#### A Behavioral Heterogeneous Agent New Keynesian Model

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#### Abstract

We develop a New Keynesian model with household heterogeneity and bounded rationality in the form of cognitive discounting. The behavioral heterogeneous agent New Keynesian model is consistent with recent empirical facts about the effectiveness and the transmission mechanisms of monetary and fiscal policy: monetary policy is amplified through indirect general equilibrium effects, fiscal multipliers on consumption are positive and the model delivers empirically-realistic intertemporal marginal propensities to consume. Simultaneously, and consistent with the data, the model resolves the forward guidance puzzle and remains stable at the effective lower bound as the model features equilibrium determinacy even under an interest-rate peg. The model is analytically tractable and nests a wide range of existing models as special cases, none of which can produce all the listed features within one model. We extend our framework to derive an equivalence result between models with bounded rationality and models with incomplete information and learning.

**Keywords:** Behavioral Macroeconomics, Heterogeneous Households, Monetary Policy, Forward Guidance, Fiscal Policy, New Keynesian Puzzles, Determinacy, Lower Bound

**JEL Codes:** E21, E52, E62, E71

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#### 1 Introduction

Recent empirical evidence on the transmission mechanisms and effectiveness of monetary and fiscal policies challenges the basic model of monetary policy, the New Keynesian model, in many dimensions: Monetary policy is transmitted to household consumption to a large extent through indirect, general equilibrium effects. Government spending increases private consumption substantially. Households' marginal propensities to consume (MPCs) out of transitory income changes are high on average in the year of the received income windfall and also remain high even in the year after. Announcements of future monetary policy changes, on the other hand, have relatively weak effects on current economic activity. Despite these weak effects of forward guidance, advanced economies have not faced large instabilities during long spells at the binding effective lower bound.<sup>1</sup>

In this paper, we propose a new framework which accounts for all these empirical facts simultaneously. To this end, we construct a New Keynesian model featuring household heterogeneity and bounded rationality in the form of cognitive discounting. The resulting behavioral heterogeneous agent New Keynesian model—or behavioral HANK—is analytically tractable which enables a clear understanding of the two frictions and how they interact. We show that it is indeed the interaction of bounded rationality and household heterogeneity that allows our model to be reconciled with the empirical evidence. Moreover, the model nests a broad spectrum of existing models—including the standard New Keynesian model, rational HANK models, and representative agent models which depart from the full-information rational expectations hypothesis (FIRE). None of these other models, however, can account for the listed empirical facts simultaneously.

To arrive at our framework, we extend the textbook New Keynesian model by household heterogeneity and bounded rationality in ways that preserve the tractability of the model. We assume that there are two groups of households, namely savers and hand-to-mouth households, and households face an uninsurable, idiosyncratic risk of switching their type. This generates heterogeneity in income, MPCs, and a precautionary savings motive of household. We introduce bounded rationality by the means of cognitive discounting. Households anchor their expectations about future macroeconomic variables to the steady state but are myopic or inattentive to future deviations from it. As a result, average expectations underreact

<sup>&</sup>lt;sup>1</sup>See, e.g., Ampudia et al. (2018), Samarina and Nguyen (2019) and Holm et al. (2021) for the empirical relevance of indirect channels in the transmission of monetary policy, Galí et al. (2007), Perotti (2007) or Dupor et al. (2021) for empirical evidence on the positive consumption response to fiscal spending, Auclert et al. (2018), Fagereng et al. (2021), Jappelli and Pistaferri (2020), Auclert (2019) and Patterson (2019) document empirical patterns of MPCs and see, for example, Del Negro et al. (2015), Miescu (2022) and Roth et al. (2021) for empirical evidence on the (in-)effectiveness of forward guidance and Cochrane (2018) on the stability at the lower bound.

to news, as empirically documented in Coibion and Gorodnichenko (2015), Bordalo et al. (2020) and Angeletos et al. (2021).<sup>2</sup>

In the behavioral HANK model, indirect general equilibrium effects account for large parts of how monetary policy is transmitted to consumption. Consistent with the data, households that exhibit high MPCs are more exposed to the business cycle in the behavioral HANK model. Thus, after an expansionary monetary policy shock (and likewise after a fiscal spending shock), high MPC households disproportionately benefit from the increase in output. This leads to an amplification of contemporaneous monetary policy through general equilibrium via a Keynesian-type multiplier. In addition, in the useful benchmark of a constant real interest rate, these general equilibrium effects also generate positive fiscal multipliers on consumption.

Even though the behavioral HANK model generates amplification of contemporaneous shocks through these indirect effects, the model does not suffer from the forward guidance puzzle: announced decreases in the interest rate in the future have weaker effects on today's output than a current decrease in the interest rate and the effectiveness on today's output decreases with the horizon of the announcement. There are two competing forces shaping the effectiveness of a forward guidance shock on today's output. First, the general equilibrium amplification channel which is at work in response to contemporaneous monetary policy shocks is ceteris paribus compounded over time. The reason is that when savers expect higher consumption in the future, they decrease their precautionary savings today as they would disproportionately benefit from the increase in output in the hand-to-mouth state. Yet, the behavioral agents cognitively discount both this indirect general equilibrium effect as well as the direct effects of the future interest rate changes. This dampens the effects of forward guidance. With every reasonable calibration, the second channel dominates, leading to a dampening of the effects of forward guidance. Additionally, the behavioral HANK model remains stable during prolonged periods at the effective lower bound (ELB), as the model features equilibrium determinacy under an interest-rate peg for a large area of the parameter space.

The intertemporal MPCs (iMPCs) have been shown to be key statistics for monetary and fiscal policy analyses (Auclert et al. (2018), Wolf (2021), Kaplan and Violante (2020)). We derive the iMPCs in the behavioral HANK model analytically. To the best of our knowledge, we are the first ones to do so in a HANK model featuring a departure from FIRE. The behavioral HANK model quantitatively matches the empirical iMPCs. Thanks to the

<sup>&</sup>lt;sup>2</sup>We show in Appendix A.7 how we can microfound our behavioral setup by the means of a noisy-signal extraction problem of otherwise rational agents. Angeletos and Lian (2017) show how other forms of bounded rationality or lack of common knowledge lead to observationally-equivalent outcomes but abstract from household heterogeneity.

closed-form solution, the model also sheds light on how the iMPCs depend on household heterogeneity frictions and bounded rationality. Boundedly-rational households tend to save more than rational households out of the income windfall as they cognitively discount the decrease in their future marginal utility which lowers the current MPC. As time progresses, however, bounded rationality increases the aggregate MPC as the behavioral savers start to consume their (higher) savings. These dynamic effects are particularly pronounced when idiosyncratic risk is relatively high.

We demonstrate that the behavioral HANK model can have qualitatively different policy implications than its rational counterpart by applying our framework to study the most effective timing of monetary policy. Consider an overheating economy which the monetary authority wants to tame by hiking interest-rates by a cumulative x%. This rate hike can be implemented immediately or by raising the rate  $\frac{x}{k}\%$  over k consecutive periods. A well-known feature of the RANK model is that monetary policy becomes more effective the more it is back-loaded. While this is also the case in the rational HANK model, the opposite is true in the behavioral HANK model: monetary policy is more effective when it is completely front-loaded, i.e., when k=1. The increased effectiveness is driven by the fact that the hand-to-mouth agents' incomes contract more strongly leading to a strong decrease in aggregate demand. Thus, the increased effectiveness of front-loading the policy comes at the cost of an increase in inequality.

We close by showing how to extend our framework to derive an equivalence result between models with bounded rationality and models of incomplete information and learning. To this end, we assume that behavioral agents anchor their beliefs to *past observations* of the respective variable instead of the respective steady state values. This extended behavioral HANK model is observationally equivalent to models featuring incomplete information and learning (see Angeletos and Huo (2021) and Gallegos (2021)).<sup>3</sup>

We calibrate this extended model to match recent findings from survey expectations data and show that the model endogenously generates hump-shaped responses of macro aggregates to monetary policy shocks. The backward-looking component in households' expectations induces endogenous persistence and thus, households respond as if contemporaneous (or future) shocks are persistent even when the shocks are actually completely transitory. This yields an endogenous behavioral-amplification mechanism that is absent in existing HANK models.

<sup>&</sup>lt;sup>3</sup>Angeletos and Huo (2021) derive an equivalence result between models with incomplete information and learning with models which include behavioral myopia and an additional friction such as habit persistence or adjustment costs. We now complement their equivalence result with a behavioral model that solely relies on one behavioral friction.

Related Literature The literature so far treats the empirical facts laid out in the Introduction mostly independently from each other. The HANK and TANK literature – both with quantitative and analytical models – have highlighted the transmission of monetary policy through indirect, general equilibrium effects (Kaplan et al. (2018), Auclert (2019), Auclert et al. (2020)), positive fiscal multipliers on consumption (Auclert et al. (2018), Galí et al. (2007)), and the role of iMPCs (Auclert et al. (2018), Cantore and Freund (2021), Kaplan and Violante (2020)). On the other side, HANK models have also been used to solve the forward guidance puzzle (McKay et al. (2016), McKay et al. (2017), Hagedorn et al. (2019)).

Werning (2015) and Bilbiie (2021) combine the themes of policy amplification and forward guidance puzzle in HANK. While these two papers focus on slightly different explanation mechanisms, both establish a trade-off inherent in models with household heterogeneity: if HANK models amplify contemporaneous monetary policy (and fiscal policy) through redistributing towards high MPC households, HANK models also dampen precautionary savings desires after a forward guidance shock which further aggravates the forward guidance puzzle.<sup>4</sup> One of our contributions is that our behavioral HANK model overcomes this so-called Catch-22 (Bilbiie (2021)).<sup>5</sup>

A mostly-detached strand of the literature suggests to relax the assumption of full-information rational expectations (FIRE) to weaken the effectiveness of future monetary policies, thereby resolving the forward guidance puzzle (Wiederholt (2015), Angeletos and Lian (2018), Andrade et al. (2019), Gabaix (2020), Pfäuti (2021) and Roth et al. (2021)). We complement these papers by introducing household heterogeneity in terms of iMPCs, asset-market participation status, and exposure to the business cycle. This way, our model cannot only resolve the forward guidance puzzle (and other NK puzzles) but also simultaneously deliver amplification of contemporaneous monetary and fiscal policy through indirect channels, as well as match empirical estimates of iMPCs.

We share the combination of household heterogeneity and some deviation from FIRE

<sup>&</sup>lt;sup>4</sup>Acharya and Dogra (2018) construct a pseudo-RANK model, in which they isolate and highlight the role of precautionary savings dynamics in explaining the solution/aggravation of the forward guidance puzzle.

<sup>&</sup>lt;sup>5</sup>Bilbiie (2021) provides two theoretical possibilities of how to sidestep the Catch-22. The first possibility is a pure risk channel which can, in theory, break the comovement of income risk and inequality. Yet to do so, it requires a calibration which seems highly at odd with the data. A second possibility is to drastically narrow down the policy space: in a world in which monetary policy is described by Wicksellian price level targeting or fiscal policy follows a nominal bond rule, there would be no Catch-22. Hagedorn et al. (2019) use a similar description of fiscal policy to solve the forward guidance puzzle in a quantitative HANK model, in which contemporaneous monetary policy is amplified. Similarly, Kaplan et al. (2016) show that in their quantitative HANK model in Kaplan et al. (2018), there is no Forward Guidance puzzle, conditional on specific fiscal policy responses to a monetary policy shock. In contrast, in our model, there is no Catch-22 independently of the exact specification of monetary and fiscal policy.

with Farhi and Werning (2019), Auclert et al. (2020), Broer et al. (2021), Angeletos and Huo (2021), Laibson et al. (2021), Gallegos (2021), and Bonciani and Oh (2022). In contrast to all these papers, we offer analytical insights into how the two frictions matter for policy analysis, and how the interaction of the two frictions is key to reconcile the model with recent empirical facts as outlined above.

**Outline.** The rest of the paper is structured as follows. We present our behavioral HANK model in Section 2 and our main analytical results in Section 3. In Section 4, we derive our equivalence result and Section 5 concludes.

## 2 A Behavioral HANK Model

In this section, we present our tractable NK model featuring household heterogeneity and bounded rationality (BR).

#### 2.1 Structure of the Model

**Households.** The economy is populated by a unit mass of households, indexed by  $j \in [0, 1]$ . Households obtain utility from (non-durable) consumption,  $C_t^j$ , and dis-utility from working  $N_t^j$ . Households discount future utility at rate  $\beta \in [0, 1]$ . We assume a standard CRRA utility function

$$U(C_t^j, N_t^j) \equiv \frac{(C_t^j)^{1-\gamma}}{1-\gamma} - \frac{(N_t^j)^{1+\varphi}}{1+\varphi},$$

where  $\varphi$  denotes the inverse Frisch elasticity and  $\gamma$  denotes the relative risk aversion. For most of the paper, we focus on  $\gamma = 1$ , that is, log-utility  $log(C_t^j)$ .

Households can save or borrow in government bonds, paying nominal interest  $i_t$ , and acquire shares of intermediate monopolistic firms. We allow for the possibility that households participate in financial markets infrequently. When they do participate, they can freely buy or sell bonds and shares and receive the intermediate firm profits,  $D_t$ . Otherwise, they simply receive the payoff from their previously acquired bonds. We denote households participating in financial markets by S as they will be S avers in equilibrium, and the non-participants by S as they will be S avers with probability S and becomes hand-to-mouth with probability S and becomes hand-to-mouth with probability S and switch with S and switch with S and S are switch with S and S are switch with S and S are switch with

We use the same simplyfing assumptions as in Bilbiie (2021) which allow for a tractable solution. In particular, we assume that households belong to a family whose utilitarian

intertemporal welfare is maximized by its family head. The head can only provide insurance within types but not across types, i.e., the head pools all the resources within types. When households switch from the saver to the hand-to-mouth type, they only keep their government bonds. Stocks cannot be used to self-insure. Using the in- and outflows between both groups and the stationary distribution, we get the following relationships between real, per-capita, beginning-of-period-t+1 bonds,  $B_{t+1}^j$  and end-of-period-t per-capita real values (before moving across types),  $Z_{t+1}^j$ :

$$(1 - \lambda)B_{t+1}^S = s(1 - \lambda)Z_{t+1}^S + (1 - h)\lambda Z_{t+1}^H$$
$$\lambda B_{t+1}^H = (1 - s)(1 - \lambda)Z_{t+1}^S + h\lambda Z_{t+1}^H,$$

which, after using the definition of  $\lambda$ , can be re-written as

$$B_{t+1}^{S} = sZ_{t+1}^{S} + (1-s)Z_{t+1}^{H}$$

$$B_{t+1}^{H} = (1-h)Z_{t+1}^{S} + hZ_{t+1}^{H}.$$
(1)

We allow for the possibility that the family head is boundedly rational (BR) in the way we describe in detail in Section 2.3.<sup>6</sup> The program of the family head is

$$W(B_t^S, B_t^H, \iota_t) = \max_{\{C_t^S, C_t^H, Z_{t+1}^S, Z_{t+1}^H, N_t^S, N_t^H, \iota_{t+1}\}} \left[ (1 - \lambda)U(C_t^S, N_t^S) + \lambda U(C_t^H, N_t^H) \right] + \beta \mathbb{E}_t^{BR} W(B_{t+1}^S, B_{t+1}^H, \iota_{t+1})$$

subject to the flow budget constraints of the savers

$$C_t^S + Z_{t+1}^S + v_t \iota_{t+1} = W_t N_t^S + \iota_t (v_t + D_t) + \frac{1 + i_{t-1}}{1 + \pi_t} B_t^S + T_t^S, \tag{2}$$

and the hand-to-mouth households

$$C_t^H + Z_{t+1}^H = W_t N_t^H + T_t^H + \frac{1 + i_{t-1}}{1 + \pi_t} B_t^H, \tag{3}$$

as well as the non-negativity constraints

$$Z_{t+1}^H, Z_{t+1}^S \ge 0,$$

<sup>&</sup>lt;sup>6</sup>We show in Appendix A.7 how the family head's expectation can be understood as an average expectation over all households' expectations within family where each household receives a noisy signal about the future state.

where  $W_t$  is the real wage,  $\iota_t$  are the shares of stocks traded at price  $v_t$ ,  $B_t$  denotes the liquid asset holdings (government bonds), and  $T_t^j$  are transfers to type-j households. As we will detail below, we assume that these transfers are financed by a proportional tax on profits,  $\tau^D$ , such that they entail a redistribution from S households (who own the firms) to H households.

The optimality conditions are given by the savers' Euler equation

$$U'(C_{t}^{S}) \ge \beta \mathbb{E}_{t}^{BR} \left[ R_{t} \left( sU'(C_{t+1}^{S}) + (1-s)U'(C_{t+1}^{H}) \right) \right]$$
and  $0 = Z_{t+1}^{S} \left[ U'(C_{t}^{S}) - \beta \mathbb{E}_{t}^{BR} \left[ R_{t} \left( sU'(C_{t+1}^{S}) + (1-s)U'(C_{t+1}^{H}) \right) \right] \right],$ 

$$(4)$$

the Euler equation of the hand-to-mouth households

$$U'(C_{t}^{H}) \ge \beta \mathbb{E}_{t}^{BR} \left[ R_{t} \left( (1 - h)U'(C_{t+1}^{S}) + hU'(C_{t+1}^{H}) \right) \right]$$
and  $0 = Z_{t+1}^{H} \left[ U'(C_{t}^{H}) - \beta \mathbb{E}_{t}^{BR} \left[ R_{t} \left( (1 - h)U'(C_{t+1}^{S}) + hU'(C_{t+1}^{H}) \right) \right] \right],$ 

$$(5)$$

and the demand for shares

$$U'(C_t^S) \ge \beta \mathbb{E}_t^{BR} \left[ \frac{v_{t+1} + D_{t+1}}{v_t} U'(C_{t+1}^S) \right] \text{ and } \iota_{t+1} = \iota_t = (1 - \lambda)^{-1},$$
 (6)

with  $R_t \equiv \frac{1+i_t}{1+\pi_{t+1}}$  being today's real interest rate. The respective labor-leisure equations of both types are given by:

$$-U'(N_t^S) = W_t U'(C_t^S) \tag{7}$$

$$-U'(N_t^H) = W_t U'(C_t^H). (8)$$

In what follows, we focus on equilibria in which the H households will always be off their Euler equation—e.g., because they do not have access to financial markets—such that equation (5) always holds with strict inequality. In addition, we follow the tradition of analytical HANK models and assume a zero liquidity equilibrium to keep our model tractable. As shares cannot be transferred to the H state, equation (4) simply prices the shares. Thus, the savers' bond Euler equation is the only Euler equation that is an equilibrium equation. Importantly, it features a self-insurance motive as savers demand bonds to self-insure their idiosyncratic risk of type-switching.

<sup>&</sup>lt;sup>7</sup>See Krusell et al. (2011), McKay et al. (2017), Ravn and Sterk (2017), and Bilbiie (2021).

**Firms.** We assume a standard NK firm side. All households consume the same aggregate basket of individual goods,  $j \in [0, 1]$ ,

$$C_t = \left(\int_0^1 C_t(j)^{\frac{\epsilon - 1}{\epsilon}} dj\right)^{\frac{\epsilon}{\epsilon - 1}}$$

where  $\epsilon > 1$  is the elasticity of substitution between the individual goods. Each firm faces demand

$$C_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon} C_t$$

where  $P_t(j)/P_t$  denotes the individual price relative to the aggregate price index,

$$P_t^{1-\epsilon} = \int_0^1 P_t(j)^{1-\epsilon} dj,$$

and produces with the linear technology

$$Y_t(j) = N_t(j).$$

The real marginal cost is given by  $W_t$ . We assume that the government pays a subsidy  $\tau^S$  on revenues to induce marginal cost pricing. The subsidy is financed by a lump-sum tax on firms  $T_t^F$ . Hence, the profit function is:

$$D_t(j) = (1 + \tau^S)[P_t(j)/P_t]Y_t(j) - W_t N_t(j) - T_t^F.$$

Total profits are then  $D_t = Y_t - W_t N_t$  and are zero in steady state. Given zero steady state profits, we have a full-insurance steady state, i.e.,  $C^H = C^S = C$ . In the log-linear dynamics around this steady state, profits vary inversely with the real wage  $\hat{d}_t = -\hat{w}_t$ . We allow for steady state inequality in Appendix C and show that our results are not driven by this assumption and are in fact barely affected even by substantial inequality in the steady state.

**Government.** Fiscal policy induces the optimal steady state subsidy and taxes profits at rate  $\tau^D$  and rebates these taxes as a transfer to H households, such that

$$T^H = \frac{\tau^D}{\lambda} D_t.$$

As will become clear later, the level of  $\tau^D$  is key for the exposure of H households to the business cycle and thus for the cyclicality of inequality. Here, we abstract from government

<sup>&</sup>lt;sup>8</sup>Throughout the paper variables with a hat on top denote log-deviations from steady state.

spending to keep it simple, but we introduce government spending in Section 3.

In most of the analysis, we assume that monetary policy follows a standard (log-linearized) Taylor rule<sup>9</sup>

$$\widehat{i_t} = \phi \pi_t + \epsilon_t^{MP}, \tag{9}$$

with  $\epsilon_t^{MP}$  being the monetary policy shock which will be specified in the sections below.

Market Clearing. Market clearing requires that the goods market clears

$$Y_t = C_t = \lambda C_t^H + (1 - \lambda)C_t^S$$

and the labor market clears

$$N_t = \lambda N_t^H + (1 - \lambda) N_t^S.$$

#### 2.2 Log-Linearized Model

We now focus on the log-linearized dynamics around the full-insurance, zero-liquidity steady state. The market clearing conditions yield  $\hat{y}_t = \hat{c}_t = \lambda \hat{c}_t^H + (1-\lambda)\hat{c}_t^S$  and  $\hat{n}_t = \lambda \hat{n}_t^H + (1-\lambda)\hat{n}_t^S$ . Importantly, we can write the consumption of the hand-to-mouth households as

$$\widehat{c}_t^H = \chi \widehat{y}_t, \tag{10}$$

with

$$\chi = 1 + \varphi \left( 1 - \frac{\tau^D}{\lambda} \right) \tag{11}$$

measuring the cyclicality of the H household's consumption.<sup>10</sup> Auclert (2019) and Patterson (2019) document that households with higher MPCs tend to be more exposed to aggregate income fluctuations, which implies  $\chi > 1$ . As  $\chi$  is a key coefficient in our model, we will vary  $\chi$  throughout the paper to understand its role in shaping our results. Different levels of  $\chi$  should then be thought of as different  $\tau^D$ , thus, different redistributive tax-transfer systems.

Combining equation (10) with the goods market clearing condition yields

$$\widehat{c}_t^S = \frac{1 - \lambda \chi}{1 - \lambda} \widehat{y}_t, \tag{12}$$

<sup>&</sup>lt;sup>9</sup>We study more general Taylor rules in Appendix A.

<sup>&</sup>lt;sup>10</sup>See Appendix A.1 for the derivation of equation (10).

which implies that consumption inequality is given by:

$$\widehat{c}_t^S - \widehat{c}_t^H = \frac{1 - \chi}{1 - \lambda} \widehat{y}_t. \tag{13}$$

Equation (13) shows that if  $\chi > 1$ , inequality is countercyclical as it varies negatively with total output, i.e., increases in recessions and decreases in booms. In line with the empirical evidence on the covariance between MPCs and business-cycle exposure, the data points toward  $\chi > 1$  when looking at the cyclicality of inequality. Coibion et al. (2017), Ampudia et al. (2018) and Samarina and Nguyen (2019) provide evidence of countercyclical inequality conditional on monetary policy shocks. Almgren et al. (2019) show that output in countries with higher shares of hand-to-mouth households responds more strongly to monetary policy shocks which, as we will see later on, implies  $\chi > 1$  in our model.

The log-linearized bond Euler equation of S households is given by

$$\widehat{c}_t^S = s \mathbb{E}_t^{BR} \left[ \widehat{c}_{t+1}^S \right] + (1 - s) \mathbb{E}_t^{BR} \left[ \widehat{c}_{t+1}^H \right] - \frac{1}{\gamma} \left( \widehat{i}_t - \mathbb{E}_t^{BR} \pi_{t+1} \right). \tag{14}$$

We will, following the assumption in Gabaix (2020), often focus on the case in which the agents are rational with respect to the real rate, i.e., we replace  $\mathbb{E}_t^{BR}\pi_{t+1}$  with  $\mathbb{E}_t\pi_{t+1}$  in equation (14). This is a conservative choice, in the following sense: we show in Appendix C that our results go through with boundedly-rational real-rate expectations and, in fact, the results become even stronger in that case. For the case without type-switching, i.e., for s = 1, equation (14) boils down to a standard Euler equation. For  $s \in [0,1)$ , however, the agent takes into account that she might switch type and self-insures against becoming hand-to-mouth next period.

**Supply Side.** We distinguish between two set-ups for the supply side: For the main part, we assume that firms are not forward-looking and, thus, we can summarize the supply side of the economy by a static Phillips Curve

$$\pi_t = \kappa \widehat{y}_t, \tag{15}$$

where  $\kappa \geq 0$  captures the slope of the Phillips Curve.<sup>11</sup> Yet, we also relax this assumption in Appendix C and show that a forward-looking (NK) Phillips Curve barely affects our results.

<sup>&</sup>lt;sup>11</sup>To arrive at this static Phillips curve, we can either assume that firms are completely myopic or that they face a Rotemberg-style adjustment cost relative to yesterday's market average price index (see Bilbiie (2021)).

#### 2.3 Bounded Rationality

We follow Gabaix (2020) and model bounded rationality as a form of cognitive discounting.<sup>12</sup> Let  $X_t$  be a random variable (or vector of variables) and let us define  $X_t^d$  as some default value the agent may have in mind and  $\tilde{X}_{t+1} \equiv X_{t+1} - X_t^d$  denotes the deviation from this default value.<sup>13</sup> The behavioral agent's expectation about  $X_{t+1}$  is then given by

$$\mathbb{E}_{t}^{BR}\left[X_{t+1}\right] = \mathbb{E}_{t}^{BR}\left[X_{t}^{d} + \tilde{X}_{t+1}\right] \equiv X_{t}^{d} + \bar{m}\mathbb{E}_{t}\left[\tilde{X}_{t+1}\right],\tag{16}$$

where  $\mathbb{E}_t[\cdot]$  is the rational expectations operator and  $\bar{m} \in [0,1]$  is the behavioral parameter which captures the degree of rationality. A higher  $\bar{m}$  denotes a smaller deviation from rational expectations and rational expectations are captured by  $\bar{m} = 1$ . We see from equation (16) that the behavioral agent anchors her expectations to the default value and cognitively discounts expected future deviations from this default value. For now, we focus on the steady state as the default value but relax this assumption in Section 4.

While we present a way to microfound  $\bar{m}$  in Appendix A.7, note, that the exact microfoundation or underlying behavioral friction is not crucial for the rest of our analysis. For example, Angeletos and Lian (2017) show how other forms of bounded rationality or lack of common knowledge lead to observationally-equivalent expectations for the case in which  $X_t^d$  denotes the steady state.

Log-linearizing equation (16) around the steady state yields

$$\mathbb{E}_{t}^{BR}\left[\widehat{x}_{t+1}\right] = (1 - \bar{m})\widehat{x}_{t}^{d} + \bar{m}\mathbb{E}_{t}\left[\widehat{x}_{t+1}\right] \tag{17}$$

and when  $X_t^d$  is the steady state value, we obtain  $\mathbb{E}_t^{BR}[\widehat{x}_{t+1}] = \bar{m}\mathbb{E}_t[\widehat{x}_{t+1}]$ . In Appendix B, we discuss empirical estimates of  $\bar{m}$  and how we can map recent evidence in Coibion and Gorodnichenko (2015) and Angeletos et al. (2021) to  $\bar{m}$ . As a benchmark, we follow Gabaix (2020) and set  $\bar{m}$  to 0.85, which is a rather conservative choice, given the empirical evidence. As one goal of our paper is to understand the role of  $\bar{m}$  for policy analysis and the interplay of  $\bar{m}$  and household heterogeneity, we will also consider different values for  $\bar{m}$ .

<sup>&</sup>lt;sup>12</sup>While Gabaix (2020) embeds bounded rationality in a NK model, the basic idea of behavioral inattention (or sparsity) has been proposed by Gabaix earlier already (see Gabaix (2014, 2016)) and a handbook treatment of behavioral inattention is given in Gabaix (2019). Benchimol and Bounader (2019) and Bonciani and Oh (2021) study optimal monetary policy in a RANK and TANK model, respectively, with this kind of behavioral frictions.

 $<sup>^{13}</sup>$ Gabaix (2020) focuses on the case in which  $X_t$  denotes the state of the economy. He shows (Lemma 1 in Gabaix (2020)) that this form of cognitive discounting also applies to all other variables. We, on the other hand, directly apply cognitive discounting to all variables. Given Lemma 1 in Gabaix (2020), our results would be unchanged, but our more direct method simplifies some of the derivations, especially in Section 4. Appendix A.6 derives our results following the approach in Gabaix (2020).

#### 3 Results

In this section, we first show how the behavioral HANK model can be summarized by three equations isomorphic to the textbook RANK model. This allows us to show how the behavioral HANK model nests a wide spectrum of existing models and show how it overcomes several challenges present in these existing models. What is more, we show how only the behavioral HANK model can account for the empirical facts recently documented in the literature, simultaneously. We then analytically characterize the intertemporal marginal propensities to consume and show how they depend on bounded rationality, household heterogeneity, and the interaction of the two. We end the section by highlighting that the behavioral HANK model leads to different policy implications than its rational counterpart.

#### 3.1 The Three-Equation Representation

The behavioral HANK model can be summarized by three equations: a Phillips curve, representing the aggregate supply side captured by equation (15), a rule for monetary policy (equation (9)), which together with the *behavioral HANK IS equation* determines aggregate demand. To obtain the behavioral HANK IS equation, we combine the hand-to-mouth households' consumption (10) with the savers' consumption (12) and their consumption Euler equation (14).<sup>14</sup>

**Proposition 1.** The behavioral HANK IS equation is given by

$$\widehat{y}_t = \psi_f \mathbb{E}_t \widehat{y}_{t+1} - \psi_c \frac{1}{\gamma} \left( \widehat{i}_t - \mathbb{E}_t \pi_{t+1} \right), \tag{18}$$

where

$$\psi_f \equiv \bar{m}\delta = \bar{m}\left[1 + (\chi - 1)\frac{1 - s}{1 - \lambda\chi}\right]$$

and

$$\psi_c \equiv \frac{1 - \lambda}{1 - \lambda \chi}.$$

Compared to RANK, two extra coefficients show up:  $\psi_c$  and  $\psi_f$ .  $\psi_c$  governs the sensitivity of today's output with respect to the contemporaneous real interest rate.  $\psi_c$  is shaped by household heterogeneity, in particular by the share of H households  $\lambda$  and their business-cycle

<sup>&</sup>lt;sup>14</sup>All derivations are in Appendix A.

exposure  $\chi$ . As the H households are more exposed to the business cycle ( $\chi > 1$ ),  $\psi_c > 1$  and contemporaneous monetary policy is amplified through general equilibrium forces.

The second new coefficient in the behavioral HANK IS equation (18),  $\psi_f$ , captures the sensitivity of today's output with respect to changes in expected future output.  $\psi_f$  is shaped by household heterogeneity and the behavioral friction as it depends on the cyclicality of income risk and the degree of bounded rationality of households as well as the interaction of the two frictions. Given countercyclical income inequality, income risk is also countercyclical which manifests itself in  $\delta > 1$ . This countercyclical risk induces compounding in the Euler equation and, thus, competes with the empirically observed underreaction of aggregate expectations ( $\bar{m} < 1$ ) which induces discounting in the Euler equation. We see in the following sections that even for a small degree of bounded rationality—much smaller than the empirics suggest—the discounting through bounded rationality dominates the compounding through countercyclical income risk. Hence, in the behavioral HANK model it holds that  $\psi_f < 1$  which makes the economy less sensitive to expectations and news about the future which is key to resolve the NK puzzles.

Equation (18) nests a wide range of existing IS equations: the IS equation in the standard rational-expectations RANK model by setting  $\psi_f = \psi_c = 1$ , RANK models deviating from FIRE by  $\delta = \psi_c = 1$ , TANK models by setting  $\bar{m} = \psi_f = 1$ , and rational HANK models by  $\bar{m} = 1.15$  We discuss this in more detail in section 3.4.

Baseline Calibration. We set the parameters close to the calibration in Bilbiie (2020) and Bilbiie (2021) which is set in order to replicate several findings on the New Keynesian cross coming from more quantitative HANK models. We set  $\chi=1.48$  which implies that H agents' income is relatively sensitive to aggregate fluctuations, in line with empirical findings in Auclert (2019) and Patterson (2019). We set the share of H agents to one third,  $\lambda=0.33$ , and the probability of an S household to become hand-to-mouth next period to 5.4%, i.e., s=0.946 (this corresponds to a s of 0.8 in annual terms). We focus on log utility,  $\gamma=1$ , and set the slope of the Phillips Curve to  $\kappa=0.02$ . The cognitive discounting parameter,  $\bar{m}$  is set to 0.85, as explained in Section 2.3. Details on the calibration and a discussion of the robustness of our findings for changing calibrations are presented in Appendix B. Note, that even when we vary certain parameters, we always keep  $\lambda<\chi^{-1}$ .

<sup>&</sup>lt;sup>15</sup>For the RANK model, see, for example, Woodford (2003) or Galí (2015), for the RANK models differing from FIRE, see, for example, Angeletos and Lian (2018), Woodford (2019), or Gabaix (2020), and for rational TANK or THANK models, see Bilbiie (2008), McKay et al. (2017) or ?

#### 3.2 Monetary Policy

We now show how the behavioral HANK model can generate amplification of contemporaneous monetary policy through indirect effects while it solves the forward guidance puzzle at the same time. Additionally, we discuss determinacy conditions of the behavioral HANK model and show that it is stable at the effective lower bound.

General equilibrium amplification and Forward Guidance. We start by showing how the behavioral HANK model generates general equilibrium amplification of current monetary policy, while simultaneously ruling out the forward guidance puzzle. The forward guidance puzzle states that announcements about future changes in the interest rate affect output today as strong (or even stronger) than contemporaneous changes in the interest rate. Miescu (2022) provides empirical evidence that conventional monetary policy is more effective than forward guidance. Consistent with these findings, Roth et al. (2021) show, by combining experimental evidence with theory, that forward guidance has relatively weak effects on consumption.

Let us now consider two different i.i.d. monetary policy experiments: we define a contemporaneous monetary policy shock as a surprise decrease in the interest rate today and a forward guidance shock as a news shock today about a decrease in the interest rate at some horizon k.

**Proposition 2.** In the behavioral HANK model, there is amplification of contemporaneous monetary policy relative to RANK if and only if

$$\chi > 1,\tag{19}$$

and the forward guidance puzzle is ruled out if

$$\bar{m}\delta + \frac{1}{\gamma} \frac{1 - \lambda}{1 - \lambda \gamma} \kappa < 1. \tag{20}$$

The behavioral HANK model generates amplification of contemporaneous monetary policy with respect to the RANK model whenver  $\chi > 1$ , that is, when high-MPC households are relatively sensitive to aggregate income fluctuations. As discussed in Section 2.2, this is consistent with empirical findings. With  $\chi > 1$ , the income of H agents moves more than one-to-one with aggregate output. Hence, after a decrease in the interest rate, a disproportionate share of the extra income is received by H agents and, thus, the high-MPC households in the

<sup>&</sup>lt;sup>16</sup>Detailed analyses of the forward guidance puzzle in RANK are provided by McKay et al. (2016) and Giannoni et al. (2015).

economy. This amplifies the increase in output through general equilibrium. The behavioral friction leaves the relative importance of direct vs. indirect effects—i.e., amplification of contemporaneous monetary policy—unaltered, as amplification of a contemporaneous i.i.d. shock is solely determined by the static redistribution towards the high MPC households.

Turning towards forward guidance, it is still the case that with  $\chi>1$  the income of H agents moves more than one-to-one with aggregate income. In this case, savers who self-insure against becoming hand-to-mouth in the future want less insurance when they expect a decrease in the interest rate since if they become hand-to-mouth they would benefit more from the increase in aggregate income. Hence, after a forward guidance shock, savers decrease their precautionary savings which compounds the increase in output today. Yet, as savers are boundedly rational, they cognitively discount these effects taking place in the future. Importantly, the savers cognitively discount both the future increase in output as well as the general equilibrium implication for their precautionary savings, thereby decreasing the effects of the forward guidance shock on today's consumption. In sum, if  $\bar{m} < 0.93$ , there is no forward guidance puzzle in the behavioral HANK model.

We now compare the behavioral HANK model to its rational counterpart to show how the behavioral HANK model overcomes a major trade-off inherent in the rational HANK model – the Catch-22. As Section 2.3 shows, we can recover the rational version of our model with  $\bar{m}=1$ . Using this, we can see how Proposition 2 nests the Catch-22 (Bilbiie (2021)). The Catch-22 describes the trade-off that the rational HANK model can either generate amplification of contemporaneous monetary policy or solve the forward guidance puzzle. To see this, note that with  $\bar{m}=1$  the forward guidance puzzle is resolved when

$$\delta + \frac{1}{\gamma} \frac{1 - \lambda}{1 - \lambda \chi} \kappa < 1$$

which requires

$$\chi < 1$$
,

as otherwise  $\delta > 1$ . Assuming  $\chi < 1$ , however, leads to dampening of contemporaneous monetary policy instead of amplification.

We graphically illustrate the Catch-22 of the rational THANK model and the resolution of it in the behavioral HANK model in Figure 1. The figure shows the response of contemporaneous output relative to the initial response in the RANK model with rational expectations for anticipated i.i.d. monetary policy shocks occurring at different times k and a Taylor coefficient of 0 (as in Bilbiie (2021)).<sup>17</sup>

<sup>&</sup>lt;sup>17</sup>Under fully-rigid prices (i.e.,  $\kappa = 0$ ), the RANK model would deliver a constant response for all k. The same is true for TANK, i.e., THANK without type switching. Whether the constant response would lie

The orange-dotted line denotes the baseline calibration of the rational THANK model. We see that this model is able to generate contemporaneous amplification of monetary policy shocks, that is, an output response that is relatively stronger than in RANK. Put differently, the GE effects are relatively strong. Yet, at the same time, it exacerbates the forward guidance puzzle as shocks occurring in the future have much stronger effects on today's output than contemporaneous shocks. The black-dashed-dotted line shows how the forward guidance puzzle can be resolved by allowing for  $\chi < 1$ . Yet, this comes at the cost that the model is unable to generate amplification of contemporaneous monetary policy shocks. Furthermore, even for the quite low  $\chi$ , the decay happens relatively slowly.<sup>18</sup>

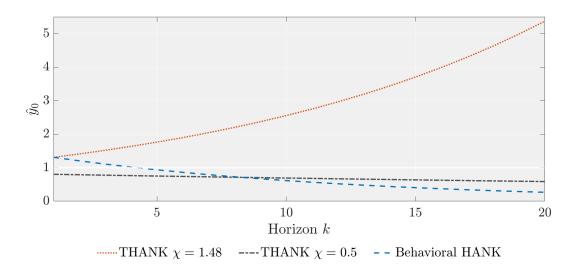


Figure 1: Resolving the Catch-22

Note: This figure shows the response of total output in period 0 to anticipated i.i.d. monetary policy shocks occurring at different horizons k, relative to the initial response in the RANK model under rational expectations (equal to 1).

The blue-dashed line shows that the behavioral HANK model generates both: amplification of contemporaneous monetary policy and a resolution of the forward guidance puzzle, as observed in the data. Note that also rational TANK models (thus, turning off type switching) or the behavioral RANK model would not deliver amplification and resolve the forward guidance puzzle simultaneously. TANK models would face the same issues as the rational RANK model in the sense that they cannot solve the forward guidance puzzle while bounded rationality in a RANK model does not deliver initial amplification.

As a direct consequence of the resolution of the Catch-22 in the behavioral HANK model, above or below its RANK counterpart depends on  $\chi \leq 1$  in the same way the initial response depends on  $\chi \leq 1$ .

<sup>&</sup>lt;sup>18</sup>Bilbiie (2020) calibrates  $\chi = 0.3$  to approximate the forward guidance dampening results in McKay et al. (2016) and McKay et al. (2017).

highly persistent monetary policy shocks have smaller effects on contemporaneous output than in RANK whereas less persistent shocks have larger effects in the behavioral HANK model. The reason is that persistent shocks also work through a forward guidance channel which is dampened in the behavioral HANK model. We elaborate this point in more detail in Appendix C.2.

**Determinacy in Behavioral HANK.** According to the Taylor principle, monetary policy needs to respond sufficiently strongly to changes in inflation in order to have a determinate equilibrium. In the rational RANK model the Taylor principle is given by  $\phi > 1$ , where  $\phi$  is the inflation-response coefficient in the Taylor rule (9). We now derive a similar determinacy condition in the behavioral HANK model and show that both household heterogeneity and bounded rationality affect this condition. The following proposition provides the behavioral HANK Taylor principle.<sup>19</sup>

**Proposition 3.** The behavioral HANK model has a determinate, locally unique equilibrium if and only if:

$$\phi > \phi^* = 1 + \frac{\bar{m}\delta - 1}{\frac{\kappa}{\gamma} \frac{1 - \lambda}{1 - \lambda \chi}}.$$
 (21)

Appendix A shows how to derive Proposition 3 and extends the result to more general Taylor rules.

To understand condition (3), consider first  $\bar{m}=1$  and, thus, focus solely on the role of household heterogeneity. With  $\chi>1$ , it follows that  $\phi^*>1$  and, hence, the threshold is higher than the RANK Taylor principle states. This insufficiency of the Taylor principle in the rational THANK model has been shown by Bilbiie (2021) and in a similar way by Acharya and Dogra (2020). As a future aggregate sunspot increases the income of households in state H disproportionately, savers cut back on precautionary savings today which further increases output today. This calls for a stronger response of the central bank to not make the sunspot self-fulfilling.

On the other hand, bounded rationality and, thus,  $\bar{m} < 1$  relaxes the condition as savers now cognitively discount both the future aggregate sunspot as well as its implication for their idiosyncratic risk. A smaller response of the central bank is needed in order to prevent the sunspot to become self-fulfilling. Given our calibration, the cutoff value for  $\bar{m}$  to restore the RANK Taylor principle in the behavioral HANK model is 0.95. What is more, given our baseline choice of  $\bar{m} = 0.85$ , we have  $\phi^* = -3.07$ . Thus, the Taylor principle is not even necessary in our behavioral HANK model as the economy features a stable unique equilibrium even under an interest rate peg. In this sense, the behavioral HANK model

<sup>&</sup>lt;sup>19</sup>We focus on local determinacy and bounded equilibria.

overcomes the famous result in Sargent and Wallace (1975) who have shown that an interest rate peg leads to equilibrium indeterminacy.<sup>20</sup>

The Lower Bound Problem. Related to the determinacy issues under a peg, the traditional New Keynesian model struggles to explain how the economy can remain stable when the effective lower bound (ELB) on nominal interest rates is binding for an extended period of time, as observed in many advanced economies over recent decades (see, e.g., Cochrane (2018)). If the ELB binds for a sufficiently long time, RANK predicts unreasonably severe recessions and, in the limit case in which the ELB binds forever, there is even indeterminacy in RANK. The intuition is directly related to our discussion about determinacy under a peg: A forever binding ELB basically implies that the Taylor coefficient is equal to zero and, thus, the nominal rate is pegged at the lower bound, thereby violating the Taylor principle.<sup>21</sup>

We now show that the behavioral HANK model resolves these issues. To this end, let us add a natural-rate shock  $r_t^n$  to the IS equation (18). We assume that in period t the natural rate decreases to a value  $\tilde{r}^n$  that is sufficiently negative such that the natural rate in levels is below the ELB. The natural rate stays at  $\tilde{r}^n$  for  $k \geq 0$  periods and after k periods the economy returns immediately back to steady state. Agents correctly anticipate the length of the binding ELB. For simplicity, we assume fully-rigid prices, i.e.,  $\kappa = 0$  and  $\pi_t = 0$  for all t, but this is not crucial for what follows. Iterating the IS equation (18) forward, it follows that output in period t is given by

$$\widehat{y}_t = -\frac{1-\lambda}{\gamma(1-\lambda\chi)} \underbrace{\left(\widehat{i}_{ELB} - \widetilde{r}^n\right)}_{>0} \sum_{j=0}^k \left(\bar{m}\delta\right)^j, \tag{22}$$

where the term  $(\hat{i}_{ELB} - \tilde{r}^n) > 0$  captures the shortfall of the policy response due to the binding ELB. Under rational expectations and countercyclical inequality,  $\chi > 1$  and, thus,  $\delta > 1$ , meaning that output implodes as  $k \to \infty$ . The same is true in the rational RANK model which is captured by  $\chi = 1$  and, thus,  $\delta = 1$ . In the behavioral HANK model, however, this is not the case. As long as  $\bar{m}\delta < 1$ , the output response in t is bounded even when  $k \to \infty$ . The condition  $\bar{m}\delta < 1$  is the same as for determinacy under a peg in the economy with fully-rigid prices. It follows that  $\bar{m} < 0.95$  is enough to rule out unboundedly-severe

<sup>&</sup>lt;sup>20</sup>Angeletos and Lian (2021) show (in a model without household heterogeneity) that small frictions in memory and intertemporal coordination lead to a unique equilibrium which is the same as the one selected by the Taylor principle but it does no longer depend on it.

<sup>&</sup>lt;sup>21</sup>Note, that this statement also extends to models featuring more elaborate monetary policy rules including Taylor rules responding to output or also the Wicksellian price-level targeting rule, as they all collapse to a constant nominal rate in a world of an ever-binding ELB.

0 -10 -20 Ŷ -30 -40 -50 5 10 15 20 25 30 35 40 Length of ELB k·······THANK - - Behavioral HANK -RANK

Figure 2: The Effective Lower Bound Problem

Note: This figure shows the contemporaneous output response for different lengths of a binding ELB k and compares the responses across different models.

recessions at the ELB even if the ELB is expected to persist forever.

We illustrate the stability of the behavioral HANK at the lower bound graphically in Figure 2. The figure shows the output response in the rational RANK, the rational THANK and the behavioral HANK to different lengths of a binding ELB (depicted on the x-axis). The shortcoming of monetary policy due to the ELB, i.e., the gap  $(\hat{i}_{ELB} - \tilde{r}^n) > 0$ , is set to a relatively small value of 0.25% (1% annually), and we set  $\bar{m} = 0.85$ . Figure 2 shows the implosion of output in the rational RANK and even more so in the rational THANK model: an ELB that is expected to bind for 40 quarters would decrease today's output in the rational RANK by 10% and in the rational THANK model by 40%. On the other hand—and consistent with recent experiences in advanced economies—output in the behavioral HANK model remains quite stable and drop by a mere 3%.

## 3.3 Fiscal Policy

We now show that the sufficient statistic for amplification of the contemporaneous monetary policy is also a sufficient statistic to generate positive consumption multipliers of fiscal policy under constant real rates. Dupor et al. (2021) provide recent empirical evidence for positive effects of government spending on private consumption. Furthermore, Nakamura and Steinsson (2014), Ramey (2019) and Chodorow-Reich (2019) document fiscal multipliers above 1, which through the lens of our model is equivalent to saying that consumption responds positively to government spending. From now on, we will use fiscal multiplier larger than one

and positive consumption response interchangeably.

To characterize fiscal multipliers, we follow Bilbiie (2021) and assume government spending  $g_t$  to follow an AR(1) with persistence  $\mu \geq 0$ , and to be 0 in steady state. The government taxes all agents uniformly to finance  $g_t$ .

We re-derive the behavioral HANK IS equation with government spending and obtain:

$$\widehat{c}_t = \bar{m}\delta \mathbb{E}_t \widehat{c}_{t+1} - \frac{1}{\gamma} \frac{1-\lambda}{1-\lambda \chi} \left( \widehat{i}_t - \mathbb{E}_t \pi_{t+1} \right) + \zeta \left[ \frac{\lambda(\chi-1)}{1-\lambda \chi} \left( g_t - \bar{m} \mathbb{E}_t g_{t+1} \right) + (\delta-1) \bar{m} \mathbb{E}_t g_{t+1} \right],$$

where  $\zeta \equiv \frac{\varphi}{\gamma(1+\frac{\varphi}{\gamma})}$ . The static Phillips Curve in this setting is given by  $\pi_t = \kappa c_t + \kappa \zeta g_t$ .

The following Proposition characterizes the fiscal multiplier in the behavioral HANK model.

**Proposition 4.** The fiscal multiplier in the behavioral HANK model is given by

$$\frac{\partial \widehat{c}_t}{\partial g_t} = \frac{1}{1 - \nu \mu} \frac{\zeta}{1 + \frac{1}{\gamma} \frac{1 - \lambda}{1 - \lambda \chi}} \phi \kappa \left[ \frac{\chi - 1}{1 - \lambda \chi} \left[ \lambda (1 - \bar{m}\mu) + \bar{m}\mu (1 - s) \right] - \kappa \frac{1}{\gamma} \frac{1 - \lambda}{1 - \lambda \chi} \left( \phi - \mu \right) \right],$$

where

$$\nu \equiv \frac{\bar{m}\delta + \frac{1}{\gamma}\kappa \frac{1-\lambda}{1-\lambda\chi}}{1 + \frac{1}{\gamma}\frac{1-\lambda}{1-\lambda\chi}\phi\kappa}.$$
 (23)

A corollary of Propositon 4 is that with persistent government spending,  $\mu > 0$ , and in the empirically-realistic case of  $\chi > 1$ , more bounded rationality, i.e., a lower  $\bar{m}$ , leads to a lower fiscal multiplier.<sup>22</sup> Bounded rationality weakens the fiscal multiplier as boundedly-rational agents discount the fact that an increase in government spending today has a positive effect on future spending as well. In the case of an i.i.d. spending shock, the fiscal multiplier is independent of  $\bar{m}$ .

To make the argument as clear as possible, we assume prices to be fully rigid,  $\kappa = 0$ , and assume that the real interest rate is held constant after the government spending shock. This is a useful benchmark as in this case, the consumption response in RANK is 0 (see Bilbiie (2011) and Woodford (2011)).<sup>23</sup>

From Proposition 4, we can directly derive the constant-real-rate multiplier in the behavioral HANK model. It shows that with  $\chi > 1$ , the fiscal multiplier is bounded from below by 0 irrespective of the persistence  $\mu$ . In other words, the constant-real-rate multiplier in the behavioral HANK model is strictly positive, regardless of the dampening of bounded

<sup>&</sup>lt;sup>22</sup>We focus on the case in which  $\nu\mu < 1$ , which holds in the behavioral HANK model even for  $\mu = 1$ , and we assume  $1 - s - \lambda < 0$ , which holds under all reasonable parameterizations.

<sup>&</sup>lt;sup>23</sup>Auclert et al. (2018) also use a constant real interest rate case to show that their HANK model can generate (output) fiscal multipliers larger than one.

rationality on the fiscal multiplier in the case of persistent spending. With  $\chi > 1$ , the high MPC households benefit disproportionately from the extra income out of the increase in government spending which increases the fiscal multiplier through a Keynesian type argument.

Figure 3 highlights the effect of bounded rationality on the fiscal multiplier by plotting the fiscal multiplier in the behavioral HANK model for varying degrees of  $\bar{m}$  (blue-solid line) and comparing it to the multiplier in THANK and RANK. For this exercise, we set the persistence parameter to an intermediate,  $\mu=0.6$ . It shows that the fiscal multiplier decreases with decreasing  $\bar{m}$ . Yet, even for the extreme case  $\bar{m}=0$ , in which households fully discount all future increases in government spending, the fiscal multiplier is still substantially above zero even though it is somewhat weaker than under rational expectations. In fact, the behavioral HANK model generates consumption responses to fiscal spending that are quantitatively close to the empirical estimates in Dupor et al. (2021) who estimate the non-durable consumption response to lie between 0.2 and 0.29. Note, that we did not target this moment.

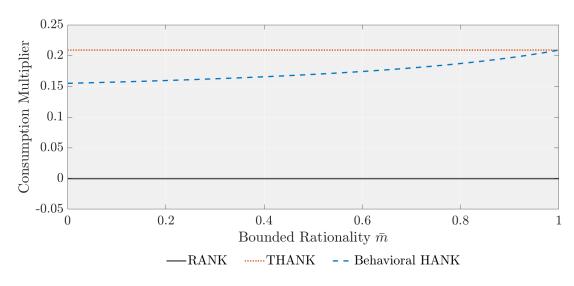


Figure 3: Consumption Response to Government Spending

Note: This figure shows the consumption multipliers (the consumption response to government spending) for different degrees of bounded rationality (blue-dashed line). The orange-dotted line plots the multiplier in the rational version of the model and the black-solid line shows the zero-multiplier in the RANK model.

It is noteworthy that the behavioral HANK model does not rely on a specific financing type to achieve positive consumption responses to fiscal spending. This is in contrast to the behavioral RANK model in Gabaix (2020). In the behavioral RANK model, bounded rationality can also increase the multiplier but only if the government delays taxing the agents to finance the contemporaneous spending as boundedly-rational agents will then discount

the future increases in taxes. In the rational THANK model, on the other hand, the fiscal multiplier can in principle be larger than one with  $\chi < 1$  if the hand-to-mouth households pay relatively less than the savers (see Bilbiie (2020) or Ferriere and Navarro (2018)). Both of these channels would also push up the multiplier in the behavioral HANK model, yet it does not depend on any of these two to achieve (output) fiscal multipliers larger than 1.

### 3.4 Behavioral HANK as a Unifying Framework

The interaction of bounded rationality and household heterogeneity is what allows the behavioral HANK model to be reconciled with the empirical facts on the transmission and effectivenss of monetary and fiscal policy. To see this, Figure 4 shows how the two frictions interact. The figure plots the parameter space for the two sufficient parameters for household heterogeneity and bounded rationality, respectively,  $(\chi, \bar{m})$ . The blue and orange dashed lines split the parameter space in the following sense: The blue line denotes the cut-off values below which the model is determinate under an interest-rate peg while above it the model is indeterminate (with the line itself belonging to the indeterminacy region). Determinacy under a peg is sufficient to rule out the forward guidance puzzle as well as the lower bound problem, and thus, is a sufficient statistic to resolve the discussed NK puzzles. The orange line denotes the cut-off values such that to the right of it, the model generates amplification while left from it—again including the line—the model does not generate amplification. Here, amplification is a stand-in for the amplification of monetary and fiscal policies through indirect, general equilibrium, effects.

This split of the parameter space into four areas allows us to distinguish the models discussed so far and to show how the behavioral HANK can overcome the limitations inherent in existing model. The RANK model is located in the "indeterminacy + no amplification" region as  $\bar{m}=1$  and  $\chi=1$ . The behavioral RANK can either be in "indeterminacy + no amplification" or in "determinacy + no amplification" depending on the degree of rationality. Attional THANK models can either be in "indeterminacy + no amplification", "determinacy + no amplification" or in "indeterminacy + amplification" while rational TANK models can only be in "indeterminacy + no amplification" or in "indeterminacy + amplification". Importantly, both cannot be in "determinacy + amplification". Furthermore, the behavioral HANK model can deliver "determinacy + amplification". Furthermore, the behavioral HANK model can in principle cover the whole parameter space as it nests all the aforementioned models

<sup>&</sup>lt;sup>24</sup>Note, this also applies to other models featuring deviations from FIRE that deliver equivalent reduced-form IS equations, e.g., Angeletos and Lian (2018) and Woodford (2019).

<sup>&</sup>lt;sup>25</sup>Note that this also applies to the models in McKay et al. (2017), Werning (2015), Ravn and Sterk (2017), Debortoli and Galí (2018), Bilbiie (2020), Bilbiie (2021) and many more.

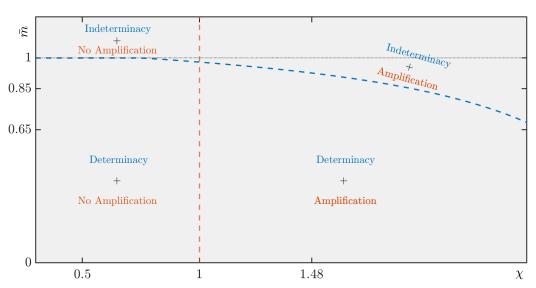


Figure 4: The Behavioral HANK as a Unifying Framework

Note: The figure characterizes four possible regions depending on whether the considered  $(\chi, \bar{m})$ -pair delivers determinacy under an interest-rate peg or not and whether the model generates amplification of contemporaneous monetary and fiscal policy or not (we only extend the y-axis above 1 for the sake of readability).

as special cases.

Having discussed the aggregate implications of the model, we now zoom in closer into the model and derive the iMPCs and show how they depend on bounded rationality, household heterogeneity, and the interaction of the two.

### 3.5 Intertemporal MPCs

The HANK literature shows that the iMPCs are a key statistic for conducting policy analysis (see, e.g., Auclert et al. (2018), Auclert et al. (2020), and Kaplan and Violante (2020)). We follow the THANK/TANK literature and define the aggregate iMPCs in the behavioral HANK model as the partial derivative of aggregate consumption at time k,  $\hat{c}_k$ , with respect to aggregate disposable income,  $\tilde{y}_0$ , keeping everything else fixed (see Bilbiie (2021), Cantore and Freund (2021), and Auclert et al. (2018)).

The following Proposition characterizes the iMPCs in the behavioral HANK model.<sup>26</sup>

**Proposition 5.** The intertemporal MPCs in the behavioral HANK model, i.e., the aggregate consumption response in period k to a one-time change in aggregate disposable income in

<sup>&</sup>lt;sup>26</sup>See Appendix D for the derivation.

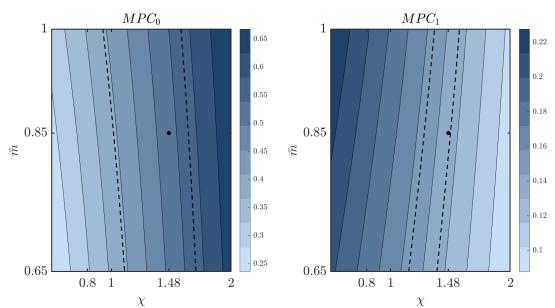


Figure 5: Intertemporal MPCs, Bounded Rationality and Household Heterogeneity

Note: This figure shows the aggregate intertemporal MPCs, i.e., the aggregate consumption response in year 0 (left) and year 1 (right) to a change in aggregate disposable income in year 0 for different  $\chi$  (x-axis) and  $\bar{m}$  (y-axis). The dashed lines show the range of empirically-estimated iMPCs and the black dot shows the model estimate given our baseline calibration. Darker colors represent higher MPCs, see the colorbars on the right side of the figures.

period 0, are given by

$$\begin{split} MPC_0 &\equiv \frac{d\widehat{c}_0}{d\widetilde{y}_0} = 1 - \frac{1 - \lambda \chi}{s\overline{m}} \mu_2^{-1} \\ MPC_1 &\equiv \frac{d\widehat{c}_k}{d\widetilde{y}_0} = \frac{1 - \lambda \chi}{s\overline{m}} \mu_2^{-1} \left(\beta^{-1} - \mu_1\right) \mu_1^{k-1}, \quad \text{for } k > 0, \end{split}$$

where the parameters  $\mu_1$  and  $\mu_2$  depend on the underlying parameters, including  $\bar{m}$  and  $\chi$  and are explicitly spelled out in Appendix D.

Figure 5 graphically depicts how the interplay of bounded rationality  $\bar{m}$  and household heterogeneity  $\chi$  determines the size of the aggregate iMPCs. Therefore, we calibrate the model annually as the empirical evidence on the iMPCs is annual (see Fagereng et al. (2021) and Auclert et al. (2018)). We set s=0.8 and  $\beta=0.95$ , and keep the rest of the calibration as above. The left panel depicts the aggregate MPCs to spend within the first year (in period 0) and the right panel shows aggregate MPCs to spend within the second year (in period 1) after the temporary increase in income in time 0. Darker colors represent higher MPCs. First, note that with our baseline calibration— $\chi=1.48$  and  $\bar{m}=0.85$  as shown by the black dots—the behavioral HANK model generates iMPCs within the first year of 0.55 and within the second year of 0.15. These values lie exactly in the estimated bounds for the iMPCs in

the data (Auclert et al. (2018)) which are between 0.42-0.6 within the first and 0.14-0.16 within the second year (see dashed lines). Away from our baseline calibration, an increase in  $\chi$  increases the MPCs in the first year but decreases them in the second year.<sup>27</sup> In contrast, an increase in  $\bar{m}$  increases the aggregate MPC in the first year and in the second year.

Let us first turn to the role of  $\chi$  for the iMPCs: Recall, the higher  $\chi$ , the more sensitive is the income of the H households to a change in aggregate income. Thus, with higher  $\chi$ , H households gain weight in relative terms for the aggregate iMPCs while the savers loose weight in relative terms. This pushes up the aggregate MPC within the first year, but pushes down the aggregate MPC within the second year as households that were hand-to-mouth in the period of the income windfall will have a MPC of 0 in the second year.

Bounded rationality, captured by  $\bar{m}$ , affects only the MPCs of the savers as only savers whether behavioral or rational—intertemporarily optimize. The savers' Euler equation dictates that the decrease in today's marginal utility of consumption—following the increase in consumption—is equalized by a decrease in tomorrow's expected marginal utility. For the behavioral saver, however, the decrease in tomorrow's marginal utility needs to be more substantial as she cognitively discounts the expectations about the future decrease. Hence, the behavioral saver saves relatively more out of the income windfall. This pushes down the aggregate MPCs in t=0. The same is true for the aggregate MPC in t=1, in which there are now two opposing forces at work: on the one hand, the saver again cognitively discounts the expectations about the future decrease in the marginal utility which depresses her consumption. On the other hand, savers have accumulated more wealth from period t=0 which tends to increase consumption. Given our calibration, in t=1 the former dominates. Figure 12 in Appendix D shows that, beginning in k = 3, the latter effect starts to dominate. If we increase the idiosyncratic risk of becoming hand-to-mouth, i.e., increase the transition probability 1-s, the aggregate MPC is already higher in t=1 for lower  $\bar{m}$ . The reason is that a smaller fraction of initial savers remains savers which pushes upwards consumption in k = 1 (see Figure 11 in Appendix D).

The effects of a change in  $\bar{m}$  are more pronounced at lower levels of  $\chi$ . Combining our discussion about the role of  $\chi$  and  $\bar{m}$ , this is intuitive: the lower  $\chi$ , the higher is the relative importance of the savers for the aggregate iMPCs and, in turn, the stronger is the effect of  $\bar{m}$  on the aggregate iMPCs. These interaction effects are quite substantial: at  $\chi = 1.48$ , a decrease of  $\bar{m}$  from 1 to 0.65 decreases the  $MPC_0$  by 7% and the  $MPC_1$  by more than 11%.

<sup>&</sup>lt;sup>27</sup>Note, that when considering micro moments like the iMPCs,  $\chi=1$  is not sufficient anymore for the model to collapse to RANK. More precisely, with  $\chi=1$  the model collapses to a THANK model which behaves in the aggregate exactly like RANK (see the incomplete-markets irrelevance result in Werning (2015)). Hence, the RANK iMPCs cannot directly be seen in Figure 5 but Proposition 5 still nests RANK for  $\chi=1$  and  $\lambda=0$ .

#### 3.6 Policy Implications: The Timing of Monetary Policy

We close this section by discussing some of the policy implications of the behavioral HANK model. In particular, we illustrate that the behavioral HANK can generate different policy implications than its rational counterpart. To this end, we analyze how the timing of monetary policy affects its effectiveness and its distributional consequences.

Consider that the central bank wants to increase the nominal interest rate by a cumulative x%, for example, to fight an overheating economy. The central bank decides whether to implement this policy within one quarter or to gradually raise the interest rate by  $\frac{x}{k}\%$  for k consecutive quarters.

**Lemma 1.** The effect of a  $\frac{x}{k}$ % interest rate hike over k consecutive periods decreases current output by

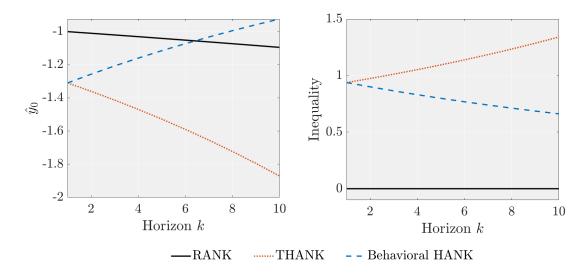
$$\widehat{y}_t = \frac{\psi_c}{\gamma} \left[ \sum_{j=0}^{k-1} \left( \psi_f + \frac{\psi_c}{\gamma} \kappa \right)^j \right] \frac{x}{k}.$$

The left panel of Figure 6 depicts the result in Lemma 1 for the behavioral HANK model and compares it to its rational counterpart and the rational RANK model. The solid-black line shows the well-known feature of RANK that the effects of monetary policy on current output become stronger when monetary policy is back-loaded: the further the interest hike is stretched out, the higher is the response on current output. The orange-dotted line shows that this feature is even more pronounced in the rational THANK model as the line is steeper than in the RANK model.

In contrast, the blue-dashed line representing the behavioral HANK model is increasing instead of decreasing in k. Thus, back-loading monetary policy decreases its effect on current output. To put it differently, monetary policy is most effective on current output if it is completely front-loaded. Hence, if the central bank wants to fight an overheating of the economy as effectively as possible, the behavioral HANK model implies front-loading the interest rate hike, while its rational counterpart suggests to rather back-load the hike.

The right panel of Figure 6 depicts the effects of the different timing of the monetary policy hikes on consumption inequality, as defined in equation (13). It shows that, according to the behavioral HANK, if monetary policy front-loads the interest rake hike, it increases inequality the most whereas a more gradual increase in the interest rate would have weaker effects on inequality. This illustrates a trade-off for the central banker: the more effectively monetary policy combats the overheating, the more it increases inequality.

Figure 6: Monetary Policy Timing: Effectiveness and Distributional Consequences



Note: This figure shows the response of current output (left panel) of a cumulative interest-rate hike by x% implemented over k consecutive periods. The right panel shows the corresponding response of inequality, defined as  $\hat{c}_t^S - \hat{c}_t^H$ .

# 4 Bounded Rationality and Incomplete Information with Learning: An Equivalence Result

In this section, we derive an equivalence result between bounded rationality and incomplete information with learning. In particular, we show how a change in the default value in the behavioral setup leads to the same IS equation as in models with incomplete information and learning (see Angeletos and Huo (2021)).

To this end, we now assume that behavioral agents anchor their expectations to their last observation instead of the steady state values. One interpretation is that agents anchor their expectations to what they read or hear in the news. Models featuring some form of backward-looking behavior indeed tend to match the expectations data coming from household surveys quite well (see, for example, Adam et al. (2017), Adam et al. (2020), Angeletos and Huo (2021), and Angeletos et al. (2021)). The backward-looking components in these models usually arise from an incomplete or noisy information setting as well as some form of (Bayesian) learning. We now show how our bounded rationality setup generates expectations that resemble these aforementioned expectations models.

**Proposition 6.** Set the boundedly-rational agents' default value to the variable's past value

$$X_t^d = X_{t-1}. (24)$$

In this case, the boundedly-rational agent's expectations of  $X_{t+1}$  becomes

$$\mathbb{E}_{t}^{BR}[X_{t+1}] = (1 - \bar{m})X_{t-1} + \bar{m}\mathbb{E}_{t}[X_{t+1}]. \tag{25}$$

These backward-looking expectations introduce a backward-looking component into the behavioral IS equation as shown in the following Proposition.

**Proposition 7.** In case the behavioral agents' default value is the past value of the respective variable, i.e.,  $X_t^d = X_{t-1}$ , the behavioral HANK IS equation is given by

$$\widehat{y}_t = \psi_f \mathbb{E}_t \widehat{y}_{t+1} - \psi_c \frac{1}{\gamma} \left( \widehat{i}_t - \mathbb{E}_t \pi_{t+1} \right) + (1 - \bar{m}) \delta \widehat{y}_{t-1}. \tag{26}$$

Proposition 7 shows that the change in the agents' default value does not change the existing behavioral and heterogeneity coefficients  $\psi_f$  and  $\psi_c$ . Yet, anchoring to past realizations introduces an additional backward-looking term in the IS equation, similar to models relying on habit persistence. Angeletos and Huo (2021) and Gallegos (2021) derive an IS equation with the same reduced form which, however, is based on an incomplete-information setting and learning. We complement their findings by showing how we can generate the equivalent outcome based on a behavioral relaxation of FIRE.

Angeletos and Huo (2021) calibrate the coefficients in front of  $\mathbb{E}_t \widehat{y}_{t+1}$  and  $\widehat{y}_{t-1}$  to match evidence from survey expectations data. By following their calibration, we can back out the implied  $\overline{m}$  and  $\chi$ . We get  $\overline{m} = 0.59$  and  $\chi = 0.72$ , thus, relatively low values compared to the calibration above. We leave the other parameters as in Section 3. We complement the backward-looking behavioral HANK IS equation with the static Phillips Curve (15).

**Determinacy.** We numerically verify that the backward-looking behavioral HANK model restores the Taylor principle. In fact, the equilibrium is determinate even under an interest-rate peg. Thus, also the backward-looking behavioral HANK model overturns the Sargent and Wallace (1975) result with this calibration.

Impulse-Response Functions. We now show how the backward-looking behavioral HANK model generates hump-shaped impulse responses and a novel behavioral amplification channel. To this end, we examine how output in the backward-looking behavioral HANK model responds to an expansionary monetary policy shock and compare the response to its rational counterpart and the RANK version of the model. We set the Taylor coefficient to 1.5, thus, guaranteeing determinacy also in the rational models and the persistence of the shock to an intermediate value,  $\rho^{MP} = 0.6$ .

Figure 7 shows the corresponding impulse-response functions. The blue-dashed line shows the results of our behavioral HANK, the orange-dotted line of its rational counterpart (THANK) and the black-solid line of RANK.

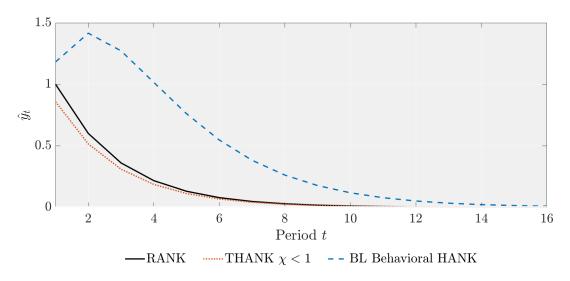


Figure 7: Output Response to a Monetary Policy Shock

Note: This figure shows the output response to a monetary policy shock for different models.

Two things stand out. First, the behavioral HANK model delivers amplification compared to RANK—even in the first period—and second, the backward-looking anchor generates hump-shaped responses. As the latter has been highlighted in Angeletos and Huo (2021), we here focus on the amplification. Figure 7 shows that the amplification stems from a behavioral amplification channel: the initial output response is amplified although the model features procyclical inequality ( $\chi < 1$ ) and, thus, the heterogeneity frictions themselves would generate dampening.

Where does the behavioral amplification come from? Given the backward-looking component in households' expectations, the increase in today's output is expected to persist as it becomes tomorrow's default value for the household's expectations. The behavioral anchor induces endogenous persistence which further increases today's output response through more optimistic expectations. Yet, there is an opposing channel at work: an exogenously persistent shock not only decreases interest rates today but also expected future interest rates. Behavioral households congitively discount these future changes and, thus, perceive the shock as less expansionary compared to a rational agent which dampens the initial response.<sup>28</sup> Given our calibration, the first channel dominates, thereby generating amplification

<sup>&</sup>lt;sup>28</sup>This is the same channel through which the fiscal multiplier of persistent government spending is

as depicted in Figure 7.

Given the two opposing forces at work, the degree of initial amplification depends on the persistence of the shock. Figure 8 shows the initial response of all three models for different degrees of persistence of the shock. As the persistence declines, the initial response becomes relatively stronger in the backward-looking behavioral HANK model compared to RANK. As a consequence, the relative amplification is largest for an i.i.d. shock.

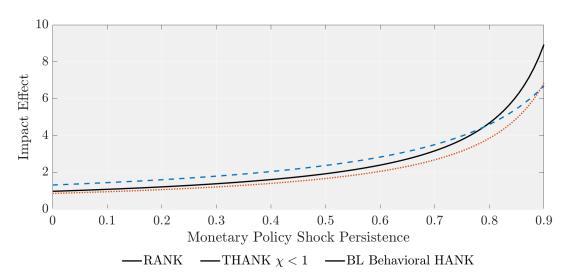


Figure 8: Initial Output Response for Varying Degrees of the Persistence

Note: This figure shows the initial output response to monetary policy shocks with different degrees of persistence.

In addition, comparing the backward-looking behavioral HANK model to its rational counterpart shows that for  $\rho^{MP} < 0.9$ , there is behavioral amplification while for more persistent shocks, there is behavioral dampening. The comparison with RANK shows that for  $\rho^{MP} < 0.80$ , the behavioral amplification dominates the heterogeneity dampening which arises because  $\chi < 1$ .

Behavioral Amplification and Forward Guidance. We now analyze analytically the behavioral-amplification mechanism and its implications for forward guidance. In the backward-looking behavioral HANK model, the output response to an interest rate change depends on the (expected) infinite future even when the shock is completely transitory.

Consider the following. The monetary authority decreases the nominal interest rate in period t to  $\tilde{i}_t < 0$  but will keep it at steady state thereafter (the argument extends to changes of the interest rate in the future). Output and inflation would be expected to go back to

dampened in our baseline model in Section 3.

zero in t+1 under rational expectations. This is, however, not true for the backward-looking behavioral HANK model.

To understand this, combine the static Phillips Curve (a static Phillips curve is again not crucial for the argument but facilitates the derivations) with the behavioral HANK IS equation to arrive at

$$\widehat{y}_t = (1 - \bar{m})\delta\widehat{y}_{t-1} - \frac{1}{\gamma} \frac{1 - \lambda}{1 - \lambda \chi} \widetilde{i}_t + \left[\delta \bar{m} + \kappa \frac{1}{\gamma} \frac{1 - \lambda}{1 - \lambda \chi}\right] \mathbb{E}_t \widehat{y}_{t+1}.$$

If households expect future output to be back to steady state – as would be the case in the rational model or the behavioral model in which the households' default value equals the steady state – a one-time, completely transitory decrease in the nominal interest rate changes contemporaneous output by

$$\frac{1}{\gamma} \frac{1 - \lambda}{1 - \lambda \chi} > 0. \tag{27}$$

Yet, in the backward-looking model, expectations in t + 1 of output in t + 2 will be above steady state when output in t increases. The more optimistic expectations feed back into output already in t.

This becomes apparent when we write the IS equation as

$$\begin{split} \widehat{y}_t \left[ 1 - (1 - \bar{m}) \delta \left[ \delta \bar{m} + \kappa \frac{1}{\gamma} \frac{1 - \lambda}{1 - \lambda \chi} \right] \right] &= \\ (1 - \bar{m}) \delta \widehat{y}_{t-1} - \frac{1}{\gamma} \frac{1 - \lambda}{1 - \lambda \chi} \left[ \widetilde{i}_t + \left[ \delta \bar{m} + \kappa \frac{1}{\gamma} \frac{1 - \lambda}{1 - \lambda \chi} \right] \mathbb{E}_t \left[ \widetilde{i}_{t+1} \right] \right] \\ &+ \left[ \delta \bar{m} + \kappa \frac{1}{\gamma} \frac{1 - \lambda}{1 - \lambda \chi} \right]^2 \mathbb{E}_t \widehat{y}_{t+2}. \end{split}$$

Thus, if households would assume that  $\hat{y}_{t+2}$  will be zero but not  $\hat{y}_{t+1}$ , the discussed interestrate change in t increases output in t by

$$\frac{\frac{1}{\gamma}\frac{1-\lambda}{1-\lambda\chi}}{1-(1-\bar{m})\delta\left[\delta\bar{m}+\kappa\frac{1}{\gamma}\frac{1-\lambda}{1-\lambda\chi}\right]},$$

which is larger than the effect for models without a backward-looking anchor as can be seen by comparing it to equation (27). Put differently, the initial output response is amplified through a behavioral channel. Iterating forward in this fashion shows how the effect increases with each iteration. However, the response is bounded, as we will see below.

Turning to forward guidance, an expected change in the nominal interest rate in period

t+1, affects output in t by

$$-\frac{\frac{1}{\gamma}\frac{1-\lambda}{1-\lambda\chi}\left[\delta\bar{m}+\kappa\frac{1}{\gamma}\frac{1-\lambda}{1-\lambda\chi}\right]}{1-(1-\bar{m})\delta\left[\delta\bar{m}+\kappa\frac{1}{\gamma}\frac{1-\lambda}{1-\lambda\chi}\right]},$$

if we assume output in t+2 to be back to zero. Given our calibration, the term  $\left|\delta\bar{m}+\kappa\frac{1}{\gamma}\frac{1-\lambda}{1-\lambda\chi}\right|$  is smaller than 1. Thus, an interest rate change tomorrow has a smaller effect on output today than a contemporaneous interest rate change such that there is no forward guidance puzzle in the backward-looking behavioral HANK model. We can continue in this fashion to show that the effects increase with the iteration but decrease with the period of the shock.

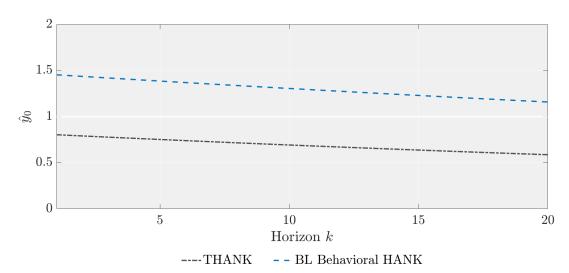


Figure 9: Forward Guidance with Backward-Looking Anchor

Note: This figure shows the period-t output response to an anticipated i.i.d. monetary policy shock in period t + k for three different economies.

Figure 9 shows these patterns graphically. First, the behavioral amplification channel discussed above is reflected in the contemporaneous effect (k=0) which is stronger than without the backward-looking expectations —reflected in the black-dashed-dotted line. Second, increasing the horizon k shows that there is no forward guidance puzzle in the backward-looking behavioral HANK model. To sum it up, also the backward-looking behavioral HANK model amplifies contemporaneous monetary policy (even for  $\chi < 1$ ) while it simultaneously dampens the effects of forward guidance.

#### 5 Conclusion

We develop a framework that accounts for recent empirical facts on the transmission and effectiveness of monetary and fiscal policy. To arrive at this framework, we introduce bounded rationality in the form of cognitive discounting and household heterogeneity into a sticky price model. We show that only when both frictions—household heterogeneity and bounded rationality—are present, the model can be reconciled with the data. Thus, it is the interaction of the two frictions that is crucial to arrive at our results. The behavioral HANK model is analytically tractable and we show how it nests a wide array of existing models—none of which can account for all the empirical patterns. What is more, we show that the behavioral HANK model can have different policy implications than its rational counterpart, e.g., when it comes to the timing of monetary policy. We also show how our framework can be used to arrive at an equivalence result of models featuring bounded rationality and models of incomplete information and learning. Altogether, the behavioral HANK model offers a tractable framework to study a broad array of questions in future research.

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# A Model Details and Derivations

# A.1 Derivation of $\chi$

In Section 2, we stated that

$$\widehat{c}_t^H = \chi \widehat{y}_t, \tag{28}$$

where  $\chi \equiv 1 + \varphi \left(1 - \frac{\tau^D}{\lambda}\right)$  is the crucial statistic coming from the household heterogeneity friction. We now show how we arrive at equation (28) from the *H*-households' budget constraint, optimality conditions and market clearing.

The labor-leisure condition of the H households is given by

$$(N_t^H)^{\varphi} = W_t(C_t^H)^{-\gamma},\tag{29}$$

and similarly for the S households. As we focus on the steady state with no inequality, we have that in steady state  $C = C^H = C^S$  and  $N = N^S = N^H$  and market clearing and the production function imply Y = C = N, which we normalize to 1.

Thus, log-linearizing the labor-leisure conditions yields

$$\varphi \widehat{n}_t^H = \widehat{w}_t - \gamma \widehat{c}_t^H$$
$$\varphi \widehat{n}_t^S = \widehat{w}_t - \gamma \widehat{c}_t^S.$$

Since both households work for the same wage, we obtain

$$\varphi \widehat{n}_t^H + \gamma \widehat{c}_t^H = \varphi \widehat{n}_t^S + \gamma \widehat{c}_t^S \tag{30}$$

Log-linearizing the market clearing conditions yields

$$\widehat{n}_t = \lambda \widehat{n}_t^H + (1 - \lambda)\widehat{n}_t^S$$

$$\widehat{c}_t = \lambda \widehat{c}_t^H + (1 - \lambda)\widehat{c}_t^S,$$

and we further have  $\hat{y}_t = \hat{c}_t = \hat{n}_t$ . Replacing  $\hat{n}_t^S$  and  $\hat{c}_t^S$  in equation (30) then gives

$$\varphi \widehat{n}_t^H + \gamma \widehat{c}_t^H = (\varphi + \gamma)\widehat{y}_t. \tag{31}$$

The budget constraint of H households (accounting for the fact that bond holdings are zero in equilibrium) is given by

$$C_t^H = W_t N_t^H + \frac{\tau^D}{\lambda} D_t, \tag{32}$$

where we replaced  $T_t^H$  with  $\frac{\tau^D}{\lambda}D_t$ . In log-linearized terms, we get

$$\widehat{c}_t^H = \widehat{w}_t + \widehat{n}_t^H + \frac{\tau^D}{\lambda} \widehat{d}_t, \tag{33}$$

and using that  $\widehat{w}_t = -\widehat{d}_t = \varphi \widehat{n}_t^H + \gamma \widehat{c}_t^H$ , we get

$$\widehat{c}_t^H = \left(\varphi \widehat{n}_t^H + \gamma \widehat{c}_t^H\right) \left(1 - \frac{\tau^D}{\lambda}\right) + \widehat{n}_t^H. \tag{34}$$

Using (31) to solve for  $\widehat{n}_t^H$  and plugging it into (34), we obtain

$$\widehat{c}_t^H = \chi \widehat{y}_t,$$

with  $\chi \equiv 1 + \varphi \left(1 - \frac{\tau^D}{\lambda}\right)$ , as stated above.

### A.2 Derivation of Proposition 1.

Combining equations (10) and (12) with the bounded-rationality setup in equation (17) for  $\hat{x}_t^d = 0$  as  $X_t^d$  is given by the steady state, we have

$$\mathbb{E}_{t}^{BR} \left[ \widehat{c}_{t+1}^{H} \right] = \bar{m} \mathbb{E}_{t} \left[ \widehat{c}_{t+1}^{H} \right] = \bar{m} \chi \mathbb{E}_{t} \left[ \widehat{y}_{t+1} \right]$$

$$\mathbb{E}_{t}^{BR} \left[ \widehat{c}_{t+1}^{S} \right] = \bar{m} \mathbb{E}_{t} \left[ \widehat{c}_{t+1}^{S} \right] = \bar{m} \frac{1 - \lambda \chi}{1 - \lambda} \mathbb{E}_{t} \left[ \widehat{y}_{t+1} \right].$$

Plugging these two equations as well as equation (12) into the savers' Euler equation (14) yields

$$\frac{1-\lambda\chi}{1-\lambda}\widehat{y}_t = s\overline{m}\frac{1-\lambda\chi}{1-\lambda}\mathbb{E}_t\left[\widehat{y}_{t+1}\right] + (1-s)\overline{m}\chi\mathbb{E}_t\left[\widehat{y}_{t+1}\right] - \frac{1}{\gamma}\left(\widehat{i}_t - \mathbb{E}_t\pi_{t+1}\right).$$

Combining the  $\mathbb{E}_t [\widehat{y}_{t+1}]$  terms and dividing by  $\frac{1-\lambda\chi}{1-\lambda}$  yields the following coefficient in front of  $\mathbb{E}_t [\widehat{y}_{t+1}]$ :

$$\psi_f \equiv \bar{m} \left[ s + (1-s)\chi \frac{1-\lambda}{1-\lambda\chi} \right]$$

$$= \bar{m} \left[ 1 - 1 + s + (1-s)\chi \frac{1-\lambda}{1-\lambda\chi} \right]$$

$$= \bar{m} \left[ 1 - \frac{1-\lambda\chi}{1-\lambda\chi} + s + (1-s)\chi \frac{1-\lambda}{1-\lambda\chi} \right]$$

$$= \bar{m} \left[ 1 - \frac{1-\lambda\chi}{1-\lambda\chi} + \frac{(1-\lambda\chi)s}{1-\lambda\chi} + (1-s)\chi \frac{1-\lambda}{1-\lambda\chi} \right]$$

$$= \bar{m} \left[ 1 + (\chi - 1) \frac{1 - s}{1 - \lambda \chi} \right].$$

Defining  $\psi_c \equiv \frac{1-\lambda}{1-\lambda\chi}$  yields the behavioral HANK IS equation in Proposition 1:

$$\widehat{y}_t = \psi_f \mathbb{E}_t \widehat{y}_{t+1} - \psi_c \frac{1}{\gamma} \left( \widehat{i}_t - \mathbb{E}_t \pi_{t+1} \right).$$

### A.3 Derivation of Proposition 2.

The first part comes from the fact that amplification is defined as

$$\frac{1-\lambda}{1-\lambda\chi} > 1,$$

which requires  $\chi > 1$ .

For the second part, recall how we model a forward guidance experiment (following Bilbiie (2021)). We assume a Taylor coefficient of 0, i.e.,  $\phi = 0$ , such that the nominal interest rate is given by  $\hat{i}_t = \varepsilon_t^{MP}$ . Replacing inflation using the Phillips curve (15), i.e.,  $\pi_t = \kappa \hat{y}_t$ , we can re-write the behavioral HANK IS equation from Proposition 1 as

$$\widehat{y}_{t} = \psi_{f} \mathbb{E}_{t} \widehat{y}_{t+1} - \psi_{c} \frac{1}{\gamma} \left( \varepsilon_{t}^{MP} - \kappa \mathbb{E}_{t} \widehat{y}_{t+1} \right)$$
$$= \left( \psi_{f} + \psi_{c} \frac{1}{\gamma} \kappa \right) \mathbb{E}_{t} \widehat{y}_{t+1} - \psi_{c} \frac{1}{\gamma} \varepsilon_{t}^{MP}$$

The forward guidance puzzle is ruled out if and only if

$$\left(\psi_f + \psi_c \frac{1}{\gamma} \kappa\right) < 1,$$

which is the same as the condition stated in Proposition 2:

$$\bar{m}\delta + \frac{1}{\gamma} \frac{1-\lambda}{1-\lambda \chi} \kappa < 1.$$

Solving this for  $\bar{m}$  yields

$$\bar{m} < \frac{1 - \frac{1 - \lambda}{\gamma(1 - \lambda \chi)} \kappa}{\delta},$$

which completes Proposition 2.

#### A.4 Derivation of Proposition 3.

Replacing  $\hat{i}_t$  by  $\phi \pi_t = \phi \kappa \hat{y}_t$  and  $\mathbb{E}_t \pi_{t+1} = \kappa \mathbb{E}_t \hat{y}_{t+1}$  in the IS equation (18), we get

$$\widehat{y}_t = \psi_f \mathbb{E}_t \widehat{y}_{t+1} - \psi_c \frac{1}{\gamma} \left( \phi \kappa \widehat{y}_t - \kappa \mathbb{E}_t \widehat{y}_{t+1} \right),$$

which can be re-written as

$$\widehat{y}_t \left( 1 + \psi_c \frac{1}{\gamma} \phi \kappa \right) = \mathbb{E}_t \widehat{y}_{t+1} \left( \psi_f + \psi_c \frac{1}{\gamma} \kappa \right).$$

Dividing by  $\left(1 + \psi_c \frac{1}{\gamma} \phi \kappa\right)$  and plugging in for  $\psi_f$  and  $\psi_c$  yields

$$\widehat{y}_t = \frac{\overline{m}\delta + \frac{(1-\lambda)\kappa}{\gamma(1-\lambda\chi)}}{1 + \kappa\phi\frac{1}{\gamma}\frac{(1-\lambda)}{1-\lambda\gamma}} \mathbb{E}_t \widehat{y}_{t+1}.$$

To obtain determinacy, the term in front of  $\mathbb{E}_t \widehat{y}_{t+1}$  has to be smaller than 1. Solving this for  $\phi$  yields

$$\phi > \phi^* = 1 + \frac{\bar{m}\delta - 1}{\frac{\kappa}{\gamma} \frac{1 - \lambda}{1 - \lambda \gamma}},\tag{35}$$

which is the condition in Proposition 3. This illustrates how bounded rationality raises the likelihood that the Taylor principle ( $\phi^* = 1$ ) is sufficient for determinacy, as the Taylor principle can only hold if

$$\bar{m}\delta < 1$$
.

In the rational model, this boils down to  $\delta \leq 1$ . However, the Taylor principle can be sufficient under bounded rationality, i.e.,  $\bar{m} < 1$ , even when  $\delta > 1$ , thus, even when allowing for amplification. Note that we could also express condition (35) as

$$\phi > \phi^* = 1 + \frac{\psi_f - 1}{\frac{\kappa}{\gamma} \psi_c}.$$

Proposition 3 can be extended to allow for Taylor rules of the form

$$\hat{i}_t = \phi_\pi \pi_t + \phi_y \hat{y}_t$$

and in which the behavioral agents do not have rational expectations about the real interest rate but rather perceive the real interest rate to be equal to

$$\widehat{r}_t^{BR} \equiv \widehat{i}_t - \bar{m}^r \mathbb{E}_t \pi_{t+1},$$

where  $\bar{m}^r$  can be equal to  $\bar{m}$  or can potentially differ from it (if it equals 1, we are back to the case in which the behavioral agent is rational with respect to real interest rates).

Combining the static Phillips Curve with the generalized Taylor rule and the behavioral HANK IS equation, it follows that

$$\widehat{y}_t = \frac{\omega_f + \frac{\kappa}{\gamma} \omega_c \bar{m}^r}{1 + \frac{\omega_c}{\gamma} \left(\kappa \phi_\pi + \phi_y\right)} \mathbb{E}_t \widehat{y}_{t+1}.$$
(36)

From equation (36), it follows that we need

$$\phi_{\pi} > \bar{m}^r - \phi_y + \frac{\omega_f - 1}{\omega_c \frac{\kappa}{\gamma}} = \bar{m}^r - \phi_y + \frac{\bar{m}\delta - 1}{\frac{1 - \lambda}{1 - \chi\lambda} \frac{\kappa}{\gamma}}$$
(37)

for the model to feature a determinate, locally unique equilibrium. Condition (37) shows that both,  $\bar{m}^r < 1$  and  $\phi_y > 0$ , weaken the condition in Proposition 3. Put differently, bounded rationality with respect to the real rate or a Taylor rule that responds to changes in output, both relax the condition on  $\phi_{\pi}$  to yield determinacy.

#### A.5 Derivation of Proposition 7

To prove Proposition 7, we start from the Euler equation (14). For simplicity, we denote  $\hat{r}_t \equiv \hat{i}_t - \mathbb{E}_t \pi_{t+1}$  as the real rate. Plugging in for  $\hat{c}_t^S$ ,  $\hat{c}_{t+1}^S$  and  $\hat{c}_{t+1}^H$  from equations (10) and (12), we get

$$\widehat{y}_t = s \mathbb{E}_t^{BR} \left[ \widehat{y}_{t+1} \right] + (1 - s) \frac{1 - \lambda}{1 - \lambda \gamma} \mathbb{E}_t^{BR} \left[ \widehat{y}_{t+1} \right] - \psi_c \widehat{r}_t,$$

which can be re-written as

$$\widehat{y}_t = \delta \mathbb{E}_t^{BR} \left[ \widehat{y}_{t+1} \right] - \psi_c \widehat{r}_t.$$

Now, using the expectations setup from Proposition 6, we get  $\delta \mathbb{E}_t^{BR} [\widehat{y}_{t+1}] = (1 - \bar{m}) \delta \widehat{y}_{t-1} + \bar{m} \delta \mathbb{E}_t [\widehat{y}_{t+1}]$  which proves Proposition 7.

# A.6 Cognitive Discounting of the State Vector

In Section 2, we assume that cognitive discounting applies to all variables, which differs slightly from the assumption in Gabaix (2020) who assumes that cognitive discounting applies to the *state* of the economy (exogenous shocks as well as announced monetary and fiscal policies). He then proves (Lemma 1 in Gabaix (2020)) how cognitive discounting applies as a result (instead of as an assumption) to all future variables, including future consumption choices. For completeness, we show in this section how our results are unaffected when

following the approach in Gabaix (2020).

Let  $X_t$  denote the (de-meaned) state vector which evolves as

$$X_{t+1} = G^X \left( X_t, \varepsilon_{t+1} \right), \tag{38}$$

where  $G^X$  denotes the transition function of X in equilibrium and  $\varepsilon$  are zero-mean innovations. Linearizing equation (38) yields

$$X_{t+1} = \Gamma X_t + \varepsilon_{t+1},\tag{39}$$

where  $\varepsilon_{t+1}$  might have been renormalized. The assumption in Gabaix (2020) is that the behavioral agent perceives the state vector to follow

$$X_{t+1} = \bar{m}G^X(X_t, \varepsilon_{t+1}), \tag{40}$$

or in linearized terms

$$X_{t+1} = \bar{m} \left( \Gamma X_t + \varepsilon_{t+1} \right). \tag{41}$$

The expectation of the boundedly-rational agent of  $X_{t+1}$  is thus  $\mathbb{E}_t^{BR}[X_{t+1}] = \bar{m}\mathbb{E}_t[X_{t+1}] = \bar{m}\Gamma X_t$ . Iterating forward, it follows that  $\mathbb{E}_t^{BR}[X_{t+k}] = \bar{m}^k\mathbb{E}_t[X_{t+k}] = \bar{m}^k\Gamma^k X_t$ .

Now, consider any variable  $z(X_t)$  with z(0) = 0 (e.g., demeaned consumption of the saver type  $C^S(X_t)$ ). Linearizing z(X), we obtain  $z(X) = b_X^z X$  for some  $b_X^z$  and thus

$$\mathbb{E}_{t}^{BR} \left[ z(X_{t+k}) \right] = \mathbb{E}_{t}^{BR} \left[ b_{X}^{z} X_{t+k} \right]$$

$$= b_{X}^{z} \mathbb{E}_{t}^{BR} \left[ X_{t+k} \right]$$

$$= b_{X}^{z} \bar{m}^{k} \mathbb{E}_{t} \left[ X_{t+k} \right]$$

$$= \bar{m}^{k} \mathbb{E}_{t} \left[ b_{X}^{z} X_{t+k} \right]$$

$$= \bar{m}^{k} \mathbb{E}_{t} \left[ z(X_{t+k}) \right].$$

For example, expected consumption of savers tomorrow (in linearized terms) is given by

$$\mathbb{E}_{t}^{BR} \left[ \widehat{c}^{S}(X_{t+1}) \right] = \bar{m} \mathbb{E}_{t} \left[ \widehat{c}^{S}(X_{t+1}) \right], \tag{42}$$

which we denote in the main text as

$$\mathbb{E}_{t}^{BR} \left[ \hat{c}_{t+1}^{S} \right] = \bar{m} \mathbb{E}_{t} \left[ \hat{c}_{t+1}^{S} \right]. \tag{43}$$

Now, take the linearized Euler equation (14) of the savers:

$$\widehat{c}_t^S = s \mathbb{E}_t^{BR} \left[ \widehat{c}_{t+1}^S \right] + (1 - s) \mathbb{E}_t^{BR} \left[ \widehat{c}_{t+1}^H \right] - \frac{1}{\gamma} \widehat{r}_t, \tag{44}$$

where  $\hat{r}_t \equiv \hat{i}_t - \mathbb{E}_t \pi_{t+1}$ .

Using the notation in Gabaix (2020), we can write the Euler equation as

$$\widehat{c}^{S}(X_{t}) = s \mathbb{E}_{t}^{BR} \left[ \widehat{c}^{S}(X_{t+1}) \right] + (1 - s) \mathbb{E}_{t}^{BR} \left[ \widehat{c}^{H}(X_{t+1}) \right] - \frac{1}{\gamma} \widehat{r}(X_{t}). \tag{45}$$

Now, applying the results above, we obtain

$$\widehat{c}^{S}(X_{t}) = s\overline{m}\mathbb{E}_{t}\left[\widehat{c}^{S}(X_{t+1})\right] + (1-s)\overline{m}\mathbb{E}_{t}\left[\widehat{c}^{H}(X_{t+1})\right] - \frac{1}{\gamma}\widehat{r}(X_{t}),\tag{46}$$

which after writing  $\hat{c}^S(X_t)$ ,  $\hat{c}^S(X_{t+1})$  and  $\hat{c}^H(X_{t+1})$  in terms of total output yields exactly the behavioral HANK IS equation in Proposition 1.

#### A.7 Microfounding $\bar{m}$

Gabaix (2020) shows how to microfound  $\bar{m}$  stemming from a noisy signal extraction problem in the case of a representative agent. Following these lines, we show how this signal-extraction problem generates a set-up in which the family head behaves as if she was boundedly rational.

The (linearized) law of motion of the state variable,  $X_t$ , is given by  $X_{t+1} = \Gamma X_t + \varepsilon_{t+1}$  (a similar reasoning extends to the non-linearized case), where X has been demeaned. Now assume that every agent j within the family of savers (the expectations of the hand-to-mouth agents are irrelevant) receives a noisy signal of  $X_{t+1}$ ,  $S_{t+1}^j$ , given by

$$S_{t+1}^{j} = \begin{cases} X_{t+1} & \text{with probability } p \\ X'_{t+1} & \text{with probability } 1 - p \end{cases}$$

where  $X'_{t+1}$  is an i.i.d. draw from the unconditional distribution of  $X_{t+1}$ , which has an unconditional mean of zero. In words, with probability p the agent j receives perfectly precise information and with probability 1-p agent j receives a signal realization that is completely uninformative. A fully-informed rational agent would have p=1.

The conditional mean of  $X_{t+1}$ , given the signal  $S_{t+1}^{j}$ , is given by

$$X_{t+1}^e \equiv \mathbb{E}\left[X_{t+1}|S_{t+1} = s_{t+1}^j\right] = p \cdot s_{t+1}^j$$
.<sup>29</sup>

The intuition is that the signal distribution is such that the agent either receives a perfectly precise signal or a completely uninformative signal. As the perfectly-precise signal arrives with probability p and the unconditional mean is zero, it follows that the agent puts a weight p on the signal.

Furthermore, we have

$$\mathbb{E}\left[S_{t+1}|X_{t+1}\right] = pX_{t+1} + (1-p)\mathbb{E}\left[X'_{t+1}\right] = pX_{t+1}.$$

So, it follows that the average expectation of  $X_{t+1}$  within the family is given by

$$\mathbb{E}\left[X_{t+1}^{e}(S_{t+1})|X_{t+1}\right] = \mathbb{E}\left[p \cdot S_{t+1}|X_{t+1}\right]$$

$$= p \cdot \mathbb{E}\left[S_{t+1}|X_{t+1}\right]$$

$$= p^{2}X_{t+1}.$$

Defining  $\bar{m} \equiv p^2$  and since  $X_{t+1} = \Gamma X_t + \varepsilon_{t+1}$ , we have that the family head perceives the law of motion of X to equal

$$X_{t+1} = \bar{m} \left( \Gamma X_t + \varepsilon_{t+1} \right), \tag{47}$$

as imposed in equation (41). The boundedly-rational expectation of  $X_{t+1}$  is then given by

$$\mathbb{E}_{t}^{BR}\left[X_{t+1}\right] = \bar{m}\mathbb{E}_{t}\left[X_{t+1}\right].$$

$$f(x_{t+1}, s_{t+1}^j) = pg(s_{t+1}^j)\delta_{s_{t+1}^j}(x_{t+1}) + (1-p)g(s_{t+1}^j)g(x_{t+1}),$$

where  $g(X_{t+1})$  denotes the distribution of  $X_{t+1}$  and  $\delta$  is the Dirac function. Given that the unconditional mean of  $X_{t+1}$  is 0, i.e.,  $\int x_{t+1}g(x_{t+1})dx_{t+1} = 0$ , it follows that

$$\mathbb{E}_{t}\left[X_{t+1}|S_{t+1}^{j} = s_{t+1}^{j}\right] = \frac{\int x_{t+1}f(x_{t+1}, s_{t+1}^{j})dx_{t+1}}{\int f(x_{t+1}, s_{t+1}^{j})dx_{t+1}}$$

$$= \frac{pg(s_{t+1}^{j})s_{t+1}^{j} + (1-p)g(s_{t+1}^{j})\int x_{t+1}g(x_{t+1})dx_{t+1}}{g(s_{t+1}^{j})}$$

$$= ps_{t+1}^{j}.$$

 $<sup>^{29}\</sup>mathrm{To}$  see this, note that the joint distribution of  $(X_{t+1},S^j_{t+1})$  is

# **B** Calibration

Parame	eter Valu	e Source/Target
THANK Parameters		
$\gamma$	1	Bilbiie (2020)
$\kappa$	0.02	Bilbiie (2020)
$\chi$	1.48	Bilbiie (2020)
$\lambda$	0.33	Bilbiie (2020)
s	$0.8^{1/2}$	<sup>'4</sup> Bilbiie (2020)
$Behavioral\ Parameter$		
$\bar{m}$	0.85	Gabaix (2020)

Table 1: Baseline calibration.

Our baseline calibration is summarized in Table 1. The values for  $\gamma$  and  $\kappa$  are directly taken from Bilbiie (2021, 2020) and are quite standard in the literature. Gabaix (2020), on the other hand, sets  $\kappa = 0.11$  and  $\gamma = 5$ . Even though these coefficients differ quite substantially from our baseline calibration, note that our results would barely be affected by this. To see this, note that amplification is only determined by  $\lambda$  and  $\chi$ , both independent of  $\kappa$  and  $\gamma$ . The determinacy condition on the other hand depends on both,  $\kappa$  and  $\gamma$ , but what ultimately matters is the fraction  $\frac{\kappa}{\gamma}$  (see Proposition 3). As  $\kappa$  and  $\gamma$  are both approximately five times larger in Gabaix (2020) compared to Bilbiie (2021) and our baseline calibration, the fraction is approximately the same and thus, the determinacy region under an interest-rate peg remains unchanged.

The household heterogeneity parameters,  $\chi$ ,  $\lambda$  and s are also standard in the THANK literature (see Bilbiie (2020)). The most important assumption for our qualitative results in Section 3 is  $\chi > 1$ , which is empirically supported. Patterson (2019) provides empirical evidence for the countercyclicality of inequality. Coibion et al. (2017), Ampudia et al. (2018) and Samarina and Nguyen (2019) provide evidence of countercyclical inequality conditional on monetary policy shocks. Almgren et al. (2019) show that output in countries with higher shares of hand-to-mouth households responds more strongly to monetary policy shocks which, through the lens of the model, implies countercyclical inequality.

For figure 5, i.e., to compute the iMPCs we choose a yearly calibration with s=0.8 and  $\beta=0.95$  (this calibration is close to the iMPC exercise in Bilbiie (2021) but while he fixes  $\chi$  to match the empirically-observed iMPCs, we vary  $\chi$  together with  $\bar{m}$  to examine their joint effects on iMPCs).

The Cognitive Discounting Parameter  $\bar{m}$ . The cognitive discounting parameter  $\bar{m}$  is set to 0.85, as in Gabaix (2020) and Benchimol and Bounader (2019). Fuhrer and Rudebusch (2004), for example, estimate an IS equation and find that  $\bar{m}\delta \approx 0.65$ , which together with  $\delta > 1$ , would imply a  $\bar{m}$  much lower than 0.85 and especially our determinacy results would be even stronger under such a calibration. Note, that the calibration of the backward-looking behavioral HANK model in Section 4, which is based on household survey expectations and taken from Angeletos and Huo (2021), is close to the estimation results from Fuhrer and Rudebusch (2004).

Another way to calibrate  $\bar{m}$  (as pointed out in Gabaix (2020)) is to interpret the estimates in Coibion and Gorodnichenko (2015) through the "cognitive-discounting lens". They regress forecast errors on forecast revisions

$$x_{t+h} - F_t x_{t+h} = c + b^{CG} (F_t x_{t+h} - F_{t-1} x_{t+h}) + u_t,$$

where  $F_t x_{t+h}$  denotes the forecast at time t of variable x, h periods ahead. Focusing on inflation, they find that  $b^{CG} > 0$  in consensus forecasts, pointing to underreaction (similar results are, for example, found in Angeletos et al. (2021) and Adam et al. (2020) for other variables).

In the model, the law of motion of x is  $x_{t+1} = \Gamma(x_t + \varepsilon_{t+1})$  whereas the behavioral agents perceive it to be  $x_{t+1} = \bar{m}\Gamma(x_t + \varepsilon_{t+1})$ . It follows that  $F_t x_{t+h} = (\bar{m}\Gamma)^h x_t$  and thus, forecast revisions are equal to

$$F_{t}x_{t+h} - F_{t-1}x_{t+h} = (\bar{m}\Gamma)^{h} x_{t} - (\bar{m}\Gamma)^{h+1} x_{t-1}$$
$$= (\bar{m}\Gamma)^{h} \Gamma(1 - \bar{m})x_{t-1} + (\bar{m}\Gamma)^{h} \varepsilon_{t}.$$

The forecast error is given by

$$x_{t+h} - F_t x_{t+h} = \Gamma^h (1 - \bar{m}^h) \Gamma x_{t-1} + \Gamma^h (1 - \bar{m}^h) \varepsilon_t + \sum_{j=0}^{h-1} \Gamma^j \varepsilon_{t+h-j},$$

where  $\sum_{j=0}^{h-1} \Gamma^j \varepsilon_{t+h-j}$  is the rational expectations forecast error. Gabaix (2020) shows that  $b^{CG}$  is bounded below  $b^{CG} \geq \frac{1-\bar{m}^h}{\bar{m}^h}$ , showing that  $\bar{m} < 1$  yields  $b^{CG} > 0$ , as found empirically. When replacing the weak inequality with an equality, we get

$$\bar{m}^h = \frac{1}{1 + b^{CG}}.$$

Most recently, Angeletos et al. (2021) estimate  $b^{CG}$  (focusing on a horizon h=3) to lie between  $b^{CG} \in [0.74, 0.81]$  for unemployment forecasts and  $b^{CG} \in [0.3, 1.53]$  for inflation, depending on the considered period (see their Table 1). These estimates imply  $\bar{m} \in [0.82, 0.83]$  for unemployment and  $\bar{m} \in [0.73, 0.92]$  for inflation, and are thus close to our preferred value of 0.85. Note, however, that these estimates pertain to professional forecasters and should therefore be seen as upper bounds on  $\bar{m}$ .

And since the focus of the paper is to understand the role of  $\bar{m}$ , we often vary  $\bar{m}$  anyway instead of focusing on one particular value.

# C Extensions

### C.1 Allowing for Steady State Inequality.

So far, we have assumed that there is no steady state inequality, i.e.,  $C^H = C^S$ . In the following, we relax this assumption and denote steady state inequality by  $\Omega \equiv \frac{C^S}{C^H}$ . Recall the savers' Euler equation

$$\left(C_{t}^{S}\right)^{-\gamma} = \beta R_{t} \mathbb{E}_{t}^{BR} \left[ s \left(C_{t}^{S}\right)^{-\gamma} + \left(1 - s\right) \left(C_{t}^{H}\right)^{-\gamma} \right],$$

from which we can derive the steady state real rate

$$R = \frac{1}{\beta(s + (1 - s)\Omega^{\gamma})}.$$

Log-linearizing the Euler equation yields

$$\widehat{c}_{t}^{S} = \beta R \bar{m} \left[ s \mathbb{E}_{t} \widehat{c}_{t+1}^{S} + (1-s) \Omega^{\gamma} \mathbb{E}_{t} \widehat{c}_{t+1}^{H} \right] - \frac{1}{\gamma} \left( \widehat{i}_{t} - \mathbb{E}_{t} \pi_{t+1} \right).$$

Combining this with the consumption functions and the steady state real rate yields the IS equation

$$\widehat{y}_t = \bar{m}\widetilde{\delta}\mathbb{E}_t\widehat{y}_{t+1} - \frac{1}{\gamma}\frac{1-\lambda}{1-\lambda\chi}\left(\widehat{i}_t - \mathbb{E}_t\pi_{t+1}\right),\tag{48}$$

with

$$\tilde{\delta} \equiv 1 + (\chi - 1) \frac{(1 - s)\Omega^{\gamma}}{s + (1 - s)\Omega^{\gamma}} \frac{1}{1 - \lambda \chi}.$$

From a qualitative perspective, the whole analysis in the paper could be carried out with  $\tilde{\delta}$  instead of  $\delta$ . Quantitatively the differences are small as well. For example, if we set  $\Omega=1.5$ , we get  $\tilde{\delta}=1.074$  instead of  $\delta=1.051$ . Thus, we need  $\bar{m}<0.91$  instead of  $\bar{m}<0.93$  for

determinacy under a peg.

# C.2 Persistent Monetary Policy Shocks

In the main text in Section 3, we illustrated the resolution of the Catch-22 by considering i.i.d. monetary policy shocks (following Bilbiie (2021)). The behavioral HANK model delivers initial amplification of these monetary shocks but the effects decrease with the horizon of the shock, i.e., the behavioral HANK model resolves the forward guidance puzzle. Another way to see this is by considering persistent shocks.

Figure 10 illustrates this. The figure shows the response of output in period t to a shock in period t for different degrees of persistence (x-axis). The black-solid line shows the output response in RANK and the blue-dashed line in the behavioral HANK. The forward guidance puzzle in RANK manifests itself in the sense that highly persistent shocks have stronger effects in RANK than in the behavioral HANK. Persistent shocks are basically a form of forward guidance and thus, with high enough persistence in the shocks, the RANK model predicts stronger effects than the behavioral HANK model.

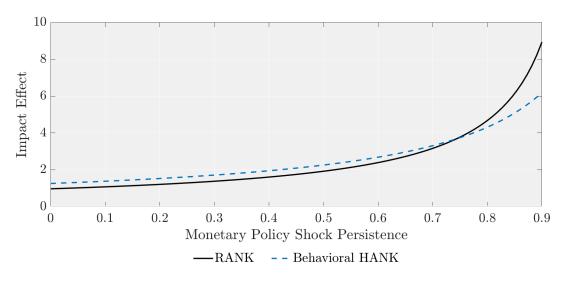


Figure 10: Initial Output Response for Varying Degrees of the Persistence

Note: This figure shows the initial output response to monetary policy shocks with different degrees of persistence.

# C.3 Forward-Looking NKPC and Real Interest Rates

In the main part of the paper, we made the assumption that agents are rational with respect to real interest rates (as in Gabaix (2020)) and assumed a static Phillips Curve (as in Bilbiie

(2021)). We now show that the results are barely affected when considering a forward-looking New Keynesian Phillips Curve (NKPC) and that agents are also boundedly rational with respect to real rates. Gabaix (2020) derives the NKPC under bounded rationality and shows that it takes the following form:

$$\pi_t = \beta M^f \mathbb{E}_t \pi_{t+1} + \kappa \widehat{y}_t.$$

with

$$M^{f} \equiv \bar{m} \left( \theta + \frac{1 - \beta \theta}{1 - \beta \theta \bar{m}} (1 - \theta) \right),\,$$

where  $1 - \theta$  captures the Calvo probability of price adjustment.

Taking everything together (including the bounded rationality with respect to real interest rates), the model can be summarized by the following three equations:

$$\widehat{y}_{t} = \psi_{f} \mathbb{E}_{t} \widehat{y}_{t+1} - \psi_{c} \frac{1}{\gamma} \left( \widehat{i}_{t} - \overline{m} \mathbb{E}_{t} \pi_{t+1} \right)$$

$$\pi_{t} = \beta M^{f} \mathbb{E}_{t} \pi_{t+1} + \kappa \widehat{y}_{t}$$

$$\widehat{i}_{t} = \phi \pi_{t}.$$

Plugging the Taylor rule into the IS equation, we can write everything in matrix form:

$$\begin{pmatrix}
\mathbb{E}_{t}\pi_{t+1} \\
\mathbb{E}_{t}\widehat{y}_{t+1}
\end{pmatrix} = \underbrace{\begin{pmatrix}
\frac{1}{\beta M^{f}} & -\frac{\kappa}{\beta M^{f}} \\
\frac{\psi_{c}}{\gamma\psi_{f}} \left(\phi - \frac{\bar{m}}{\beta M^{f}}\right) & \frac{1}{\psi_{f}} \left(1 + \frac{\psi_{c}\bar{m}\kappa}{\gamma\beta M^{f}}\right) \\
-\frac{\kappa}{2}
\end{pmatrix}}_{-\Lambda} \begin{pmatrix}
\pi_{t} \\
\widehat{y}_{t}
\end{pmatrix}.$$
(49)

For determinacy, we need

$$det(A) > 1; \quad det(A) - tr(A) > -1; \quad det(A) + tr(A) > -1.$$

The last condition is always satisfied. The first two conditions are satisfied if and only if

$$\phi > \max \left\{ \frac{\beta \delta M^f \bar{m} - 1}{\frac{\kappa}{\gamma} \frac{1 - \lambda}{1 - \lambda \chi}}, \bar{m} + \frac{(\delta \bar{m} - 1)(1 - \beta M^f)}{\frac{\kappa}{\gamma} \frac{1 - \lambda}{1 - \lambda \chi}} \right\}.$$

In the case of a static Phillips curve but bounded rationality with respect to the real rate, the second condition is the crucial one. To capture the static Phillips curve, we can simply set  $M^f = 0$ . In this case, it follows that we have a uniquely-determined (bounded) equilibrium for  $\phi > -3.22$ . Thus, the condition is even weaker than in the main part of the paper.

If we allow for a forward-looking Phillips curve and using the same calibration as in the main text and relying on Gabaix (2020) for the two newly-introduced parameters,  $\theta = 0.875$  and  $\beta = 0.99$ , it follows that we have determinacy even under an interest rate peg for our baseline calibration with  $\bar{m} = 0.85$ .

# D Details on Intertemporal MPCs

In this section, we derive the iMPCs discussed in Section 3.5. Defining  $Y_t^j$  as type j's disposable income, we can write the households' budget constraints as

$$C_{t}^{H} = Y_{t}^{H} + \frac{1-s}{\lambda} R_{t} B_{t}$$

$$C_{t}^{S} + \frac{1}{1-\lambda} B_{t+1} = Y_{t}^{S} + \frac{s}{1-\lambda} R_{t} B_{t},$$

where  $R_t$  denotes the real interest rate and  $B_t$  real bonds. Log-linearizing the two budget constraints around the zero-liquidity steady state and  $R = \beta^{-1}$  yields

$$\widehat{c}_t^H = \widehat{y}_t^H + \frac{1-s}{\lambda} \beta^{-1} b_t \tag{50}$$

$$\widehat{c}_{t}^{S} + \frac{1}{1 - \lambda} b_{t+1} = \widehat{y}_{t}^{S} + \frac{s}{1 - \lambda} \beta^{-1} b_{t}, \tag{51}$$

where  $b_t$  denotes real bonds in shares of steady state output. Aggregating (50) and (51) delivers

$$\hat{c}_t = \tilde{y}_t + \beta^{-1} b_t - b_{t+1}, \tag{52}$$

where  $\tilde{y}_t$  denotes aggregate disposable income.

By plugging equations (50) and (51) into the savers' Euler equation (14), we can derive the dynamics of liquid assets  $b_t$  (ignoring changes in the real rate as this is a partial equilibrium exercise):

$$\mathbb{E}_{t}b_{t+2} - b_{t+1} \left[ \frac{1}{s\bar{m}} + \beta^{-1}s + \frac{(1-s)^{2}\beta^{-1}(1-\lambda)}{s\lambda} \right] + \frac{\beta^{-1}}{\bar{m}}b_{t} =$$

$$(1-\lambda)\mathbb{E}_{t}\widehat{y}_{t+1}^{S} + \frac{1-s}{s}(1-\lambda)\mathbb{E}_{t}\widehat{y}_{t+1}^{H} - \frac{1-\lambda}{s\bar{m}}\widehat{y}_{t}^{S}.$$
(53)

Note that a change in total disposable income by one changes the hand-to-mouth households' disposable income by  $\chi$  and the savers' disposable income by  $\frac{1-\lambda\chi}{1-\lambda}$ .

Let us denote the right-hand side of equation (53) by  $-\mathbb{E}_t \hat{z}_t$ . Factorizing the left-hand side and letting F denote the forward-operator, it follows that

$$(F - \mu_1)(F - \mu_2)\mathbb{E}_t b_t = -\mathbb{E}_t \widehat{z}_t, \tag{54}$$

where  $\mu_1$  and  $\mu_2$  denote the roots of the characteristic equation

$$\mathbb{E}_t b_{t+2} - \phi_1 b_{t+1} - \phi_2 b_t = 0, \tag{55}$$

where

$$\phi_1 \equiv \left[ \frac{1}{s\overline{m}} + \beta^{-1}s + \frac{(1-s)^2\beta^{-1}(1-\lambda)}{s\lambda} \right]$$
 (56)

and

$$\phi_2 \equiv -\frac{\beta^{-1}}{\bar{m}}.\tag{57}$$

Thus, the roots are given by

$$\mu_{1,2} = \frac{\phi_1 \pm \sqrt{\phi_1^2 + 4\phi_2}}{2}.\tag{58}$$

It follows that

$$b_{t+1} = \mu_1 b_t - (F - \mu_2)^{-1} \mathbb{E}_t \widehat{z}_t$$
  
=  $\mu_1 b_t + \frac{\mu_2^{-1}}{1 - F \mu_2^{-1}} \mathbb{E}_t \widehat{z}_t$ .

Note that  $\mathbb{E}_t \widehat{z}_t$  can be written as  $\frac{1-\lambda\chi}{s} \left(\delta \mathbb{E}_t \widehat{y}_{t+1} - \frac{1}{\bar{m}} \widehat{y}_t\right)$ . Without loss of generality, we let  $\mu_2 > \mu_1$  and we have  $\mu_2 > 1$ . We have  $(1 - F\mu_2^{-1})^{-1} = \sum_{l=0}^{\infty} \mu_2^{-l} F^l$ . Thus, we end up with

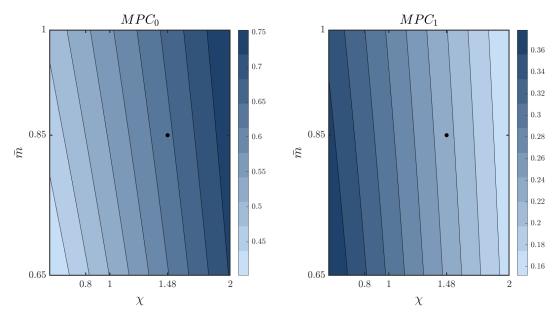
$$b_{t+1} = \mu_1 b_t + \frac{1 - \lambda \chi}{s} \sum_{l=0}^{\infty} \mu_2^{-(l+1)} \mathbb{E}_t \left( \frac{1}{\bar{m}} \widehat{y}_{t+l} - \delta \widehat{y}_{t+1+l} \right).$$
 (59)

Taking derivatives with respect to  $\hat{y}_{t+k}$  yields Proposition 5.

iMPCs and the Role of Idiosyncratic Risk. In Figure 11, we plot he MPCs in the year of the income windfall (left panel) and the first year after the windfall (right panel) for a relatively high idiosyncratic risk of 1-s=0.5. The high probability of becoming hand-to-mouth flips the role of  $\bar{m}$  for the  $MPC_1$  compared to our baseline calibration as discussed in Section 3.5. The reason being that the behavioral savers save a relatively large amount of the received income windfall in period 0 as they cognitively discount the decrease in their future marginal utility. Thus, they end up with relatively more disposable income in year 1. Now, given the relatively high probability of type switching, there are many savers who end up being hand-to-mouth in year 1 after the income windfall. As they are hand-to-mouth, they consume their previously-accumulated savings which increases the  $MPC_1$ . The more behavioral the savers are, i.e., the lower  $\bar{m}$  is, the more pronounced this effect and hence, a lower  $\bar{m}$  increases the  $MPC_1$  in the case of a relatively high 1-s.

**iMPCs for more than two periods.** Figure 12 plots the MPCs for the year of the income windfall as well as the five consecutive years for different degrees of rationality. As discussed in section 3.5, under our benchmark calibration, the rational model predicts somewhat larger

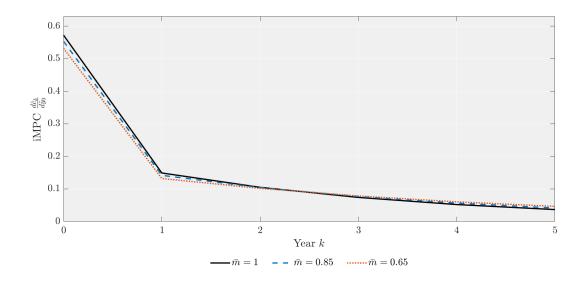
Figure 11: Intertemporal MPCs, Bounded Rationality and Household Heterogeneity



Note: This figure shows the aggregate intertemporal MPCs, i.e., the aggregate consumption response in year 0 (left) and year 1 (right) to a change in aggregate disposable income in year 0 for a transition probability 1 - s = 0.5.

initial MPCs as the behavioral savers save relatively more. Over time, however, the MPCs in the behavioral model lie above their rational counterparts due to the fact that more and more of the initial savers become hand-to-mouth and start consuming their (higher) savings. As Figure 11 shows, the probability of type switching, 1 - s, matters for when exactly the behavioral model starts to generate larger MPCs compared to the rational model.

Figure 12: Intertemporal MPCs



Note: This figure shows the aggregate intertemporal MPCs, i.e., the aggregate consumption response in year k to a change in aggregate disposable income in year 0 for different  $\bar{m}$ .