_{t+1}}{v_t}U'\left(C_{t+1}^S\right)\right] \text{ and } \iota_{t+1}=\iota_t=(1-\lambda)^{-1}, \tag{7}$$

with $R_t \equiv \frac{1+i_t}{1+\pi_{t+1}}$ being today's real interest rate. The respective labor-leisure equations of both types are given by:

$$-U'\left(N_t^S\right) = W_t U'(C_t^S) \tag{8}$$

$$-U'\left(N_t^H\right) = W_t U'(C_t^H). \tag{9}$$

In what follows, we focus on equilibria in which the H households will always be off their Euler equation—e.g., because they do not have access to financial markets—such that equation (6) always holds with strict inequality. In addition, we follow the tradition of analytical HANK models and assume a zero liquidity equilibrium to keep our model tractable.[7] As shares cannot be transferred to the H state, equation (5) simply prices the shares. Thus, the savers' bond Euler equation is the only Euler equation that is an equilibrium equation. Importantly, it features a self-insurance motive as savers demand bonds to self-insure their idiosyncratic risk of type-switching.

Firms. We assume a standard NK firm side.All households consume the same aggregate basket of individual goods, $j \in [0,1]$,

$$C_t = \left(\int_0^1 C_t(j)^{\frac{\epsilon-1}{\epsilon}}dj\right)^{\frac{\epsilon}{\epsilon-1}}$$

where $\epsilon > 1$ is the elasticity of substitution between the individual goods. Each firm faces demand

$$C_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon} C_t$$

where $P_t(j)/P_t$ denotes the individual price relative to the aggregate price index,

$$P_t^{1-\epsilon} = \int_0^1 P_t(j)^{1-\epsilon}dj,$$

and produces with the linear technology

$$Y_t(j) = N_t(j).$$

[7]See Krusell et al. (2011), McKay et al. (2017), Ravn and Sterk (2017), and Bilbiie (2021).

The real marginal cost is given by W_t. We assume that the government pays a subsidy τ^S on revenues to induce marginal cost pricing. The subsidy is financed by a lump-sum tax on firms T_t^F. Hence, the profit function is:

$$D_t(j) = (1 + \tau^S)[P_t(j)/P_t]Y_t(j) - W_t N_t(j) - T_t^F.$$

Total profits are then $D_t = Y_t - W_t N_t$ and are zero in steady state. Given zero steady state profits, we have a full-insurance steady state, i.e., $C^H = C^S = C$. In the log-linear dynamics around this steady state, profits vary inversely with the real wage $\widehat{d}_t = -\widehat{w}_t$.[8] We allow for steady state inequality in Appendix C and show that our results are not driven by this assumption and are in fact barely affected even by substantial inequality in the steady state.

Government. Fiscal policy induces the optimal steady state subsidy and taxes profits at rate τ^D and rebates these taxes as a transfer to H households, such that

$$T^H = \frac{\tau^D}{\lambda} D_t.$$

As will become clear later, the level of τ^D is key for the exposure of H households to the business cycle and thus for the cyclicality of inequality. Here, we abstract from government spending to keep it simple, but we introduce government spending in Section 3.

In most of the analysis, we assume that monetary policy follows a standard (log-linearized) Taylor rule[9]

$$\widehat{i}_t = \phi \pi_t + \epsilon_t^{MP}, \tag{10}$$

with ϵ_t^{MP} being the monetary policy shock which will be specified in the sections below.

Market Clearing. Market clearing requires that the goods market clears

$$Y_t = C_t = \lambda C_t^H + (1 - \lambda) C_t^S$$

and the labor market clears

$$N_t = \lambda N_t^H + (1 - \lambda) N_t^S.$$

[8]Throughout the paper variables with a hat on top denote log-deviations from steady state.
[9]We study more general Taylor rules in Appendix A.

2.2 Log-Linearized Model

We now focus on the log-linearized dynamics around the full-insurance, zero-liquidity steady state. The market clearing conditions yield $\widehat{y}_t = \widehat{c}_t = \lambda\widehat{c}_t^H + (1-\lambda)\widehat{c}_t^S$ and $\widehat{n}_t = \lambda\widehat{n}_t^H + (1-\lambda)\widehat{n}_t^S$. Importantly, we can write the consumption of the hand-to-mouth households as

$$\widehat{c}_t^H = \chi\widehat{y}_t, \tag{11}$$

with

$$\chi = 1 + \varphi\left(1 - \frac{\tau^D}{\lambda}\right) \tag{12}$$

measuring the cyclicality of the H household's consumption.[10] Auclert (2019) and Patterson (2019) document that households with higher MPCs tend to be more exposed to aggregate income fluctuations, which implies $\chi > 1$. As χ is a key coefficient in our model, we will vary χ throughout the paper to understand its role in shaping our results. Different levels of χ should then be thought of as different τ^D, thus, different redistributive tax-transfer systems.

Combining equation (11) with the goods market clearing condition yields

$$\widehat{c}_t^S = \frac{1-\lambda\chi}{1-\lambda}\widehat{y}_t, \tag{13}$$

which implies that consumption inequality is given by:

$$\widehat{c}_t^S - \widehat{c}_t^H = \frac{1-\chi}{1-\lambda}\widehat{y}_t. \tag{14}$$

Equation (14) shows that if $\chi > 1$, inequality is countercyclical as it varies negatively with total output, i.e., increases in recessions and decreases in booms. In line with the empirical evidence on the covariance between MPCs and business-cycle exposure, the data points toward $\chi > 1$ when looking at the cyclicality of inequality. Coibion et al. (2017), Mumtaz and Theophilopoulou (2017), Ampudia et al. (2018) and Samarina and Nguyen (2019) provide evidence of countercyclical inequality conditional on monetary policy shocks. Almgren et al. (2019) show that output in countries with higher shares of hand-to-mouth households responds more strongly to monetary policy shocks which, as we will see later on, implies $\chi > 1$ in our model.

The log-linearized bond Euler equation of S households is given by

$$\widehat{c}_t^S = s\mathbb{E}_t^{BR}\left[\widehat{c}_{t+1}^S\right] + (1-s)\mathbb{E}_t^{BR}\left[\widehat{c}_{t+1}^H\right] - \frac{1}{\gamma}\left(\widehat{i}_t - \mathbb{E}_t^{BR}\pi_{t+1}\right). \tag{15}$$

[10] See Appendix A.1 for the derivation of equation (11).

We will, following the assumption in Gabaix (2020), often focus on the case in which the agents are rational with respect to the real rate, i.e., we replace $\mathbb{E}_t^{BR}\pi_{t+1}$ with $\mathbb{E}_t\pi_{t+1}$ in equation (15). This is a conservative choice, in the following sense: we show in Appendix C that our results go through with boundedly-rational real-rate expectations and, in fact, the results become even stronger in that case. For the case without type-switching, i.e., for $s = 1$, equation (15) boils down to a standard Euler equation. For $s \in [0, 1)$, however, the agent takes into account that she might switch type and self-insures against becoming hand-to-mouth next period.

Supply Side. We distinguish between two set-ups for the supply side: For the main part, we assume that firms are not forward-looking and, thus, we can summarize the supply side of the economy by a static Phillips Curve

$$\pi_t = \kappa \widehat{y}_t, \tag{16}$$

where $\kappa \geq 0$ captures the slope of the Phillips Curve.[11] Yet, we also relax this assumption in Appendix C and show that a forward-looking (NK) Phillips Curve barely affects our results.

2.3 Bounded Rationality

We follow Gabaix (2020) and model bounded rationality as a form of cognitive discounting.[12] Let X_t be a random variable (or vector of variables) and let us define X_t^d as some default value the agent may have in mind and $\tilde{X}_{t+1} \equiv X_{t+1} - X_t^d$ denotes the deviation from this default value.[13] The behavioral agent's expectation about X_{t+1} is then given by

$$\mathbb{E}_t^{BR}\left[X_{t+1}\right] = \mathbb{E}_t^{BR}\left[X_t^d + \tilde{X}_{t+1}\right] \equiv X_t^d + \bar{m}\mathbb{E}_t\left[\tilde{X}_{t+1}\right], \tag{17}$$

[11] To arrive at this static Phillips curve, we can either assume that firms are completely myopic or that they face a Rotemberg-style adjustment cost relative to yesterday's market average price index (see Bilbiie (2021)).

[12] While Gabaix (2020) embeds bounded rationality in a NK model, the basic idea of behavioral inattention (or sparsity) has been proposed by Gabaix earlier already (see Gabaix (2014, 2016)) and a handbook treatment of behavioral inattention is given in Gabaix (2019). Benchimol and Bounader (2019) and Bonciani and Oh (2021) study optimal monetary policy in a RANK and TANK model, respectively, with this kind of behavioral frictions.

[13] Gabaix (2020) focuses on the case in which X_t denotes the state of the economy. He shows (Lemma 1 in Gabaix (2020)) that this form of cognitive discounting also applies to all other variables. We, on the other hand, directly apply cognitive discounting to all variables. Given Lemma 1 in Gabaix (2020), our results would be unchanged, but our more direct method simplifies some of the derivations, especially in Section 4.2. Appendix A.6 derives our results following the approach in Gabaix (2020).

where $\mathbb{E}_t[\cdot]$ is the rational expectations operator and $\bar{m} \in [0,1]$ is the behavioral parameter which captures the degree of rationality. A higher $\bar{m}$ denotes a smaller deviation from rational expectations and rational expectations are captured by $\bar{m} = 1$. We see from equation (17) that the behavioral agent anchors her expectations to the default value and cognitively discounts expected future deviations from this default value. For now, we focus on the steady state as the default value but relax this assumption in Section 4.2.

While we present a way to microfound $\bar{m}$ in Appendix A.7, note, that the exact microfoundation or underlying behavioral friction is not crucial for the rest of our analysis. For example, Angeletos and Lian (2017) show how other forms of bounded rationality or lack of common knowledge lead to observationally-equivalent expectations for the case in which X_t^d denotes the steady state.

Log-linearizing equation (17) around the steady state yields

$$\mathbb{E}_t^{BR}[\widehat{x}_{t+1}] = (1-\bar{m})\widehat{x}_t^d + \bar{m}\mathbb{E}_t[\widehat{x}_{t+1}] \tag{18}$$

and when X_t^d is the steady state value, we obtain $\mathbb{E}_t^{BR}[\widehat{x}_{t+1}] = \bar{m}\mathbb{E}_t[\widehat{x}_{t+1}]$. In Appendix B, we discuss empirical estimates of $\bar{m}$ and how we can map recent evidence in Coibion and Gorodnichenko (2015) and Angeletos et al. (2021) to $\bar{m}$. As a benchmark, we follow Gabaix (2020) and set $\bar{m}$ to 0.85, which is a rather conservative choice, given the empirical evidence. As one goal of our paper is to understand the role of $\bar{m}$ for policy analysis and the interplay of $\bar{m}$ and household heterogeneity, we will also consider different values for $\bar{m}$.

3 Results

In this section, we first show how the behavioral HANK model can be summarized by three equations isomorphic to the textbook RANK model. This allows us to show how the behavioral HANK model nests a wide spectrum of existing models and show how it overcomes several challenges present in these existing models. What is more, we show how only the behavioral HANK model can account for the empirical facts recently documented in the literature, simultaneously. We then analytically characterize the intertemporal marginal propensities to consume and show how they depend on bounded rationality, household heterogeneity, and the interaction of the two. We then show that the behavioral HANK model leads to different policy implications than its rational counterpart. We end the section by introducing bounded rationality in a quantitative HANK model and show that the main insights from the tractable model carry over to more quantitative models.

3.1 The Three-Equation Representation

The behavioral HANK model can be summarized by three equations: a Phillips curve, representing the aggregate supply side captured by equation (16), a rule for monetary policy (equation (10)), which together with the *behavioral HANK IS equation* determines aggregate demand. To obtain the behavioral HANK IS equation, we combine the hand-to-mouth households' consumption (11) with the savers' consumption (13) and their consumption Euler equation (15).[14]

Proposition 1. *The behavioral HANK IS equation is given by*

$$\widehat{y}_t = \psi_f \mathbb{E}_t \widehat{y}_{t+1} - \psi_c \frac{1}{\gamma} \left(\widehat{i}_t - \mathbb{E}_t \pi_{t+1} \right), \tag{19}$$

where

$$\psi_f \equiv \bar{m}\delta = \bar{m} \left[1 + (\chi - 1) \frac{1-s}{1-\lambda\chi} \right]$$

and

$$\psi_c \equiv \frac{1-\lambda}{1-\lambda\chi}.$$

Compared to RANK, two extra coefficients show up: ψ_c and ψ_f. ψ_c governs the sensitivity of today's output with respect to the contemporaneous real interest rate. ψ_c is shaped by household heterogeneity, in particular by the share of H households λ and their business-cycle exposure χ. As the H households are more exposed to the business cycle ($\chi > 1$), $\psi_c > 1$ and contemporaneous monetary policy is amplified through general equilibrium forces.

The second new coefficient in the behavioral HANK IS equation (19), ψ_f, captures the sensitivity of today's output with respect to changes in expected future output. ψ_f is shaped by household heterogeneity *and* the behavioral friction as it depends on the cyclicality of income risk *and* the degree of bounded rationality of households as well as the interaction of the two frictions. Given countercyclical income inequality, income risk is also countercyclical which manifests itself in $\delta > 1$. This countercyclical risk induces compounding in the Euler equation and, thus, competes with the empirically observed underreaction of aggregate expectations ($\bar{m} < 1$) which induces discounting in the Euler equation. We see in the following sections that even for a small degree of bounded rationality—much smaller than the empirics suggest—the discounting through bounded rationality dominates the compounding through countercyclical income risk. Hence, in the behavioral HANK model it holds that $\psi_f < 1$

[14] All derivations are in Appendix A.

which makes the economy less sensitive to expectations and news about the future which is key to resolve the NK puzzles.

Equation (19) nests a wide range of existing IS equations: the IS equation in the standard rational-expectations RANK model by setting $\psi_f = \psi_c = 1$, RANK models deviating from FIRE by $\delta = \psi_c = 1$, TANK models by setting $\bar{m} = \psi_f = 1$, and rational HANK models by $\bar{m} = 1$.[15] We discuss this in more detail in section 3.4.

Baseline Calibration. We set the parameters close to the calibration in Bilbiie (2020) and Bilbiie (2021) which is set in order to replicate several findings on the New Keynesian cross coming from more quantitative HANK models. We set $\chi = 1.48$ which implies that H agents' income is relatively sensitive to aggregate fluctuations, in line with empirical findings in Auclert (2019) and Patterson (2019). We set the share of H agents to one third, $\lambda = 0.33$, and the probability of an S household to become hand-to-mouth next period to 5.4%, i.e., $s = 0.946$ (this corresponds to a s of 0.8 in annual terms). We focus on log utility, $\gamma = 1$, and set the slope of the Phillips Curve to $\kappa = 0.02$. The cognitive discounting parameter, $\bar{m}$ is set to 0.85, as explained in Section 2.3. Details on the calibration and a discussion of the robustness of our findings for changing calibrations are presented in Appendix B. Note, that even when we vary certain parameters, we always keep $\lambda < \chi^{-1}$.

3.2 Monetary Policy

We now show how the behavioral HANK model can generate amplification of contemporaneous monetary policy through indirect effects while it solves the forward guidance puzzle at the same time. Additionally, we discuss determinacy conditions of the behavioral HANK model and show that it is stable at the effective lower bound.

General equilibrium amplification and Forward Guidance. We start by showing how the behavioral HANK model generates general equilibrium amplification of current monetary policy, while simultaneously ruling out the forward guidance puzzle. The forward guidance puzzle states that announcements about future changes in the interest rate affect output today as strong (or even stronger) than contemporaneous changes in the interest rate.[16] Miescu (2022) provides empirical evidence that conventional monetary policy is more effective than forward guidance. Consistent with these findings, Roth et al. (2021) show,

[15]For the RANK model, see, for example, Woodford (2003) or Galí (2015), for the RANK models differing from FIRE, see, for example, Angeletos and Lian (2018), Woodford (2019), or Gabaix (2020), and for rational TANK or THANK models, see Bilbiie (2008), McKay et al. (2017) or **?**

[16]Detailed analyses of the forward guidance puzzle in RANK are provided by McKay et al. (2016) and Giannoni et al. (2015).

by combining experimental evidence with theory, that forward guidance has relatively weak effects on consumption.

Let us now consider two different i.i.d. monetary policy experiments: we define a contemporaneous monetary policy shock as a surprise decrease in the interest rate today and a forward guidance shock as a news shock today about a decrease in the interest rate at some horizon k.

Proposition 2. *In the behavioral HANK model, there is amplification of contemporaneous monetary policy relative to RANK if and only if*

$$\chi > 1, \tag{20}$$

and the forward guidance puzzle is ruled out if

$$\bar{m}\delta + \frac{1}{\gamma}\frac{1-\lambda}{1-\lambda\chi}\kappa < 1. \tag{21}$$

The behavioral HANK model generates *amplification* of contemporaneous monetary policy with respect to the RANK model whenver $\chi > 1$, that is, when high-MPC households are relatively sensitive to aggregate income fluctuations. As discussed in Section 2.2, this is consistent with empirical findings. With $\chi > 1$, the income of H agents moves more than one-to-one with aggregate output. Hence, after a decrease in the interest rate, a disproportionate share of the extra income is received by H agents and, thus, the high-MPC households in the economy. This amplifies the increase in output through general equilibrium. The behavioral friction leaves the relative importance of direct vs. indirect effects—i.e., amplification of contemporaneous monetary policy—unaltered, as amplification of a contemporaneous i.i.d. shock is solely determined by the static redistribution towards the high MPC households.

Turning towards forward guidance, it is still the case that with $\chi > 1$ the income of H agents moves more than one-to-one with aggregate income. In this case, savers who self-insure against becoming hand-to-mouth in the future want less insurance when they expect a decrease in the interest rate since if they become hand-to-mouth they would benefit more from the increase in aggregate income. Hence, after a forward guidance shock, savers decrease their precautionary savings which compounds the increase in output today. Yet, as savers are boundedly rational, they cognitively discount these effects taking place in the future. Importantly, the savers cognitively discount both the future increase in output as well as the general equilibrium implication for their precautionary savings, thereby decreasing the effects of the forward guidance shock on today's consumption. In sum, if $\bar{m} < 0.93$, there is no forward guidance puzzle in the behavioral HANK model.

We now compare the behavioral HANK model to its rational counterpart to show how the behavioral HANK model overcomes a major trade-off inherent in the rational HANK model – the Catch-22. As Section 2.3 shows, we can recover the rational version of our model with $\bar{m} = 1$. Using this, we can see how Proposition 2 nests the *Catch-22* (Bilbiie (2021)). The Catch-22 describes the trade-off that the rational HANK model can either generate amplification of contemporaneous monetary policy *or* solve the forward guidance puzzle. To see this, note that with $\bar{m} = 1$ the forward guidance puzzle is resolved when

$$\delta + \frac{1}{\gamma}\frac{1-\lambda}{1-\lambda\chi}\kappa < 1$$

which requires

$$\chi < 1,$$

as otherwise $\delta > 1$. Assuming $\chi < 1$, however, leads to *dampening* of contemporaneous monetary policy instead of amplification.

We graphically illustrate the Catch-22 of the rational THANK model and the resolution of it in the behavioral HANK model in Figure 1. The figure shows the response of contemporaneous output relative to the initial response in the RANK model with rational expectations for anticipated i.i.d. monetary policy shocks occurring at different times k and a Taylor coefficient of 0 (as in Bilbiie (2021)).[17]

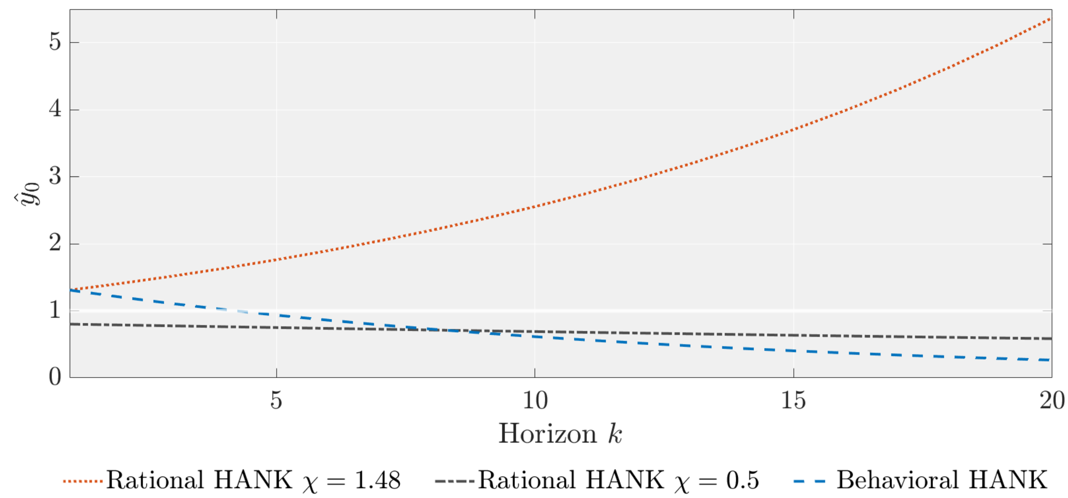
The orange-dotted line denotes the baseline calibration of the rational THANK model. We see that this model is able to generate contemporaneous amplification of monetary policy shocks, that is, an output response that is relatively stronger than in RANK. Put differently, the GE effects are relatively strong. Yet, at the same time, it exacerbates the forward guidance puzzle as shocks occurring in the future have much stronger effects on today's output than contemporaneous shocks. The black-dashed-dotted line shows how the forward guidance puzzle can be resolved by allowing for $\chi < 1$. Yet, this comes at the cost that the model is unable to generate amplification of contemporaneous monetary policy shocks. Furthermore, even for the quite low χ, the decay happens relatively slowly.[18]

The blue-dashed line shows that the behavioral HANK model generates both: amplification of contemporaneous monetary policy and a resolution of the forward guidance puzzle, as observed in the data. Note that also rational TANK models (thus, turning off type switch-

[17]Under fully-rigid prices (i.e., $\kappa = 0$), the RANK model would deliver a constant response for all k. The same is true for TANK, i.e., THANK without type switching. Whether the constant response would lie above or below its RANK counterpart depends on $\chi \lesseqgtr 1$ in the same way the initial response depends on $\chi \lesseqgtr 1$.

[18]Bilbiie (2020) calibrates $\chi = 0.3$ to approximate the forward guidance dampening results in McKay et al. (2016) and McKay et al. (2017).

Figure 1: Resolving the Catch-22



Note: This figure shows the response of total output in period 0 to anticipated i.i.d. monetary policy shocks occurring at different horizons k, relative to the initial response in the RANK model under rational expectations (equal to 1).

ing) or the behavioral RANK model would not deliver amplification and resolve the forward guidance puzzle simultaneously. TANK models would face the same issues as the rational RANK model in the sense that they cannot solve the forward guidance puzzle while bounded rationality in a RANK model does not deliver initial amplification.

As a direct consequence of the resolution of the Catch-22 in the behavioral HANK model, highly persistent monetary policy shocks have smaller effects on contemporaneous output than in RANK whereas less persistent shocks have larger effects in the behavioral HANK model. The reason is that persistent shocks also work through a forward guidance channel which is dampened in the behavioral HANK model. We elaborate this point in more detail in Appendix C.2.

Determinacy in Behavioral HANK. According to the Taylor principle, monetary policy needs to respond sufficiently strongly to changes in inflation in order to have a determinate equilibrium. In the rational RANK model the Taylor principle is given by $\phi > 1$, where ϕ is the inflation-response coefficient in the Taylor rule (10). We now derive a similar determinacy condition in the behavioral HANK model and show that both household heterogeneity and bounded rationality affect this condition. The following proposition provides the behavioral HANK Taylor principle.[19]

[19] We focus on local determinacy and bounded equilibria.

Proposition 3. *The behavioral HANK model has a determinate, locally unique equilibrium if and only if:*

$$\phi > \phi^* = 1 + \frac{\bar{m}\delta - 1}{\frac{\kappa}{\gamma}\frac{1-\lambda}{1-\lambda\chi}}. \tag{22}$$

Appendix A shows how to derive Proposition 3 and extends the result to more general Taylor rules.

To understand condition (3), consider first $\bar{m} = 1$ and, thus, focus solely on the role of household heterogeneity. With $\chi > 1$, it follows that $\phi^* > 1$ and, hence, the threshold is higher than the RANK Taylor principle states. This insufficiency of the Taylor principle in the rational THANK model has been shown by Bilbiie (2021) and in a similar way by Acharya and Dogra (2020). As a future aggregate sunspot increases the income of households in state H disproportionately, savers cut back on precautionary savings today which further increases output today. This calls for a stronger response of the central bank to not make the sunspot self-fulfilling.

On the other hand, bounded rationality and, thus, $\bar{m} < 1$ relaxes the condition as savers now cognitively discount both the future aggregate sunspot as well as its implication for their idiosyncratic risk. A smaller response of the central bank is needed in order to prevent the sunspot to become self-fulfilling. Given our calibration, the cutoff value for $\bar{m}$ to restore the RANK Taylor principle in the behavioral HANK model is 0.95. What is more, given our baseline choice of $\bar{m} = 0.85$, we have $\phi^* = -3.07$. Thus, the Taylor principle is not even necessary in our behavioral HANK model as the economy features a stable unique equilibrium even under an interest rate peg. In this sense, the behavioral HANK model overcomes the famous result in Sargent and Wallace (1975) who have shown that an interest rate peg leads to equilibrium indeterminacy.[20]

Stability at the Effective Lower Bound. Related to the determinacy issues under a peg, the traditional New Keynesian model struggles to explain how the economy can remain stable when the effective lower bound (ELB) on nominal interest rates is binding for an extended period of time, as observed in many advanced economies over recent decades (see, e.g., Cochrane (2018)). If the ELB binds for a sufficiently long time, RANK predicts unreasonably severe recessions and, in the limit case in which the ELB binds forever, there is even indeterminacy in RANK. The intuition is directly related to our discussion about determinacy under a peg: A forever binding ELB basically implies that the Taylor coefficient

[20] Angeletos and Lian (2021) show (in a model without household heterogeneity) that small frictions in memory and intertemporal coordination lead to a unique equilibrium which is the same as the one selected by the Taylor principle but it does no longer depend on it.

is equal to zero and, thus, the nominal rate is pegged at the lower bound, thereby violating the Taylor principle.[21]

We now show that the behavioral HANK model resolves these issues. To this end, let us add a natural-rate shock r_t^n to the IS equation (19). We assume that in period t the natural rate decreases to a value $\widetilde{r}^n$ that is sufficiently negative such that the natural rate in levels is below the ELB. The natural rate stays at $\widetilde{r}^n$ for $k \geq 0$ periods and after k periods the economy returns immediately back to steady state. Agents correctly anticipate the length of the binding ELB. For simplicity, we assume fully-rigid prices, i.e., $\kappa = 0$ and $\pi_t = 0$ for all t, but this is not crucial for what follows. Iterating the IS equation (19) forward, it follows that output in period t is given by

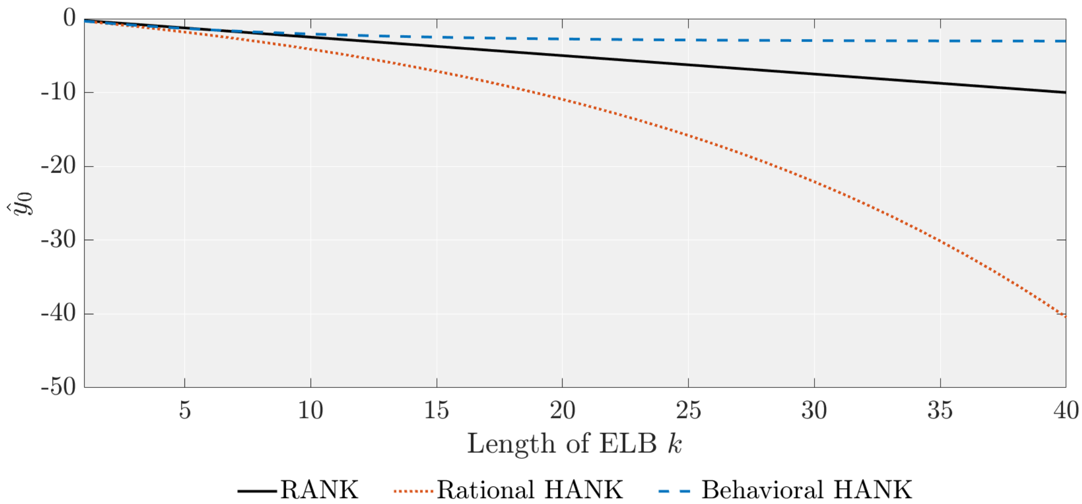
$$\widehat{y}_t = -\frac{1-\lambda}{\gamma(1-\lambda\chi)} \underbrace{\left(\widehat{i}_{ELB} - \widetilde{r}^n\right)}_{>0} \sum_{j=0}^{k} \left(\bar{m}\delta\right)^j , \tag{23}$$

where the term $\left(\widehat{i}_{ELB} - \widetilde{r}^n\right) > 0$ captures the shortfall of the policy response due to the binding ELB. Under rational expectations and countercyclical inequality, $\chi > 1$ and, thus, $\delta > 1$, meaning that output implodes as $k \to \infty$. The same is true in the rational RANK model which is captured by $\chi = 1$ and, thus, $\delta = 1$. In the behavioral HANK model, however, this is not the case. As long as $\bar{m}\delta < 1$, the output response in t is bounded even when $k \to \infty$. The condition $\bar{m}\delta < 1$ is the same as for determinacy under a peg in the economy with fully-rigid prices. It follows that $\bar{m} < 0.95$ is enough to rule out unboundedly-severe recessions at the ELB even if the ELB is expected to persist forever.

We illustrate the stability of the behavioral HANK at the lower bound graphically in Figure 2. The figure shows the output response in the rational RANK, the rational THANK and the behavioral HANK to different lengths of a binding ELB (depicted on the x-axis). The shortcoming of monetary policy due to the ELB, i.e., the gap $\left(\widehat{i}_{ELB} - \widetilde{r}^n\right) > 0$, is set to a relatively small value of 0.25% (1% annually), and we set $\bar{m} = 0.85$. Figure 2 shows the implosion of output in the rational RANK and even more so in the rational THANK model: an ELB that is expected to bind for 40 quarters would decrease today's output in the rational RANK by 10% and in the rational THANK model by 40%. On the other hand—and consistent with recent experiences in advanced economies—output in the behavioral HANK model remains quite stable and drop by a mere 3%.

[21] Note, that this statement also extends to models featuring more elaborate monetary policy rules including Taylor rules responding to output or also the Wicksellian price-level targeting rule, as they all collapse to a constant nominal rate in a world of an ever-binding ELB.

Figure 2: The Effective Lower Bound Problem



Note: This figure shows the contemporaneous output response for different lengths of a binding ELB k and compares the responses across different models.

3.3 Fiscal Policy

We now show that the sufficient statistic for amplification of the contemporaneous monetary policy is also a sufficient statistic to generate positive consumption multipliers of fiscal policy under constant real rates. Dupor et al. (2021) provide recent empirical evidence for positive effects of government spending on private consumption. Furthermore, Nakamura and Steinsson (2014), Ramey (2019) and Chodorow-Reich (2019) document fiscal multipliers above 1, which through the lens of our model is equivalent to saying that consumption responds positively to government spending. From now on, we will use *fiscal multiplier larger than one* and *positive consumption response* interchangeably.

To characterize fiscal multipliers, we follow Bilbiie (2021) and assume government spending g_t to follow an AR(1) with persistence $\rho_g \geq 0$, and to be 0 in steady state. The government taxes all agents uniformly to finance g_t.

We re-derive the behavioral HANK IS equation with government spending and obtain:

$$\widehat{c}_t = \bar{m}\delta\mathbb{E}_t\widehat{c}_{t+1} - \frac{1}{\gamma}\frac{1-\lambda}{1-\lambda\chi}\left(\widehat{i}_t - \mathbb{E}_t\pi_{t+1}\right) + \zeta\left[\frac{\lambda(\chi-1)}{1-\lambda\chi}\left(g_t - \bar{m}\mathbb{E}_t g_{t+1}\right) + (\delta-1)\bar{m}\mathbb{E}_t g_{t+1}\right],$$

where $\zeta \equiv \frac{\varphi}{\gamma\left(1+\frac{\varphi}{\gamma}\right)}$. The static Phillips Curve in this setting is given by $\pi_t = \kappa c_t + \kappa\zeta g_t$.

The following Proposition characterizes the fiscal multiplier in the behavioral HANK model.

Proposition 4. *The fiscal multiplier in the behavioral HANK model is given by*

$$\frac{\partial \widehat{c}_t}{\partial g_t} = \frac{1}{1-\nu\rho_g} \frac{\zeta}{1+\frac{1}{\gamma}\frac{1-\lambda}{1-\lambda\chi}\phi\kappa} \left[\frac{\chi-1}{1-\lambda\chi} \left[\lambda(1-\bar{m}\rho_g) + \bar{m}\rho_g(1-s)\right] - \kappa \frac{1}{\gamma} \frac{1-\lambda}{1-\lambda\chi} (\phi - \rho_g) \right],$$

where

$$\nu \equiv \frac{\bar{m}\delta + \frac{1}{\gamma}\kappa\frac{1-\lambda}{1-\lambda\chi}}{1+\frac{1}{\gamma}\frac{1-\lambda}{1-\lambda\chi}\phi\kappa}. \tag{24}$$

A corollary of Propositon 4 is that with persistent government spending, $\rho_g > 0$, and in the empirically-realistic case of $\chi > 1$, more bounded rationality, i.e., a lower $\bar{m}$, leads to a lower fiscal multiplier.[22] Bounded rationality weakens the fiscal multiplier as boundedly-rational agents discount the fact that an increase in government spending today has a positive effect on future spending as well. In the case of an i.i.d. spending shock, the fiscal multiplier is independent of $\bar{m}$.

To make the argument as clear as possible, we assume prices to be fully rigid, $\kappa = 0$, and assume that the real interest rate is held constant after the government spending shock. This is a useful benchmark as in this case, the consumption response in RANK is 0 (see Bilbiie (2011) and Woodford (2011)).[23]

From Proposition 4, we can directly derive the constant-real-rate multiplier in the behavioral HANK model. It shows that with $\chi > 1$, the fiscal multiplier is bounded from below by 0 irrespective of the persistence ρ_g. In other words, the constant-real-rate multiplier in the behavioral HANK model is strictly positive, regardless of the dampening of bounded rationality on the fiscal multiplier in the case of persistent spending. With $\chi > 1$, the high MPC households benefit disproportionately from the extra income out of the increase in government spending which increases the fiscal multiplier through a Keynesian type argument.

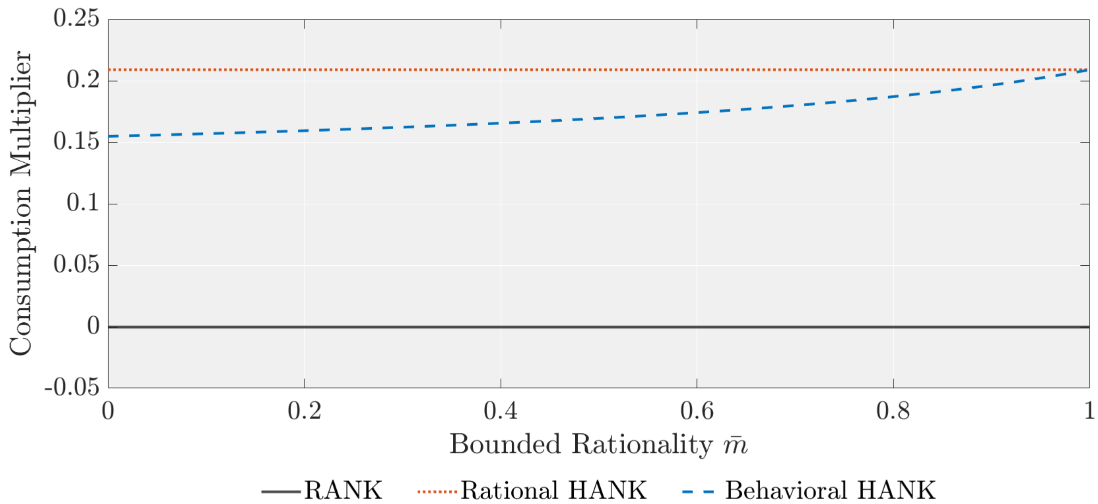
Figure 3 highlights the effect of bounded rationality on the fiscal multiplier by plotting the fiscal multiplier in the behavioral HANK model for varying degrees of $\bar{m}$ (blue-solid line) and comparing it to the multiplier in THANK and RANK. For this exercise, we set the persistence parameter to an intermediate, $\rho_g = 0.6$. It shows that the fiscal multiplier decreases with decreasing $\bar{m}$. Yet, even for the extreme case $\bar{m} = 0$, in which households fully discount all future increases in government spending, the fiscal multiplier is still substantially above zero even though it is somewhat weaker than under rational expectations. In fact, the behavioral HANK model generates consumption responses to fiscal spending that are

[22] We focus on the case in which $\nu\rho_g < 1$, which holds in the behavioral HANK model even for $\rho_g = 1$, and we assume $1 - s - \lambda < 0$, which holds under all reasonable parameterizations.

[23] Auclert et al. (2018) also use a constant real interest rate case to show that their HANK model can generate (output) fiscal multipliers larger than one.

quantitatively close to the empirical estimates in Dupor et al. (2021) who estimate the non-durable consumption response to lie between 0.2 and 0.29. Note, that we did not target this moment.

Figure 3: Consumption Response to Government Spending



Note: This figure shows the consumption multipliers (the consumption response to government spending) for different degrees of bounded rationality (blue-dashed line). The orange-dotted line plots the multiplier in the rational version of the model and the black-solid line shows the zero-multiplier in the RANK model.

It is noteworthy that the behavioral HANK model does not rely on a specific financing type to achieve positive consumption responses to fiscal spending. This is in contrast to the behavioral RANK model in Gabaix (2020). In the behavioral RANK model, bounded rationality can also increase the multiplier but only if the government delays taxing the agents to finance the contemporaneous spending as boundedly-rational agents will then discount the future increases in taxes. In the rational THANK model, on the other hand, the fiscal multiplier can in principle be larger than one with $\chi < 1$ if the hand-to-mouth households pay relatively less than the savers (see Bilbiie (2020) or Ferriere and Navarro (2018)). Both of these channels would also push up the multiplier in the behavioral HANK model, yet it does not depend on any of these two to achieve (output) fiscal multipliers larger than 1.

3.4 Behavioral HANK as a Unifying Framework

The interaction of bounded rationality and household heterogeneity is what allows the behavioral HANK model to be reconciled with the empirical facts on the transmission and effectivenss of monetary and fiscal policy. To see this, Figure 4 shows how the two frictions interact. The figure plots the parameter space for the two sufficient parameters for household heterogeneity and bounded rationality, respectively, $(\chi, \bar{m})$. The blue and orange dashed lines split the parameter space in the following sense: The blue line denotes the cut-off values below which the model is determinate under an interest-rate peg while above it the model is indeterminate (with the line itself belonging to the indeterminacy region). Determinacy under a peg is sufficient to rule out the forward guidance puzzle as well as the lower bound problem, and thus, is a sufficient statistic to resolve the discussed NK puzzles. The orange line denotes the cut-off values such that to the right of it, the model generates amplification while left from it—again including the line—the model does not generate amplification. Here, *amplification* is a stand-in for the amplification of monetary and fiscal policies through indirect, general equilibrium, effects.

This split of the parameter space into four areas allows us to distinguish the models discussed so far and to show how the behavioral HANK can overcome the limitations inherent in existing model. The RANK model is located in the "indeterminacy + no amplification" region as $\bar{m} = 1$ and $\chi = 1$. The behavioral RANK can either be in "indeterminacy + no amplification" or in "determinacy + no amplification" depending on the degree of rationality.[24] Rational THANK models can either be in "indeterminacy + no amplification", "determinacy + no amplification" or in "indeterminacy + amplification" while rational TANK models can only be in "indeterminacy + no amplification" or in "indeterminacy + amplification". Importantly, both cannot be in "determinacy + amplification".[25] Only the behavioral HANK model can deliver "determinacy + amplification". Furthermore, the behavioral HANK model can in principle cover the whole parameter space as it nests all the aforementioned models as special cases.

Having discussed the aggregate implications of the model, we now zoom in closer into the model and derive the iMPCs and show how they depend on bounded rationality, household heterogeneity, and the interaction of the two.

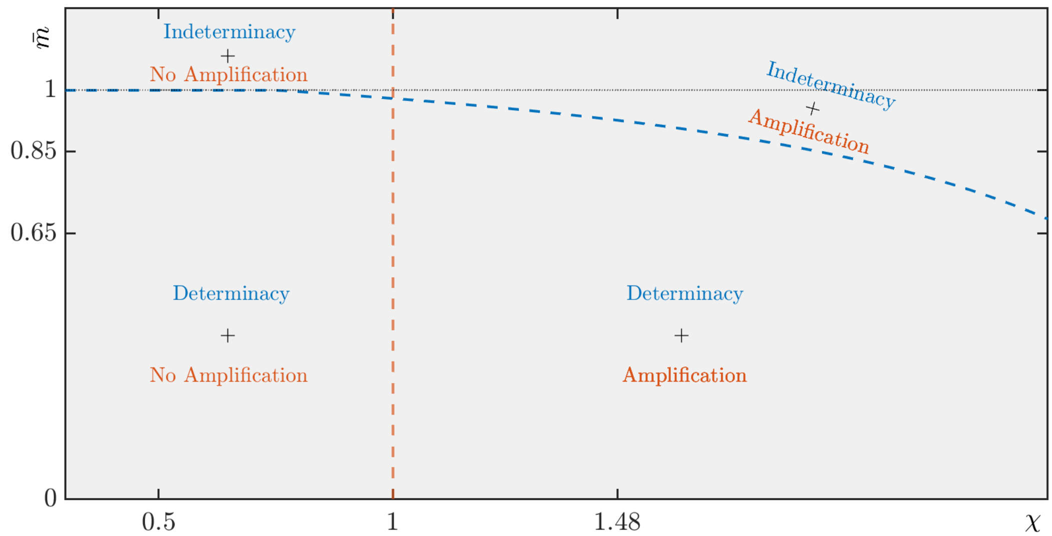
3.5 Intertemporal MPCs

The HANK literature shows that the iMPCs are a key statistic for conducting policy analysis (see, e.g., Auclert et al. (2018), Auclert et al. (2020), and Kaplan and Violante (2020)). We follow the THANK/TANK literature and define the aggregate iMPCs in the behavioral HANK model as the partial derivative of aggregate consumption at time k, $\widehat{c}_k$, with respect to aggregate disposable income, $\tilde{y}_0$, keeping everything else fixed (see Bilbiie (2021), Cantore

[24] Note, this also applies to other models featuring deviations from FIRE that deliver equivalent reduced-form IS equations, e.g., Angeletos and Lian (2018) and Woodford (2019).

[25] Note that this also applies to the models in McKay et al. (2017), Werning (2015), Ravn and Sterk (2017), Debortoli and Galí (2018), Bilbiie (2020), Bilbiie (2021) and many more.

Figure 4: The Behavioral HANK as a Unifying Framework



Note: The figure characterizes four possible regions depending on whether the considered $(\chi, \bar{m})$-pair delivers determinacy under an interest-rate peg or not and whether the model generates amplification of contemporaneous monetary and fiscal policy or not (we only extend the y-axis above 1 for the sake of readability).

and Freund (2021), and Auclert et al. (2018)).

The following Proposition characterizes the iMPCs in the behavioral HANK model.[26]

Proposition 5. *The intertemporal MPCs in the behavioral HANK model, i.e., the aggregate consumption response in period k to a one-time change in aggregate disposable income in period 0, are given by*

$$MPC_0 \equiv \frac{d\widehat{c}_0}{d\widetilde{y}_0} = 1 - \frac{1-\lambda\chi}{s\bar{m}}\mu_2^{-1}$$

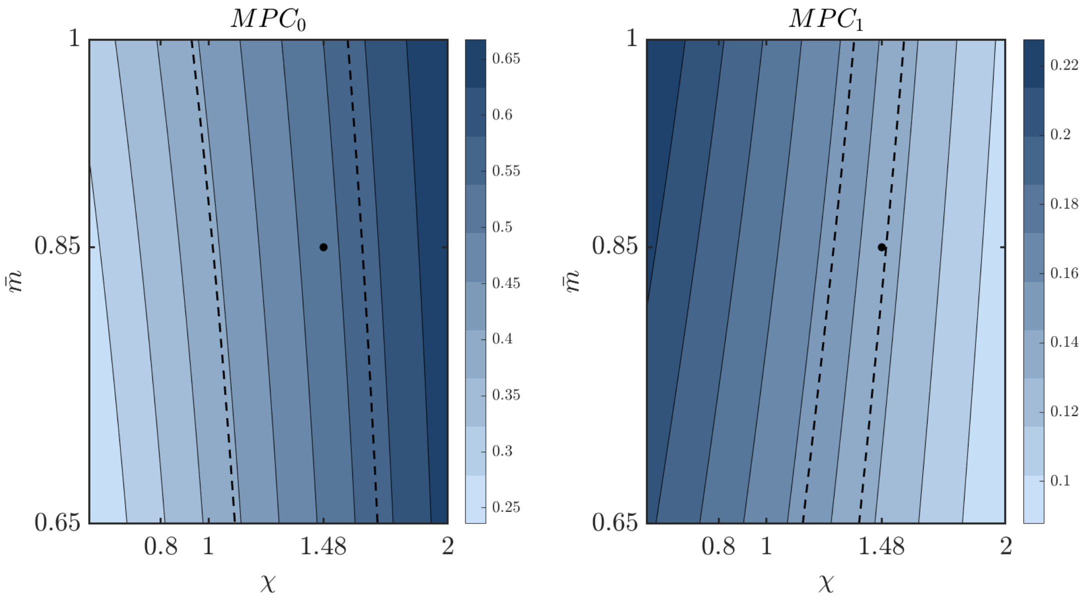
$$MPC_1 \equiv \frac{d\widehat{c}_k}{d\widetilde{y}_0} = \frac{1-\lambda\chi}{s\bar{m}}\mu_2^{-1}\left(\beta^{-1} - \mu_1\right)\mu_1^{k-1}, \qquad \text{for } k > 0,$$

where the parameters μ_1 and μ_2 depend on the underlying parameters, including $\bar{m}$ and χ and are explicitly spelled out in Appendix D.

Figure 5 graphically depicts how the interplay of bounded rationality $\bar{m}$ and household heterogeneity χ determines the size of the aggregate iMPCs. Therefore, we calibrate the model annually as the empirical evidence on the iMPCs is annual (see Fagereng et al. (2021) and Auclert et al. (2018)). We set $s = 0.8$ and $\beta = 0.95$, and keep the rest of the calibration as above. The left panel depicts the aggregate MPCs to spend within the first year (in period 0) and the right panel shows aggregate MPCs to spend within the second year (in period

[26]See Appendix D for the derivation.

Figure 5: Intertemporal MPCs, Bounded Rationality and Household Heterogeneity



Note: This figure shows the aggregate intertemporal MPCs, i.e., the aggregate consumption response in year 0 (left) and year 1 (right) to a change in aggregate disposable income in year 0 for different χ (x-axis) and $\bar{m}$ (y-axis). The dashed lines show the range of empirically-estimated iMPCs and the black dot shows the model estimate given our baseline calibration. Darker colors represent higher MPCs, see the colorbars on the right side of the figures.

1) after the temporary increase in income in time 0. Darker colors represent higher MPCs. First, note that with our baseline calibration—$\chi = 1.48$ and $\bar{m} = 0.85$ as shown by the black dots—the behavioral HANK model generates iMPCs within the first year of 0.55 and within the second year of 0.15. These values lie exactly in the estimated bounds for the iMPCs in the data (Auclert et al. (2018)) which are between $0.42 - 0.6$ within the first and $0.14 - 0.16$ within the second year (see dashed lines). Away from our baseline calibration, an increase in χ increases the MPCs in the first year but decreases them in the second year.[27] In contrast, an increase in $\bar{m}$ increases the aggregate MPC in the first year and in the second year.

Let us first turn to the role of χ for the iMPCs: Recall, the higher χ, the more sensitive is the income of the H households to a change in aggregate income. Thus, with higher χ, H households gain weight in relative terms for the aggregate iMPCs while the savers loose weight in relative terms. This pushes up the aggregate MPC within the first year, but pushes down the aggregate MPC within the second year as households that were hand-to-mouth in the period of the income windfall will have a MPC of 0 in the second year.

[27]Note, that when considering micro moments like the iMPCs, $\chi = 1$ is not sufficient anymore for the model to collapse to RANK. More precisely, with $\chi = 1$ the model collapses to a THANK model which behaves in the aggregate exactly like RANK (see the incomplete-markets irrelevance result in Werning (2015)). Hence, the RANK iMPCs cannot directly be seen in Figure 5 but Proposition 5 still nests RANK for $\chi = 1$ and $\lambda = 0$.

Bounded rationality, captured by $\bar{m}$, affects only the MPCs of the savers as only savers—whether behavioral or rational—intertemporarily optimize. The savers' Euler equation dictates that the decrease in today's marginal utility of consumption—following the increase in consumption—is equalized by a decrease in tomorrow's expected marginal utility. For the behavioral saver, however, the decrease in tomorrow's marginal utility needs to be more substantial as she cognitively discounts the expectations about the future decrease. Hence, the behavioral saver saves relatively more out of the income windfall. This pushes down the aggregate MPCs in $t = 0$. The same is true for the aggregate MPC in $t = 1$, in which there are now two opposing forces at work: on the one hand, the saver again cognitively discounts the expectations about the future decrease in the marginal utility which depresses her consumption. On the other hand, savers have accumulated more wealth from period $t = 0$ which tends to increase consumption. Given our calibration, in $t = 1$ the former dominates. Figure 12 in Appendix D shows that, beginning in $k = 3$, the latter effect starts to dominate. If we increase the idiosyncratic risk of becoming hand-to-mouth, i.e., increase the transition probability $1 - s$, the aggregate MPC is already higher in $t = 1$ for lower $\bar{m}$. The reason is that a smaller fraction of initial savers remains savers which pushes upwards consumption in $k = 1$ (see Figure 11 in Appendix D).

The effects of a change in $\bar{m}$ are more pronounced at lower levels of χ. Combining our discussion about the role of χ and $\bar{m}$, this is intuitive: the lower χ, the higher is the relative importance of the savers for the aggregate iMPCs and, in turn, the stronger is the effect of $\bar{m}$ on the aggregate iMPCs. These interaction effects are quite substantial: at $\chi = 1.48$, a decrease of $\bar{m}$ from 1 to 0.65 decreases the MPC_0 by 7% and the MPC_1 by more than 11%.

3.6 Policy Implications: The Timing of Monetary Policy

We close this section by discussing some of the policy implications of the behavioral HANK model. In particular, we illustrate that the behavioral HANK can generate different policy implications than its rational counterpart. To this end, we analyze how the timing of monetary policy affects its effectiveness and its distributional consequences.

Consider that the central bank wants to increase the nominal interest rate by a cumulative x%, for example, to fight an overheating economy. The central bank decides whether to implement this policy within one quarter or to gradually raise the interest rate by $\frac{x}{k}$% for k consecutive quarters.

Lemma 1. *The effect of a $\frac{x}{k}$% interest rate hike over k consecutive periods decreases current* *output by*

$$\widehat{y}_t = \frac{\psi_c}{\gamma}\left[\sum_{j=0}^{k-1}\left(\psi_f + \frac{\psi_c}{\gamma}\kappa\right)^j\right]\frac{x}{k}.$$

The left panel of Figure 6 depicts the result in Lemma 1 for the behavioral HANK model and compares it to its rational counterpart and the rational RANK model. The solid-black line shows the well-known feature of RANK that the effects of monetary policy on current output become stronger when monetary policy is back-loaded: the further the interest hike is stretched out, the higher is the response on current output. The orange-dotted line shows that this feature is even more pronounced in the rational THANK model as the line is steeper than in the RANK model.

In contrast, the blue-dashed line representing the behavioral HANK model is increasing instead of decreasing in k. Thus, back-loading monetary policy decreases its effect on current output. To put it differently, monetary policy is most effective on current output if it is completely front-loaded. Hence, if the central bank wants to fight an overheating of the economy as effectively as possible, the behavioral HANK model implies front-loading the interest rate hike, while its rational counterpart suggests to rather back-load the hike.

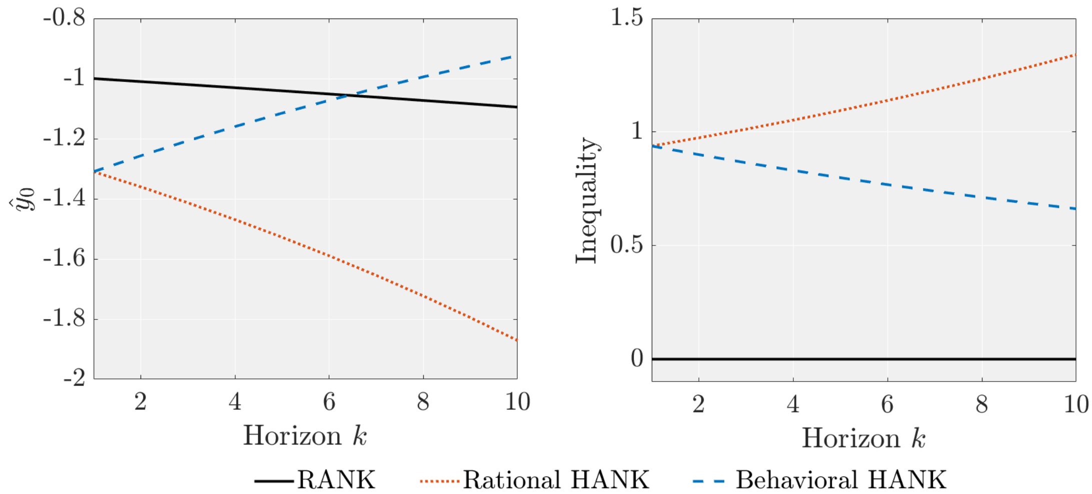
The right panel of Figure 6 depicts the effects of the different timing of the monetary policy hikes on consumption inequality, as defined in equation (14). It shows that, according to the behavioral HANK, if monetary policy front-loads the interest rake hike, it increases inequality the most whereas a more gradual increase in the interest rate would have weaker effects on inequality. This illustrates a trade-off for the central banker: the more effectively monetary policy combats the overheating, the more it increases inequality.

3.7 A Quantitative Behavioral HANK Model

In this section, we develop a quantitative behavioral heterogeneous agent New Keynesian model and show that the main insights of our tractable three-equation model carry over to more quantitative models.

Our quantitative model employs the standard HANK set-up in which households are ex-ante identical but face an idiosyncratic productivity risk. Households self-insure against this idiosyncratic risk by accumulating liquid bonds issued by the government. The fiscal authority issues a constant amount of real debt, B, and collects tax payments from households to finance its interest rate payments. Hence, ex-post households differ in their current productivity level, z, and their wealth b. Households' utility function is the same as considered in the tractable model (equation (1)).

Figure 6: Monetary Policy Timing: Effectiveness and Distributional Consequences



Note: This figure shows the response of current output (left panel) of a cumulative interest-rate hike by $x\%$ implemented over k consecutive periods. The right panel shows the corresponding response of inequality, defined as $\widehat{c}_t^S - \widehat{c}_t^H$.

The budget constraint of a household with current state $h = (b, z)$ is given by

$$c_{h,t} + \frac{b_{h,t+1}}{1 + r_t} = b_{h,t} + w_t z_{h,t} l_{h,t} + D_t d(z) - \tau_t(z)$$
$$b_{h,t+1} \geq \underline{b},$$

where the second equation denotes the household's (exogenous) borrowing constraint. As in McKay et al. (2016), we assume that households pay taxes conditional on their productivity level, $\tau_t(z)$. In addition, we assume that households receive a share of the dividends, $D_t d(z)$ conditional to their productivity level. Similar to the setup in the tractable model, we assume that the high productivity households receive a larger share of the dividends than low-productivity households. As dividends are countercyclical in the model, this assumption makes sure that households with higher MPCs (which are highly correlated with the low-productivity state) tend to be more exposed to the business cycle, in line with the tractable model and the empirical evidence (Auclert (2019), Patterson (2019)). This is different from the model in McKay et al. (2016) who assume that every household receives the same share of the dividends which leads to procyclical inequality.[28]

We introduce bounded rationality in the same way as in our tractable model. Households are fully rational with respect to their idiosyncratic risk, but they cognitively discount

[28] We show how our quantitative behavioral HANK nests McKay et al. (2016) in Appendix F.

the expected deviations of future aggregates (including prices) from their respective values in the stationary equilibrium. As a household's individual consumption depends on these aggregates, she cognitively discounts expected future *deviations* of her marginal utility in each state from its stationary equilibrium counterpart.

Households have perfect foresight about the path of the real interest rate. The Euler equation of a household with current state $h = (b, z)$ thus reads

$$c_{h,t}^{-\gamma} \geq \beta R_t \mathbb{E}_t^{BR} \left[c_{h',t+1}^{-\gamma} \right], \tag{25}$$

which holds with equality for non-constrained households, while it holds with strict inequality for households that are pushed to their borrowing constraint. The labor-leisure condition is identical to the one in the tractable model and holds for every household. In the case of rational expectations, the model collapses to a standard one-asset HANK model, similar to McKay et al. (2016), Hagedorn et al. (2019), or Debortoli and Galí (2018). We relegate the details of the model and the parameterization to Appendix F.

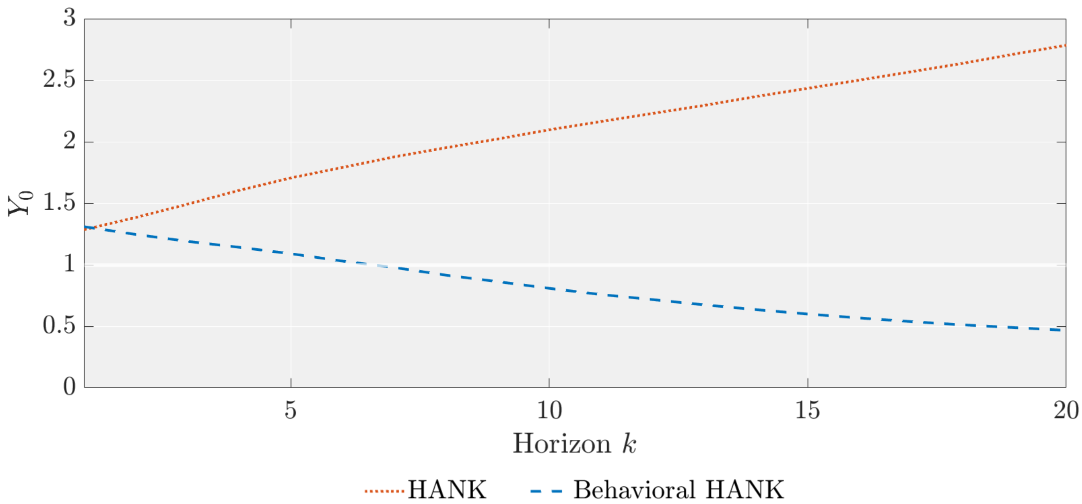
Monetary Policy Experiment. Let us now consider the following experiment. The monetary authority announces in period 0 to decrease the nominal interest rate by 10 basis points in period k and keeps the nominal rate at its steady state value in all other periods. Following Farhi and Werning (2019), we focus on the case with fully rigid prices and thus, the change in the nominal rate translates one for one to changes in the real rate and is thus also consistent with the exercise in McKay et al. (2016). What is the effect of such an interest-rate change on total output in period 0?

Figure 7 provides the answer. It shows on the vertical axis the response of output in period 0, Y_0, to an announced real rate change implemented in period k (horizontal axis). The white, horizontal line at 1 represents the response in the complete-markets model, i.e., in the rational RANK model.[29] The constant response in RANK is a consequence of the assumption that forward guidance is implemented through changes in the real rate.

The orange-dotted line shows the results for the rational HANK model. We see how the fact that households with higher MPCs tend to be more exposed to the business cycle leads to an increase in the effectiveness of contemporaneous monetary policy. This amplification through indirect effects, however, extends intertemporally and results in an aggravation of the forward guidance puzzle. Indeed, we see that the further away the announced interest rate change takes place, the stronger the response of output today. A change that is announced to take place in twenty quarters leads to a response of today's output that is almost three

[29]Note that for an easier interpretation, we normalized the y-axis by dividing through the response in the rational RANK model which is 0.05% after a shock of 10 basis points.

Figure 7: Monetary Policy in the Quantitative Model



Note: This figure shows the response of total output in period 0 to anticipated i.i.d. monetary policy shocks occurring at different horizons k, relative to the response in the RANK model under rational expectations (equal to 1).

times as strong as in RANK.

The behavioral HANK model, on the other hand, does not suffer from the forward guidance puzzle, as shown by the blue-dashed line. While the effect of a contemporaneous interest rate change is almost identical as in the rational HANK model, interest rate changes announced to take place in the future have relatively weak effects on contemporaneous output and the effects decrease with the horizon. In Appendix F, we discuss how our resolution of the forward guidance puzzle contrasts with other resolutions in the HANK literature.

Overall, Figure 7 illustrates that the main insights of the tractable behavioral HANK model carry over to more quantitative models. Contemporaneous monetary policy is amplified through indirect, general equilibrium, channels whereas announced future policies have relatively weaker effects on today's economy. Furthermore, the results are not only qualitatively similar in the tractable and the quantitative models but also in terms of their magnitude.

4 Extensions

We now extend our baseline tractable model along two dimensions. First, we allow for sticky wages and show how the interplay of sticky wages, household heterogeneity and bounded rationality leads to hump-shaped responses of macroeconomic variables in response to aggregate shocks, as well as forecast-error dynamics consistent with recent findings from survey data. Second, we derive an equivalence result between HANK models with bounded rationality and HANK models with incomplete information and learning.

4.1 Sticky Wages

Recent HANK models have relaxed the assumption of fully-flexible wages and rather assume wages to be sticky, bringing these models closer to the data (see, e.g., Auclert et al. (2020) or Broer et al. (2020)). We first show that the behavioral HANK model extended by sticky wages generates hump-shaped responses of aggregate variables to a monetary policy shock. We then show that the sticky-wage behavioral HANK model also generates dynamic forecast errors that are in line with recent empirical evidence.

Modelling sticky wages. To introduce sticky wages, we follow Colciago (2011) and assume a centralized labor market in which a labor union allocates the hours of households to firms and makes sure that S and H households work the same amount. The labor union faces the typical Calvo (1983) constraint, such that it can re-optimize the wage within a given period only with a certain probability, giving rise to a wage Phillips Curve. We assume that the labor union sets wages based on rational expectations to focus on bounded rationality solely on the household side.

The wage Phillips curve is given by

$$\pi_t^w = \beta \mathbb{E}_t \pi_{t+1}^w + \kappa_w \mu_t^w,$$

where π_t^w denotes wage inflation, κ_w the slope of the wage Phillips curve and μ_t^w is a time-varying wage markup, given by

$$\widehat{\mu}_t^w = \gamma \widehat{c}_t + \varphi \widehat{n}_t - \widehat{w}_t.$$

We set $\kappa_w = 0.075$ as in Bilbiie et al. (2021). The rest of the model is as above. We relegate the details and the parameterization to Appendix E.

We follow Auclert et al. (2020) and introduce interest-rate smoothing in the Taylor rule:

$$\widehat{i}_t = \rho_i \widehat{i}_{t-1} + (1-\rho_i)\phi\pi_t + \varepsilon_t^{MP}$$

and we set $\rho_i = 0.89$ and $\phi = 1.5$ as estimated by Auclert et al. (2020) and assume the shocks ε_t^{MP} to be completely transitory. Furthermore, we assume price-setting firm managers to be fully rational, giving rise to the standard New Keynesian Phillips curve

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa_\pi \widehat{mc}_t,$$

where $\widehat{mc}_t$ denotes the time-varying price markup.

Hump-shaped responses to monetary policy shock. Figure 8 shows the impulse-response functions of output, real wages and consumption of the two household types to a monetary policy shock for the behavioral HANK model (blue-dashed lines), the behavioral RANK model (orange-dashed-dotted lines), and the rational HANK model (black-solid lines).

The main-takeaway is that the output response to a monetary policy shock is hump-shaped in the behavioral HANK model but neither in its representative agent nor in its rational counterpart. The hump-shaped response is quite remarkable as the model neither features backward-looking expectations nor any form of consumption habits, investment or investment adjustment costs. So, where do these hump-shaped responses come from in the behavioral HANK model?

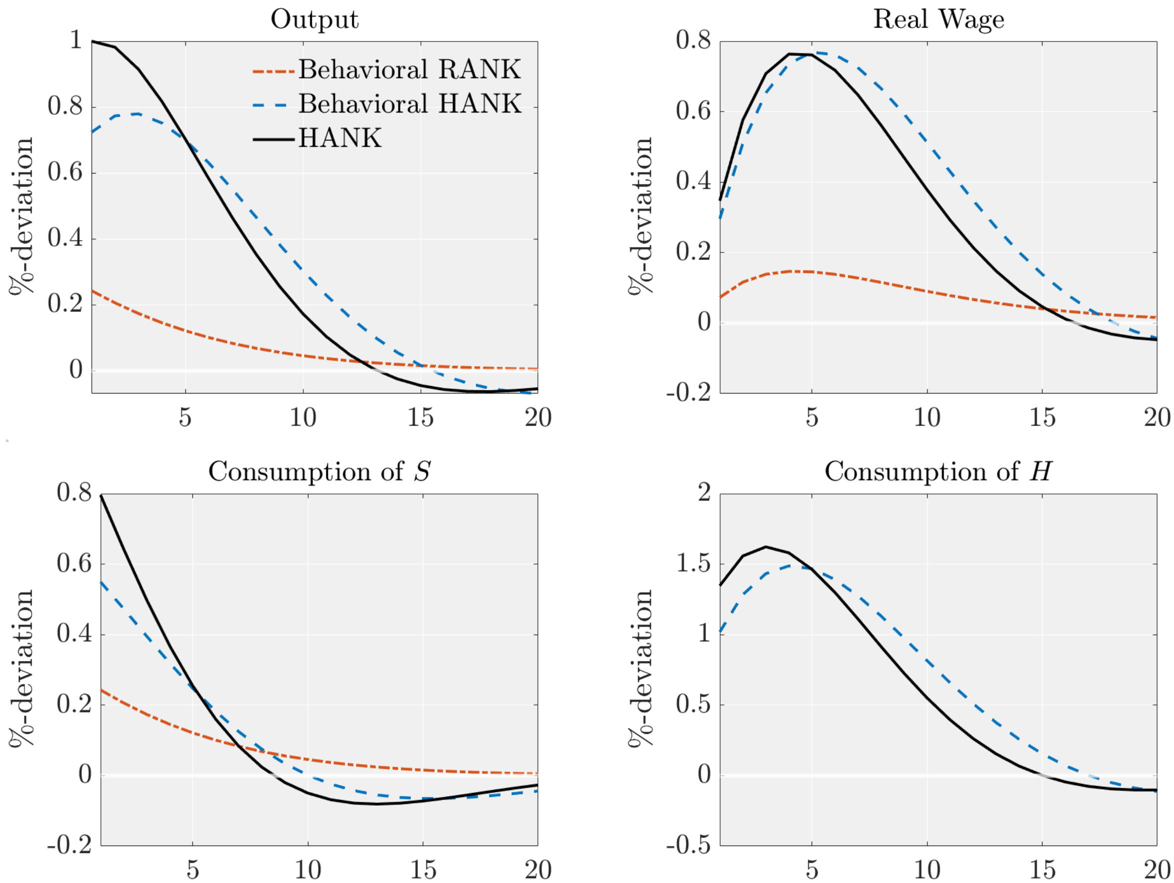
First, note that the introduction of wage rigidity leads to a hump-shaped response in real wages, which is the case in all three models. Since wages determine the H households' income in the rational and the behavioral HANK, their consumption also follows a hump-shape (see lower right figure).[30] Crucial for the overall response, however, are not only the H agents but also the savers.

Under rational expectations, savers perfectly understand how the consumption of H agents responds and what this implies for their idiosyncratic risk induced by type switching. In particular, they understand already on impact that their self-insurance motive will be relaxed for some periods. Thus, the savers immediately cut back on precautionary savings and, thus, their consumption response is already relatively strong on impact. Under bounded rationality, however, savers cognitively discount the future and thus, underreact to the expected increase in wages and, thus, the relaxation of their idiosyncratic risk.[31] Hence, on impact, they do not cut back on precautionary savings as strong as if they would be rational. Going forward, they learn that their self-insurance motive is still (or even more) relaxed. As a consequence, their consumption decreases slower and they have a flatter consumption profile compared to the rationale savers. It is the combination of the flatter consumption

[30] We show the H households' consumption response only for the rational and behavioral HANK model, as the representative agent model does not feature H agents.

[31] The cognitive discounting of the future is what resolves the forward-guidance puzzle in the baseline model. Here, this manifests itself in the fact that consumption responds less on impact in the behavioral model because sticky wages and interest-rate smoothing generate endogenous persistence even though the shock is completely transitory.

Figure 8: Monetary Policy Shock



Note: This figure shows the impulse-response functions of output, real wages and consumption of the two household types to a monetary policy shock in the behavioral HANK model for different $\bar{m}$ and the rational HANK model with sticky wages. The shock size is normalized such that output in the rational model increases by 1pp on impact.

profile of savers and the hump-shaped consumption profile of hand-to-mouth that generates the hump-shaped response of consumption in the aggregate.

As in the rational HANK model, the model with a representative (behavioral) agent does not generate the hump-shaped response. The reason is that without constrained agents, the wage profile does not translate into hump-shaped consumption of (a sub population of) households to begin with. It is thus indeed the *interaction* of household heterogeneity and bounded rationality that produces these hump-shaped responses.

Auclert et al. (2020) argue that many macroeconomic models fail to generate the *micro jumps and macro humps* that we observe in the data, i.e., iMPCs that respond strongly on impact and hump-shaped responses of macroeconomic variables to aggregate shocks. Our results on iMPCs in Section 3.5 as well as the results presented in Figure 8 show how the behavioral HANK model offers a tractable analogue to the full-blown HANK model presented in Auclert et al. (2020).[32]

Forecast-errors dynamics. We now show that the sticky-wage behavioral HANK model generates dynamic forecast errors as observed in survey data. We look at the dynamics of the forecast errors from the behavioral agents' perspective after the monetary policy shock and focus on one-period ahead forecast errors and how they evolve over time. For a variable $\widehat{x}$, the forecast error is defined as

$$FE^{\widehat{x}}_{t+h+1|t+h} \equiv \widehat{x}_{t+h} - \bar{m}\mathbb{E}_{t+h}\left[\widehat{x}_{t+h+1}\right].$$

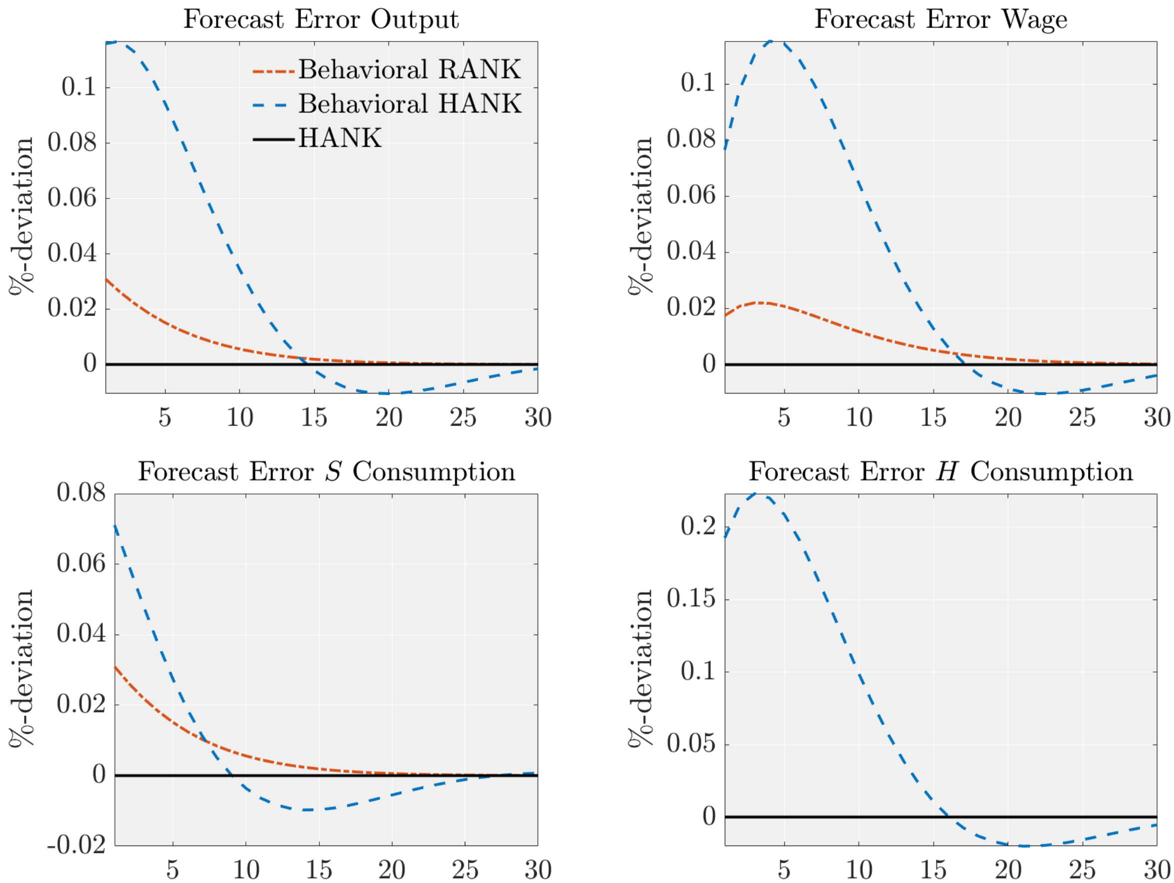
A positive forecast error thus means that the agent's forecast was lower than the actual outcome.

Figure 9 shows the forecast errors of output, the real wage and consumption of the two household types, starting in the first period after the shock. For completeness, the black line at zero shows that under rational expectations, i.e., $\bar{m} = 1$, forecast errors are equal to 0. In the behavioral model, however, this is not the case. In fact, forecast errors are positive in the first quarters after the shock, illustrating the underreaction of the agents' expectations to the shock. This underreaction is what drives the hump-shaped response to a large degree and what is responsible for the dynamics of the savers' consumption response discussed in Figure 8.

After about 15 quarters, however, forecast errors turn negative. Put differently, the behavioral agents' expectations show patterns of delayed overreaction. These dynamic patterns of initial underreaction followed by a delayed overreaction has recently been documented empirically in Angeletos et al. (2021) for unemployment and inflation and in Adam et al. (2020) for housing prices. In fact, Angeletos et al. (2021) argue that looking at the dynamics of forecast errors in response to structural shocks is more informative than other tests considered in earlier papers. The dynamic responses furthermore reconcile seemingly conflicting evidence on underreaction (as in Coibion and Gorodnichenko (2015)) and overreaction (as in Adam et al. (2017) or Kohlhas and Walther (2021)). In contrast to Angeletos et al. (2021) or Adam et al. (2020), the behavioral HANK model with sticky wages generates these dynamic patterns of forecast errors even though the behavioral agents' expectations are purely forward looking. Also the behavioral RANK model as in Gabaix (2020) cannot generate these delayed overreactions. Hence, the interaction of household heterogeneity, cognitive discounting and sticky wages cannot only generates hump-shaped responses of macroeconomic aggregates but

[32] Another way to generate hump-shaped responses of output to monetary policy shocks in the behavioral HANK model is to keep wages fully flexible and to allow for persistence in the monetary policy shocks. In this way, the iMPCs presented in Figure 5 are completely unaltered.

Figure 9: Forecast Error Dynamics



Note: This figure shows the forecast error dynamics of output, the real wage, consumption of savers and of hand-to-mouth households after an expansionary monetary policy shock.

also forecast error dynamics that are fully consistent with recent evidence from household survey expectations.

4.2 Bounded Rationality and Incomplete Information with Learning: An Equivalence Result

In this section, we derive an equivalence result between bounded rationality and incomplete information with learning. In particular, we show how a change in the default value in the behavioral setup leads to the same IS equation as in models with incomplete information and learning (see Angeletos and Huo (2021) and Gallegos (2021)).

To this end, we now assume that behavioral agents anchor their expectations to their *last observation* instead of the steady state values. Models featuring some form of backward-looking behavior indeed tend to match the expectations data coming from household surveys quite well (see, for example, Adam et al. (2017), Adam et al. (2020), Angeletos and Huo (2021), and Angeletos et al. (2021)). The backward-looking components in these models usually arise from an incomplete or noisy information setting as well as some form of (Bayesian) learning. We now show how our bounded rationality setup generates expectations that resemble these aforementioned expectations models.

Proposition 6. *Set the boundedly-rational agents' default value to the variable's past value*

$$X_t^d = X_{t-1}. \tag{26}$$

In this case, the boundedly-rational agent's expectations of X_{t+1} becomes

$$\mathbb{E}_t^{BR}\left[X_{t+1}\right] = (1-\bar{m})X_{t-1} + \bar{m}\mathbb{E}_t\left[X_{t+1}\right]. \tag{27}$$

These backward-looking expectations introduce a backward-looking component into the behavioral IS equation as shown in the following Proposition.

Proposition 7. *In case the behavioral agents' default value is the past value of the respective variable, i.e., $X_t^d = X_{t-1}$, the behavioral HANK IS equation is given by*

$$\widehat{y}_t = \psi_f \mathbb{E}_t \widehat{y}_{t+1} - \psi_c \frac{1}{\gamma}\left(\widehat{i}_t - \mathbb{E}_t \pi_{t+1}\right) + (1-\bar{m})\delta \widehat{y}_{t-1}. \tag{28}$$

Proposition 7 shows that the change in the agents' default value does not change the existing behavioral and heterogeneity coefficients ψ_f and ψ_c. Yet, anchoring to past realizations introduces an additional backward-looking term in the IS equation, similar to models relying on habit persistence.[33] Angeletos and Huo (2021) and Gallegos (2021) derive an IS equation with the same reduced form which, however, is based on an incomplete-information setting and learning. We complement their findings by showing how we can generate the equivalent outcome based on a *behavioral* relaxation of FIRE.

5 Conclusion

We develop a framework that accounts for recent empirical facts on the transmission and effectiveness of monetary and fiscal policy. To arrive at this framework, we introduce bounded rationality in the form of cognitive discounting and household heterogeneity into a sticky

[33] In Appendix G, we discuss how we can calibrate the model to match recent evidence from survey expectations and that the backward-looking model features determinacy under an interest-rate peg and delivers hump-shaped responses of macroeconomic aggregates to monetary shocks through a behavioral channel.

price model. We show that only when both frictions—household heterogeneity and bounded rationality—are present, the model can be reconciled with the data. Thus, it is the interaction of the two frictions that is crucial to arrive at our results. The behavioral HANK model is analytically tractable and we show how it nests a wide array of existing models—none of which can account for all the empirical patterns. What is more, we show that the behavioral HANK model can have different policy implications than its rational counterpart, e.g., when it comes to the timing of monetary policy. The main insights carry over to a quantitative version of our behavioral HANK model. Extending the model by allowing for sticky wages generates hump-shaped responses of macroeconomic aggregates to monetary policy shocks and delivers forecast error dynamics that are consistent with recent survey evidence. We also show how our framework can be used to arrive at an equivalence result of models featuring bounded rationality and models of incomplete information and learning. Altogether, the behavioral HANK model offers a tractable framework to study a broad array of questions in future research.

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A Model Details and Derivations

A.1 Derivation of χ

In Section 2, we stated that

$$\widehat{c}_t^H = \chi \widehat{y}_t, \tag{29}$$

where $\chi \equiv 1 + \varphi \left(1 - \frac{\tau^D}{\lambda}\right)$ is *the* crucial statistic coming from the household heterogeneity friction. We now show how we arrive at equation (29) from the H-households' budget constraint, optimality conditions and market clearing.

The labor-leisure condition ofthe H households is given by

$$(N_t^H)^{\varphi} = W_t (C_t^H)^{-\gamma}, \tag{30}$$

and similarly for the S households. As we focus on the steady state with no inequality, we have that in steady state $C = C^H = C^S$ and $N = N^S = N^H$ and market clearing and the production function imply $Y = C = N$, which we normalize to 1.

Thus, log-linearizing the labor-leisure conditions yields

$$\begin{aligned} \varphi \widehat{n}_t^H &= \widehat{w}_t - \gamma \widehat{c}_t^H \\ \varphi \widehat{n}_t^S &= \widehat{w}_t - \gamma \widehat{c}_t^S. \end{aligned}$$

Since both households work for the same wage, we obtain

$$\varphi \widehat{n}_t^H + \gamma \widehat{c}_t^H = \varphi \widehat{n}_t^S + \gamma \widehat{c}_t^S \tag{31}$$

Log-linearizing the market clearing conditions yields

$$\begin{aligned} \widehat{n}_t &= \lambda \widehat{n}_t^H + (1-\lambda) \widehat{n}_t^S \\ \widehat{c}_t &= \lambda \widehat{c}_t^H + (1-\lambda) \widehat{c}_t^S, \end{aligned}$$

and we further have $\widehat{y}_t = \widehat{c}_t = \widehat{n}_t$. Replacing $\widehat{n}_t^S$ and $\widehat{c}_t^S$ in equation (31) then gives

$$\varphi \widehat{n}_t^H + \gamma \widehat{c}_t^H = (\varphi + \gamma) \widehat{y}_t. \tag{32}$$

The budget constraint of H households (accounting for the fact that bond holdings are zero in equilibrium) is given by

$$C_t^H = W_t N_t^H + \frac{\tau^D}{\lambda} D_t, \tag{33}$$

where we replaced T_t^H with $\frac{\tau^D}{\lambda} D_t$. In log-linearized terms, we get

$$\widehat{c}_t^H = \widehat{w}_t + \widehat{n}_t^H + \frac{\tau^D}{\lambda} \widehat{d}_t, \tag{34}$$

and using that $\widehat{w}_t = -\widehat{d}_t = \varphi \widehat{n}_t^H + \gamma \widehat{c}_t^H$, we get

$$\widehat{c}_t^H = \left(\varphi \widehat{n}_t^H + \gamma \widehat{c}_t^H \right) \left(1 - \frac{\tau^D}{\lambda} \right) + \widehat{n}_t^H. \tag{35}$$

Using (32) to solve for $\widehat{n}_t^H$ and plugging it into (35), we obtain

$$\widehat{c}_t^H = \chi \widehat{y}_t,$$

with $\chi \equiv 1 + \varphi \left(1 - \frac{\tau^D}{\lambda} \right)$, as stated above.

A.2 Derivation of Proposition 1.

Combining equations (11) and (13) with the bounded-rationality setup in equation (18) for $\widehat{x}_t^d = 0$ as X_t^d is given by the steady state, we have

$$\begin{aligned}
\mathbb{E}_t^{BR} \left[\widehat{c}_{t+1}^H \right] &= \bar{m} \mathbb{E}_t \left[\widehat{c}_{t+1}^H \right] = \bar{m} \chi \mathbb{E}_t \left[\widehat{y}_{t+1} \right] \\
\mathbb{E}_t^{BR} \left[\widehat{c}_{t+1}^S \right] &= \bar{m} \mathbb{E}_t \left[\widehat{c}_{t+1}^S \right] = \bar{m} \frac{1 - \lambda \chi}{1 - \lambda} \mathbb{E}_t \left[\widehat{y}_{t+1} \right].
\end{aligned}$$

Plugging these two equations as well as equation (13) into the savers' Euler equation (15) yields

$$\frac{1 - \lambda \chi}{1 - \lambda} \widehat{y}_t = s \bar{m} \frac{1 - \lambda \chi}{1 - \lambda} \mathbb{E}_t \left[\widehat{y}_{t+1} \right] + (1 - s) \bar{m} \chi \mathbb{E}_t \left[\widehat{y}_{t+1} \right] - \frac{1}{\gamma} \left(\widehat{i}_t - \mathbb{E}_t \pi_{t+1} \right).$$

Combining the $\mathbb{E}_t \left[\widehat{y}_{t+1} \right]$ terms and dividing by $\frac{1-\lambda\chi}{1-\lambda}$ yields the following coefficient in front of $\mathbb{E}_t \left[\widehat{y}_{t+1} \right]$:

$$\begin{aligned}
\psi_f \equiv \bar{m} & \left[s + (1 - s) \chi \frac{1 - \lambda}{1 - \lambda \chi} \right] \\
= \bar{m} & \left[1 - 1 + s + (1 - s) \chi \frac{1 - \lambda}{1 - \lambda \chi} \right] \\
= \bar{m} & \left[1 - \frac{1 - \lambda \chi}{1 - \lambda \chi} + s + (1 - s) \chi \frac{1 - \lambda}{1 - \lambda \chi} \right] \\
= \bar{m} & \left[1 - \frac{1 - \lambda \chi}{1 - \lambda \chi} + \frac{(1 - \lambda \chi) s}{1 - \lambda \chi} + (1 - s) \chi \frac{1 - \lambda}{1 - \lambda \chi} \right]
\end{aligned}$$

$$= \bar{m}\left[1 + (\chi - 1)\frac{1-s}{1-\lambda\chi}\right].$$

Defining $\psi_c \equiv \frac{1-\lambda}{1-\lambda\chi}$ yields the behavioral HANK IS equation in Proposition 1:

$$\widehat{y}_t = \psi_f \mathbb{E}_t \widehat{y}_{t+1} - \psi_c \frac{1}{\gamma}\left(\widehat{i}_t - \mathbb{E}_t \pi_{t+1}\right).$$

A.3 Derivation of Proposition 2.

The first part comes from the fact that amplification is defined as

$$\frac{1-\lambda}{1-\lambda\chi} > 1,$$

which requires $\chi > 1$.

For the second part, recall how we model a forward guidance experiment (following Bilbiie (2021)). We assume a Taylor coefficient of 0, i.e., $\phi = 0$, such that the nominal interest rate is given by $\widehat{i}_t = \varepsilon_t^{MP}$. Replacing inflation using the Phillips curve (16), i.e., $\pi_t = \kappa \widehat{y}_t$, we can re-write the behavioral HANK IS equation from Proposition 1 as

$$\begin{aligned}
\widehat{y}_t &= \psi_f \mathbb{E}_t \widehat{y}_{t+1} - \psi_c \frac{1}{\gamma}\left(\varepsilon_t^{MP} - \kappa \mathbb{E}_t \widehat{y}_{t+1}\right) \\
&= \left(\psi_f + \psi_c \frac{1}{\gamma}\kappa\right) \mathbb{E}_t \widehat{y}_{t+1} - \psi_c \frac{1}{\gamma}\varepsilon_t^{MP}
\end{aligned}$$

The forward guidance puzzle is ruled out if and only if

$$\left(\psi_f + \psi_c \frac{1}{\gamma}\kappa\right) < 1,$$

which is the same as the condition stated in Proposition 2:

$$\bar{m}\delta + \frac{1}{\gamma}\frac{1-\lambda}{1-\lambda\chi}\kappa < 1.$$

Solving this for $\bar{m}$ yields

$$\bar{m} < \frac{1 - \frac{1-\lambda}{\gamma(1-\lambda\chi)}\kappa}{\delta},$$

which completes Proposition 2.

A.4 Derivation of Proposition 3.

Replacing $\widehat{i}_t$ by $\phi\pi_t = \phi\kappa\widehat{y}_t$ and $\mathbb{E}_t\pi_{t+1} = \kappa\mathbb{E}_t\widehat{y}_{t+1}$ in the IS equation (19), we get

$$\widehat{y}_t = \psi_f \mathbb{E}_t \widehat{y}_{t+1} - \psi_c \frac{1}{\gamma}\left(\phi\kappa\widehat{y}_t - \kappa\mathbb{E}_t\widehat{y}_{t+1}\right),$$

which can be re-written as

$$\widehat{y}_t \left(1 + \psi_c \frac{1}{\gamma}\phi\kappa\right) = \mathbb{E}_t\widehat{y}_{t+1}\left(\psi_f + \psi_c \frac{1}{\gamma}\kappa\right).$$

Dividing by $\left(1 + \psi_c \frac{1}{\gamma}\phi\kappa\right)$ and plugging in for ψ_f and ψ_c yields

$$\widehat{y}_t = \frac{\bar{m}\delta + \frac{(1-\lambda)\kappa}{\gamma(1-\lambda\chi)}}{1 + \kappa\phi\frac{1}{\gamma}\frac{(1-\lambda)}{1-\lambda\chi}}\mathbb{E}_t\widehat{y}_{t+1}.$$

To obtain determinacy, the term in front of $\mathbb{E}_t\widehat{y}_{t+1}$ has to be smaller than 1. Solving this for ϕ yields

$$\phi > \phi^* = 1 + \frac{\bar{m}\delta - 1}{\frac{\kappa}{\gamma}\frac{1-\lambda}{1-\lambda\chi}}, \tag{36}$$

which is the condition in Proposition 3. This illustrates how bounded rationality raises the likelihood that the Taylor principle ($\phi^* = 1$) is sufficient for determinacy, as the Taylor principle can only hold if

$$\bar{m}\delta \leq 1.$$

In the rational model, this boils down to $\delta \leq 1$. However, the Taylor principle can be sufficient under bounded rationality, i.e., $\bar{m} < 1$, even when $\delta > 1$, thus, even when allowing for amplification. Note that we could also express condition (36) as

$$\phi > \phi^* = 1 + \frac{\psi_f - 1}{\frac{\kappa}{\gamma}\psi_c}.$$

Proposition 3 can be extended to allow for Taylor rules of the form

$$\widehat{i}_t = \phi_\pi \pi_t + \phi_y \widehat{y}_t$$

and in which the behavioral agents do not have rational expectations about the real interest rate but rather perceive the real interest rate to be equal to

$$\widehat{r}_t^{BR} \equiv \widehat{i}_t - \bar{m}^r \mathbb{E}_t \pi_{t+1},$$

where $\bar{m}^r$ can be equal to $\bar{m}$ or can potentially differ from it (if it equals 1, we are back to the case in which the behavioral agent is rational with respect to real interest rates).

Combining the static Phillips Curve with the generalized Taylor rule and the behavioral HANK IS equation, it follows that

$$\widehat{y}_t = \frac{\omega_f + \frac{\kappa}{\gamma}\omega_c \bar{m}^r}{1 + \frac{\omega_c}{\gamma}\left(\kappa\phi_\pi + \phi_y\right)} \mathbb{E}_t \widehat{y}_{t+1}. \tag{37}$$

From equation (37), it follows that we need

$$\phi_\pi > \bar{m}^r - \phi_y + \frac{\omega_f - 1}{\omega_c \frac{\kappa}{\gamma}} = \bar{m}^r - \phi_y + \frac{\bar{m}\delta - 1}{\frac{1-\lambda}{1-\chi\lambda}\frac{\kappa}{\gamma}} \tag{38}$$

for the model to feature a determinate, locally unique equilibrium. Condition (38) shows that both, $\bar{m}^r < 1$ and $\phi_y > 0$, weaken the condition in Proposition 3. Put differently, bounded rationality with respect to the real rate or a Taylor rule that responds to changes in output, both relax the condition on ϕ_π to yield determinacy.

A.5 Derivation of Proposition 7

To prove Proposition 7, we start from the Euler equation (15). For simplicity, we denote $\widehat{r}_t \equiv \widehat{i}_t - \mathbb{E}_t \pi_{t+1}$ as the real rate. Plugging in for $\widehat{c}_t^S$, $\widehat{c}_{t+1}^S$ and $\widehat{c}_{t+1}^H$ from equations (11) and (13), we get

$$\widehat{y}_t = s\mathbb{E}_t^{BR}\left[\widehat{y}_{t+1}\right] + (1-s)\frac{1-\lambda}{1-\lambda\chi}\mathbb{E}_t^{BR}\left[\widehat{y}_{t+1}\right] - \psi_c \widehat{r}_t,$$

which can be re-written as

$$\widehat{y}_t = \delta\mathbb{E}_t^{BR}\left[\widehat{y}_{t+1}\right] - \psi_c \widehat{r}_t.$$

Now, using the expectations setup from Proposition 6, we get $\delta\mathbb{E}_t^{BR}\left[\widehat{y}_{t+1}\right] = (1-\bar{m})\delta\widehat{y}_{t-1} + \bar{m}\delta\mathbb{E}_t\left[\widehat{y}_{t+1}\right]$ which proves Proposition 7.

A.6 Cognitive Discounting of the State Vector

In Section 2, we assume that cognitive discounting applies to all variables, which differs slightly from the assumption in Gabaix (2020) who assumes that cognitive discounting applies to the *state* of the economy (exogenous shocks as well as announced monetary and fiscal policies). He then proves (Lemma 1 in Gabaix (2020)) how cognitive discounting applies as a result (instead of as an assumption) to all future variables, including future consumption choices. For completeness, we show in this section how our results are unaffected when following the approach in Gabaix (2020).

Let X_t denote the (de-meaned) state vector which evolves as

$$X_{t+1} = G^X \left(X_t, \varepsilon_{t+1} \right), \tag{39}$$

where G^X denotes the transition function of X in equilibrium and ε are zero-mean innovations. Linearizing equation (39) yields

$$X_{t+1} = \Gamma X_t + \varepsilon_{t+1}, \tag{40}$$

where ε_{t+1} might have been renormalized. The assumption in Gabaix (2020) is that the behavioral agent perceives the state vector to follow

$$X_{t+1} = \bar{m} G^X (X_t, \varepsilon_{t+1}), \tag{41}$$

or in linearized terms

$$X_{t+1} = \bar{m} \left(\Gamma X_t + \varepsilon_{t+1} \right). \tag{42}$$

The expectation of the boundedly-rational agent of X_{t+1} is thus $\mathbb{E}_t^{BR} \left[X_{t+1} \right] = \bar{m} \mathbb{E}_t \left[X_{t+1} \right] = \bar{m} \Gamma X_t$. Iterating forward, it follows that $\mathbb{E}_t^{BR} \left[X_{t+k} \right] = \bar{m}^k \mathbb{E}_t \left[X_{t+k} \right] = \bar{m}^k \Gamma^k X_t$.

Now, consider any variable $z(X_t)$ with $z(0) = 0$ (e.g., demeaned consumption of the saver type $C^S(X_t)$). Linearizing $z(X)$, we obtain $z(X) = b_X^z X$ for some b_X^z and thus

$$\begin{aligned}
\mathbb{E}_t^{BR} \left[z(X_{t+k}) \right] &= \mathbb{E}_t^{BR} \left[b_X^z X_{t+k} \right] \\
&= b_X^z \mathbb{E}_t^{BR} \left[X_{t+k} \right] \\
&= b_X^z \bar{m}^k \mathbb{E}_t \left[X_{t+k} \right] \\
&= \bar{m}^k \mathbb{E}_t \left[b_X^z X_{t+k} \right] \\
&= \bar{m}^k \mathbb{E}_t \left[z(X_{t+k}) \right].
\end{aligned}$$

For example, expected consumption of savers tomorrow (in linearized terms) is given by

$$\mathbb{E}_t^{BR} \left[\widehat{c}^S (X_{t+1}) \right] = \bar{m} \mathbb{E}_t \left[\widehat{c}^S (X_{t+1}) \right], \tag{43}$$

which we denote in the main text as

$$\mathbb{E}_t^{BR} \left[\widehat{c}_{t+1}^S \right] = \bar{m} \mathbb{E}_t \left[\widehat{c}_{t+1}^S \right]. \tag{44}$$

Now, take the linearized Euler equation (15) of the savers:

$$\widehat{c}_t^S = s\mathbb{E}_t^{BR}\left[\widehat{c}_{t+1}^S\right] + (1-s)\mathbb{E}_t^{BR}\left[\widehat{c}_{t+1}^H\right] - \frac{1}{\gamma}\widehat{r}_t, \tag{45}$$

where $\widehat{r}_t \equiv \widehat{i}_t - \mathbb{E}_t\pi_{t+1}$.

Using the notation in Gabaix (2020), we can write the Euler equation as

$$\widehat{c}^S(X_t) = s\mathbb{E}_t^{BR}\left[\widehat{c}^S(X_{t+1})\right] + (1-s)\mathbb{E}_t^{BR}\left[\widehat{c}^H(X_{t+1})\right] - \frac{1}{\gamma}\widehat{r}(X_t). \tag{46}$$

Now, applying the results above, we obtain

$$\widehat{c}^S(X_t) = s\bar{m}\mathbb{E}_t\left[\widehat{c}^S(X_{t+1})\right] + (1-s)\bar{m}\mathbb{E}_t\left[\widehat{c}^H(X_{t+1})\right] - \frac{1}{\gamma}\widehat{r}(X_t), \tag{47}$$

which after writing $\widehat{c}^S(X_t)$, $\widehat{c}^S(X_{t+1})$ and $\widehat{c}^H(X_{t+1})$ in terms of total output yields exactly the behavioral HANK IS equation in Proposition 1.

A.7 Microfounding $\bar{m}$

Gabaix (2020) shows how to microfound $\bar{m}$ stemming from a noisy signal extraction problem in the case of a representative agent. Following these lines, we show how this signal-extraction problem generates a set-up in which the family head behaves as if she was boundedly rational.

The (linearized) law of motion of the state variable, X_t, is given by $X_{t+1} = \Gamma X_t + \varepsilon_{t+1}$ (a similar reasoning extends to the non-linearized case), where X has been demeaned. Now assume that every agent j within the family of savers (the expectations of the hand-to-mouth agents are irrelevant) receives a noisy signal of X_{t+1}, S_{t+1}^j, given by

$$S_{t+1}^j = \begin{cases} X_{t+1} & \text{with probability } p \\ X'_{t+1} & \text{with probability } 1-p \end{cases}$$

where X'_{t+1} is an i.i.d. draw from the unconditional distribution of X_{t+1}, which has an unconditional mean of zero. In words, with probability p the agent j receives perfectly precise information and with probability $1-p$ agent j receives a signal realization that is completely uninformative. A fully-informed rational agent would have $p = 1$.

The conditional mean of X_{t+1}, given the signal S^j_{t+1}, is given by

$$X^e_{t+1} \equiv \mathbb{E}\left[X_{t+1}|S_{t+1} = s^j_{t+1}\right] = p \cdot s^j_{t+1}.^{34}$$

The intuition is that the signal distribution is such that the agent either receives a perfectly precise signal or a completely uninformative signal. As the perfectly-precise signal arrives with probability p and the unconditional mean is zero, it follows that the agent puts a weight p on the signal.

Furthermore, we have

$$\mathbb{E}\left[S_{t+1}|X_{t+1}\right] = pX_{t+1} + (1-p)\mathbb{E}\left[X'_{t+1}\right] = pX_{t+1}.$$

So, it follows that the *average* expectation of X_{t+1} within the family is given by

$$\begin{aligned}\mathbb{E}\left[X^e_{t+1}(S_{t+1})|X_{t+1}\right] &= \mathbb{E}\left[p \cdot S_{t+1}|X_{t+1}\right] \\ &= p \cdot \mathbb{E}\left[S_{t+1}|X_{t+1}\right] \\ &= p^2 X_{t+1}.\end{aligned}$$

Defining $\bar{m} \equiv p^2$ and since $X_{t+1} = \Gamma X_t + \varepsilon_{t+1}$, we have that the family head perceives the law of motion of X to equal

$$X_{t+1} = \bar{m}\left(\Gamma X_t + \varepsilon_{t+1}\right), \tag{48}$$

as imposed in equation (42). The boundedly-rational expectation of X_{t+1} is then given by

$$\mathbb{E}^{BR}_t\left[X_{t+1}\right] = \bar{m}\mathbb{E}_t\left[X_{t+1}\right].$$

[34] To see this, note that the joint distribution of (X_{t+1}, S^j_{t+1}) is

$$f(x_{t+1}, s^j_{t+1}) = pg(s^j_{t+1})\delta_{s^j_{t+1}}(x_{t+1}) + (1-p)g(s^j_{t+1})g(x_{t+1}),$$

where $g(X_{t+1})$ denotes the distribution of X_{t+1} and δ is the Dirac function. Given that the unconditional mean of X_{t+1} is 0, i.e., $\int x_{t+1}g(x_{t+1})dx_{t+1} = 0$, it follows that

$$\begin{aligned}\mathbb{E}_t\left[X_{t+1}|S^j_{t+1} = s^j_{t+1}\right] &= \frac{\int x_{t+1}f(x_{t+1}, s^j_{t+1})dx_{t+1}}{\int f(x_{t+1}, s^j_{t+1})dx_{t+1}} \\ &= \frac{pg(s^j_{t+1})s^j_{t+1} + (1-p)g(s^j_{t+1})\int x_{t+1}g(x_{t+1})dx_{t+1}}{g(s^j_{t+1})} \\ &= ps^j_{t+1}.\end{aligned}$$

B Calibration

Parameter	Value	Source/Target
	HANK Parameters	
γ	1	Bilbiie (2020)
κ	0.02	Bilbiie (2020)
χ	1.48	Bilbiie (2020)
λ	0.33	Bilbiie (2020)
s	$0.8^{1/4}$	Bilbiie (2020)
	Behavioral Parameter	
$\bar{m}$	0.85	Gabaix (2020)

Table 1: Baseline calibration.

Our baseline calibration is summarized in Table 1. The values for γ and κ are directly taken from Bilbiie (2021, 2020) and are quite standard in the literature. Gabaix (2020), on the other hand, sets $\kappa = 0.11$ and $\gamma = 5$. Even though these coefficients differ quite substantially from our baseline calibration, note that our results would barely be affected by this. To see this, note that *amplification* is only determined by λ and χ, both independent of κ and γ. The determinacy condition on the other hand depends on both, κ and γ, but what ultimately matters is the fraction $\frac{\kappa}{\gamma}$ (see Proposition 3). As κ and γ are both approximately five times larger in Gabaix (2020) compared to Bilbiie (2021) and our baseline calibration, the fraction is approximately the same and thus, the determinacy region under an interest-rate peg remains unchanged.

The household heterogeneity parameters, χ, λ and s are also standard in the analytical HANK literature (see Bilbiie (2020)). The most important assumption for our qualitative results in Section 3 is $\chi > 1$, which is empirically supported. Patterson (2019) provides empirical evidence for the countercyclicality of inequality. Coibion et al. (2017), Mumtaz and Theophilopoulou (2017), Ampudia et al. (2018) and Samarina and Nguyen (2019) provide evidence of countercyclical inequality conditional on monetary policy shocks. Almgren et al. (2019) show that output in countries with higher shares of hand-to-mouth households responds more strongly to monetary policy shocks which, through the lens of the model, implies countercyclical inequality.

For figure 5, i.e., to compute the iMPCs we choose a yearly calibration with $s = 0.8$ and $\beta = 0.95$ (this calibration is close to the iMPC exercise in Bilbiie (2021) but while he fixes χ to match the empirically-observed iMPCs, we vary χ together with $\bar{m}$ to examine their joint effects on iMPCs).

The Cognitive Discounting Parameter $\bar{m}$. The cognitive discounting parameter $\bar{m}$ is set to 0.85, as in Gabaix (2020) and Benchimol and Bounader (2019). Fuhrer and Rudebusch (2004), for example, estimate an IS equation and find that $\bar{m}\delta \approx 0.65$, which together with $\delta > 1$, would imply a $\bar{m}$ much lower than 0.85 and especially our determinacy results would be even stronger under such a calibration. Note, that the calibration of the backward-looking behavioral HANK model in Section 4.2, which is based on household survey expectations and taken from Angeletos and Huo (2021), is close to the estimation results from Fuhrer and Rudebusch (2004).

Another way to calibrate $\bar{m}$ (as pointed out in Gabaix (2020)) is to interpret the estimates in Coibion and Gorodnichenko (2015) through the "cognitive-discounting lens". They regress forecast errors on forecast revisions

$$x_{t+h} - F_t x_{t+h} = c + b^{CG}\left(F_t x_{t+h} - F_{t-1} x_{t+h}\right) + u_t,$$

where $F_t x_{t+h}$ denotes the forecast at time t of variable x, h periods ahead. Focusing on inflation, they find that $b^{CG} > 0$ in consensus forecasts, pointing to *underreaction* (similar results are, for example, found in Angeletos et al. (2021) and Adam et al. (2020) for other variables).

In the model, the law of motion of x is $x_{t+1} = \Gamma\left(x_t + \varepsilon_{t+1}\right)$ whereas the behavioral agents perceive it to be $x_{t+1} = \bar{m}\Gamma\left(x_t + \varepsilon_{t+1}\right)$. It follows that $F_t x_{t+h} = \left(\bar{m}\Gamma\right)^h x_t$ and thus, forecast revisions are equal to

$$\begin{aligned} F_t x_{t+h} - F_{t-1} x_{t+h} &= \left(\bar{m}\Gamma\right)^h x_t - \left(\bar{m}\Gamma\right)^{h+1} x_{t-1} \\ &= \left(\bar{m}\Gamma\right)^h \Gamma(1-\bar{m}) x_{t-1} + \left(\bar{m}\Gamma\right)^h \varepsilon_t. \end{aligned}$$

The forecast error is given by

$$x_{t+h} - F_t x_{t+h} = \Gamma^h(1-\bar{m}^h)\Gamma x_{t-1} + \Gamma^h(1-\bar{m}^h)\varepsilon_t + \sum_{j=0}^{h-1} \Gamma^j \varepsilon_{t+h-j},$$

where $\sum_{j=0}^{h-1} \Gamma^j \varepsilon_{t+h-j}$ is the rational expectations forecast error. Gabaix (2020) shows that b^{CG} is bounded below $b^{CG} \geq \frac{1-\bar{m}^h}{\bar{m}^h}$, showing that $\bar{m} < 1$ yields $b^{CG} > 0$, as found empirically. When replacing the weak inequality with an equality, we get

$$\bar{m}^h = \frac{1}{1+b^{CG}}.$$

Most recently, Angeletos et al. (2021) estimate b^{CG} (focusing on a horizon $h = 3$) to lie between $b^{CG} \in [0.74, 0.81]$ for unemployment forecasts and $b^{CG} \in [0.3, 1.53]$ for inflation, depending on the considered period (see their Table 1). These estimates imply $\bar{m} \in [0.82, 0.83]$ for unemployment and $\bar{m} \in [0.73, 0.92]$ for inflation, and are thus close to our preferred value of 0.85. Note, however, that these estimates pertain to professional forecasters and should therefore be seen as upper bounds on $\bar{m}$.

And since the focus of the paper is to understand the role of $\bar{m}$, we often vary $\bar{m}$ anyway instead of focusing on one particular value.

C Extensions

C.1 Allowing for Steady State Inequality.

So far, we have assumed that there is no steady state inequality, i.e., $C^H = C^S$. In the following, we relax this assumption and denote steady state inequality by $\Omega \equiv \frac{C^S}{C^H}$. Recall the savers' Euler equation

$$\left(C_t^S\right)^{-\gamma} = \beta R_t \mathbb{E}_t^{BR}\left[s\left(C_t^S\right)^{-\gamma} + (1-s)\left(C_t^H\right)^{-\gamma}\right],$$

from which we can derive the steady state real rate

$$R = \frac{1}{\beta(s + (1-s)\Omega^\gamma)}.$$

Log-linearizing the Euler equation yields

$$\widehat{c}_t^S = \beta R \bar{m}\left[s\mathbb{E}_t\widehat{c}_{t+1}^S + (1-s)\Omega^\gamma \mathbb{E}_t\widehat{c}_{t+1}^H\right] - \frac{1}{\gamma}\left(\widehat{i}_t - \mathbb{E}_t\pi_{t+1}\right).$$

Combining this with the consumption functions and the steady state real rate yields the IS equation

$$\widehat{y}_t = \bar{m}\tilde{\delta}\mathbb{E}_t\widehat{y}_{t+1} - \frac{1}{\gamma}\frac{1-\lambda}{1-\lambda\chi}\left(\widehat{i}_t - \mathbb{E}_t\pi_{t+1}\right), \tag{49}$$

with

$$\tilde{\delta} \equiv 1 + (\chi - 1)\frac{(1-s)\Omega^\gamma}{s + (1-s)\Omega^\gamma}\frac{1}{1-\lambda\chi}.$$

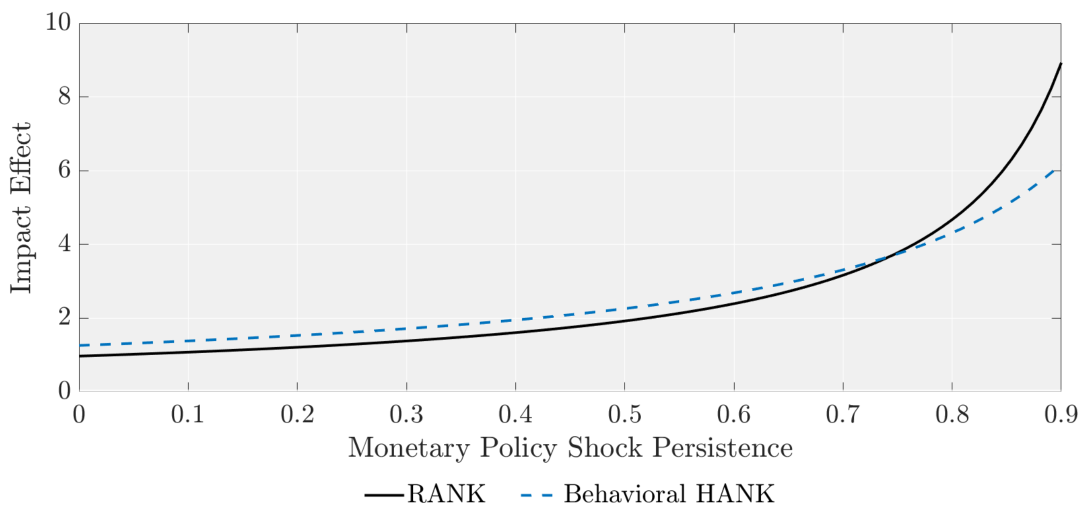
From a qualitative perspective, the whole analysis in the paper could be carried out with $\tilde{\delta}$ instead of δ. Quantitatively the differences are small as well. For example, if we set $\Omega = 1.5$, we get $\tilde{\delta} = 1.074$ instead of $\delta = 1.051$. Thus, we need $\bar{m} < 0.91$ instead of $\bar{m} < 0.93$ for determinacy under a peg.

C.2 Persistent Monetary Policy Shocks

In the main text in Section 3, we illustrated the resolution of the Catch-22 by considering i.i.d. monetary policy shocks (following Bilbiie (2021)). The behavioral HANK model delivers initial amplification of these monetary shocks but the effects decrease with the horizon of the shock, i.e., the behavioral HANK model resolves the forward guidance puzzle. Another way to see this is by considering persistent shocks.

Figure 10 illustrates this. The figure shows the response of output in period t to a shock in period t for different degrees of persistence (x-axis). The black-solid line shows the output response in RANK and the blue-dashed line in the behavioral HANK. The forward guidance puzzle in RANK manifests itself in the sense that highly persistent shocks have stronger effects in RANK than in the behavioral HANK. Persistent shocks are basically a form of forward guidance and thus, with high enough persistence in the shocks, the RANK model predicts stronger effects than the behavioral HANK model.

Figure 10: Initial Output Response for Varying Degrees of the Persistence



Note: This figure shows the initial output response to monetary policy shocks with different degrees of persistence.

C.3 Forward-Looking NKPC and Real Interest Rates

In the main part of the paper, we made the assumption that agents are rational with respect to real interest rates (as in Gabaix (2020)) and assumed a static Phillips Curve (as in Bilbiie

(2021)). We now show that the results are barely affected when considering a forward-looking New Keynesian Phillips Curve (NKPC) and that agents are also boundedly rational with respect to real rates. Gabaix (2020) derives the NKPC under bounded rationality and shows that it takes the following form:

$$\pi_t = \beta M^f \mathbb{E}_t \pi_{t+1} + \kappa \widehat{y}_t,$$

with

$$M^f \equiv \bar{m}\left(\theta + \frac{1-\beta\theta}{1-\beta\theta\bar{m}}(1-\theta)\right),$$

where $1-\theta$ captures the Calvo probability of price adjustment.

Taking everything together (including the bounded rationality with respect to real interest rates), the model can be summarized by the following three equations:

$$\begin{aligned}
\widehat{y}_t &= \psi_f \mathbb{E}_t \widehat{y}_{t+1} - \psi_c \frac{1}{\gamma}\left(\widehat{i}_t - \bar{m}\mathbb{E}_t \pi_{t+1}\right) \\
\pi_t &= \beta M^f \mathbb{E}_t \pi_{t+1} + \kappa \widehat{y}_t \\
\widehat{i}_t &= \phi \pi_t.
\end{aligned}$$

Plugging the Taylor rule into the IS equation, we can write everything in matrix form:

$$\begin{pmatrix} \mathbb{E}_t \pi_{t+1} \\ \mathbb{E}_t \widehat{y}_{t+1} \end{pmatrix} = \underbrace{\begin{pmatrix} \frac{1}{\beta M^f} & -\frac{\kappa}{\beta M^f} \\ \frac{\psi_c}{\gamma\psi_f}\left(\phi - \frac{\bar{m}}{\beta M^f}\right) & \frac{1}{\psi_f}\left(1 + \frac{\psi_c \bar{m}\kappa}{\gamma\beta M^f}\right) \end{pmatrix}}_{\equiv A} \begin{pmatrix} \pi_t \\ \widehat{y}_t \end{pmatrix}. \tag{50}$$

For determinacy, we need

$$det(A) > 1; \quad det(A) - tr(A) > -1; \quad det(A) + tr(A) > -1.$$

The last condition is always satisfied. The first two conditions are satisfied if and only if

$$\phi > \max\left\{ \frac{\beta\delta M^f \bar{m} - 1}{\frac{\kappa}{\gamma}\frac{1-\lambda}{1-\lambda\chi}}, \bar{m} + \frac{(\delta\bar{m}-1)(1-\beta M^f)}{\frac{\kappa}{\gamma}\frac{1-\lambda}{1-\lambda\chi}} \right\}.$$

In the case of a static Phillips curve but bounded rationality with respect to the real rate, the second condition is the crucial one. To capture the static Phillips curve, we can simply set $M^f = 0$. In this case, it follows that we have a uniquely-determined (bounded) equilibrium for $\phi > -3.22$. Thus, the condition is even weaker than in the main part of the paper.

If we allow for a forward-looking Phillips curve and using the same calibration as in the main text and relying on Gabaix (2020) for the two newly-introduced parameters, $\theta = 0.875$ and $\beta = 0.99$, it follows that we have determinacy even under an interest rate peg for our baseline calibration with $\bar{m} = 0.85$.

D Details on Intertemporal MPCs

In this section, we derive the iMPCs discussed in Section 3.5. Defining Y_t^j as type j's disposable income, we can write the households' budget constraints as

$$C_t^H = Y_t^H + \frac{1-s}{\lambda} R_t B_t$$
$$C_t^S + \frac{1}{1-\lambda} B_{t+1} = Y_t^S + \frac{s}{1-\lambda} R_t B_t,$$

where R_t denotes the real interest rate and B_t real bonds. Log-linearizing the two budget constraints around the zero-liquidity steady state and $R = \beta^{-1}$ yields

$$\widehat{c}_t^H = \widehat{y}_t^H + \frac{1-s}{\lambda} \beta^{-1} b_t \tag{51}$$
$$\widehat{c}_t^S + \frac{1}{1-\lambda} b_{t+1} = \widehat{y}_t^S + \frac{s}{1-\lambda} \beta^{-1} b_t, \tag{52}$$

where b_t denotes real bonds in shares of steady state output. Aggregating (51) and (52) delivers

$$\widehat{c}_t = \tilde{y}_t + \beta^{-1} b_t - b_{t+1}, \tag{53}$$

where $\tilde{y}_t$ denotes aggregate disposable income.

By plugging equations (51) and (52) into the savers' Euler equation (15), we can derive the dynamics of liquid assets b_t (ignoring changes in the real rate as this is a partial equilibrium exercise):

$$\begin{aligned} &\mathbb{E}_t b_{t+2} - b_{t+1} \left[\frac{1}{s\bar{m}} + \beta^{-1} s + \frac{(1-s)^2 \beta^{-1} (1-\lambda)}{s\lambda} \right] + \frac{\beta^{-1}}{\bar{m}} b_t = \\ &(1-\lambda) \mathbb{E}_t \widehat{y}_{t+1}^S + \frac{1-s}{s} (1-\lambda) \mathbb{E}_t \widehat{y}_{t+1}^H - \frac{1-\lambda}{s\bar{m}} \widehat{y}_t^S. \end{aligned} \tag{54}$$

Note that a change in total disposable income by one changes the hand-to-mouth households' disposable income by χ and the savers' disposable income by $\frac{1-\lambda\chi}{1-\lambda}$.

Let us denote the right-hand side of equation (54) by $-\mathbb{E}_t \widehat{z}_t$. Factorizing the left-hand side and letting F denote the forward-operator, it follows that

$$(F - \mu_1)(F - \mu_2) \mathbb{E}_t b_t = -\mathbb{E}_t \widehat{z}_t, \tag{55}$$

where μ_1 and μ_2 denote the roots of the characteristic equation

$$\mathbb{E}_t b_{t+2} - \phi_1 b_{t+1} - \phi_2 b_t = 0, \tag{56}$$

where

$$\phi_1 \equiv \left[\frac{1}{s\bar{m}} + \beta^{-1}s + \frac{(1-s)^2\beta^{-1}(1-\lambda)}{s\lambda}\right] \tag{57}$$

and

$$\phi_2 \equiv -\frac{\beta^{-1}}{\bar{m}}. \tag{58}$$

Thus, the roots are given by

$$\mu_{1,2} = \frac{\phi_1 \pm \sqrt{\phi_1^2 + 4\phi_2}}{2}. \tag{59}$$

It follows that

$$\begin{aligned} b_{t+1} &= \mu_1 b_t - (F - \mu_2)^{-1}\mathbb{E}_t \widehat{z}_t \\ &= \mu_1 b_t + \frac{\mu_2^{-1}}{1 - F\mu_2^{-1}}\mathbb{E}_t \widehat{z}_t. \end{aligned}$$

Note that $\mathbb{E}_t \widehat{z}_t$ can be written as $\frac{1-\lambda\chi}{s}\left(\delta\mathbb{E}_t\widehat{y}_{t+1} - \frac{1}{\bar{m}}\widehat{y}_t\right)$. Without loss of generality, we let $\mu_2 > \mu_1$ and we have $\mu_2 > 1$. We have $(1 - F\mu_2^{-1})^{-1} = \sum_{l=0}^{\infty}\mu_2^{-l}F^l$. Thus, we end up with

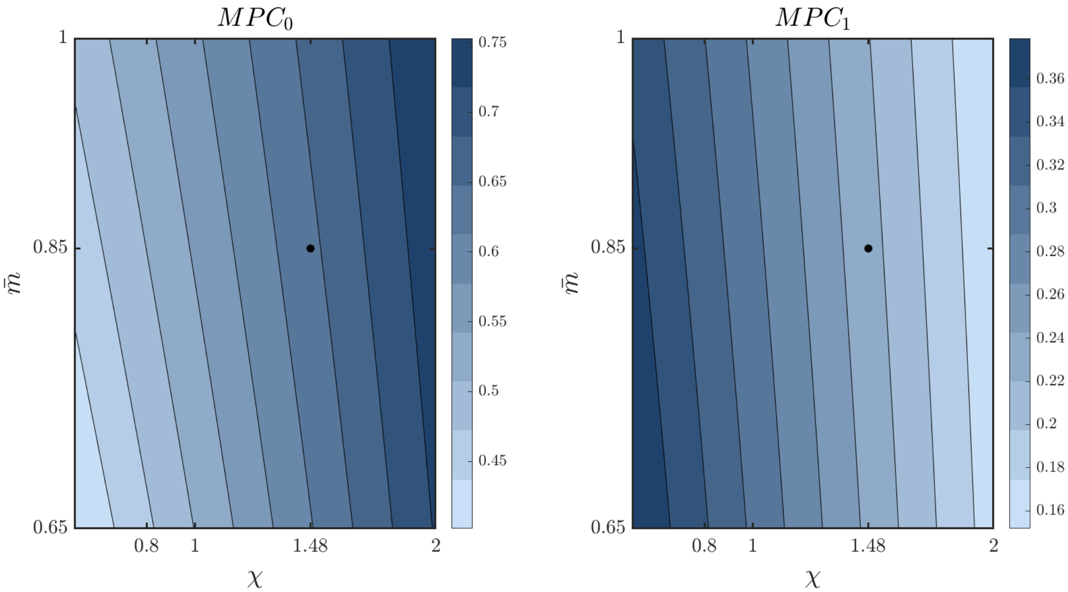
$$b_{t+1} = \mu_1 b_t + \frac{1-\lambda\chi}{s}\sum_{l=0}^{\infty}\mu_2^{-(l+1)}\mathbb{E}_t\left(\frac{1}{\bar{m}}\widehat{y}_{t+l} - \delta\widehat{y}_{t+1+l}\right). \tag{60}$$

Taking derivatives with respect to $\widehat{y}_{t+k}$ yields Proposition 5.

iMPCs and the Role of Idiosyncratic Risk. In Figure 11, we plot he MPCs in the year of the income windfall (left panel) and the first year after the windfall (right panel) for a relatively high idiosyncratic risk of $1-s = 0.5$. The high probability of becoming hand-to-mouth flips the role of $\bar{m}$ for the MPC_1 compared to our baseline calibration as discussed in Section 3.5. The reason being that the behavioral savers save a relatively large amount of the received income windfall in period 0 as they cognitively discount the decrease in their future marginal utility. Thus, they end up with relatively more disposable income in year 1. Now, given the relatively high probability of type switching, there are many savers who end up being hand-to-mouth in year 1 after the income windfall. As they are hand-to-mouth, they consume their previously-accumulated savings which increases the MPC_1. The more behavioral the savers are, i.e., the lower $\bar{m}$ is, the more pronounced this effect and hence, a lower $\bar{m}$ increases the MPC_1 in the case of a relatively high $1-s$.

iMPCs for more than two periods. Figure 12 plots the MPCs for the year of the income windfall as well as the five consecutive years for different degrees of rationality. As discussed in section 3.5, under our benchmark calibration, the rational model predicts somewhat larger

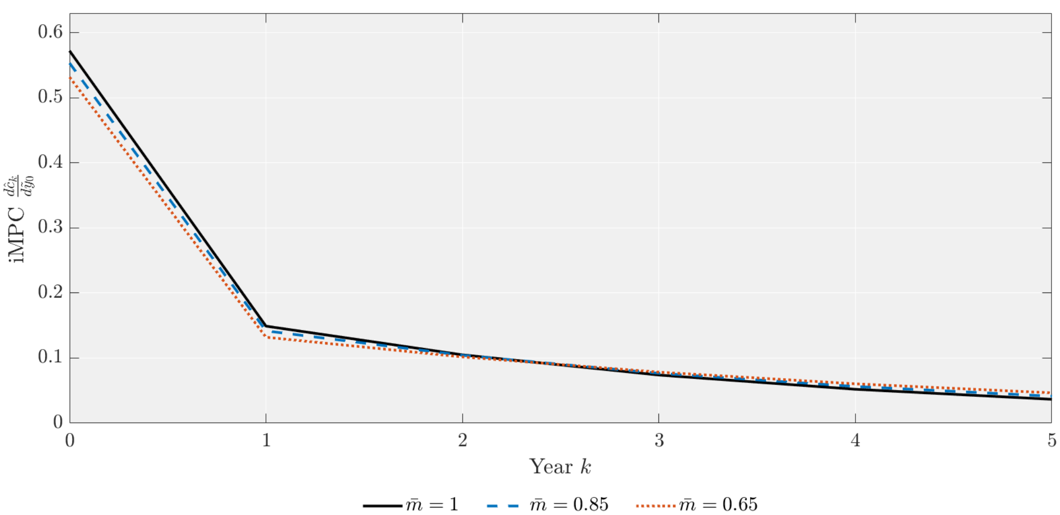
Figure 11: Intertemporal MPCs, Bounded Rationality and Household Heterogeneity



Note: This figure shows the aggregate intertemporal MPCs, i.e., the aggregate consumption response in year 0 (left) and year 1 (right) to a change in aggregate disposable income in year 0 for a transition probability $1 - s = 0.5$.

initial MPCs as the behavioral savers save relatively more. Over time, however, the MPCs in the behavioral model lie above their rational counterparts due to the fact that more and more of the initial savers become hand-to-mouth and start consuming their (higher) savings. As Figure 11 shows, the probability of type switching, $1 - s$, matters for when exactly the behavioral model starts to generate larger MPCs compared to the rational model.

Figure 12: Intertemporal MPCs



Note: This figure shows the aggregate intertemporal MPCs, i.e., the aggregate consumption response in year k to a change in aggregate disposable income in year 0 for different $\bar{m}$.

E Sticky Wages

In this section, we provide details on the sticky-wage extension presented in Section **??** as well as the calibration used to produce Figure 8. The way we introduce sticky wages follows Colciago (2011) and recently adopted by Bilbiie et al. (2021).

In the household block, the only difference to our benchmark model is that we assume that there is a labor union pooling labor and setting wages on behalf of households. This leads to a condition similar to the labor-leisure conditions in Section 2. But instead of individual conditions, the condition is the same for every household:

$$\varphi \widehat{n}_t = \widehat{w}_t - \gamma \widehat{c}_t,$$

and $\widehat{n}_t = \widehat{n}_t^S = \widehat{n}_t^H$.

The labor union, however, is subject to wage rigidities. The nominal wage can only be re-optimized with a constant probability, which leads to a time-varying wage markup

$$\mu_t^w = \varphi \widehat{n}_t - \widehat{w}_t + \gamma \widehat{c}_t,$$

a wage Phillips Curve

$$\pi_t^w = \beta \mathbb{E}_t \pi_{t+1}^w + \kappa_w \mu_t^w.$$

Wage inflation equals

$$\pi_t^w = \widehat{w}_t - \widehat{w}_{t-1} + \pi_t.$$

The firm side is exactly the same as in the main text but we focus on the case with rational firms, which gives rise to a standard Phillips Curve:

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa_\pi \widehat{mc}_t,$$

where $\widehat{mc}_t$ is a time-varying price markup. Table 2 summarizes all equilibrium equations.

The calibration of this extended model is presented in Table 3. The parameters γ, φ, s, β and $\bar{m}$ are as in our baseline calibration. The parameters of the Taylor rule, ρ_i and ϕ, are set as estimated in Auclert et al. (2020).

The slope of the wage Phillips curve, κ_w, is set as in Bilbiie et al. (2021) and we focus on the *no-redistribution* case $\tau^D = 0$. Note, that this leads to impact responses of consumption of the two household types that are very close to the ones in our baseline model: $\widehat{c}_t^H$ increases by about 1.42, whereas output increases by 1. The baseline calibration of $\chi = 1.48$ would predict that in the model without sticky wages, $\widehat{c}_t^H$ increases by 1.48 when output increases

Table 2: Sticky Wages, Equilibrium Equations

Name	Equation
Wage Markup	$\widehat{\mu}_t^w = \gamma\widehat{c}_t + \varphi\widehat{n}_t - \widehat{w}_t$
Wage Phillips Curve	$\pi_t^w = \beta\mathbb{E}_t\pi_{t+1}^w + \kappa_w\mu_t^w$
Wage Inflation	$\pi_t^w = \widehat{w}_t - \widehat{w}_{t-1} + \pi_t$
Bond Euler	$\widehat{c}_t^S = s\bar{m}\mathbb{E}_t\widehat{c}_{t+1}^S + (1-s)\bar{m}\mathbb{E}_t\widehat{c}_{t+1}^H - \frac{1}{\gamma}(\widehat{i}_t - \mathbb{E}_t\pi_{t+1})$
H Budget Constraint	$\widehat{c}_t^H = \widehat{w}_t + \widehat{n}_t + \widehat{t}_t^H$
H Transfer	$\widehat{t}_t^H = \frac{\tau^D}{\lambda}D_t$
Profits	$\widehat{d}_t = \widehat{y}_t - (\widehat{w}_t + \widehat{n}_t)$
Labor Demand	$\widehat{w}_t = \widehat{mc}_t + \widehat{y}_t - \widehat{n}_t$
Phillips Curve	$\pi_t = \beta\mathbb{E}_t\pi_{t+1} + \kappa_\pi\widehat{mc}_t$
Production	$\widehat{y}_t = \widehat{n}_t$
Consumption	$\widehat{c}_t = \lambda\widehat{c}_t^H + (1-\lambda)\widehat{c}_t^S$
Resource Constraint	$\widehat{y}_t = \widehat{c}_t$
Taylor Rule	$\widehat{i}_t = \rho_i\widehat{i}_{t-1} + (1-\rho_i)\phi\pi_t + \varepsilon_t^{MP}$

Table 3: Sticky Wage Model Calibration.

Parameter	γ	κ_π	λ	s	φ	τ^D	κ_w	β	ρ_i	ϕ
Value	1	0.01	0.37	$0.8^{1/4}$	1	0	0.075	0.99	0.89	1.5

by 1. We focus on a relatively stable inflation and set κ_π to 0.01.

The only parameter that we change with respect to our baseline calibration is λ which we set to 0.37 instead of 0.33. A value of 0.37 is still in the range of often used values (see, for example Bilbiie (2020)). We increase λ somewhat compared to our baseline calibration in order to increase the role of hand-to-mouth households in the response to monetary policy shocks and thus, allows the model to generate the pronounced hump-shaped responses. Setting $\lambda = 0.33$ still produces hump-shaped responses but those are somewhat less pronounced.

F A Quantitative Behavioral HANK Model

Table 4 shows how we calibrate the quantitative model introduced in Section 3.7.

The calibration closely follows the parameterization in McKay et al. (2016). As in McKay et al. (2016), we assume that high productivity households pay all the taxes. The main difference to their calibration is that they assume that every household receives an equal share of the dividends whereas we assume that the high productivity households receive 80% of the dividend payments, while the middle productivity class receive 20% of it. The low productivity households do not receive any dividend payments. We choose this calibration such that the contemporaneous amplification in the quantitative HANK model matches the one from the tractable model, outlined in Section 2.

Parameter	Description	Value
R	Steady State Real Rate	0.5%
γ	Risk aversion	2
φ	Inverse of Frisch elasticity	2
μ	Markup	1.2
θ	Calvo Price Stickiness	0.15
ρ_z	Autocorrelation of idiosyncratic risk	0.966
σ_z^2	Variance of idiosyncratic risk	0.0384
$\tau(z)$	Tax shares	[0, 1, 1]
$d(z)$	Dividend shares	[0, 0.2, 0.8]
$\frac{B}{4Y}$	Total wealth	0.625

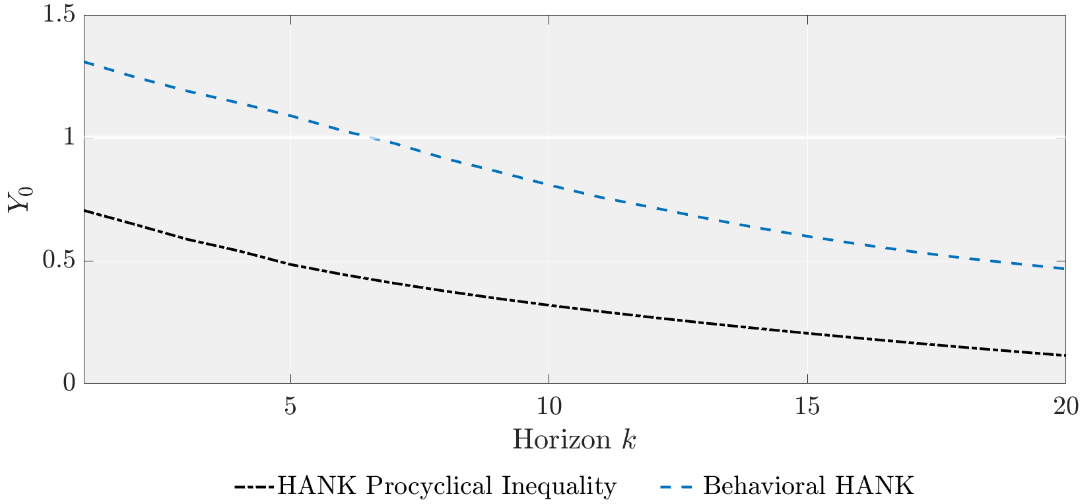
Table 4: Baseline calibration of quantitative HANK model.

Other resolutions of the forward-guidance puzzle in HANK. How does our quantitative behavioral HANK model compare to other resolutions of the forward guidance puzzle within one-asset HANK models? McKay et al. (2016) resolve the forward guidance puzzle by assuming that every household receives an equal share of the dividends, leading to procyclical inequality. Thus, the low-productivity households—who also exhibit larger MPCs on average than households with higher productivities—are less exposed to monetary policy. Therefore, the effectiveness of monetary policy is dampened overall, leading to a resolution of the forward guidance puzzle but also ruling out the amplification of contemporaneous shocks, as shown Figure 13.

Second, Hagedorn et al. (2019) solve the forward guidance puzzle by introducing a nominal anchor into their model. In particular, they impose a nominal steady state government debt level, which implies that the model has a steady state price level. This allows them to resolve the forward guidance and generate amplification of contemporaneous monetary policy. We show how introducing bounded rationality also sidesteps the Catch-22 without relying on a nominal anchor.

Third, Farhi and Werning (2019) suggest a similar resolution to the forward guidance puzzle as our model by combining incomplete markets and bounded rationality. Our behavioral HANK model differs from theirs in two dimension: first, we introduce bounded rationality in the form of cognitive discounting whereas Farhi and Werning (2019) assumes level-k thinking. Second, in our model contemporaneous monetary policy is amplified whereas it is not in Farhi and Werning (2019).

Figure 13: Resolving the Forward Guidance Puzzle in HANK



Note: This figure shows the response of total output in period 0 to anticipated i.i.d. monetary policy shocks occurring at different horizons k, relative to the response in the RANK model under rational expectations (equal to 1).

G Details on the Backward-Looking Behavioral HANK Model

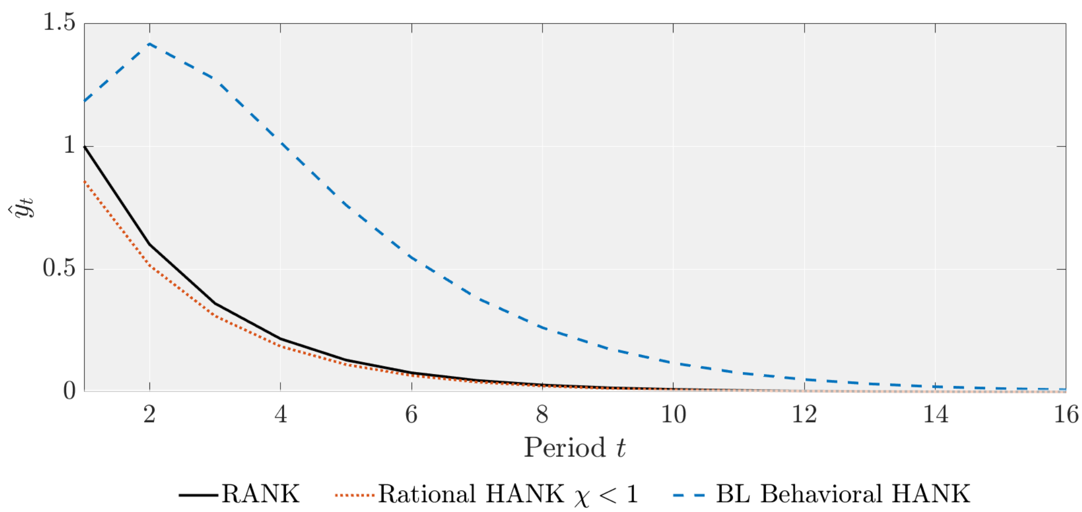
Here, we discuss how we can calibrate the backward-looking behavioral HANK model from Section 4.2 to match data coming from survey expectations. To do so, we follow Angeletos and Huo (2021) who calibrate the coefficients in front of $\mathbb{E}_t\widehat{y}_{t+1}$ and $\widehat{y}_{t-1}$ to match exactly this kind of evidence from survey expectations data. By following their calibration, we can back out the implied $\bar{m}$ and χ. We get $\bar{m} = 0.59$ and $\chi = 0.72$, thus, relatively low values compared to the calibration above. We leave the other parameters as in Section 3. We complement the backward-looking behavioral HANK IS equation with the static Phillips Curve (16).

Determinacy. We numerically verify that the backward-looking behavioral HANK model restores the Taylor principle. In fact, the equilibrium is determinate even under an interest-rate peg. Thus, also the backward-looking behavioral HANK model overturns the Sargent and Wallace (1975) result with this calibration.

Impulse-Response Functions. We now show how the backward-looking behavioral HANK model generates hump-shaped impulse responses and a novel behavioral amplification channel. To this end, we examine how output in the backward-looking behavioral HANK model responds to an expansionary monetary policy shock and compare the response to its rational counterpart and the RANK version of the model. We set the Taylor coefficient to 1.5, thus, guaranteeing determinacy also in the rational models and the persistence of the shock to an intermediate value, $\rho^{MP} = 0.6$.

Figure 14 shows the corresponding impulse-response functions. The blue-dashed line shows the results of our behavioral HANK, the orange-dotted line of its rational counterpart (THANK) and the black-solid line of RANK.

Figure 14: Output Response to a Monetary Policy Shock



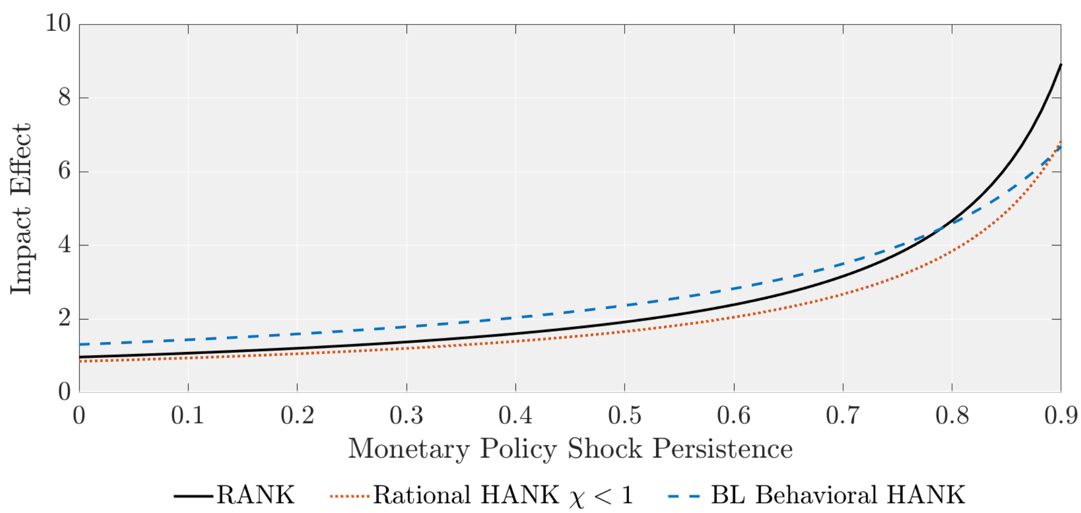
Note: This figure shows the output response to a monetary policy shock for different models.

Two things stand out. First, the behavioral HANK model delivers amplification compared to RANK—even in the first period—and second, the backward-looking anchor generates hump-shaped responses. As the latter has been highlighted in Angeletos and Huo (2021), we here focus on the amplification. Figure 14 shows that the amplification stems from a *behavioral amplification channel*: the initial output response is amplified although the model features procyclical inequality ($\chi < 1$) and, thus, the heterogeneity frictions themselves would generate dampening.

Where does the behavioral amplification come from? Given the backward-looking component in households' expectations, the increase in today's output is expected to persist as it becomes tomorrow's default value for the household's expectations. The behavioral anchor induces *endogenous* persistence which further increases today's output response through more optimistic expectations. Yet, there is an opposing channel at work: an *exogenously* persistent shock not only decreases interest rates today but also expected future interest rates. Behavioral households congitively discount these future changes and, thus, perceive the shock as less expansionary compared to a rational agent which dampens the initial response.[35] Given our calibration, the first channel dominates, thereby generating amplification as depicted in Figure 14.

Given the two opposing forces at work, the degree of initial amplification depends on the persistence of the shock. Figure 15 shows the initial response of all three models for different degrees of persistence of the shock. As the persistence declines, the initial response becomes relatively stronger in the backward-looking behavioral HANK model compared to RANK. As a consequence, the relative amplification is largest for an i.i.d. shock.

Figure 15: Initial Output Response for Varying Degrees of the Persistence



Note: This figure shows the initial output response to monetary policy shocks with different degrees of persistence.

In addition, comparing the backward-looking behavioral HANK model to its rational counterpart shows that for $\rho^{MP} < 0.9$, there is behavioral amplification while for more persistent shocks, there is behavioral dampening. The comparison with RANK shows that for $\rho^{MP} < 0.80$, the behavioral amplification dominates the heterogeneity dampening which arises because $\chi < 1$.

Behavioral Amplification and Forward Guidance. We now analyze analytically the behavioral-amplification mechanism and its implications for forward guidance. In the backward-looking behavioral HANK model, the output response to an interest rate change depends on the (expected) infinite future even when the shock is completely transitory.

[35]This is the same channel through which the fiscal multiplier of persistent government spending is dampened in our baseline model in Section 3.

Consider the following. The monetary authority decreases the nominal interest rate in period t to $\widetilde{i}_t < 0$ but will keep it at steady state thereafter (the argument extends to changes of the interest rate in the future). Output and inflation would be expected to go back to zero in $t+1$ under rational expectations. This is, however, not true for the backward-looking behavioral HANK model.

To understand this, combine the static Phillips Curve (a static Phillips curve is again not crucial for the argument but facilitates the derivations) with the behavioral HANK IS equation to arrive at

$$\widehat{y}_t = (1-\bar{m})\delta\widehat{y}_{t-1} - \frac{1}{\gamma}\frac{1-\lambda}{1-\lambda\chi}\widetilde{i}_t + \left[\delta\bar{m} + \kappa\frac{1}{\gamma}\frac{1-\lambda}{1-\lambda\chi}\right]\mathbb{E}_t\widehat{y}_{t+1}.$$

If households expect future output to be back to steady state – as would be the case in the rational model or the behavioral model in which the households' default value equals the steady state – a one-time, completely transitory decrease in the nominal interest rate changes contemporaneous output by

$$\frac{1}{\gamma}\frac{1-\lambda}{1-\lambda\chi} > 0. \tag{61}$$

Yet, in the backward-looking model, expectations in $t+1$ of output in $t+2$ will be above steady state when output in t increases. The more optimistic expectations feed back into output already in t.

This becomes apparent when we write the IS equation as

$$\begin{aligned}
&\widehat{y}_t\left[1-(1-\bar{m})\delta\left[\delta\bar{m}+\kappa\frac{1}{\gamma}\frac{1-\lambda}{1-\lambda\chi}\right]\right] = \\
&(1-\bar{m})\delta\widehat{y}_{t-1} - \frac{1}{\gamma}\frac{1-\lambda}{1-\lambda\chi}\left[\widetilde{i}_t + \left[\delta\bar{m}+\kappa\frac{1}{\gamma}\frac{1-\lambda}{1-\lambda\chi}\right]\mathbb{E}_t\left[\widetilde{i}_{t+1}\right]\right] \\
&+\left[\delta\bar{m}+\kappa\frac{1}{\gamma}\frac{1-\lambda}{1-\lambda\chi}\right]^2\mathbb{E}_t\widehat{y}_{t+2}.
\end{aligned}$$

Thus, if households would assume that $\widehat{y}_{t+2}$ will be zero but not $\widehat{y}_{t+1}$, the discussed interest-rate change in t increases output in t by

$$\frac{\frac{1}{\gamma}\frac{1-\lambda}{1-\lambda\chi}}{1-(1-\bar{m})\delta\left[\delta\bar{m}+\kappa\frac{1}{\gamma}\frac{1-\lambda}{1-\lambda\chi}\right]},$$

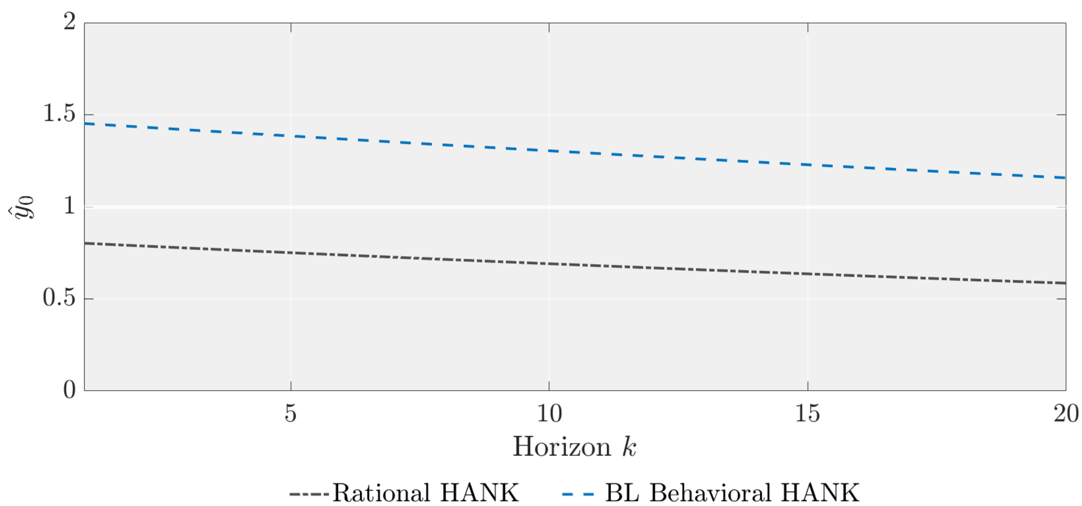
which is larger than the effect for models without a backward-looking anchor as can be seen by comparing it to equation (61). Put differently, the initial output response is amplified through a behavioral channel. Iterating forward in this fashion shows how the effect increases with each iteration. However, the response is bounded, as we will see below.

Turning to forward guidance, an expected change in the nominal interest rate in period $t+1$, affects output in t by

$$-\frac{\frac{1}{\gamma}\frac{1-\lambda}{1-\lambda\chi}\left[\delta\bar{m}+\kappa\frac{1}{\gamma}\frac{1-\lambda}{1-\lambda\chi}\right]}{1-(1-\bar{m})\delta\left[\delta\bar{m}+\kappa\frac{1}{\gamma}\frac{1-\lambda}{1-\lambda\chi}\right]},$$

if we assume output in $t+2$ to be back to zero. Given our calibration, the term $\left[\delta\bar{m}+\kappa\frac{1}{\gamma}\frac{1-\lambda}{1-\lambda\chi}\right]$ is smaller than 1. Thus, an interest rate change tomorrow has a smaller effect on output today than a contemporaneous interest rate change such that there is no forward guidance puzzle in the backward-looking behavioral HANK model. We can continue in this fashion to show that the effects increase with the iteration but decrease with the period of the shock.

Figure 16: Forward Guidance with Backward-Looking Anchor



Note: This figure shows the period-t output response to an anticipated i.i.d. monetary policy shock in period $t+k$ for three different economies.

Figure 16 shows these patterns graphically. First, the behavioral amplification channel discussed above is reflected in the contemporaneous effect ($k=0$) which is stronger than without the backward-looking expectations —reflected in the black-dashed-dotted line. Second, increasing the horizon k shows that there is no forward guidance puzzle in the backward-looking behavioral HANK model. To sum it up, also the backward-looking behavioral HANK model amplifies contemporaneous monetary policy (even for $\chi<1$) while it simultaneously dampens the effects of forward guidance.