Subjective Housing Price Expectations, Falling Natural Rates and the Optimal Inflation Target

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Abstract

U.S. households' housing price expectations deviate systematically from rational expectations: (i) expectations are updated on average too sluggishly; (ii) following housing price changes, expectations initially underreact but subsequently overreact; (iii) households are overly optimistic (pessimistic) about capital gains when the price-to-rent ratio is high (low). We show that weak forms of capital gain extrapolation allow to simultaneously replicate the behavior of housing prices and these deviations from rational expectations as an equilibrium outcome. Embedding capital gain extrapolation into a sticky price model featuring a lower-bound constraint on nominal interest rates, we show that lower natural rates of interest increase the volatility of housing prices and thereby the volatility of the natural rate of interest. This exacerbates the relevance of the lower bound constraint and causes the optimal inflation target to increase strongly as the natural rate falls.

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1 Introduction

The large and sustained booms and busts in housing prices in advanced economies are often attributed to households holding excessively optimistic or pessimistic beliefs about future housing prices (Piazzesi and Schneider (2006), Kaplan, Mitman, and Violante (2020)). This view is supported by a nascent literature that documents puzzling facts about the behavior of housing price expectations. Survey measures of expected future housing prices have been found to be influenced by past housing price changes, but appear to underreact to these changes, and they also miss the tendency of housing prices to mean revert over time (Kuchler and Zafar (2019), Case, Shiller, and Thompson (2012), Ma (2020) and Armona, Fuster, and Zafar (2019)).

Documenting in which ways households' housing price expectations deviate from the rational expectations benchmark is an important task but remains in itself uninformative about how important the observed deviations are for economic outcomes in housing markets and for the conduct of monetary policy. Understanding these features requires a structural equilibrium model that quantitatively replicates how households' expectations deviate from rational expectations. Constructing such an equilibrium model, calibrating it to the behavior of household beliefs in survey data, and understanding its implications for the optimal design of monetary policy is the main objective of the present paper. To the best of our knowledge, it is the first paper pursuing this task.

We begin our analysis by comprehensively quantifying the dimensions along which households' housing price expectations deviate from the full-information rational expectations benchmark. To this end, we consider the Michigan Survey of Consumers, which provides the longest available time series of quantitative housing price expectations for the United States, covering the years 2007-2021.

We document three dimensions along which household expectations deviate from rational expectations. First, expectations about future housing prices are revised too sluggishly over time, a feature that housing price expectations share with other household expectations (Coibion and Gorodnichenko (2015)). Second, households' capital gain expectations covary positively with market valuation, i.e., the price-to-rent ratio, while actual future capital gains covary negatively with market valuation. We show that the difference is striking, highly statistically significant, and in line with findings on investor expectations in stock markets (Adam, Marcet, and Beutel (2017)).[1] Third, in a dynamic sense, households' capital gain expectations initially underreact to observed capital gains, i.e., households are too pessimistic in the first few quarters following a positive capital gain, but later on overreact, i.e., households hold too optimistic expectations after about twelve quarters. The pattern of initial underreaction and subsequent overreaction is similarly present in other macroeconomic expectations, see Angeletos, Huo, and Sastry (2020).

While the first and third deviation from rational housing price expectations have been documented before using different data sets, see Armona, Fuster, and Zafar (2019), the second deviation from rational expectations is new to the housing literature. We quantify here all three deviations using a single data set, so as to obtain a coherent set of quantitative

[1]For stock markets, Adam, Matveev, and Nagel (2021) show that this cannot be explained by investors reporting risk-adjusted expectations.

targets for our structural equilibrium model with subjective housing expectations.

Equipped with these facts, we construct first a simple housing model with optimizing households that hold subjective beliefs about housing price behavior. Bayesian belief updating implies that households weakly extrapolate past capital gains into the future. The model reproduces – as an equilibrium outcome – important patterns of the behavior of U.S. housing prices, in particular, the large and protracted swings in the price-to-rent ratio over time, as well as the three dimensions mentioned above along which household expectations deviate from the rational expectations benchmark. The quantitative fit is surprisingly good, despite the simplicity of the model.

The simple model generates two important insights. First, it shows that the standard deviation for the price-to-rent ratio would be much lower in the presence of rational housing expectations. This suggests that the observed volatility of housing prices is to a significant extent due to the presence of subjective beliefs. This lends credence to the view that the observed deviations from rational expectations substantially contribute to booms and busts in housing markets.

Second, the simple model connects the secular decline in natural rates of interest with higher volatility of housing prices. Specifically, the model predicts that lower real interest rates imply larger effects of belief fluctuations on equilibrium housing prices. This prediction does not emerge in the presence of rational housing price expectations, but is consistent with the data. We show that in a number of advanced economies, including the United States, the volatility of housing prices has increased considerably at the same time as the level of the natural rate of interest has fallen.

The most important objective of paper is to understand the monetary policy implications generated by a setting where households (weakly) extrapolate capital gains into the future. We are particularly interested in the optimal policy response to increased housing price volatility that is induced by falling natural rates of interest in a setting where policy rates cannot move into negative territory. To this end, we introduce capital gain extrapolation into an otherwise standard New Keynesian model featuring a housing sector and a lower bound constraint on nominal interest rates.

The sticky price model has a number of attractive features. First, it shares the implications for housing price behavior and household beliefs with the simpler model considered before and thus quantitatively replicates the patterns of belief deviations and housing prices. Second, it is immune to the critique by Barsky, House, and Kimball (2007) regarding the behavior of sticky price models featuring durable goods. In line with the data, the model implies that housing demand reacts more strongly to monetary disturbances than non-housing demand, despite the fact that housing prices are fully flexible. Third, the model introduces subjective housing beliefs in a way that monetary policy is unable to manipulate household beliefs to its own advantage. This allows for a meaningful discussion of Ramsey optimal monetary policy in the presence of subjective beliefs. Finally, the model makes a minimal departure from rational expectations: expectations about non-housing related variables are rational and all agents maximize given their (subjective) beliefs about the future.

To gain analytic insights, we derive a linear-quadratic approximation to the optimal policy problem and show how it is affected by the presence of subjective housing beliefs. We find that housing price gaps, i.e., deviations of housing prices from their efficient level, affect the economy via two channels. First, inefficiently high housing prices, driven by capital gain optimism, give rise to negative cost-push terms in the Phillips curve.[2] This feature allows the model to potentially generate a non-inflationary housing boom. Yet, a second channel is more important: rising housing price volatility increases the volatility of the natural rate of interest. Since increased housing price volatility is itself triggered by a fall in the average level of the natural rate, this dramatically exacerbates the lower-bound problem for a monetary policy authority confronted with falling natural rates.

The natural rate is affected by housing prices, because higher housing prices make it optimal to allocate more resources to housing investment. This exerts positive pressure on the output gap and counteracting these – so as to keep the output gap stable – requires policy to increase the real interest rate. Under rational expectations, housing prices never deviate from their efficient value, so that policy never has to work against inefficient investment pressures. With rational expectations, the volatility of the natural rate is thus independent of the average level of the natural rate.

These contrasting predictions of the model under rational and subjective housing beliefs also lead to rather different policy messages on how the optimal inflation target, i.e., the average inflation rate implied by optimal monetary policy, should respond to a fall in the natural rate of interest. Under rational expectations, the optimal inflation target is nearly invariant to the average level of the natural rate.

In the presence of capital gain extrapolation, the optimal inflation target increases considerably in response to a fall in the average natural rate. This is due to the increased volatility in the natural rate and cost-push shocks, which causes the lower bound on the nominal rate to become more restrictive. A more restrictive lower bound forces monetary policy to rely more strongly on promising future inflation in order to lower the real interest rate. This increases the average inflation rate under optimal policy. For our calibrated model, we find that the optimal inflation target should increase approximately by one third of a percent in response to a one percent fall in the natural rate with the increase becoming non-linear for very low levels of the natural rate.

We also investigate the optimal policy response to housing demand shocks. While inflation and the output gap do not respond to these shocks under rational expectations, capital gains induced by housing demand shocks get amplified by capital gain extrapolation and thereby generate persistent housing price gaps to which monetary policy optimally responds. Housing price gaps, however, generate opposing effects. On the one hand, inefficiently high housing prices generate negative cost-push pressures, which calls for a decrease in the policy rate; on the other hand, inefficiently high housing prices trigger a housing investment boom, which puts upward pressure on the output gap. Counteracting this second effect requires hiking policy rates.

In our calibrated model, the second effect quantitatively dominates. Optimal monetary policy thus 'leans against' housing price movements, but the optimal strength of the reaction depends on the direction of the shock. Following a positive housing preference shock, the increase in the interest rate (nominal and real) is more pronounced than the interest rate decrease following a negative housing demand shock. The presence of the

[2] Conversely, inefficiently low housing prices, driven by capital gain pessimism, cause positive cost-push terms.

lower-bound constraint thus attenuates the degree to which monetary policy leans against negative housing demand shocks.

We also consider whether macroprudential policies could address the housing market inefficiencies generated by capital gain extrapolation. We do so by considering housing taxes that might be levied on households in order to insulate monetary policymakers from the fluctuations in the housing price gap. We find that the required taxes would have to be large and very volatile. For our calibrated model, taxes must often exceed 20% of the rental value of housing per period and also often require equally sized or even larger housing subsidies. It appears somewhat unlikely that any of the existing macroprudential tools are capable of generating effects of such magnitude. And to the best of our knowledge, none of the available macroprudential tools allows subsidizing private sector behavior. Less aggressive tax policies turn out to be considerably less effective in bringing down the volatility of the housing price gap. This suggests that macroprudential policies are unable to substantially reduce the monetary policy trade-offs arising from subjective housing price expectations.

This paper is related to work by Andrade, Galí, Le Bihan, and Matheron (2019, 2021) who study how the optimal inflation target depends on the natural rate of interest in a setting with a lower bound constraint. In line with our findings, they show that an increase in the inflation target is a promising approach to deal with the lower-bound problem. While their work considers optimized Taylor rules in a medium-scale sticky price model without a housing sector and rational expectations, the present paper studies Ramsey optimal policy in a model featuring a housing sector and subjective housing expectations.

A number of papers consider Ramsey optimal policy in the presence of a lower-bound constraint, but also abstract from housing markets and the presence of subjective beliefs (Eggertsson and Woodford (2003), Adam and Billi (2006), Coibion, Gorodnichenko, and Wieland (2012)). This literature finds that lower bound episodes tend to be short and infrequent under optimal policy, so that average inflation is very close to zero under optimal policy. The present paper shows that this conclusion is substantially altered in the presence of subjective housing price expectations.

Optimal monetary policy with subjective beliefs has previously been analyzed in Caines and Winkler (2021) and Adam and Woodford (2021). These papers abstract from the lower-bound constraint and consider different belief setups that are not calibrated to replicate patterns of deviations from rational housing price expectations as observed in survey data.[3] We show that taking into account the existence of a lower-bound constraint on nominal rates is quantitatively important for understanding how the optimal inflation target responds to lower natural rates.

The rest of the paper is structured as follows. Section 2 documents how survey expectations about future housing prices deviate from rational expectations. Section 3 presents a simple housing model in which households extrapolate capital gains. It shows how this simple model can jointly replicate in equilibrium the behavior of housing prices and the pattern of deviations from rational expectations. Section 4 then presents the full housing model with sticky prices, subjective housing beliefs, and a lower-bound constraint

[3] Adam and Woodford (2021) consider 'worst-case' belief distortions, while Caines and Winkler (2021) consider a setting with 'conditionally model-consistent beliefs'. Both setups generate deviations from rational expectations for variables other than housing prices.

on nominal rates. Section 5 derives a quadratic approximation to the monetary policy problem, which allows obtaining important analytic insights into the new economic forces arising from the presence of subjective housing price beliefs. We calibrate the model in Section 6 and present quantitative results about the optimal inflation target and the optimal policy response to housing shocks in Section 7. Section 8 discusses macroprudential policies and Section 9 concludes.

2 Cyclical Properties of Housing Price Expectations

This section documents that households' housing price expectations deviate in systematic ways from the full-information rational expectations (RE) benchmark. We consider three rationality tests that have recently been proposed in the literature (Coibion and Gorodnichenko (2015), Adam, Marcet, and Beutel (2017) and Angeletos, Huo, and Sastry (2020)). These tests cover different dimensions along which subjective expectations deviate from RE.

We measure housing prices using the S&P/Case-Shiller U.S. National Home Price Index and let q_t denote the quarterly average of the monthly housing price index. We consider both nominal and real housing prices with real housing prices being obtained by deflating nominal housing prices with the CPI.[4][5]

Expectations about housing capital gains are taken from the Michigan household survey. The survey provides subjective expectations about nominal four-quarter-ahead housing price growth, $E_t^{\mathcal{P}}[q_{t+4}/q_t]$, for the period 2007-2021. The survey also provides housing price growth expectations over the next five years. We focus on the shorter horizon because these expectations determine housing prices according to our model. The shorter horizon also allows performing a dynamic decomposition of forecast errors over time in response to realized capital gains.[6]

We consider both mean and median household expectations.[7] When considering real housing price expectations, we deflate the nominal mean (median) capital gain expectations with the mean (median) inflation expectation over the same period, as obtained from the Michigan survey.[8]

[4]The simplified model in the next section makes predictions about real housing prices only, while the survey data contains information about nominal capital gain expectations. This leads us to consider nominal and real housing prices.

[5]We use the "Consumer Price Index for All Urban Consumers: All Items in U.S. City Average"obtained from FRED.

[6]Data limitations make such a decomposition difficult for five-year-ahead forecasts: with only 15 years of data, the dynamic decompositions become largely insignificant. Appendix A.1 shows, however, that all other patterns documented below are equally present in five-year-ahead expectations.

[7]Analyzing the dynamics of individual expectations over time is difficult because households in the Michigan survey are sampled at most twice. In general, cross-sectional disagreement between households might partly reflect heterogeneous information on the part of households, see Kohlhas and Walther (2021).

[8]As is well-known, these inflation expectations feature an upward bias relative to actual inflation outcomes. This, however, will not be the source of rejection of the RE hypothesis: all our tests focus on the cyclical properties of capital gain expectations and eliminate mean differences between forecasts and realizations using appropriate regression constants that will not be used in our rationality tests.

Table 1: Sluggish adjustment of housing price expectations

	Mean Expectations	Median Expectations
Nominal Housing Prices		
$\widehat{b}^{CG}$	2.22^{***}	2.85^{***}
	(0.507)	(0.513)
Real Housing Prices		
$\widehat{b}^{CG}$	2.00^{***}	2.47^{***}
	(0.332)	(0.366)

Notes: This figure shows the empirical estimates of regression (1) for nominal and real housings price and considers mean and median expectations. The reported standard errors are robust with respect to heteroskedasticity and serial correlation (Newey-West with four lags). Significance levels: *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

Sluggish Updating About the Expected Housing Price Level. We start by documenting that the mean/median household expectation about the future level of housing prices is updated too sluggishly. This can be tested following the approach of Coibion and Gorodnichenko (2015), which involves considering regressions of the form

$$q_{t+4} - E_t^{\mathcal{P}}\left[q_{t+4}\right] = a^{CG} + b^{CG} \cdot \left(E_t^{\mathcal{P}}\left[q_{t+4}\right] - E_{t-1}^{\mathcal{P}}\left[q_{t+3}\right]\right) + \varepsilon_t. \tag{1}$$

The regression projects forecast errors about the future housing price level on past forecast revisions. Under the RE hypothesis, information that is contained in agents' information set, i.e., past forecasts and their revisions, should not predict future forecast errors ($H_0 : b^{CG} = 0$).

We estimate equation (1) for nominal and real capital gains, using mean and median expectations, respectively. Expectations of the future house price level are computed as $E_t^{\mathcal{P}}\left[q_{t+4}\right] = E_t^{\mathcal{P}}\left[q_{t+4}/q_t\right] q_t$, where $E_t^{\mathcal{P}}\left[q_{t+4}/q_t\right]$ denotes the capital gain expectations from the Michigan survey and q_t the S&P/Case-Shiller Index.[9]

Table 1 reports the estimated b^{CG} from regression (1). We find that $\widehat{b}^{CG} > 0$, which is inconsistent with the RE hypothesis. The regression coefficient is positive and statistically significant at the 1% level in all considered specifications. This implies that the mean/median agent updates beliefs too sluggishly: future realizations move (on average) by more than what is suggested by past forecast revisions. The magnitude of the estimates is also large in economic terms: a coefficient estimate of two suggests that forecast revisions should approximately be three times as strong than they actually are.

Overall, sluggish belief updating is consistent with previous findings on the behavior of survey expectations about output, inflation and unemployment (Coibion and Gorodnichenko (2015), Angeletos, Huo, and Sastry (2020), Kohlhas and Walther (2021)). Furthermore, Bordalo, Gennaioli, Ma, and Shleifer (2020) provide evidence of sluggish belief adjustment in consensus forecasts for other housing variables, such as residential investment and new housing starts.

[9]When considering real housing prices, nominal capital gain expectations from the Michigan survey are deflated using the subjective (mean or median) inflation expectations from the Michigan survey.

Appendix A.2 shows that our findings are robust to using an instrumental-variable approach for estimating regression (1), in which forecast revisions are instrumented with monetary policy shocks obtained via high-frequency identification. Appendix A.3 shows that similar results emerge when using capital gains and expected capital gains in equation 1 instead of the level and expected level of the housing price.

Opposing Cyclicality of Actual and Expected Capital Gains. Our second test documents the different cyclicality of actual and expected capital gains in housing markets. Differences between the cyclicality of actual and expected capital gains have previously been documented for stock markets, where actual and expected stock market capital gains covary differently with the price-to-dividend ratio (Greenwood and Shleifer (2014), Adam, Marcet, and Beutel (2017)). We consider here the cyclicality of expected and actual capital gains in the housing market with the price-to-rent ratio PR:

$$E_t^{\mathcal{P}}\left[\frac{q_{t+4}}{q_t}\right] = a + c \cdot PR_{t-1} + u_t \tag{2}$$

$$\frac{q_{t+4}}{q_t} = \mathbf{a} + \mathbf{c} \cdot PR_{t-1} + \mathbf{u}_t. \tag{3}$$

The rational expectations hypothesis implies $H_0 : c = \mathbf{c}$, whenever the agents' information set contains the past price-to-rent ratio as an observable.[10] Since the predictor variable used on the right-hand side of the preceding regressions equations is highly persistent, we correct for small sample bias in the coefficient estimates (Stambaugh (1999)).[11]

Table 2 reports the regression results. It shows that expected capital gains are positively associated with the PR-ratio, while realized capital gains are negatively associated. Expected capital gains are pro-cyclical, i.e., are high when market valuation is high, while realized capital gains are counter-cyclical, i.e., are low when market valuation is high. This pattern of is akin to the one documented in stock markets.

Quantitatively, the results imply that a two standard deviation increase of the PR-ratio by 15.5 units increases the mean household expectations about four-quarter-ahead real capital gains by around 0.5%. Actual four-quarter ahead capital gains, however, fall by around 1.5%, so that the forecast error is approximately 2%.

The last column in Table 2 performs a test of the rational expectations hypothesis that the cyclicality of actual and expected returns are equal ($H_0 : c = \mathbf{c}$). The test corrects for small sample bias, which is reported in the second to last column. We find that the difference in the cyclicality of actual and expected capital gains is highly statistically significant in all cases. Appendix A.4 shows that similar results are obtained when first subtracting equation (2) from (3) and estimating the resulting equation with forecast errors on the left-hand side, as in Kohlhas and Walther (2021) who do not consider housing related variables.

[10] In the regressions, we use the lagged PR-ratio, PR_{t-1}, instead of the current value, because the PR-ratio is computed using the average price over a quarter. In Adam, Marcet, and Beutel (2017) the price-to-dividend ratio was computed using the beginning of quarter stock price, which allowed using the current value in the regression.

[11] The small sample bias correction in Table 2 follows the same approach as the one in Table 1A in Adam, Marcet, and Beutel (2017).

Table 2: Cyclicality of expected vs. actual capital gains

	$\hat{c}$ (in %)	$\hat{\mathbf{c}}$ (in %)	bias (in %) $-E(\hat{\mathbf{c}}-\hat{c})$	p-value $H_0 : c = \mathbf{c}$
Nominal Housing Prices				
Mean Expectations	0.033	-0.102	0.006	0.000
	(0.008)	(0.007)		
Median Expectations	0.014	-0.102	0.009	0.000
	(0.001)	(0.007)		
Real Housing Prices				
Mean Expectations	0.030	-0.113	-0.003	0.000
	(0.017)	(0.009)		
Median Expectations	0.010	-0.113	0.006	0.000
	(0.004)	(0.009)		

Notes: $\widehat{c}$ is the estimate of c in equation (2) and $\widehat{\mathbf{c}}$ the estimate of $\mathbf{c}$ in equation (3). The Stambaugh (1999) small sample bias correction is reported in the second-to-last column and the last column reports the p-values for the null hypothesis $c = \mathbf{c}$. Newey-West standard errors using four lags are in parentheses.

Initial under- and subsequent over-reaction of housing price expectations. While the results in Table 1 show that households adjust short-term housing price beliefs on average too sluggishly, the results in Table 2 indicate over-optimism (over-pessimism) in housing price expectations when the current market valuation is high (low), which points to some form of overreaction to past housing price increases. It turns out that both patterns can be jointly understood by considering the dynamic response of actual and expected capital gains to housing price changes.

Following the approach in Angeletos, Huo, and Sastry (2020), who analyze forecast errors about unemployment and inflation, we investigate how capital gains and forecast errors about these capital gains evolve over time in response to realized capital gains.[12] Provided households observe realized capital gains, the RE hypothesis implies that it should not be possible to predict future forecast errors with current capital gains.

We estimate the dynamic responses using local projections (Jorda (2005)) of the form

$$X_{t+h} = a^h + b^h \frac{q_{t-1}}{q_{t-2}} + u_t^h, \tag{4}$$

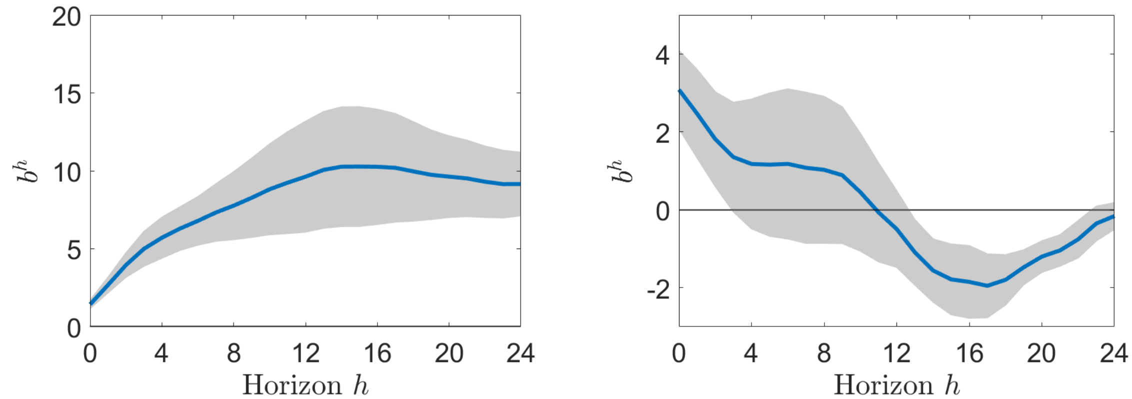
where the left-hand side variable X_{t+h} is either the cumulative capital gain (q_{t+h+4}/q_t), or the forecast error about the four-quarter-ahead capital gain $(q_{t+h+4}/q_{t+h} - E_{t+h}^{\mathcal{P}}[q_{t+h+4}/q_{t+h}])$, and u_t^h a serially correlated and heteroskedastic error term. Note that forecast errors are positive when households are overly pessimistic about capital gains and negative if households are overly optimistic.

Figure 1 reports the estimated coefficients b^h from local projection (4). Panel (a) depicts the response of cumulative capital gains. It shows that the initial capital gains is not only

[12] These dynamic responses are well-defined in econometric terms, even if they cannot be given a structural interpretation, because past capital gains are likely driven by a combination of past shocks.

Figure 1: Dynamic responses to a realized capital gain

(a) Cumulative Capital Gains (b) Capital Gain Forecast Errors



Notes: Panel (a) shows the dynamic response of cumulative real capital gains at horizon h to a one standard deviation innovation in the housing capital gain. Panel (b) reports the dynamic response of housing-price forecast errors at horizon h of one-year ahead expectations to a one standard deviation innovation in the housing capital gain. Positive (negative) values indicate that realized capital gains exceed (fall short of) expected capital gains. The shaded area shows the 90%-confidence intervals, standard errors are robust with respect to serial correlation and heteroskedasticity (Newey-West with $h+1$ lags).

persistent, but increases further over time, reaching a plateau after around twelve quarters. Given the high serial correlation displayed by capital gains in housing markets, this feature is perhaps not too surprising.

Panel (b) depicts the dynamic response of forecast errors. Forecast errors are initially positive but later on – once cumulative capital gains reach their plateau – become negative before eventually disappearing. The positive values initial periods indicates that agents' expectations react too sluggishly: realized capital gains are persistently larger than the expected gains. This also implies an underreaction in terms of the expected level of housing prices. Subsequently, when all actual capital gains have materialized and housing prices start to slightly mean revert, agents are too optimistic about future capital gains. This aligns well with the prior observation that capital gain expectations display the wrong cyclicality with housing market valuation.[13] It also implies that households entirely miss the mean-reversion in capital gains: forecast errors turn negative once housing prices level off and start to slightly mean-revert. This pattern is consistent with the experimental evidence provided in Armona, Fuster, and Zafar (2019).

In Appendix A.5, we show that the nominal forecast error responses look very similar. Likewise, using median expectations instead of mean expectations makes no noticeable difference of the results. In Appendix A.6, we show all our results obtained thus far are robust to excluding the Corona Virus period, i.e., to letting the sample period end in the last quarter of 2019.

[13]Since rents move only very slowly over time, changes in housing prices capture changes in the price-to-rent ratio rather well.

Analysis Using Regional Data. As is well known, housing prices often display considerable regional variation across the United States. We thus check whether the three deviations from RE documented above are also present in regional housing prices and housing price beliefs. Appendix A.7 uses regional housing price indices and exploits local information contained in the Michigan survey that allows grouping survey respondents into four different U.S. regions (North East, North Central/Midwest, South, and West). Repeating the above analyses at the regional level, it shows that one obtains quantitatively similar results.

The next section presents a simple housing model that can quantitatively replicate the forecast error deviations documented in this section.

3 Simple Model with Capital Gain Extrapolation

This section presents a bare-bones housing model in which households (weakly) extrapolate past capital gains. The model makes equilibrium predictions for the joint dynamics of housing prices and housing price beliefs. Housing prices in the model depend on households' housing price beliefs, with the latter being influenced by past housing price behavior. We show that equilibrium dynamics of housing prices and housing price beliefs quantitatively replicate key features of housing price behavior in the U.S., as well as the deviations from rational expectations documented in the previous section. The simple model also predicts that low levels of the natural rate of interest give rise to increased housing price volatility. As we show, this prediction is consistent with the evolution of natural rates and housing prices in advanced economies over the past decades.

The full model in Section 4 additionally features nominal rigidities, a lower bound constraint on nominal rates, generalized preferences, and endogenous production of consumption goods and housing. The present section abstracts from these features, but nevertheless shares its implications for housing price behavior and housing price beliefs with the full model.

The Household Problem. There is a measure one of identical households.[14] Households are internally rational, as in Adam and Marcet (2011), i.e., maximize utility holding potentially subjective beliefs about variables beyond their control. The representative household chooses consumption C_t, housing units to own D_t, and housing units to rent D_t^R, to maximize

$$\begin{aligned} \max_{\left\{C_t \geq 0, D_t \in [0, D^{\max}], D_t^R \geq 0\right\}_{t=0}^{\infty}} \quad & E_t^{\mathcal{P}} \sum_{t=0}^{\infty} \beta^t \left[C_t + \xi_t^d \left(D_t + D_t^R\right)\right] \\ \text{s.t.} \quad : \quad & C_t + \left(D_t - (1-\delta) D_{t-1}\right) q_t + R_t D_t^R = Y_t \text{ for all } t \geq 0, \end{aligned}$$

where Y_t is an exogenous (and sufficiently large) endowment, q_t the real price of housing, R_t the real rental price and $\delta > 0$ the housing depreciation rate. Rental units and housing units owned are perfect substitutes and $\xi_t^d \geq 0$ denotes a housing preference shock. The

[14] The fact that households are identical is not common knowledge among households.

household's subjective probability measure $\mathcal{P}$ allows for subjective housing price beliefs. For simplicity, we assume beliefs about other variables beyond the household's control, $\{Y_t, \xi_t^d, R_t\}_{t=1}^{\infty}$, to be rational. The latter assumption is not important for the results derived in this section.

Housing choices are subject to a short-selling constraint $D_t \geq 0$, which is standard, and to a long constraint $D_t \leq D^{\max}$. The latter insures existence of optimal plans in the presence of distorted housing beliefs. The long constraint is chosen such that it will never bind in equilibrium, i.e., $D^{\max} > D$, where D denotes the exogenously fixed housing supply. Without loss of generality, rental units are assumed to be in zero net supply.

The household first-order conditions imply that rents are given by

$$R_t = \xi_t^d \tag{5}$$

and that equilibrium housing prices satisfy[15]

$$q_t = \xi_t^d + \beta(1-\delta)E_t^{\mathcal{P}} q_{t+1}. \tag{6}$$

Capital Gain Extrapolation. We now introduce subjective price beliefs that give rise to capital gain extrapolation, using the setup in Adam, Marcet, and Nicolini (2016). Importantly, the precise details generating capital gain extrapolation are not essential for the results in this section and we could have used alternative belief assumptions, e.g., learning from life-time experience as in Nagel and Xu (2018) and Malmendier and Nagel (2011, 2015), or could have directly assumed extrapolative behavior as in Barberis, Greenwood, Jin, and Shleifer (2015).

Households perceive capital gains to evolve according to

$$\frac{q_t}{q_{t-1}} = b_t + \varepsilon_t, \tag{7}$$

where $\varepsilon_t \sim iiN(0, \sigma_\varepsilon^2)$ is a transitory component of capital gains and b_t a persistent component, which itself evolves according to $b_t = b_{t-1} + \nu_t$, with $\nu_t \sim iiN(0, \sigma_\nu^2)$.[16] Households observe the realized capital gains (q_t/q_{t-1}) and use Bayesian belief updating to decompose observed capital gains into their persistent and transitory components. With conjugate prior beliefs, the subjective conditional one-step-ahead capital gain expectations

$$\beta_t \equiv E_t^{\mathcal{P}}\left(q_{t+1}/q_t\right) \tag{8}$$

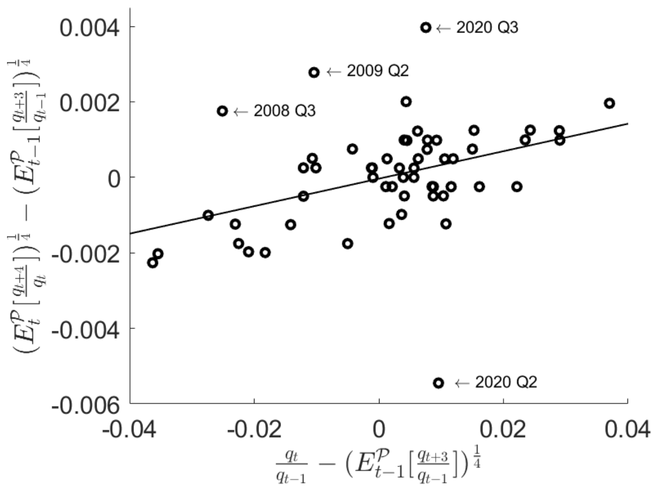
evolve according to

$$\beta_t = \min\left\{\beta_{t-1} + \frac{1}{\alpha}\left(\frac{q_{t-1}}{q_{t-2}} - \beta_{t-1}\right), \beta^U\right\}, \tag{9}$$

[15]This holds true in equilibrium because $0 < D < D^{\max}$. For the household, however, first-order conditions may hold only with inequality under the subjectively optimal plans, due to the presence of short and long constraints. The latter explains why rational households can hold price expectations that differ from the discounted sum of future rents, see Adam and Marcet (2011) for details and Adam and Nagel (2022) for related arguments.

[16]In the full model in Section 4, we will assume the same beliefs for risk-adjusted house price growth. With risk neutrality, the two coincide.

Figure 2: Capital gain surprises and revisions



Notes: This figure plots the capital gain surprises against capital gain revisions in the Michigan survey (2007-2021), along with a linear regression line.

where $1/\alpha$ is the Kalman gain determining how strongly households' capital gain expectations respond to past capital gain surprises.[17] The Kalman gain thus captures the degree to which past capital gain surprises are extrapolated into the future. The upper bound β^U on the beliefs in equation (9) is there to insure that capital gain optimism is bounded from above, so as to keep subjectively expected utility finite.[18]

Figure 2 illustrates the relationship between belief revisions and forecast errors implied by equation (9) using the Michigan survey data. The figure plots on the vertical axis a measure of the quarterly revision in capital gain expectations, $\left(E_t^{\mathcal{P}}\left(q_{t+4}/q_t\right)\right)^{\frac{1}{4}} - \left(E_{t-1}^{\mathcal{P}}\left(q_{t+3}/q_{t-1}\right)\right)^{\frac{1}{4}}$, and on the horizontal axis a measure of the forecast error in quarterly capital gains, $\frac{q_t}{q_{t-1}} - \left(E_{t-1}^{\mathcal{P}}\left(q_{t+3}/q_{t-1}\right)\right)^{\frac{1}{4}}$, for all quarters in the Michigan survey. Consistent with equation (9), there is a clear positive and approximately linear relationship between capital gain surprises and belief revisions in Figure 2. The most notable deviations from the regression line are the ones around the Great Recession (2008Q3 and 2009Q2) and the Covid Recession (2020Q2 and 2020Q3).

Equilibrium Dynamics of Housing Prices and Capital Gain Expectations. From equation (6) and the definition of subjective beliefs β_t it follows that the equilibrium housing price is given by

$$q_t = \frac{1}{1-\beta(1-\delta)\beta_t}\xi_t^d, \tag{10}$$

where β_t evolves according to (9). Equations (9) and (10) thus jointly characterize the equilibrium dynamics of housing prices and subjective beliefs. From equations (5) and (10)

[17] The (steady-state) Kalman gain depends on the subjectively perceived values for $(\sigma_\varepsilon^2, \sigma_\nu^2)$.

[18] The bound can be interpreted as a short-cut for a truncated prior support or b_t. The bounding function in (9) is a special case of the bounding function used in Adam, Marcet, and Nicolini (2016), obtained by setting $\beta^L = \beta^U$.

Table 3: Housing price moments: data versus model

	Data	Subjective Belief Model	RE Model
$std\,(PR_t)$	8.6	8.6	2.69
$corr(PR_t, PR_{t-1})$	0.99	0.99	0.99
$std(q_t/q_{t-1})$	0.06	0.04	0.003
$corr(q_t/q_{t-1}, q_{t-1}/q_{t-2})$	0.97	0.94	-0.01

Notes: The table reports the standard deviation and first-order autocorrelation of price-to-rent ratios and capital gains in the data, for the model under subjective housing beliefs and the model under rational expectations.

follows that the equilibrium price-to-rent ratio is given by

$$PR_t \equiv \frac{q_t}{R_t} = \frac{1}{1-\beta(1-\delta)\beta_t}. \tag{11}$$

Calibration. The simple model just described can generate empirically plausible housing price behavior and the resulting housing price beliefs quantitatively match the deviations from rational expectations presented in the previous section. The calibration in this section is identical to the one for the full model, with the exception for the standard deviation of the innovations to housing preferences.[19] We consider housing preference shocks evolving according to

$$\log \xi_t^d = (1-\rho_\xi)\log \underline{\xi}^d + \rho_\xi \log \xi_{t-1}^d + \varepsilon_t^d, \tag{12}$$

where $\varepsilon_t^d \sim iiN$ satisfies $E[e^{\varepsilon_t^d}] = 1$. Following Adam and Woodford (2021), we set $\rho_\xi = 0.99$ and $\delta = 0.03/4$. The standard deviation of ε_t^d is set to 0.0067, so that the model replicates the empirical standard deviation of the price-to-rent ratio, expressed in percent deviation from its mean, over the period for which we have survey data on housing expectations (2007-2021). The average value of the housing preference $\underline{\xi}^d > 0$ is irrelevant, as we are only concerned with moments characterizing cyclical properties (deviations from average values).

For the subjective belief process, we completely tie our hands and set $1/\alpha = 0.007$, which is the value estimated in Adam, Marcet, and Nicolini (2016) using stock market expectations. The low value for the Kalman gain implies that agents extrapolate only weakly, as they believe most of the realized capital gains being due to transitory components. The value for the upper belief bound β^U is set as in the full model, where it matches the maximum observed deviation of the price-to-rent ratio from its mean. Finally, the quarterly discount factor β is set such that the real interest rate is equal to 0.75%, which is the average value of the estimated U.S. natural rate over the period 2007-2021, according to estimates using the approach of Holston, Laubach, and Williams (2017).

Housing Price Behavior. Table 3 illustrates that the subjective belief model replicates surprisingly well the empirical behavior of the price-to-rent ratio and of capital gains. While

[19] This is so because the present section matches moments for a different time period than the full model, i.e., the period for which we have subjective expectations data (2007-2021).

Table 4: Patterns of deviations from rational expectations: data versus model

	Subjective Belief Model	Data	
		Mean Expectations	Median Expectations
b^{CG} from (1)	2.09	1.68	2.12
		(0.355)	(0.394)
c (in %) from (2)	0.03	0.030	0.010
		(0.172)	(0.043)
$\mathbf{c}$ (in %) from (3)	-0.063	-0.113	-0.113
		(0.009)	(0.009)

Notes: This table shows the model-implied regression coefficients of regressions (1), (2) and (3) for a natural rate of 0.75% (annualized) in the first column and the empirical results (for real housing prices) in the second and third column.

the standard deviation of the price-to-rent ratio is a targeted moment, all other moments are untargeted. The model matches very well the high quarterly autocorrelation of the price-to-rent ratio and the fairly high quarterly autocorrelation of capital gains. It undershoots somewhat the standard deviation of quarterly capital gains, illustrating that it features perhaps too little high-frequency variation in prices.[20]

Table 3 also reports the rational expectations (RE) outcome using the same calibration as for the subjective belief model. It shows that the about 70% of the fluctuations in the price-dividend ratio in the subjective belief model is due to capital gain extrapolation. Adam, Marcet, and Nicolini (2016) explain how capital gain extrapolation generates momentum and mean reversion in prices and thus contributes to asset price volatility.

While the ability of capital gain extrapolation to increase the price volatility is well-known, we now turn to the new question of whether the model with capital gain extrapolation matches the structure of forecast errors documented in Section 2.

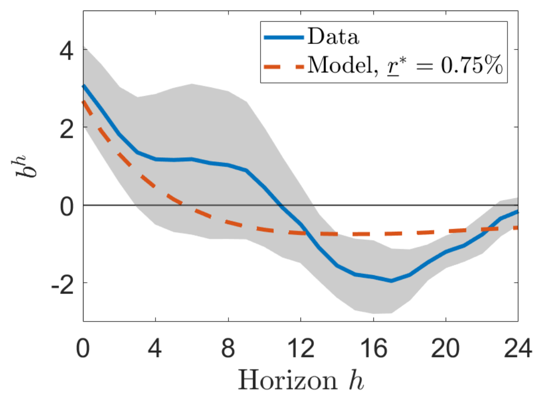
Belief revisions and forecast errors. The simple model quantitatively matches the three deviations from rational housing price expectations documented in Section 2.

Table 4 reports the outcomes of population regressions of equations (1), (2) and (3) for the calibrated subjective belief model. The results shows the model matches sluggish updating about expected housing prices ($b^{CG} > 0$) and the opposing cyclicality of actual and expected capital gains ($c > 0$ and $\mathbf{c} < 0$). For better comparison, Table 4 also reports also the empirical estimates of the corresponding coefficients from Tables 1 and 2. The magnitude of the coefficients generated by the model closely match the ones obtained using survey data, with the exception that the model underpredicts the counter-cyclicality of actual capital gains.

Figure 3 shows that the simple model is able to match the dynamic response of forecast errors documented empirically in Figure 1(b). We compute model-implied forecast errors as $FE^{model}_{t+h} = \frac{q_{t+4+h}}{q_{t+h}} - (\beta_{t+h})^4$ and compute the population local projections (4). Consistent

[20]This could easily be remedied by adding some iid shocks, say iid shocks to the discount factor β.

Figure 3: Dynamic forecast error response: data versus model



Notes: The figure shows impulse-response functions of housing-price forecast errors of one-year ahead expectations to a one standard deviation innovation in the housing capital gain from the model and the data. The shaded area shows the 90%-confidence intervals of the empirical estimates, standard errors are robust with respect to serial correlation and heteroskedasticity (Newey-West with $h+1$ lags).

with the data, the model generates initial underprediction of capital gains (over-pessimism) and subsequently overprediction (over-optimism).

Appendix B.1 reports the dynamic forecast error responses for the model and in the data about the expected housing price level (rather than the expected capital gain). It shows that the model matches equally well the patterns of forecasts errors about the future housing price level.

Falling Natural Rates and Rising Housing Price Volatility. The simple housing model also predicts that falling natural rates of interest will give rise to higher volatility for housing prices. Such a relationship between natural rates and housing volatility is present in the data. It can be seen by considering regressions of the form

$$Std(PR_{t-\frac{h}{2}}, ..., PR_{t+\frac{h}{2}}) = a_h^* - b_h^* \cdot r_t^* + u_{t,h}, \tag{13}$$

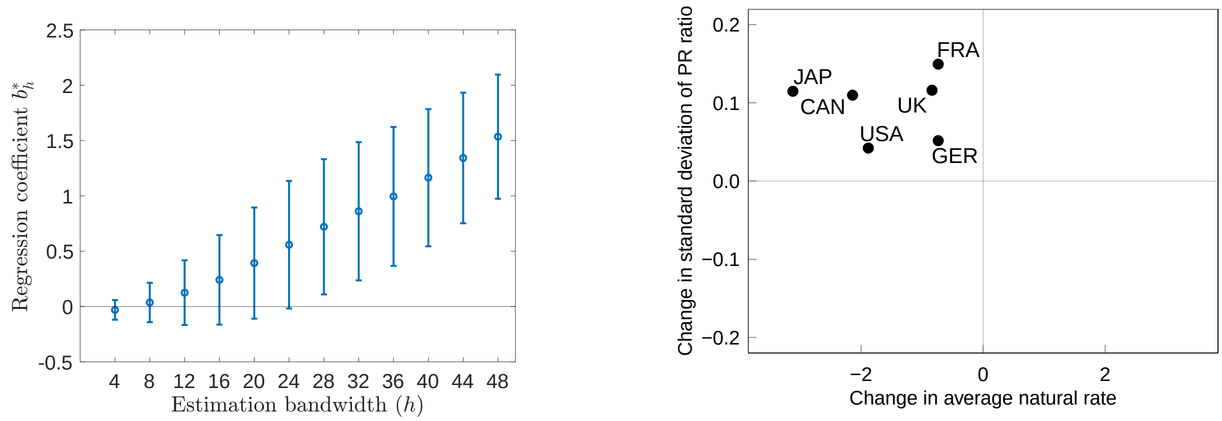
where r_t^* denotes the natural rate of interest from Holston, Laubach, and Williams (2017) and $Std(PR_{t-\frac{h}{2}}, ..., PR_{t+\frac{h}{2}})$ the standard deviation of the price rent ratio using a window of $h+1$ quarters centered around period t. Under the standard assumption that the natural rate of interest is only a function of exogenous fundamentals, the regression coefficients b_h^* can be interpreted as capturing a causal relationship.

Panel (a) in Figure 4 reports the coefficients b_h^* for the United States using various estimation bandwidths h. While narrow bandwidths generate insignificant outcomes, most likely due to the difficulty associated with reliably estimating the standard deviation of the PR-ratio, the coefficient becomes positive and significant for larger bandwidth and is quite large when using a bandwidth of 48 quarters (+/- 2 years). We can thus conclude that the standard deviation of U.S. housing prices is rising as the natural rate falls.

Panel (b) in Figure 4 shows that this relationship is also present in other countries: it plots the change in the average level of the natural rate from the period before 1990 to the

Figure 4: The natural rate and housing price fluctuations

(a) Relationship between lower U.S. natural rates & housing price volatility (b_h^*) (b) Changes in the natural rate & housing price volatility in advanced economies



Notes: Panel (a) reports the regression coefficient b_h^* from equation (13) together with 68% Newey-West error bands using h lags. Panel (b) plots the pre-/post-1990 changes in the average natural rate against the changes in the volatility of the price-to-rent ratio for different advanced economies. The volatilities of the price-to-rent ratios in the pre-/post-1990 periods are the standard deviations relative to the period-specific mean values.

period after 1990 for the U.S., Canada, France, Germany, and the United Kingdom, against the change in the standard deviation of the price-to-rent ratio over the same periods. To take possible shifts in the mean of the PR-ratio over time into account, e.g., due to falling real interest rates, the standard deviation of the PR-ratio is computed in each of the two sub-periods for the *percent* deviation of the PR-ratio from its period-specific mean.[21] In all six advanced economies, the PR-ratio has become more volatile as the average level of the natural rate has declined.

Equations (9) and (10) reveal how housing prices in our simple model are affected by the level of the natural rate of interest. The natural rate of interest is given by $\underline{r}^* = 1/\beta - 1$ and only depends on the discount factor $\beta \in (0, 1)$. A discount factor closer to one thus lowers the natural rate of interest.

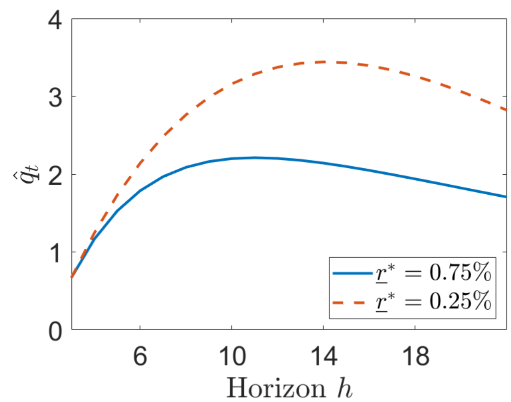
Figure 5 illustrates that the model in fact generates a negative relationship between the level of the natural rate and housing price volatility.[22] It presents the impulse response of real housing prices to a positive housing preference shock ξ_t^d, which is the only shock driving housing prices in the model. It considers this response for the calibrated level of the natural rate of 0.75% and for a lower natural rate level equal to 0.25%.[23] The key message of Figure 5 is that housing prices respond stronger to housing demand shocks when natural rates are

[21]The empirical results become even stronger if one considers instead the absolute standard deviation of the PR-ratio.

[22]The full model presented later on will also be able to quantiatively replicate this relationship, see section 6.

[23]To account for the higher housing price levels associated with lower natural rates, we show impulse responses in terms of percent deviations from their respective steady state values. The model-implied response for the PR-ratio to a housing preference shock looks very similar and is shown in Appendix B.2.

Figure 5: Model-implied housing price response for different natural rates



lower: the same shock gives rise to an approximately 75% stronger housing price response when the natural rate is at its lower level.

This surprising model outcome can be explained as follows. The capital gain increase triggered by the fundamental shock in the initial period leads to an upward revision of capital gain expectations. Equation (10) implies, however, that these higher capital gain expectations produce larger realized capital gains, the higher is the value for β, i.e., the lower is the natural rate of interest. Higher realized capital gains produce stronger upward revisions in beliefs in the future and thus feed stronger capital gains in the subsequent period. Through this feedback loop, low natural rates generate more momentum in housing price changes following fundamental shocks, allowing the model to replicate the relationship between natural rates and the volatility of housing prices.

4 Full Model with Capital Gain Extrapolation

This section studies the monetary policy implications of falling natural rates of interest and rising housing price volatility. To this end, we embed capital gain extrapolation into a sticky price model with a housing sector. The model features endogenous production of consumption goods and housing and generalizes the setup in Adam and Woodford (2021) by allowing for belief distortions that are not absolutely continuous with respect to the beliefs held by the policymaker. This permits analyzing the subjective housing beliefs as in equation (7), which give rise to capital gain extrapolation and deviations from rational expectations matching patterns in the survey data. In addition, we consider a lower-bound constraint on nominal rates, which we show to be quantitatively important for understanding how the optimal inflation target responds to lower natural rates in the presence of subjective housing beliefs.

We consider an economy populated by internally rational decision makers (Adam and Marcet (2011)): households maximize utility and firms maximize profits, but both do so using a potentially subjective probability measure $\mathcal{P}$, which assigns probabilities to all external variables, i.e., to all variables that are beyond agents' control. These variables include fundamental shocks, as well as competitive market prices (wages, goods prices, housing prices and rents). The setup delivers rational expectations in the special case where $\mathcal{P}$ is the objective probability measure.

The economy is made up of identical infinitely-lived households, each of which maximizes the following objective function[24]

$$U \equiv E_0^{\mathcal{P}} \sum_{t=0}^{\infty} \beta^t \left[\tilde{u}(C_t;\xi_t) - \int_0^1 \tilde{v}(H_t(j);\xi_t)dj + \tilde{\omega}(D_t + D_t^R;\xi_t) \right], \tag{14}$$

subject to the sequence of flow budget constraints

$$\begin{aligned} &C_t + B_t + (D_t - (1-\delta)D_{t-1}) \frac{q_t^u}{\tilde{u}_C(C_t;\xi_t)} + k_t + R_t D_t^R = \\ &\tilde{d}(k_t;\xi_t)\frac{q_t^u}{\tilde{u}_C(C_t;\xi_t)} + \int_0^1 w_t(j)H_t(j)dj + \frac{B_{t-1}}{\Pi_t}(1+i_{t-1}) + \frac{\Sigma_t}{P_t} + \frac{T_t}{P_t}, \end{aligned} \tag{15}$$

where C_t is an aggregate consumption good, $H_t(j)$ is the quantity supplied of labor of type j and $w_t(j)$ the associated real wage, D_t the stock of owned houses, D_t^R the units of rented houses, $\delta \in [0,1]$ the housing depreciation rate, and q_t^u the real price of houses in marginal utility units, defined as

$$q_t^u \equiv q_t \tilde{u}_C(C_t;\xi_t),$$

where q_t is the real house price in units of consumption.[25] The variable q_t^u provides a measure of whether housing is currently expensive or inexpensive, in units that are particularly relevant for determining housing demand. The variable k_t denotes investment in new houses and $\tilde{d}(k_t;\xi_t)$ the resulting production of new houses.[26] $B_t \equiv \widetilde{B}_t/P_t$ denotes the real value of nominal government bond holdings $\widetilde{B}_t$ and P_t the nominal price of consumption. $\Pi_t = P_t/P_{t-1}$ is the inflation rate, i_t the nominal interest rate, R_t the real rental rate for housing units, and ξ_t is a vector of exogenous disturbances, which may induce random shifts in the functions $\tilde{u}$, $\tilde{v}$, $\tilde{\omega}$ and $\tilde{d}$. T_t denotes nominal lump sum transfers (taxes if negative) from the government and Σ_t nominal profits accruing to households from the ownership of firms.

Households discount future payoffs at the rate $\beta \in (0,1)$. Since our model is formulated in terms of growth-detrended variables, the discount rate β jointly captures the time preference rate $\widetilde{\beta} \in (0,1)$ and the steady-state growth rate of marginal utility. Letting $g_c \geq 0$ denote the steady-state growth rate of consumption in non-detrended terms, we have

$$\beta \equiv \widetilde{\beta}\frac{\tilde{u}_C(C(1+g_c);\underline{\xi})}{\tilde{u}_C(C;\underline{\xi})}, \tag{16}$$

where $\underline{\xi}$ denotes the steady state value of the disturbance ξ_t. When the growth rate g_c of the economy falls, the discount rate β increases because marginal utility falls less strongly. We

[24]It cannot be common knowledge to households that they are representative whenever $\mathcal{P}$ deviates from the rational measure.

[25]In Section 3, q_t^u and q_t coincide due to risk-neutrality.

[26]We consolidate housing production into the household budget constraint. It would be equivalent to have instead a separate housing production sector that is owned by households.

can thus capture a fall in the trend growth rate of the economy simply via an increase in the time discount rate β. Declining trend growth causes the steady-state real interest rate and thus the average natural rate of interest to fall, which is in line with the estimates provided in Holston, Laubach, and Williams (2017) (see Appendix G).

The aggregate consumption good is a Dixit-Stiglitz aggregate of each of a continuum of differentiated goods,

$$C_t \equiv \left[\int_0^1 c_t(i)^{\frac{\eta-1}{\eta}} di \right]^{\frac{\eta}{\eta-1}}, \tag{17}$$

with an elasticity of substitution $\eta > 1$. We further assume isoelastic functional forms

$$\begin{aligned} \tilde{u}(C_t;\xi_t) &\equiv \frac{C_t^{1-\tilde{\sigma}^{-1}} \bar{C}_t^{\tilde{\sigma}^{-1}}}{1-\tilde{\sigma}^{-1}}, \\ \tilde{v}(H_t(j);\xi_t) &\equiv \frac{\lambda}{1+\nu} \left(H_t(j)\right)^{1+\nu} \bar{H}_t^{-\nu}, \\ \tilde{\omega}(D_t + D_t^R;\xi_t) &\equiv \xi_t^d \left(D_t + D_t^R\right), \\ \tilde{d}(k_t;\xi_t) &\equiv \frac{A_t^d}{\tilde{\alpha}} k_t^{\tilde{\alpha}}, \end{aligned} \tag{18}$$

where $\tilde{\sigma}, \nu > 0$, $\tilde{\alpha} \in (0,1)$ and $\{\bar{C}_t, \bar{H}_t, \xi_t^d, A_t^d\}$ are bounded, exogenous and positive disturbance processes which are among the exogenous disturbances included in the vector ξ_t.

Our specification includes two housing-related disturbances, namely ξ_t^d, which captures shocks to housing preferences, and A_t^d, which captures shocks to the productivity in the construction of new houses. We impose linearity in the utility function (18), because it greatly facilitates the characterization of optimal policy, with rented and owned housing units being perfect substitutes. Introducing a weight on rental units relative to housing units would allow us to perfectly match the average price-to-rent ratio we observe in the data. However, since this does not change any other results, we abstract from such a scaling parameter and assign equal weight to housing and renting in the utility.

Each differentiated good is supplied by a single monopolistically competitive producer; there is a common technology for the production of all goods, in which (industry-specific) labor is the only variable input,

$$y_t(i) = A_t f(h_t(i)) = A_t h_t(i)^{1/\phi}, \tag{19}$$

where A_t is an exogenously varying technology factor, and $\phi > 1$. The Dixit-Stiglitz preferences (17) imply that the quantity demanded of each individual good i will equal[27]

$$y_t(i) = Y_t \left(\frac{p_t(i)}{P_t} \right)^{-\eta}, \tag{20}$$

[27]In addition to assuming that household utility depends only on the quantity obtained of C_t, we assume that the government also cares only about the quantity obtained of the composite good defined by (17), and that it seeks to obtain this good through a minimum-cost combination of purchases of individual goods.

where Y_t is the total demand for the composite good defined in (17), $p_t(i)$ is the price of the individual good, and P_t is the price index,

$$P_t \equiv \left[\int_0^1 p_t(i)^{1-\eta} di \right]^{\frac{1}{1-\eta}}, \tag{21}$$

corresponding to the minimum cost for which a unit of the composite good can be purchased in period t. Total demand is given by

$$Y_t = C_t + k_t + g_t Y_t, \tag{22}$$

where g_t is the share of the total amount of composite goods purchased by the government, treated here as an exogenous disturbance process.

4.1 Household Optimality Conditions

Internally rational households choose state-contingent sequences for the choice variables $\left\{C_t, H_t(j), D_t, D_t^R, k_t, B_t\right\}$ so as to maximize (14), subject to the budget constraints (15), taking as given their beliefs about the processes $\{P_t, w_t(j), q_t^u, R_t, i_t, \Sigma_t/P_t, T_t/P_t\}$, as determined by the (subjective) measure $\mathcal{P}$.

We shall be particularly interested in the policy implications generated by subjective housing price beliefs. To insure that an optimum exists in the presence of potentially distorted beliefs about the housing price q_t^u, we require housing choices to lie in some compact choice set $D_t \in [0, D^{\max}]$, as discussed in Section 3, where the upper bound can be arbitrarily large.

The first order conditions give rise to an optimal labor supply relation

$$w_t(j) = \frac{\tilde{v}_H(H_t(j); \xi_t)}{\tilde{u}_C(C_t; \xi_t)}, \tag{23}$$

a consumption Euler equation

$$\tilde{u}_C(C_t; \xi_t) = \beta E_t^{\mathcal{P}} \left[\tilde{u}_C(C_{t+1}; \xi_{t+1}) \frac{1+i_t}{P_{t+1}/P_t} \right], \tag{24}$$

an equation characterizing optimal investment in new houses

$$k_t = \left(A_t^d q_t^u \frac{C_t^{\tilde{\sigma}^{-1}}}{\bar{C}_t^{\tilde{\sigma}^{-1}}} \right)^{\frac{1}{1-\tilde{\alpha}}}, \tag{25}$$

an optimality condition for rental units

$$\xi_t^d = R_t \tilde{u}_C(C_t, \xi_t), \tag{26}$$

and a set of conditions determining the optimal housing demand D_t:

$$\begin{array}{ll} q_t^u < \xi_t^d + \beta(1-\delta) E_t^{\mathcal{P}} q_{t+1}^u & \text{if } D_t = D^{\max} \\ q_t^u = \xi_t^d + \beta(1-\delta) E_t^{\mathcal{P}} q_{t+1}^u & \text{if } D_t \in (0, D^{\max}) \\ q_t^u > \xi_t^d + \beta(1-\delta) E_t^{\mathcal{P}} q_{t+1}^u & \text{if } D_t = 0. \end{array} \tag{27}$$

With rational expectations, the upper and lower holding bounds never bind.[28] Since we are interested in how the presence of belief distortions about future housing values affect equilibrium outcomes, the bounds in equation (27) can potentially bind under the *subjectively* optimal plans. This explains why an internally rational household can hold subjective housing price expectations, even if she holds rational expectations about the preference shocks ξ_t^d in equation (27).

Forward-iterating on equation (24), which holds with equality under all belief-specifications, delivers a present-value formulation of the consumption Euler equation

$$\tilde{u}_C(C_t;\xi_t) = \lim_{T\to\infty} E_t^{\mathcal{P}}\left[\tilde{u}_C(C_T;\xi_T)\beta^T \prod_{k=0}^{T-t} \frac{1+i_{t+k}}{P_{t+k+1}/P_{t+k}}\right], \tag{28}$$

which will be convenient to work with, especially under subjective belief specifications. Household choices must also satisfy the transversality constraint

$$\lim_{T\to\infty} \beta^T E_t^{\mathcal{P}}\left[\tilde{u}_C(C_T;\xi_T)B_T + D_T q_T^u\right] = 0. \tag{29}$$

Optimal household behavior under potentially distorted beliefs is jointly characterized by equations (23) and (25)-(29).

4.2 Optimal Price Setting by Firms

The producers in each industry fix the prices of their goods in monetary units for a random interval of time, as in the model of staggered pricing introduced by Calvo (1983) and Yun (1996). Producers use the representative households' subjectively optimal consumption plans to discount profits and are assumed to know the product demand function (20). They need to formulate beliefs about the future price levels P_T, industry-specific wages $w_T(j)$, aggregate demand Y_T, and productivity A_T.

Let $0 \le \alpha < 1$ be the fraction of prices that remain unchanged in any period. A supplier i in industry j that changes its price in period t chooses its new price $p_t(i)$ to maximize

$$E_t^{\mathcal{P}} \sum_{T=t}^{\infty} \alpha^{T-t} Q_{t,T} \Pi\left(p_t(i), P_T, w_T(j), Y_T, A_T\right), \tag{30}$$

where $E_t^{\mathcal{P}}$ denotes the expectations of price setters conditional on time t information, which are identical to the expectations held by consumers. Firms discount random nominal income in period T using households' subjective stochastic discount factor $Q_{t,T}$, which is given by

$$Q_{t,T} = \beta^{T-t} \frac{\tilde{u}_C\left(C_T, \xi_T\right)}{\tilde{u}_C\left(C_t, \xi_t\right)} \frac{P_t}{P_T}.$$

The term α^{T-t} in equation (30) captures the probability that a price chosen in period t will not have been revised by period T, and the function $\Pi\left(p_t(i), ...\right)$ indicates the nominal profits of the firm in period t, as discussed next.

[28]The upper bound $D^{\max}$ has been chosen sufficiently large for this to be true. The lower bound is never reached because the housing production function satisfies Inada conditions.

Profits are equal to after-tax sales revenues net of the wage bill. Sales revenues are determined by the demand function (20), so that (nominal) after-tax revenue equals

$$(1-\tau_t)\, p_t(i) Y_t \left(\frac{p_t(i)}{P_t}\right)^{-\eta}.$$

Here τ_t is a proportional tax on sales revenues in period t, $\{\tau_t\}$ is treated as an exogenous disturbance process, taken as given by the monetary policymaker. We assume that τ_t fluctuates over a small interval around a non-zero steady state level $\underline{\tau}$. We allow for exogenous variations in the tax rate in order to include the possibility of "pure cost-push shocks" that affect the equilibrium pricing behavior while implying no change in the efficient allocation of resources.

The labor demand of firm i at a given industry-specific wage $w_t(j)$ can be written as

$$h_t(i) = \left(\frac{Y_t}{A_t}\right)^{\phi} p_t(i)^{-\eta\phi} P_t^{\eta\phi}, \tag{31}$$

which follows from (19) and (20). Using this, the nominal wage bill is given by

$$P_t w_t(j) h_t(i) = P_t w_t(j) \left(\frac{Y_t}{A_t}\right)^{\phi} p_t(i)^{-\eta\phi} P_t^{\eta\phi}.$$

Subtracting the nominal wage bill from the above expression for nominal after tax revenue, we obtain the function $\Pi\left(p_t(i), P_T, w_T(j), Y_T, A_T\right)$ used in (30).

Each of the suppliers that revise their prices in period t chooses the same new price p_t^*, that maximizes (30). The first-order condition with respect to $p_t(i)$ is given by[29]

$$E_t^{\mathcal{P}} \sum_{T=t}^{\infty} \alpha^{T-t} Q_{t,T} \Pi_1\left(p_t(i), P_T, w_T(j), Y_T, A_T\right) = 0.$$

The equilibrium choice p_t^*, which is the same for each firm i in industry j, is the solution to this equation. Letting p_t^j denote the price charged by firms in industry j at time t, we have $p_t^j = p_t^*$ in periods in which industry j resets its prices and $p_t^j = p_{t-1}^j$ otherwise.

Under the assumed isoelastic functional forms, the optimal choice has a closed-form solution

$$\left(\frac{p_t^*}{P_t}\right)^{1+\eta(\phi-1)} = \frac{E_t^{\mathcal{P}} \sum_{T=t}^{\infty} \alpha^{T-t} Q_{t,T} \frac{\eta}{\eta-1} \phi w_T(j) \left(\frac{Y_T}{A_T}\right)^{\phi} \left(\frac{P_T}{P_t}\right)^{\eta\phi+1}}{E_t^{\mathcal{P}} \sum_{T=t}^{\infty} \alpha^{T-t} Q_{t,T} (1-\tau_T) Y_T \left(\frac{P_T}{P_t}\right)^{\eta}}. \tag{32}$$

The price index evolves according to a law of motion

$$P_t = \left[(1-\alpha)\, p_t^{*1-\eta} + \alpha P_{t-1}^{1-\eta}\right]^{\frac{1}{1-\eta}}, \tag{33}$$

[29] Note that supplier i's profits in (30) are a concave function of the quantity sold $y_t(i)$, since revenues are proportional to $y_t(i)^{\frac{\eta-1}{\eta}}$ and hence concave in $y_t(i)$, while costs are convex in $y_t(i)$. Moreover, since $y_t(i)$ is proportional to $p_t(i)^{-\eta}$, the profit function is also concave in $p_t(i)^{-\eta}$. The first-order condition for the optimal choice of the price $p_t(i)$ is the same as the one with respect to $p_t(i)^{-\eta}$; hence the first-order condition with respect to $p_t(i)$ is both necessary and sufficient for an optimum.

as a consequence of (21). The equilibrium inflation in any period is characterized by

$$\left(\frac{P_t}{P_{t-1}}\right)^{\eta-1} = \frac{1-(1-\alpha)\left(\frac{p_t^*}{P_t}\right)^{1-\eta}}{\alpha}. \tag{34}$$

The welfare loss from price adjustment frictions can be captured by price dispersion, which is defined as

$$\Delta_t \equiv \int_0^1 \left(\frac{p_t^j}{P_t}\right)^{-\eta(1+\omega)} dj \geq 1, \tag{35}$$

where

$$\omega \equiv \phi(1+\nu) - 1 > 0$$

is the elasticity of real marginal cost in an industry with respect to industry output.

Using equation (33) together with the fact that the relative prices of the industries that do not change their prices in period t remain the same, one can derive a law of motion for the price dispersion term Δ_t of the form

$$\Delta_t = h(\Delta_{t-1}, P_t/P_{t-1}), \tag{36}$$

with

$$h(\Delta_t, P_t/P_{t-1}) \;\equiv\; \alpha\Delta_t\left(\frac{P_t}{P_{t-1}}\right)^{\eta(1+\omega)} + (1-\alpha)\left(\frac{1-\alpha\left(\frac{P_t}{P_{t-1}}\right)^{\eta-1}}{1-\alpha}\right)^{\frac{\eta(1+\omega)}{\eta-1}}.$$

As is commonly done, we assume that the initial degree of price dispersion is small ($\Delta_{-1} \sim O(2)$).

Equations (32), (34), and (36) jointly define a short-run aggregate supply relation between inflation, output and house prices (via the aggregate demand equation (22) and (25)), given the current disturbances ξ_t, and expectations regarding future wages, prices, output, consumption and disturbances. Equation (36) describes the evolution of the costs of price dispersion over time.

For future reference, we remark that all firms together make total profits equal to

$$\frac{\Sigma_t}{P_t} = (1-\tau_t)Y_t - w_t H_t, \tag{37}$$

where $w_t H_t = \int_0^1 w_t(j) H_t(j) dj$.

4.3 Government Budget Constraint and Market Clearing Conditions

The government consumes goods $g_t Y_t$, imposes a sales tax τ_t, issues nominal bonds $\widetilde{B}_t \equiv P_t B_t$, and pays lump sum transfers T_t to households. The government budget constraint is given by

$$B_t = B_{t-1}\frac{1+i_{t-1}}{P_t/P_{t-1}} + \frac{T_t}{P_t} + (g_t - \tau_t)\, Y_t.$$

For simplicity, we assume that lump sum transfers (taxes if negative) are set such that they keep real government debt constant at some initial level B_{-1}. This implies that government transfers are given by

$$\frac{T_t}{P_t} = -(g_t - \tau_t)Y_t + B_{t-1}\left(1 - \frac{1+i_{t-1}}{P_t/P_{t-1}}\right). \tag{38}$$

Using (22) and (25), one can express the market clearing condition for the consumption/investment good as

$$Y_t = \frac{C_t + \Omega_t C_t^{\frac{\tilde{\sigma}^{-1}}{1-\tilde{\alpha}}}}{1-g_t}, \tag{39}$$

where

$$\Omega_t \equiv \left(A_t^d \bar{C}_t^{-\tilde{\sigma}^{-1}} q_t^u\right)^{\frac{1}{1-\tilde{\alpha}}} > 0 \tag{40}$$

is a term that depends on exogenous shocks and belief distortions in the housing market only, see equation (27). The previous two equations implicitly define a function

$$C_t = C(Y_t, q_t^u, \xi_t), \tag{41}$$

which delivers the market clearing consumption level, for a given output level Y_t, given housing prices q_t^u and given exogenous disturbances ξ_t.

The market clearing condition for housing is

$$D_t = (1-\delta)D_{t-1} + \tilde{d}(k_t; \xi_t), \tag{42}$$

and rental market clearing requires

$$D_t^R = 0. \tag{43}$$

Labor market clearing requires that the supply of labor of type j in (23) is equal to labor demand of industry j, which is given by (31), as all firms in the industry charge the same price. This delivers

$$w_t(j) = \frac{\tilde{v}_H(H_t(j);\xi_t)}{\tilde{u}_C(C_t;\xi_t)} = \frac{\lambda\,(H_t(j))^{\nu}\,\bar{H}_t^{-\nu}}{C_t^{-\tilde{\sigma}^{-1}}\bar{C}_t^{\tilde{\sigma}^{-1}}} = \lambda\frac{\bar{H}_t^{-\nu}}{\bar{C}_t^{\tilde{\sigma}^{-1}}}\left(\frac{Y_t}{A_t}\right)^{\nu\phi} C_t^{\tilde{\sigma}^{-1}}\left(\frac{p_t^j}{P_t}\right)^{-\nu\eta\phi}, \tag{44}$$

where $p_t^j = p_t^*$ in periods where industry j can adjust prices and $p_t^j = p_{t-1}^j$ otherwise.

4.4 Equilibrium and Ramsey Problem with Subjective Beliefs

We now define the equilibrium in the presence of subjective beliefs, as well as the nonlinear Ramsey problem characterizing the monetary policymaker's optimization problem in the presence of subjective beliefs.

We start by defining an *Internally Rational Expectations Equilibrium (IREE)*, which is a generalization of the notion of a Rational Expectations Equilibrium (REE) to settings with subjective private sector beliefs:

Definition 1 *An internally rational expectations equilibrium (IREE) is a bounded stochastic process for* $\{Y_t, C_t, k_t, D_t, \{w_t(j)\}, p_t^*, P_t, \Delta_t, q_t^u, i_t\}_{t=0}^{\infty}$ *satisfying the aggregate supply equations (32), and (34), the law of motion for the evolution of price distortions (36), the household optimality conditions (25), (27), (28), and the market clearing conditions (39), (42) and (44) for all* j.

The equilibrium features ten variables (counting the continuum of wages as a single variable) that must satisfy nine conditions, leaving one degree of freedom to be determined by monetary policy.[30] In the special case with rational beliefs ($E_t^{\mathcal{P}}[\cdot] = E_t[\cdot]$), the IREE is a Rational Expectations Equilibrium (REE).

Given the equilibrium outcome, the remaining model variables can be determined as follows. Equilibrium profits are given by equation (37) and equilibrium taxes by equation (38). Equilibrium labor supply $H_t(j)$ follows from equation (23) for each labor type j. Equilibrium bond holdings satisfy $B_t = B_{-1}$ and equilibrium inflation is $\Pi_t \equiv P_t/P_{t-1}$. Equilibrium rental units are given by equation (43) and equilibrium rental prices by equation (26).

The Ramsey problem allows the policymaker to choose the sequence of policy rates, prices and allocations to maximize household utility, subject to the constraint that prices and allocations constitute an IREE. The policymaker thereby maximizes household utility under rational expectations, i.e., under a probability measure that is different from the one entertained by households, whenever the latter hold distorted beliefs. Benigno and Paciello (2014) refer to such a policymaker as a 'paternalistic' policymaker. The non-linear Ramsey problem is spelled out in Appendix C. To gain economic insights into the forces shaping the policy problem, the next section considers a quadratic approximation to the nonlinear problem.

5 The Monetary Policy Problem: Analytic Insights

This section derives analytic insights into the monetary policy problem. In particular, it presents a quadratic approximation to the policymaker's Ramsey problem that highlights the new economic forces arising from the presence of capital gain extrapolation.[31] It shows how subjective capital gain expectations shift the Phillips curve and affect the natural rate of interest in the IS equation.

The quadratic approximation derived below is valid for two alternative belief settings.[32] The first setting is standard and assumes rational expectations. While constituting a useful

[30] The transversality condition (29) must also be satisfied in equilibrium, but is not imposed as an equilibrium condition, as it will hold for all belief specifications considered below.

[31] The nonlinear problem can be found in Appendix C. The quadratic approximation delivers a valid second-order approximation to the problem, whenever (i) the steady-state Lagrange multipliers associated with the nonlinear constraints are of order $O(1)$, which is the case when the steady state output distortion $\Theta \equiv \log\left(\frac{\eta}{\eta-1}\frac{1-g}{1-\tau}\right)$ is of order $O(1)$, and (ii) the gap between the steady-state interest rate and the lower bound, i.e., $\frac{1}{\beta} - 1$, is also of $O(1)$, i.e., when steady state real interest rates/natural rates are low.

[32] Recall from our earlier discussion that firms must hold beliefs about future values of P_t, $w_t(j)$, Y_t and that households must hold beliefs about future values of $(P_t, w_t(j), q_t^u, R_t, i_t, \Sigma_t/P_t, T_t/P_t)$. Both actors must additionally hold beliefs about the fundamental shocks entering their decision problem.

benchmark, the assumption of rational housing price expectations is strongly rejected by the survey evidence in Section 2.

The second setting considers subjective housing beliefs. In particular, it considers capital gain extrapolation according to equations (7)-(9) introduced in the simple model in Section 3, but with the variable q_t being replaced by q_t^u. The latter implies that households extrapolate capital gains in units of marginal utility rather than in units of consumption. Specifying subjective beliefs in units of marginal utility leaves the ability of the learning rule to replicate the survey evidence unchanged[33], but has three advantages.

First, the dynamics of housing prices in units of marginal utility is unaffected by monetary policy, even if housing prices in units of consumption do depend on policy. As a result, the object about which agents learn does not depend on policy. The policymaker thus cannot 'manipulate' households' subjective housing price beliefs in a way to achieve outcomes that are potentially better than under rational expectations.[34] In addition, it allows side-stepping the otherwise thorny issue of how the learning rule should respond to the conduct of monetary policy.

Second, the belief setup allows replicating the fact that housing demand/investment responds more strongly to monetary policy disturbances than non-housing demand, thereby avoiding the pitfalls described in Barsky, House, and Kimball (2007). Appendix D shows that in response to an exogenous shift in the path of nominal interest rates $\boldsymbol{i}$, the change in housing investment and consumption satisfies at all times

$$\frac{d \log k_t}{d\boldsymbol{i}} = \frac{1}{1-\tilde{\alpha}} \frac{1}{\tilde{\sigma}} \cdot \frac{d \log C_t}{d\boldsymbol{i}}, \tag{45}$$

where $1/(1-\tilde{\alpha})$ is the price elasticity of housing supply and $1/\tilde{\sigma}$ the coefficient of relative risk aversion in consumption. The calibrated model considered later on features $1/((1-\tilde{\alpha})\tilde{\sigma}) > 1$.[35]

Third, the belief specification greatly simplifies the algebra involved in deriving the second-order approximation to the Ramsey problem, because it allows for a relatively straightforward determination of the equilibrium path of subjectively optimal consumption choices.

Overall, we wish to consider a minimal deviation from rational expectations, therefore keep expectations about all other variables rational to the extent possible.[36] Finally, to insure that households' subjectively optimal plans satisfy the transversality condition, we assume that households hold rational capital gain expectations in the very long run, i.e.,

[33]This is so because we consider log consumption preferences which imply that contributions from fluctuations in marginal utility are orders of magnitude smaller than those generated by subjective beliefs.

[34]This is a key distinction to the setups analyzed in Molnar and Santoro (2014), Mele, Molnar, and Santoro (2020), and Caines and Winkler (2021).

[35]The calibration use log utility in consumption ($1/\tilde{\sigma} = 1$) and a supply elasticity of $1/(1-\tilde{\alpha}) = 5$.

[36]In particular, household continue to hold rational expectations about all other prices, i.e., about $\{P_t, w_t(j), i_t\}$ and firms hold rational expectations about $\{P_t, w_t(j), Y_t\}$. Furthermore, all actors continue to hold rational expectations about the exogenous fundamentals. Beliefs about profits and lump sum taxes, $\{\Sigma_t/P_t, T_t/P_t\}$ continue to be determined by equations (37) and (38), evaluated with rational output expectations and the state-contingent optimal choices for $\{H_t, k_t, B_t\}$. Rental price expectations, however, cannot be kept rational: they need to satisfy equation (26), which shows that they are influenced by the subjectively optimal consumption plans implied by equation (28).

after some arbitrarily large but finite period $\bar{T} < \infty$.[37] We then consider the policy problem with subjective beliefs in periods $t \ll \overline{T}$.

For the two belief settings just described, the quadratic approximation of the Ramsey problem is given by[38]

$$\max_{\{\pi_t, y_t^{gap}, \widehat{q}_t^u, i_t \geq \underline{i}\}} -E_0 \sum_{t=0}^{\infty} \beta^t \frac{1}{2} \left(\Lambda_\pi \pi_t^2 + \Lambda_y \left(y_t^{gap}\right)^2 + \Lambda_q \left(\widehat{q}_t^u - \widehat{q}_t^{u*}\right)^2 \right) \tag{46}$$

s.t.:

$$\pi_t = \beta E_t \pi_{t+1} + \kappa_y y_t^{gap} + \kappa_q \left(\widehat{q}_t^u - \widehat{q}_t^{u*}\right) + u_t \text{ for } t \geq 0 \tag{47}$$

$$y_t^{gap} = \lim_{T \to \infty} E_t y_T^{gap} - \varphi E_t \sum_{k=0}^{\infty} \left(i_{t+k} - \pi_{t+1+k} - r_{t+k}^{*,RE} \right) - \frac{C_q}{C_Y} \left(\widehat{q}_t^u - \widehat{q}_t^{u*}\right) \text{ for } t \geq 0 \tag{48}$$

as well as equations determining $\left(\widehat{q}_t^u - \widehat{q}_t^{u*}\right)$ and initial pre-commitments,

where $\pi_t = \log \Pi_t$ denotes inflation and y_t^{gap} the output gap, which is defined as $y_t^{gap} = \log Y_t - \log Y_t^*$, with Y_t^* denoting the efficient level of output, as defined in equation (F.1) in appendix F. The housing price gap $\widehat{q}_t^u - \widehat{q}_t^{u*}$ is the difference between the housing price $\widehat{q}_t^u = \log q_t^u$ and its efficient welfare-maximizing level $\widehat{q}_t^{u*}$, which is given by[39]

$$q_t^{u*} = \overline{\xi}_t^d, \tag{49}$$

where $\overline{\xi}_t^d \equiv \sum_{T=t}^{\infty} E_t[(1-\delta)^{T-t} \beta^{T-t} \xi_T^d]$.

The policymaker's objective (46) involves the standard terms of squared inflation and the squared output gap, but also depends on the squared housing price gap. The latter arises because any deviation of housing prices from their efficient level distorts – for a given level of the output gap – housing investment, as we explain below. The equilibrium value of the housing price gap will depend on the belief specification and will be discussed in detail in the next two sections.

Constraint (47) is the New Keynesian Phillips Curve and depends on the housing price gap. The coefficients $\kappa_q < 0$ and $\kappa_y > 0$ are defined in Appendix F.3 and imply that positive housing price gaps exert *negative* cost-push effects: high housing prices increase housing investment and – for a given output gap – decrease non-housing consumption. The latter raises the marginal utility of non-housing consumption and thereby depresses wages and marginal costs. This allows the model to potentially produce a non-inflationary boom in housing prices and housing investment. The mark-up disturbance u_t is a function of exogenous disturbances only.

Constraint (48) is the linearized and forward-iterated IS equation. A key new insight here is that the IS equation also depends on the housing price gap. This implies that the housing price gap affects the natural rate of interest, as discussed in detail below. The

[37] Appendix E shows that this is sufficient to insure that subjectively optimal plans satisfy the transversality constraint (29).

[38] See Appendix F for a derivation.

[39] See the derivation in Appendix H.4. All variables in the approximation are expressed in terms of log deviations from the efficient steady state.

coefficients $C_q < 0$ and $C_Y > 0$ are the derivatives of the function $C(\cdot)$ defined in (41) with respect to q^u and Y, respectively, evaluated at the efficient steady state. The variable $r_t^{*,RE}$ in equation (48) denotes the natural interest rate under RE and is a function of exogenous disturbances only.[40] The long-run output gap expectations $\lim_T E_t y_T^{gap}$ in equation (48) are the ones associated with a setting in which agents hold rational housing expectations.[41]

Note, that the policymaker's choice of the nominal interest rate i_t is subject to an effective lower bound $i_t \geq \underline{i}$, where the bound $\underline{i} < 0$ is expressed in terms of deviations from the interest rate in a zero-inflation steady state. For the special case with a zero lower bound, we have $\underline{i} = -(1-\beta)/\beta$. In the absence of a lower bound constraint or when economic shocks never cause the bound to become binding, the IS equation (48) can be dropped from the policy problem.

Interestingly, the expectations showing up in the monetary policy problem (F.11) are all rational. The way subjective housing price expectations affect the monetary policy problem are thus fully captured through their effects on the housing price gap. The next two sections determine the housing price gaps under rational and subjective beliefs and what they imply for optimal policy.

5.1 Rational Housing Price Expectations

With fully rational expectations we have

$$\widehat{q}_t^{u,RE} = \widehat{q}_t^{u*}, \tag{50}$$

which shows that the housing price *gap* is zero at all times, independently of monetary policy and independently of the economic disturbances hitting the economy.[42] Under RE, the Ramsey problem with a lower bound constraint (46) is thus isomorphic to the Ramsey problem in a standard New Keynesian model without a housing sector, as considered for instance in Adam and Billi (2006). This result may appear surprising because monetary policy decisions do affect the housing price in units of consumption $\widehat{q}_t$. Yet, as the policy problem (46) makes clear, it is only the housing price gap in units of marginal utility, $\widehat{q}_t^u - \widehat{q}_t^{u*}$, that is relevant from a welfare perspective. Under RE, the presence of a housing sector thus generates no fundamentally new economic insights into the monetary policy problem.[43]

The RE setup also has difficulties in making a connection between the average natural rate of interest and the volatility of the price-to-rent (PR) ratio. Under RE, the equilibrium PR-ratio is

$$PR_t^{RE} = \frac{q_t^u}{\xi_t^d}, \tag{51}$$

which to a first-order approximation is given by

$$\widehat{PR}_t^{RE} = Z \cdot \widehat{\xi}_t^d, \tag{52}$$

[40]More precisely, $r_t^{*,RE}$ is the real interest rate consistent with the optimal consumption level in a setting with flexible prices and fully rational expectations, see Appendix F.4 for details.

[41]Recall that housing expectations are assumed rational in the long-run in both belief settings.

[42]See Appendix H.1 for proofs on the results about housing prices and the price-rent ratio presented in this section.

[43]The inclusion of a housing sector only affects the definition of the output gap, which now also depends on housing sector disturbances.

with $Z \equiv \beta(1-\delta)(\rho_\xi - 1)/(1-\beta(1-\delta)\rho_\xi)$. Equation (52) shows that the PR-ratio displays persistent variation under RE, if and only if housing demand shocks $\widetilde{\xi}_t^d$ are persistent. In fact, replicating the high quarterly auto-correlation of the PR-ratio in Table 3 requires choosing a shock persistence ρ_ξ very close to one. Yet, in the limit $\rho_\xi \to 1$, the derivative $\partial Z/\partial\beta$ uniformly converges to zero for all $\beta \in [0,1]$. This implies that the volatility of the PR ratio will be largely independent of the natural rate of interest when housing demand shocks are sufficiently persistent. Under RE, there is thus no quantitatively important relationship between the average natural rate of interest and the volatility of the PR ratio, unlike in the case with capital gain extrapolation.

Given equation (50), the IS equation (48) implies that setting

$$i_t - E_t\pi_{t+1} = r_t^{*,RE} \quad \text{for all } t \geq 0 \tag{53}$$

is consistent with a constant output gap, i.e.,

$$y_t^{gap} = \lim_T E_t y_T^{gap} \quad \text{for all } t \geq 0.$$

This justifies our interpretation of $r_t^{*,RE}$ as the natural rate of interest under RE.[44] It also shows that the volatility of the natural rate of interest is independent of the average value of the natural rate under RE. This will cease to be the case under subjective housing beliefs.

5.2 Subjective Housing Price Expectations

This section discusses three new economic forces showing up in the monetary policy problem in the presence of subjective housing price beliefs. It shows (i) how housing price fluctuations are affected by the average level of the natural rate of interest, (ii) how these fluctuations affect the volatility of the natural rate of interest, and (iii) how these fluctuations distort the allocation of output.

Housing prices under subjective beliefs are jointly determined by equations (9) and (10), where q_t should again be replaced by q_t^u. Since these equations do not depend on policy, the policymaker can treat the housing price gap as exogenous, as is the case with RE.[45] Yet, the housing price gap will now generally differ from zero, as the housing price gap can become positive or negative depending on the degree of capital gain optimism/pessimism.

The average natural rate and housing price volatility. With subjective housing price expectations, the equilibrium housing price is given by[46]

$$q_t^{u,\mathcal{P}} = \frac{1}{1-\beta(1-\delta)\beta_t}\xi_t^d \tag{54}$$

[44] In the presence of a lower bound constraint on nominal rates, it might not be feasible to implement (53) at all times.

[45] This does not imply that the housing price q_t is invariant to monetary policy: monetary can determine how variations in q_t^u get split up into variations of the housing price q_t and variations in marginal utility $\widetilde{u}_C(C_t;\xi_t)$.

[46] See Appendix H.1 for a derivation of this and subsequent results, including the generalized expressions for the case with $\rho_\xi < 1$.

and the price-to-rent ratio by

$$PR_t^{\mathcal{P}} = \frac{q_t^{u,\mathcal{P}}}{\xi_t^d}. \tag{55}$$

For the limit with persistent housing demand shocks ($\rho_\xi \to 1$), we can derive the first-order approximation

$$\widehat{q}_t^{u,\mathcal{P}} = \widehat{q}_t^{u,RE} + (\beta_t - 1)\frac{\beta(1-\delta)}{1-\beta(1-\delta)\beta_t}\left(1+\widehat{\xi}_t^d\right), \tag{56}$$

which decomposes the equilibrium housing price into its RE value plus a contribution coming from the presence of subjective beliefs. We then also have

$$E_t^{\mathcal{P}}\left[\widehat{q}_{t+1}^{u,\mathcal{P}}\right] = E_t\left[\widehat{q}_{t+1}^{u,RE}\right] + (\beta_t - 1)\left[1 + \frac{\beta(1-\delta)}{1-\beta(1-\delta)\beta_t}\left(1+\widehat{\xi}_t^d\right)\right], \tag{57}$$

which shows that subjective housing price expectations are equal to their RE equilibrium value whenever expected capital gains are equal to one ($\beta_t = 1$). Capital gain extrapolation, however, will induce fluctuations of β_t around one and thus drive a wedge between the housing price under learning and RE.[47]

As explained for the simple model in Section 3, lower values for the average natural rate (discount factors β closer to one), will induce stronger fluctuations in capital gain expectations (β_t), because housing prices are more sensitive to belief revisions, see equation (54). Lower average values for the natural rates will thus be associated with increased fluctuations in housing prices and the PR-ratio, in line with empirical evidence presented in Section 3.

Housing price fluctuations and the natural rate of interest. The presence of non-zero housing price gaps also affects the natural rate of interest. This can be seen by considering a policy that sets real interest rates equal to the RE natural real rate ($r_t^{*,RE}$). Such a policy now ceases to deliver a constant output gap, instead implies

$$y_t^{gap} = \lim_T E_t y_T^{gap} - \frac{C_q}{C_Y}\left(\widehat{q}_t^u - \widehat{q}_t^{u*}\right). \tag{58}$$

Since $C_q/C_Y < 0$, a positive (negative) housing price gap is then associated with a positive (negative) output gap: high housing prices stimulate housing investment and thereby output. Since the output expansion is inefficient, the policymaker might find it optimal to *lean against housing prices*. The extent to which this is optimal will be explored quantitatively in Section 7 below.

The following lemma derives the natural rate $r_t^{*,\mathcal{P}}$ for our setting with subjective housing price beliefs:[48]

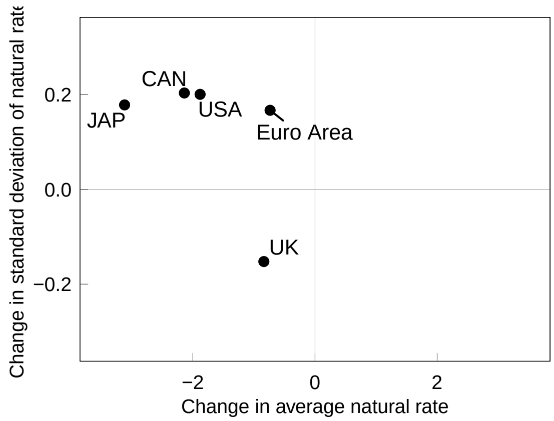
Lemma 1 *Let the natural rate of interest under subjective beliefs be given by*

$$r_t^{*,\mathcal{P}} \equiv r_t^{*,RE} - \frac{1}{\varphi}\frac{C_q}{C_Y}\left(\left(\widehat{q}_t^u - \widehat{q}_t^{u*}\right) - E_t\left(\widehat{q}_{t+1}^u - \widehat{q}_{t+1}^{u*}\right)\right) \qquad \textit{for all } t. \tag{59}$$

[47] In the limit where the Kalman gain ($1/\alpha$) in the updating equation (9) approaches zero, the model with capital gain extrapolation converges to the RE model.

[48] As is the case with RE, it will generally not be optimal (or not even feasible) to set interest rates equal to the natural rate at all times due to the presence of a lower bound constraint on nominal rates

Figure 6: Changes in the average natural rate vs. changes in the volatility of the natural rate



Notes: This figure plots the pre-/post-1990 changes in the average natural rates against the changes in the natural rate volatility for several advanced economies. The volatilities of the natural rates in the pre-/post-1990 periods are the standard deviations of the linearly detrended series.

When real interest rates are equal to $r_t^{*,\mathcal{P}}$ for all $t \geq 0$, then the IS equation (48) is consistent with

$$y_t^{gap} = \lim_T E_t y_T^{gap} \qquad \textit{for all } t. \tag{60}$$

The proof can be found in Appendix H.1. Equation (59) generalizes the natural interest rate definition under RE to a setting with potentially subjective beliefs. In the special case with a constant housing price gap, we have $r_t^{*,\mathcal{P}} = r_t^{*,RE}$, even when the constant housing price gap differs from zero. This shows that the natural rate under subjective beliefs differs from it RE value if and only if the housing price gap is expected to go up or down. Since $C_q/C_Y < 0$, the natural rate will exceed (fall short of) its RE level, when the current housing price gap is higher (lower) than tomorrow's (expected) gap.

Since fluctuations in housing prices become larger when the average natural rate falls, the expected changes in the housing price gap will also become more volatile. A lower average level of the natural rate is thus not only associated with more volatile housing prices but also with more volatile natural rates of interest.

Figure 6 shows that this model prediction is consistent with the data. The figure plots the changes in the average level of the natural rate from the period before 1990 to the period after 1990 on the horizontal axis and the corresponding change in the natural rate *volatility* on the vertical axis. The volatilities of the natural rates in the pre-/post-1990 periods are the standard deviations of the linearly detrended series. The figure is again based on the estimates in Holston, Laubach, and Williams (2017). While the level of the natural rate decreased, the volatility of it increased in four out of the five advanced economies. Appendix G discusses the robustness of these results.

Housing price fluctuations and the misallocation of output. We now show that fluctuations in the housing price gap distort the allocation of output between its alternative uses, i.e., between housing investment and non-housing consumption. The housing investment gap, i.e., the difference between actual investment $\widehat{k}_t$ and its efficient level $\widehat{k}_t^*$, is – to a first-order approximation – given by

$$\widehat{k}_t - \widehat{k}_t^* = \frac{\tilde{\sigma}^{-1} C_Y}{1-\tilde{\alpha}} y_t^{gap} + \frac{1+\tilde{\sigma}^{-1} C_q}{1-\tilde{\alpha}} \left(\widehat{q}_t^u - \widehat{q}_t^{u*}\right). \tag{61}$$

Under rational expectations, the housing price gap is zero and the investment gap is only distorted to the extent that the output gap is not closed. Additional output then gets allocated in constant proportions to housing investment and non-housing consumption, as $(\tilde{\sigma}^{-1} C_Y)/(1-\tilde{\alpha}) > 0$. In the presence of subjective beliefs, however, an additional distortion arises: the housing investment gap is then also driven by the housing price gap. Given the calibration considered later on, we have $(1+\tilde{\sigma}^{-1} C_q)/(1-\tilde{\alpha}) > 0$, so that a positive housing price gap ($\widehat{q}_t^u - \widehat{q}_t^{u*} > 0$) reinforces the investment distortions generated by a positive output gap.[49] This explains why the squared housing price gap shows up in the policymaker's objective function (46). While monetary policy cannot affect the housing price gap within our belief setup, it is the case that larger housing price gap fluctuations, as induced by lower natural rates, contribute to increased welfare losses.

6 Model Calibration

To explore the quantitative implications for monetary policy arising from the presence of capital gain extrapolation, we consider a calibrated model. The calibration strategy consists of choosing a set of standard parameter values previously considered in the literature and of matching salient features of the behavior of natural interest rates and housing prices in the United States in the pre-1990 period. We then test the model by considering its predictions for the lower natural rate levels observed in the post-1990 period up to 2021. We compare across long time spans of 30 years each to obtain more reliable estimates of housing price volatility, which is difficult to estimate given the high degree of serial correlation of housing prices.

Calibration to the pre-1990 period. Table 5 summarizes the model parameterization. The quarterly discount factor β is chosen such that the steady-state natural rate equals the pre-1990 average of the U.S. natural rate of 3.34%, as estimated by Holston, Laubach, and Williams (2017). The interest rate elasticity of output φ, the slope of the Phillips curve κ_y, and the welfare weight $\frac{\Lambda_y}{\Lambda_\pi}$ are taken from Table 2 in Adam and Billi (2006). The Phillips Curve coefficient κ_q and the ratio C_q/C_y are set as in Adam and Woodford (2021).[50]

We now discuss the parameterization of the exogenous shock processes. The persistence of the housing preference shock ρ_ξ is set such that the RE model captures the high serial

[49] This distortion in the allocation of output between housing investment and non-housing consumption is present independently of other frictions such as sticky prices or the lower-bound constraint on nominal interest rates.

[50] The calibration target for the ratio C_q/C_y is the ratio of residential fixed investment over the sum of nonresidential fixed investment and personal consumption expenditure, which is on average approximately equal to 6.3% in the US. This and the remaining parameters then imply $\kappa_q = -0.0023$, see Appendix H.8 for details.

Table 5: Model calibration

Parameter	Value	Source/Target
Preferences and technology		
β	0.9917	Average U.S. natural rate pre 1990
φ	1	Adam and Billi (2006)
κ_y	0.057	Adam and Billi (2006)
$\frac{\Lambda_y}{\Lambda_\pi}$	0.007	Adam and Billi (2006)
κ_q	-0.0023	Adam and Woodford (2021)
$\frac{C_q}{C_Y}$	-0.29633	Adam and Woodford (2021)
δ	0.03/4	Adam and Woodford (2021)
Exogenous shock processes		
ρ_{r^*}	0.8	Adam and Billi (2006)
σ_{r^*}	0.2940% (RE)	Adam and Billi (2006)
	0.1394% (subj. beliefs)	
ρ_ξ	0.99	Quarterly autocorrel. of the PR-ratio of 0.99
σ_{ξ^d}	0.0233 (RE)	Std. dev. of price-to-rent ratio pre 1990
	0.0165 (subj. beliefs)	
Subjective belief parameters		
α	1/0.007	Adam, Marcet and Nicolini (2016)
β^U	1.0031	Max. percentage deviation of PR-ratio from mean

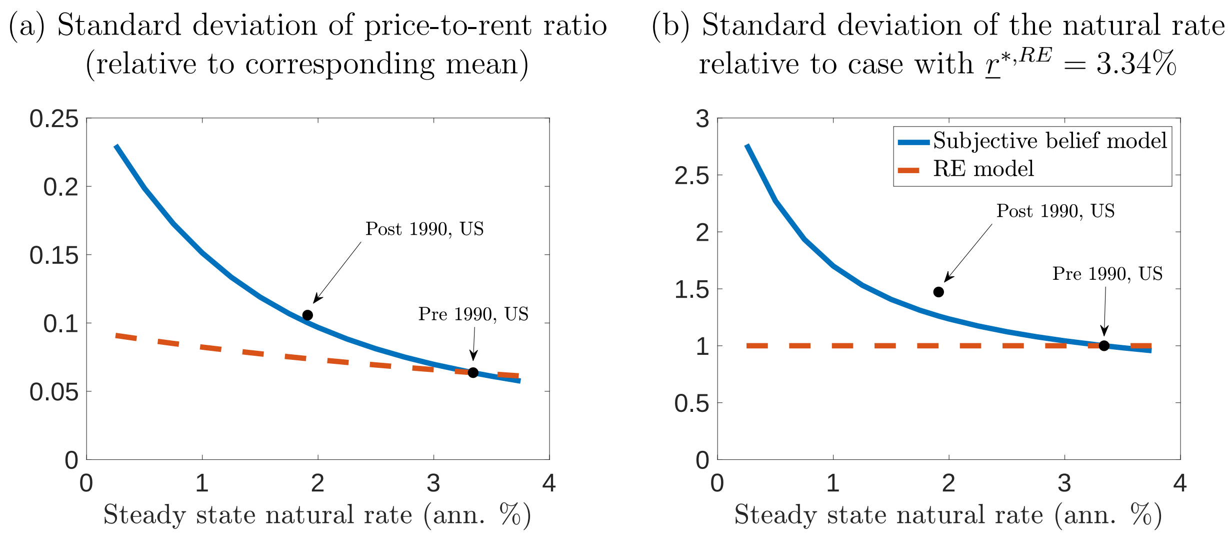
autocorrelation of the PR ratio in the data. The standard deviation of the innovations to the housing preferences σ_ξ are set such that the rational expectations and subjective belief models both replicate the pre-1990 standard deviation of the PR-ratio. For the subjective belief model, this is achieved by simulating equations (9) and (11), which requires specifying the belief updating parameters α and β^U. We set $\alpha = 1/0.007$ following Adam, Marcet, and Nicolini (2016) and determine σ_{ξ^d} and β^U jointly such that (i) we match the volatility of the price-to-rent ratio and (ii) the simulated data matches the maximum deviation of the price-to-rent ratio in the data from its sample mean. The latter statistic identifies β^u. This procedure yields $\beta^U = 1.0031$ and $\sigma_{\xi^d} = 0.0165$. Housing demand disturbances are less volatile than under RE because fluctuations in subjective beliefs contribute to the fluctuations in housing prices. In fact, the calibration implies that about 50% of housing price fluctuations are due to subjective beliefs.

We consider the natural rate process

$$r_t^{*,RE} = \rho_{r*} r_{t-1}^{*,RE} + \varepsilon_t^r, \tag{62}$$

where $\varepsilon_t^r \sim iiN(0, \sigma_{r^*}^2)$. For the RE model, we set ρ_{r^*} and σ_{r^*} equal to the values in Adam and Billi (2006). For the subjective believe model, we use the same value for ρ_{r^*} but choose σ_{r^*} such that the generalized natural rate for the subjective belief model, defined in equation (59), has the same volatility as the natural rate in the RE model. This yields $\sigma_{r^*,RE} = 0.1393\%$, which is lower than under RE, because fluctuations in the housing price gap contribute to fluctuations in the natural rate in the presence of subjective beliefs. To

Figure 7: Standard deviation of price-to-rent ratio and natural rate



Notes: This figure plots, for different steady state levels of the natural rate, the standard deviation of the price-to-rent ratio (relative to its mean) and the standard deviation of the natural rate.

economize on the number of state variables in the policy problem, we abstract from the presence of mark-up shocks.[51]

Evaluation of the model in the post-1990 period. Figure 7 illustrates the predictions of the RE model (dashed line) and subjective belief model (solid line) for the standard deviation of the price-to-rent ratio (panel a) and the standard deviation of the natural rate of interest (panel b). The panels depict these outcomes, which are independent of monetary policy, on the vertical axis for various levels of the steady-state natural rate on the horizontal axis. Variations in the steady-state level of the natural rate are achieved via appropriate variations in the discount factor.[52] The dots in Figure 7 report the average values for the pre- and post-1990 U.S. sample periods, where the average natural rate was equal to 3.34% and 1.91%, respectively.[53]

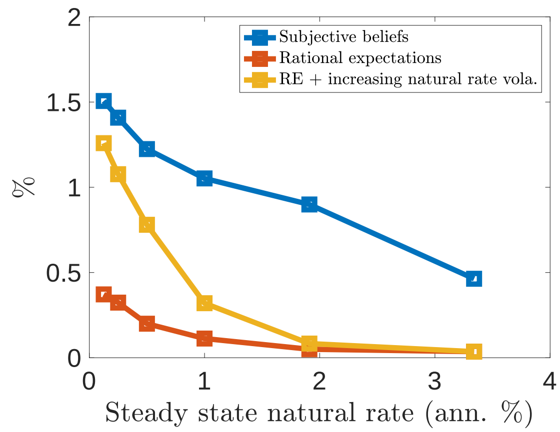
Since the model has been calibrated to the pre-1990 period, the RE and subjective belief model both match the pre-1990 data points in Figure 7. The subjective belief model also performs quite well in matching the post-1990 outcomes, despite the fact that these outcomes are not calibration targets. In particular, the standard deviation of the price-to-rent ratio and the standard deviation of the natural rate endogenously increase as the natural rate falls, with the magnitudes roughly matching the increase observed in the data. In contrast, the RE model produces no increase in the volatility of the natural rate and only a weak increase

[51] Adam and Billi (2006) show that mark-up shocks are too small and display too little persistence to cause the lower-bound constraint to become binding.

[52] As discussed before, variations in the discount factor may be driven by variations in the long-term growth rate and/or by variations in time-preferences.

[53] The reported increase in the standard deviation of the natural rate is again based on the estimates in Holston, Laubach, and Williams (2017).

Figure 8: Average inflation under optimal monetary policy



Notes: The figure reports the optimal inflation target for different average levels of the natural rate in the presence of a zero lower bound constraint. The red line depicts the optimal target for the case with rational housing price beliefs and the blue line the one with subjective housing price beliefs. The yellow line shows the optimal average inflation under RE where the exogenous volatility of the natural rate is adjusted such that it matches the endogenous volatility increase under subjective beliefs.

in the volatility of the price-to-rent ratio, for reasons discussed in Section 5.1. Matching the increase in housing price volatility under RE requires increasing the volatility of housing demand shocks. Since such an increase is irrelevant for monetary policy under RE, we leave the volatility of housing preference shocks unchanged. Similarly, matching the increase in the natural rate volatility under RE would require increasing σ_{r^*}. We will consider such increases when discussing our quantitative results.

7 Quantitative Implications for Monetary Policy

This section illustrates the quantitative implications of falling natural rates and rising housing price volatility for the conduct of optimal monetary policy. It starts by determining the implications of falling natural rates for the optimal inflation target, i.e., for the average inflation rate implied by optimal monetary policy. It then illustrates the dramatically different optimal response to housing demand shocks under subjective and objective housing beliefs. Details of the nonlinear numerical solution procedure underlying the results in this section can be found in Appendix H.6.

7.1 The Optimal Inflation Target

Figure 8 depicts the optimal inflation target for different steady-state levels of the natural rate of interest, i.e., the average inflation rate implied by optimal monetary policy. It shows the optimal target for the setup with subjective housing beliefs (upper line), for the case with rational housing price beliefs (lower line), and for a third case that we discuss below.

We find that the optimal target is close to zero, whenever housing expectations are rational. This holds quite independently of the average level of the natural rate, confirming earlier findings in Adam and Billi (2006) who considered the value for the average natural rate at the upper end of the range shown in Figure 8. This may appear surprising given that it is optimal for monetary policy to promise future inflation, so as to lower real interest rates, whenever adverse natural rate shocks cause the lower-bound constraint on nominal rates to bind. While the lower bound is reached more often when the average natural rate is low, inflation promises still have to be made relatively infrequently and can be quite modest. Hence, they do not significantly affect the average rate of inflation.

This result differs quite substantially from the ones reported in Andrade, Galí, Le Bihan, and Matheron (2019), who find that the optimal target should move up approximately one-to-one with a fall in the natural rate under rational expectations. Besides that Andrade, Galí, Le Bihan, and Matheron (2019) consider a medium-scale sticky price model without housing, the main difference to our approach is that they study Taylor rules with optimized intercepts rather than optimal monetary policy. As shown in Coibion, Gorodnichenko, and Wieland (2012) it makes a big difference for the optimal inflation target whether the monetary policy maker follows a Taylor rule or Ramsey optimal policy.

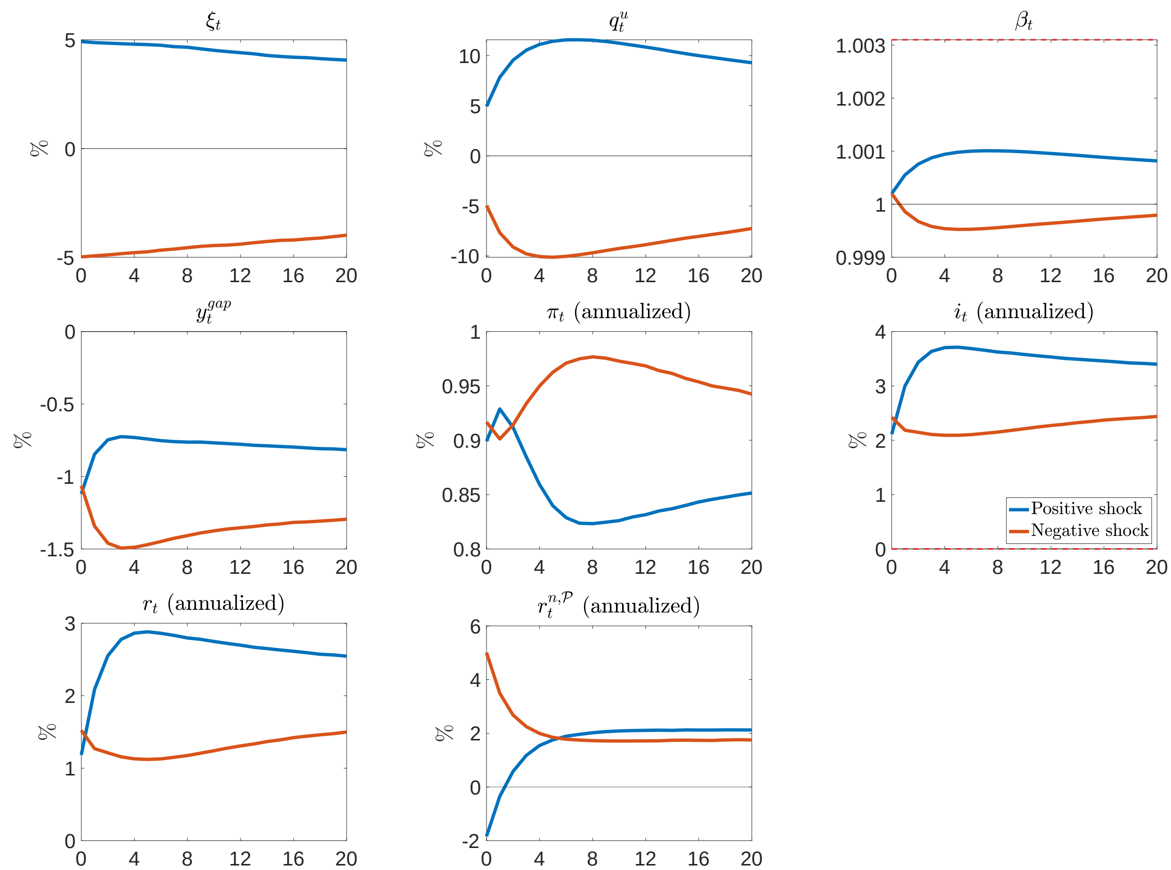
While lower natural rates trigger (slightly) larger housing price fluctuations under rational expectations, increased volatility is fully efficient and does not affect the natural rate of interest. Under rational expectations, the optimal inflation target is thus unaffected by housing price fluctuations, including for very low levels of the natural rate.

The upper line in Figure 8 shows that the situation is quite different with subjective housing beliefs. The optimal inflation target is overall substantially higher and also reacts more strongly to a fall in the average natural rate of interest. In fact, a fall in the steady-state natural rate from its pre-1990 average (3.34%) to its post-1990 average (1.9%) causes the optimal inflation target to increase by almost 0.5%. The corresponding increase under rational expectations is less than 0.05%. This difference is due to the fact that the endogenous volatility component of the natural rate increases once the natural rate drops. This reinforces the stringency of the zero lower bound, but is an effect that is absent under RE. It requires that the central bank engages more often in inflation promises, as it faces the lower bound constraint.

The optimal inflation target with subjective housing beliefs is substantially higher than the optimal target with RE, even at the pre-1990 average level of the natural rate. This is the case although the volatility of the natural rate is calibrated at this point to be equal across both models. This is due to two reasons: First, fluctuations in the housing price gap also generate cost-push term in the Phillips curve. Second, belief fluctuations induce more persistent variations in the natural rate than the exogenous natural rate shocks. This puts further upward pressure on the optimal inflation rate, as it requires larger and more persistent inflation promises by the central bank.

To illustrate this last point, the middle line in Figure 8 depicts the optimal inflation rate under rational expectations, when we set the volatility of the (exogenous) natural rate in the RE model such that it matches the volatility of the natural rate in the subjective belief model, for each considered level of the natural rate. While the optimal inflation rate increases relative to the benchmark RE setting, the level of the optimal inflation target still falls short of the one implied by subjective beliefs.

Figure 9: Impulse responses to a housing preference shock



Notes: The figure reports the average impulse responses of the economy under subjective beliefs (at $\underline{r}^* = 1.91\%$) after a three-standard-deviation housing demand shock. The blue lines show the responses after a positive shock and the red lines after a negative shock.

7.2 Leaning Against Housing Demand Shocks

We now examine the optimal monetary policy response to housing demand shocks. Under RE, housing demand shocks affect the housing price and the efficient housing price identically, so that the housing price gap remains at zero. As a result, neither the output gap nor inflation respond to housing demand shocks. In contrast, it becomes optimal to "lean against" housing demand shocks in the presence of subjective beliefs. Yet, due to the lower bound constraint, the optimal response to positive and negative housing demand shocks displays considerable asymmetry.

The top row in Figure 9 shows the response of housing-related variables to a persistent positive/negative housing demand shock of 5%.[54] On impact, the shock triggers capital gains of an equal amount, which then trigger belief revisions that fuel further upward movements

[54]We initialize the economy at its ergodic mean and then hit the economy with a one-time shock of three standard deviations. We then average the subsequent response over the possible future shock realizations. We assume a steady-state natural rate equal to its post-1990 mean (1.91%).

of the housing price in the same direction. The positive shock, for instance, pushes housing prices up by about 5% on impact, with belief momentum generating approximately another 5% in the first six quarters after the shock. This causes the housing price gap to become significantly positive (not shown in the figure). Once actual housing price increases start to fall short of the expected housing price increases, the housing boom reverts direction.

Higher housing prices push up housing investment, which causes upward pressure on the output gap. Optimal monetary policy leans strongly against the housing price and increases nominal and real interest rates. It does so despite the fact that the natural rate of interest falls in response to the shock. The policy response causes a fall in inflation, which is amplified by the fact that the increase in housing prices and investment increases the marginal utility of consumption, hence, dampens wages and marginal costs. A positive housing demand shock thus results – in the presence of subjective housing beliefs – in a disinflationary housing boom episode under optimal monetary policy.

The policy response to a positive housing demand shock is much stronger than that to a negative housing demand shock. In particular, nominal and real interest rates fall considerably less following a negative shock realization. This is so because a negative housing price gap is inflationary and inflation is already high to start with. Negative housing demand shocks thus move inflation further away from its optimal level of zero.[55] Yet, policy still "leans against" the housing price decrease: real interest rates fall despite the fact that the natural rate increases.

The fact that leaning against housing prices can be optimal in the presence of capital gain extrapolation is in line with results in Caines and Winkler (2021), who consider a setting with 'conditionally model consistent beliefs' in which expectations differ for many variables from rational expectations, and with results in Adam and Woodford (2021), who consider a setting where the policymaker fears 'worst-case' belief distortions about inflation and housing price expectations. As none of these papers consider a lower-bound constraint, the policy response to positive and negative shocks is symmetric in their settings.

8 The Role of Macroprudential Policy

It is often argued that macroprudential policies can be used to stabilize financial markets and that this would allow monetary policy to ignore disturbances coming from the housing sector, see Svensson (2018) for a prominent exposition of this view. In this section, we evaluate the quantitative plausibility of this view within our setup with subjective housing beliefs.

We show below that fully eliminating fluctuations in the housing price gap requires imposing large and volatile macroprudential taxes. None of the macroprudential instruments thus far available in advanced economies appear suited to achieve economic effects anywhere near the required size. In addition, it is often necessary for macroprudential policy to pay substantial subsidies. To the best of our knowledge, none of the available macroprudential instruments acts in a way that subsidizes actions by economic actors. Less aggressive policies, that aim at only partly eliminating the housing

[55]While the output gap is moved closer to its optimal level, the weight on the output gap in the welfare function is two orders so magnitude smaller than that on inflation, see Table 5.

price gap, still require considerable tax volatility, because fluctuations in subjective beliefs turn out not to be independent of tax policy pursued.

We analyze the issue by considering a setup in which the policymaker can tax or subsidize the ownership of housing. While actual macroprudential policies often operate via constraints imposed on the banking sector, their ultimate effect is to make housing more or less expensive to households. For this reason, we consider taxes and subsidies at the household level.

Specifically, we analyze a proportional and time-varying tax τ_t^D that is applied to the rental value of housing in every period t. A household owning D_t units of houses, then has to pay taxes of

$$\tau_t^D D_t R_t \tag{63}$$

units of consumption.[56] We find this specification more plausible than a policy that taxes the market value of housing, as it is difficult to determine market values in real time. A setup that taxes the physical housing units, i.e., where taxes are equal to $\tau_t^D D_t$, delivers very similar results, but is analytically more cumbersome. Furthermore, the tax setup in equation (63) is equivalent to a setup where taxes directly affect household utility, i.e., where the utility contribution from owning houses would instead be given by $\xi_t^d \left(1-\tau_t^D\right) D_t$ and no monetary taxes would have to be paid. We prefer the formulation in equation (63) because it allows expressing taxes in monetary units.

In the presence of these taxes, housing prices under subjective beliefs are given by

$$q_t^u = \frac{\left(1-\tau_t^D\right)\xi_t^d}{1-\beta(1-\delta)\beta_t}, \tag{64}$$

and the housing-price *gap* in percentage deviations from the steady state (where $\underline{\tau}^D = 0$) is

$$\widehat{q}_t^u - \widehat{q}_t^{u*} = \frac{(1-\beta(1-\delta))\left(1-\tau_t^D\right)\widehat{\xi}_t^d}{1-\beta(1-\delta)\beta_t} + \frac{\beta(1-\delta)(\beta_t-1)}{1-\beta(1-\delta)\beta_t} - \frac{1-\beta(1-\delta)}{1-\beta(1-\delta)\beta_t}\tau_t^D - \frac{1-\beta(1-\delta)}{1-\beta(1-\delta)\rho_\xi}\widehat{\xi}_t^d.$$

The previous equation shows that macroprudential policy must eliminate housing price gap fluctuations that are due to housing demand shocks ($\widehat{\xi}_t^d$) and due to fluctuations in subjective capital gain expectations (β_t). Doing so requires setting the tax according to

$$\tau_t^{D*} = \frac{\beta(1-\delta)}{1+\widehat{\xi}_t^d}\left[\frac{(\beta_t-\rho_\xi)}{1-\beta(1-\delta)\rho_\xi}\widehat{\xi}_t^d + \frac{1}{1-\beta(1-\delta)}(\beta_t-1)\right]. \tag{65}$$

To understand what the preceding equation implies for the behavior of taxes, one has to take into account that the fluctuations in subjective beliefs (β_t) depend themselves on the tax: the tax influences housing prices, see equation (64), and thus – via capital gain extrapolation – the evolution of subjective beliefs.

To analyze the behavior of taxes, we consider the calibrated subjective belief model from Section 6 for the case where the average natural rate is equal to its post-1990 average

[56] To keep the rest of the model unchanged, the household also needs to expect lump sum tax rebates that are equal to the amount of subjectively expected tax payments.

Table 6: Taxes and housing price gap fluctuations for alternative tax sensitivities λ^D

Tax sensitivity λ^D	Housing Price Gap $\widehat{q}_t^u - \widehat{q}_t^{u*}$	Housing Taxes τ_t^D		
Value	Std. dev.	Std. dev.	Maximum	Minimum
0.0	14.2%	0.0%	0.0%	0.0%
0.2	9.8%	2.4%	7.0%	-12.1%
0.4	6.4%	4.2%	13.8%	-21.7%
0.6	3.7%	5.7%	18.0%	-30.0%
0.8	1.7%	7.0%	21.3%	-36.2%
1.0	0.0%	8.0%	23.9%	-41.8%

Notes: The table reports the standard deviation of the housing gap, $\widehat{q}_t^u - \widehat{q}_t^{u*}$, as well as the standard deviation, minimum value and maximum value of the macroprudential tax τ^D, for different tax sensitivities λ^D.

(1.9%). We consider also intermediate forms of taxation that do not aim at fully eliminating the housing gap, by specifying taxes as

$$\tau_t^D = \lambda^D \tau_t^{D*},$$

where $\lambda^D \in [0, 1]$ is a sensitivity parameter. Our prior setup assumed $\lambda^D = 0$, while fully eliminating the housing price gap using macroprudential policy requires setting $\lambda^D = 1$. We then simulate the dynamics of housing prices, beliefs and taxes for alternative values of λ^D.

Table 6 reports the main outcomes. It shows that a higher tax sensitivity (λ^D) steadily reduces the standard deviation of the housing price gap (second column). However, the standard deviation of taxes has to steadily increase. For a policy that fully eliminates the housing price gap ($\lambda^D = 1$), the standard deviation of taxes is a staggering 8% of the rental value of housing. Taxes reach maximum values up to 24% and minimum values deeply in negative territory, with subsidies above 40% of the rental value. These taxes fully stabilize the housing price gap but still induce substantial variation in subjective beliefs. The latter explains why taxes have to remain rather volatile. Intermediate policies, say ones that set $\lambda^D = 0.4$, substantially reduce the volatility of the housing gap, but still require rather volatile taxes and often very large subsidies.

Given the outcomes in Table 6, we conclude that the currently available macroprudential instruments will unlikely be able to insulate the monetary authority from disturbances in the housing sector arising from capital gain extrapolation.

9 Conclusions

This paper documents systematic deviations from rational housing price expectations and constructs a structural equilibrium model that jointly replicates the behavior of housing prices and the patterns of deviations from rational expectations. The model shows that subjective housing price beliefs significantly contribute to housing price fluctuations and that lower natural rates of interest generate increased volatility for housing prices and the natural rate.

Optimal monetary policy responds to falling and more volatile natural rates by implementing higher average rates of inflation. Monetary policy should also lean against housing price fluctuations induced by housing demand shocks, with reactions to housing price increases being more forceful than the reaction to housing price downturns. None of these features is optimal if households hold rational housing price expectations. This highlights the importance of basing policy advice on economic models featuring empirically plausible specifications for household beliefs.

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A Additional Results for Section 2

A.1 Five-Year-Ahead Capital Gain Expectations

While for our baseline results in Section 2 we focus on short-term housing price expectations, our findings equally hold for medium-term five-year-ahead expectations. We estimate the five-year analogue of regression (1) as follows:

$$q_{t+20} - E_t^{\mathcal{P}}[q_{t+20}] = a^{CG} + b^{CG} \cdot \left(E_t^{\mathcal{P}}[q_{t+20}] - E_{t-1}^{\mathcal{P}}[q_{t+19}]\right) + \varepsilon_t. \tag{A.1}$$

Table A.1 reports the estimates of b^{CG} showing that five-year expectations are updated sluggishly.

Table A.1: Sluggish adjustment of five-year-ahead housing price expectations

	Mean Expectations	Median Expectations
$\widehat{b}^{CG}$	6.95***	6.89***
	(1.703)	(1.680)

Notes: This table reports the empirical estimates of regression (A.1) using nominal housing-price expectations. The reported standard errors are robust with respect to heteroskedasticity and serial correlation (Newey-West with four lags). Significance levels: *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

We also run five-year-ahead versions of the regressions (2) and (3):

$$E_t^{\mathcal{P}}\left[\frac{q_{t+20}}{q_t}\right] = a + c \cdot PR_{t-1} + u_t \tag{A.2}$$

$$\frac{q_{t+20}}{q_t} = \mathbf{a} + \mathbf{c} \cdot PR_{t-1} + \mathbf{u}_t. \tag{A.3}$$

Table A.2 shows that five-year-ahead capital gain expectations covary positively with the price-to-rent ratio, whereas actual capital gains covary negatively.

Table A.2: Expected vs. actual capital gains using five-year-ahead housing price expectations

	$\hat{c}$ (in %)	$\hat{\mathbf{c}}$ (in %)	bias (in %) $-E(\hat{\mathbf{c}} - \hat{c})$	p-value $H_0: c = \mathbf{c}$
Mean Expectations	0.045	-1.889	0.0159	0.000
	(0.0001)	(0.01997)		
Median Expectations	0.044	-1.889	0.0155	0.000
	(0.00024)	(0.01997)		

Notes: $\widehat{c}$ is the estimate of c in equation (A.2) and $\widehat{\mathbf{c}}$ the estimate of $\mathbf{c}$ in equation (A.3). The Stambaugh (1999) small sample bias correction is reported in the second-to-last column and the last column reports the p-values for the null hypothesis $c = \mathbf{c}$. Newey-West standard errors using four lags in parentheses.

A.2 IV Estimation of Sluggish Belief Updating

To insure that the results obtained from regression (1) in Section 2 are not driven by forecast revisions being correlated with the error term, we follow Coibion and Gorodnichenko (2015) by adopting an Instrumental Variable approach. Specifically, we consider monetary policy shocks as an instrument for forecast revisions. We identify daily monetary policy shocks as changes of the current-month federal funds future in a 30-minute window around scheduled FOMC announcements (following the approach in Gürkaynak, Sack, and Swanson (2005) and Gorodnichenko and Weber (2016)). We then aggregate shocks to quarterly frequency by assigning daily shocks partly to the current quarter and partly to the consecutive quarter, based on the number of remaining days in the current quarter. Table A.3 reports the results of the IV regression. The coefficients are positive and statistically significant, with point estimates that are even larger than the ones reported in Section 2.

Table A.3: Instrumental variable regression

	Mean Expectations	Median Expectations
Nominal Housing Prices		
$\widehat{b}^{CG}$	2.85**	3.84***
	(1.259)	(1.497)
First-stage F-statistic	21.88	17.78
Real Housing Prices		
$\widehat{b}^{CG}$	2.62***	3.45***
	(0.745)	(0.649)
First-stage F-statistic	44.49	34.13

Notes: $\widehat{b}^{CG}$ report the results from regression (1), instrumenting forecast revisions using monetary policy shocks, obtained via high-frequency identification. Significance levels: *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

A.3 Sluggish Adjustment of Capital Gain Expectations

Regression (1) in Section 2 studies sluggish adjustment of expectations about the housing price level. Similar results can be obtained when considering expectations about capital gains. *Specification 1* in Table A.4 reports the regression coefficient when one replaces actual and expected housing price levels on the left-hand side of equation (1) with actual and expected capital gains. The coefficient estimates remain positive and highly statistically significant. *Specification 2* in Table A.4 reports results when replacing expectations about housing price levels with expectations about capital gains on the right-hand side of equation (1) and *Specification 3* reports results when replacing levels by (actual and expected) capital gains on both sides of equation (1). The coefficient estimates remain positive, but the significance levels are lower for Specifications 2 and 3.

Table A.4: Sluggish adjustment of housing price growth expectations

	Mean Expectations	Median Expectations
Specification 1		
Nominal Housing Prices		
$\widehat{b}^{CG}$	0.023***	0.030***
	(0.005)	(0.005)
Real Housing Prices		
$\widehat{b}^{CG}$	0.024***	0.031***
	(0.004)	(0.004)
Specification 2		
Nominal Housing Prices		
$\widehat{b}^{CG}$	492*	182
	(279)	(210)
Real Housing Prices		
$\widehat{b}^{CG}$	302*	158
	(164)	(168)
Specification 3		
Nominal Housing Prices		
$\widehat{b}^{CG}$	5.20*	2.16
	(2.896)	(2.06)
Real Housing Prices		
$\widehat{b}^{CG}$	3.23*	2.06
	(1.678)	(1.835)

Notes: This table shows the results of regression (1) in terms of house-price growth rates instead of house-price levels. *Specification 1* denotes the case in which we replace housing-price levels with capital gains on the left-hand side of regression (1), *Specification 2* the case in which we replace the right-hand side and *Specification 3* denotes the case in which we replace levels with capital gains on both sides of regression (1). The reported standard errors are robust with respect to heteroskedasticity and serial correlation (Newey-West with four lags). Significance levels: *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

A.4 Cyclicality of Housing Price Forecast Errors

A similar version of the test from Adam, Marcet, and Beutel (2017) presented in Section 2, which considers the cyclicality of expected gains, is proposed by Kohlhas and Walther (2021). In this case, we regress forecast errors about housing prices on the price-to-rent ratio. Formally, we estimate

$$\frac{q_{t+4}}{q_t} - E_t^{\mathcal{P}}\left[\frac{q_{t+4}}{q_t}\right] = \alpha + \gamma \cdot PR_{t-1} + \varepsilon_t. \tag{A.4}$$

Table A.5 shows the results. We find a negative and statistically significant coefficient in all cases. Thus, consumers tend to become too optimistic (pessimistic) when they observe high (low) housing valuations, inconsistent with rational expectations.

Table A.5: Forecast errors and price-to-rent ratios

	Mean Expectations	Median Expectations
Nominal Housing Prices		
$\widehat{\gamma}$	-0.5^{***}	-0.5^{***}
	(0.09)	(0.10)
Real Housing Prices		
$\widehat{\gamma}$	-0.5^{***}	-0.5^{***}
	(0.08)	(0.10)

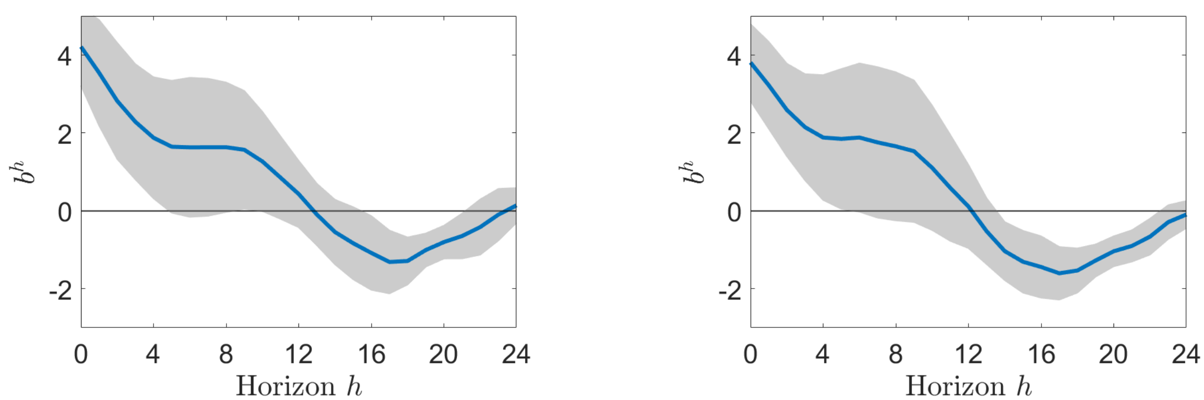
Notes: This table shows the results of regression (A.4), whereas the estimated regression coefficients (and standard errors) are multiplied by one hundred for better readability. The reported standard errors are robust with respect to heteroskedasticity and serial correlation (Newey-West with four lags). Significance levels: *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

A.5 Dynamics of forecast errors with median and nominal housing price expectations

Figure A.1 shows alternative specifications of the dynamic forecast error responses presented in Section 2. Panel (a) presents the response of forecast errors for nominal housing prices. Panel (b) shows the response of forecast errors for real housing prices (as in Section 2) but considering median expectations. The figure shows that these responses are very close to the baseline specification shown in Section 2.

Figure A.1: Dynamic Forecast error response to realized capital gains

(a) Nominal (Mean) Capital Gain Expectations (b) Median (Real) Capital Gain Expectations



Notes: Panel (a) shows impulse-response functions of nominal capital gain forecast errors to a one standard deviation innovation in the housing capital gain. Panel (b) shows the impulse-response functions of median (real) capital gain forecast errors of one-year ahead expectations to a one standard deviation innovation in the housing capital gain. The shaded area shows the 90%-confidence intervals, standard errors are robust with respect to serial correlation and heteroskedasticity (Newey-West with $h + 1$ lags).

A.6 Results when Excluding the Corona Virus Period

The empirical results reported in Section 2 are based on the entire period for which household-survey expectations are available, i.e., 2007-2021. This section reports results obtained when ending the sample in 2019, thereby excluding the recent Corona Virus crisis period. This is motivated by the fact that the two largest outliers in Figure 2 fall into the period after 2019. Tables A.6 and A.7 show, however, that our results are qualitatively and quantitatively robust to excluding observations from the years 2020 and 2021.

Table A.6: Sluggish adjustment of housing price expectations: excluding coronavirus crisis

	Mean Expectations	Median Expectations
Nominal Housing Prices		
$\widehat{b}^{CG}$	2.18***	2.80***
	(0.503)	(0.502)
Real Housing Prices		
$\widehat{b}^{CG}$	1.97***	2.43***
	(0.332)	(0.360)

Notes: This table shows the results of regression (1) excluding the coronavirus crisis, i.e., we exclude the years 2020 and 2021. The reported standard errors are robust with respect to heteroskedasticity and serial correlation (Newey-West with four lags). Significance levels: *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

Table A.7: Expected vs. actual capital gains: excluding coronavirus crisis

	$\hat{c}$ (in %)	$\hat{\mathbf{c}}$ (in %)	bias (in %) $-E(\hat{\mathbf{c}} - \hat{c})$	p-value $H_0 : c = \mathbf{c}$
Nominal Housing Prices				
Mean Expectations	0.058	-0.065	0.0036	0.000
	(0.0066)	(0.0126)		
Median Expectations	0.018	-0.065	0.0118	0.042
	(0.0010)	(0.0126)		
Real Housing Prices				
Mean Expectations	0.0614	-0.0483	-0.0009	0.000
	(0.0136)	(0.0090)		
Median Expectations	0.196	-0.483	0.076	0.017
	(0.0034)	(0.0090)		

Notes: This table shows the results of regressions (2) and (3) excluding the coronavirus period, i.e., we exclude the years 2020 and 2021. $\widehat{c}$ is the estimate of c in equation (2) and $\widehat{\mathbf{c}}$ the estimate of $\mathbf{c}$ in equation (3). The small sample bias correction is reported in the second to last column and the last column reports the p-values for the null hypothesis $c = \mathbf{c}$ in the fifth column. Newey-West standard errors using four lags in parentheses.

A.7 Regional Housing Prices and Expectations

This appendix considers regional variation in housing prices and housing price expectations. This is possible because the Michigan Survey reports the location of respondents using four different regions: West, North East, North Central (or Midwest) and South. While the Case-Shiller Price Index is not available at this level of regional disaggregation, we can construct a regional housing price index using the Case-Shiller Index that is available for twenty large U.S. cities. Following the definition of the regions in the Michigan Survey, we assign the twenty cities to the four regions and then aggregate city price indices to a regional index using two alternative approaches. The first approach weighs cities by population (as of 2019) within each region, while the second approach uses equal weights for all cities within a region.

Table A.8 lists all twenty cities, the region to which we allocate them and their regional population weights.[57] As in our baseline approach using aggregate data, we deflate nominal housing price indices by the aggregate CPI to obtain a real housing price index. We obtain real housing price expectations by deflating nominal (mean) expectations with region-specific (mean) inflation expectations.

Table A.8: Regions, cities and their weights

City	Region	Weight	City	Region	Weight
Denver	West	$\frac{0.705}{10.595}$	Chicago	North Central	$\frac{2.71}{4.189}$
Las Vegas	West	$\frac{0.634}{10.595}$	Cleveland	North Central	$\frac{0.385}{4.189}$
Los Angeles	West	$\frac{3.97}{10.595}$	Detroit	North Central	$\frac{0.674}{4.189}$
Phoenix	West	$\frac{1.633}{10.595}$	Minneapolis	North Central	$\frac{0.42}{4.189}$
Portland	West	$\frac{0.645}{10.595}$	Atlanta	South	$\frac{0.488}{4.209}$
San Diego	West	$\frac{1.41}{10.595}$	Charlotte	South	$\frac{0.857}{4.209}$
San Francisco	West	$\frac{0.874}{10.595}$	Dallas	South	$\frac{1.331}{4.209}$
Seattle	West	$\frac{0.724}{10.595}$	Miami	South	$\frac{0.454}{4.209}$
Boston	North East	$\frac{0.68}{9.1}$	Tampa	South	$\frac{0.387}{4.209}$
New York	North East	$\frac{8.42}{9.1}$	Washington DC	South	$\frac{0.692}{4.209}$

Notes: This table lists the twenty cities for which the Case-Shiller Home Price Index is available, the region to which the cities are allocated based on the Michigan Survey and their respective weights within region.

Table A.9 reports the region-specific estimates of b^{CG} in regression equation (1). All point estimates are positive with magnitudes that are broadly in line with the estimates at the national level. Furthermore, all regional estimates are significant at the 1% level. This shows that households update expectations sluggishly in all regions, consistent with the findings reported for the national level reported in the main text.

Table A.10 reports the region-specific estimates of c and $\mathbf{c}$ from regressions (2) and (3). Since regional price-to-rent ratios are not available, the regression uses real housing prices

[57] The weights are calculated as the ratio of the population in the considered city, divided by the sum of populations in all cities in the respective region.

Table A.9: Sluggish adjustment of housing price expectations across regions crisis

	Weighted	Unweighted
$\widehat{b}^{CG,W}$	2.00***	1.95***
	(0.411)	(0.374)
$\widehat{b}^{CG,NE}$	1.24***	1.15***
	(0.385)	(0.441)
$\widehat{b}^{CG,NC}$	1.97***	1.95***
	(0.461)	(0.459)
$\widehat{b}^{CG,S}$	1.74***	1.94***
	(0.385)	(0.393)

Notes: This table shows the results of regression (1) using regional housing prices and expectations. The superscripts W, NE, NC and S denote the regions West, North East, North Central (or Midwest) and South, respectively. The reported standard errors are robust with respect to heteroskedasticity and serial correlation (Newey-West with four lags). Significance levels: *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

on the right-hand side. In line with our findings at the aggregate level, we find $c > 0$ and $\mathbf{c} < \mathbf{0}$ in all the regions with the differences being largely highly statistically significant.

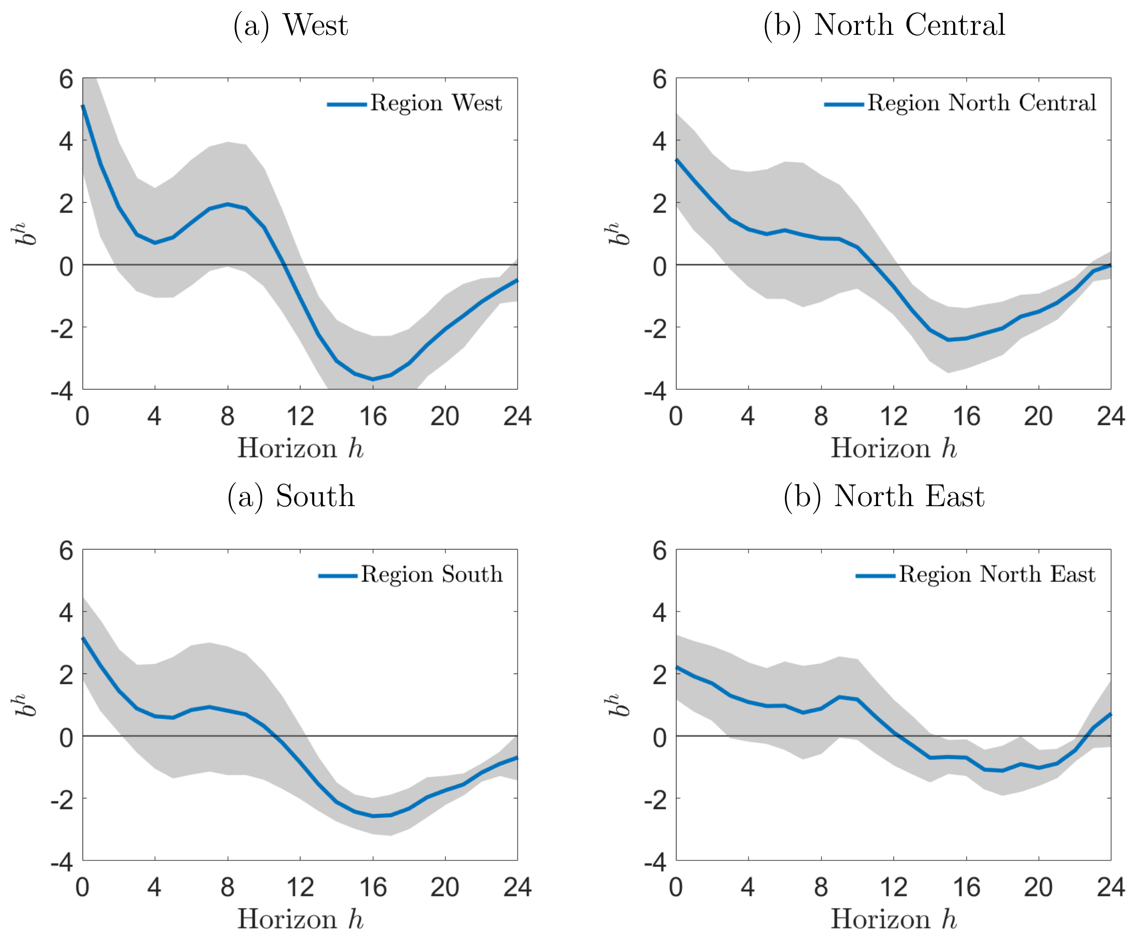
Figure A.2 shows the dynamic forecast errors responses to a one standard deviation innovation in the real housing capital gain in each of the four regions. Figure A.3 shows the results for the case in which the cities within regions are equally weighted. In line with the findings reported in the main text, households' housing capital gain expectations initially underreact but overshoot after some time.

Table A.10: Expected vs. actual capital gains across regions

	$\hat{c}$ (in %)	$\hat{\mathbf{c}}$ (in %)	bias (in %) $-E(\hat{\mathbf{c}} - \hat{c})$	p-value $H_0 : c = \mathbf{c}$
West				
Population-weighted	0.109	-0.216	0.090	0.083
	(0.0036)	(0.1360)		
Equally weighted	0.132	-0132	0.137	0.183
	(0.0034)	(0.1197)		
North Central				
Population-weighted	0.045	-0.544	0.008	0.000
	(0.0089)	(0.0256)		
Equally weighted	0.088	-0.458	0.0191	0.000
	(0.0118)	(0.0769)		
North East				
Population-weighted	0.013	-0.474	0.001	0.000
	(0.0089)	(0.0072)		
Equally weighted	0.126	-0.315	0.023	0.000
	(0.0187)	(0.0838)		
South				
Population-weighted	0.210	-0.008	0.137	0.144
	(0.0023)	(0.1067)		
Equally weighted	0.163	-0.238	0.055	0.014
	(0.0044)	(0.1250)		

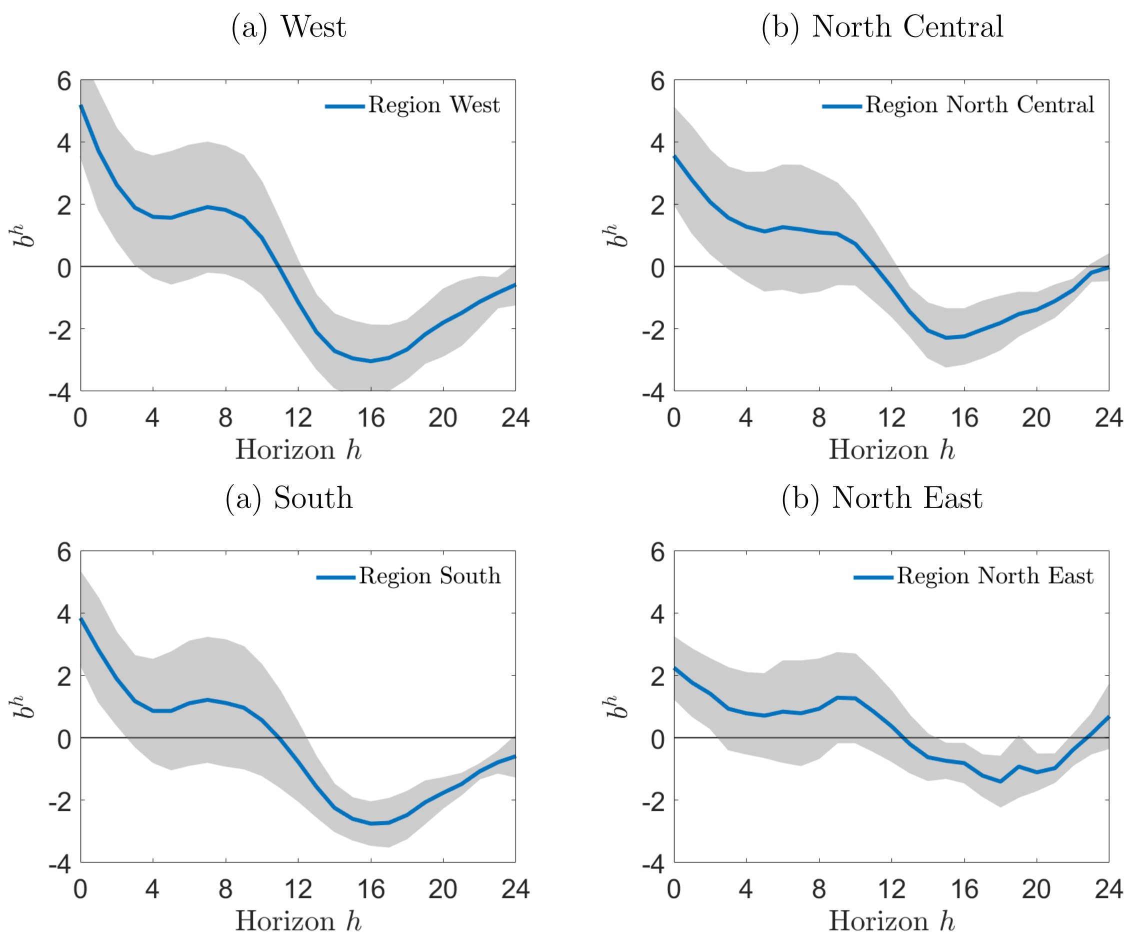
Notes: This table shows the results of regressions (2) and (3) for different regions. $\widehat{c}$ is the estimate of c in equation (2) and $\widehat{\mathbf{c}}$ the estimate of $\mathbf{c}$ in equation (3). The small sample bias correction is reported in the second to last column and the last column reports the p-values for the null hypothesis $c = \mathbf{c}$ in the fifth column. Newey-West standard errors using four lags in parentheses.

Figure A.2: Regional dynamic forecast error responses to realized capital gains (population-weighted city housing price index)



Notes: The figure shows the dynamic response of real capital gain forecast errors across the four different regions (in which cities' housing indices are weighted by their population share) to a one standard deviation innovation in the housing capital gain. The shaded area shows the 90%-confidence intervals, standard errors are robust with respect to serial correlation and heteroskedasticity (Newey-West with $h + 1$ lags).

Figure A.3: Regional dynamic forecast error response to realized capital gains (equal weights)



Notes: The figure shows the dynamic response of real capital gain forecast errors across the four different regions (in which cities are equally weighted) to a one standard deviation innovation in the housing capital gain. The shaded area shows the 90%-confidence intervals, standard errors are robust with respect to serial correlation and heteroskedasticity (Newey-West with $h + 1$ lags).

B Additional Results for Section 3

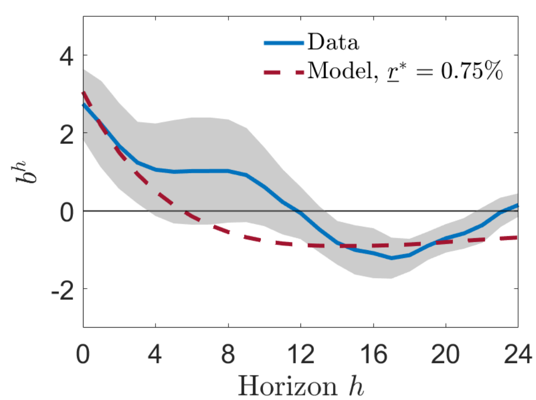
B.1 Dynamic Forecast Error Responses: Housing Price Level

Figure B.1 shows that the simple housing model also not only matches the empirical dynamic forecast error response about capital gains well, but also does a good job in matching the forecast errors about the level of future housing prices. The results are obtained by defining the forecast error X_{t+h} in equation (4) as

$$X_{t+h} \equiv q_{t+4+h} - E^{\mathcal{P}}_{t+h}\left[q_{t+4+h}\right] \tag{B.1}$$

and estimating the resulting local projections in the data and the population local projection for the model. Figure B.1 shows that households' expectations about the future level of housing prices initially undershoot and subsequently overshoot, as is the case with expected capital gains.

Figure B.1: Dynamic forecast error responses: housing price levels

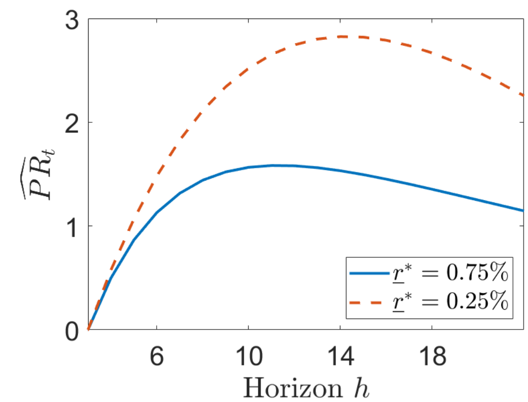


Notes: The figure shows impulse-response functions of housing-price level forecast errors of one-year ahead expectations to a one standard deviation innovation in the housing capital gain from the data and in the data. The shaded area shows the 90%-confidence intervals of the empirical estimates, standard errors are robust with respect to serial correlation and heteroskedasticity (Newey-West with $h+1$ lags).

B.2 Model Response of the PR-Ratio to Housing Demand Shocks

Section 3 shows that real housing prices are more sensitive to housing demand shocks at lower levels of the natural rate. Figure B.2 illustrates that the same holds true for the model-implied price-to-rent ratio. The figure depicts the structural impulse response of the price-to-rent ratio (in percent deviations from steady state) to a one standard deviation housing-preference shock. It shows the response for a natural rate of 0.75% (blue line) and 0.25% (red line). The IRFs for the price-to-rent ration look very similar to the ones for real housing prices, shown in Figure 4(a).

Figure B.2: Impulse response functions



Notes: This figure shows the structural impulse response functions of the price-to-rent ratio (in percent deviations from steady state) to a one standard deviation housing-preference shock for different natural rates.

C The Nonlinear Optimal Policy Problem

We shall consider Ramsey optimal policies in which the policymaker chooses the sequence of policy rates, prices, and allocations to maximize rationally expected household utility, subject to the constraint that prices and allocations constitute an Internally Rational Expectations Equilibrium. Note that the policymaker maximizes utility under a probability measure that is different from the one entertained by households, whenever the latter hold subjective beliefs. Benigno and Paciello (2014) refer to such a policymaker as being 'paternalistic'.

The objective of the policymaker is to maximize household utility. Using equation (20) to express the relative quantities demanded of the differentiated goods each period as a function of their relative prices and the linear dependence of utility on the stock of assets, we can write the utility flow to the representative household in the form

$$u(Y_t, q_t^u; \xi_t) - v(Y_t; \xi_t)\Delta_t + \bar{\xi}_t^d \frac{A_t^d}{\tilde{\alpha}} k_t^{\tilde{\alpha}},$$

with

$$\begin{aligned} u(Y_t, q_t^u; \xi_t) &\equiv \tilde{u}(C(Y_t, q_t^u, \xi_t); \xi_t) \\ v(y_t^j; \xi_t) &\equiv \tilde{v}(f^{-1}(y_t^j / A_t); \xi_t), \end{aligned}$$

where Δ_t, defined in equation (35), captures the misallocations from price dispersion. The term

$$\bar{\xi}_t^d \equiv \sum_{T=t}^{\infty} E_t[(1-\delta)^{T-t} \beta^{T-t} \xi_T^d]$$

captures the present value contribution from new housing investment. We can use (25) and (41) to express k_t in terms of Y_t, q_t^u and exogenous shocks. Hence, we can express the policy maker's objective of maximizing (14) under rational expectations, as maximizing

$$U = E_0 \sum_{t=0}^{\infty} \beta^t U(Y_t, \Delta_t, q_t^u; \xi_t),$$

where the flow utility is given by

$$\begin{aligned} U(Y_t, \Delta_t, q_t^u; \xi_t) \equiv\ & \frac{\bar{C}_t^{\tilde{\sigma}^{-1}} C(Y_t, q_t^u, \xi_t)^{1-\tilde{\sigma}^{-1}}}{1-\tilde{\sigma}^{-1}} \\ & - \frac{\lambda}{1+\nu} \bar{H}_t^{-\nu} \left(\frac{Y_t}{A_t}\right)^{1+\omega} \Delta_t \\ & + \frac{A_t^d \bar{\xi}_t^d}{\tilde{\alpha}} \Omega(q_t^u, \xi_t)^{\tilde{\alpha}}\ C(Y_t, q_t^u, \xi_t)^{\frac{\tilde{\alpha}}{1-\tilde{\alpha}} \tilde{\sigma}^{-1}}, \end{aligned} \tag{C.1}$$

which is a monotonically decreasing function of Δ given Y, q^u and ξ, and where $\Omega(q^u, \xi)$ is the function defined in (40). The only endogenous variables that are relevant for evaluating the policymaker's objective function are thus Y_t, Δ_t and q_t^u.

The non-linear optimal monetary policy problem is then given by

$$\max_{\{Y_t, q_t^u, p_t^*, w_t(j), P_t, \Delta_t, i_t \geq 0\}} E_0 \sum_{t=0}^{\infty} \beta^t U(Y_t, \Delta_t, q_t^u; \xi_t) \tag{C.2}$$

subject to

$$\left(\frac{p_t^*}{P_t}\right)^{1+\eta(\phi-1)} = \frac{E_t^{\mathcal{P}} \sum_{T=t}^{\infty} (\alpha)^{T-t} Q_{t,T} \frac{\eta}{\eta-1} \phi w_T(j) \left(\frac{Y_T}{A_T}\right)^{\phi} \left(\frac{P_T}{P_t}\right)^{\eta\phi+1}}{E_t^{\mathcal{P}} \sum_{T=t}^{\infty} (\alpha)^{T-t} Q_{t,T}(1-\tau_T) Y_T \left(\frac{P_T}{P_t}\right)^{\eta}} \tag{C.3}$$

$$w_t(j) = \lambda \frac{\bar{H}_t^{-\nu}}{\bar{C}_t^{\tilde{\sigma}^{-1}}} \left(\frac{Y_t}{A_t}\right)^{\phi\nu} C\left(Y_t, q_t^u, \xi_t\right)^{\tilde{\sigma}^{-1}} \left(\frac{p_t^*}{P_t}\right)^{-\eta\phi\nu} \tag{C.4}$$

$$(P_t/P_{t-1})^{\eta-1} = \frac{1-(1-\alpha)\left(\frac{p_t^*}{P_t}\right)^{1-\eta}}{\alpha} \tag{C.5}$$

$$\Delta_t = h(\Delta_{t-1}, P_t/P_{t-1}) \tag{C.6}$$

$$\tilde{u}_C(C(Y_t, q_t^u, \xi_t); \xi_t) = \lim_{T\to\infty} E_t^{\mathcal{P}} \left[\tilde{u}_C(C_T; \xi_T) \beta^T \prod_{k=0}^{T-t} \frac{1+i_{t+k}}{P_{t+k+1}/P_{t+k}}\right] \tag{C.7}$$

$$q_t^u = \xi_t^d + \beta(1-\delta) E_t^{\mathcal{P}} q_{t+1}^u, \tag{C.8}$$

where the initial price level P_{-1} and initial price dispersion Δ_{-1} are given. Equation (C.4) insures that wages clear current labor markets. Similarly, by setting $C_t = C(Y_t, q_t^u, \xi_t)$ on the left-hand side of the consumption Euler equation (C.7), we impose market clearing for output goods in period t. Similarly, setting q_t^u equal to the value defined in (C.8) insures market clearing in the housing market.[58] Firms' subjective expectations about future wages and households' subjectively optimal consumption plans for the future, however, will generally not be consistent with labor market or goods market clearing in the future in all subjectively perceived contingencies, when beliefs deviate from rational ones.

To be able to analyze the policy problem further, it is necessary to be more specific about the beliefs $\mathcal{P}$ entertained by households and firms.

D Derivation of Equation (45)

Recall the definition of q_t^u which implies

$$\log q_t^u = \log q_t + \log \widetilde{u}_c(C_t; \xi_t)$$

Under the considered belief setup in which agents learn about risk-adjusted capital gains, the dynamics of risk-adjusted capital gains and beliefs are independent of monetary policy. The response of $\log q_t^u$ to a unexpected change in the path of nominal rates $\boldsymbol{i}$ is thus $\frac{d \log q_t^u}{d\boldsymbol{i}} = 0$,

[58]This holds as long as $D^{\max}$ is chosen sufficiently large, such that it never binds along the equilibrium path.

so that

$$
\begin{aligned}
\frac{d \log q_t}{d\boldsymbol{i}} &= -\frac{d \log \widetilde{u}_c(C_t;\xi_t)}{d\boldsymbol{i}} \\
&= -\frac{d \log \widetilde{u}_c(C_t;\xi_t)}{d \log C_t} \frac{d \log C_t}{d\boldsymbol{i}} \\
&= -\frac{\widetilde{u}_{cc}(C_t;\xi_t) C_t}{\widetilde{u}_c(C_t;\xi_t)} \frac{d \log C_t}{d\boldsymbol{i}} \\
&= \frac{1}{\widetilde{\widetilde{\sigma}}} \frac{d \log C_t}{d\boldsymbol{i}} \qquad \text{(D.1)}
\end{aligned}
$$

The optimal housing supply equation (25) can be written as

$$
\log k_t = \frac{1}{1-\tilde{\alpha}} \left(\log A_t^d + \log q_t \right).
$$

Taking derivatives with respect to $\boldsymbol{i}$ in the previous equation and using (D.1) delivers (45).

E Assumptions about Long-Run Beliefs

To insure that the subjectively optimal consumption plans satisfy the transversality condition (29), we impose that equation (7) describes subjective housing price beliefs for an arbitrarily long but finite amount of time $t < \bar{T} < \infty$ and that households hold rational expectations in the long-run, i.e. for all periods $t \geq \overline{T}$. Agents thus perceive

$$
q_t^u = q_t^{u*} \quad \text{for all } t \geq \bar{T}, \mathcal{P} \text{ almost surely,}
$$

where $q_t^{u*} = \bar{\xi}_t^d \equiv \sum_{T=t}^{\infty} E_t[(1-\delta)^{T-t} \beta^{T-t} \xi_T^d]$ is the rational expectations housing price. Appendix H.3 shows that this assumption is sufficient to insure that the transversality condition is satisfied. The transversality condition may also hold under weaker conditions, but actually showing this turns out to be difficult. The fact that agents will eventually hold rational housing and rental price expectations could be interpreted as agents learning to make rational predictions in the long-run.

F Quadratic Approximation of the Policy Problem

This appendix derives the linear-quadratic approximation to the nonlinear policy problem in Appendix C.

F.1 Optimal Dynamics and the Housing Price Gap

It will be convenient to determine the welfare-maximizing level of output and the welfare-maximizing housing price under flexible prices, so as to express output and housing prices in terms of gaps relative to these maximizing values. We thus define (Y_t^*, q_t^{u*}) as the values (Y_t, q_t^u) that maximize $U(Y_t, 1, q_t^u; \xi_t)$, which are implicitly defined by[59]

$$U_Y(Y_t^*, 1, q_t^{u*}; \xi_t) = U_{q^u}(Y_t^*, 1, q_t^{u*}; \xi_t) = 0.$$

In particular, we have

$$q_t^{u*} = \overline{\xi}_t^d, \tag{F.1}$$

as shown in Appendix H.4. We have

$$\widehat{q}_t^{u,RE} = \widehat{q}_t^{u*}, \tag{F.2}$$

which shows that housing price fluctuations are indeed efficient under RE.

The output gap is defined as

$$y_t^{gap} \equiv \log(Y_t) - \log(Y_t^*) = \widehat{y}_t - \widehat{y}_t^*, \tag{F.3}$$

i.e. the log-difference of output from its dynamically optimal value.

Under subjective beliefs, it follows from equations (56) and the linearization of (F.1) (see Appendix H.1 below) that

$$\widehat{q}_t^{u,\mathcal{P}} - \widehat{q}_t^{u*} = \left(\frac{1 - \beta(1-\delta)}{1 - \beta(1-\delta)\beta_t} - \frac{1 - \beta(1-\delta)}{1 - \beta(1-\delta)\rho_\xi} \right) \widehat{\xi}_t^d + \frac{\beta(1-\delta)(\beta_t - 1)}{1 - \beta(1-\delta)\beta_t}. \tag{F.4}$$

Again, for the case where $\beta_t = 1$ and with persistent housing demand shocks ($\rho_\xi \to 1$), the housing price gap under subjective beliefs is equal to the housing price gap under RE. Belief fluctuations, however, now contribute to fluctuations in the housing price gap.

For the real housing price gap, $\widehat{q}_t - \widehat{q}_t^*$, this implies

$$\widehat{q}_t - \widehat{q}_t^* = \left(1 + \widetilde{\sigma}^{-1} C_q\right) \left(\widehat{q}_t^u - \widehat{q}_t^{u*}\right) + \widetilde{\sigma}^{-1} C_Y y_t^{gap}. \tag{F.5}$$

F.2 Quadratically Approximated Welfare Objective

A second-order approximation to the utility function delivers

$$\frac{1}{2} U_{\widehat{Y}\widehat{Y}} \left(\widehat{y}_t - \widehat{y}_t^*\right)^2 + \frac{1}{2} U_{\widehat{q}^u \widehat{q}^u} \left(\widehat{q}_t^u - \widehat{q}_t^{u*}\right)^2 + \frac{1}{2} \underline{\gamma}^* h_{22} \pi_t^2 + t.i.p.,$$

where $t.i.p.$ denotes terms independent of policy and $\underline{\gamma}^*$ is the Lagrange multiplier associated with equation (C.6) at the optimal steady state. See Appendix H.5 for a detailed derivation. The dependence of the objective function on inflation follows from a second-order approximation of the constraint (C.6), which allows expressing the second-order utility losses associated with price distortions Δ_t as a function of squared inflation terms.

[59] The optimal path for $\{Y_t^*, q_t^{u*}\}$ can then be used to determine optimal dynamics for the remaining variables. In particular, equation (41) determines C_t^*, equation (25) determines k_t^* and thus D_t^*, and equation (19) determines H_t^*.

Since the fluctuations in the housing price gap, $\widehat{q}_t^u - \widehat{q}_t^{u*}$, are either constant (with RE) or determined independently of policy (under subjective beliefs, see Equation (F.4)), the endogenous part of the loss function can be written as

$$\sum_{t=0}^{\infty} \beta^t \frac{1}{2} \left(\Lambda_\pi \pi_t^2 + \Lambda_y \left(y_t^{gap} \right)^2 \right).$$

F.3 New Keynesian Phillips Curve

Linearizing Equations (C.3)-(C.5) delivers the linearized Phillips curve. The condition for the equilibrium wage (C.4) in period T in industry j in which firms last updated their prices in period t is given by

$$w_T(j) = \tilde{w}_T(j) \left(\frac{p_t^j}{P_t} \right)^{-\eta\phi\nu} \left(\frac{P_T}{P_t} \right)^{\eta\phi\nu},$$

where

$$\tilde{w}_T(j) \equiv \lambda \frac{\bar{H}_T^{-\nu}}{\bar{C}_T^{\tilde{\sigma}^{-1}}} \left(\frac{Y_T}{A_T} \right)^{\phi\nu} C\left(Y_T, q_T^u, \xi_T\right)^{\tilde{\sigma}^{-1}}.$$

Since firms' expectations about $w_T(j)$ and P_T are rational, their expectations about $\tilde{w}_T(j)$ are rational as well. Using the expression for $w_T(j)$, noting that $p_t(i) = p_t^j = p_t^*$, and writing out $Q_{t,T}$, it follows that

$$\left(\frac{p_t^*}{P_t} \right) = \left(\frac{E_t^{\mathcal{P}} \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \frac{\eta}{\eta-1} \phi \bar{C}_T^{\tilde{\sigma}^{-1}} C_T^{-\tilde{\sigma}^{-1}} \tilde{w}_T(j) \left(\frac{Y_T}{A_T} \right)^{\phi} \left(\frac{P_T}{P_t} \right)^{\eta(1+\omega)}}{E_t^{\mathcal{P}} \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \bar{C}_T^{\tilde{\sigma}^{-1}} C_T^{-\tilde{\sigma}^{-1}} (1-\tau_T) Y_T \left(\frac{P_T}{P_t} \right)^{\eta-1}} \right)^{\frac{1}{1+\omega\eta}}. \tag{F.6}$$

Log-linearizing equation (F.6) delivers[60]

$$\widehat{p}_t^* - \widehat{P}_t = \frac{1-\alpha\beta}{1+\omega\eta} \left\{ \widehat{\tilde{w}}_t(j) + \phi \left(\widehat{y}_t - \widehat{A}_t \right) - \widehat{\tau}_t - \widehat{y}_t + \alpha\beta E_t^{\mathcal{P}} \left[\frac{1+\omega\eta}{1-\alpha\beta} \left(\widehat{p}_{t+1}^* - \widehat{P}_{t+1} + \pi_{t+1} \right) \right] \right\}. \tag{F.7}$$

As the expectation in (F.7) is only about variables about which the private agents hold rational expectations, we can replace $E_t^{\mathcal{P}}[\cdot]$ with $E_t[\cdot]$.[61] Therefore, (C.5) can be used in period t and $t+1$, which in its linearized form is given by

$$\widehat{p}_t^* - \widehat{P}_t = \frac{\alpha}{1-\alpha} \pi_t.$$

[60] This follows from the the fact that in steady state, we have $p^* = P$, so that

$$\frac{\eta}{\eta-1} \phi \bar{C}^{\tilde{\sigma}^{-1}} C^{-\tilde{\sigma}^{-1}} \tilde{w}(j) \left(\frac{Y}{A} \right)^{\phi} = \bar{C}^{\tilde{\sigma}^{-1}} C^{-\tilde{\sigma}^{-1}} (1-\tau) Y.$$

The steady state value of the numerator in (F.6) is thus given by $\frac{1}{1-\alpha\beta} \frac{\eta}{\eta-1} \phi \bar{C}^{\tilde{\sigma}^{-1}} C^{-\tilde{\sigma}^{-1}} \tilde{w}(j) \left(\frac{Y}{A} \right)^{\phi}$ and the steady state value of the denominator by $\frac{1}{1-\alpha\beta} \bar{C}^{\tilde{\sigma}^{-1}} C^{-\tilde{\sigma}^{-1}} (1-\tau) Y$.

[61] The subjective consumption plans showing up in the stochastic discount factor drop out at this order of approximation.

Substituting $\widehat{w}_t(j)$ by the linearized version of the equilibrium condition (C.4) delivers the linearized New Keynesian Phillips Curve:

$$\pi_t = \kappa_y y_t^{gap} + \kappa_q \left(\widehat{q}_t^u - \widehat{q}_t^{u*}\right) + \beta E_t \pi_{t+1} + u_t, \tag{F.8}$$

where the coefficients κ are given by

$$\kappa_y = \frac{1-\alpha}{\alpha}\frac{1-\alpha\beta}{1+\omega\eta}\left(k_y - f_y\right) > 0$$
$$\kappa_q = -\frac{1-\alpha}{\alpha}\frac{1-\alpha\beta}{1+\omega\eta} f_q < 0,$$

with $k_y = \partial \log k / \partial \log y,\ f_y = \partial \log f / \partial \log y,\ f_q = \partial \log f / \partial \log q^u$, such that

$$k_y - f_y = \omega + \tilde{\sigma}^{-1} \frac{\left(1 - \underline{g}\right)\underline{Y}}{\underline{C} + \frac{\tilde{\sigma}^{-1}}{1-\tilde{\alpha}}\underline{k}} = \omega + \tilde{\sigma}^{-1} C_Y > 0$$
$$f_q = \tilde{\sigma}^{-1} \frac{\frac{\underline{k}}{1-\tilde{\alpha}}}{\underline{C} + \frac{\tilde{\sigma}^{-1}}{1-\tilde{\alpha}}\underline{k}} = -\tilde{\sigma}^{-1} C_q > 0,$$

where $C_q \equiv \frac{q^u}{C}\frac{\partial C}{\partial q^u}$ and $C_Y \equiv \frac{Y}{C}\frac{\partial C}{\partial Y}$, and where the functions $f\left(Y, q^u; \xi\right) \equiv \left(1-\tau\right) \bar{C}^{\tilde{\sigma}^{-1}} Y C\left(Y, q^u; \xi\right)^{-\tilde{\sigma}^{-1}}$ and $k\left(y; \xi\right) \equiv \frac{\eta}{\eta-1}\lambda\phi\frac{\bar{H}^{-\nu}}{A^{1+\omega}} Y^{1+\omega}$ are the same as in Adam and Woodford (2021), for the current period in which markets clear and the internally rational agents observe this.

The cost-push shock u_t is given by

$$u_t = \frac{\left(1-\alpha\right)\left(1-\alpha\beta\right)}{\alpha\left(1+\omega\eta\right)}\left(\Theta + \widehat{\tau}_t - \widehat{g}_t\right),$$

where

$$\widehat{\tau}_t = -\log\left(\frac{1-\tau_t}{1-\bar{\tau}_t}\right)$$
$$\widehat{g}_t = -\log\left(\frac{1-g_t}{1-\bar{g}_t}\right)$$

define deviations of τ_t and g_t from their second-best steady state values.

As in the standard New Keynesian model, a linearization of (C.6) implies that the state variable Δ_t is zero to first order under the maintained assumption that initial price dispersion satisfies $\Delta_{-1} \sim O(2)$. This constraint, together with the assumption that the Lagrange multipliers are of order $O(1)$, thus drops out of the quadratic formulation of the optimal policy problem. The second-order approximation of (C.6) is, however, important to express the quadratic approximation of utility in terms of inflation.

F.4 Linearized IS Equation with Potentially Non-Rational Housing Price Beliefs

We here linearize the constraint (C.7). One difficulty with this constraint is that it features the limiting expectations of the subjectively optimal consumption plan on the right hand side. Generally, this would require solving for the subjectively optimal consumption paths, which is generally difficult.

Under our beliefs specifications, housing prices beliefs are rational in the limit. This insures that we do not have to solve for the subjectively optimal consumption plan, instead can derive the IS equation directly in terms of the output gap.

We can now define the natural rate of interest:

Definition 2 *The natural rate $r_t^{*,RE}$ is the one implied by the consumption Euler equation (24) or (C.7), rational expectations, and the welfare-maximizing consumption levels under flexible prices $\{C_t^*\}$. It satisfies*

$$\tilde{u}_C(C_t^*;\xi_t) = \beta E_t\left[u_C(C_{t+1}^*;\xi_t)(1+r_{t+k}^{*,RE})\right]. \tag{F.9}$$

Using the previous definition, we obtain the linearized Euler equation under potentially subjective housing prices beliefs:

Lemma 2 *For the considered belief specifications, the log-linearized household optimality condition (C.7) implies for all t*

$$y_t^{gap} = \lim_T E_t y_T^{gap} - E_t\left(\sum_{k=0}^{\infty}\varphi\left(i_{t+k}-\pi_{t+1+k}-r_{t+k}^{*,RE}\right)\right) - \frac{C_q}{C_Y}\left(\widehat{q}_t^u - \widehat{q}_t^{u*}\right), \tag{F.10}$$

where $\lim_T E_t y_T^{gap}$ is the (rational) long-run expectation of the output gap, and $\varphi \equiv -\frac{\tilde{u}_c}{\tilde{u}_{cc}C}\frac{1}{C_Y} > 0$. The coefficients $C_q < 0$ and $C_Y > 0$ are the ones defined in the derivation of the linearized Phillips Curve.

The proof can be found in Appendix H.2

F.5 Lagrangian Formulation of the Approximated Ramsey Problem

Collecting results from the previous sections, we obtain the following Lagrangian formulation of the Ramsey problem

$$\max_{\{\pi_t, y_t^{gap}, i_t \geq \underline{i}\}} \min_{\{\varphi_t, \lambda_t\}} \tag{F.11}$$

$$E_0 \sum_{t=0}^{\infty}\beta^t\Bigg\{-\frac{1}{2}\left(\Lambda_\pi \pi_t^2 + \Lambda_y\left(y_t^{gap}\right)^2\right)$$

$$+\varphi_t\left[\pi_t - \kappa_y y_t^{gap} - \kappa_q\left(\widehat{q}_t^u - \widehat{q}_t^{u*}\right) - u_t - \beta E_t \pi_{t+1}\right] \tag{F.12}$$

$$+\lambda_t\left[y_t^{gap} - \lim_T E_t y_T^{gap} + \varphi E_t\sum_{k=0}^{\infty}\left(i_{t+k}-\pi_{t+1+k}-r_{t+k}^{*,RE}\right) + \frac{C_q}{C_Y}\left(\widehat{q}_t^u - \widehat{q}_t^{u*}\right)\right] \tag{F.13}$$

$$-\varphi_{-1}\pi_0 - \lambda_{-1}\left(\varphi\pi_0 - y_0^{gap} - \frac{C_q}{C_Y}\left(\widehat{q}_0^u - \widehat{q}_0^{u*}\right)\right)\Bigg\},$$

where the process for $(\widehat{q}_t^u - \widehat{q}_t^{u*})$ can be treated as exogenous for the purpose of monetary policy and where the initial Lagrange multipliers $(\varphi_{-1}, \lambda_{-1})$ capture initial pre-commitments. In order to numerically solve the optimal policy problem in (F.11), we recursify the problem as proposed in Marcet and Marimon (2019) and solve for the associated value functions and optimal policies. Details of the recursive formulation can be found in Appendix H.6.

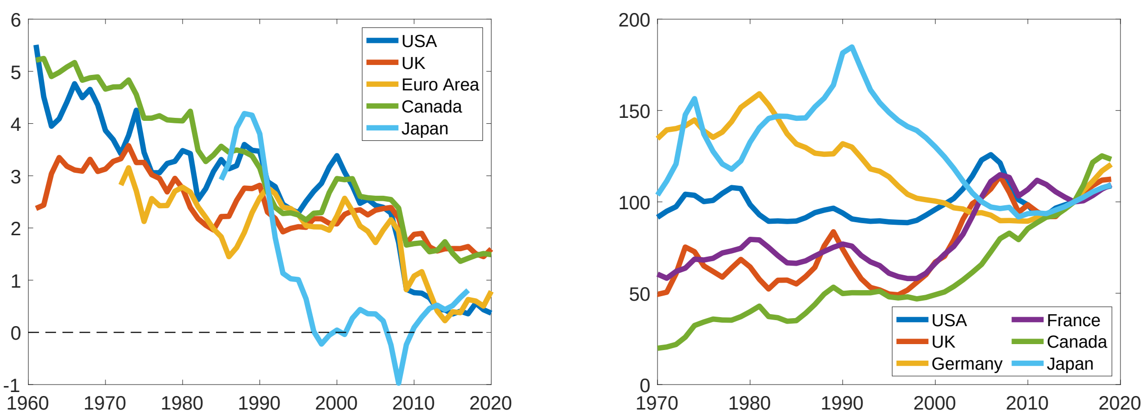
Online Appendix
Not intended for publication

G The Volatility of PR-Ratio and of the Natural Rate

Figure G.1 shows the evolution of natural rates of interest and price-to-rent ratios the U.S., Canada, France, Germany, and the United Kingdom, which we use in Section 2. The natural rates are estimated by Holston, Laubach, and Williams (2017) and Fujiwara, Iwasaki, Muto, Nishizaki, and Sudo (2016). The price-to-rent ratios are taken from the OECD. We convert the quarterly series of natural rates to annual series by taking arithmetic averages and the quarterly series or PR-ratios to annual series by taking harmonic averages.

Figure G.1: Natural rates and price-to-rent ratios

(a) Natural Rates of Interest (b) Price-to-rent ratios



Notes: This figure shows the evolution of the natural rate of interest (left panel) and price-to-rent ratios (right panel) for different advanced economies over the period 1961-2020 and 1970-2019, respectively.

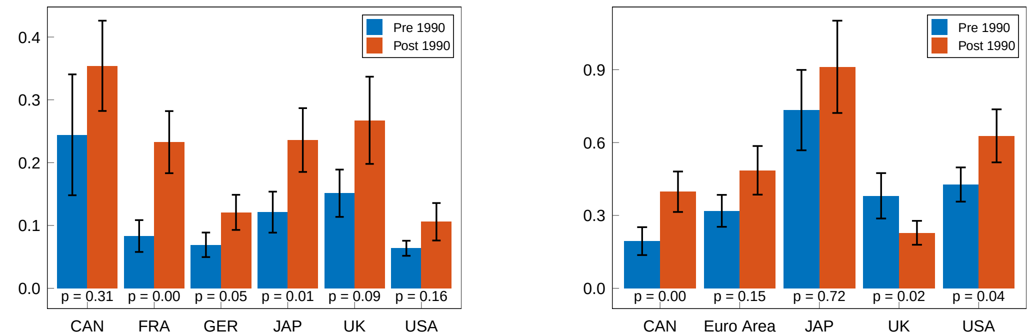
Figures 4(b) and 6 in the Section 2 document that the fall in the level of the natural rates of interest across several advanced economies was accompanied by an increase in the volatility of the price-to-rent ratio and in the volatility of natural rates. These trends are consistent with the subjective belief model, outlined in Sections 3 and 4.

Figure G.2 plots the volatility of the price-to-rent ratio (left panel) and the standard deviation of the natural rate (right panel), respectively before 1990 (blue bars) and after 1990 (red bars), along with 90% confidence bands. The reported volatilities of the price-to-rent ratios are the standard deviations relative to the period-specific mean values, in line with the model. The reported volatilities of the natural rates of interest are the standard deviations of the fluctuations around a linear time trend, in order to isolate high-frequency volatility that can be related to natural rate fluctuations in the model around a fixed steady state value of the natural rate. Figure G.4 shows the volatility price-to-rent ratio using the same linear detrending approach. The p-values below the respective bars are for the null hypothesis of no change in the volatility. The increase in the volatility of the PR ratio and the natural rate were statistically significant in most of the advanced economies. The evidence is not always statistically significant due to the high serial correlation of the price-to-rent ratio and the natural rate, which makes it difficult to estimate standard deviations precisely. Figure G.3 shows that the reported volatility increases are not driven by the exact point where we split the data, instead looks often similar for other split points.

Figure G.2: Volatility of the PR ratio and natural rates pre and post 1990.

(a) Standard Deviation of the Price-to-Rent Ratio Pre and Post 1990

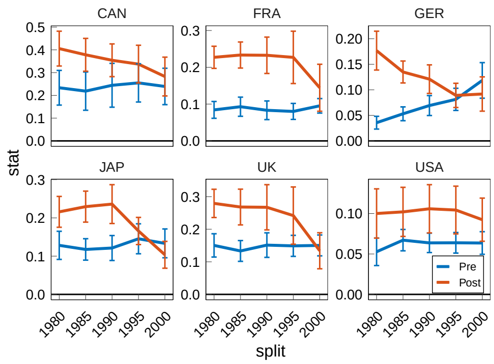
(b) Standard Deviation of Natural Rate Pre and Post 1990



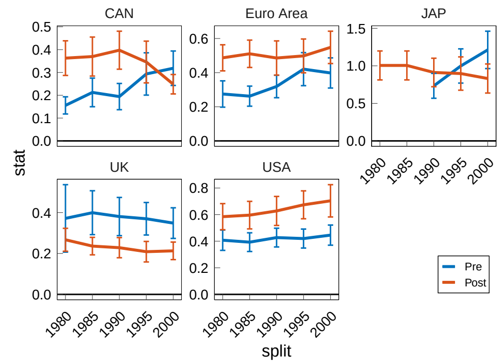
Notes: The black lines denote the 90%-confidence bands. The p-value corresponds to the test whether or not the values changed from pre to post 1990. The reported volatilities of the price-to-rent ratios are the standard deviations relative to the period-specific mean values. The reported volatilities of the natural rates of interest are the standard deviations of the fluctuations around a linear time trend.

Figure G.3: Robustness of housing and natural rate volatility increases with different sample splits

(a) Volatility of the Price-to-Rent ratios

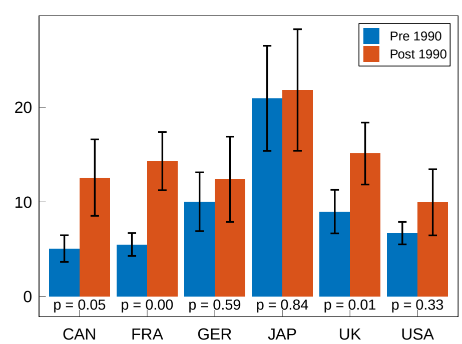


(b) Volatility of natural rates



Notes: Panel (a) shows the standard deviation of the price-to-rent ratio, and panel (b) shows the standard deviation of the natural rate for different advanced economies, computed for varied subsamples. The blue lines show the estimates for the pre-period, and the red lines for the post-period, when the sample is split at the year marked on the horizontal axis. The whiskers denote 90%-confidence bands.

Figure G.4: Standard deviation of the detrended PR ratio pre and post 1990



Note: The black lines denote the 90%-confidence bands. The p-value corresponds to the test whether or not the values changed from pre to post 1990.

H Proofs

H.1 Results in Section 5

Proof of Results in Section 5.1. Result (50) follows from iterating forward on (27). Log linearizing (50), we have

$$\widehat{q}_t^u = \widehat{\overline{\xi}}_t^d,$$

and log-linearizing (12) delivers

$$\widehat{\xi}_t^d = \rho_\xi \widehat{\xi}_{t-1}^d + \varepsilon_t^d.$$

Since the steady-state value of $\underline{\overline{\xi}}^d$ is

$$\underline{\overline{\xi}}^d = \frac{\underline{\xi}^d}{1-\beta(1-\delta)},$$

the log-linearization of $\overline{\xi}_t^d$ delivers

$$\begin{aligned}
\widehat{\overline{\xi}}_t^d &= (1-\beta(1-\delta))\left[\widehat{\xi}_t^d + \beta(1-\delta)E_t\widehat{\xi}_{t+1}^d + ...\right] \\
&= (1-\beta(1-\delta))\left[\widehat{\xi}_t^d + \beta(1-\delta)\rho_\xi\widehat{\xi}_t^d + ...\right] \\
&= (1-\beta(1-\delta))\sum_{T=t}^{\infty}(\beta(1-\delta)\rho_\xi)^{T-t}\widehat{\xi}_t^d \\
&= \widehat{\xi}_t^d\frac{1-\beta(1-\delta)}{1-\beta(1-\delta)\rho_\xi}.
\end{aligned}$$

The results for the price-to rent ration follow by noticing that equation (26) implies

$$PR_t \equiv \frac{q_t}{R_t} = \frac{q_t^u}{\xi_t^d}. \tag{H.1}$$

■

Proof of Results in Section 5.2. From equation (27), which has to hold with equality in equilibrium, and equation (8) we get

$$q_t^{u,\mathcal{P}} = \frac{1}{1-\beta(1-\delta)\beta_t}\xi_t^d$$

The percent deviation of housing prices from the steady state, in which $\beta_t = 1$ and $\xi_t^d = \underline{\xi}^d$, is then given by

$$\begin{aligned}
\widehat{q}_t^{u,\mathcal{P}} &= \frac{\frac{1}{1-\beta(1-\delta)\beta_t}\xi_t^d - \frac{1}{1-\beta(1-\delta)}\underline{\xi}^d}{\frac{1}{1-\beta(1-\delta)}\underline{\xi}^d} \\
&= \frac{1-\beta(1-\delta)}{1-\beta(1-\delta)\beta_t}\frac{\xi_t^d}{\underline{\xi}^d} - 1 \\
&= \frac{1-\beta(1-\delta)}{1-\beta(1-\delta)\beta_t}\left(1+\widehat{\xi}_t^d\right) - 1 \\
&= \frac{1-\beta(1-\delta)}{1-\beta(1-\delta)\beta_t}\widehat{\xi}_t^d + \frac{\beta(1-\delta)(\beta_t-1)}{1-\beta(1-\delta)\beta_t} \qquad \text{(H.2)}
\end{aligned}$$

Note, that we can decompose the housing price under subjective beliefs into the housing price under RE and terms that are driven by beliefs:

$$\widehat{q}_t^{u,\mathcal{P}} = \widehat{q}_t^{u,RE} + \frac{\beta(1-\delta)\left(\beta_t-1\right)}{1-\beta(1-\delta)\beta_t} + \frac{\left(1-\beta(1-\delta)\right)\left(\beta(1-\delta)\left(\beta_t-\rho_\xi\right)\right)}{\left(1-\beta(1-\delta)\beta_t\right)\left(1-\beta(1-\delta)\rho_\xi\right)}\widehat{\xi}_t^d. \qquad \text{(H.3)}$$

Note, that

$$E_t^{\mathcal{P}}\left[q_{t+1}^{u,\mathcal{P}}\right] = \beta_t q_t^{u,\mathcal{P}}.$$

Therefore, a log-linear approximation around the optimal steady state, in which $\underline{\beta}=1$, yields

$$E_t^{\mathcal{P}}\left[\widehat{q}_{t+1}^{u,\mathcal{P}}\right] = \widehat{q}_t^{u,\mathcal{P}} + \left(\beta_t-1\right).$$

From this, we can add and subtract on the right-hand side

$$E_t\left[\widehat{q}_{t+1}^{u,RE}\right] = \rho_\xi\widehat{\xi}_t^d\frac{1-\beta(1-\delta)}{1-\beta(1-\delta)\rho_\xi},$$

which, after plugging in the expression from (H.2), delivers

$$\begin{aligned}
E_t^{\mathcal{P}}\left[\widehat{q}_{t+1}^{u,\mathcal{P}}\right] &= E_t\left[\widehat{q}_{t+1}^{u,RE}\right] + \left(\beta_t-1\right)\left[1+\frac{\beta(1-\delta)}{1-\beta(1-\delta)\beta_t}\right] \\
&\quad + \left(1-\beta(1-\delta)\rho_\xi - \left(1-\beta(1-\delta)\beta_t\right)\rho_\xi\right)\frac{\left(1-\beta(1-\delta)\right)}{\left(1-\beta(1-\delta)\beta_t\right)\left(1-\beta(1-\delta)\rho_\xi\right)}\widehat{\xi}_t^d.
\end{aligned}$$

In the limit $\rho_\xi \to 1$, this boils down to

$$E_t^{\mathcal{P}}\left[\widehat{q}_{t+1}^{u,\mathcal{P}}\right] = E_t\left[\widehat{q}_{t+1}^{u,RE}\right] + \left(\beta_t-1\right)\left[1+\frac{\beta(1-\delta)}{1-\beta(1-\delta)\beta_t}\left(1+\widehat{\xi}_t^d\right)\right]$$

Log-linearizing equation (H.1), which holds true independent of the belief specification, yields

$$\widehat{PR}_t^{\mathcal{P}} = \widehat{q}_t^{u,\mathcal{P}} - \widehat{\xi}_t^d.$$

■

Proof of Lemma 1. Under the proposed policy that sets $i_t - E_t\pi_{t+1}$ equal to the natural rate defined in equation (59), we have

$$\begin{aligned}
y_t^{gap} &= \lim_T E_t y_T^{gap} - E_t\left(\sum_{k=0}^{\infty}\varphi\left(i_{t+k} - \pi_{t+1+k} - r_{t+k}^{*,RE}\right)\right) - \frac{C_q}{C_Y}\left(\widehat{q}_t^u - \widehat{q}_t^{u*}\right) \\
&= \lim_T E_t y_T^{gap} - E_t\left(\sum_{k=0}^{\infty}\varphi\left(r_{t+k}^{*,RE} - \frac{1}{\varphi}\frac{C_q}{C_Y}\left(\left(\widehat{q}_{t+k}^u - \widehat{q}_{t+k}^{u*}\right) - E_{t+k}\left(\widehat{q}_{t+k+1}^u - \widehat{q}_{t+k+1}^{u*}\right)\right) - r_{t+k}^{*,RE}\right)\right) \\
&\quad - \frac{C_q}{C_Y}\left(\widehat{q}_t^u - \widehat{q}_t^{u*}\right) \\
&= \lim_T E_t y_T^{gap} + E_t\left(\sum_{k=0}^{\infty}\left(\frac{C_q}{C_Y}\left(\left(\widehat{q}_{t+k}^u - \widehat{q}_{t+k}^{u*}\right) - \left(\widehat{q}_{t+k+1}^u - \widehat{q}_{t+k+1}^{u*}\right)\right)\right)\right) - \frac{C_q}{C_Y}\left(\widehat{q}_t^u - \widehat{q}_t^{u*}\right) \\
&= \lim_T E_t y_T^{gap} + E_t\left(\frac{C_q}{C_Y}\left(\left(\widehat{q}_t^u - \widehat{q}_t^{u*}\right) - \lim_k E_t\left(\widehat{q}_{t+k+1}^u - \widehat{q}_{t+k+1}^{u*}\right)\right)\right) - \frac{C_q}{C_Y}\left(\widehat{q}_t^u - \widehat{q}_t^{u*}\right) \\
&= \lim_T E_t y_T^{gap} + \left(\frac{C_q}{C_Y}\left(\widehat{q}_t^u - \widehat{q}_t^{u*}\right)\right) - \frac{C_q}{C_Y}\left(\widehat{q}_t^u - \widehat{q}_t^{u*}\right) \\
&= \lim_T E_t y_T^{gap},
\end{aligned}$$

which proves that with this policy, the output gap is indeed constant, and $r^{*,\mathcal{P}}$ is the real rate that implies a constant output gap. ■

H.2 Log-linearized Euler equation

Proof of Lemma 2. Log-linearizing equation (C.7) around the optimal steady state delivers

$$\tilde{u}_{CC}C\widehat{c}_t + \tilde{u}_{C\xi}\underline{\xi}\widehat{\xi}_t = E_t^{\mathcal{P}}\sum_{k=0}^{\infty}\tilde{u}_C\left(i_{t+k} - \pi_{t+1+k}\right) + \lim_{T\to\infty}E_t^{\mathcal{P}}\left(\tilde{u}_{CC}C\widehat{c}_T + \tilde{u}_{C\xi}\underline{\xi}\widehat{\xi}_T\right),$$

and log-linearizing (F.9) gives

$$\tilde{u}_{CC}C\widehat{c}_t^* + \tilde{u}_{C\xi}\underline{\xi}\widehat{\xi}_t = E_t\sum_{k=0}^{\infty}\tilde{u}_C r_{t+k}^{*,RE} + \lim_{T\to\infty}E_t\left(\tilde{u}_{CC}C\widehat{c}_T^* + \tilde{u}_{C\xi}\underline{\xi}\widehat{\xi}_T\right).$$

Subtracting the previous equation from (H.4) delivers

$$\widehat{c}_t - \widehat{c}_t^* = E_t^{\mathcal{P}}\sum_{k=0}^{\infty}\frac{\tilde{u}_C}{\tilde{u}_{CC}C}\left(i_{t+k} - \pi_{t+1+k} - r_{t+k}^{*,RE}\right) + \lim_{T\to\infty}E_t^{\mathcal{P}}\left(\widehat{c}_{T+1} - \widehat{c}_{T+1}^*\right), \tag{H.4}$$

where we used $E_t^{\mathcal{P}}\xi_T = E_t\xi_T$ and $E_t^{\mathcal{P}}\widehat{c}_{T+1}^* = E_t\widehat{c}_{T+1}^*$, which hold because agents hold rational expectations about fundamentals.

In all periods in which the subjectively optimal plan is consistent with market clearing in the goods sector, the plan satisfies equation (41). Log-linearizing equation (41) delivers

$$\widehat{c}_t = C_Y\widehat{y}_t + C_q\widehat{q}_t^u + C_\xi\widehat{\xi}_t, \tag{H.5}$$

where $\widehat{\xi}_t$ is a vector of exogenous disturbances (involving $A_t^d, \bar{C}_t, g_t$). Evaluating this equation at the optimal dynamics defines the optimal consumption gap $\widehat{c}_t^*$:

$$\widehat{c}_t^* \equiv C_Y \widehat{y}_t^* + C_q \widehat{q}_t^{u*} + C_\xi \widehat{\xi}_t.$$

Subtracting the previous equation from (H.5) delivers

$$\begin{aligned} \widehat{c}_t - \widehat{c}_t^* &= C_Y \left(\widehat{y}_t - \widehat{y}_t^*\right) + C_q \left(\widehat{q}_t^u - \widehat{q}_t^{u*}\right) \\ &= C_Y y_t^{gap} + C_q \left(\widehat{q}_t^u - \widehat{q}_t^{u*}\right) \end{aligned} \tag{H.6}$$

Since the current consumption market in period t clears, equation (H.6) holds in period t and can be used to substitute the consumption gap on the left-hand side of equation (H.4). Similarly, since housing price expectations are rational in the limit, the consumption market also clears in the limit under the subjectively optimal plans, i.e., equation (41) holds for $t \geq T'$. We can thus use equation (H.6) also to substitute the consumption gap on the r.h.s. of equation (H.4). Using the fact that housing price expectations are rational in the limit ($\lim_T E_t^P \left(\widehat{q}_t^u - \widehat{q}_t^{u*}\right) = 0$), we obtain

$$y_t^{gap} = \lim_T E_t^{\mathcal{P}} y_T^{gap} - E_t \left(\sum_{k=0}^{\infty} \varphi \left(i_{t+k} - \pi_{t+1+k} - r_{t+k}^{*,RE} \right) \right) - \frac{C_q}{C_Y} \left(\widehat{q}_t^u - \widehat{q}_t^{u*}\right).$$

Since we assumed that agents' beliefs about profits and taxes are given by equations (37) and (38), respectively, evaluated using rational income expectations, the household holds rational expectations about total income. This can be seen by substituting (37) and (38) into the budget constraint (15). We thus have $\lim_T E_t^{\mathcal{P}} y_T^{gap} = \lim_T E_t y_T^{gap}$ in the previous equation, which delivers (F.10). ■

H.3 Transversality Condition Satisfied with Subjective Housing Price Beliefs

This appendix shows that under the considered subjective belief specifications, the optimal plans satisfy the transversality constraint (29). Since $D_t \in [0, D^{\max}]$ and $E_t^{\mathcal{P}} q_T^u = E_t \overline{\xi}_T^d$ for $T \geq T'$, we have $\lim_{T\to\infty} \beta^T E_t^{\mathcal{P}} \left(D_T q_T^u\right) = 0$. We thus only need to show that $\lim_{T\to\infty} \beta^T E_t^{\mathcal{P}} \frac{\bar{C}_T^{\tilde{\sigma}^{-1}}}{C_T^{\tilde{\sigma}^{-1}}} B_T = 0$. Combining the budget constraint (15) with (37) and (38) we obtain

$$C_t + B_t + \Big(D_t - (1-\delta)D_{t-1} - \tilde{d}(k_t;\xi_t)\Big) q_t^u \frac{C_t^{\tilde{\sigma}^{-1}}}{\bar{C}_t^{\tilde{\sigma}^{-1}}} + k_t = (1-g_t)\, Y_t + B_{t-1}.$$

For $t \geq T'$ the subjectively optimal plans satisfy market clearing in the housing market, i.e.,

$$D_t - (1-\delta)D_{t-1} - \tilde{d}(k_t;\xi_t) = 0$$

so that the budget constraint implies

$$C_t + B_t + k_t = (1-g_t)\, Y_t + B_{t-1}. \tag{H.7}$$

Furthermore, for $t \geq T'$ subjectively optimal plans also satisfy market clearing for consumption goods, i.e.,

$$C_t + k_t = (1 - g_t)\, Y_t.$$

It thus follows that the subjectively optimal debt level B_t in the budget constraint (H.7) is constant under the subjectively optimal plan, after period $t \geq T'$. Furthermore, the expectations about Y_t in the budget constraint (H.7) is rational under the assumed lump sum transfer expectations, so that the household's subjective consumption expectations are the same as in a rational expectations equilibrium. (The subjectively optimal investment decisions k_t are driven by rational housing price expectations). Since the limit expectations $\bar{C}_T^{\tilde{\sigma}^{-1}}/C_T^{\tilde{\sigma}^{-1}}$ are bounded in the rational expectations equilibrium, it follows that $\lim_{T\to\infty} \beta^T E_t^{\mathcal{P}} \frac{\bar{C}_T^{\tilde{\sigma}^{-1}}}{C_T^{\tilde{\sigma}^{-1}}} B_T = 0$.

H.4 Optimal House Price Absent Price Rigidities

The following derivation closely follows Adam and Woodford (2021). We obtain $U_{q^u}\left(Y_t, \Delta_t, q_t^u, \xi_t\right)$ from differentiating equation (C.1) with respect to q_t^u and set it equal to 0:

$$\begin{aligned}
U_{q^u}\left(Y_t, \Delta_t, q_t^u, \xi_t\right) = {} & \bar{C}_t^{\tilde{\sigma}^{-1}} C_{q^u}\left(Y_t, q_t^u, \xi_t\right) C\left(Y_t, q_t^u, \xi_t\right)^{-\tilde{\sigma}^{-1}} \\
& + A_t^d \overline{\xi}_t^d \frac{\partial \Omega\left(q_t^u, \xi_t\right)}{\partial q_t^u} \Omega\left(q_t^u, \xi_t\right)^{\tilde{\alpha}-1} C\left(Y_t, q_t^u, \xi_t\right)^{\frac{\tilde{\alpha}}{1-\tilde{\alpha}}\tilde{\sigma}^{-1}} \\
& + \frac{\tilde{\sigma}}{1-\tilde{\alpha}} A_t^d \overline{\xi}_t^d \Omega\left(q_t^u, \xi_t\right)^{\tilde{\alpha}} C\left(Y_t, q_t^u, \xi_t\right)^{\frac{\tilde{\alpha}}{1-\tilde{\alpha}}\tilde{\sigma}^{-1}-1} C_{q^u}\left(Y_t, q_t^u, \xi_t\right) = 0,
\end{aligned}$$

where

$$\frac{\partial \Omega\left(q_t^u, \xi_t\right)}{\partial q_t^u} = \frac{1}{q_t^u} \frac{1}{1-\tilde{\alpha}} \Omega\left(q_t^u, \xi_t\right),$$

and when defining $\chi \equiv \frac{\tilde{\sigma}^{-1}}{1-\tilde{\alpha}} - 1$, we get

$$C_{q^u}(Y_t, q_t^u; \xi_t) \equiv \frac{\partial C(Y_t, q_t^u; \xi_t)}{\partial q^u} = \frac{-\frac{1}{q_t^u}\frac{1}{1-\tilde{\alpha}} \Omega(q_t^u, \xi_t) C(Y_t, q_t^u, \xi_t)^{\chi+1}}{1 + (1+\chi)\, \Omega(q_t^u, \xi_t) C(Y_t, q_t^u, \xi_t)^{\chi}}.$$

Taking everything together, we get

$$U_{q^u}\left(Y_t, \Delta_t, q_t^u, \xi_t\right) = \frac{\frac{1}{q_t^u}\frac{1}{1-\tilde{\alpha}} \Omega(q_t^u, \xi_t) C(Y_t, q_t^u, \xi_t)^{\chi+1}}{1 + (1+\chi)\, \Omega(q_t^u, \xi_t) C(Y_t, q_t^u, \xi_t)^{\chi}} \bar{C}_t^{\tilde{\sigma}-1} \left(\frac{\overline{\xi}_t^d}{q_t^u} - 1 \right).$$

In order for U_{q^u} to be zero, we need to have that

$$q_t^{u*} = \overline{\xi}_t^d,$$

as stated in equation (F.1).

H.5 Quadratically Approximated Welfare Objective

This derivation follows Adam and Woodford (2021). In the optimal steady state, we have $U_Y = U_{q^u} = U_{Yq^u} = 0$, as well as $U_\Delta + \underline{\gamma}\,(\beta h_1 - 1) = 0$. Given the assumption $\Delta_{-1} \sim O(2)$, it follows $\Delta_t \sim O(2)$ for all $t \geq 0$. Additionally, we have $h_2 \equiv \frac{\partial h(\Delta,\Pi)}{\partial \Pi} = 0$ at the optimal steady state. Therefore, a second-order approximation of the contribution of the variables $(Y_t, \Delta_t, q_t^u, \Pi_t, \xi_t)$ to the utility of the household yields

$$\frac{1}{2}U_{\widehat{Y}\widehat{Y}}\left(\widehat{y}_t - \widehat{y}_t^*\right) + \frac{1}{2}U_{\widehat{q}^u\widehat{q}^u}\left(\widehat{q}_t^u - \widehat{q}_t^{u*}\right) + \frac{1}{2}\underline{\gamma}^* h_{22}\pi_t^2 + t.i.p.,$$

where $t.i.p.$ contains all terms independent of policy. Under rational expectations, we have that $\left(\widehat{q}_t^u - \widehat{q}_t^{u*}\right) = 0$ and is thus constant and independent of (monetary) policy. Under subjective beliefs, $\left(\widehat{q}_t^u - \widehat{q}_t^{u*}\right)$ is purely driven by beliefs β_t and housing demand shocks ξ_t^d, see equation (F.4), both independent of policy. Therefore, we include $\frac{1}{2}U_{\widehat{q}^u\widehat{q}^u}\left(\widehat{q}_t^u - \widehat{q}_t^{u*}\right)$ in $t.i.p.$.

The term $U_{\widehat{Y}\widehat{Y}}$ is given by $U_{\widehat{Y}\widehat{Y}} \equiv Y\frac{\partial}{\partial Y}\left(U_{\widehat{Y}}\right) \equiv Y\frac{\partial}{\partial Y}\left(YU_Y\right) = \underline{Y}^* U_Y + \left(\underline{Y}^*\right)^2 U_{YY}$. At the optimal steady state, we have

$$\begin{aligned}
\Lambda_\pi &= -\frac{1}{2}\underline{\gamma}^* h_{22} > 0 \\
\Lambda_y &= -\frac{1}{2}\left(\underline{Y}^*\right)^2 U_{YY} > 0,
\end{aligned}$$

where

$$\begin{aligned}
U_{YY} &= -\tilde{\sigma}^{-1}\left(1-\underline{g}\right)\underline{\bar{C}}^{\tilde{\sigma}^{-1}} C\left(\underline{Y}, \underline{q}^u, \underline{\xi}\right)^{-\tilde{\sigma}^{-1}-1} C_Y \frac{\underline{Y}^*}{C\left(\underline{Y}, \underline{q}^u, \underline{\xi}\right)} \\
&\quad - \frac{\lambda}{1+\nu}\left(1+\omega\right)\omega\frac{\underline{\bar{H}}^{-\nu}}{\underline{A}^{1+\omega}}\underline{Y}^{\omega-1} < 0 \\
h_{22} &= \frac{\alpha\eta\left(1+\omega\right)\left(1+\omega\eta\right)}{1-\alpha} > 0 \\
\underline{\gamma}^* &= \frac{U_\Delta}{1-\alpha\beta} < 0,
\end{aligned}$$

with

$$U_\Delta = -\frac{\underline{Y}^*\left(1-\underline{g}\right)}{1+\omega}\left(\frac{\bar{C}^{\tilde{\sigma}^{-1}}}{C\left(\underline{Y}^*, \underline{q}^{u*}, \underline{\xi}\right)}\right)^{\tilde{\sigma}^{-1}} < 0.$$

H.6 Recursified Optimal Policy Problem with Lower Bound

We numerically solve the quadratically approximated optimal policy problem with forward-looking constraints (F.11). While it would be preferable to solve the fully nonlinear Ramsey problem, as spelled out in Appendix C, this is computationally not feasible with sufficient degree of numerical accuracy because the problem features 9 state variables and an occasionally binding constraint. The quadratically approximated problem features 2 state variables less because price dispersion Δ_t is to first order independent of policy and because the Phillips curve reduces from a system involving two forward-looking infinite sums, see equation (F.6), to a system involving only a single infinite sum, see (F.7).

Eggertsson and Singh (2020) compare the exact solution of the standard New Keynesian model with lower bound to the solution of the linear-quadratic approximation with lower bound and show that the quantitative deviations are modest, even for extreme shocks of the size capturing the 2008 recession in the U.S..

To obtain a recursive problem, we apply the approach of Marcet and Marimon (2019) to the problem with forward-looking constraints (F.11). We thereby assume that the Lagrangian defined by problem (F.11) satisfies the usual duality properties that allow interchanging the order of maximization and minimization, which we verify ex-post using the computed value function. We set the terminal value function for $t = T'$ to its RE value function $W^{RE}(\cdot)$. For $t \leq T'$ we have a value function $W_t(\cdot)$ satisfying the following recursion:

$$
\begin{aligned}
& W_t(\varphi_{t-1}, \mu_{t-1}, u_t, r_t^{*,RE}, \beta_t, \xi_t^d, q_{t-1}^u) \\
= \quad & \max_{\left(\pi_t, y_t^{gap}, i_t \geq \underline{i}\right)} \min_{(\varphi_t, \lambda_t)} -\frac{1}{2}\left(\Lambda_\pi \pi_t^2 + \Lambda_y \left(y_t^{gap}\right)^2\right) \\
& + (\varphi_t - \varphi_{t-1})\, \pi_t - \varphi_t \left(\kappa_y y_t^{gap} + \kappa_q \left(\widehat{q}_t^u - \widehat{q}_t^{u*}\right) + u_t\right) \\
& + \lambda_t \left[y_t^{gap} - \lim_T E_t y_T^{gap} + \varphi \left(i_t - E_t \sum_{k=0}^{\infty} r_{t+k}^{*,RE} \right) + \frac{C_q}{C_Y} \left(\widehat{q}_t^u - \widehat{q}_t^{u*}\right) \right] \\
& + \mu_{t-1} \varphi \left(i_t - \pi_t\right) + \gamma_t \left(i_t - \underline{i}\right) \\
& + \beta E_t \left[W_{t+1}(\varphi_t, \underbrace{\beta^{-1}\left(\lambda_t + \mu_{t-1}\right)}_{=\mu_t}, u_{t+1}, r_{t+1}^{*,RE}, \beta_{t+1}, \xi_{t+1}^d, q_t^u \right]
\end{aligned} \tag{H.8}
$$

where the next period state variables (β_{t+1}, q_t^u) are determined by equations (9) and (54) and $(\widehat{q}_t^u - \widehat{q}_t^{u*})$ is determined by equation (F.4). Here we assume that $r_t^{*,RE}$ follows a Markov process, such that the term $E_t \sum_{k=0}^{\infty} r_{t+k}^{*,RE}$ showing up in the current-period return can be expressed as a function of the current state $r_t^{*,RE}$. The future state variables $(\varphi_t, \mu_t, \beta_{t+1}, q_t^u)$ are predetermined in period t. The expectation about the continuation value is thus only over the exogenous states $(u_{t+1}, r_{t+1}^{*,RE}, \xi_{t+1}^d)$. The endogenous state variable φ_{t-1} is simply the lagged Lagrange multiplier on the New Keynesian Phillips curve with housing. The endogenous state variable μ_{t-1} is given for all $t \geq 0$ by

$$
\mu_t = \beta^{-(t+1)} \left(\lambda_0 + \mu_{-1}\right) + \beta^{-t} \lambda_1 + ... + \beta^{-1} \lambda_t.
$$

The initial values (φ_{-1}, μ_{-1}) are given at time zero and equal to zero in the case of time-zero-optimal monetary policy.

For periods $t < T'$, where T' is the period from which housing price expectations are rational and the lower bound constraint ceases to bind, the value functions depend on time, thereafter they are time-invariant. Likewise for sufficiently large T', the value functions $W_t(\cdot)$ and $W_{t+1}(\cdot)$ will become very similar.

We can numerically solve for the value function $W_t(\cdot)$ by value function iteration, starting with $W_{T'}$ which is the value function associated with the linear-quadratic problem with RE.

H.7 Optimal Targeting Rule

Differentiating (H.8) with respect to $\{\pi_t, y_t^{gap}, i_t\}$ yields:

$$\begin{aligned}
\frac{\partial W_t}{\partial \pi_t} &= -\Lambda_\pi \pi_t + (\varphi_t - \varphi_{t-1}) - \mu_{t-1}\varphi = 0 \\
\frac{\partial W_t}{\partial y_t^{gap}} &= -\Lambda_y y_t^{gap} - \varphi_t \kappa_y + \lambda_t = 0 \\
\frac{\partial W_t}{\partial i_t} &= \gamma_t + \lambda_t \varphi + \mu_{t-1}\varphi = 0 \text{ and } \gamma_t \left(i_t - \underline{i}\right) = 0.
\end{aligned}$$

Combining these first-order conditions, we can derive the following targeting rule which characterizes optimal monetary policy

$$\Lambda_\pi \pi_t + \frac{\Lambda_y}{\kappa_y}\left(y_t^{gap} - y_{t-1}^{gap}\right) + \frac{\lambda_{t-1}}{\kappa_y} + \mu_{t-1}\left(\varphi + \frac{1}{\kappa_y}\right) + \frac{\gamma_t}{\varphi \kappa_y} = 0,$$

where γ_t is the Lagrange multiplier associated with the lower bound on interest rates. If the lower bound on the nominal interest rate does not bind in the current period, we have $\gamma_t = 0$. Furthermore, if the lower bound has not been binding up to period t, the IS equation has not posed a constraint for the monetary policymaker. Thus, $\lambda_{t-1} = \lambda_{t-k} = 0$ for all $k = 0, 1, ..., t$. For an initial value of $\mu_{-1} = 0$, it follows that $\mu_{t-1} = 0$. The targeting rule then collapses to

$$\Lambda_\pi \pi_t + \frac{\Lambda_y}{\kappa_y}\left(y_t^{gap} - y_{t-1}^{gap}\right) = 0,$$

which is the same as in Clarida, Galí, and Gertler (1999).

The Lagrange multiplier $\gamma_t \leq 0$ captures the cost of a currently binding lower bound. If $\gamma_t < 0$, the optimal policy requires a compensation in the form of a positive output gap or inflation. The multipliers λ_{t-1} and μ_{t-1} capture promises from past commitments when the lower bound was binding.

Another way to express equation (H.9) is to write it as

$$\Lambda_\pi \pi_t + \frac{\Lambda_y}{\kappa_y}\left(y_t^{gap} - y_{t-1}^{gap}\right) + \frac{1}{\varphi \kappa_y}\left[\gamma_t - \frac{1 + \beta + \varphi \kappa_y}{\beta}\gamma_{t-1} + \frac{\gamma_{t-2}}{\beta}\right] = 0. \tag{H.9}$$

House prices do not enter the optimal target criterion directly but larger fluctuations in house prices make the lower bound bind more often and for a longer period of time. The optimal policy, thus, requires larger compensations in terms of positive output gaps and inflation. To implement this, the nominal interest rate needs to be kept longer at the lower bound.

H.8 Calibration of C_q/C_Y

To calibrate C_q/C_Y, the ratio of the consumption elasticities to housing prices and income, respectively, note that from appendix "Second-Order Conditions for Optimal Allocation" in Adam and Woodford (2021), we have

$$C_{q^u}(Y_t, q_t^u; \xi_t) \equiv \frac{\partial C(Y_t, q_t^u; \xi_t)}{\partial q^u} = \frac{-\frac{1}{q_t^u}\frac{1}{1-\widetilde{\alpha}}\Omega(q_t^u, \xi_t) C(Y_t, q_t^u, \xi_t)^{\chi+1}}{1 + (1+\chi)\,\Omega(q_t^u, \xi_t) C(Y_t, q_t^u, \xi_t)^{\chi}}$$

where $\chi \equiv \frac{\tilde{\sigma}^{-1}}{1-\tilde{\alpha}} - 1$. In our formulation, we have defined

$$C_q \quad \equiv \quad \frac{\partial C(Y_t, q_t^u; \xi_t)}{\partial \ln q_t^u} = \frac{\partial C(Y_t, q_t^u; \xi_t)}{\partial q_t^u} \frac{\partial q_t^u}{\partial \ln q_t^u} = C_{q^u}(Y_t, q_t^u; \xi_t) \frac{q_t^u}{C_t}$$

so that we have

$$C_q = -\frac{\frac{1}{1-\widetilde{\alpha}} \Omega(q_t^u, \xi_t) C(Y_t, q_t^u, \xi_t)^{\chi+1}}{C(Y_t, q_t^u, \xi_t) + (1+\chi)\, \Omega(q_t^u, \xi_t) C(Y_t, q_t^u, \xi_t)^{\chi+1}}.$$

From the appendix in Adam and Woodford (2021) we also have

$$C_Y(Y_t, q_t^u, \xi_t) \equiv \frac{\partial C_Y(Y_t, q_t^u, \xi_t)}{\partial Y_t} = \frac{1-g_t}{1 + \Omega(q_t^u, \xi_t)\,(1+\chi)\, C(Y_t, q_t^u, \xi_t)^{\chi}}$$

so that in our notation

$$C_Y \equiv \frac{\partial C_Y(Y_t, q_t^u, \xi_t)}{\partial \ln Y_t} = \frac{(1-g_t)\, Y_t}{C(Y_t, q_t^u, \xi_t) + \Omega(q_t^u, \xi_t)\,(1+\chi)\, C(Y_t, q_t^u, \xi_t)^{\chi+1}}.$$

We then have

$$\frac{C_q}{C_Y} \quad = \quad \frac{\frac{-\frac{1}{1-\widetilde{\alpha}} \Omega(q_t^u, \xi_t) C(Y_t, q_t^u, \xi_t)^{\chi+1}}{C(Y_t, q_t^u, \xi_t) + (1+\chi) \Omega(q_t^u, \xi_t) C(Y_t, q_t^u, \xi_t)^{\chi+1}}}{\frac{(1-g_t) Y_t}{C(Y_t, q_t^u, \xi_t) + \Omega(q_t^u, \xi_t)(1+\chi) C(Y_t, q_t^u, \xi_t)^{\chi+1}}} = -\frac{1}{1-\widetilde{\alpha}} \frac{\Omega(q_t^u, \xi_t) C(Y_t, q_t^u, \xi_t)^{\chi+1}}{(1-g_t)\, Y_t}.$$

In the steady state, we have $\overline{Y}(1-\overline{g}) = \overline{C} + \overline{\Omega C}^{\chi+1}$, which says that privately consumed output $\overline{Y}(1-\overline{g})$ is divided up into consumption $\overline{C}$ and resources invested in the housing sector, $\overline{\Omega C}^{1+\chi}$. We thus have that

$$\frac{\overline{\Omega C}^{\chi+1}}{\overline{Y}(1-\overline{g})} \quad = \quad 1 - \frac{\overline{C}}{\overline{Y}(1-\overline{g})} = 1 - \frac{\overline{C}}{\overline{C} + \overline{\Omega C}^{\chi+1}} = 1 - \frac{1}{1 + \overline{\Omega C}^{\chi}}.$$

Following Adam and Woodford (2021), we set this to the share of housing investment to total consumption, $\overline{\Omega C}^{\chi}$, equal to 6.3%, so that in steady state we have

$$\frac{C_q}{C_Y} = -\frac{1}{1-\widetilde{\alpha}} \left(1 - \frac{1}{1.063}.\right)$$

Finally, following Adam and Woodford (2021), we set the long-run elasticity of housing supply equal to five, which implies $\widetilde{\alpha} = 0.8$, so that

$$\frac{C_q}{C_Y} = -5 \left(1 - \frac{1}{1.063}\right) \approx -0.29633.$$

From this, it follows that

$$C_Y = \frac{(1-g)Y}{C + (1+\chi)\, \Omega C^{\chi+1}} = \frac{C + k}{C + \frac{\tilde{\sigma}^{-1}}{1-\tilde{\alpha}} k} = \frac{1 + \frac{k}{C}}{1 + \frac{\tilde{\sigma}^{-1}}{1-\tilde{\alpha}} \frac{k}{C}} = \frac{1 + 0.063}{1 + 5 \cdot 0.063} = 0.80836$$

and

$$C_q = -0.29633 \cdot 0.80836 = -0.23954.$$