A Behavioral Heterogeneous Agent New Keynesian Model

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Abstract

We propose a behavioral heterogeneous agent New Keynesian model in which monetary policy is amplified through indirect general equilibrium effects, fiscal multipliers can be larger than one and which delivers empirically-realistic intertemporal marginal propensities to consume. Simultaneously, the model resolves the forward guidance puzzle, remains stable at the effective lower bound and determinate under an interest-rate peg. The model is analytically tractable and nests a wide range of existing models as special cases, none of which can produce all the listed features within one model. We further show how our framework can be extended to derive an equivalence result with models featuring incomplete information and learning. In doing so, we show how the model generates hump-shaped responses of aggregate variables and a novel behavioral amplification channel that is absent in existing HANK models.

Keywords: Behavioral Macroeconomics, Heterogeneous Households, Monetary Policy, Forward Guidance, Fiscal Policy, New Keynesian Puzzles, Determinacy, Lower Bound

JEL Codes: E21, E52, E62, E71

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1 Introduction

New Keynesian models with household heterogeneity have become popular for analyzing monetary policy, fiscal policy, and business cycles.[1] Among other features, these Heterogeneous Agent New Keynesian (HANK) models can generate intertemporal Marginal Propensities to Consume (iMPCs) that are in line with the data (Auclert et al. (2018), Kaplan and Violante (2020)), monetary policy that is amplified through indirect general equilibrium effects (Kaplan et al. (2018), Auclert (2019), Auclert et al. (2020)), and fiscal multipliers that are larger than one even under constant real rates (Auclert et al. (2018)).[2] The quantitative nature and complexity of these models has motivated the development of analytically-tractable HANK (THANK) models who provide a clearer understanding of the HANK transmission mechanisms. Thereby, these models uncovered a major challenge inherit in models featuring household heterogeneity: when generating the aforementioned "desirable" HANK features, HANK models tend to aggravate major NK puzzles such as the forward guidance puzzle, unreasonably large recessions at the Effective Lower Bound (ELB) and the Taylor principle fails to be sufficient for determinacy (see Werning (2015), Bilbiie (2021) and Acharya and Dogra (2020)).[3] This trade-off prevents an overarching analysis of monetary policy and fiscal policy within one single framework.

We propose such a framework by constructing a New Keynesian model which incorporates household heterogeneity and behavioral frictions in the form of cognitive discounting. The resulting *behavioral HANK* model generates the desirable HANK features and *simultaneously* offers a resolution to the NK puzzles, thereby providing a unifying framework for monetary and fiscal policy analysis. In the behavioral HANK model, indirect general equilibrium effects account for large parts of the transmission of monetary policy to consumption and fiscal policy is amplified. In addition, the model matches estimated iMPCs in the data which are a crucial statistic to discipline HANK models (Auclert et al. (2018)). At the same

[1]For monetary policy see, e.g., Kaplan et al. (2018), Auclert (2019), Auclert et al. (2020), Luetticke (2021), McKay et al. (2016), Hagedorn et al. (2019a), Kyriazis (2022). For fiscal policy see, e.g., Auclert et al. (2018), Hagedorn et al. (2019b), Ferriere and Navarro (2018), Oh and Reis (2012), Wolf (2021), Bayer et al. (2020), Seidl and Seyrich (2021), McKay and Reis (2016). For business cycle analyses see, Guerrieri and Lorenzoni (2017), Bayer et al. (2019).

[2]These model features are supported by recent empirical evidence: Auclert et al. (2018) provide empirical estimates of iMPCs. The importance of indirect channels for monetary policy is empirically supported in Ampudia et al. (2018), Samarina and Nguyen (2019) and Holm et al. (2021). Nakamura and Steinsson (2014) and Chodorow-Reich (2019) provide recent evidence on fiscal multipliers above one. Ramey (2019) also shows that fiscal multipliers can be substantially above one under accommodative monetary policy.

[3]While these issues have been highlighted in tractable models mainly, an earlier version of Auclert et al. (2018) and the discussion of these issues in Acharya and Dogra (2020) show similar indeterminacy problems of quantitative HANK models. In addition and in line with the THANK literature, Hagedorn et al. (2019a) show that whether forward guidance is dampened *or* amplified in the standard one asset quantitative HANK model depends on the cyclicality of inequality.

time, the model resolves the forward guidance puzzle as the effectiveness of future monetary policy is weaker than contemporaneous monetary policy and the response of current output declines with the horizon of the announced interest-rate change. Additionally, we show that the behavioral HANK model restores the Taylor principle. In fact, it features determinacy even under an interest-rate peg for a large area of the parameter space. Relatedly, the behavioral HANK model remains stable during prolonged periods at the ELB.

We highlight how the behavioral friction interacts with household heterogeneity and show that both are necessary for our results. What is more, our framework nests a wide range of existing models such that we can cleanly compare these models to the behavioral HANK model. None of the competing models can generate the desirable HANK features while simultaneously offering a resolution to the NK puzzles.

To arrive at our framework, we extend the textbook Representative Agent New Keynesian model (RANK) in two dimensions. First, we introduce household heterogeneity following the THANK literature, as summarized below. There are two groups of households, savers and hand-to-mouth households, and households face an exogenous probability to switch their type. This uninsurable idiosyncratic risk leads to precautionary-savings motives of households together with heterogeneity in income and MPCs. Second, we introduce bounded rationality by the means of cognitive discounting as in Gabaix (2020). Households anchor their expectations about future macroeconomic variables to the steady state but are myopic or inattentive to future deviations from it.[4]

Despite these two departures from the textbook RANK, we can describe the entire model dynamics around the steady state by three equations isomorphic to the textbook model: an IS curve, a Phillips curve, and a rule for monetary policy. Key to our results is the behavioral HANK IS equation:

$$\widehat{y}_t = \psi_f \mathbb{E}_t \widehat{y}_{t+1} - \psi_c \frac{1}{\gamma} \left(\widehat{i}_t - \mathbb{E}_t \pi_{t+1} \right),$$

where $\widehat{y}_t$ denotes total output (in log-deviations from its steady state), $\mathbb{E}_t$ is the *rational* expectations operator, $\widehat{i}_t$ denotes the nominal interest rate, π_t the inflation rate, and $\frac{1}{\gamma}$ the intertemporal elasticity of substitution. Compared to RANK, two extra coefficients show up: ψ_c and ψ_f.

ψ_c governs the sensitivity of today's output with respect to the contemporaneous real interest rate. ψ_c is shaped by household heterogeneity and crucially depends on the cyclicality of income inequality: if income inequality is countercyclical, which seems to be the empirical consensus, $\psi_c > 1$ and contemporaneous monetary policy is amplified through

[4] Appendix B and Gabaix (2019, 2020) discuss the empirical evidence of cognitive discounting and Angeletos and Lian (2017) show how other forms of bounded rationality or lack of common knowledge lead to observationally-equivalent outcomes.

general equilibrium forces.[5,6] In addition, $\psi_c > 1$ is also sufficient for fiscal multipliers to be larger than one conditional on the real interest rate being constant. The other coefficient, ψ_f, captures the sensitivity of today's output with respect to changes in expected future output. ψ_f is shaped by household heterogeneity *and* the behavioral friction as it depends on the cyclicality of income risk *and* the degree of bounded rationality of households as well as the interaction of the two frictions. Given countercyclical income inequality, income risk is also countercyclical. Countercyclical risk induces compounding in the Euler equation and, thus, competes with cognitive discounting which induces discounting in the Euler equation. However, even for a small degree of bounded rationality—much smaller than the empirics suggest—the discounting through bounded rationality dominates the compounding through countercyclical income risk. Hence, in the behavioral HANK model it holds that $\psi_f < 1$ which makes the economy less sensitive to expectations and news about the future which is key to resolve the NK puzzles.

Furthermore, we characterize the iMPCs in the behavioral HANK model analytically and analyze how household heterogeneity and bounded rationality affect these iMPCs. If income risk is more countercyclical, i.e., hand-to-mouth households are more exposed to the business cycle, the aggregate MPC in the year of the income windfall increases, especially when households are less behavioral. Boundedly-rational households tend to save more than rational households out of the windfall as they cognitively discount the decrease in their future marginal utility which lowers the current MPC. As time progresses, however, bounded rationality increases the aggregate MPC as the behavioral savers start to consume their (higher) savings. These dynamic effects are particularly pronounced when idiosyncratic risk is relatively high.

We demonstrate that the behavioral HANK model can have *qualitatively different* policy implications than its rational counterpart by applying our framework to study the most effective timing of monetary policy. Consider an *overheating* economy which the monetary authority wants to tame by hiking interest-rates by a cumulative $x\%$. This rate hike can be implemented immediately or by raising the rate $\frac{x}{k}\%$ over k consecutive periods. A well-known feature of the RANK model is that monetary policy becomes more effective the more it is back-loaded. While this is also the case in THANK under countercyclical inequality,

[5]Patterson (2019) provides empirical evidence for the countercyclicality of inequality. Coibion et al. (2017), Ampudia et al. (2018) and Samarina and Nguyen (2019) provide evidence of countercyclical inequality conditional on monetary policy shocks. Almgren et al. (2019) show that output in countries with higher shares of hand-to-mouth households responds more strongly to monetary policy shocks which, through the lens of the model, implies countercyclical inequality.

[6]"Amplification" does not need to be interpreted literally as γ can always be adjusted to scale the impact effect. Hence, it should rather be understood as a high importance of general equilibrium (indirect) effects relative to direct effects.

the opposite is true in the behavioral HANK model: monetary policy is more effective when it is completely *front-loaded*, i.e., when $k = 1$. The increased effectiveness, however, comes at the cost of an increase in inequality which is more pronounced in the behavioral HANK model when monetary policy is front-loaded.

We close by showing how to extend our framework to derive an equivalence result between models with bounded rationality and models of incomplete information and learning. To this end, we assume that behavioral agents anchor their beliefs to *past observations* of the respective variable instead of the respective steady state values. This extended behavioral HANK model is observationally equivalent to models featuring incomplete information and learning (see Angeletos and Huo (2021) and Gallegos (2021)).[7]

We calibrate the extended model to match recent findings from survey expectations data and show that the model endogenously generates hump-shaped responses of macro aggregates to monetary policy shocks. The backward-looking component in households' expectations induces endogenous persistence and thus, households respond *as if* contemporaneous (or future) shocks are persistent even when the shocks are actually completely transitory. This yields an endogenous *behavioral-amplification mechanism* that is absent in existing HANK models. A similar reasoning extends to future interest rate changes even though their effects become smaller as the interest rate cut is expected to take place at a later date. Thus, the extended behavioral HANK model also rules out the forward guidance puzzle. In addition, it again delivers determinacy under an interest rate peg.

Outline. The rest of the paper is structured as follows. In Section 2, we summarize the related literature. We present our behavioral HANK model in Section 3 and our main analytical results in Section 4. In Section 5, we derive our equivalence result and Section 6 concludes.

2 Related Literature

Tractable HANK models have been used to either deliver amplification of shocks and policies *or* to deliver dampening of the effects of forward guidance.[8] McKay et al. (2017) use a

[7] Angeletos and Huo (2021) derive an equivalence result between models with incomplete information and learning with models which include behavioral myopia and an additional friction such as habit persistence or adjustment costs. We now complement their equivalence result with a behavioral model that solely relies on one behavioral friction.

[8] Examples of tractable HANK models that do not focus on amplification or resolving puzzles include Nistico (2016), Cúrdia and Woodford (2016), Challe and Ragot (2016), Acharya et al. (2020), Challe (2020), Bilbiie and Ragot (2021), Bilbiie et al. (2021), Broer et al. (2020) and see, e.g., Caballero and Simsek (2019), Caballero and Simsek (2020) or Hommes et al. (2019) for tractable models of *belief heterogeneity*.

tractable HANK model with in-built procyclical risk to approximate their finding in McKay et al. (2016) in which, again, procyclical risk provides a solution to the forward guidance puzzle. Ravn and Sterk (2017) and Ravn and Sterk (2021) show that incorporating search-and-matching frictions into a tractable HANK model delivers countercyclical risk and amplification of business cycle shocks. Debortoli and Galí (2018) approximate the amplification of monetary policy of their HANK model by a Two Agent NK model (TANK)—which can be thought of as a special case of a THANK model.[9] Werning (2015) provides an incomplete-markets irrelevance benchmark which shows that contemporaneous monetary policy and forward guidance is as strong as in RANK if income risk is acyclical. Acharya and Dogra (2020) show similarly to Werning (2015) that the resolution to NK puzzles such as the forward guidance puzzle depends on the cyclicality of risk by constructing a THANK model in which the precautionary savings motive of households is the only difference to RANK. While procyclical risk as in McKay et al. (2016) and McKay et al. (2017) resolves the forward guidance puzzle and allows for sufficiency of the Taylor principle, countercyclical risk aggravates these puzzles. Bilbiie (2020) and Bilbiie (2021) go one step further and show that in THANK models, income risk co-moves with income inequality. Since contemporaneous monetary and fiscal policy is amplified with countercyclical inequality and dampened with procyclical inequality, Bilbiie (2021) shows that tractable HANK models can either solve the NK puzzles or generate policy amplification but not both at the same time—a *Catch-22*. One of our contributions is to show how HANK models can overcome this Catch-22.[10]

A mostly-detached strand of the literature has suggested to relax the assumption of full-information rational expectations (FIRE) to weaken the effectiveness of future monetary policies, thereby resolving the forward guidance puzzle (Wiederholt (2015), Angeletos and Lian (2018), Andrade et al. (2019), Gabaix (2020), Pfäuti (2021) and Roth et al. (2021)). We complement these papers by introducing household heterogeneity in terms of iMPCs, asset-market participation status, and exposure to the business cycle. This way, our model cannot only resolve the forward guidance puzzle (and other NK puzzles) but also simultane-

[9] While abstracting from the cyclicality of income risk, TANK models which date back to Campbell and Mankiw (1989), Mankiw (2000), Galí et al. (2007), and Bilbiie (2008) can generate monetary and fiscal amplification. Cantore and Freund (2021) use a TANK model to match empirically-observed iMPCs and Maliar and Naubert (2019) provide a recent in-depth analysis of TANK models.

[10] Bilbiie (2021) provides two theoretical possibilities of how to sidestep the Catch-22. The first possibility is a pure risk channel which can, in theory, break the comovement of income risk and inequality. Yet to do so, it requires a calibration which seems highly at odd with the data. A second possibility is to drastically narrow down the policy space: in a world in which monetary policy is described by Wicksellian price level targeting or fiscal policy follows a nominal bond rule, there would be no Catch-22. Hagedorn et al. (2019a) use a similar description of fiscal policy to solve the forward guidance puzzle in a HANK model, in which contemporaneous monetary policy is amplified. In contrast, in our model, there is no Catch-22 *independently* of the exact specification of monetary and fiscal policy.

ously deliver amplification of contemporaneous monetary and fiscal policy as well as match empirical estimates of iMPCs.

We share the combination of household heterogeneity and some deviation from FIRE with Farhi and Werning (2019), Auclert et al. (2020), Broer et al. (2021), Angeletos and Huo (2021), Laibson et al. (2021), Gallegos (2021), and Bonciani and Oh (2022). In contrast to all these papers, we offer analytical insights into how the two frictions matter for policy analysis, and how bounded rationality can resolve several puzzles present in NK models while it at the same time allows the model to keep desirable HANK features, such as amplification of monetary policy and fiscal multipliers above one. Auclert et al. (2020) derive iMPCs in a HANK model with sticky information. We complement their analysis by providing closed-form solutions. To the best of our knowledge, we are the first to provide analytical iMPCs in a HANK model with some departure from FIRE.

Angeletos and Huo (2021) derive an IS equation in a HANK model featuring incomplete information and show how this generates hump-shaped responses of macro aggregates to monetary policy shocks. We derive an equivalent IS equation by extending the behavioral framework in Gabaix (2020). We thus bridge the gap between the literature that relaxes the *full-information* part of FIRE and the one that relaxes the *rational-expectations* part. We further highlight how bounded rationality can generate a behavioral amplification mechanism in addition to the HANK amplification mechanism.

3 A Behavioral HANK Model

In this section, we present our tractable NK model featuring household heterogeneity and bounded rationality (BR).

3.1 Structure of the Model

Households. The economy is populated by a unit mass of households, indexed by $j \in [0,1]$. Households obtain utility from (non-durable) consumption, C_t^j, and dis-utility from working N_t^j. Households discount future utility at rate $\beta \in [0,1]$. Assuming a standard, separable, CRRA utility function, households' lifetime utility is given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{(C_t^j)^{1-\gamma}}{1-\gamma} - \frac{(N_t^j)^{1+\varphi}}{1+\varphi} \right),$$

where φ denotes the inverse Frisch elasticity and γ denotes the relative risk aversion. For most of the paper, we focus on $\gamma = 1$, that is, log-utility $log(C_t^j)$.

Households can save or borrow in government bonds, paying nominal interest i_t, and acquire shares of intermediate monopolistic firms. We introduce household heterogeneity following Bilbiie (2021) and allow for the possibility that households participate in financial markets infrequently. When they do participate, they can freely buy or sell bonds and shares and receive all the profits, D_t, from the monopolistic firms. Otherwise, they simply receive the payoff from their previously acquired bonds. We denote households participating in financial markets by S as they will be Savers in equilibrium, and the non-participants by H as they will be $H$and-to-mouth. A saver remains a saver with probability s and becomes hand-to-mouth with probability $1-s$. Hand-to-mouth households remain hand-to-mouth with probability h and switch with probability $1-h$. In what follows, we focus on stationary equilibria where $\lambda \equiv \frac{1-s}{2-s-h}$ denotes the constant share of hand-to-mouths.

We use the same simplyfing assumptions as in Bilbiie (2021) which allow for a tractable solution. In particular, we assume that households belong to a family whose utilitarian intertemporal welfare is maximized by its family head. The head can only provide insurance within types but not across types, i.e., the head pools all the resources within types. When households switch from the saver to the hand-to-mouth type, they only keep their government bonds. Stocks cannot be used to self-insure. Using the in- and outflows between both groups and the stationary distribution, we get the following relationships between real, per-capita, beginning-of-period-t+1 bonds, B_{t+1}^j and end-of-period-t per-capita real values (before moving across types), Z_{t+1}^j:

$$\begin{aligned} B_{t+1}^S &= sZ_{t+1}^S + (1-s)Z_{t+1}^H \\ B_{t+1}^H &= (1-h)Z_{t+1}^S + hZ_{t+1}^H. \end{aligned} \tag{1}$$

We allow for the possibility that the family head is boundedly rational (BR) in the way we describe in detail in Section 3.3.[11] The program of the family head is

$$W\left(B_t^S, B_t^H, \iota_t\right) = \max_{\{C_t^S, C_t^H, Z_{t+1}^S, Z_{t+1}^H, N_t^S, N_t^H, \iota_{t+1}\}} \left[(1-\lambda)U\left(C_t^S, N_t^S\right) + \lambda U\left(C_t^H, N_t^H\right)\right] + \beta \mathbb{E}_t^{BR} W\left(B_{t+1}^S, B_{t+1}^H, \iota_{t+1}\right)$$

subject to the respective budget constraints

[11] We show in Appendix A.6 how the family head's expectation can be understood as an average expectation over all households' expectations within family where each household receives a noisy signal about the future state.

$$C_t^S + Z_{t+1}^S + v_t \iota_{t+1} = W_t N_t^S + \iota_t (v_t + D_t) + \frac{1+i_{t-1}}{1+\pi_t} B_t^S + T_t^S \tag{2}$$

$$C_t^H + Z_{t+1}^H = W_t N_t^H + T_t^H + \frac{1+i_{t-1}}{1+\pi_t} B_t^H, \tag{3}$$

$$Z_{t+1}^H, Z_{t+1}^S \geq 0$$

where W_t is the real wage, ι_t are the shares of stocks traded at price v_t, B_t denotes the liquid asset holdings (government bonds), and T_t^j are transfers to type-j households. We assume that these transfers are financed by a proportional tax on profits, τ^D, such that they entail a redistribution from S households (who own the firms) to H households.

The optimality conditions are given by the savers' Euler equation

$$U'\big(C_t^S\big) \geq \beta \mathbb{E}_t^{BR}\Big[R_t\left(sU'\big(C_{t+1}^S\big) + (1-s)U'\big(C_{t+1}^H\big)\right)\Big] \tag{4}$$

$$\text{and } 0 = Z_{t+1}^S \Big[U'\big(C_t^S\big) - \mathbb{E}_t^{BR}\Big[R_t\left(sU'\big(C_{t+1}^S\big) + (1-s)U'\big(C_{t+1}^H\big)\right)\Big]\Big],$$

the Euler equation of the hand-to-mouth households

$$U'\big(C_t^H\big) \geq \beta \mathbb{E}_t^{BR}\Big[R_t\left((1-h)U'\big(C_{t+1}^S\big) + hU'\big(C_{t+1}^H\big)\right)\Big] \tag{5}$$

$$\text{and } 0 = Z_{t+1}^H \Big[U'\big(C_t^H\big) - \mathbb{E}_t^{BR}\Big[R_t\left((1-h)U'\big(C_{t+1}^S\big) + hU'\big(C_{t+1}^H\big)\right)\Big]\Big],$$

and the demand for shares

$$U'\big(C_t^S\big) \geq \beta \mathbb{E}_t^{BR}\left[\frac{v_{t+1}+D_{t+1}}{v_t} U'\big(C_{t+1}^S\big)\right] \text{ and } \iota_{t+1} = \iota_t = (1-\lambda)^{-1}, \tag{6}$$

with $R_t \equiv \frac{1+i_t}{1+\pi_{t+1}}$ being today's real interest rate. The respective labor-leisure equations of both types are given by:

$$-U'\big(N_t^S\big) = W_t U'(C_t^S) \tag{7}$$

$$-U'\big(N_t^H\big) = W_t U'(C_t^H). \tag{8}$$

In what follows, we focus on equilibria in which the H households will always be off their Euler equation—e.g., because they do not have access to financial markets—such that equation (5) always holds with strict inequality. In addition, we follow the THANK tradition and assume a zero liquidity equilibrium.[12] As shares cannot be transferred to the H state, equation (4) simply prices the shares. Thus, the savers' bond Euler equation is the only Euler equation that is an equilibrium equation. Importantly, it features a self-insurance motive as savers demand bonds to self-insure their idiosyncratic risk of type-switching.

Firms. We assume a standard NK firm side as in Bilbiie (2020). All households consume the same aggregate basket of goods, $j \in [0,1]$, $C_t = (\int_0^1 C_t(j)^{(\epsilon-1)/\epsilon} dj)^{\epsilon/(\epsilon-1)}$ where $\epsilon > 1$ is the elasticity of substitution between the individual goods. Each firm faces demand $C_t(j) = (P_t(j)/P_t)^{-\epsilon} C_t$ where $P_t(j)/P_t$ denotes the individual price relative to the aggregate price index, $P_t^{1-\epsilon} = \int_0^1 P_t(j)^{1-\epsilon} dj$, and produces with the linear technology $Y_t(j) = N_t(j)$. The real marginal cost is given by W_t. We assume that the government pays the standard NK optimal subsidy τ^S financed by a lump-sum tax on firms T_t^F. Hence, the profit function is: $D_t(j) = (1+\tau^S)[P_t(j)/P_t]Y_t(j) - W_t N_t(j) - T_t^F$. Total profits are then $D_t = Y_t - W_t N_t$ and are zero in steady state. Given zero steady state profits, we have a full-insurance steady state, i.e., $C^H = C^S = C$. In the log-linear dynamics around this steady state, profits vary inversely with the real wage $\widehat{d}_t = -\widehat{w}_t$.[13] We allow for steady state inequality in Appendix C and show that our results are not driven by this assumption and are in fact barely affected even by substantial inequality in the steady state.

Government. Fiscal policy induces the optimal steady state subsidy and taxes profits at rate τ^D and rebates these taxes as a transfer to H households, such that $T^H = \frac{\tau^D}{\lambda} D_t$. As will become clear later, the level of τ^D is key for the exposure of H households to the business cycle and thus for the cyclicality of inequality. Here, we abstract from government spending to keep it simple, but we introduce government spending in Section 4.

In most of the analysis, we assume that monetary policy follows a standard (log-linearized) Taylor rule[14]

$$\widehat{i_t} = \phi \pi_t + \epsilon_t^{MP}, \tag{9}$$

with ϵ_t^{MP} being the monetary policy shock which will be specified in the sections below.

Market Clearing. Market clearing requires $Y_t = C_t = \lambda C_t^H + (1-\lambda) C_t^S$ and $N_t = \lambda N_t^H + (1-\lambda) N_t^S$.

[12] See Krusell et al. (2011), McKay et al. (2017), Ravn and Sterk (2017), and Bilbiie (2021).

[13] Throughout the paper variables with a hat on top denote log-deviations from steady state.

[14] We study more general Taylor rules in Appendix A.

3.2 Log-Linearized Model

We now focus on the log-linearized dynamics around the full-insurance, zero-liquidity steady state. The market clearing conditions yield $\widehat{y}_t = \widehat{c}_t = \lambda\widehat{c}_t^H + (1-\lambda)\widehat{c}_t^S$ and $\widehat{n}_t = \lambda\widehat{n}_t^H + (1-\lambda)\widehat{n}_t^S$. Importantly, we can write the consumption of the hand-to-mouth households as

$$\widehat{c}_t^H = \chi\widehat{y}_t, \tag{10}$$

with

$$\chi = 1 + \varphi\left(1 - \frac{\tau^D}{\lambda}\right) \tag{11}$$

measuring the cyclicality of the H household's consumption. As χ is the key coefficient from our household heterogeneity set-up, we will vary χ throughout the paper. Different levels of χ should then be thought of as different τ^D, thus, different redistributive tax-transfer systems.

Combining equation (10) with the goods market clearing condition yields

$$\widehat{c}_t^S = \frac{1-\lambda\chi}{1-\lambda}\widehat{y}_t, \tag{12}$$

which implies that consumption inequality is given by:

$$\widehat{c}_t^S - \widehat{c}_t^H = \frac{1-\chi}{1-\lambda}\widehat{y}_t. \tag{13}$$

Equation (13) shows that if $\chi > 1$, inequality is countercyclical as it varies negatively with total output, i.e., increases in recessions and decreases in booms.

The log-linearized bond Euler equation of S households is given by

$$\widehat{c}_t^S = s\mathbb{E}_t^{BR}\left[\widehat{c}_{t+1}^S\right] + (1-s)\mathbb{E}_t^{BR}\left[\widehat{c}_{t+1}^H\right] - \frac{1}{\gamma}\left(\widehat{i}_t - \mathbb{E}_t^{BR}\pi_{t+1}\right). \tag{14}$$

We will, following the assumption in Gabaix (2020), often focus on the case in which the agents are rational with respect to the real rate, i.e., we replace $\mathbb{E}_t^{BR}\pi_{t+1}$ with $\mathbb{E}_t\pi_{t+1}$ in equation (14). In Appendix C, we show how our results go through with boundedly-rational real-rate expectations. In fact, the results become even stronger when considering boundedly-rational real-rate expectations. For the case without type-switching, i.e., for $s = 1$, equation (14) boils down to a standard Euler equation. For $s \in [0, 1)$, however, the agent takes into account that she might switch type and self-insures against becoming hand-to-mouth next period.

Supply Side. We distinguish between two set-ups for the supply side: For the main part, we follow Bilbiie (2021) and assume that firms are not forward-looking and, thus, we can summarize the supply side of the economy by a static Phillips Curve

$$\pi_t = \kappa \widehat{y}_t, \tag{15}$$

where $\kappa \geq 0$ captures the slope of the Phillips Curve.[15] Yet, we also relax this assumption in Appendix C and show that a forward-looking (NK) Phillips Curve barely affects our results.

3.3 Bounded Rationality

We follow Gabaix (2020) and model bounded rationality as a form of cognitive discounting.[16] Let X_t be a random variable (or vector of variables) and let us define X_t^d as some default value the agent may have in mind, e.g., the steady state value of X, and $\tilde{X}_{t+1} \equiv X_{t+1} - X_t^d$ denotes the deviation from this default value.[17] The behavioral agent's expectation about X_{t+1} is then given by

$$\mathbb{E}_t^{BR}\left[X_{t+1}\right] = \mathbb{E}_t^{BR}\left[\tilde{X}_{t+1} + X_t^d\right] \equiv \bar{m}\mathbb{E}_t\left[\tilde{X}_{t+1}\right] + X_t^d, \tag{16}$$

where $\mathbb{E}_t\left[\cdot\right]$ is the rational expectations operator and $\bar{m} \in [0,1]$ is the behavioral parameter which captures the degree of rationality. A higher $\bar{m}$ denotes a smaller deviation from rational expectations and rational expectations are captured by $\bar{m} = 1$. We see from equation (16) that the behavioral agent anchors her expectations to the default value and cognitively discounts future deviations from this default value. For now, we focus on the steady state as the default value but relax this assumption in Section 5.

While we present a way to microfound $\bar{m}$ in Appendix A.6, note, that the exact microfoundation or underlying behavioral friction is not crucial for the rest of our analysis. For example, Angeletos and Lian (2017) show how other forms of bounded rationality or lack of

[15] To arrive at this static Phillips curve, we can either assume that firms are completely myopic or that they face a Rotemberg-style adjustment cost relative to yesterday's market average price index (see Bilbiie (2021)).

[16] While Gabaix (2020) embeds bounded rationality in a NK model, the basic idea of behavioral inattention (or sparsity) has been proposed by Gabaix earlier already (see Gabaix (2014, 2016)) and a handbook treatment of behavioral inattention is given in Gabaix (2019). Benchimol and Bounader (2019) and Bonciani and Oh (2021) study optimal monetary policy in a RANK and TANK model, respectively, with this kind of behavioral frictions.

[17] Gabaix (2020) focuses on the case in which X_t denotes the state of the economy. He shows (Lemma 1 in Gabaix (2020)) that this form of cognitive discounting also applies to all other variables. We, on the other hand, directly apply cognitive discounting to all variables. Given Lemma 1 in Gabaix (2020), our results would be unchanged, but our more direct method simplifies some of the derivations, especially in Section 5. Appendix A.5 derives our results following the approach in Gabaix (2020).

common knowledge lead to observationally-equivalent expectations for the case in which X_t^d denotes the steady state.

Log-linearizing equation (16) around the steady state yields

$$\mathbb{E}_t^{BR}\left[\widehat{x}_{t+1}\right] = (1-\bar{m})\widehat{x}_t^d + \bar{m}\mathbb{E}_t\left[\widehat{x}_{t+1}\right] \tag{17}$$

and when X_t^d is the steady state value, we obtain $\mathbb{E}_t^{BR}\left[\widehat{x}_{t+1}\right] = \bar{m}\mathbb{E}_t\left[\widehat{x}_{t+1}\right]$.

To calibrate $\bar{m}$, we follow Gabaix (2020) who states that empirical estimates of $\bar{m}$ point towards values of about 0.65. Nevertheless, he chooses 0.85 as a conservative choice for his benchmark value which we also take as our benchmark calibration.[18] As one goal of our paper is to understand the role of $\bar{m}$ for policy analysis and the interplay of $\bar{m}$ and household heterogeneity, we will also consider different values for $\bar{m}$.

4 Results

In this section, we first show how the behavioral HANK model can be summarized by three equations isomorphic to the textbook RANK model. We highlight how the behavioral HANK model nests a wide spectrum of existing models and show how it overcomes several challenges present in these existing models. What is more, we show how only the behavioral HANK model can overcome all of these challenges at the same time. We then analytically characterize the intertemporal marginal propensities to consume and show how they depend on bounded rationality, household heterogeneity, and the interaction of the two. We end the section by highlighting that the behavioral HANK model generates different policy implications than its rational counterpart.

4.1 The Three-Equation Representation

The behavioral HANK model can be summarized by three equations: a Phillips curve, representing the aggregate supply side captured by equation (15), a Taylor rule for monetary policy (equation (9)), and the *behavioral HANK IS equation* which together determine aggregate demand.

We obtain the behavioral HANK IS equation by combining the hand-to-mouth households' consumption (10) with the savers' consumption (12) and their consumption Euler equation (14).

[18] We discuss empirical estimates of $\bar{m}$ and how we can map recent evidence in Coibion and Gorodnichenko (2015) and Angeletos et al. (2021) to $\bar{m}$ in Appendix B.

Proposition 1. *The behavioral HANK IS equation is given by*

$$\widehat{y}_t = \psi_f \mathbb{E}_t \widehat{y}_{t+1} - \psi_c \frac{1}{\gamma} \left(\widehat{i}_t - \mathbb{E}_t \pi_{t+1} \right), \tag{18}$$

where

$$\psi_f \equiv \bar{m}\delta = \bar{m} \left[1 + (\chi - 1) \frac{1-s}{1-\lambda\chi} \right]$$

and

$$\psi_c \equiv \frac{1-\lambda}{1-\lambda\chi}.$$

Compared to RANK, two extra coefficients show up: ψ_c and ψ_f. ψ_c governs the sensitivity of today's output with respect to the contemporaneous real interest rate. ψ_c is shaped by household heterogeneity, λ and χ, and crucially depends on the cyclicality of income inequality: if income inequality is countercyclical ($\chi > 1$), which seems to be the empirical consensus, $\psi_c > 1$ and contemporaneous monetary policy is amplified through general equilibrium forces.[19]

The second new coefficient, ψ_f, captures the sensitivity of today's output with respect to changes in expected future output. ψ_f is shaped by household heterogeneity *and* the behavioral friction as it depends on the cyclicality of income risk *and* the degree of bounded rationality of households as well as the interaction of the two frictions. Given countercyclical income inequality, income risk is also countercyclical which manifests itself in $\delta > 1$. This countercyclical risk induces compounding in the Euler equation and, thus, competes with cognitive discounting ($\bar{m} < 1$) which induces discounting in the Euler equation. We see in the following secions that even for a small degree of bounded rationality—much smaller than the empirics suggest—the discounting through bounded rationality dominates the compounding through countercyclical income risk. Hence, in the behavioral HANK model it holds that $\psi_f < 1$ which makes the economy less sensitive to expectations and news about the future which is key to resolve the NK puzzles.

Equation (18) nests a wide range of existing IS equations: the IS equation in the standard rational-expectations RANK (see, e.g., Woodford (2003) or Galí (2015) for a textbook treatment) by setting $\psi_f = \psi_c = 1$, the *behavioral* RANK of Gabaix (2020) by $\delta = \psi_c = 1$,

[19] Empirical evidence for countercyclical inequality (in particular, conditional on monetary policy shocks), see, for example, Patterson (2019), Coibion et al. (2017), Ampudia et al. (2018) and Samarina and Nguyen (2019). Almgren et al. (2019) show that output in countries with higher shares of hand-to-mouth households responds more strongly to monetary policy shocks which, through the lens of the model, requires $\chi > 1$.

the TANK model of Bilbiie (2008) by $\bar{m} = \psi_f = 1$, and the rational THANK by Bilbiie (2021) by $\bar{m} = 1$.

4.2 Policy Amplification and Puzzles

We first show how our behavioral HANK model can generate the desirable HANK features such as fiscal multipliers larger than one and monetary policy that mainly works through indirect, general equilibrium, channels and how it simultaneously resolves NK puzzles.

Overcoming the Catch-22. The Catch-22 describes the trade-off that rational THANK models can either generate amplification of contemporaneous monetary policy and fiscal policy or solve the forward guidance puzzle (Bilbiie (2021)). *Amplification* can be understood as a steeper Keynesian cross (see Bilbiie (2020)) which also implies a more prominent role for indirect (general equilibrium) channels of monetary policy.[20] Amplification of contemporaneous monetary policy compared to the representative-agent model ($\psi_c > 1$) requires

$$\chi > 1, \tag{19}$$

whereas the solution of the forward guidance puzzle[21] requires

$$\chi < 1. \tag{20}$$

With $\chi > 1$, the income of H agents moves more than one-to-one with aggregate output. Hence, after a decrease in the interest rate, a disproportionate share of the extra income is received by H agents. Given their high MPC out of transitory increases in income, this amplifies the increase in output through general equilibrium.

If $\chi < 1$ the income of H agents moves less than one-to-one with aggregate income. In this case, savers who self-insure against becoming hand-to-mouth in the future want more insurance when they expect a decrease in the interest rate since if they become hand-to-mouth they would benefit less from the increase in aggregate income. Hence, after a forward

[20] The importance of indirect effects in HANK models has been extensively discussed in Kaplan et al. (2018) and is empirically supported in Ampudia et al. (2018), Samarina and Nguyen (2019) and Holm et al. (2021). Thus, we think the focus on the role of general equilibrium amplification is somewhat cleaner as the *magnitude* of the impact effect of the shock can always be scaled by the intertemporal elasticity of substitution $\frac{1}{\gamma}$.

[21] We define the forward guidance puzzle as the model feature that announcements about future changes in the interest rate affect output today as strong (or even stronger) than contemporaneous changes in the interest rate. For detailed analyses of the forward guidance puzzle in RANK, see McKay et al. (2016) and Giannoni et al. (2015). Miescu (2022) provides empirical evidence that conventional monetary policy is more effective than forward guidance.

guidance shock, savers increase their precautionary savings which dampens the increase in output today offering a possible solution to the forward guidance puzzle.[22] On the contrary, if $\chi > 1$ savers want less insurance after a forward guidance shock since if they become hand-to-mouth they would disproportionately benefit from the increase in aggregate income. The decrease in precautionary savings by the savers then further increases output today aggravating the forward guidance puzzle.

We now show how the behavioral HANK model resolves the Catch-22, i.e., it achieves amplification of contemporaneous monetary policy and a solution of the forward guidance puzzle, simultaneously.

Proposition 2. *In the behavioral HANK model, there is amplification of monetary policy relative to RANK if and only if*

$$\chi > 1, \tag{21}$$

and the forward guidance puzzle is ruled out if

$$\bar{m}\delta + \frac{1}{\gamma}\frac{1-\lambda}{1-\lambda\chi}\kappa < 1, \tag{22}$$

or in terms of $\bar{m}$*:*

$$\bar{m} < \frac{1 - \frac{1-\lambda}{\gamma(1-\lambda\chi)}\kappa}{\delta}. \tag{23}$$

Proposition 2 shows that for a sufficiently low $\bar{m}$, the forward guidance puzzle is resolved with $\chi > 1$ and, hence, the behavioral HANK model can resolve the Catch-22. The reason is the following: The behavioral friction leaves the relative importance of direct vs. indirect effects—i.e., amplification of current monetary policy—unaltered, as amplification is solely determined by a contemporaneous redistribution towards the high MPC households. In contrast, bounded rationality affects how households perceive their idiosyncratic risk. In particular, it opposes the compounding effects stemming from the expected countercyclical income risk. If the behavioral friction dominates, i.e., when condition (23) holds, the behavioral HANK model delivers a discounted Euler equation. Given our calibration, it follows, that $\bar{m} < 0.93$ is sufficient.[23] Thus, already a small deviation from rational expectations is

[22] Note that condition (20) is necessary but not sufficient for solving the forward guidance puzzle. The sufficient condition takes the inflation response into account and is given in THANK by $\delta + \frac{1}{\gamma}\frac{1-\lambda}{1-\lambda\chi}\kappa < 1$.

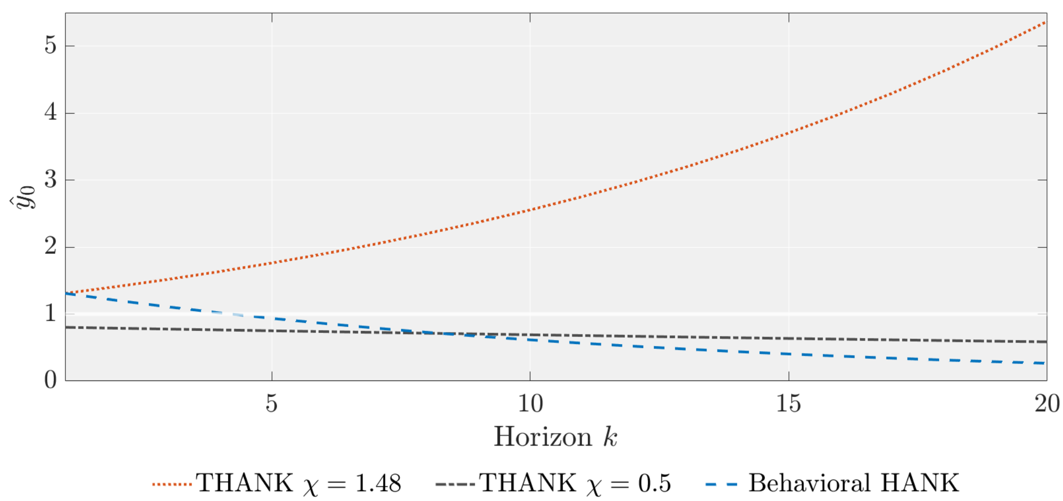
[23] The calibration is as follows: $\chi = 1.48$, $\lambda = 0.33$, $s = 0.9457$ (this corresponds to a s of 0.8 in annual terms), $\gamma = 1$, $\kappa = 0.02$ and we set the Taylor coefficient to 0 for the forward guidance exercise. This calibration is close to the calibration in Bilbiie (2021) and Bilbiie (2020) and is set in order to replicate several findings on the New Keynesian cross coming from more quantitative HANK models. If we exactly take the calibration in Bilbiie (2021), the condition for $\bar{m}$ would be even weaker. Details on the calibration and a discussion of the robustness of our findings for changing calibrations are shown in Appendix B. Note, that even when we vary certain parameters, we always focus on cases with $\lambda < \chi^{-1}$.

enough to resolve the Catch-22 and our conservative calibration of $\bar{m} = 0.85$ is sufficient.

We graphically illustrate the Catch-22 of the rational THANK model and the resolution of it in the behavioral HANK model in Figure 1. The figure shows the response of contemporaneous output relative to the initial response in the RANK model with rational expectations for anticipated i.i.d. monetary policy shocks occurring at different times k and a Taylor coefficient of 0 (as in Bilbiie (2021)).[24]

The orange-dotted line denotes the baseline calibration of the rational THANK model. We see that this model is able to generate contemporaneous amplification of monetary policy shocks, that is, an output response that is relatively stronger than in RANK. Put differently, the GE effects are relatively strong. Yet, at the same time, it exacerbates the forward guidance puzzle as shocks occurring in the future have much stronger effects on today's output than shocks today. The black-dashed-dotted line shows how the forward guidance puzzle can be resolved by allowing for $\chi < 1$. Yet, this comes at the cost that the model is unable to generate amplification of contemporaneous monetary policy shocks. Furthermore, even for the quite low χ, the decay happens relatively slowly.[25]

Figure 1: Resolving the Catch-22



Note: This figure shows the response of total output in period 0 to anticipated i.i.d. monetary policy shocks occurring at different horizons k, relative to the initial response in the RANK model under rational expectations (equal to 1). The parameters are set to $\lambda = 0.33$, $s = 0.9457$, $\gamma = 1$, $\kappa = 0.02$.

The blue-dashed line shows that the behavioral HANK model generates both: amplifica-

[24]Under fully-rigid prices (i.e., $\kappa = 0$), the RANK model would deliver a constant response for all k. The same is true for TANK, i.e., THANK without type switching. Whether the constant response would lie above or below its RANK counterpart depends on $\chi \lesseqgtr 1$ in the same way the initial response depends on $\chi \lesseqgtr 1$.

[25]Bilbiie (2020) calibrates $\chi = 0.3$ to approximate the forward guidance dampening results in McKay et al. (2016) and McKay et al. (2017).

tion of contemporaneous monetary policy and a resolution of the forward guidance puzzle. Note that also rational TANK models (thus, turning off type switching) or the behavioral RANK model would not deliver amplification and resolve the forward guidance puzzle simultaneously. TANK models would face the same issues as the rational RANK model in the sense that they cannot solve the forward guidance puzzle while bounded rationality in a RANK model does not deliver initial amplification.

As a direct consequence of the resolution of the Catch-22 in the behavioral HANK model, highly persistent monetary policy shocks have smaller effects on contemporaneous output than in RANK whereas less persistent shocks have larger effects in the behavioral HANK model. The reason is that persistent shocks also work through a forward guidance channel which is dampened in the behavioral HANK model. We elaborate this point in more detail in Appendix C.2.

Revisiting the Taylor Principle. According to the Taylor principle, monetary policy needs to respond sufficiently to changes in inflation in order to have a determinate equilibrium. In the rational RANK model the Taylor principle is given by $\phi > 1$. We now derive a behavioral HANK Taylor principle and show that both household heterogeneity and bounded rationality affect this condition. The following proposition shows the behavioral HANK Taylor principle.[26]

Proposition 3. *The behavioral HANK model has a determinate, locally unique equilibrium if and only if:*

$$\phi > \phi^* = 1 + \frac{\bar{m}\delta - 1}{\frac{\kappa}{\gamma}\frac{1-\lambda}{1-\lambda\chi}}. \tag{24}$$

Appendix A shows how to derive Proposition 3 and extends the result to more general Taylor rules.

Let us first set $\bar{m} = 1$ and, thus, focus on the role of household heterogeneity. With $\chi > 1$, $\phi^* > 1$ and, hence, the threshold is above the RANK Taylor principle. This insufficiency of the Taylor principle in the rational THANK model has been shown by Bilbiie (2021) and in a similar way by Acharya and Dogra (2020). As a future aggregate sunspot increases the income of households in state H disproportionately, savers cut back on precautionary savings today which further increases output today. This calls for a stronger response of the central bank to not make the sunspot self-fulfilling.

On the other hand, bounded rationality and, thus, $\bar{m} < 1$ relaxes the condition as savers now cognitively discount both the future aggregate sunspot as well as its implication for their idiosyncratic risk. Now a smaller response of the central bank is needed in order to prevent

[26] We focus on local determinacy and bounded equilibria.

the sunspot to become self-fulfilling. Given our calibration, the cutoff value for $\bar{m}$ to restore the RANK Taylor principle in the behavioral HANK model is 0.95. What is more, given our baseline choice of $\bar{m} = 0.85$, $\phi^* = -3.07$. Thus, the Taylor principle is not even necessary in our behavioral HANK model as the economy features a stable unique equilibrium even under an interest rate peg. In this sense, the behavioral HANK model overcomes the famous result in Sargent and Wallace (1975) who have shown that an interest rate peg leads to equilibrium indeterminacy.[27]

The Lower Bound Problem. Related to the determinacy issues under a peg, the traditional New Keynesian model struggles to explain how the economy can remain stable when the effective lower bound (ELB) on nominal interest rates is binding for an extended period of time. If this is the case, RANK predicts unreasonably severe recessions and, in the limit case in which the ELB binds forever, there is even indeterminacy in RANK. The intuition is directly related to our discussion about determinacy under a peg: A forever binding ELB basically implies that the Taylor coefficient is equal to zero and, thus, the nominal rate is pegged at the lower bound thereby violating the Taylor principle.[28]

We now show that the behavioral HANK model resolves these issues. To this end, let us add a natural-rate shock r_t^n to the IS equation (18). We assume that in period t the natural rate decreases to a value $\widetilde{r}^n$ that is sufficiently negative such that the natural rate in levels is below the ELB. The natural rate stays at $\widetilde{r}^n$ for $k \geq 0$ periods and after k periods, the economy returns immediately back to steady state. Agents correctly anticipate the length of the binding ELB. For simplicity, we assume fully-rigid prices, i.e., $\kappa = 0$ and $\pi_t = 0$ for all t, but this is not crucial for what follows. Iterating the IS equation (18) forward, it follows that output in period t is given by

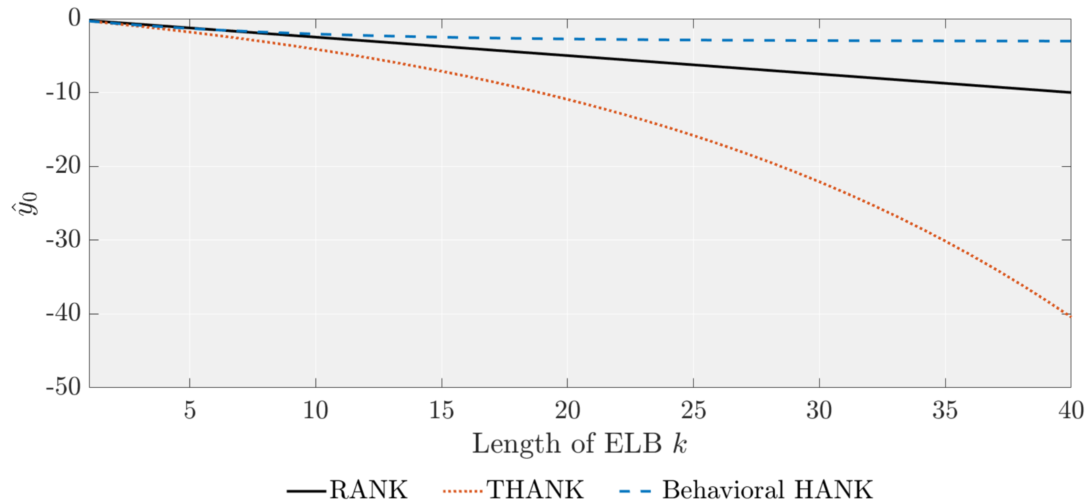
$$\widehat{y}_t = -\frac{1-\lambda}{\gamma(1-\lambda\chi)} \underbrace{\left(\widehat{i}_{ELB} - \widetilde{r}^n\right)}_{>0} \sum_{j=0}^{k} (\bar{m}\delta)^j \,, \tag{25}$$

where the term $\left(\widehat{i}_{ELB} - \widetilde{r}^n\right) > 0$ captures the shortfall of the policy response due to the binding ELB. Under rational expectations and countercyclical inequality, $\chi > 1$ and, thus, $\delta > 1$, output implodes as $k \to \infty$. The same is true in the rational RANK model which is

[27] Angeletos and Lian (2021) show (in a model without household heterogeneity) that small frictions in memory and intertemporal coordination lead to a unique equilibrium which is the same as the one selected by the Taylor principle but it does no longer depend on it.

[28] Note, that this statement also extends to models featuring more elaborate monetary policy rules including Taylor rules responding to output or also the Wicksellian price-level targeting rule, as they all collapse to a constant nominal rate in a world of an ever-binding ELB.

Figure 2: The Effective Lower Bound Problem



Note: This figure shows the contemporaneous output response for different lengths of a binding ELB k and compares the responses across different models.

captured by $\chi = 1$ and, thus, $\delta = 1$. In the behavioral HANK model, however, this is not the case. As long as $\bar{m}\delta < 1$, the output response in t is bounded even when $k \to \infty$. The condition $\bar{m}\delta < 1$ is the same as for determinacy under a peg in the economy with fully-rigid prices. It follows that $\bar{m} < 0.95$ is enough to rule out unboundedly-severe recessions at the ELB even if the ELB is expected to persist forever.

We illustrate the resolution of the lower bound problem graphically in Figure 2. The figure shows the output response in the rational RANK, the rational THANK and the behavioral HANK to different lengths of a binding ELB. The shortcoming of monetary policy due to the ELB, i.e., the gap $\left(\widehat{i}_{ELB} - \widetilde{r}^n\right) > 0$, is set to a relatively small value of 0.25% (1% annually), and we set $\bar{m} = 0.85$. Figure 2 shows the implosion of output in the rational RANK and even more so in the rational THANK model: an ELB that is expected to bind for 40 quarters would decrease today's output in the rational RANK by 10% and in the rational THANK model by 40%. On the other hand, output in the behavioral HANK model would drop by a mere 3%.

Fiscal Multipliers. We now show that the sufficient statistic for amplification of the contemporaneous monetary policy is also a sufficient statistic to generate constant real rate multiplier above 1 in our behavioral HANK model. To characterize fiscal multipliers, we follow Bilbiie (2021) and assume government spending g_t to follow an AR(1) with persistence $\mu \geq 0$, and to be 0 in steady state. The government taxes all agents uniformly to finance g_t.

We re-derive the behavioral HANK IS equation with government spending and obtain:

$$\widehat{c}_t = \bar{m}\delta\mathbb{E}_t\widehat{c}_{t+1} - \frac{1}{\gamma}\frac{1-\lambda}{1-\lambda\chi}\left(\widehat{i}_t - \mathbb{E}_t\pi_{t+1}\right) + \zeta\left[\frac{\lambda(\chi-1)}{1-\lambda\chi}\left(g_t - \bar{m}\mathbb{E}_t g_{t+1}\right) + (\delta-1)\bar{m}\mathbb{E}_t g_{t+1}\right],$$

where $\zeta \equiv \frac{\varphi}{\gamma\left(1+\frac{\varphi}{\gamma}\right)}$. The static Phillips Curve in this setting is given by $\pi_t = \kappa c_t + \kappa\zeta g_t$.

The following Proposition characterizes the fiscal multiplier in the behavioral HANK model.

Proposition 4. *The fiscal multiplier in the behavioral HANK model is given by*

$$\frac{\partial\widehat{y}_t}{\partial g_t} = 1 + \frac{1}{1-\nu\mu}\frac{\zeta}{1+\frac{1}{\gamma}\frac{1-\lambda}{1-\lambda\chi}\phi\kappa}\left[\frac{\chi-1}{1-\lambda\chi}\left[\lambda + \bar{m}\mu(1-s-\lambda)\right] - \kappa\frac{1}{\gamma}\frac{1-\lambda}{1-\lambda\chi}(\phi-\mu)\right],$$

where

$$\nu \equiv \frac{\bar{m}\delta + \frac{1}{\gamma}\kappa\frac{1-\lambda}{1-\lambda\chi}}{1+\frac{1}{\gamma}\frac{1-\lambda}{1-\lambda\chi}\phi\kappa}. \tag{26}$$

A corollary of Propositon 4 is that in the case of persistent government spending, $\mu > 0$, and in the amplification case ($\chi > 1$), more bounded rationality, i.e., a lower $\bar{m}$, leads to a lower fiscal multiplier.[29] Bounded rationality weakens the fiscal multiplier as boundedly-rational agents discount the fact that an increase in government spending today has a positive effect on future spending as well. In the case of an i.i.d. spending shock, the fiscal multiplier is independent of $\bar{m}$.

We follow Bilbiie (2021) and set $\kappa = 0$. Together with our Taylor rule (equation 9), this implies that the real interest rate is constant after the government spending shock. This is a useful benchmark as in this case, the fiscal multiplier in the RANK model is 1 and the *consumption* response is 0 (see Bilbiie (2011) and Woodford (2011)).[30]

From Proposition 4, we can directly derive the constant-real-rate multiplier in the behavioral HANK model. It shows that with $\chi > 1$, the fiscal multiplier is bounded from below by 1 irrespective of the persistence μ. In other words, the constant-real-rate multiplier in the behavioral HANK model is strictly above 1, regardless of the dampening of bounded rationality on the fiscal multiplier in the case of persistent spending. This shows that the intuition in Bilbiie (2021) and in Auclert et al. (2018) is not reversed by the introduction of bounded rationality: with $\chi > 1$, the high MPC households benefit disproportionately

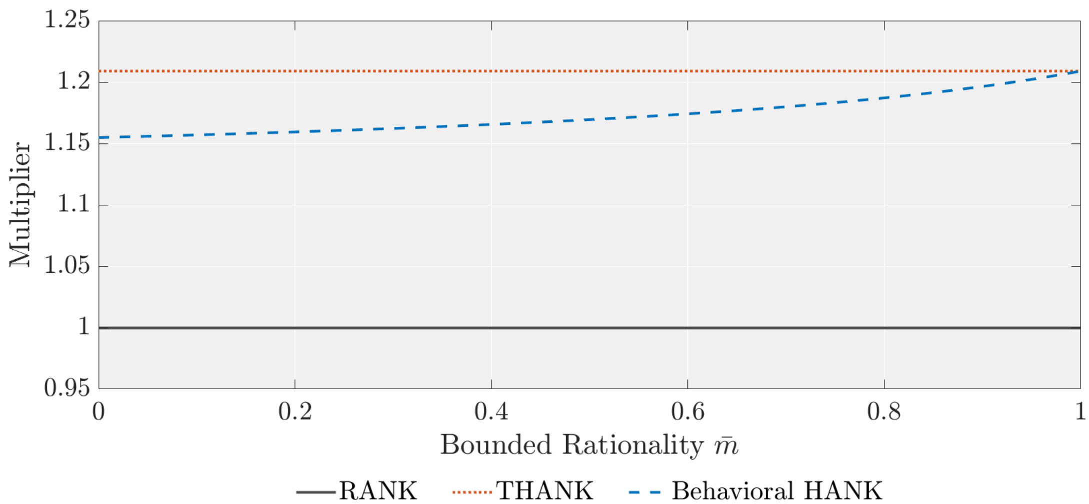
[29]This also assumes that the risk of becoming hand-to-mouth is not excessively high, i.e., $1-s > \lambda$, which is the case under any reasonable parameterization. We also focus on the case in which $\nu\mu < 1$, which holds in the behavioral HANK model even for $\mu = 1$.

[30]Auclert et al. (2018) also use a constant real interest rate case to show that their HANK model can generate fiscal multipliers larger than one.

from the extra income out of the increase in government spending which increases the fiscal multiplier through a Keynesian type argument.

Figure 3 takes a more thorough look at the role of the bounded rationality on the fiscal multiplier by showing the fiscal multiplier in the behavioral HANK model for varying degrees of $\bar{m}$ (blue-solid line) and comparing it to the multiplier in the THANK and the RANK model. For this exercise, we set the persistence to a medium value, $\mu = 0.6$. It shows that the fiscal multiplier decreases with decreasing $\bar{m}$. Yet, even for the extreme case $\bar{m} = 0$, in which households fully discount all future increases in government spending, the fiscal multiplier is still substantially above one even though it is somewhat weaker than under rational expectations.

Figure 3: Fiscal Multipliers



Note: This figure shows the fiscal multipliers for different degrees of bounded rationality (blue-dashed line). The orange-dotted line plots the fiscal multiplier in the rational version of the model and the black-solid line shows the zero-multiplier in the RANK model.

It is noteworthy that the behavioral HANK model does not rely on a specific financing type to achieve fiscal multipliers larger than one. This is in contrast to the behavioral RANK model in Gabaix (2020). In the behavioral RANK model, bounded rationality can also increase the multiplier but only if the government delays taxing the agents to finance the contemporaneous spending as boundedly-rational agents will then discount the future increases in taxes. In the rational THANK model, on the other hand, the fiscal multiplier can in principle be larger than one with $\chi < 1$ if the hand-to-mouth households pay relatively less than the savers (see Bilbiie (2020) or Ferriere and Navarro (2018)). Both of these channels would also push up the multiplier in the behavioral HANK model, yet it does not depend on any of these two to achieve fiscal multipliers larger than 1.

Behavioral HANK as a Unifying Framework. Figure 4 shows how bounded rationality and household heterogeneity interact to generate the desirable HANK features and to simultaneously resolve the NK puzzles. The figure shows the parameter space for the two sufficient parameters for household heterogeneity and bounded rationality, respectively, $(\chi, \bar{m})$. The blue and orange dashed lines split the parameter space in the following sense: The blue line denotes the cut-off values below which the model is determinate under an interest-rate peg while above it the model is indeterminate (with the line itself belonging to the indeterminacy region). Determinacy under a peg is sufficient to rule out the forward guidance puzzle as well as the lower bound problem, and thus, is a sufficient statistic to resolve the discussed NK puzzles. The orange line denotes the cut-off values such that to the right of it, the model generates amplification while left from it—again including the line—the model does not generate amplification. Here, "amplification" is a stand-in for the desirable HANK features: monetary policy amplification through indirect (GE) effects and fiscal multipliers above 1 under constant real rates.

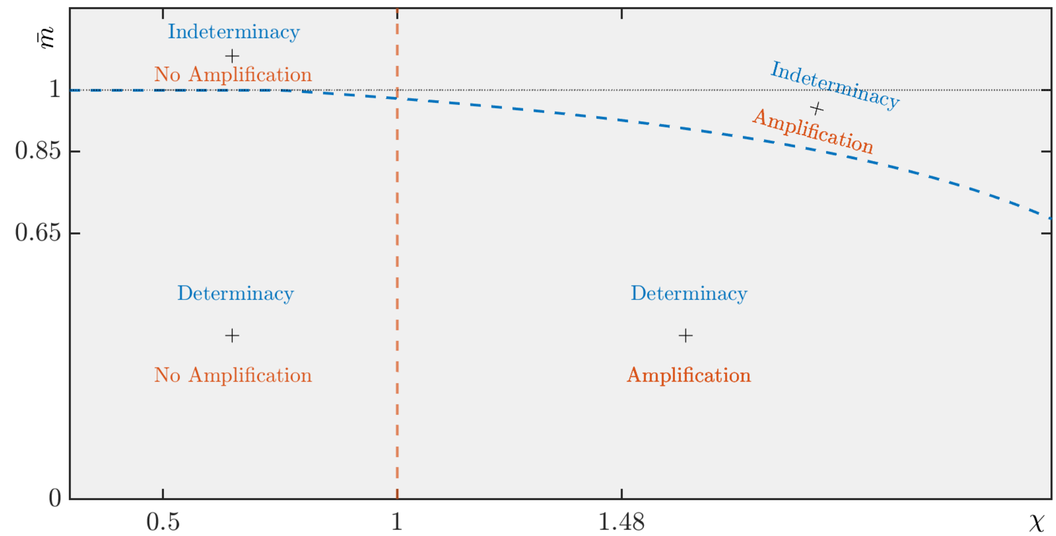
This split of the parameter space into four areas allows us to distinguish the models discussed so far and to show how the behavioral HANK can overcome the limitations inherent in existing model. The RANK model is located in the "indeterminacy + no amplification" region as $\bar{m} = 1$ and $\chi = 1$. The behavioral RANK can either be in "indeterminacy + no amplification" or in "determinacy + no amplification" depending on the degree of rationality.[31] Rational THANK models can either be in "indeterminacy + no amplification", "determinacy + no amplification" or in "indeterminacy + amplification" while rational TANK models can only be in "indeterminacy + no amplification" or in "indeterminacy + amplification". Importantly, both cannot be in "determinacy + amplification".[32] Only the behavioral HANK model can deliver "determinacy + amplification". Furthermore, the behavioral HANK model can in principle cover the whole parameter space as it nests all the aforementioned models as special cases.

Having discussed the aggregate implications of the model, we now zoom in closer into the model and derive the iMPCs and show how they depend on bounded rationality, household heterogeneity, and the interaction of the two.

[31] Note, this also applies to other models featuring deviations from FIRE that deliver equivalent reduced-for IS equations, e.g., Angeletos and Lian (2018) and Woodford (2019).

[32] Note that this applies to all the THANK models as summarized in Section 2.

Figure 4: The Behavioral HANK as a Unifying Framework



Note: The figure characterizes four possible regions depending on whether the considered $(\chi, \bar{m})$-pair delivers determinacy under an interest-rate peg or not and whether the model generates amplification of contemporaneous monetary and fiscal policy or not (we only extend the y-axis above 1 for the sake of readability).

4.3 Intertemporal MPCs

The HANK literature shows that the iMPCs are a key statistic for conducting policy analysis (see, e.g., Auclert et al. (2018), Auclert et al. (2020), and Kaplan and Violante (2020)). We follow the THANK/TANK literature and define the aggregate iMPCs in the behavioral HANK model as the partial derivative of aggregate consumption at time k, $\widehat{c}_k$, with respect to aggregate disposable income, $\tilde{y}_0$, keeping everything else fixed (see Bilbiie (2021), Cantore and Freund (2021), and Auclert et al. (2018)).

The following Proposition characterizes the iMPCs in the behavioral HANK model.[33]

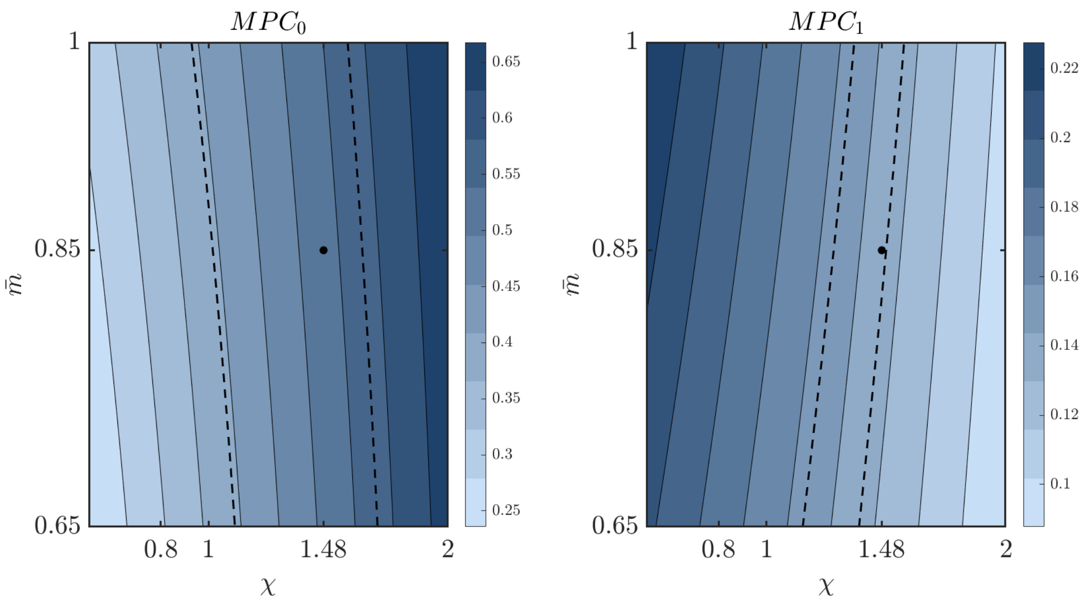
Proposition 5. *The intertemporal MPCs in the behavioral HANK model, i.e., the aggregate consumption response in period k to a one-time change in aggregate disposable income in period 0, are given by*

$$\frac{d\widehat{c}_0}{d\widetilde{y}_0} = 1 - \frac{1-\lambda\chi}{s\bar{m}}\mu_2^{-1}$$
$$\frac{d\widehat{c}_k}{d\widetilde{y}_0} = \frac{1-\lambda\chi}{s\bar{m}}\mu_2^{-1}\left(\beta^{-1}-\mu_1\right)\mu_1^{k-1}, \qquad \textit{for } k>0,$$

where the parameters μ_1 and μ_2 depend on the underlying parameters, including $\bar{m}$ and χ and are explicitly spelled out in Appendix D.

[33]See Appendix D for the derivation.

Figure 5: Intertemporal MPCs, Bounded Rationality and Household Heterogeneity



Note: This figure shows the aggregate intertemporal MPCs, i.e., the aggregate consumption response in year 0 (left) and year 1 (right) to a change in aggregate disposable income in year 0 for different χ (x-axis) and $\bar{m}$ (y-axis). The dashed lines show the range of empirically-estimated iMPCs and the black dot shows the model estimate given our baseline calibration. Darker colors represent higher MPCs, see the colorbars on the right side of the figures.

Figure 5 graphically depicts how the interplay of bounded rationality $\bar{m}$ and household heterogeneity χ determines the size of the aggregate iMPCs. Therefore, we calibrate the model annually as the empirical evidence on the iMPCs is annual (see Fagereng et al. (2021) and Auclert et al. (2018)). We set $s = 0.8$, $\lambda = 0.33$, $\gamma = 1$ and $\beta = 0.95$. These values lie within the standard range of values used in the THANK literature (see Bilbiie (2020) or Bilbiie (2021)). The left panel depicts the aggregate MPCs to spend within the first year (in period 0) and the right panel shows aggregate MPCs to spend within the second year (in period 1) after the temporary increase in income in time 0. Darker colors represent higher MPCs. First, note that with our baseline calibration—$\chi = 1.48$ and $\bar{m} = 0.85$ as shown by the black dots—the behavioral HANK model generates iMPCs within the first year of 0.55 and within the second year of 0.15. This lies exactly in the estimated bounds for the iMPCs in the data which are between $0.42 - 0.6$ within the first and $0.14 - 0.16$ within the second year (see dashed lines). Away from our baseline calibration, an increase in χ increases the MPCs in the first year but decreases them in the second year.[34] In contrast, an increase in

[34]Note, that when considering micro moments like the iMPCs, $\chi = 1$ is not sufficient anymore for the model to collapse to RANK. More precisely, with $\chi = 1$ the model collapses to a THANK model which behaves in the aggregate exactly like RANK (see the incomplete-markets irrelevance result in Werning (2015)). Hence, the RANK iMPCs are not pictured in Figure 5 but Proposition 5 still nests RANK for $\chi = 1$ and $\lambda = 0$.

$\bar{m}$ increases the aggregate MPC in the first year and in the second year.

Let us first turn to the role of χ for the iMPCs: Recall, the higher χ, the more sensitive is the income of the H households to a change in aggregate income. Thus, with higher χ, H households gain weight in relative terms for the aggregate iMPCs while the savers loose weight in relative terms. This pushes up the aggregate MPC within the first year, but pushes down the aggregate MPC within the second year as households that were hand-to-mouth in the period of the income windfall will have a MPC of 0 in the second year.

Bounded rationality, captured by $\bar{m}$, affects only the MPCs of the savers as only savers—whether behavioral or rational—intertemporarily optimize. The savers' Euler equation dictates that the decrease in today's marginal utility of consumption—following the increase in consumption—is equalized by a decrease in tomorrow's expected marginal utility. For the behavioral saver, however, the decrease in tomorrow's marginal utility needs to be more substantial as she cognitively discounts the expectations about the future decrease. Hence, the behavioral saver saves relatively more out of the income windfall. This pushes down the aggregate MPCs in $t = 0$. The same is true for the aggregate MPC in $t = 1$, in which there are now two opposing forces at work: on the one hand, the saver again cognitively discounts the expectations about the future decrease in the marginal utility which depresses her consumption. On the other hand, savers have accumulated more wealth from period $t = 0$ which tends to increase consumption. Given our calibration, in $t = 1$ the former dominates. Figure 12 in Appendix D shows that, beginning in $k = 3$, the latter effect starts to dominate. If we increase the idiosyncratic risk of becoming hand-to-mouth, i.e., increase the transition probability $1 - s$, the aggregate MPC is already higher in $t = 1$ for lower $\bar{m}$. The reason is that a smaller fraction of initial savers remains savers which pushes upwards consumption in $k = 1$ (see Figure 11 in Appendix D).

The effects of a change in $\bar{m}$ are more pronounced at lower levels of χ. Combining our discussion about the role of χ and $\bar{m}$, this is intuitive: the lower χ, the higher is the relative importance of the savers for the aggregate iMPCs and, in turn, the stronger is the effect of $\bar{m}$ on the aggregate iMPCs. These interaction effects are quite substantial: at $\chi = 1.48$, a decrease of $\bar{m}$ from 1 to 0.65 decreases the MPC_0 by 7% and the MPC_1 by more than 11%.

4.4 Policy Implications: The Timing of Monetary Policy

We close this section by discussing some of the policy implications of the behavioral HANK model. In particular, we illustrate that the behavioral HANK can generate different policy implications than its rational counterpart. To this end, we analyze how the timing of monetary policy affects its effectiveness and its distributional consequences.

Consider that the central bank wants to increase the nominal interest rate by a cumulative $x\%$, for example, to fight an overheating economy. The central bank decides whether to implement this policy within one quarter or to gradually raise the interest rate by $\frac{x}{k}\%$ for k consecutive quarters.

Lemma 1. *The effect of a $\frac{x}{k}\%$ interest rate hike over k consecutive periods decreases current output by*

$$\widehat{y}_t = \frac{\psi_c}{\gamma}\left[\sum_{j=0}^{k-1}\left(\psi_f + \frac{\psi_c}{\gamma}\kappa\right)^j\right]\frac{x}{k}.$$

The left panel of Figure 6 depicts the result in Lemma 1 for the behavioral HANK model and compares it to its rational counterpart and the rational RANK model. The solid-black line shows the well-known feature of RANK that the effects of monetary policy on current output become stronger when monetary policy is back-loaded: the further the interest hike is stretched out, the higher is the response on current output. The orange-dotted line shows that this feature is even more pronounced in the rational THANK model as the line is steeper than in the RANK model.

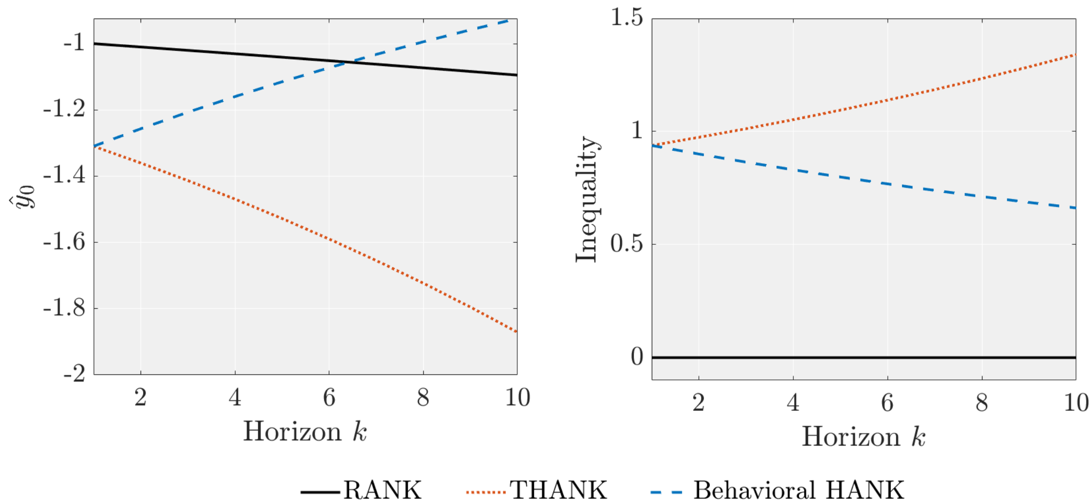
In contrast, the blue-dashed line of the behavioral HANK model is increasing instead of decreasing in k. Thus, back-loading monetary policy decreases its effect on current output. To put it differently, monetary policy is most effective on current output if it is completely front-loaded. Hence, if the central bank wants to fight an overheating of the economy as effectively as possible, the behavioral HANK model implies front-loading the interest rate hike, while its rational counterpart suggests to rather back-load the hike.

The right panel of Figure 6 depicts the effects of the different timing of the monetary policy hikes on consumption inequality, as defined in equation (13). It shows that, according to the behavioral HANK, if monetary policy front-loads the interest rake hike, it increases inequality the most whereas a more gradual increase in the interest rate would have weaker effects on inequality. This illustrates a trade-off for the central banker: the more effectively monetary policy combats the overheating, the more it increases inequality.

5 Bounded Rationality and Incomplete Information with Learning: An Equivalence Result

In this section, we derive an equivalence result between bounded rationality and incomplete information with learning. In particular, we show how a change in the default value in the behavioral setup leads to the same IS equation as in models with incomplete information and learning (see Angeletos and Huo (2021)).

Figure 6: Monetary Policy Timing: Effectiveness and Distributional Consequences



Note: This figure shows the response of current output (left panel) of a cumulative interest-rate hike by $x\%$ implemented over k consecutive periods. The right panel shows the corresponding response of inequality, defined as $\widehat{c}_t^S - \widehat{c}_t^H$.

To this end, we now assume that behavioral agents anchor their expectations to their *last observation* instead of to the steady state values. A possible interpretation is that agents anchor their expectations to what they read or hear in the news. Models featuring some form of backward-looking behavior indeed tend to match the expectations data coming from household surveys quite well (see, for example, Adam et al. (2017), Adam et al. (2020), Angeletos and Huo (2021), and Angeletos et al. (2021)). The backward-looking components in these models usually arise from an incomplete or noisy information setting as well as some form of (Bayesian) learning. We now show how our bounded rationality setup generates expectations that resemble these aforementioned expectations models.

Proposition 6. *Set the boundedly-rational agents' default value to the variable's past value*

$$X_t^d = X_{t-1}. \tag{27}$$

In this case, the boundedly-rational agent's expectations of X_{t+1} becomes

$$\mathbb{E}_t^{BR}\left[X_{t+1}\right] = (1-\bar{m})X_{t-1} + \bar{m}\mathbb{E}_t\left[X_{t+1}\right]. \tag{28}$$

These backward-looking expectations introduce a backward-looking component into the behavioral IS equation as summarized in the following Proposition.

Proposition 7. *In case the behavioral agents' default value is the past value of the respective variable, i.e., $X_t^d = X_{t-1}$, the behavioral HANK IS equation is given by*

$$\widehat{y}_t = \psi_f \mathbb{E}_t \widehat{y}_{t+1} - \psi_c \frac{1}{\gamma} \left(\widehat{i}_t - \mathbb{E}_t \pi_{t+1} \right) + (1 - \bar{m}) \delta \widehat{y}_{t-1}. \tag{29}$$

Proposition 7 shows that the change in the agents' default value does not change the existing behavioral and heterogeneity coefficients ψ_f and ψ_c. Yet, anchoring to past realizations introduces an additional backward-looking term in the IS equation, similar to models relying on habit persistence. Angeletos and Huo (2021) and Gallegos (2021) derive an IS equation with the same reduced form which, however, is based on an incomplete-information setting and learning. We complement their findings by showing how we can generate the equivalent outcome based on a *behavioral* relaxation of FIRE.

Angeletos and Huo (2021) calibrate the coefficients in front of $\mathbb{E}_t \widehat{y}_{t+1}$ and $\widehat{y}_{t-1}$ to match evidence from survey expectations data. By following their calibration, we can back out the implied $\bar{m}$ and χ. We get $\bar{m} = 0.59$ and $\chi = 0.72$, thus, relatively low values compared to the calibration above. We leave the other parameters as in Section 4. We complement the backward-looking behavioral HANK IS equation with the static Phillips Curve (15).

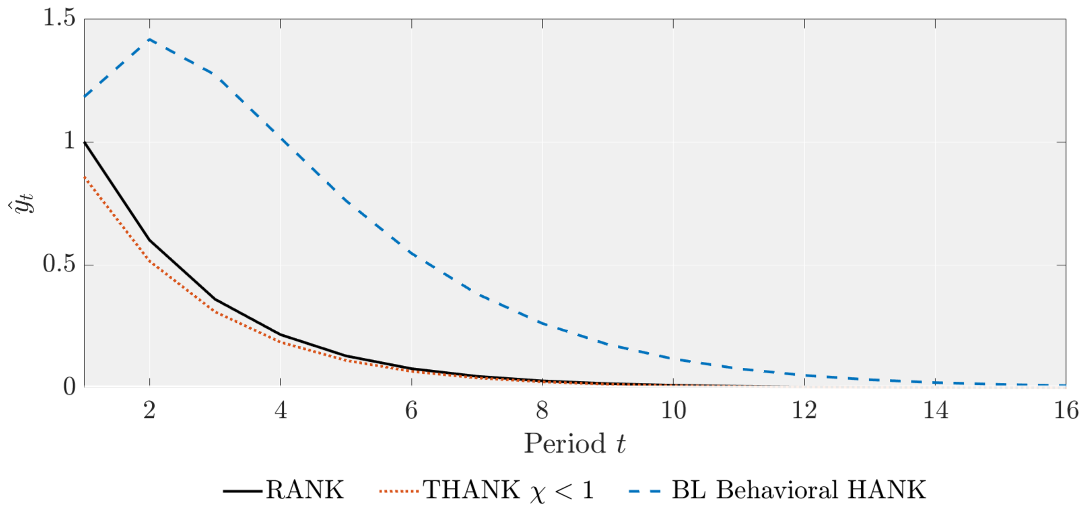
Determinacy. We numerically verify that the backward-looking behavioral HANK model restores the Taylor principle. In fact, the equilibrium is determinate even under an interest-rate peg. Thus, also the backward-looking behavioral HANK model overturns the Sargent and Wallace (1975) result with this calibration.

Impulse-Response Functions. We now show how the backward-looking behavioral HANK model generates hump-shaped impulse responses and a novel behavioral amplification channel. To this end, we examine how output in the backward-looking behavioral HANK model responds to an expansionary monetary policy shock and compare the response to its rational counterpart and the RANK version of the model. We set the Taylor coefficient to 1.5, thus, guaranteeing determinacy also in the rational models and the persistence of the shock to an intermediate value, $\rho^{MP} = 0.6$.

Figure 7 shows the corresponding impulse-response functions. The blue-dashed line shows the results of our behavioral HANK, the orange-dotted line of its rational counterpart (THANK) and the black-solid line of RANK.

Two things stand out. First, the behavioral HANK model delivers amplification compared to RANK—even in the first period—and second, the backward-looking anchor generates hump-shaped responses. As the latter has been highlighted in Angeletos and Huo (2021),

Figure 7: Output Response to a Monetary Policy Shock



Note: This figure shows the output response to a monetary policy shock for different models.

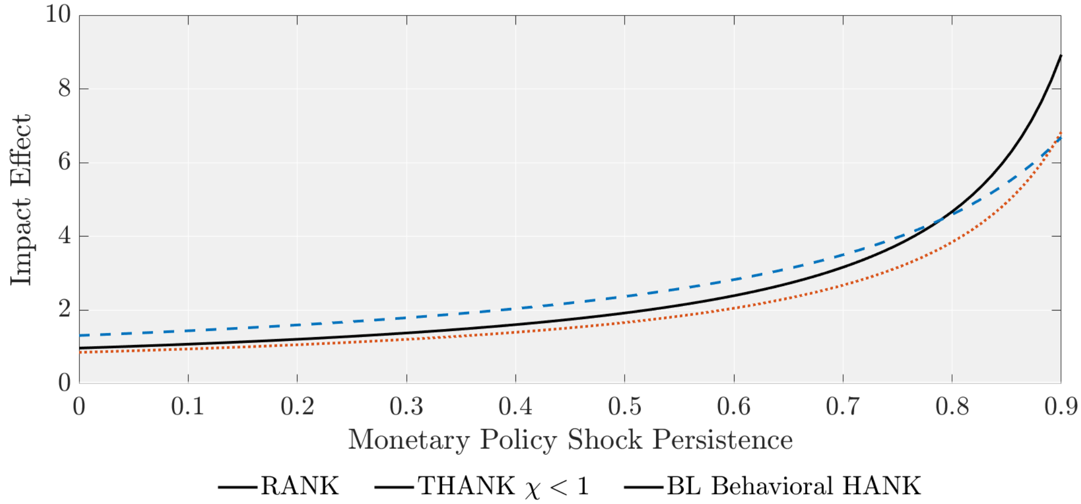
we here focus on the amplification. Figure 7 shows that the amplification stems from a *behavioral amplification channel*: the initial output response is amplified although the model features procyclical inequality ($\chi < 1$) and, thus, the heterogeneity frictions themselves would generate dampening.

Where does the behavioral amplification come from? Given the backward-looking component in households' expectations, the increase in today's output is expected to persist as it becomes tomorrow's default value for the household's expectations. The behavioral anchor induces *endogenous* persistence which further increases today's output response through more optimistic expectations. Yet, there is an opposing channel at work: an *exogenously* persistent shock not only decreases interest rates today but also expected future interest rates. Behavioral households congitively discount these future changes and, thus, perceive the shock as less expansionary compared to a rational agent which dampens the initial response.[35] Given our calibration, the first channel dominates, thereby generating amplification as depicted in Figure 7.

Given the two opposing forces at work, the degree of initial amplification depends on the persistence of the shock. Figure 8 shows the initial response of all three models for different degrees of persistence of the shock. As the persistence declines, the initial response becomes relatively stronger in the backward-looking behavioral HANK model compared to RANK. As a consequence, the relative amplification is largest for an i.i.d. shock.

[35]This is the same channel through which the fiscal multiplier of persistent government spending is dampened in our baseline model in Section 4.

Figure 8: Initial Output Response for Varying Degrees of the Persistence



Note: This figure shows the initial output response to monetary policy shocks with different degrees of persistence.

In addition, comparing the backward-looking behavioral HANK model to its rational counterpart shows that for $\rho^{MP} < 0.9$, there is behavioral amplification while for more persistent shocks, there is behavioral dampening. The comparison with RANK shows that for $\rho^{MP} < 0.80$, the behavioral amplification dominates the heterogeneity dampening which arises because $\chi < 1$.

Behavioral Amplification and Forward Guidance. We now analyze analytically the behavioral-amplification mechanism and its implications for forward guidance. In the backward-looking behavioral HANK model, the output response to an interest rate change depends on the (expected) infinite future even when the shock is completely transitory.

Consider the following. The monetary authority decreases the nominal interest rate in period t to $\widetilde{i}_t < 0$ but will keep it at steady state thereafter (the argument extends to changes of the interest rate in the future). Output and inflation would be expected to go back to zero in $t+1$ under rational expectations. This is, however, not true for the backward-looking behavioral HANK model.

To understand this, combine the static Phillips Curve (a static Phillips curve is again not crucial for the argument but facilitates the derivations) with the behavioral HANK IS equation to arrive at

$$\widehat{y}_t = (1-\bar{m})\delta\widehat{y}_{t-1} - \frac{1}{\gamma}\frac{1-\lambda}{1-\lambda\chi}\widetilde{i}_t + \left[\delta\bar{m} + \kappa\frac{1}{\gamma}\frac{1-\lambda}{1-\lambda\chi}\right]\mathbb{E}_t\widehat{y}_{t+1}.$$

If households expect future output to be back to steady state – as would be the case in the rational model or the behavioral model in which the households' default value equals the steady state – a one-time, completely transitory decrease in the nominal interest rate changes contemporaneous output by

$$\frac{1}{\gamma}\frac{1-\lambda}{1-\lambda\chi} > 0. \tag{30}$$

Yet, in the backward-looking model, expectations in $t+1$ of output in $t+2$ will be above steady state when output in t increases. The more optimistic expectations feed back into output already in t.

This becomes apparent when we write the IS equation as

$$\begin{aligned}
&\widehat{y}_t\left[1-(1-\bar{m})\delta\left[\delta\bar{m}+\kappa\frac{1}{\gamma}\frac{1-\lambda}{1-\lambda\chi}\right]\right] = \\
&(1-\bar{m})\delta\widehat{y}_{t-1}-\frac{1}{\gamma}\frac{1-\lambda}{1-\lambda\chi}\left[\widetilde{i}_t+\left[\delta\bar{m}+\kappa\frac{1}{\gamma}\frac{1-\lambda}{1-\lambda\chi}\right]\mathbb{E}_t\left[\widetilde{i}_{t+1}\right]\right] \\
&+\left[\delta\bar{m}+\kappa\frac{1}{\gamma}\frac{1-\lambda}{1-\lambda\chi}\right]^2\mathbb{E}_t\widehat{y}_{t+2}.
\end{aligned}$$

Thus, if households would assume that $\widehat{y}_{t+2}$ will be zero but not $\widehat{y}_{t+1}$, the discussed interest-rate change in t increases output in t by

$$\frac{\frac{1}{\gamma}\frac{1-\lambda}{1-\lambda\chi}}{1-(1-\bar{m})\delta\left[\delta\bar{m}+\kappa\frac{1}{\gamma}\frac{1-\lambda}{1-\lambda\chi}\right]},$$

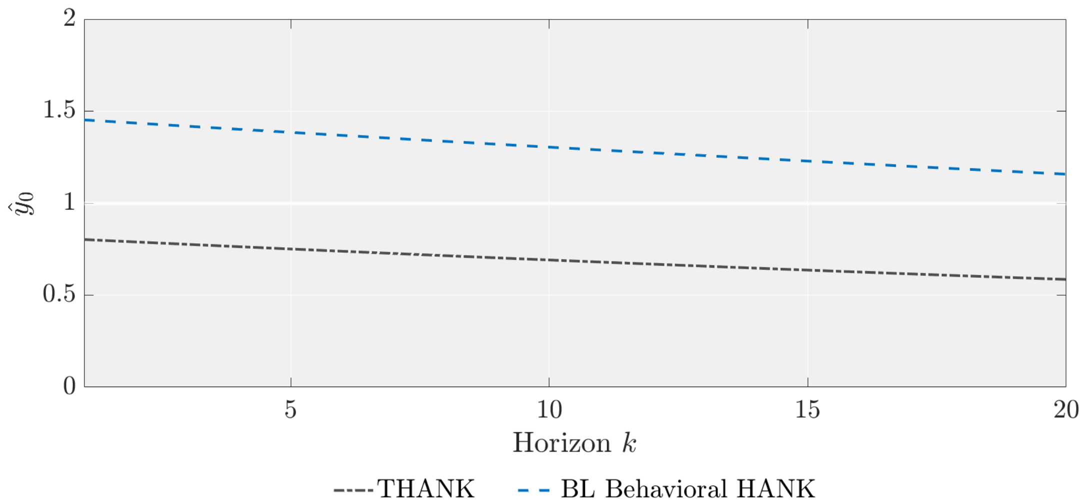
which is larger than the effect for models without a backward-looking anchor as can be seen by comparing it to equation ((30)). Put differently, the initial output response is amplified through a behavioral channel. Iterating forward in this fashion shows how the effect increases with each iteration. However, the response is bounded, as we will see below.

Turning to forward guidance, an expected change in the nominal interest rate in period $t+1$, affects output in t by

$$-\frac{\frac{1}{\gamma}\frac{1-\lambda}{1-\lambda\chi}\left[\delta\bar{m}+\kappa\frac{1}{\gamma}\frac{1-\lambda}{1-\lambda\chi}\right]}{1-(1-\bar{m})\delta\left[\delta\bar{m}+\kappa\frac{1}{\gamma}\frac{1-\lambda}{1-\lambda\chi}\right]},$$

if we assume output in $t+2$ to be back to zero. Given our calibration, the term $\left[\delta\bar{m}+\kappa\frac{1}{\gamma}\frac{1-\lambda}{1-\lambda\chi}\right]$ is smaller than 1. Thus, an interest rate change tomorrow has a smaller effect on output today than a contemporaneous interest rate change such that there is no forward guidance puzzle in the backward-looking behavioral HANK model. We can continue in this fashion to show that the effects increase with the iteration but decrease with the period of the shock.

Figure 9: Forward Guidance with Backward-Looking Anchor



Note: This figure shows the period-t output response to an anticipated i.i.d. monetary policy shock in period $t+k$ for three different economies.

Figure 9 shows these patterns graphically. First, the behavioral amplification channel discussed above is reflected in the contemporaneous effect ($k = 0$) which is stronger than without the backward-looking expectations —reflected in the black-dashed-dotted line. Second, increasing the horizon k shows that there is no forward guidance puzzle in the backward-looking behavioral HANK model. To sum it up, also the backward-looking behavioral HANK model amplifies contemporaneous monetary policy (even for $\chi < 1$) while it simultaneously dampens the effects of forward guidance.

6 Conclusion

We propose a framework that generates both desirable HANK features and resolves NK puzzles: in our behavioral HANK model, monetary policy mainly works through indirect effects, fiscal multipliers are larger than one, and the model generates empirically realistic intertemporal marginal propensities to consume. At the same time, there is no forward guidance puzzle, output remains stable even for (infinitely) long spells at the ELB and the model is determinate under a peg. This is in stark contrast to existing models that are nested within our framework but cannot deliver all these features simultaneously. Therefore, we think that the behavioral HANK model provides a suitable framework for an overarching policy analysis. What is more, we show that the behavioral HANK model can have different policy implications than its rational counterpart, e.g., when it comes to the timing of monetary policy.

We further show that by a small change in the agents' default value to which they anchor their expectations, the resulting backward-looking behavioral HANK model endogenously generates hump-shaped responses of macroeconomic aggregates to monetary policy shocks. In addition, it gives rise to a behavioral amplification channel which allows the model to deliver amplification compared to RANK under conditions in which the rational model would imply dampening. Importantly, the behavioral HANK model achieves all these features while remaining analytically tractable. Thus, it offers a simple framework to study a broad array of questions in future research.

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A Model Details and Derivations

A.1 Derivation of Proposition 1.

Combining equations (10) and (12) with the bounded-rationality setup in equation (17) for $\widehat{x}_t^d = 0$ as X_t^d is given by the steady state, we have

$$\begin{aligned}
\mathbb{E}_t^{BR}\left[\widehat{c}_{t+1}^H\right] &= \bar{m}\mathbb{E}_t\left[\widehat{c}_{t+1}^H\right] = \bar{m}\chi\mathbb{E}_t\left[\widehat{y}_{t+1}\right] \\
\mathbb{E}_t^{BR}\left[\widehat{c}_{t+1}^S\right] &= \bar{m}\mathbb{E}_t\left[\widehat{c}_{t+1}^S\right] = \bar{m}\frac{1-\lambda\chi}{1-\lambda}\mathbb{E}_t\left[\widehat{y}_{t+1}\right].
\end{aligned}$$

Plugging these two equations as well as equation (12) into the savers' Euler equation (14) yields

$$\frac{1-\lambda\chi}{1-\lambda}\widehat{y}_t = s\bar{m}\frac{1-\lambda\chi}{1-\lambda}\mathbb{E}_t\left[\widehat{y}_{t+1}\right] + (1-s)\bar{m}\chi\mathbb{E}_t\left[\widehat{y}_{t+1}\right] - \frac{1}{\gamma}\left(\widehat{i}_t - \mathbb{E}_t\pi_{t+1}\right).$$

Combining the $\mathbb{E}_t\left[\widehat{y}_{t+1}\right]$ terms and dividing by $\frac{1-\lambda\chi}{1-\lambda}$ yields the following coefficient in front of $\mathbb{E}_t\left[\widehat{y}_{t+1}\right]$:

$$\begin{aligned}
\psi_f &\equiv \bar{m}\left[s + (1-s)\chi\frac{1-\lambda}{1-\lambda\chi}\right] \\
&= \bar{m}\left[1 - 1 + s + (1-s)\chi\frac{1-\lambda}{1-\lambda\chi}\right] \\
&= \bar{m}\left[1 - \frac{1-\lambda\chi}{1-\lambda\chi} + s + (1-s)\chi\frac{1-\lambda}{1-\lambda\chi}\right] \\
&= \bar{m}\left[1 - \frac{1-\lambda\chi}{1-\lambda\chi} + \frac{(1-\lambda\chi)s}{1-\lambda\chi} + (1-s)\chi\frac{1-\lambda}{1-\lambda\chi}\right] \\
&= \bar{m}\left[1 + (\chi-1)\frac{1-s}{1-\lambda\chi}\right].
\end{aligned}$$

Defining $\psi_c \equiv \frac{1-\lambda}{1-\lambda\chi}$ yields the behavioral HANK IS equation in Proposition 1:

$$\widehat{y}_t = \psi_f\mathbb{E}_t\widehat{y}_{t+1} - \psi_c\frac{1}{\gamma}\left(\widehat{i}_t - \mathbb{E}_t\pi_{t+1}\right).$$

A.2 Derivation of Proposition 2.

The first part comes from the fact that amplification is defined as

$$\frac{1-\lambda}{1-\lambda\chi} > 1,$$

which requires $\chi > 1$.

For the second part, recall how we model a forward guidance experiment (following Bilbiie (2021)). We assume a Taylor coefficient of 0, i.e., $\phi = 0$, such that the nominal interest rate is given by $\widehat{i}_t = \varepsilon_t^{MP}$. Replacing inflation using the Phillips curve (15), i.e., $\pi_t = \kappa \widehat{y}_t$, we can re-write the behavioral HANK IS equation from Proposition 1 as

$$\begin{aligned}\widehat{y}_t &= \psi_f \mathbb{E}_t \widehat{y}_{t+1} - \psi_c \frac{1}{\gamma}\left(\varepsilon_t^{MP} - \kappa \mathbb{E}_t \widehat{y}_{t+1}\right) \\ &= \left(\psi_f + \psi_c \frac{1}{\gamma}\kappa\right) \mathbb{E}_t \widehat{y}_{t+1} - \psi_c \frac{1}{\gamma} \varepsilon_t^{MP}\end{aligned}$$

The forward guidance puzzle is ruled out if and only if

$$\left(\psi_f + \psi_c \frac{1}{\gamma}\kappa\right) < 1,$$

which is the same as the condition stated in Proposition 2:

$$\bar{m}\delta + \frac{1}{\gamma}\frac{1-\lambda}{1-\lambda\chi}\kappa < 1.$$

Solving this for $\bar{m}$ yields

$$\bar{m} < \frac{1 - \frac{1-\lambda}{\gamma(1-\lambda\chi)}\kappa}{\delta},$$

which completes Proposition 2.

A.3 Derivation of Proposition 3.

Replacing $\widehat{i}_t$ by $\phi\pi_t = \phi\kappa\widehat{y}_t$ and $\mathbb{E}_t\pi_{t+1} = \kappa\mathbb{E}_t\widehat{y}_{t+1}$ in the IS equation (18), we get

$$\widehat{y}_t = \psi_f \mathbb{E}_t \widehat{y}_{t+1} - \psi_c \frac{1}{\gamma}\left(\phi\kappa\widehat{y}_t - \kappa \mathbb{E}_t \widehat{y}_{t+1}\right),$$

which can be re-written as

$$\widehat{y}_t \left(1 + \psi_c \frac{1}{\gamma}\phi\kappa\right) = \mathbb{E}_t \widehat{y}_{t+1} \left(\psi_f + \psi_c \frac{1}{\gamma}\kappa\right).$$

Dividing by $\left(1 + \psi_c \frac{1}{\gamma}\phi\kappa\right)$ and plugging in for ψ_f and ψ_c yields

$$\widehat{y}_t = \frac{\bar{m}\delta + \frac{(1-\lambda)\kappa}{\gamma(1-\lambda\chi)}}{1 + \kappa\phi\frac{1}{\gamma}\frac{(1-\lambda)}{1-\lambda\chi}} \mathbb{E}_t \widehat{y}_{t+1}.$$

To obtain determinacy, the term in front of $\mathbb{E}_t \widehat{y}_{t+1}$ has to be smaller than 1. Solving this for ϕ yields

$$\phi > \phi^* = 1 + \frac{\bar{m}\delta - 1}{\frac{\kappa}{\gamma}\frac{1-\lambda}{1-\lambda\chi}}, \tag{31}$$

which is the condition in Proposition 3. This illustrates how bounded rationality raises the likelihood that the Taylor principle ($\phi^* = 1$) is sufficient for determinacy, as the Taylor principle can only hold if

$$\bar{m}\delta \leq 1.$$

In the rational model, this boils down to $\delta \leq 1$. However, the Taylor principle can be sufficient under bounded rationality, i.e., $\bar{m} < 1$, even when $\delta > 1$, thus, even when allowing for amplification. Note that we could also express condition (31) as

$$\phi > \phi^* = 1 + \frac{\psi_f - 1}{\frac{\kappa}{\gamma}\psi_c}.$$

Proposition 3 can be extended to allow for Taylor rules of the form

$$\widehat{i}_t = \phi_\pi \pi_t + \phi_y \widehat{y}_t$$

and in which the behavioral agents do not have rational expectations about the real interest rate but rather perceive the real interest rate to be equal to

$$\widehat{r}_t^{BR} \equiv \widehat{i}_t - \bar{m}^r \mathbb{E}_t \pi_{t+1},$$

where $\bar{m}^r$ can be equal to $\bar{m}$ or can potentially differ from it (if it equals 1, we are back to the case in which the behavioral agent is rational with respect to real interest rates).

Combining the static Phillips Curve with the generalized Taylor rule and the behavioral HANK IS equation, it follows that

$$\widehat{y}_t = \frac{\omega_f + \frac{\kappa}{\gamma}\omega_c \bar{m}^r}{1 + \frac{\omega_c}{\gamma}\left(\kappa\phi_\pi + \phi_y\right)} \mathbb{E}_t \widehat{y}_{t+1}. \tag{32}$$

From equation (32), it follows that we need

$$\phi_\pi > \bar{m}^r - \phi_y + \frac{\omega_f - 1}{\omega_c \frac{\kappa}{\gamma}} = \bar{m}^r - \phi_y + \frac{\bar{m}\delta - 1}{\frac{1-\lambda}{1-\chi\lambda}\frac{\kappa}{\gamma}} \tag{33}$$

for the model to feature a determinate, locally unique equilibrium. Condition (33) shows that both, $\bar{m}^r < 1$ and $\phi_y > 0$, weaken the condition in Proposition 3. Put differently, bounded rationality with respect to the real rate or a Taylor rule that responds to changes in output, both relax the condition on ϕ_π to yield determinacy.

A.4 Derivation of Proposition 7

To prove Proposition 7, we start from the Euler equation (14). For simplicity, we denote $\widehat{r}_t \equiv \widehat{i}_t - \mathbb{E}_t \pi_{t+1}$ as the real rate. Plugging in for $\widehat{c}_t^S$, $\widehat{c}_{t+1}^S$ and $\widehat{c}_{t+1}^H$ from equations (10) and (12), we get

$$\widehat{y}_t = s\mathbb{E}_t^{BR}\left[\widehat{y}_{t+1}\right] + (1-s)\frac{1-\lambda}{1-\lambda\chi}\mathbb{E}_t^{BR}\left[\widehat{y}_{t+1}\right] - \psi_c \widehat{r}_t,$$

which can be re-written as

$$\widehat{y}_t = \delta\mathbb{E}_t^{BR}\left[\widehat{y}_{t+1}\right] - \psi_c \widehat{r}_t.$$

Now, using the expectations setup from Proposition 6, we get $\delta\mathbb{E}_t^{BR}\left[\widehat{y}_{t+1}\right] = (1-\bar{m})\delta\widehat{y}_{t-1} + \bar{m}\delta\mathbb{E}_t\left[\widehat{y}_{t+1}\right]$ which proves Proposition 7.

A.5 Cognitive Discounting of the State Vector

In Section 3, we assume that cognitive discounting applies to all variables, which differs slightly from the assumption in Gabaix (2020) who assumes that cognitive discounting applies to the *state* of the economy (exogenous shocks as well as announced monetary and fiscal policies). He then proves (Lemma 1 in Gabaix (2020)) how cognitive discounting applies as a result (instead of as an assumption) to all future variables, including future consumption choices. For completeness, we show in this section how our results are unaffected when following the approach in Gabaix (2020).

Let X_t denote the (de-meaned) state vector which evolves as

$$X_{t+1} = G^X\left(X_t, \varepsilon_{t+1}\right), \tag{34}$$

where G^X denotes the transition function of X in equilibrium and ε are zero-mean innovations. Linearizing equation (34) yields

$$X_{t+1} = \Gamma X_t + \varepsilon_{t+1}, \tag{35}$$

where ε_{t+1} might have been renormalized. The assumption in Gabaix (2020) is that the behavioral agent perceives the state vector to follow

$$X_{t+1} = \bar{m}G^X(X_t, \varepsilon_{t+1}), \tag{36}$$

or in linearized terms

$$X_{t+1} = \bar{m}\left(\Gamma X_t + \varepsilon_{t+1}\right). \tag{37}$$

The expectation of the boundedly-rational agent of X_{t+1} is thus $\mathbb{E}_t^{BR}\left[X_{t+1}\right] = \bar{m}\mathbb{E}_t\left[X_{t+1}\right] = \bar{m}\Gamma X_t$. Iterating forward, it follows that $\mathbb{E}_t^{BR}\left[X_{t+k}\right] = \bar{m}^k\mathbb{E}_t\left[X_{t+k}\right] = \bar{m}^k\Gamma^k X_t$.

Now, consider any variable $z(X_t)$ with $z(0) = 0$ (e.g., demeaned consumption of the saver type $C^S(X_t)$). Linearizing $z(X)$, we obtain $z(X) = b_X^z X$ for some b_X^z and thus

$$\begin{aligned}
\mathbb{E}_t^{BR}\left[z(X_{t+k})\right] &= \mathbb{E}_t^{BR}\left[b_X^z X_{t+k}\right] \\
&= b_X^z \mathbb{E}_t^{BR}\left[X_{t+k}\right] \\
&= b_X^z \bar{m}^k \mathbb{E}_t\left[X_{t+k}\right] \\
&= \bar{m}^k \mathbb{E}_t\left[b_X^z X_{t+k}\right] \\
&= \bar{m}^k \mathbb{E}_t\left[z(X_{t+k})\right].
\end{aligned}$$

For example, expected consumption of savers tomorrow (in linearized terms) is given by

$$\mathbb{E}_t^{BR}\left[\widehat{c}^S(X_{t+1})\right] = \bar{m}\mathbb{E}_t\left[\widehat{c}^S(X_{t+1})\right], \tag{38}$$

which we denote in the main text as

$$\mathbb{E}_t^{BR}\left[\widehat{c}_{t+1}^S\right] = \bar{m}\mathbb{E}_t\left[\widehat{c}_{t+1}^S\right]. \tag{39}$$

Now, take the linearized Euler equation (14) of the savers:

$$\widehat{c}_t^S = s\mathbb{E}_t^{BR}\left[\widehat{c}_{t+1}^S\right] + (1-s)\mathbb{E}_t^{BR}\left[\widehat{c}_{t+1}^H\right] - \frac{1}{\gamma}\widehat{r}_t, \tag{40}$$

where $\widehat{r}_t \equiv \widehat{i}_t - \mathbb{E}_t\pi_{t+1}$.

Using the notation in Gabaix (2020), we can write the Euler equation as

$$\widehat{c}^S(X_t) = s\mathbb{E}_t^{BR}\left[\widehat{c}^S(X_{t+1})\right] + (1-s)\mathbb{E}_t^{BR}\left[\widehat{c}^H(X_{t+1})\right] - \frac{1}{\gamma}\widehat{r}(X_t). \tag{41}$$

Now, applying the results above, we obtain

$$\widehat{c}^S(X_t) = s\bar{m}\mathbb{E}_t\left[\widehat{c}^S(X_{t+1})\right] + (1-s)\bar{m}\mathbb{E}_t\left[\widehat{c}^H(X_{t+1})\right] - \frac{1}{\gamma}\widehat{r}(X_t), \tag{42}$$

which after writing $\widehat{c}^S(X_t)$, $\widehat{c}^S(X_{t+1})$ and $\widehat{c}^H(X_{t+1})$ in terms of total output yields exactly the behavioral HANK IS equation in Proposition 1.

A.6 Microfounding $\bar{m}$

Gabaix (2020) shows how to microfound $\bar{m}$ stemming from a noisy signal extraction problem in the case of a representative agent. Following these lines, we show how this signal-extraction problem generates a set-up in which the family head behaves as if she was boundedly rational.

The (linearized) law of motion of the state variable, X_t, is given by $X_{t+1} = \Gamma X_t + \varepsilon_{t+1}$ (a similar reasoning extends to the non-linearized case), where X has been demeaned. Now assume that every agent j within the family of savers (the expectations of the hand-to-mouth agents are irrelevant) receives a noisy signal of X_{t+1}, S^j_{t+1}, given by

$$S^j_{t+1} = \begin{cases} X_{t+1} & \text{with probability } p \\ X'_{t+1} & \text{with probability } 1-p \end{cases}$$

where X'_{t+1} is an i.i.d. draw from the unconditional distribution of X_{t+1}, which has an unconditional mean of zero. In words, with probability p the agent j receives perfectly precise information and with probability $1-p$ agent j receives a signal realization that is completely uninformative. A fully-informed rational agent would have $p = 1$.

The conditional mean of X_{t+1}, given the signal S^j_{t+1}, is given by

$$X^e_{t+1} \equiv \mathbb{E}\left[X_{t+1} | S_{t+1} = s^j_{t+1}\right] = p \cdot s^j_{t+1}.^{36}$$

The intuition is that the signal distribution is such that the agent either receives a perfectly precise signal or a completely uninformative signal. As the perfectly-precise signal arrives with probability p and the unconditional mean is zero, it follows that the agent puts a weight p on the signal.

[36] To see this, note that the joint distribution of (X_{t+1}, S^j_{t+1}) is

$$f(x_{t+1}, s^j_{t+1}) = pg(s^j_{t+1})\delta_{s^j_{t+1}}(x_{t+1}) + (1-p)g(s^j_{t+1})g(x_{t+1}),$$

where $g(X_{t+1})$ denotes the distribution of X_{t+1} and δ is the Dirac function. Given that the unconditional mean of X_{t+1} is 0, i.e., $\int x_{t+1} g(x_{t+1}) dx_{t+1} = 0$, it follows that

$$\begin{aligned} \mathbb{E}_t\left[X_{t+1} | S^j_{t+1} = s^j_{t+1}\right] &= \frac{\int x_{t+1} f(x_{t+1}, s^j_{t+1}) dx_{t+1}}{\int f(x_{t+1}, s^j_{t+1}) dx_{t+1}} \\ &= \frac{pg(s^j_{t+1}) s^j_{t+1} + (1-p) g(s^j_{t+1}) \int x_{t+1} g(x_{t+1}) dx_{t+1}}{g(s^j_{t+1})} \\ &= p s^j_{t+1}. \end{aligned}$$

Furthermore, we have

$$\mathbb{E}\left[S_{t+1}|X_{t+1}\right] = pX_{t+1} + (1-p)\mathbb{E}\left[X'_{t+1}\right] = pX_{t+1}.$$

So, it follows that the *average* expectation of X_{t+1} within the family is given by

$$\begin{aligned}\mathbb{E}\left[X^e_{t+1}(S_{t+1})|X_{t+1}\right] &= \mathbb{E}\left[p\cdot S_{t+1}|X_{t+1}\right]\\ &= p\cdot\mathbb{E}\left[S_{t+1}|X_{t+1}\right]\\ &= p^2X_{t+1}.\end{aligned}$$

Defining $\bar{m}\equiv p^2$ and since $X_{t+1}=\Gamma X_t+\varepsilon_{t+1}$, we have that the family head perceives the law of motion of X to equal

$$X_{t+1} = \bar{m}\left(\Gamma X_t + \varepsilon_{t+1}\right), \tag{43}$$

as imposed in equation (37). The boundedly-rational expectation of X_{t+1} is then given by

$$\mathbb{E}^{BR}_t\left[X_{t+1}\right] = \bar{m}\mathbb{E}_t\left[X_{t+1}\right].$$

B Calibration

Parameter	Value	Source/Target
	THANK Parameters	
γ	1	Bilbiie (2020)
κ	0.02	Bilbiie (2020)
χ	1.48	Bilbiie (2020)
λ	0.33	Bilbiie (2020)
s	$0.8^{1/4}$	Bilbiie (2020)
	Behavioral Parameter	
$\bar{m}$	0.85	Gabaix (2020)

Table 1: Baseline calibration.

Our baseline calibration is summarized in Table 1. The values for γ and κ are directly taken from Bilbiie (2021, 2020) and are quite standard in the literature. Gabaix (2020), on the other hand, sets $\kappa = 0.11$ and $\gamma = 5$. Even though these coefficients differ quite substantially from our baseline calibration, note that our results would barely be affected by this. To see this, note that *amplification* is only determined by λ and χ, both independent of κ and γ. The determinacy condition on the other hand depends on both, κ and γ, but what ultimately matters is the fraction $\frac{\kappa}{\gamma}$ (see Proposition 3). As κ and γ are both approximately five times larger in Gabaix (2020) compared to Bilbiie (2021) and our baseline calibration, the fraction is approximately the same and thus, the determinacy region under an interest-rate peg remains unchanged.

The household heterogeneity parameters, χ, λ and s are also standard in the THANK literature (see Bilbiie (2020)). The most important assumption for our qualitative results in Section 4 is $\chi > 1$, which is empirically supported. Patterson (2019) provides empirical evidence for the countercyclicality of inequality. Coibion et al. (2017), Ampudia et al. (2018) and Samarina and Nguyen (2019) provide evidence of countercyclical inequality conditional on monetary policy shocks. Almgren et al. (2019) show that output in countries with higher shares of hand-to-mouth households responds more strongly to monetary policy shocks which, through the lens of the model, implies countercyclical inequality.

For figure 5, i.e., to compute the iMPCs we choose a yearly calibration with $s = 0.8$ and $\beta = 0.95$ (this calibration is close to the iMPC exercise in Bilbiie (2021) but while he fixes χ to match the empirically-observed iMPCs, we vary χ together with $\bar{m}$ to examine their joint effects on iMPCs).

The Cognitive Discounting Parameter $\bar{m}$. The cognitive discounting parameter $\bar{m}$ is set to 0.85, as in Gabaix (2020) and Benchimol and Bounader (2019). Fuhrer and Rudebusch (2004), for example, estimate an IS equation and find that $\bar{m}\delta \approx 0.65$, which together with $\delta > 1$, would imply a $\bar{m}$ much lower than 0.85 and especially our determinacy results would be even stronger under such a calibration. Note, that the calibration of the backward-looking behavioral HANK model in Section 5, which is based on household survey expectations and taken from Angeletos and Huo (2021), is close to the estimation results from Fuhrer and Rudebusch (2004).

Another way to calibrate $\bar{m}$ (as pointed out in Gabaix (2020)) is to interpret the estimates in Coibion and Gorodnichenko (2015) through the "cognitive-discounting lens". They regress forecast errors on forecast revisions

$$x_{t+h} - F_t x_{t+h} = c + b^{CG} \left(F_t x_{t+h} - F_{t-1} x_{t+h} \right) + u_t,$$

where $F_t x_{t+h}$ denotes the forecast at time t of variable x, h periods ahead. Focusing on inflation, they find that $b^{CG} > 0$ in consensus forecasts, pointing to *underreaction* (similar results are, for example, found in Angeletos et al. (2021) and Adam et al. (2020) for other variables).

In the model, the law of motion of x is $x_{t+1} = \Gamma\left(x_t + \varepsilon_{t+1}\right)$ whereas the behavioral agents perceive it to be $x_{t+1} = \bar{m}\Gamma\left(x_t + \varepsilon_{t+1}\right)$. It follows that $F_t x_{t+h} = \left(\bar{m}\Gamma\right)^h x_t$ and thus, forecast revisions are equal to

$$\begin{aligned} F_t x_{t+h} - F_{t-1} x_{t+h} &= \left(\bar{m}\Gamma\right)^h x_t - \left(\bar{m}\Gamma\right)^{h+1} x_{t-1} \\ &= \left(\bar{m}\Gamma\right)^h \Gamma(1-\bar{m}) x_{t-1} + \left(\bar{m}\Gamma\right)^h \varepsilon_t. \end{aligned}$$

The forecast error is given by

$$x_{t+h} - F_t x_{t+h} = \Gamma^h (1-\bar{m}^h)\Gamma x_{t-1} + \Gamma^h (1-\bar{m}^h)\varepsilon_t + \sum_{j=0}^{h-1} \Gamma^j \varepsilon_{t+h-j},$$

where $\sum_{j=0}^{h-1} \Gamma^j \varepsilon_{t+h-j}$ is the rational expectations forecast error. Gabaix (2020) shows that b^{CG} is bounded below $b^{CG} \geq \frac{1-\bar{m}^h}{\bar{m}^h}$, showing that $\bar{m} < 1$ yields $b^{CG} > 0$, as found empirically. When replacing the weak inequality with an equality, we get

$$\bar{m}^h = \frac{1}{1+b^{CG}}.$$

Most recently, Angeletos et al. (2021) estimate b^{CG} (focusing on a horizon $h = 3$) to lie between $b^{CG} \in [0.74, 0.81]$ for unemployment forecasts and $b^{CG} \in [0.3, 1.53]$ for inflation, depending on the considered period (see their Table 1). These estimates imply $\bar{m} \in [0.82, 0.83]$ for unemployment and $\bar{m} \in [0.73, 0.92]$ for inflation, and are thus close to our preferred value of 0.85. Note, however, that these estimates pertain to professional forecasters and should therefore be seen as upper bounds on $\bar{m}$.

And since the focus of the paper is to understand the role of $\bar{m}$, we often vary $\bar{m}$ anyway instead of focusing on one particular value.

C Extensions

C.1 Allowing for Steady State Inequality.

So far, we have assumed that there is no steady state inequality, i.e., $C^H = C^S$. In the following, we relax this assumption and denote steady state inequality by $\Omega \equiv \frac{C^S}{C^H}$. Recall the savers' Euler equation

$$\left(C_t^S\right)^{-\gamma} = \beta R_t \mathbb{E}_t^{BR} \left[s \left(C_t^S\right)^{-\gamma} + (1-s)\left(C_t^H\right)^{-\gamma} \right],$$

from which we can derive the steady state real rate

$$R = \frac{1}{\beta(s + (1-s)\Omega^\gamma)}.$$

Log-linearizing the Euler equation yields

$$\widehat{c}_t^S = \beta R \bar{m} \left[s \mathbb{E}_t \widehat{c}_{t+1}^S + (1-s)\Omega^\gamma \mathbb{E}_t \widehat{c}_{t+1}^H \right] - \frac{1}{\gamma}\left(\widehat{i}_t - \mathbb{E}_t \pi_{t+1}\right).$$

Combining this with the consumption functions and the steady state real rate yields the IS equation

$$\widehat{y}_t = \bar{m}\tilde{\delta}\mathbb{E}_t \widehat{y}_{t+1} - \frac{1}{\gamma}\frac{1-\lambda}{1-\lambda\chi}\left(\widehat{i}_t - \mathbb{E}_t \pi_{t+1}\right), \tag{44}$$

with

$$\tilde{\delta} \equiv 1 + (\chi - 1)\frac{(1-s)\Omega^\gamma}{s + (1-s)\Omega^\gamma}\frac{1}{1-\lambda\chi}.$$

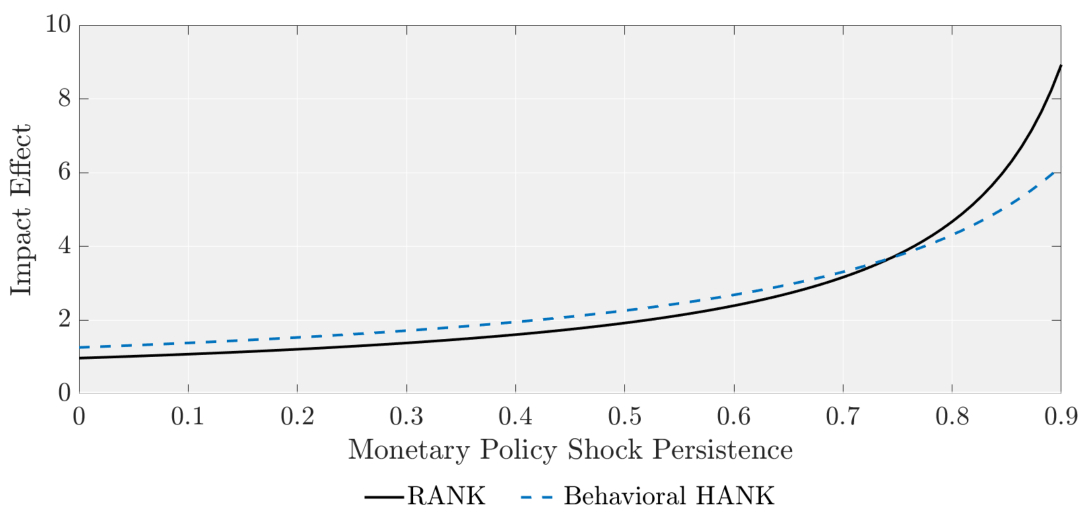
From a qualitative perspective, the whole analysis in the paper could be carried out with $\tilde{\delta}$ instead of δ. Quantitatively the differences are small as well. For example, if we set $\Omega = 1.5$, we get $\tilde{\delta} = 1.074$ instead of $\delta = 1.051$. Thus, we need $\bar{m} < 0.91$ instead of $\bar{m} < 0.93$ for determinacy under a peg.

C.2 Persistent Monetary Policy Shocks

In the main text in Section 4, we illustrated the resolution of the Catch-22 by considering i.i.d. monetary policy shocks (following Bilbiie (2021)). The behavioral HANK model delivers initial amplification of these monetary shocks but the effects decrease with the horizon of the shock, i.e., the behavioral HANK model resolves the forward guidance puzzle. Another way to see this is by considering persistent shocks.

Figure 10 illustrates this. The figure shows the response of output in period t to a shock in period t for different degrees of persistence (x-axis). The black-solid line shows the output response in RANK and the blue-dashed line in the behavioral HANK. The forward guidance puzzle in RANK manifests itself in the sense that highly persistent shocks have stronger effects in RANK than in the behavioral HANK. Persistent shocks are basically a form of forward guidance and thus, with high enough persistence in the shocks, the RANK model predicts stronger effects than the behavioral HANK model.

Figure 10: Initial Output Response for Varying Degrees of the Persistence



Note: This figure shows the initial output response to monetary policy shocks with different degrees of persistence.

C.3 Forward-Looking NKPC and Real Interest Rates

In the main part of the paper, we made the assumption that agents are rational with respect to real interest rates (as in Gabaix (2020)) and assumed a static Phillips Curve (as in Bilbiie

(2021)). We now show that the results are barely affected when considering a forward-looking New Keynesian Phillips Curve (NKPC) and that agents are also boundedly rational with respect to real rates. Gabaix (2020) derives the NKPC under bounded rationality and shows that it takes the following form:

$$\pi_t = \beta M^f \mathbb{E}_t \pi_{t+1} + \kappa \widehat{y}_t,$$

with

$$M^f \equiv \bar{m} \left(\theta + \frac{1-\beta\theta}{1-\beta\theta\bar{m}} (1-\theta) \right),$$

where $1-\theta$ captures the Calvo probability of price adjustment.

Taking everything together (including the bounded rationality with respect to real interest rates), the model can be summarized by the following three equations:

$$\begin{aligned}
\widehat{y}_t &= \psi_f \mathbb{E}_t \widehat{y}_{t+1} - \psi_c \frac{1}{\gamma} \left(\widehat{i}_t - \bar{m} \mathbb{E}_t \pi_{t+1} \right) \\
\pi_t &= \beta M^f \mathbb{E}_t \pi_{t+1} + \kappa \widehat{y}_t \\
\widehat{i}_t &= \phi \pi_t.
\end{aligned}$$

Plugging the Taylor rule into the IS equation, we can write everything in matrix form:

$$\begin{pmatrix} \mathbb{E}_t \pi_{t+1} \\ \mathbb{E}_t \widehat{y}_{t+1} \end{pmatrix} = \underbrace{\begin{pmatrix} \frac{1}{\beta M^f} & -\frac{\kappa}{\beta M^f} \\ \frac{\psi_c}{\gamma \psi_f} \left(\phi - \frac{\bar{m}}{\beta M^f} \right) & \frac{1}{\psi_f} \left(1 + \frac{\psi_c \bar{m} \kappa}{\gamma \beta M^f} \right) \end{pmatrix}}_{\equiv A} \begin{pmatrix} \pi_t \\ \widehat{y}_t \end{pmatrix}. \tag{45}$$

For determinacy, we need

$$det(A) > 1; \quad det(A) - tr(A) > -1; \quad det(A) + tr(A) > -1.$$

The last condition is always satisfied. The first two conditions are satisfied if and only if

$$\phi > \max \left\{ \frac{\beta \delta M^f \bar{m} - 1}{\frac{\kappa}{\gamma} \frac{1-\lambda}{1-\lambda\chi}}, \bar{m} + \frac{(\delta \bar{m} - 1)(1 - \beta M^f)}{\frac{\kappa}{\gamma} \frac{1-\lambda}{1-\lambda\chi}} \right\}.$$

In the case of a static Phillips curve but bounded rationality with respect to the real rate, the second condition is the crucial one. To capture the static Phillips curve, we can simply set $M^f = 0$. In this case, it follows that we have a uniquely-determined (bounded) equilibrium for $\phi > -3.22$. Thus, the condition is even weaker than in the main part of the paper.

If we allow for a forward-looking Phillips curve and using the same calibration as in the main text and relying on Gabaix (2020) for the two newly-introduced parameters, $\theta = 0.875$ and $\beta = 0.99$, it follows that we have determinacy even under an interest rate peg for our baseline calibration with $\bar{m} = 0.85$.

D Details on Intertemporal MPCs

In this section, we derive the iMPCs discussed in Section 4.3. Defining Y_t^j as type j's disposable income, we can write the households' budget constraints as

$$C_t^H = Y_t^H + \frac{1-s}{\lambda} R_t B_t$$
$$C_t^S + \frac{1}{1-\lambda} B_{t+1} = Y_t^S + \frac{s}{1-\lambda} R_t B_t,$$

where R_t denotes the real interest rate and B_t real bonds. Log-linearizing the two budget constraints around the zero-liquidity steady state and $R = \beta^{-1}$ yields

$$\widehat{c}_t^H = \widehat{y}_t^H + \frac{1-s}{\lambda} \beta^{-1} b_t \tag{46}$$
$$\widehat{c}_t^S + \frac{1}{1-\lambda} b_{t+1} = \widehat{y}_t^S + \frac{s}{1-\lambda} \beta^{-1} b_t, \tag{47}$$

where b_t denotes real bonds in shares of steady state output. Aggregating (46) and (47) delivers

$$\widehat{c}_t = \tilde{y}_t + \beta^{-1} b_t - b_{t+1}, \tag{48}$$

where $\tilde{y}_t$ denotes aggregate disposable income.

By plugging equations (46) and (47) into the savers' Euler equation (14), we can derive the dynamics of liquid assets b_t (ignoring changes in the real rate as this is a partial equilibrium exercise):

$$\begin{aligned} &\mathbb{E}_t b_{t+2} - b_{t+1} \left[\frac{1}{s\bar{m}} + \beta^{-1} s + \frac{(1-s)^2 \beta^{-1} (1-\lambda)}{s\lambda} \right] + \frac{\beta^{-1}}{\bar{m}} b_t = \\ &(1-\lambda) \mathbb{E}_t \widehat{y}_{t+1}^S + \frac{1-s}{s} (1-\lambda) \mathbb{E}_t \widehat{y}_{t+1}^H - \frac{1-\lambda}{s\bar{m}} \widehat{y}_t^S. \end{aligned} \tag{49}$$

Note that a change in total disposable income by one changes the hand-to-mouth households' disposable income by χ and the savers' disposable income by $\frac{1-\lambda\chi}{1-\lambda}$.

Let us denote the right-hand side of equation (49) by $-\mathbb{E}_t \widehat{z}_t$. Factorizing the left-hand side and letting F denote the forward-operator, it follows that

$$(F - \mu_1)(F - \mu_2) \mathbb{E}_t b_t = -\mathbb{E}_t \widehat{z}_t, \tag{50}$$

where μ_1 and μ_2 denote the roots of the characteristic equation

$$\mathbb{E}_t b_{t+2} - \phi_1 b_{t+1} - \phi_2 b_t = 0, \tag{51}$$

where

$$\phi_1 \equiv \left[\frac{1}{s\bar{m}} + \beta^{-1}s + \frac{(1-s)^2\beta^{-1}(1-\lambda)}{s\lambda}\right] \tag{52}$$

and

$$\phi_2 \equiv -\frac{\beta^{-1}}{\bar{m}}. \tag{53}$$

Thus, the roots are given by

$$\mu_{1,2} = \frac{\phi_1 \pm \sqrt{\phi_1^2 + 4\phi_2}}{2}. \tag{54}$$

It follows that

$$\begin{aligned} b_{t+1} &= \mu_1 b_t - (F - \mu_2)^{-1}\mathbb{E}_t\widehat{z}_t \\ &= \mu_1 b_t + \frac{\mu_2^{-1}}{1 - F\mu_2^{-1}}\mathbb{E}_t\widehat{z}_t. \end{aligned}$$

Note that $\mathbb{E}_t\widehat{z}_t$ can be written as $\frac{1-\lambda\chi}{s}\left(\delta\mathbb{E}_t\widehat{y}_{t+1} - \frac{1}{\bar{m}}\widehat{y}_t\right)$. Without loss of generality, we let $\mu_2 > \mu_1$ and we have $\mu_2 > 1$. We have $(1 - F\mu_2^{-1})^{-1} = \sum_{l=0}^{\infty}\mu_2^{-l}F^l$. Thus, we end up with

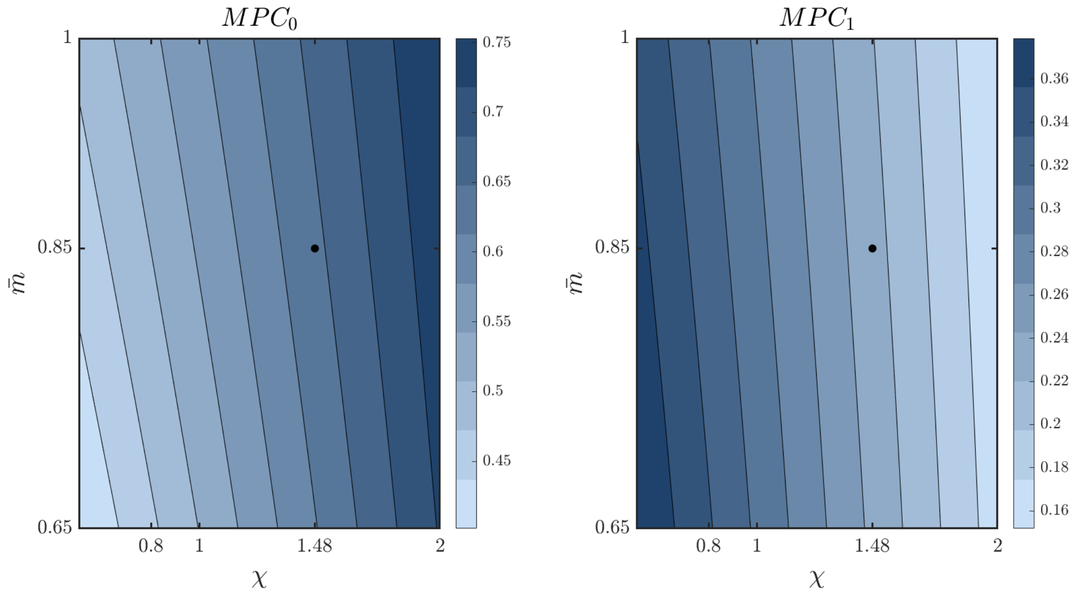
$$b_{t+1} = \mu_1 b_t + \frac{1-\lambda\chi}{s}\sum_{l=0}^{\infty}\mu_2^{-(l+1)}\mathbb{E}_t\left(\frac{1}{\bar{m}}\widehat{y}_{t+l} - \delta\widehat{y}_{t+1+l}\right). \tag{55}$$

Taking derivatives with respect to $\widehat{y}_{t+k}$ yields Proposition 5.

iMPCs and the Role of Idiosyncratic Risk. In Figure 11, we plot he MPCs in the year of the income windfall (left panel) and the first year after the windfall (right panel) for a relatively high idiosyncratic risk of $1-s = 0.5$. The high probability of becoming hand-to-mouth flips the role of $\bar{m}$ for the MPC_1 compared to our baseline calibration as discussed in Section 4.3. The reason being that the behavioral savers save a relatively large amount of the received income windfall in period 0 as they cognitively discount the decrease in their future marginal utility. Thus, they end up with relatively more disposable income in year 1. Now, given the relatively high probability of type switching, there are many savers who end up being hand-to-mouth in year 1 after the income windfall. As they are hand-to-mouth, they consume their previously-accumulated savings which increases the MPC_1. The more behavioral the savers are, i.e., the lower $\bar{m}$ is, the more pronounced this effect and hence, a lower $\bar{m}$ increases the MPC_1 in the case of a relatively high $1-s$.

iMPCs for more than two periods. Figure 12 plots the MPCs for the year of the income windfall as well as the five consecutive years for different degrees of rationality. As discussed in section 4.3, under our benchmark calibration, the rational model predicts somewhat larger

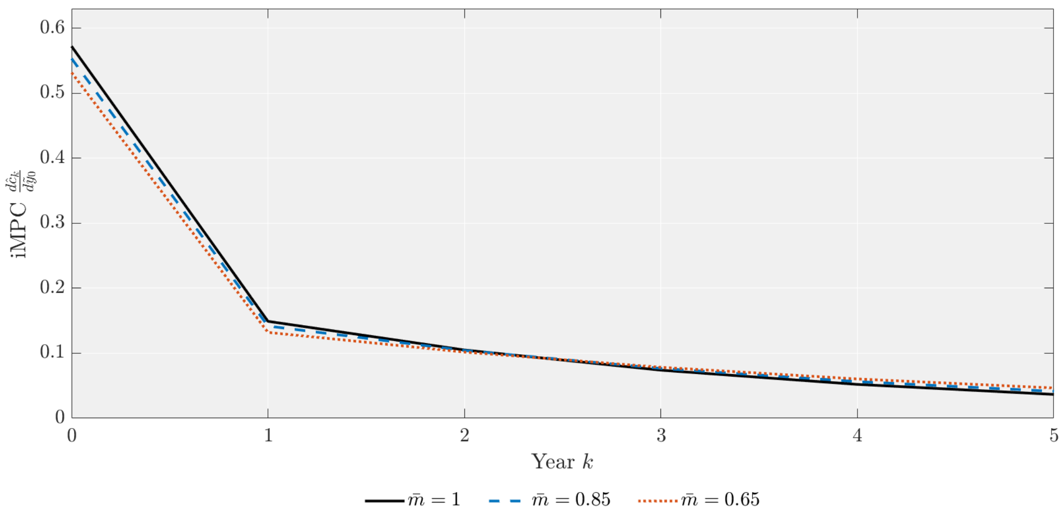
Figure 11: Intertemporal MPCs, Bounded Rationality and Household Heterogeneity



Note: This figure shows the aggregate intertemporal MPCs, i.e., the aggregate consumption response in year 0 (left) and year 1 (right) to a change in aggregate disposable income in year 0 for a transition probability $1 - s = 0.5$.

initial MPCs as the behavioral savers save relatively more. Over time, however, the MPCs in the behavioral model lie above their rational counterparts due to the fact that more and more of the initial savers become hand-to-mouth and start consuming their (higher) savings. As Figure 11 shows, the probability of type switching, $1 - s$, matters for when exactly the behavioral model starts to generate larger MPCs compared to the rational model.

Figure 12: Intertemporal MPCs



Note: This figure shows the aggregate intertemporal MPCs, i.e., the aggregate consumption response in year k to a change in aggregate disposable income in year 0 for different $\bar{m}$.