Inflation – who cares?
Monetary Policy in Times of Low Attention

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Abstract

The decrease in the volatility and persistence of US inflation was accompanied by a decline in the public's *attention to inflation*. This decline in attention weakens the effectiveness of make-up policies and negative interest rates and can lead to *inflation-attention traps*: prolonged periods of a binding lower bound and low inflation due to slowly-adjusting inflation expectations. To mitigate the drawbacks of lower attention, it is optimal to increase the inflation target as attention declines. Accounting for the lower bound fundamentally changes the normative implications of declining attention. While lower attention raises welfare absent the lower-bound constraint, it decreases welfare when accounting for the lower bound.

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1 Introduction

Managing inflation expectations is an important instrument for monetary policy. It offers a powerful substitute to conventional tools if the nominal interest rate is constrained by the lower bound. By making promises about the future conduct of its policy, the monetary authority can shape inflation expectations, and thus, steer the real interest rate. At least this is how it works in theory.[1] But while traditional analyses assume that agents have full-information rational expectations (FIRE), recent empirical evidence suggests that the general public is usually poorly informed about and inattentive to monetary policy and inflation.[2] But has the public always been so inattentive to inflation? And what do these low levels of attention imply for monetary policy? The present paper seeks to answer these questions.

Figure 1 gives a first idea of how the public's attention to inflation changed over the last five decades. It shows the frequency of the word "inflation" in two major US newspapers (blue-dashed lines), the New York Times (left panel) and the Washington Post (right panel), together with the annual US CPI inflation (black-solid lines). It is evident that news coverage is higher in times of high and volatile inflation as was the case during the 1970s and early 80s. Moreover, the figure suggests that the public's attention to inflation—proxied here by news coverage—has not always been as low as in recent years, but declined over time.[3]

In this paper, I propose an approach to quantify *attention to inflation* in the data. This approach is based on a model of optimal attention choice subject to information acquisition costs. The result is a law of motion for inflation expectations in which attention governs how strongly agents update their expectations following an inflation surprise. The optimal degree of attention depends positively on how high the stakes are, how volatile and persistent inflation is, and it depends negatively on the cost of information. Consistent with the recent literature (e.g., Candia et al. (2021) and Coibion et al. (2020)), as well as Figure 1, I find that attention was very low in recent years. In the 1970s and 1980s, however, attention to inflation was substantially higher. In line with the underlying model, there is a positive

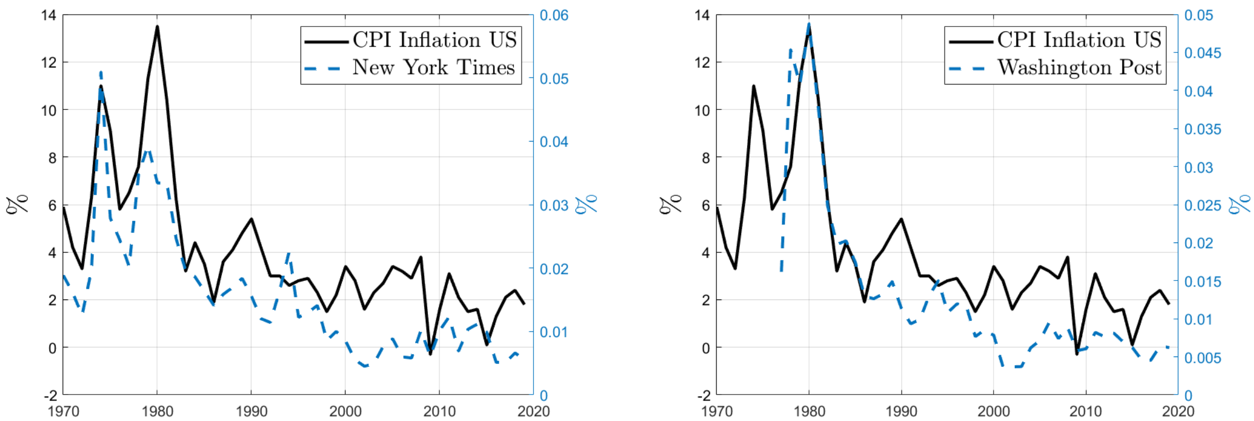
[1]For textbook analysis of optimal monetary policy under rational expectations, see e.g., Woodford (2003) or Galí (2015). See Eggertsson and Woodford (2003), Adam and Billi (2006) or Coibion, Gorodnichenko, and Wieland (2012) for early analyses including the lower bound on nominal interest rates.

[2]Most recently, Candia, Coibion, and Gorodnichenko (2021) and Coibion, Gorodnichenko, Knotek, and Schoenle (2020) show that US firms as well as households are usually poorly informed about and quite inattentive to monetary policy. Coibion and Gorodnichenko (2012, 2015a), for example, show that models of limited attention more closely align with empirical patterns of inflation expectations, compared to models of FIRE. In line with limited attention, D'Acunto, Hoang, and Weber (2020) show that forward guidance is quite ineffective in stimulating inflation expectations.

[3]Figure 7 (in Appendix A) shows a similar pattern when looking at book coverage of inflation and that this not just a US phenomenon, but applies to other countries as well.

Figure 1: News Coverage of Inflation

(a) New York Times (b) Washington Post



Notes: This figure shows the relative frequency (blue dashed lines, right axis) of the word "inflation" in the New York Times (left) and the Washington Post (right). The black solid line shows annual US CPI inflation (left axis).

relationship between attention and inflation volatility, as well as between attention and inflation persistence. This is true for professional forecasters, consumers and holds for a number of specifications.

How does the decline in attention matter for monetary policy? To answer this question, I augment the standard New Keynesian model with an effective lower bound (ELB) on the nominal interest rate and with inflation expectations that are characterized by limited attention. Incorporating limited attention considerably affects the conduct of monetary policy. On the plus side, lower attention has a stabilizing effect on inflation expectations and actual inflation, resembling more anchored (short-run) expectations.[4] I show analytically how a decline in attention mutes the reaction of inflation to shocks and to fluctuations in the output gap due to a muted response of inflation expectations. Put differently, a decline in attention resembles a flatter Phillips Curve, as in Afrouzi and Yang (2021), and as empirically documented, e.g., in Del Negro, Lenza, Primiceri, and Tambalotti (2020). Lower attention, however, also renders managing expectations more difficult, which is particularly relevant if the nominal interest rate is constrained by the lower bound.[5] This paper is the first to study this trade off of lower attention in a fully stochastic model with an occasionally

[4] I focus on *short-run* expectations, whereas anchoring usually refers to the stabilization of *long-run* expectations, see for example Gáti (2020) or Jørgensen and Lansing (2021).

[5] Coibion, Gorodnichenko, and Weber (2019) and Coibion et al. (2020) show that managing expectations by the central bank is indeed a difficult task and the effects of monetary policy are much smaller than in most theoretical models. D'Acunto et al. (2020) find small effects of forward guidance on inflation expectations and durable consumption.

binding lower-bound constraint and to characterize Ramsey optimal monetary policy in such a setting.

I show analytically how lower attention to inflation weakens the effectiveness of make-up policies such as forward guidance.[6] On top of that, a decrease in the lower bound—e.g., going from a zero lower bound to a negative lower bound—is less stimulating if agents' attention is low. A promising alternative to make-up policies is to increase the inflation target. A higher average inflation rate increases the average nominal rate, thus, provides the policymaker with additional space when faced with an adverse shock such that the lower bound binds less frequently. On top of that, even if the lower bound becomes binding after an adverse shock, the real rate is lower under a higher inflation target due to the slow adjustment of inflation expectations.

I then illustrate how under sub-otpimal policy limited attention can lead to substantially longer periods at the ELB. While lower levels of attention attenuate the initial response of inflation expectations to a given shock, the decline in expectations becomes more persistent which hinders actual inflation from recovering quickly. Due to the persistently-low inflation, the monetary authority keeps the interest rate at the ELB for longer. I refer to these periods of long spells at the ELB and persistent declines in inflation and inflation expectations as *inflation-attention traps*. The response of the output gap, on the other hand, is very similar to the one under rational expectations. Thus, low attention offers a potential explanation for why inflation was relatively stable during the Great Recession but was persistently low during the subsequent recovery, seemingly disconnected from output (as documented in Del Negro et al. (2020)).

To understand how monetary policy should *optimally* respond to inflation-attention traps and low attention in general, I then derive the Ramsey optimal policy in the presence of limited attention and the ELB. As attention declines, monetary policy becomes less powerful in managing inflation expectations, which is critical at the lower bound. To mitigate this loss of control and to prevent attention traps from happening, the optimal inflation target—the average inflation rate under Ramsey optimal policy—substantially increases as attention falls. As foreshadowed in the discussion of the analytical results on the effectiveness of make-up policies, increasing the optimal inflation target is a precautionary response to the ineffectiveness of forward guidance and other make-up policies under limited attention. The raise in the inflation target provides additional space when cutting the interest rate following adverse shocks, and therefore helps to prevent long spells at the ELB. Thus, the reason for

[6] My paper thus adds to the literature on the *forward-guidance puzzle* (see, e.g., Del Negro, Giannoni, and Patterson (2012), McKay, Nakamura, and Steinsson (2016) and Andrade, Gaballo, Mengus, and Mojon (2019)) and shows how low attention can resolve this puzzle.

a strictly positive inflation target is different from earlier papers that also considered an occasionally-binding ELB (e.g., Adam and Billi (2006), Adam, Pfäuti, and Reinelt (2020)). In these papers, the higher average inflation rate arises due to promises the policymaker makes *at the lower bound*. In the present paper, on the other hand, the higher inflation rate arises due to considerations *before the lower bound* binds, *foreseeing what will happen* at the lower bound. Given current estimates of attention, the optimal inflation target under limited attention is roughly 2-3 percentage points higher than under rational expectations.

Another instrument to combat the drawbacks of limited attention are negative interest rate policies. Allowing for negative interest rates up to -0.5% (annualized) lowers the necessary increase in the optimal inflation target. As attention declines, however, the effectiveness of negative interest rate policies weakens and the optimal increase in the inflation target is close to the one without negative rates.

The paper concludes by shedding light on the normative implications of lower attention while accounting for the lower-bound constraint. Absent the ELB constraint, lower attention is welfare improving through its stabilization effects on actual inflation (similar to the findings in Paciello and Wiederholt (2014)). When taking the ELB into account, however, welfare decreases as attention declines due to the increase in the average *level* of inflation. This level effect dominates the stabilization benefits. Thus, accounting for the ELB and its interactions with low levels of attention drastically changes the welfare implications of declining attention. Overall, these findings highlight potential drawbacks of stabilizing inflation expectations that arise once the effective lower bound becomes binding.

Related Literature Related to the empirical part of the present paper is Jørgensen and Lansing (2021) who show that inflation expectations have become more anchored over the last decades.[7] My measure of attention is inversely related to their definition of anchoring, but attention is concerned with short-run expectations whereas anchoring usually refers to the stabilization of long-run expectations. I complement their empirical analysis in several dimensions which are clearly spelled out in Section 2. Overall, my paper aligns well with their empirical findings and offers new insights in how stabilized expectations matter when nominal interest rates are constrained by a lower-bound constraint and what this implies for forward guidance and optimal monetary policy.

My limited-attention model of inflation expectations is closely related to the general information choice problem in Mackowiak, Matejka, and Wiederholt (2020). In contrast to their model, and the rational inattention literature more generally, agents in my model use

[7] Similarly, Gáti (2020) documents that anchoring of long-run inflation expectations is time varying and has substantially increased recently. Gallegos (2021), on the other hand, finds evidence for less anchored expectations due to more open communication of the Federal Reserve since the 1980s.

a simplified view about the variable to forecast.[8] This model is close to the one in Vellekoop and Wiederholt (2019). I complement their work by examining how attention changed over the last fifty years and by showing that attention tends to be higher in times of volatile and persistent inflation.

Ball, Mankiw, and Reis (2005), Adam (2007) and Paciello and Wiederholt (2014) characterize optimal monetary policy in a model with limited attention. Gáti (2020) also studies optimal monetary policy when agents slowly update their expectations and the updating gain is endogenous. In contrast to these papers, I introduce a lower bound on the nominal interest rate.

Wiederholt (2015) and Gabaix (2020) examine how information rigidities and inattention matter at the zero lower bound. Angeletos and Lian (2018) study the implications of relaxing the common knowledge assumption for forward guidance and show that the effects of forward guidance are attenuated in such a setting. My paper complements these three papers by studying the Ramsey optimal policy in a fully stochastic setup and focuses on the implications for the optimal inflation target.

The early literature on the optimal inflation rate usually finds an optimal inflation rate of close to zero (see Schmitt-Grohé and Uribe (2010)), even when accounting for the effective lower bound (Adam and Billi (2006), Coibion et al. (2012)). More recent studies challenge this view. Eggertsson, Mehrotra, and Robbins (2019), Andrade, Galí, Le Bihan, and Matheron (2019, 2021) and Adam et al. (2020), for example, all find that a higher inflation target is desirable in a world of falling natural rates and nominal rates that are constrained by the ELB. Other reasons to increase the inflation target are, for example, nominal wage rigidities (Benigno and Ricci (2011)) or relative price trends (Adam and Weber (2019, 2020)). I show in the present paper that limited attention provides an additional reason to consider a higher inflation target.

Road Map The rest of the paper is structured as follows. The empirical strategy to quantify attention, the description of the data and the empirical results are presented in Section 2. In Section 3, I show how limited attention renders make-up policies ineffective and how it can lead to inflation-attention traps, before I then study optimal policy in Section 4. Section 5 concludes.

[8] Mackowiak et al. (2020) provide a recent overview of this literature, which was inspired by the seminal paper Sims (2003). For further developments in this literature, see, among others, Mackowiak and Wiederholt (2009), Paciello and Wiederholt (2014), Maćkowiak, Matějka, and Wiederholt (2018), Afrouzi and Yang (2021), and see Gabaix (2019) for an overview of *behavioral* inattention.

2 Quantifying Attention

In this section, I derive an expectations-formation process under limited attention that provides a straightforward approach to measure attention to inflation empirically. The model is an application of Mackowiak et al. (2020), who study a general problem of optimal information acquisition. I sketch the model here and relegate all the details and derivations to appendix B.

To form her expectations, the agent acquires information. Even if all necessary information is in principle available, acquiring and processing information is costly, given the limited cognitive abilities and limited amount of time available. This means that the amount of attention the agent pays to relevant information is limited, and will depend on how costly acquiring information is, how high the stakes are, as well as the properties of inflation itself.

The main difference to Mackowiak et al. (2020) is that agents in my model do not exactly know the underlying process of inflation but have a simplified view of how inflation evolves. In particular, the agent believes that (demeaned) inflation tomorrow, π', depends on (demeaned) inflation today, π, as follows

$$\pi' = \rho_\pi \pi + \nu,$$

where $\rho_\pi \in [0, 1]$ denotes the perceived persistence of inflation and $\nu \sim i.i.N.(0, \sigma_\nu^2)$. This assumption is supported by empirical evidence (see e.g., Faust and Wright (2013) or Canova (2007)).[9] Note, that the perceived volatility and persistence need not be the same as their actual counterparts, consistent with the empirical evidence on inflation expectations (see, e.g., Table 5 in the Appendix). Inflation in the current period is unobservable, so before forming an expectation about future inflation, the agent needs to form an expectation about today's inflation. I denote this nowcast $\widetilde{\pi}$, and the resulting forecast about next period's inflation $\pi^e = \rho_\pi \widetilde{\pi}$. Given her beliefs, the full-information forecast π^{e*} is

$$\pi^{e*} \equiv \rho_\pi \pi.$$

But since π is not perfectly observable, the actual forecast will deviate from the full-information forecast. Deviating, however, is costly, as this causes the agent to make mistakes in her decisions.

The agent's choice is not only about how to form her expectations given certain information, but about how to choose this information optimally, while taking into account how

[9] Fulton and Hubrich (2021) show that simple models such as AR(1) models are hard to beat when forecasting inflation in real time.

this will later affect her forecast. That is, she chooses the form of the signal s she receives about current inflation. Since acquiring information is costly, it cannot be optimal to acquire different signals that lead to an identical forecast. Due to this one-to-one relation of signal and forecast, we can directly work with the joint distribution of π^e and π, $f(\pi^e, \pi)$, instead of working with the signal.

Let $U(\pi^e, \pi)$ denote the negative of the loss that is incurred when the agent's forecast deviates from the forecast under full information, and assume it to be quadratic

$$U(\pi^e, \pi) = -r\left(\rho_\pi \pi - \pi^e\right)^2,$$

where r measures the stakes of making a mistake.[10,11]

Acquiring information is costly, and I assume that the cost is linear in *mutual information* $I(\pi; \pi^e)$, i.e., the expected reduction in entropy of π due to knowledge of π^e:

$$C(f) = \lambda I(\pi; \pi^e) = \lambda\left(H(\pi) - E\left[H(\pi|\pi^e)\right]\right),$$

where $H(x) = -\int f(x) log(f(x))dx$ is the entropy of x and λ is a parameter that measures the cost of information.

In this setup and with a normal prior, Gaussian signals are optimal (and in fact the unique solution, see Matějka and McKay (2015)). The optimal signal thus has the form

$$s = \pi + \varepsilon,$$

with $\varepsilon \sim i.i.N.(0, \sigma_\varepsilon^2)$.

The optimal forecast is given by $\pi^e = \rho_\pi E\left[\pi|s\right]$, and Bayesian updating implies

$$\pi^e = \rho_\pi\left(1-\gamma\right)\hat{\pi} + \rho_\pi \gamma s, \tag{1}$$

where $\gamma = 1 - \frac{\sigma^2_{\pi|s}}{\sigma^2_\pi} \in [0, 1]$ measures how much attention the agent pays to inflation, and $\hat{\pi}$ denotes the prior mean of π.

Solving for the optimal γ and writing the cost of information relative to the stakes, $\tilde{\lambda} \equiv \frac{\lambda}{r}$

[10] A quadratic loss function is usually derived from a second-order approximation of the household's utility function or the firm's profit function (see, e.g., Mackowiak and Wiederholt (2009)).

[11] These stakes (or also the information cost parameter λ) can be interpreted as a way to incorporate other variables to which the agent might pay attention. For example, a household might not only want to forecast inflation but also her own income stream going forward. In this case, a smaller r could capture an increase in her idiosyncratic income volatility. Thus, paying attention to inflation is relatively less beneficial, as the relative importance of her idiosyncratic income increases. Such an interpretation also explains why professional forecasters might not be fully informed about inflation, given that they usually forecast a whole array of variables.

yields the *optimal* level of attention, summarized in the following Lemma.[12]

Lemma 1 *The optimal level of attention is given by*

$$\gamma = max\left(0, 1 - \frac{\tilde{\lambda}}{2\rho_\pi^2\sigma_\pi^2}\right), \tag{2}$$

which shows that the optimal level of attention is

(i) decreasing in the relative cost of information acquisition, $\tilde{\lambda} \equiv \frac{\lambda}{r}$*,*

(ii) increasing in inflation volatility, σ_π*, and*

(iii) increasing in inflation persistence, ρ_π*.*

Attention in equation (1) captures how much the agent revises her expectations after making a forecast error. An inattentive agent does not put a lot of weight on her received information and rather sticks to her prior beliefs, whereas an attentive agent updates her expectations strongly since her received signals are more precise. From Lemma 1, we see that aside from the relative information cost, $\tilde{\lambda}$, the persistence, ρ_π, and the volatility of inflation, σ_π, are crucial drivers of attention. The model predicts a positive relationship between attention and σ_π, as well as between attention and ρ_π. In the following, I will first estimate attention γ, asses how it changed over time and then test whether there is indeed evidence for these positive relations.

2.1 Bringing the Model to the Data

I now lay out the empirical strategy to estimate attention in the data. To do so, I basically extend the static version just outlined to a dynamic setting. The agent believes that inflation π follows

$$\pi_t = (1 - \rho_\pi)\bar{\pi} + \rho_\pi \pi_{t-1} + \nu_t,$$

where $\bar{\pi}$ is the agent's long-run belief about inflation and ρ_π is the perceived persistence of inflation. I assume that the error term ν_t is normally distributed with mean zero and variance σ_ν^2.

The agent receives a signal about inflation of the form

$$s_{it} = \pi_t + \varepsilon_{it},$$

[12] All derivation details are in Appendix B, but this is identical with the example in Mackowiak et al. (2020) with a quadratic loss function and Gaussian shocks.

where the noise ε_{it} is assumed to be normally distributed with variance σ_ε^2.

Given these assumptions, it follows from the (steady state) Kalman filter that optimal updating is given by

$$\pi^e_{t+1|t,i} = (1-\rho_\pi)\bar{\pi} + \rho_\pi \pi^e_{t|t-1,i} + \rho_\pi \gamma \left(\pi_t - \pi^e_{t|t-1,i}\right) + u_{i,t}. \tag{3}$$

From equation (3), we observe that lower attention implies that the agent updates her expectations to a given forecast error, $(\pi_t - \pi^e_{t|t-1,i})$, less strongly. Lower attention is reflected in more noisy signals. More noise means the agent trusts her received signals less and thus, puts less weight on these signals. Hence, her expectations remain more strongly anchored at her prior beliefs.

In the estimation of equation (3), I allow for individual-specific intercepts. This can either reflect a mean bias in the perceived inflation rate, $\bar{\pi}_i \neq \bar{\pi}$, or that the agent believes her signals are biased on average, as in Vellekoop and Wiederholt (2019).

2.2 Data

I focus on the Survey of Professional Forecasters (SPF) from the Federal Reserve Bank of Philadelphia, as well as the Survey of Consumers from the University of Michigan (SoC). In the Appendix, I show that the findings extend to other data sets as well. For the SPF, I consider individual and aggregate forecasts. The main focus is on expectations about the quarter-on-quarter percentage change in the GDP deflator, which is available since 1969. I drop forecasters for which I have less than eight observations. As a robustness check, I will show that the results are robust to using expectations about the consumer price index, CPI. This data series, however, is only available since 1979.

While the SPF provides data on expectations about the next quarter, the SoC only provides one-year-ahead expectations. Therefore, I will compare them to the actual year-on-year changes in the CPI.[13] A drawback of the SoC is that it includes the same survey respondents at most twice. Therefore, estimating how individuals update their expectations is infeasible. Instead, I consider average (and median) expectations. Additionally, I estimate attention using the Survey of Consumer Expectations from the Federal Reserve Bank of New York (SCE). The SCE, launched in 2013, has a panel structure and thus allows me to estimate attention using individual-consumer data, at least for the period after 2013. Data on actual inflation comes from the FRED database from the Federal Reserve Bank of St. Louis. Appendix C provides summary statistics and plots the discussed time series.

[13]The question in the SoC is not explicitly about the CPI but about “prices”. It is standard in the literature to compare it to the CPI.

As discussed in the literature review above, Jørgensen and Lansing (2021) estimate how strongly anchored inflation expectations in the US are and how this changed over the last fifty years. I extend their empirical strategy in several dimensions. First, I allow the persistence of perceived inflation to change over time and do not restrict it to follow a random walk. Second, I show that not only aggregate professional forecasters' expectations have become more anchored, but also consider individual-specific expectations, as well as consumers' inflation expectations. Third, I do not impose the structure of the New Keynesian Phillips Curve on the data but directly estimate attention simply based on the proposed law of motion for inflation expectations.

2.3 Estimation Results

Before estimating attention, I rewrite the updating equation (3) as

$$\pi^e_{t+1|t,i} = \beta_i + \beta_1 \pi^e_{t|t-1,i} + \beta_2 \left(\pi_t - \pi^e_{t|t-1,i}\right) + u_{i,t}, \tag{4}$$

where $\beta_i = (1-\rho_\pi)\bar{\pi}_i$, $\beta_1 = \rho_\pi$ and $\frac{\beta_2}{\beta_1} = \gamma$. I estimate (4) using a forecaster-fixed-effects regression. Since the dependent variable shows up with a lag on the right-hand side, however, the exogeneity assumption is violated. Therefore, I apply the estimator proposed by Blundell and Bond (1998) (BB for short) and I use all available lags of the dependent variable as instruments.[14] All reported standard errors are robust with respect to heteroskedasticity and serial correlation. As an alternative to the BB estimator, I also estimate (4) using pooled OLS. For the Survey of Consumers, I apply the Newey-West estimator using four lags (Newey and West (1987)).

To examine how attention changed over time, I run regression (4) for the period before and after 1990, separately. The results are robust to different split points (see Appendix D). Later on, I will estimate (4) using rolling-windows of ten years each.

Table 1 shows the results. We see that the estimated attention parameter γ is substantially lower after 1990 compared to the period before 1990.[15] This is true for professional forecasters (first two columns) and for consumers (third and fourth column), and as I show in Appendix D, also for the Livingston Survey and the Federal Reserve Bank's Greenbook forecasts.[16] The point estimates after 1990 are basically half of what they were before the 1990s.

[14]Appendix D shows that the results are robust to using fewer lags.

[15]I test the validity of the instruments in the Blundell-Bond estimation by testing for autocorrelation of order one and two in the first-differenced error terms. The respective p-values are 0.000 (order 1) and 0.973 (order 2) for the period before 1990 and 0.000 (order 1) and 0.737 (order 2) for the period after 1990. This indicates that the instruments used in the estimation are valid.

[16]I show in Appendix D that these results also hold for other split points of the sample as well as other changes in the exact setup.

Table 1: Regression Results of Equation (4)

	Survey of Professional Forecasters				Survey of Consumers			
	Blundell Bond		Pooled OLS		Averages		Median	
	< 1990	≥ 1990	< 1990	≥ 1990	< 1990	≥ 1990	< 1990	≥ 1990
$\widehat{\gamma}$	0.70	0.41	0.44	0.22	0.75	0.31	0.43	0.24
s.e.	(0.1005)	(0.0522)	(0.0397)	(0.0290)	(0.1574)	(0.0881)	(0.0970)	(0.0601)
N	2235	3566	2235	3566	84	120	47	120

Note: This table shows the results from regression (4) for professional forecasters (SPF) as well as for consumers. For the SPF, I use the Blundell and Bond (1998) (BB) estimator (first two columns), as well as pooled OLS (columns 3-4). For the Survey of Consumer, I consider average expectations (columns 5-6) and median expectations (columns 7-8). The standard errors, reported in parentheses, are robust with respect to heteroskedasticity and serial correlation.

To interpret these numbers, let's consider the estimates from the Blundell-Bond estimator. We see that before the 1990s forecasters adjusted their expectations after a one percentage point forecast error by 0.70 percentage points. After 1990, however, the adjustment was a mere 0.41 percentage points on average.

The decline in attention is even more pronounced when focusing on the most recent decade. To show this, I run regression (4) for the period between 2010 and 2020. Additionally, I also use data from the New York Fed Survey of Consumer Expectations, starting in 2013. The advantage of this survey compared to the Michigan Survey is that it surveys the same consumers up to twelve times in a row, providing a sample size magnitudes larger. Table 2 shows the results. We see that overall, attention declined significantly compared to earlier periods and is between 0.04 and 0.17 during this period of low and stable inflation. Furthermore, the results from the SCE lie in the same ballpark as the ones from the Michigan Survey, which indicates that using average (or median) consumer expectations does not fundamentally affect the results. In fact, the estimated attention parameter for the average expectations from the Michigan Survey is 0.04 when restricting the sample to 2013-2020, which is exactly the same as the estimate obtained from the New York Fed Survey.

2.3.1 What Drives Attention?

Lemma 1 states that two key drivers of attention are inflation volatility and inflation persistence. To examine these relationships empirically, I estimate regression (4), using a rolling-window approach in which every window is 10 years long, and obtain one attention parameter for every window, $\widehat{\gamma}_t$, as well as the period-specific inflation volatility, $\widehat{\sigma}_{\pi,t}$ and the persistence parameter, $\widehat{\rho}_{\pi,t}$. In particular, I use the window-specific standard deviation of inflation as my measure of $\widehat{\sigma}_{\pi,t}$ and the first-order autocorrelation of inflation for $\widehat{\rho}_{\pi,t}$. Appendix D shows

Table 2: Attention since 2010

	Survey of Professional Forecasters		Michigan Survey		NY Fed Survey
	Blundell Bond	Pooled OLS	Averages	Median	Pooled OLS
$\widehat{\gamma}$	0.17	0.07	0.12	0.09	0.04
s.e.	(0.0729)	(0.0333)	(0.0658)	(0.0616)	(0.0316)
N	1322	1322	40	40	74229

Note: This table shows the results from regression (4) for the period between 2010 and 2020 for professional forecasters (SPF) as well as for consumers. For the SPF, I use the Blundell and Bond (1998) (BB) estimator (first column), as well as pooled OLS (column 2). For the Survey of Consumer, I consider average expectations (column 3) and median expectations (columns 4). The standard errors, reported in parentheses, are robust with respect to heteroskedasticity and serial correlation. Additonally, column 5 shows the results for consumer inflation expectations from the New York Fed Survey of Consumer Expectations.

that the following results also hold for different window lengths or when using the standard deviation and persistence of *expected* inflation for $\widehat{\sigma}_{\pi,t}$ and $\widehat{\rho}_{\pi,t}$.

Figure 2 summarizes the results graphically. The left scatterplot shows the inflation volatility on the x-axis and the estimated attention parameter, $\widehat{\gamma}$, on the y-axis. The attention parameters shown in the figure are the ones for individual professional forecasters, obtained via pooled OLS. The right panel shows the relationship between attention and inflation persistence (on the x-axis). In both cases, we see that there is a clear positive relationship, just as the limited-attention model predicts.[17]

To check if these findings are statistically significant, I regress attention on inflation volatility or on inflation persistence as follows

$$\widehat{\gamma}_t = \alpha_1 + \beta\widehat{\sigma}_{\pi,t} + u_t \tag{5}$$

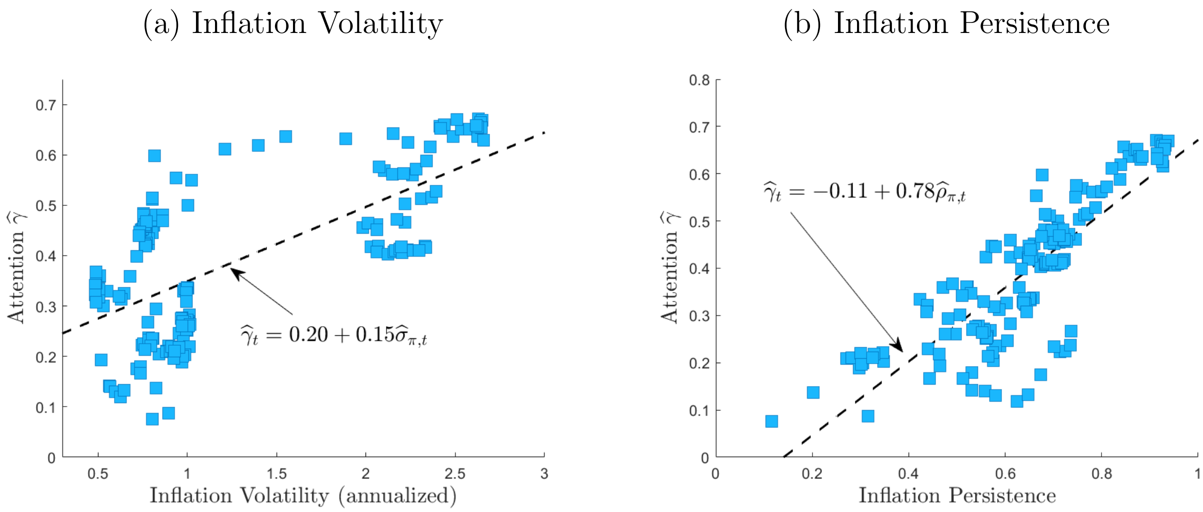
$$\widehat{\gamma}_t = \alpha_2 + \zeta\widehat{\sigma}_{\pi,t} + u_t. \tag{6}$$

Table 3 reports the results. Standard errors are robust with respect to heteroskedasticity. We see that the observed patterns in Figure 2 are indeed statistically significant. Attention to inflation is positively correlated with inflation volatility and inflation persistence, as Lemma 5 predicts. This is true for professional forecasters as well as for consumers. Overall, the magnitudes of the estimates indicate that the results are somewhat stronger for professional forecasters. Appendix D shows that these results hold when regressing attention on inflation volatility and persistence jointly.

Given the decline in inflation volatility and inflation persistence over the last fifty years (see Table 5 in Appendix C), the positive correlation with attention supports the findings

[17] Figure 9 in Appendix D shows the estimated attention levels for the SPF together with inflation volatility graphically over time.

Figure 2: Attention, Inflation Volatility and Inflation Persistence



Notes: The right panel shows the relationship between inflation volatility and the estimated attention parameter, $\widehat{\gamma}$. The left panel shows the relationship between the persistence of inflation and the estimated attention parameter. Both panels report the results for individual professional forecasters where the attention parameter was estimated via pooled OLS.

Table 3: Attention, Inflation Volatility and Inflation Persistence

	Survey of Professional Forecasters		Michigan Survey
Estimator	Blundell-Bond	Pooled OLS	OLS
$\widehat{\beta}$	0.13***	0.15***	0.09***
s.e.	(0.0155)	(0.0105)	(0.0272)
$\widehat{\zeta}$	0.71***	0.78***	0.56***
s.e.	(0.0568)	(0.0349)	(0.0714)
N	165	165	163

Note: This table shows the results of regression (5) and (6). Standard errors are robust with respect to heteroskedasticity. *** : p-value < 0.01, ** : p-value < 0.05, * : p-value < 0.1.

in Table 1. Namely, that attention declined during the same time period. Benati (2008) documents a decline in inflation persistence in advanced economies, especially for countries that introduced inflation targeting regimes. Just as the limited-attention model predicts, while inflation volatility and persistence declined, so did attention. The question is, how does this matter for monetary policy?

2.3.2 Robustness

The previous results are robust to a wide array of specifications. I briefly summarize them here, and relegate the details to Appendix D.

Different Data Sources Instead of looking at professional forecasters and consumers, I show that attention also decreased when considering the Fed's Greenbook forecasts or the Livingston Survey. Furthermore, the SPF also provides forecasts about the CPI instead of the GDP deflator. Because this series is only available since 1979, the pre-1990 level of attention is smaller than for GDP-deflator forecasts. Nevertheless, there is a decline in attention.

Different Sample Splits I show that the decline in attention is not specific to the sample split point in 1990, but is a more general feature observed in the data. When splitting the sample in 1985 or 1995 instead of 1990, the decline in attention is similar to the baseline split. In fact, when splitting the sample already in 1985, the decline in attention is even more pronounced as the period between 1985 and 1990 was a time of already relatively low and stable in inflation.

Different Specifications of the BB Estimator In the baseline specification of the estimator proposed in Blundell and Bond (1998), I include all possible lags as instruments. I vary the maximum number of lags and show that in all cases there is a clear decline in attention.

Trend Inflation Since different levels of trend inflation might affect the results, I include time-fixed effects. This also takes care of unobserved changes in the underlying persistence and/or volatility of inflation. The decline in attention is somewhat muted, but still present. Furthermore, I find a significantly positive relationship between attention and inflation volatility, as well as between attention and inflation persistence. Additional support for the fact that the results are not driven by changes in trend inflation is that the outlined rolling-window approach takes (partly) care of changes in trend inflation by allowing for window-specific intercepts.

SPF Aggregates In the baseline specification, I consider individual forecasts for the SPF. Using average expectations instead shows that the observed decline in attention is robust to this.

Joint Regression Instead of regressing attention on inflation volatility and inflation persistence separately, I show that the results are robust to regressing on them jointly.

Quasi Panel of Consumers I group consumers, from the Survey of Consumers, in four groups based on their income and can thus allow for income-group-specific fixed effects and estimate attention using the Blundell and Bond (1998) estimator, as I did for professional forecasters. The results are robust and barely change compared to the case with average consumer expectations.

Volatility and Persistence of Inflation Expectations Instead of looking at the relation between attention and inflation volatility and persistence, I consider the volatility and persistence of inflation expectations. The results hold.

Window Length The previous section showed that there is a positive relationship between attention to inflation and inflation volatility. To show this, I relied on a rolling-windows approach in which every window had a length of ten years. In the Appendix, I show that these findings are robust to using different window lengths.

3 A New Keynesian Model with Limited Attention

How does the decline in attention to inflation affect the conduct of monetary policy? To answer this question, I augment the standard New Keynesian model with inflation expectations that are characterized by limited attention, and a lower-bound constraint on the nominal interest rate. To do this, I replace the inflation expectations under FIRE with their limited-attention counterpart derived in the previous section.[18] This way of incorporating limited attention in the model allows me to study the Ramsey optimal policy in the presence of an occasionally-binding lower bound on the policy rate.[19] Apart from the formation of inflation expectations, I build on the standard New Keynesian model without capital, with rigid prices in the spirit of Calvo (1983) and Yun (1996) and with a lower bound on the nominal interest rate.[20] The government pays a subsidy to intermediate-goods producers to eliminate steady state distortions arising from market power.

[18]One way to *micro-found* this would be similar to Section V in Angeletos and La'o (2010), where, for example, firms have two managers that do not communicate with each other. In my setting, this would require that one manager sets the firm's price and takes its employment decision for some *given inflation expectation* and the other manager provides this inflation forecast, according to the steps outlined in Section 2, without taking the other manager's decisions into account (and similary for the representative household). An approach similar to mine, namely replacing the expectations after the derivation of the equilibrium conditions, is taken in Jørgensen and Lansing (2021) or parts of the *learning* literature (see, for example, Evans and Honkapohja (2003) or Milani (2007) for early contributions to the monetary policy literature).

[19]See, e.g., Nimark (2008), Mackowiak and Wiederholt (2009), Angeletos and Huo (2021), Gáti (2020), Afrouzi and Yang (2021), and Gallegos (2021)), for papers that derive the New Keynesian model (or certain blocks of it) under some form of non-FIRE assumptions without this ad-hoc assumption. These papers, however, do not account for the occasionally binding ELB.

[20]See, e.g., Woodford (2003) or Galí (2015) for the derivation of this model, and Adam and Billi (2006) for the analysis of how the optimal monetary policy changes in the presence of an ELB constraint.

The linearized model yields an aggregate supply equation, the so-called *New Keynesian Phillips Curve*, and an aggregate Euler equation.

$$\pi_t = \beta \pi^e_{t+1|} + \kappa y^{gap}_t + u_t, \tag{7}$$
$$y^{gap}_t = E_t y^{gap}_{t+1} - \varphi \left(i_t - \pi^e_{t+1|t} - r^n_t \right), \tag{8}$$

where κ measures the sensitivity of aggregate inflation to changes in the output gap, y^{gap}, $\beta \in (0,1)$ is the time discount factor of the representative household, and u_t are cost-push shocks, following an AR(1) process with persistence $\rho_u \in [0,1]$ and innovations $\varepsilon^u \sim i.i.N.(0, \sigma^2_u)$. The output gap is the log deviation of output from its efficient counterpart that would prevail under flexible prices. Altogether, equation (7) summarizes the aggregate supply side of the economy. Equation (8), together with monetary policy, determines aggregate demand in this model. Here, $\varphi > 0$ measures the real rate elasticity of output, i_t is the nominal interest rate which is set by the monetary authority, and r^n_t is the natural interest rate. The natural interest rate is the real rate that prevails in the economy with fully flexible prices and is exogenous. It follows an AR(1) process with persistence $\rho_r \in [0,1]$ and innovations $\varepsilon^r \sim i.i.N.(0, \sigma^2_r)$, independent of ε^u. The nominal interest rate and the natural rate are both expressed in absolute deviations of their respective steady state values, $\bar{i}$ and $\bar{r}^n$, with $\bar{i} = \bar{r}^n$, as I linearize the model around the zero-inflation steady state. E_t denotes the full-information rational expectations operator.

Inflation expectations are characterized by limited attention and are given by

$$\pi^e_{t+1|t} = (1 - \rho_\pi)\bar{\pi} + \rho_\pi \pi^e_{t|t-1} + \rho_\pi \gamma \left(\pi_t - \pi^e_{t|t-1} \right), \tag{9}$$

where the notation is the same as in Section 2. Given the representative agent assumption I abstract from noise shocks in (9). For the most part, I will focus on the case with $\rho_\pi = 1$ in which average inflation expectations align with actual average inflation and long-run beliefs $\bar{\pi}$ are irrelevant. This belief formation process is empirically plausible, in the sense that it is consistent with recent empirical findings, documented in Angeletos, Huo, and Sastry (2020): after a shock, expectations initially underreact, followed by a delayed overreaction (see Appendix F.5).

As is standard in the rational inattention literature, I assume that the attention parameter γ is constant.[21] The usual assumption to obtain this is that in period $t = 0$ the agent chooses her level of attention and then obtains all future signals at this point. This leaves conditional second moments time-invariant and thus, the optimal level of attention constant. I will,

[21] See, e.g., Mackowiak and Wiederholt (2009), Maćkowiak et al. (2018); Mackowiak et al. (2020).

however, compare economies with different levels of attention.

Table 4: Model Parameterization

Parameter	Value	Source/Target
	Preferences and technology	
β	0.9975	Average natural rate of 1%
φ	1	Adam and Billi (2006)
κ	0.057	Adam and Billi (2006)
	Exogenous shock processes	
ρ_r	0.8	Adam and Billi (2006)
σ_r	0.2940%	Adam and Billi (2006)
ρ_u	0	Adam and Billi (2006)
σ_u	0.154%	Adam and Billi (2006)

Calibration I calibrate the model to quarterly frequency. I assume an annualized steady state natural rate of 1%. The rest of the calibration is taken from Adam and Billi (2006). Table 4 summarizes the calibration.

Attention and the Phillips Curve With the aforementioned inflation expectations in the case of $\rho_\pi = 1$, the following proposition shows that inflation becomes less sensitive to cost-push shocks and fluctuations in the output gap as attention declines.

Proposition 1 *The New Keynesian Phillips Curve under limited attention is given by*

$$\pi_t = \frac{\beta(1-\gamma)}{1-\beta\gamma}\pi^e_{t|t-1} + \frac{\kappa}{1-\beta\gamma}y^{gap}_t + \frac{1}{1-\beta\gamma}u_t.$$

A decrease in attention γ leads to a muted reaction of inflation

(i) to fluctuations in the output gap, i.e., it resembles a flatter Phillips Curve,

(ii) to cost-push shocks, u_t, when keeping y^{gap}_t constant, and

(iii) to changes in prior beliefs, $\pi^e_{t|t-1}$, when keeping y^{gap}_t constant.

Proof. See Appendix E. ■

Proposition 1 captures the stabilizing effects of lower attention. As attention declines, firms' inflation expectations react less to changes in actual inflation. Through the Phillips Curve, this muted reaction of expectations in turn stabilizes inflation itself. What cannot be seen from Proposition 1, but what will be crucial in the subsequent analysis, is that lower attention not only affects the initial response of inflation and inflation expectations but the dynamics as well. Changes in inflation and inflation expectations become more persistent at low levels of attention, even though the initial response is muted.

Combining Proposition 1 with the Euler equation (8) yields the following lemma.

Lemma 2 *Inflation under limited attention can be written as*

$$\pi_t = \frac{\kappa}{1-\gamma(\beta+\kappa\varphi)} E_t y^{gap}_{t+1} - \frac{\kappa\varphi}{1-\gamma(\beta+\kappa\varphi)} i_t + \frac{\beta+\varphi\kappa}{1-\gamma(\beta+\kappa\varphi)}(1-\gamma)\pi^e_{t|t-1}$$
$$+ \frac{1}{1-\gamma(\beta+\kappa\varphi)} u_t + \frac{\kappa\varphi}{1-\gamma(\beta+\kappa\varphi)} r^n_t.$$

Under the assumption $1-\gamma(\beta+\kappa\varphi) > 0$, which is the case with my calibration for all estimated values of γ, Lemma 2 shows that lower attention mutes the reaction of inflation to shocks, u_t and r^n_t. Lower attention, however, also weakens the inflation response to changes in current nominal interest rates, i_t, as well as expected future output gaps, $E_t y^{gap}_{t+1}$. The last point already hints at the fact that a given promise about the future outlook of the economy, for example an announced *make-up policy* after a binding ELB, becomes less effective in stimulating the current level of inflation. In the following, I explore this in more detail.

3.1 Forward Guidance and Attention

To see how lower attention weakens the effectiveness of forward guidance, consider the following stylized experiment. The economy is hit by a negative natural rate shock in period $t = 0$, $r^n_0 < 0$, that pushes the nominal interest rate to the effective lower bound, i.e., $i_0 = -\underline{i}$.[22] In $t = 1$, the natural rate is back at its steady state value and stays there indefinitely, $r^n_t = 0$ for all $t \geq 1$. I further assume that from period $t = 2$ onwards, the output gap y^{gap}_t, and the real rate, $i_t - \pi^e_{t+1|t}$, are back at their steady states, $y^{gap}_t = 0$ and $i_t - \pi^e_{t+1|t} = 0$ for all $t \geq 2$.

To model forward guidance, the real rate is assumed to be below the natural rate in $t = 1$. To make it comparable across different degrees of attention, I impose that

$$r_1 \equiv i_1 - \pi^e_{2|1} < 0$$

is the same for all γ and known in advance.[23] Hence, forward guidance here means to announce a certain value for the *real* rate. Later on, I discuss the implications of forward guidance via the *nominal* rate. In the following, I assume that $(-\underline{i} - r^n_0 + r_1)$ is negative,

[22] Here, $-\underline{i}$ denotes the ELB in absolute deviations. The ELB in levels is then $-\underline{i} + \bar{i}$.

[23] This is different to Angeletos and Lian (2018), where private agents are uncertain about future policies.

which means that the announced policy, captured by $r_1 < 0$, *makes up* for the binding lower bound in $t = 0$, captured by $-\underline{i} - r_0^n > 0$.

Given the real rate r_1, the Euler equation determines the output gap in period 1 as

$$y_1^{gap} = -\varphi\left(r_1\right) > 0. \tag{10}$$

Equation (10) captures the *make-up policy:* by keeping the real rate below the natural rate, output is (expected to be) above potential.

In $t = 0$, the ELB binds and the natural rate is negative. Thus, the Euler equation in $t = 0$ yields

$$y_0^{gap} = \underbrace{-\varphi\left(r_1\right)}_{=E_0 y_1^{gap}} - \varphi\left(-\underline{i} - \pi_{1|0}^e - r_0^n\right).$$

The law of motion for inflation expectations is given by

$$\pi_{1|0}^e = (1-\rho_\pi)\bar{\pi} + \rho_\pi(1-\gamma)\pi_{0|-1}^e + \rho_\pi\gamma\pi_0,$$

which can be substituted into the Phillips Curve:

$$\pi_0 = \frac{\beta}{1-\beta\rho_\pi\gamma}\left((1-\rho_\pi)\bar{\pi} + \rho_\pi(1-\gamma)\pi_{0|-1}^e\right) + \frac{\kappa}{1-\beta\rho_\pi\gamma}y_0^{gap}.$$

Thus, inflation expectations are given by

$$\pi_{1|0}^e = \frac{1-\rho_\pi}{1-\beta\rho_\pi\gamma}\bar{\pi} + \frac{\rho_\pi(1-\gamma)}{1-\beta\rho_\pi\gamma}\pi_{0|-1}^e + \frac{\kappa\rho_\pi\gamma}{1-\beta\rho_\pi\gamma}y_0^{gap}.$$

Putting everything together, we arrive at the following Proposition.

Proposition 2 *The output gap in the period when the shock hits, $t = 0$, is given by*

$$\begin{aligned} y_0^{gap} = &-\frac{\varphi\left(1-\beta\rho_\pi\gamma\right)}{1-\rho_\pi\gamma(\beta+\varphi\kappa)}\left[-\underline{i} - r_0^n + r_1\right] \\ &+ \frac{\varphi}{1-\rho_\pi\gamma(\beta+\varphi\kappa)}\left[(1-\rho_\pi)\bar{\pi} + \rho_\pi(1-\gamma)\pi_{0|-1}^e\right] \end{aligned}$$

and inflation in $t = 0$ is given by

$$\begin{aligned} \pi_0 = &-\frac{\kappa\varphi}{1-\rho_\pi\gamma(\beta+\varphi\kappa)}\left[-\underline{i} - r_0^n + r_1\right] + (1-\rho_\pi)\left[\frac{\beta}{1-\beta\rho_\pi\gamma} + \frac{\varphi}{1-\rho_\pi\gamma(\beta+\varphi\kappa)}\right]\bar{\pi} \\ &+ \rho_\pi(1-\gamma)\left[\frac{\beta}{1-\beta\rho_\pi\gamma} + \frac{\varphi}{1-\rho_\pi\gamma(\beta+\varphi\kappa)}\right]\pi_{0|-1}^e. \end{aligned} \tag{11}$$

Proposition 2 captures the effectiveness of forward guidance on the output gap and inflation in the period when the shock hits. The assumption that $(1-\rho_\pi\gamma(\beta+\varphi\kappa))$ is positive, makes sure that forward guidance, i.e, a lower r_1 has a stimulating effect on output and inflation in $t=0$. Proposition 2 captures several channels how a change in attention affects the economy's response to forward guidance, which I now collect in a series of corollaries.

Corollary 1 *Lower attention*

(i) weakens the negative effect of the shock on impact,

(ii) weakens the effects of forward guidance on the output gap and inflation,

(iii) weakens the stimulative effects of a decrease in the lower bound $-\underline{i}$.

Corollary 1 follows from the fact that the terms $\frac{\varphi(1-\beta\rho_\pi\gamma)}{1-\rho_\pi\gamma(\beta+\varphi\kappa)}$ and $\frac{\kappa\varphi}{1-\rho_\pi\gamma(\beta+\varphi\kappa)}$ in front of $[-\underline{i}-r_0^n+r_1]$ are both increasing in γ. Points (i) and (ii) capture the main trade off of lower attention. While lower attention has a stabilizing effect via more anchored inflation expectations (point (i)), it renders forward guidance less effective (point (ii)). The reason why forward guidance becomes less effective as attention declines is because inflation expectations increase less in response to the announced policy, and thus, the real rate remains higher. Point (iii) illustrates an additional drawback of lower attention. A reduction of the effective lower bound, $-\underline{i}$, is less stimulating if agents in the economy are less attentive. Thus, going from a zero lower bound to a lower bound in negative territory, as conducted in several advanced economies over the last ten years, becomes less effective in terms of stimulating output and inflation if the public is inattentive. Note, that a decrease in the perceived inflation persistence, ρ_π, has the exact same implications as a decrease in γ.

The next corollary discusses how changes in attention affect the role of long-run inflation beliefs on the output gap and inflation.

Corollary 2 *Lower attention weakens the positive effects of higher long-run inflation beliefs* $\bar{\pi}$ *on output and inflation,*

Corollary 2 says that higher long-run beliefs have a positive effect on inflation and the output gap, but lower attention weakens these effects. However, as long as $\gamma(\beta+\varphi\kappa)<1$, a higher ρ_π mutes the effects of $\bar{\pi}$ on the output gap. Since this condition is usually satisfied and because ρ_π is in general close to 1, the role of high long-run inflation beliefs is quite weak. In the limit case $\rho_\pi\to 1$, long-run beliefs become irrelevant.

How attention matters for the transmission of prior inflation expectations on the output gap and inflation is ambiguous, as the following Corollary shows.

Corollary 3 *Lower attention*

(i) weakens the positive effect of higher prior inflation beliefs, $\pi^e_{0|-1}$, *on the output gap if and only if,*

$$\rho_\pi \left(\beta + \varphi\kappa\right) > 1, \tag{12}$$

(ii) weakens the positive effect of higher prior inflation beliefs on inflation if and only if

$$\frac{\rho_\pi\beta\left(\rho_\pi\beta - 1\right)}{\left(1-\beta\rho_\pi\gamma\right)^2} + \frac{\rho_\pi\varphi\left(\rho_\pi(\beta+\varphi\kappa)-1\right)}{\left(1-\rho_\pi\gamma(\beta+\varphi\kappa)\right)^2} > 0. \tag{13}$$

Overall, the role of attention for the effects of higher prior beliefs on output and inflation is ambiguous. This is mainly the case because, on the one hand, lower attention implies that agents put more weight on their prior beliefs. On the other hand, as discussed previously, lower attention leads to more stable inflation overall, thus, weakening the effects of prior beliefs. This can also be seen in the discussion of the Phillips Curve, see Proposition 1.

Given the calibration in Table 4, conditions (12) and (13) both hold. The effects of changes in γ, however, are numerically small. Thus, an increase in the average inflation rate—which also increases average prior beliefs—is a promising monetary instrument to combat the loss of control via forward guidance as attention declines. By *ex-ante* increasing the average inflation rate, the policymaker not only supports higher inflation expectations and thus, lower real rates for a given nominal rate, but also gains additional policy space through the increase in the average nominal rate. Higher average inflation, however, is also costly. In the analysis of optimal policy, later on, I will explore this trade off and characterize the optimal inflation target for different levels of attention.

Forward Guidance via Nominal Interest Rates So far, forward guidance was characterized as a promise to keep the *real* rate low. Now, assume that forward guidance is conducted via promising lower *nominal* rates instead. Thus, i_1 will be fixed across different γ. For simplicity, I focus on the case with $\rho_\pi = 1$ and $\pi^e_{0|-1} = 0$. It follows from the Euler equation in $t = 1$ that

$$y_1^{gap} = -\varphi\left(i_1 - \left(1-\gamma\right)\gamma\pi_0 - \gamma\pi_1\right).$$

The Phillips Curve in $t = 1$ yields

$$\pi_1 = \frac{(1-\gamma)\gamma}{1-\beta\gamma}\pi_0 + \frac{\kappa}{1-\beta\gamma}y_1^{gap},$$

so that we get an expression for y_1^{gap} in terms of π_0:

$$y_1^{gap} = -\frac{\varphi\left(1-\beta\gamma\right)}{1-\gamma\left(\beta+\varphi\kappa\right)} i_1 + \varphi(1-\gamma)\gamma\frac{1+\gamma(1-\beta)}{1-\gamma(\beta+\varphi\kappa)}\pi_0. \tag{14}$$

Given $\pi^e_{1|0} = \gamma\pi_0$, the Phillips Curve in $t=0$ yields

$$\pi_0 = \frac{\kappa}{1-\beta\gamma} y_0^{gap},$$

and hence, $\pi^e_{1|0} = \frac{\kappa\gamma}{1-\beta\gamma} y_0^{gap}$. Plugging this into the Euler equation in $t=0$ gives

$$y_0^{gap} = \mathbb{E}_0 y_1^{gap} - \varphi\left(-\underline{i} - \frac{\kappa\gamma}{1-\beta\gamma} y_0^{gap} - r_0^n\right).$$

Solving for y_0^{gap} leads to the following Lemma.

Lemma 3 *Forward guidance via the nominal interest rate yields the following output gap*

$$y_0^{gap} = A_1\left[-\frac{\varphi\left(1-\beta\gamma\right)}{1-\gamma\left(\beta+\varphi\kappa\right)} i_1 - \varphi\left(-\underline{i} - r_0^n\right)\right], \tag{15}$$

and inflation

$$\pi_0 = \frac{\kappa}{1-\beta\gamma} A_1\left[-\frac{\varphi\left(1-\beta\gamma\right)}{1-\gamma\left(\beta+\varphi\kappa\right)} i_1 - \varphi\left(-\underline{i} - r_0^n\right)\right], \tag{16}$$

where

$$A_1 \equiv \frac{1}{1-\varphi(1-\gamma)\gamma\frac{1+\gamma(1-\beta)}{1-\gamma(\beta+\varphi\kappa)}\frac{\kappa}{1-\beta\gamma} - \frac{\varphi\kappa\gamma}{1-\beta\gamma}}. \tag{17}$$

Given the calibration in Table 4, A_1 is positive and increasing in γ. Thus, promising lower future nominal interest rates can indeed stimulate the economy. But similar to the case in which the policy maker commits to a certain future *real* rate, forward guidance becomes less effective when agents are less attentive. In fact, all three results from Corollary 1 go through.

Recall equation (14):

$$y_1^{gap} = -\frac{\varphi\left(1-\beta\gamma\right)}{1-\gamma\left(\beta+\varphi\kappa\right)} i_1 + \varphi(1-\gamma)\gamma\frac{1+\gamma(1-\beta)}{1-\gamma(\beta+\varphi\kappa)}\pi_0.$$

Note, that the first term becomes less negative as γ declines. Given the calibration in Table 4, also the second term decreases as attention declines. Thus, for a given π_0, a particular i_1 has weaker effects on the output gap in $t=1$ at lower levels of attention. Since lower attention also weakens the positive effects of forward guidance on π_0, the output gap (and inflation) stay lower also in $t=1$.

Since inflation in $t = 0$ and $t = 1$ is lower at smaller values of γ, also $\pi^e_{2|1}$ will be lower and thus, for a given nominal rate i_1, the *real* rate, $r_1 \equiv i_1 - \pi^e_{2|1}$, will be higher. Hence, to achieve a certain forward guidance in terms of the real interest rate, the promise in terms of the nominal rate needs to be larger when firms and households are inattentive. Combining this with the findings on the effectiveness of forward guidance via the *real* rate (Proposition 2) shows how lower attention renders forward guidance less powerful *even though* the promise in terms of the *nominal rate* is stronger.

Heterogeneous Attention So far, I assumed that firms and households are equally attentive. But what if firms and households differ in their attention to inflation? Let us denote firms' attention by γ_F and households' attention by γ_H with $\gamma_F \neq \gamma_H$. For clarity, I focus on the case with $\rho_\pi = 1$ and $\pi^{e,j}_{0|-1} = 0$ for $j \in \{F, H\}$.

Lemma 4 *With heterogeneous attention to inflation, the output gap in $t = 0$ is given by*

$$y_0^{gap} = \frac{-\varphi\left(1 - \beta\gamma_F\right)}{1 - \beta\gamma_F - \kappa\varphi\gamma_H}\left[-\underline{i} + r_1 - r_0^n\right], \tag{18}$$

and inflation by

$$\pi_0 = \frac{-\varphi\kappa}{1 - \beta\gamma_F - \kappa\varphi\gamma_H}\left[-\underline{i} + r_1 - r_0^n\right], \tag{19}$$

where $r_1 \equiv i_1 - \pi^{e,H}_{2|1}$ is the real rate given the households' expectations.

Lemma 4 shows that a similar result as in Corollary 1 holds under heterogeneous attention levels.

Corollary 4 *Lower attention of either firms or households*

(i) weakens the negative effect of the shock on the output gap and inflation on impact,

(ii) weakens the effects of forward guidance on the output gap and inflation,

(iii) weakens the stimulative effects of a decrease in the lower bound $-\underline{i}$ on the output gap and inflation.

The parts concerning the output gap in Corollary 4 follow because the term in front of the brackets in equation (18) becomes more negative as either of $\{\gamma_F, \gamma_H\}$ increases:

$$\frac{\partial\left[\frac{-\varphi(1-\beta\gamma_F)}{1-\beta\gamma_F-\kappa\varphi\gamma_H}\right]}{\partial\gamma_F} = -\frac{\beta\kappa\varphi^2\gamma_H}{(1 - \beta\gamma_F - \kappa\varphi\gamma_H)^2} < 0$$
$$\frac{\partial\left[\frac{-\varphi(1-\beta\gamma_F)}{1-\beta\gamma_F-\kappa\varphi\gamma_H}\right]}{\partial\gamma_H} = -\frac{\varphi^2(1 - \beta\gamma_F)}{(1 - \beta\gamma_F - \kappa\varphi\gamma_H)^2} < 0,$$

and the parts concerning inflation because the term $\frac{-\varphi\kappa}{1-\beta\gamma_F-\kappa\varphi\gamma_H}$ in equation (19) becomes more negative as either of $\{\gamma_F, \gamma_H\}$ increases, too.

Thus, if either firms or households (or both) become less attentive, forward guidance becomes less effective. In fact, the two degrees of attention reinforce each other, as the following Corollary shows.

Corollary 5 *Lower levels of households' attention to inflation weaken the effectiveness of forward guidance, especially when firms' attention to inflation is low, and vice-versa.*

To see this, note that

$$\frac{\partial^2\left[-\frac{\varphi\kappa}{1-\beta\gamma_F-\kappa\varphi\gamma_H}\right]}{\partial\gamma_F\partial\gamma_H} = \frac{-2\varphi^2\kappa^2\beta}{\left(1-\beta\gamma_F-\kappa\varphi\gamma_H\right)^3} < 0,$$

$$\frac{\partial^2\left[\frac{-\varphi(1-\beta\gamma_F)}{1-\beta\gamma_F-\kappa\varphi\gamma_H}\right]}{\partial\gamma_F\partial\gamma_H} = \frac{-\beta\kappa\varphi^2\left[1-\beta\gamma_F+\kappa\varphi\gamma_H\right]}{\left(1-\beta\gamma_F-\kappa\varphi\gamma_H\right)^3} < 0.$$

Given these stylized examples on the effectiveness of forward guidance and its dependence on the public's attention, I now show in a numerical example an additional drawback of lower attention, namely that the economy can get stuck in an *inflation-attention trap.*

3.2 Inflation-Attention Traps

To close the model from Section 3, I assume for now that, away from the lower bound, the monetary authority sets the nominal interest rate according to a Taylor rule

$$\tilde{i}_t = \rho_i\tilde{i}_{t-1} + (1-\rho_i)\left(\phi_\pi\pi_t + \phi_y y_t^{gap}\right), \tag{20}$$

where $\rho_i \in [0,1)$ captures persistence in the interest rate, $\phi_\pi > 1$ and $\phi_y \geq 0$ denote the reaction coefficients to inflation and the output gap, respectively. The actual interest rate, i_t, is given by

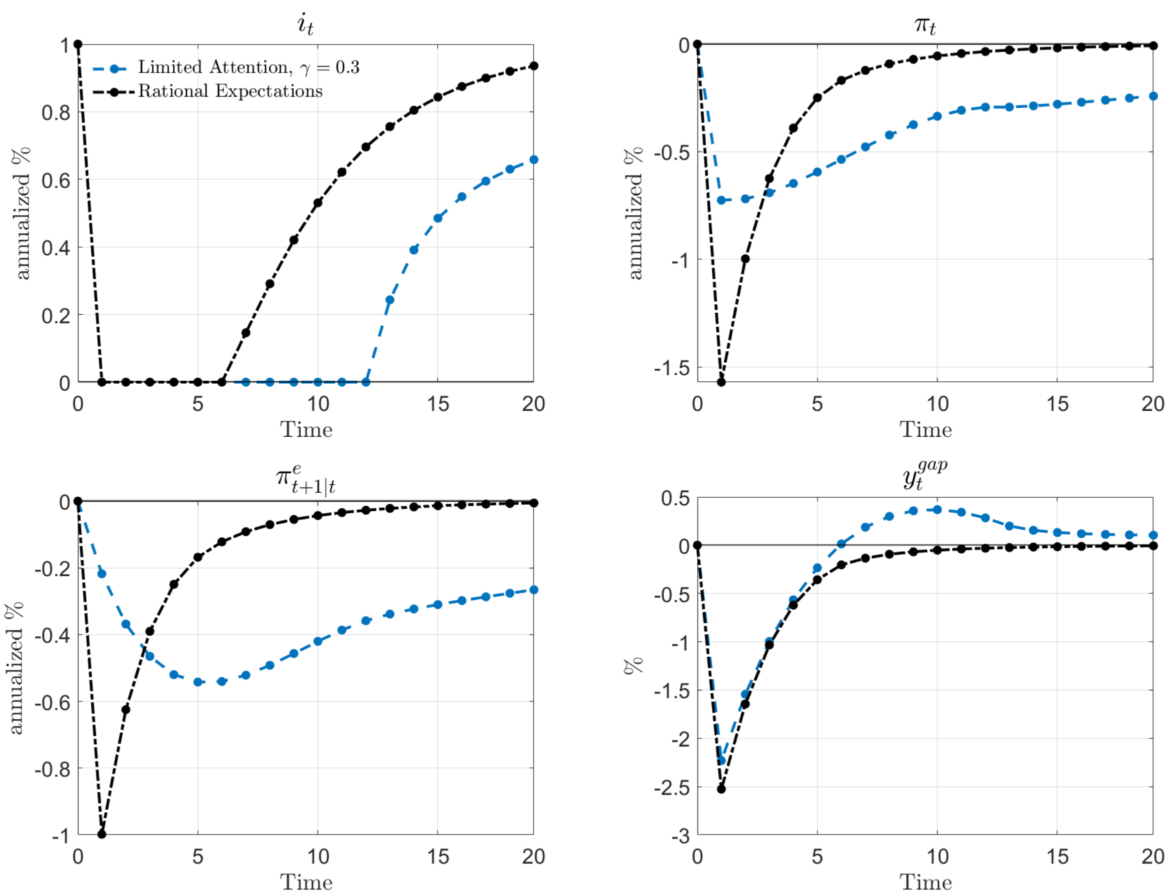
$$i_t = max\{\tilde{i}_t, -\bar{i}\}, \tag{21}$$

where I set the lower bound (in levels) to zero. Thus, as long as the Taylor rule (20) implies a policy rate in levels, $\tilde{i}_t + \bar{i}$, above zero, i_t equals the one obtained from (20). If the lower bound becomes binding, on the other hand, the policy rate in levels is 0. This implies that in terms of deviations from the steady state i_t is equal to $-\bar{i}$. Later on, I will replace this *ad hoc* monetary policy rule with the Ramsey optimal policy.

I set the persistence parameter of the nominal interest rate of 0.7, and the reaction coefficients $\phi_\pi = 2$ and $\phi_y = 0.5$, as in Andrade et al. (2019). In Appendix F.2, I show that the exact specification of the Taylor rule is inconsequential for the following results.

What happens if a negative natural rate shock hits the economy and pushes the nominal rate to the lower bound? To answer this, Figure 3 plots the impulse response functions of the model's main variables to a negative natural rate shock of three standard deviations. The black-dashed-dotted lines are the IRFs in the model under FIRE and the blue-dashed lines are the ones under limited attention for the case $\gamma = 0.3$.

Figure 3: Impulse Response Functions to a Negative Natural Rate Shock



Note: This figure shows the impulse-response functions of the nominal interest rate (upper-left panel), inflation (upper-right panel), inflation expectations (lower-left) and the output gap (lower-right) to a negative natural rate shock of three standard deviations. The blue-dashed lines show the case for the limited-attention model and the black-dashed-dotted lines for the rational expectations model. Everything is in terms of percentage deviations from the respective steady state levels, except the nominal rate is in levels.

In both cases, the shock is large enough to push the economy to the lower bound. While the reaction of the output gap is very similar in both economies, the responses of inflation and inflation expectations are strikingly different. Initially, the muted response of inflation expectations to the adverse shock is reflected in a smaller downturn of inflation itself under limited attention. This captures the stabilizing effects that come with lower attention.

The sluggish adjustment of inflation expectations in the following, however, leads to a very persistent undershooting of inflation. Even five years after the shock, inflation and inflation expectations are still 0.2-0.3 percentage points below their steady state levels of zero. The result is a prolonged period of a binding lower bound. While the economy under rational expectations escapes the ELB five periods after the shock, the economy under limited attention is stuck for more than twice as long. This is what I label *inflation-attention trap*. A side-effect of these traps is that the long ELB period can lead to an output boom. As discussed earlier, this (expected) output boom in the future has small effects on the economy today if people are inattentive.

Overall, limited attention to inflation offers a possible explanation for why several advanced economies were stuck at the ELB after the financial crisis, as well as inflation undershooting the central banks' inflation targets, even though the initial decrease was smaller than in previous recessions and output declined significantly (Del Negro et al. (2020)). In other words, the limited-attention model can explain the *missing deflation puzzle* as well as the *missing inflation puzzle* (Coibion and Gorodnichenko (2015b), Constancio (2015)).

4 Implications for Optimal Monetary Policy

How does declining attention affect the conduct of optimal monetary policy? In this section, I characterize the Ramsey optimal monetary policy in this economy. I focus on the case of $\rho_\pi = 1$, in which average inflation expectations coincide with the actual inflation average. Appendix F.6 reports the results when we introduce a mean bias and relax the random-walk assumption.

The policymaker's objective is to maximize the representative household's utility, taking the household's and firms' optimal behavior, including their attention choice, as given. Thus, the policymaker cannot exploit the private agent's lack of information. Nevertheless, the policymaker can affect inflation expectations by influencing inflation itself and he can set the average inflation expectations by setting the average inflation rate.

The policymaker evaluates the household's utility under rational expectations. A second-order approximation to the household's utility function yields the policymaker's objective

$$-\frac{1}{2}E_0\sum_{t=0}^{\infty}\beta^t\left[\pi_t^2+\chi\left(y_t^{gap}\right)^2\right], \tag{22}$$

where χ is the relative weight of the output gap. Following Adam and Billi (2006), I set $\chi = 0.007$. In the following, I refer to (22) as *welfare*.

In sum, the optimal policy problem is given by

$$\max_{\pi_t, y_t^{gap}, i_t} \quad -\frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left[\pi_t^2 + \chi \left(y_t^{gap} \right)^2 \right] \tag{23}$$

subject to

$$\pi_t = \beta \pi_{t+1|t}^e + \kappa y_t^{gap} + u_t \tag{24}$$
$$y_t^{gap} = E_t y_{t+1}^{gap} - \varphi \left(i_t - \pi_{t+1|t}^e - r_t^n \right) \tag{25}$$
$$\pi_{t+1|t}^e = \pi_{t|t-1}^e + \gamma \left(\pi_t - \pi_{t|t-1}^e \right) \tag{26}$$
$$u_t = \rho_u u_{t-1} + \varepsilon_t^u \tag{27}$$
$$r_t^n = \rho_r r_{t-1}^n + \varepsilon_t^r \tag{28}$$
$$i_t \geq -\bar{i}, \tag{29}$$

with $\varepsilon_t^u \sim i.i.N. \left(0, \sigma_u^2\right)$ and $\varepsilon_t^r \sim i.i.N. \left(0, \sigma_{r^n}^2\right)$ and (29) is the lower-bound constraint.[24] All variables are in percent deviations from their respective steady state, except the nominal interest rate and the natural rate which are in absolute deviations. In Appendix F.1, I derive the optimal target rule in this economy.

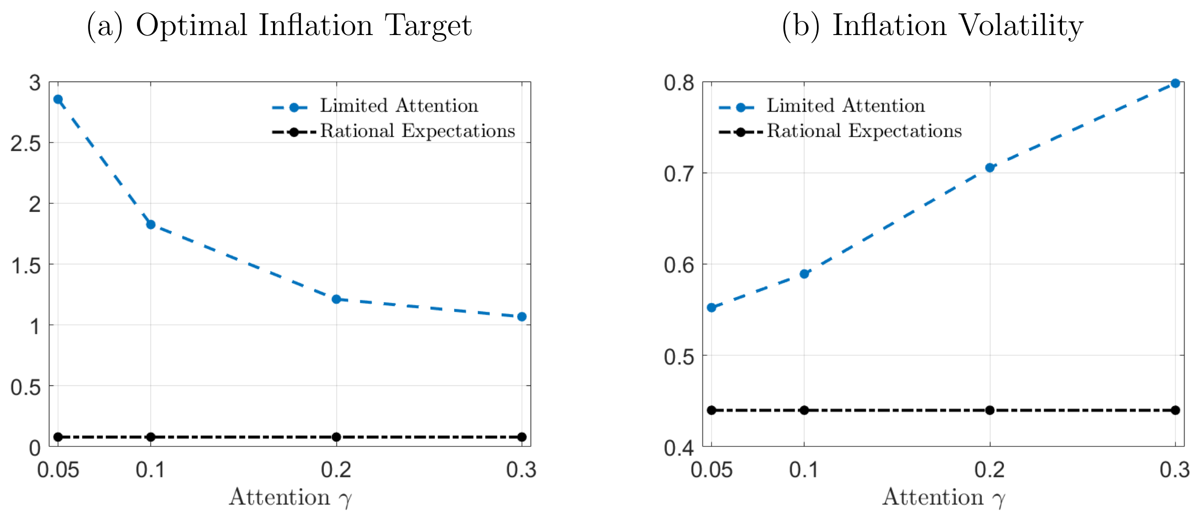
4.1 The Optimal Inflation Target

What does limited attention, especially low levels of attention, imply for inflation volatility and the optimal inflation target? For this, I solve the Ramsey problem for different levels of attention, namely $\gamma \in \{0.05, 0.1, 0.2, 0.3\}$. An attention parameter of 0.3 is close to the estimates for consumers' attention after 1990, and the lower levels of 0.05 and 0.1 are close to the ones observed since 2010.

Figure 4 shows the results. The average inflation rate under Ramsey optimal policy—the optimal inflation target—is plotted in the left panel and the corresponding inflation volatility in the right panel. The blue-dashed lines show the results for the model under limited attention, and the black-dashed-dotted lines for the rational-expectations model. We see that the optimal inflation target increases substantially as attention declines. At current estimates of attention, $\gamma \in \{0.05, 0.1\}$, the inflation target is about 2-3 percentage points higher than under rational expectations due to the discussed inflation-attention traps. As the nominal interest rate is pushed to the lower bound, monetary policy loses much of its power, which can result in prolonged periods of a binding ELB, as well as inflation

[24] I solve this numerically by recursifying the Lagrangian, as in Marcet and Marimon (2019). See Appendix F.7 for details.

Figure 4: Optimal Inflation and Inflation Volatility



Notes: This figure shows the average inflation rate under Ramsey optimal policy (left panel) and the corresponding inflation volatility for different attention levels. The blue-dashed lines show the results for the model under limited attention, and the black-dashed-dotted lines for the rational-expectations model.

undershooting its target. To counteract this, it is optimal to increase the average level of inflation, which increases the average nominal interest rate and thus makes it less likely that the ELB becomes binding. Indeed, the frequency of a binding ELB decreases substantially. For $\gamma = 0.3$, the ELB is binding 22% of the time, whereas this value shrinks to 1.4% for an attention level of 0.05.

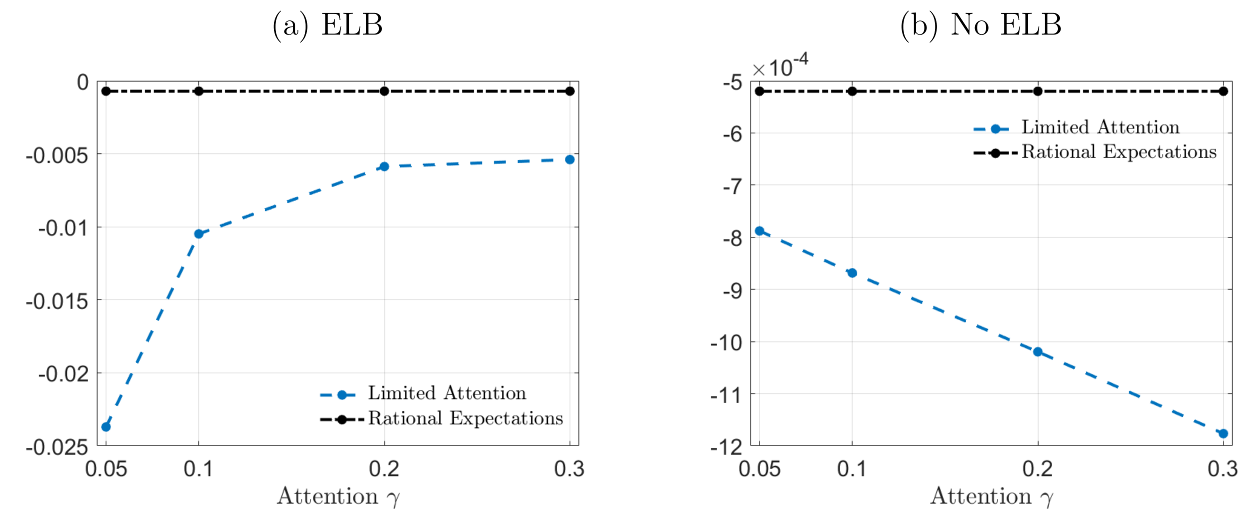
While lower attention renders forward guidance, make-up policies and other policies that work (partly) through inflation expectations ineffective, lower attention also stabilizes inflation, as can be seen from the right panel in Figure 4. First, because lower attention mutes the inflation response to shocks and output (see Proposition 1). Second, the lower ELB frequency further stabilizes the economy. Thus, lower attention to inflation can help stabilizing actual inflation and reduces the number of binding-ELB periods. The lower inflation volatility at lower levels of attention in fact justifies these low attention levels, as optimal attention depends positively on inflation volatility (see Section (B)). This low volatility, however, requires an increase in the inflation target, which is costly. Thus, it is not clear *a priori* whether lower attention leads to welfare gains or not.

4.2 Welfare

What are the effects of declining attention on overall welfare? Welfare is given by equation (22) and from the previous discussion, we know that lower attention poses a trade off. On the one hand, inflation volatility decreases and the ELB binds less frequently, when attention is low. This raises welfare. On the other hand, lower attention complicates managing inflation expectations and thus, the optimal average *level* of inflation increases, which is costly. Which effect dominates?

Panel (a) in Figure 5 shows that the cost of the level effect outweighs the stabilization benefits. As attention falls, welfare decreases. This is especially pronounced at low levels of attention, where the optimal inflation target increases substantially (see Figure 4).

Figure 5: Welfare and Attention



Notes: This figure shows welfare (22) under Ramsey optimal policy for different levels of attention. The left panel shows the results for the case with an occasionally-binding ELB, and the right panel without an ELB. The blue-dashed lines show the results for the model under limited attention, and the black-dashed-dotted lines for the rational-expectations model.

Absent the lower-bound constraint, the complication in managing inflation expectations due to limited attention is much less pronounced since managing expectations is particularly important at the lower bound. In fact, lower attention is welfare improving in the case without an ELB. Panel (b) in Figure 5 shows this graphically. The stabilization benefits that arise from lower attention—which is reflected in more anchored expectations—lead to an increase in welfare.

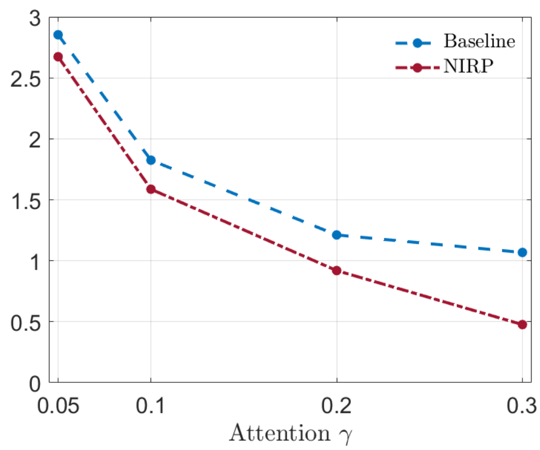
These findings show that accounting for the ELB is crucial for making a normative statement about costs and benefits of stabilizing inflation expectations. The ELB highlights the drawbacks that arise from the stabilization of expectations due to the fall in attention, as the management of expectations becomes particularly relevant when the ELB binds.

4.3 Negative Interest Rate Policies

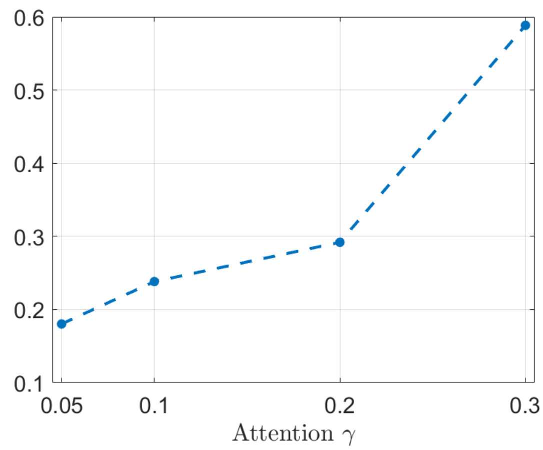
In recent years, several central banks in advanced economies have implemented negative interest rate policies (NIRP).[25] Could negative rates limit the negative consequences of declining attention? In order to answer this question, I solve the same Ramsey optimal policy problem as above, but set the effective lower bound to -0.5% (annualized).

Figure 6: Negative Interest Rate Policies and Attention

(a) Optimal Inflation Target



(b) Change in Inflation Target



Notes: The left panel shows the average inflation rate under optimal policy for different degrees of attention γ. The blue-dashed lines show the results for the benchmark model where the lower bound is at 0, and the red-dashed-dotted lines show the results when allowing for negative interest rates up to -0.5% (annualized). The right panel shows the difference in the optimal inflation targets, defined as $\pi^{*,ZLB} - \pi^{*,NIRP}$, where π^* denotes the optimal inflation target and the superscripts ZLB and $NIRP$ denote the two cases where the ELB is at 0% or -0.5%, respectively.

Figure 6 reports the outcomes. Panel (a) shows the optimal inflation target (red-dashed-dotted line) and compares it to the case with an ELB at 0 (blue-dashed line). We see that the additional policy space due to the negative lower bound indeed calls for a lower inflation target. As the discussion in the analytical section 3 foreshadowed, however, the decline in attention also weakens the effectiveness of NIRP. We see this by observing that the optimal inflation target under NIRP gets closer to the one without negative rates as attention declines. To see these gaps clearly, panel (b) shows the difference in the optimal inflation targets, defined as $\pi^{*,ZLB} - \pi^{*,NIRP}$, where π^* denotes the optimal inflation target and the superscripts ZLB and $NIRP$ denote the two cases where the ELB is at 0% or -0.5%, respectively. Overall, allowing for negative policy rates can help limiting the drawbacks of low attention but these policies itself become less effective as attention declines.

[25]See Brandão-Marques, Casiraghi, Gelos, Kamber, and Meeks (2021) for a recent survey on negative interest rate policies and its effectiveness.

5 Conclusion

With the stabilization of inflation over the last fifty years in advanced economies, inflation became less important in people's everyday lives. In this paper, I quantify this using a limited-attention model of inflation expectations. In line with this model, I show that attention to inflation decreased together with inflation volatility and inflation persistence since the 1970s. Especially in the period between 2010 and 2020, the general public's attention to inflation was close to zero.

For monetary policy the decline in attention was desirable at first, since lower attention stabilizes inflation expectations and hence, stabilizes actual inflation. With the outbreak of the Great Recession and nominal rates at their lower bound, however, managing inflation expectations became a central tool for monetary policy. But managing inflation expectations is difficult when people are inattentive.

In this paper, I study how lower attention matters for monetary policy. After an adverse shock that pushes the nominal rate to the lower bound, inflation expectations and inflation remain persistently low and thus, the lower bound binds for a prolonged period. To mitigate these drawbacks that arise from low attention, the optimal policy response is a substantial increase in the inflation target. This increases the average nominal rate and thus, binding ELB periods become less likely. The costs of this increase in inflation, however, outweigh the stabilization benefits of lower attention. Lower attention, therefore, decreases welfare if we account for the lower bound. This stands in stark contrast to the case without an ELB in which case lower attention leads to welfare gains through the stabilization of inflation expectations and inflation. Overall, my paper shows that accounting for the ELB is crucial when assessing the role of the public's attention to inflation.

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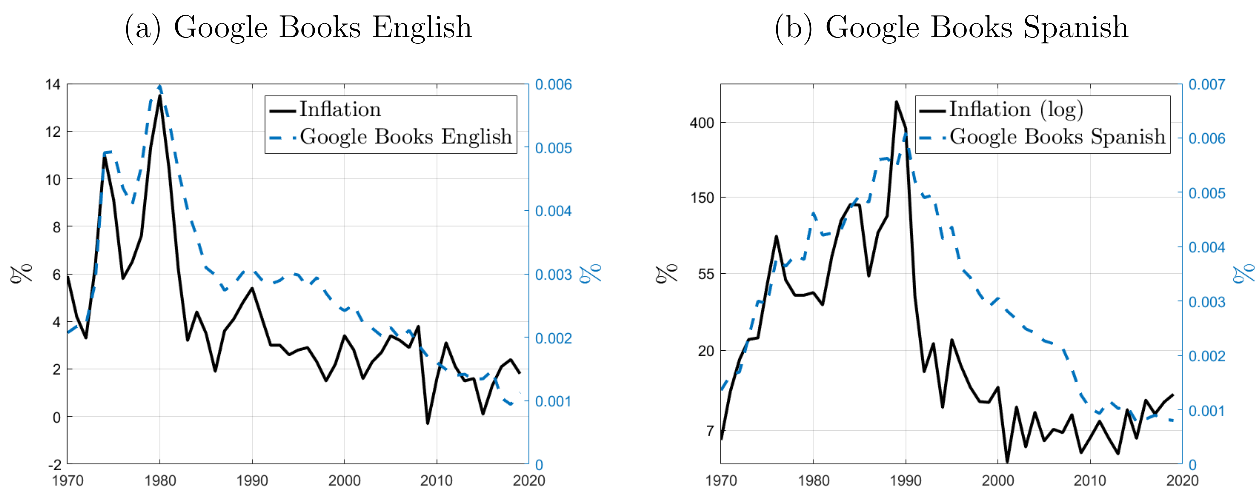
A Book Coverage of Inflation

I showed in the main text that news coverage of inflation in the US substantially decreased since the Great Inflation period and that inflation is especially prominent in the news when inflation is high and volatile. In Figure 7, we see that a similar picture emerges when looking at the coverage of "inflation" in books, according to *Google Books Ngram Viewer*. In the left panel, we see that "inflation" is covered more frequently in English books written in times of high inflation. But this is not simply a US phenomenon. To see this, I show the same statistic for books written in Spanish for the word "inflación". To contrast this with inflation, the black solid line shows the average inflation (in logs) of the four largest Spanish-speaking countries, weighted by their 2020 population size. These are Argentina, Colombia, Mexico and Spain. Again, we observe that attention to inflation—measured by book coverage—is higher in times of high and volatile inflation.

Figure 7: Book Coverage of Inflation



Notes: The blue dashed lines show the frequency of the words "inflation" and "inflación", respectively, in English and Spanish books, according to Google Books Ngram Viewer. The black solid line shows the corresponding inflation rates.

B A Limited-Attention Model of Inflation Expectations

In this section, I derive an expectations-formation process under limited attention that provides a straightforward approach to measure attention to inflation empirically. The model is an application of Mackowiak et al. (2020), who study a general problem of optimal information acquisition. Instead of forecasting the current state of the economy, however, the agent in my model wants to forecast *future* inflation. The agent could be any economic agent, such as a household, a firm manager or a professional forecaster.

To form her expectations, the agent acquires information. Even if all necessary information is in principle available, acquiring and processing information is costly, given the limited cognitive abilities and limited amount of time available. This means that the amount of attention the agent pays to relevant information is limited, and will depend on how costly acquiring information is, how high the stakes are, as well as the properties of inflation itself.

The main difference to Mackowiak et al. (2020) is that agents in my model do not exactly know the underlying process of inflation but have a simplified view of how inflation evolves. In particular, the agent believes that (demeaned) inflation tomorrow, π', depends on (demeaned) inflation today, π, as follows

$$\pi' = \rho_\pi \pi + \nu,$$

where $\rho_\pi \in [0,1]$ denotes the perceived persistence of inflation and $\nu \sim i.i.N.(0, \sigma_\nu^2)$. This assumption is supported by empirical evidence (see e.g., Faust and Wright (2013) or Canova (2007)).[26] Inflation in the current period is unobservable, so before forming an expectation about future inflation, the agent needs to form an expectation about today's inflation. I denote this nowcast $\widetilde{\pi}$, and the resulting forecast about next period's inflation $\pi^e = \rho_\pi \widetilde{\pi}$. Given her beliefs, the full-information forecast π^{e*} is

$$\pi^{e*} \equiv \rho_\pi \pi.$$

But since π is not perfectly observable, the actual forecast will deviate from the full-information forecast. Deviating, however, is costly, as this causes the agent to make mistakes in her decisions.

The agent's choice is not only about how to form her expectations given certain information, but about how to choose this information optimally, while taking into account how

[26] Fulton and Hubrich (2021) show that simple models such as AR(1) models are hard to beat when forecasting inflation in real time.

this will later affect her forecast. That is, she chooses the form of the signal s she receives about current inflation. Since acquiring information is costly, it cannot be optimal to acquire different signals that lead to an identical forecast. Due to this one-to-one relation of signal and forecast, we can directly work with the joint distribution of π^e and π, $f(\pi^e, \pi)$, instead of working with the signal.

Let $U(\pi^e, \pi)$ denote the negative of the loss that is incurred when the agent's forecast deviates from the forecast under full information, and $C(f)$ the cost of information. Then, the agent's problem is given by

$$\max_f \int U(\pi^e, \pi) f(\pi^e, \pi) d\pi d\pi^e - C(f) \tag{30}$$
$$\text{subject to } \int f(\pi^e, \pi) d\pi^e = g(\pi), \text{ for all } \pi,$$

where $g(\pi)$ is the agent's prior, which is assumed to be Gaussian; $\pi \sim N(\hat{\pi}, \sigma_\pi^2)$. $C(.)$ is the cost function that captures how costly information acquisition is. It is linear in *mutual information* $I(\pi; \pi^e)$, i.e., the expected reduction in entropy of π due to knowledge of π^e:

$$C(f) = \lambda I(\pi; \pi^e) = \lambda \left(H(\pi) - E\left[H(\pi|\pi^e) \right] \right),$$

where $H(x) = -\int f(x) log(f(x)) dx$ is the entropy of x and λ is a parameter that measures the cost of information.

The objective function $U(.)$ is assumed to be quadratic:

$$U(\pi^e, \pi) = -r \left(\rho_\pi \pi - \pi^e \right)^2,$$

where r measures the stakes of making a mistake.[27,28]

In this setup, Gaussian signals are optimal (and in fact the unique solution, see Matějka and McKay (2015)). The optimal signal thus has the form

$$s = \pi + \varepsilon,$$

[27] A quadratic loss function is usually derived from a second-order approximation of the household's utility function or the firm's profit function (see, e.g., Mackowiak and Wiederholt (2009)).

[28] These stakes (or also the information cost parameter λ) can be interpreted as a way to incorporate other variables to which the agent might pay attention. For example, a household might not only want to forecast inflation but also her own income stream going forward. In this case, a smaller r could capture an increase in her idiosyncratic income volatility. Thus, paying attention to inflation is relatively less beneficial, as the relative importance of her idiosyncratic income increases. Such an interpretation also explains why professional forecasters might not be fully informed about inflation, given that they usually forecast a whole array of variables.

with $\varepsilon \sim i.i.N.(0, \sigma_\varepsilon^2)$.[29] The problem (30) now reads

$$\max_{\sigma^2_{\pi|s} \leq \sigma^2_\pi} E_\pi \left[E_s \left[-r\rho_\pi^2 \left(\pi - E[\pi|s] \right)^2 \right] \right] - \lambda I(\pi; \pi^e) = \max_{\sigma^2_{\pi|s} \leq \sigma^2_\pi} \left(-r\rho_\pi^2 \sigma^2_{\pi|s} - \frac{\lambda}{2} log \frac{\sigma_\pi^2}{\sigma^2_{\pi|s}} \right). \quad (31)$$

The optimal forecast is given by $\pi^e = \rho_\pi E\left[\pi|s\right]$, and Bayesian updating implies

$$\pi^e = \rho_\pi \left(1 - \gamma\right) \hat{\pi} + \rho_\pi \gamma s, \quad (32)$$

where $\gamma = 1 - \frac{\sigma^2_{\pi|s}}{\sigma^2_\pi} \in [0, 1]$ measures how much attention the agent pays to inflation, and $\hat{\pi}$ denotes the prior mean of π.

An equivalent way of writing γ is

$$\gamma = \frac{\sigma_\pi^2}{\sigma_\pi^2 + \sigma_\varepsilon^2}. \quad (33)$$

Now, since the agent *chooses* the level of attention, we can re-formulate (31) as

$$\max_{\gamma \in [0,1]} \left(-r\rho_\pi^2 (1-\gamma)\sigma_\pi^2 - \frac{\lambda}{2} log \frac{1}{1-\gamma} \right). \quad (34)$$

Writing the cost of information relative to the stakes, $\tilde{\lambda} \equiv \frac{\lambda}{r}$, and solving the optimization problem (34) yields the *optimal* level of attention, summarized in the following Lemma.

Lemma 5 *The optimal level of attention is given by*

$$\gamma = max \left(0, 1 - \frac{\tilde{\lambda}}{2\rho_\pi^2 \sigma_\pi^2} \right), \quad (35)$$

which shows that the optimal level of attention is

(i) decreasing in the relative cost of information acquisition, $\tilde{\lambda} \equiv \frac{\lambda}{r}$*,*

(ii) increasing in inflation volatility, σ_π*, and*

(iii) increasing in inflation persistence, ρ_π*.*

Attention in equation (32) captures how much the agent revises her expectations after making a forecast error. An inattentive agent does not put a lot of weight on her received information and rather sticks to her prior beliefs, whereas an attentive agent updates her

[29] In this case, the entropy becomes $H(x) = \frac{1}{2} log(2\pi e \sigma_x^2)$, where σ_x^2 is the variance of x. Note, that here π denotes the number "pi" and not inflation.

expectations strongly since her received signals are more precise. From Lemma 5, we see that aside from the relative information cost, $\tilde{\lambda}$, the persistence, ρ_π, and the volatility of inflation, σ_π, are crucial drivers of attention. The model predicts a positive relationship between attention and σ_π, as well as between attention and ρ_π. In the following, I will first estimate attention γ, asses how it changed over time and then test whether there is indeed evidence for these positive relations.

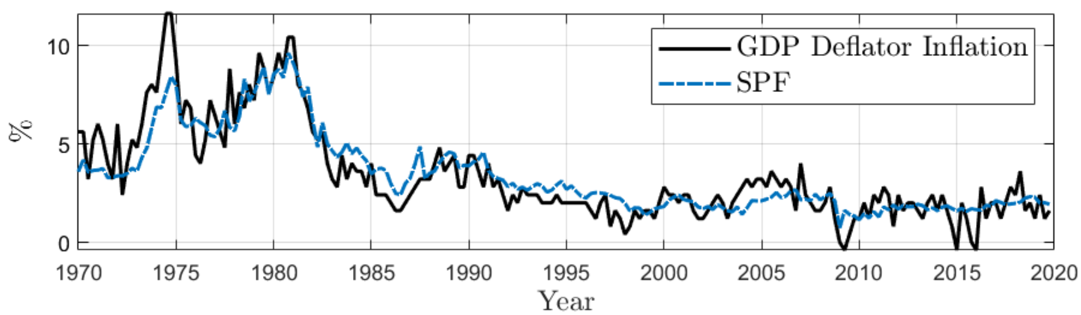
C Data and Summary Statistics

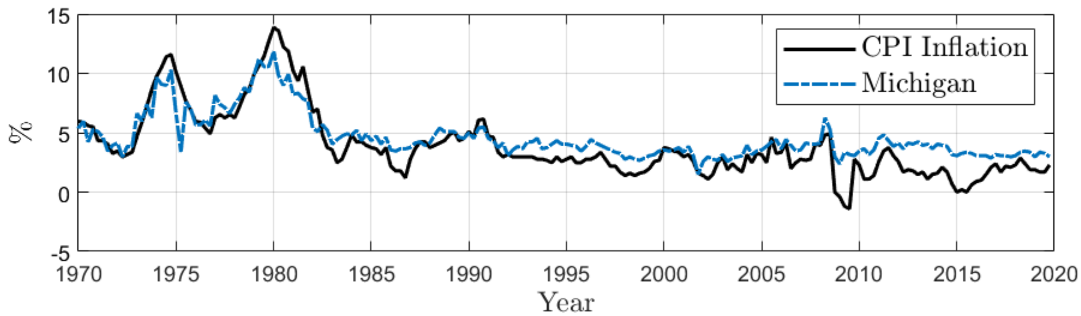
Figure 8 shows the main time series that are used in Section 2. Apart from the apparent decrease in the level and volatility of inflation as well as inflation expectations, we see that expectations became more and more detached from actual inflation. First, consumer expectations seem to be biased on average in the most recent decades, as can be seen in the lower panel. While these expectations closely tracked inflation in the 70s and 80s, this is not the case anymore.[30] Second, professional forecasters' expectations seem to perform quite well on average. In the last twenty years, however, they barely react to actual changes in inflation anymore. Overall, these observations suggest that attention decreased in the last decades.

Table 5 shows the summary statistics, for the period before and after the 1990s, separately. For professional forecasters, the perceived persistence is higher than the actual one. This is especially the case when the actual persistence is relatively low, as was the case after 1990. Afrouzi, Kwon, Landier, Ma, and Thesmar (2020) document a similar finding in an experimental setting. This might point towards lower attention since the 1990s. Note, that in the empirical analysis I account for changes in the perceived persistence.

[30] In the empirical analysis I account for this mean bias.

Figure 8: Inflation and Inflation Expectations





Note: This figure shows the raw time series of inflation, as well as survey expectations about future inflation. Everything is in annualized percentages.

Table 5: Summary Statistics

	GDP Deflator Inflation		SPF Expectations	
	1968-1990	1990-2020	1968-1990	1990-2020
Mean (%)	5.44	2.00	5.18	2.15
Std. Dev. (%)	2.43	0.90	1.87	0.63
Persistence	0.84	0.55	0.93	0.92
	CPI Inflation		Consumer Expectations	
	1968-1990	1990-2020	1968-1990	1990-2020
Mean (%)	6.09	2.42	6.00	3.62
Std. Dev. (%)	3.00	1.26	2.17	0.68
Persistence	0.96	0.77	0.85	0.70

Note: This table shows the summary statistics of the data. The upper panel shows the statistics for the quarter-on-quarter GDP deflator inflation (left) and the corresponding inflation expectations from the Survey of Professional Forecasters (right). The lower panel shows the year-on-year CPI inflation (left) and the corresponding inflation expectations from the Survey of Consumers from the University of Michigan. All data are annualized.

D Robustness

In this section, I show that the empirical results are robust along several dimensions.

Additional Data Sources

In Table 6, I show how attention changed over time for different data sources. The first two columns show the results for the Greenbook forecasts, columns 3-4 for the Livingston Survey, and columns 5-6 and 7-8 are for CPI forecasts from the SPF instead of forecasts about the GDP deflator. As in the main text, I use two different estimators. First, the Blundell-Bond estimator (columns 5-6) and pooled OLS (columns 7-8). All standard errors are robust with respect to heteroskedasticity and serial correlation. We see that the main finding of lower attention in inflation expectations in the period after 1990 compared to the period before is robust to these changes in the data source and/or exact variable.

Table 6: Regression Results of Equation (4)

	Greenbook		Livingston		SPF CPI BB		SPF CPI OLS	
	< 1990	≥ 1990	< 1990	≥ 1990	< 1990	≥ 1990	< 1990	≥ 1990
$\widehat{\gamma}$	0.39	0.24	0.28	0.17	0.36	0.23	0.17	0.13
s.e.	(0.0851)	(0.0715)	(0.0554)	(0.0624)	(0.1444)	(0.0328)	(0.0409)	(0.0142)
N	84	100	83	61	550	3,577	550	3,577

Note: This table shows the results from regression (4) for different data sources. The standard errors, reported in parentheses, are robust with respect to heteroskedasticity and serial correlation.

Different Sample Splits

Table 1 in the main body of the paper shows that attention to inflation declined by focusing on a sample split in 1990. To show that this is robust to the exact split point, Tables 7 and 8 show that the result holds when splitting the sample in 1985 or 1995, respectively. In fact, the decline in attention is even somewhat more pronounced when splitting the sample in 1985. This is in line with the theoretical prediction of the limited-attention model. Namely, the period between 1985 and 1990 was a period of relatively low and stable inflation compared to the period pre 1985 (see Figure 8), and thus, a period in which the model would predict a relatively low level of attention.

Different Specifications of the BB Estimator

In the baseline estimation, reported in Table 1, I included all potential lags for the Blundell-Bond estimation. To show that the results are robust to this specification, I show in Table

Table 7: Regression Results of Equation (4), pre 1985 vs. post 1985

	Survey of Professional Forecasters				Survey of Consumers			
	Blundell Bond		Pooled OLS		Averages		Median	
	< 1985	≥ 1985	< 1985	≥ 1985	< 1985	≥ 1985	< 1985	≥ 1985
$\widehat{\gamma}$	0.75	0.37	0.45	0.25	0.77	0.31	0.50	0.26
s.e.	(0.1247)	(0.0399)	(0.0403)	(0.0338)	(0.1688)	(0.0811)	(0.0955)	(0.0561)
N	1914	3887	1914	3887	64	140	27	140

Note: This table shows the results from regression (4) for professional forecasters (SPF) as well as for consumers. For the SPF, I use the Blundell and Bond (1998) (BB) estimator (first two columns), as well as pooled OLS (columns 3-4). For the Survey of Consumer, I consider average expectations (columns 5-6) and median expectations (columns 7-8). The standard errors, reported in parentheses, are robust with respect to heteroskedasticity and serial correlation.

Table 8: Regression Results of Equation (4), pre 1995 vs. post 1995

	Survey of Professional Forecasters				Survey of Consumers			
	Blundell Bond		Pooled OLS		Averages		Median	
	< 1995	≥ 1995	< 1995	≥ 1995	< 1995	≥ 1995	< 1995	≥ 1995
$\widehat{\gamma}$	0.70	0.41	0.44	0.21	0.72	0.27	0.43	0.22
s.e.	(0.0907)	(0.0654)	(0.0379)	(0.0344)	(0.1473)	(0.0962)	(0.0819)	(0.0654)
N	2708	3093	2708	3093	104	100	67	100

Note: This table shows the results from regression (4) for professional forecasters (SPF) as well as for consumers. For the SPF, I use the Blundell and Bond (1998) (BB) estimator (first two columns), as well as pooled OLS (columns 3-4). For the Survey of Consumer, I consider average expectations (columns 5-6) and median expectations (columns 7-8). The standard errors, reported in parentheses, are robust with respect to heteroskedasticity and serial correlation.

9 that for maximum lag lengths of 20, 10, and 5 periods, the estimated attention parameter $\widehat{\gamma}$ is in all cases higher before 1990 compared to the period after 1990.

Time Fixed Effects

There are potential concerns that differences in trend inflation may affect the empirical results in Section 2. To account for this, I include time-fixed effects in regression (4). To see how this works, recall that (4) is given by

$$\pi^e_{t+1|t,i} = \beta_i + \beta_1 \pi^e_{t|t-1,i} + \beta_2 \left(\pi_t - \pi^e_{t|t-1,i}\right) + u_{i,t}. \tag{36}$$

With time-fixed effects, a problem of multicollinearity arises. To deal with this, I first compute a period-specific persistence parameter, ρ_π. Note, that in (36), β_1 measures this persistence. Therefore, I subtract $\widehat{\rho}_\pi \pi^e_{t|t-1,i}$ from both sides and then to directly estimate γ,

Table 9: Different Maximum Lag Lengths

	All Lags		20 Lags		10 Lags		5 Lags	
	< 1990	≥ 1990	< 1990	≥ 1990	< 1990	≥ 1990	< 1990	≥ 1990
$\widehat{\gamma}$	0.70	0.41	0.74	0.51	0.84	0.69	0.94	0.82
s.e.	(0.1005)	(0.0522)	(0.1086)	(0.0632)	(0.1247)	(0.1127)	(0.1520)	(0.1570)
N	2235	3566	2235	3566	2235	3566	2235	3566

Note: This table shows the results from regression (4) for different numbers of lags included in the BB estimation. The standard errors, reported in parentheses, are robust with respect to heteroskedasticity and serial correlation.

I further divide both sides by $\widehat{\rho}_\pi$:

$$\frac{\pi^e_{t+1|t,i} - \widehat{\rho}_\pi \pi^e_{t|t-1,i}}{\widehat{\rho}_\pi} = \delta_i + d_t + \gamma \left(\pi_t - \pi^e_{t|t-1,i}\right) + v_{i,t}, \tag{37}$$

where d_t captures time-fixed effects, $\delta_i = \frac{\beta_i}{\widehat{\rho}_\pi}$ and $v_{i,t} = \frac{u_{i,t}}{\widehat{\rho}_\pi}$. I do this transformation for the period before and after 1990 separately. Note, that this transformation also deals with the endogeneity problem explained in Section 2.

The estimated attention levels are 0.75 (s.e. 0.0327) for the period before 1990 and 0.61 (s.e. 0.0295) after 1990 if I use the first-order autocorrelation of expected inflation as my measure of ρ_π. If I use the estimate of β_1 from equation (37) as my measure of ρ_π, the estimated attention before the 1990s is 0.68 (s.e. 0.0252) and the one after the 1990s is 0.46 (s.e. 0.0242). Thus, we see that the decrease in attention is robust to controlling for time-fixed effects, even though the decline is somewhat muted.

When using the first-order autocorrelation of expected inflation as my measure of ρ_π, estimating equation (5) in this way, delivers a point estimate of 0.06 (s.e. 0.0111) that is statistically significant on all conventional significance levels. The estimate for ζ in regression (6) is 0.23 (s.e. 0.0220), statistically significant on all conventional significance levels. When using $\hat{\beta}_1$ from (37) as the measure of ρ_π, the point estimate of β in equation (5) is 0.06 (s.e. 0.0074) and the estimate of ζ in (6) is 0.29 (s.e. 0.0306), both statistically significant on all conventional levels of significance. Thus, the positive relationships between attention and volatility, as well as between attention and inflation persistence, are robust to controlling for time fixed effects.

Professional Forecasters in the Aggregate

When estimating attention of professional forecastors' average expectations instead of individual ones, we obtain a value of 0.24 (s.e. 0.0481) for the period before 1990 and of 0.09 (s.e. 0.0353) after 1990. Consistent with the main results, attention substantially decreased in recent decades and is about half after 1990 compared to before.

Estimating regression (5) on aggregate SPF data delivers a coefficient of 0.15 (p-value of 0.000) and the estimate of ζ in regression (6) is 0.69 (p-value of 0.000). Thus, the results reported in the main text are robust.

Joint Regressions

Regress attention on inflation volatility and persistence jointly:

$$\widehat{\gamma}_t = \alpha + \beta\widehat{\sigma}_{\pi,t} + \zeta\widehat{\rho}_{\pi,t} + u_t. \tag{38}$$

Table 10 shows that the results are robust to this change in specification.

Table 10: Attention, Inflation Volatility and Inflation Persistence

	Survey of Professional Forecasters		Michigan Survey
Estimator	Blundell-Bond	Pooled OLS	OLS
$\widehat{\beta}$	0.04***	0.05***	0.06***
s.e.	(0.0153)	(0.0128)	(0.0150)
$\widehat{\zeta}$	0.59***	0.65***	0.31***
s.e.	(0.0597)	(0.0499)	(0.0772)
N	165	165	163

Note: This table shows the results of regression (38). Standard errors are robust with respect to heteroskedasticity. *** : p-value < 0.01, ** : p-value < 0.05, * : p-value < 0.1.

Quasi-Panel of Consumers

The Survey of Consumers does not follow consumers over time. Therefore, I could not allow for individual-specific fixed effects but rather looked at average and/or median inflation expectations. I now group the survey respondents into four groups, based on their income. The SoC provides data on this starting in the last quarter of 1979.

Table 11 shows the results. The first two columns report the results for the split point in 1990, and the third and fourth column for the split point in 1995. We see that the estimated attention levels using this quasi panel are similar to the ones obtained using average expectations (Table 1).

Table 12 shows the results of regressions (5) and (6) (first column), as well as of the joint regression (38), using this quasi panel of consumers. We see that the results are robust and that there is indeed a significantly positive relation between attention and inflation volatility, as well as between attention and inflation persistence.

Table 11: Regression Results of Equation (4), Quasi-Panel

	Survey of Consumers			
	< 1990	≥ 1990	< 1995	≥ 1995
$\widehat{\gamma}$	0.77	0.33	0.70	0.29
s.e.	(0.0933)	(0.0263)	(0.1078)	(0.0289)
N	160	480	240	400

Note: This table shows the results from regression (4), estimated using the Blundell and Bond (1998) estimator, for consumers grouped into four groups, based on their income. The standard errors, reported in parentheses, are robust with respect to heteroskedasticity and serial correlation.

Table 12: Attention, Inflation Volatility and Inflation Persistence

	Survey of Consumers	
Estimator	Separate	Joint
$\widehat{\beta}$	0.13***	0.13***
s.e.	(0.0106)	(0.0126)
$\widehat{\zeta}$	0.20***	0.12***
s.e.	(0.0787)	(0.0620)
N	121	121

Note: This table shows the results of regressions (5), (6) (first column) and (38) (second column) using a quasi panel of consumers. The attention parameters have been estimated using the BB-estimator. Standard errors are robust with respect to heteroskedasticity. *** : p-value < 0.01, ** : p-value < 0.05, * : p-value $<$ 0.1.

Volatility and Persistence of Inflation Expectations

Table 13 shows the results of regressions 5 and 6 using the volatility and persistence of inflation expectations instead of actual inflation as dependent variables. Standard errors are robust with respect to heteroskedasticity.

Table 13: Attention, Inflation Volatility and Inflation Persistence

	Survey of Professional Forecasters		Michigan Survey
Estimator	Blundell-Bond	Pooled OLS	OLS
$\widehat{\beta}$	0.14***	0.16***	0.13***
s.e.	(0.0153)	(0.0098)	(0.0172)
$\widehat{\zeta}$	1.06***	1.21***	0.23***
s.e.	(0.1272)	(0.0838)	(0.0685)
N	165	165	163

Note: This table shows the results of regressions 5 and 6 using the volatility and persistence of inflation expectations instead of actual inflation as dependent variables. Standard errors are robust with respect to heteroskedasticity. *** : p-value < 0.01, ** : p-value < 0.05, * : p-value < 0.1.

Window Length

As predicted by the underlying model of optimal information acquisition, I showed that there is indeed a positive relationship between attention to inflation and inflation volatility, as well as between attention and inflation persistence. In the baseline specification, I relied on a rolling-window approach in which every window was 10 years. Tables 14 and 15 show that these results are robust to using different window lengths, namely 5 and 15 years.

Table 14: Attention, Inflation Volatility and Inflation Persistence

	Survey of Professional Forecasters		Michigan Survey
Estimator	Blundell-Bond	Pooled OLS	OLS
$\widehat{\beta}$	-0.01	0.06***	0.13***
s.e.	(0.1643)	(0.0185)	(0.0411)
$\widehat{\zeta}$	0.73	0.44***	0.40***
s.e.	(0.6731)	(0.0551)	(0.1547)
N	185	185	183

Note: This table shows the results of regression (38) using windows of 5 years each. Standard errors are robust with respect to heteroskedasticity. *** : p-value < 0.01, ** : p-value < 0.05, * : p-value < 0.1.

Table 15: Attention, Inflation Volatility and Inflation Persistence

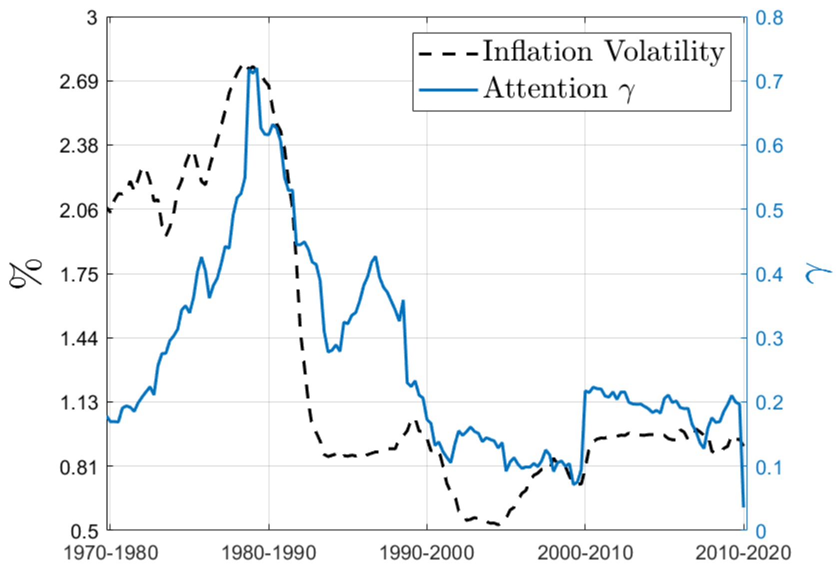
	Survey of Professional Forecasters		Michigan Survey
Estimator	Blundell-Bond	Pooled OLS	OLS
$\widehat{\beta}$	0.01	0.01	0.07***
s.e.	(0.0124)	(0.0115)	(0.0136)
$\widehat{\zeta}$	0.90***	1.00***	0.43***
s.e.	(0.0603)	(0.0552)	(0.0706)
N	145	145	143

Note: This table shows the results of regression (38) using windows of 15 years each. Standard errors are robust with respect to heteroskedasticity. *** : p-value < 0.01, ** : p-value < 0.05, * : p-value < 0.1.

Attention over Time

Figure 9 shows the estimated attention levels, γ, (black-solid line) from the SPF consensus forecasts, together with the volatility of GDP deflator inflation (blue-dashed lines). We clearly see the aforementioned decrease in attention over time, as well as the positive correlation of attention and inflation volatility.

Figure 9: Attention and Inflation Volatility over Time



Notes: This figure shows the estimated attention levels, γ, (black-solid line) from the SPF consensus forecasts, together with the volatility of GDP deflator inflation (blue-dashed lines).

E Proofs

E.1 Proof of Proposition 1

Proof. The New Keynesian Phillips Curve is given by

$$\pi_t = \beta \pi^e_{t+1|t} + \kappa y^{gap}_t + u_t.$$

Substituting

$$\pi^e_{t+1|t} = \pi^e_{t|t-1} + \gamma \left(\pi_t - \pi^e_{t|t-1}\right)$$

for $\pi^e_{t+1|t}$ yields

$$\begin{aligned}
&\pi_t = \beta \left(\pi^e_{t|t-1} + \gamma \left(\pi_t - \pi^e_{t|t-1}\right)\right) + \kappa y^{gap}_t + u_t \\
\Leftrightarrow &\pi_t(1-\beta\gamma) = \beta \pi^e_{t|t-1}\left(1-\gamma\right) + \kappa y^{gap}_t + u_t \\
\Leftrightarrow &\pi_t = \frac{\beta \pi^e_{t|t-1}\left(1-\gamma\right) + \kappa y^{gap}_t + u_t}{(1-\beta\gamma)} \\
\Leftrightarrow &\pi_t = \frac{\beta\left(1-\gamma\right)}{(1-\beta\gamma)}\pi^e_{t|t-1} + \frac{\kappa}{(1-\beta\gamma)} y^{gap}_t + \frac{u_t}{(1-\beta\gamma)}.
\end{aligned}$$

Now, taking derivatives with respect to y_t^{gap}, u_t, and $\pi^e_{t|t-1}$, respectively, yields the results (i), (ii), and (iii). ■

F Additional Model Insights

F.1 Optimal Target Rule

The following proposition derives the optimal target rule that emerges from this problem.

Proposition 3 *With limited attention, the optimal target rule under full commitment with an effective lower bound constraint is given by*

$$\pi_t + \frac{\chi}{\kappa}\left(1-\beta\gamma\right) y_t^{gap} = \phi_t \left[\frac{1-\beta\gamma}{\varphi\kappa} - \gamma\right] - \frac{1-\beta\gamma}{\kappa\beta\varphi}\phi_{t-1}, \tag{39}$$

where $\phi \geq 0$ is the Lagrange multiplier associated with the ELB constraint. Without the ELB and in the limit case of no attention, $\gamma \to 0$, the optimal target rule is given by

$$\pi_t + \frac{\chi}{\kappa} y_t^{gap} = 0, \tag{40}$$

which is the same as under discretion and rational expectations.

Proof. In the following, I focus on the more general case with an occasionally-binding ELB. The optimal target rule without an ELB then follows directly. First, let me write the optimal policy problem from Section 4 as a Lagrangian:

$$\begin{aligned}\mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta^t \Bigg\{ & -\frac{1}{2}\left[\pi_t^2 + \chi\left(y_t^{gap}\right)^2\right] \\ & + \mu_{1,t}\left[\pi_t - \beta\left(\pi^e_{t|t-1} + \gamma\left(\pi_t - \pi^e_{t|t-1}\right)\right) - \kappa y_t^{gap} - u_t\right] \\ & + \mu_{2,t}\left[y_t^{gap} - y_{t+1}^{gap} + \varphi\left(i_t - \left(\pi^e_{t|t-1} + \gamma\left(\pi_t - \pi^e_{t|t-1}\right)\right) - r_t^n\right)\right] \\ & - \phi_t\left(i_t - \bar{i}\right) \Bigg\},\end{aligned}$$

where I plugged in the expression for inflation expectations (26). The associated optimality conditions are given by

$$\begin{aligned}\{\pi_t\}: &\quad -\pi_t + \mu_{1,t}(1-\beta\gamma) - \gamma\varphi\mu_{2,t} = 0 \\ \{y_t^{gap}\}: &\quad -\chi y_t^{gap} - \kappa\mu_{1,t} + \mu_{2,t} - \beta^{-1}\mu_{2,t-1} = 0 \\ \{i_t\}: &\quad \mu_{2,t}\varphi - \phi_t = 0\end{aligned}$$

plus the complementary slackness condition $\phi_t \left(i_t - \bar{i}\right) = 0$.

Combining the second and third conditions and solving for $\mu_{1,t}$ and $\mu_{2,t}$ in terms of π_t, y_t^{gap} and ϕ_t and plugging them into the first one delivers the optimal target rule.

Given that absent the ELB constraint, we have $\phi_{t+j} = 0$ for all $j \in \{..., -1, 0, 1, ...\}$ and setting γ to zero, proves the second part. ■

We observe that the optimal target rule is history dependent. This time dependency captures promises the monetary authority makes when the ELB binds (see Eggertsson and Woodford (2003)). The main difference to the model under rational expectations is that now these promises are about future output gaps only and not about inflation. Thus, the policymaker can still stabilize the economy when the lower bound is binding. By promising lower future interest rates, output gap expectations increase and thus support current aggregate demand via the Euler equation. But the overall effectiveness of these promises is muted compared to the FIRE case as there is no additional effect via inflation expectations.

In the case without an ELB, the Euler equation is not a constraint anymore, as the nominal rate can always be set ex-post such that the Euler equation holds. Mathematically, this is the case where $\phi_{t+j} = 0$ for all $j \in \{..., -1, 0, 1, ...\}$. Hence, the policymaker focuses on managing inflation expectations. The optimal target rule is given by

$$\pi_t + \frac{\chi}{\kappa} \left(1 - \beta\gamma\right) y_t^{gap} = 0. \tag{41}$$

The result is a target rule concerned with present inflation and output only, even under full commitment. This stands in stark contrast to the case under rational expectations where the optimal policy is history dependent, thus, accounting for its effects on inflation expectations (Clarida, Gali, and Gertler (1999)). In fact, the optimal target rule under limited attention resembles the optimal policy under discretion which is solely concerned with the current state of the economy as well (Clarida et al. (1999)). In the limit case of zero attention, inflation expectations are fully stabilized and the optimal target rule coincides with the one under discretion (similar to Gáti (2020)). In that case, inflation expectations are fully anchored and the policymaker does not need to account for its policy's effect on inflation expectations.

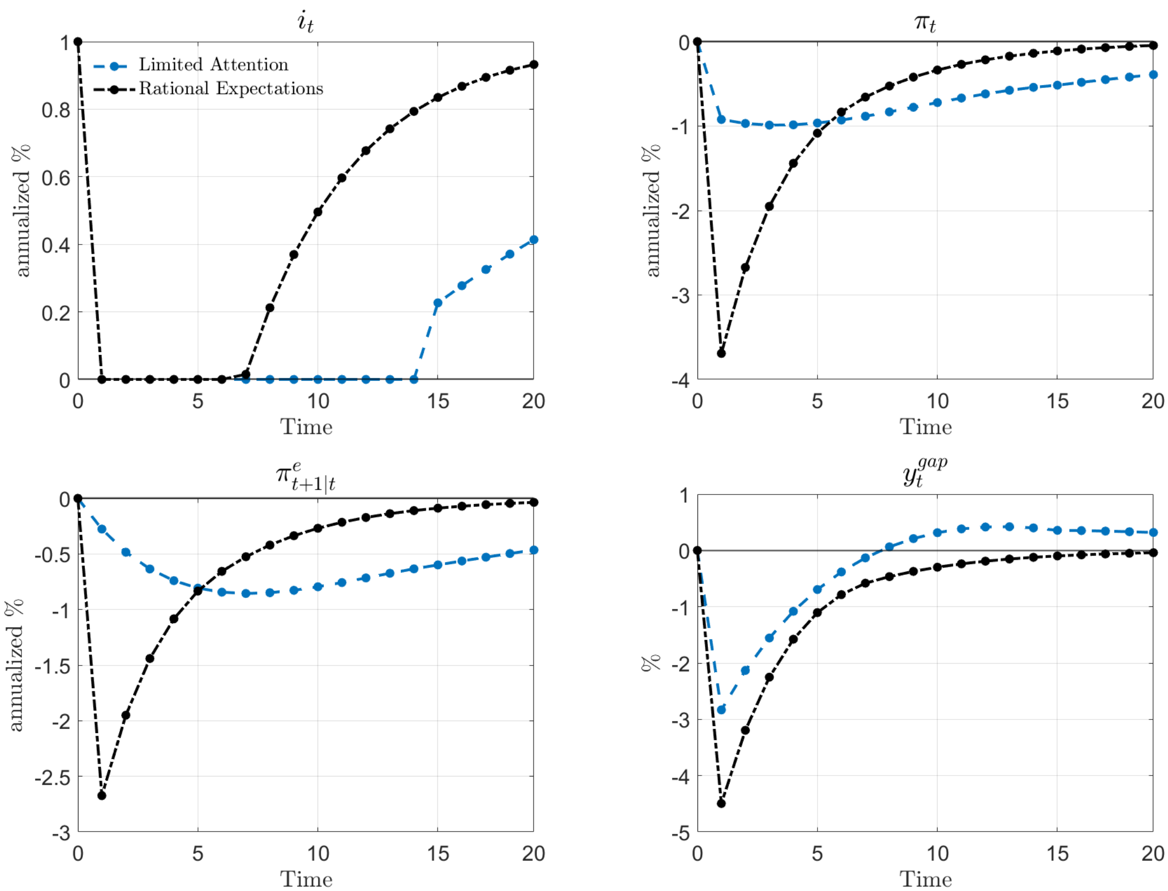
F.2 Different Taylor Rule

To show that the exact specification of the Taylor rule is not essential for the occurence of inflation-attention traps, Figure 10 shows the impulse-response functions of the nominal interest rate, inflation, inflation expectations and the output gap for the model in which the

Taylor rule absent the ELB is given by

$$i_t = 1.5\pi_t. \tag{42}$$

Figure 10: IRFs to Natural Rate Shock for Taylor rule (42)



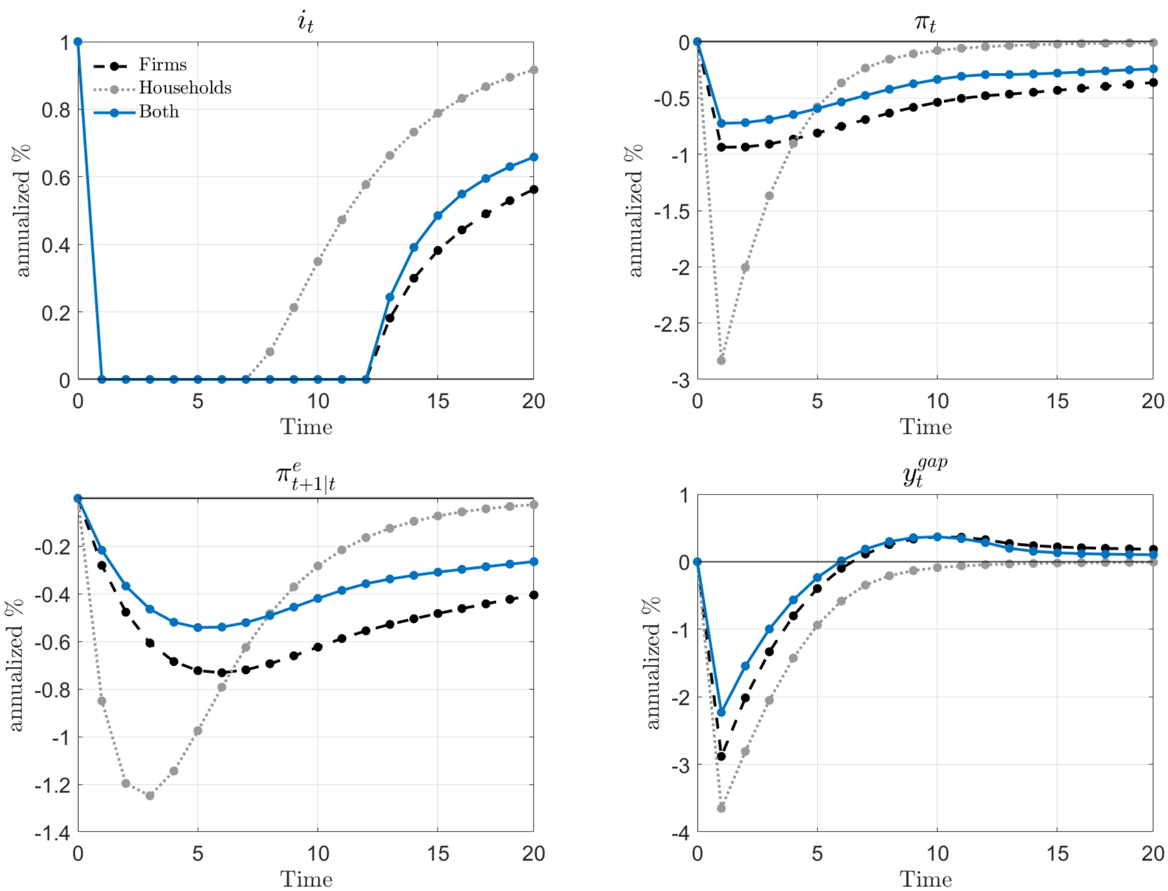
Note: This figure shows the impulse-response functions of the nominal interest rate (upper-left panel), inflation (upper-right panel), inflation expectations (lower-left) and the output gap (lower-right) to a negative natural rate shock of three standard deviations. The blue-dashed lines show the case for the limited-attention model and the black-dashed-dotted lines for the rational expectations model. Everything is in terms of percentage deviations from the respective steady state levels, except the nominal rate is in levels.

F.3 Supply vs. Demand

In the main body of the paper, I assumed that households and firms share the same inflation expectations. What if one of them has rational expectations about inflation and the other one is subject to limited attention? Figure 11 plots the impulse-response functions to a negative natural rate shock of three standard deviations for the three different scenarios. The blue-solid lines show the baseline results, the gray-dotted lines show the results for the scenario in which firms have rational expectations but households only pay limited attention to inflation, and the black-dashed lines show the results for the case in which only firms pay limited attention and households are rational.

What is evident from these graphs is that the price-setting behavior of firms is crucial for attention traps to happen. If firms have rational expectations (gray-dotted lines), then firms understand that the shock is temporary and expect that the economy will recover relatively quickly. Thus, they expect inflation to increase shortly after the shock and thus, will set their prices accordingly. So, inflation recovers much faster, as in the case with rational expectations.

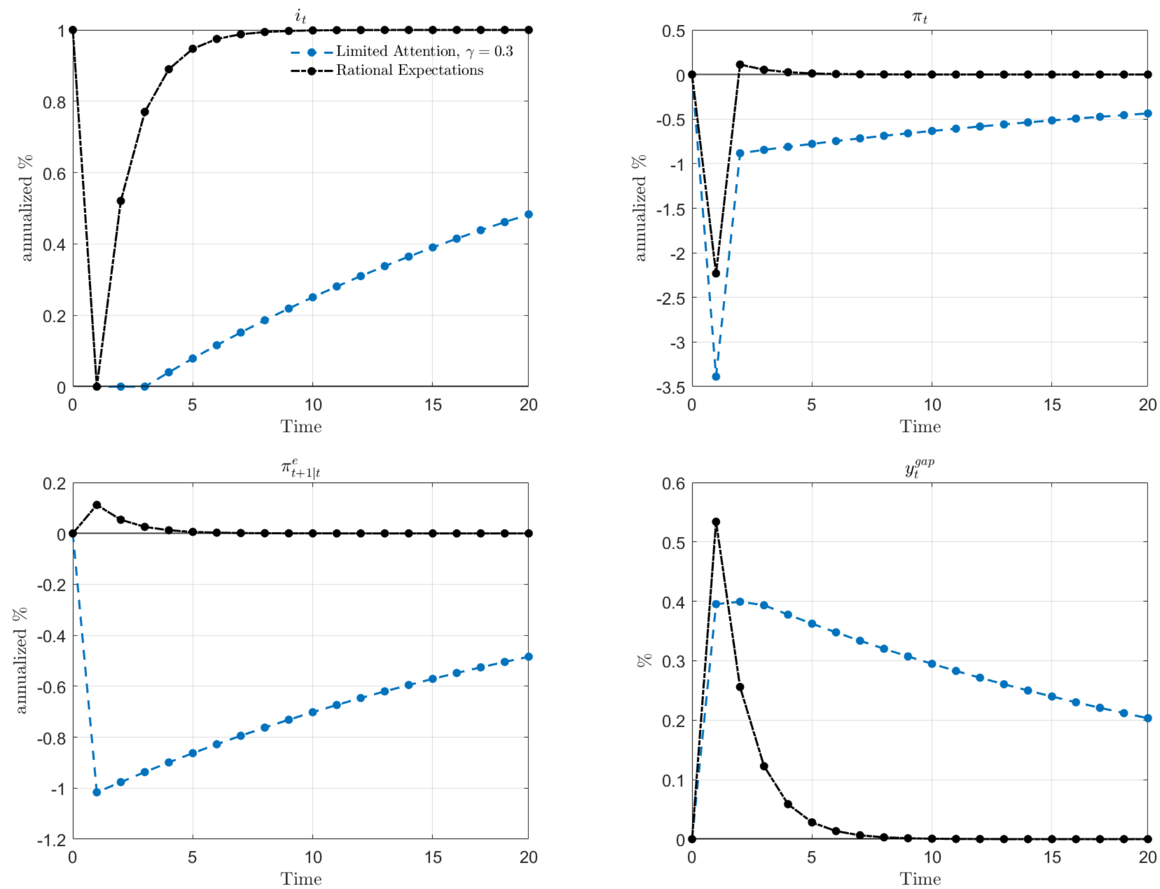
Figure 11: Impulse Response Functions to a Negative Natural Rate Shock



Note: This figure shows the impulse-response functions of the nominal interest rate (upper-left panel), inflation (upper-right panel), inflation expectations (lower-left) and the output gap (lower-right) to a negative natural rate shock of three standard deviations. The blue-dashed lines show the case for the limited-attention model and the black-dashed-dotted lines for the rational expectations model. Everything is in terms of percentage deviations from the respective steady state levels, except the nominal rate is in levels.

F.4 Cost-Push Shock

Figure 12: Impulse Response Functions to a Negative Cost-Push Shock

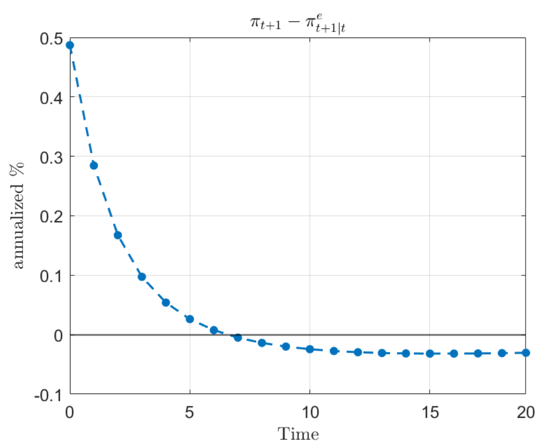


Note: This figure shows the impulse-response functions of the nominal interest rate (upper-left panel), inflation (upper-right panel), inflation expectations (lower-left) and the output gap (lower-right) to a negative cost-push shock of four standard deviations. The blue-dashed lines show the case for the limited-attention model and the black-dashed-dotted lines for the rational expectations model. Everything is in terms of percentage deviations from the respective steady state levels, expect the nominal rate is in levels.

F.5 Forecast Errors

Angeletos et al. (2020) propose a new test of models that deviate from FIRE. Namely, that expectations should initially underreact but overshoot eventually. A straightforward way to test this is to look at the model-implied impulse response functions of the forecast error, $\pi_{t+1} - \pi^e_{t+1|t}$, to an exogenous shock. Figure 13 shows these IRFs. The left panel shows the IRF of the forecast error after a positive natural rate shock and the right panel shows the corresponding IRF to a negative natural rate shock. In both cases, we see an underreaction in expectations, which manifests itself in a positive forecast error after a shock that increases the forecasted variable, and vice-versa following a negative shock. After about 5-6 periods, the forecast error response, however, flips sign. This is exactly the eventual overreaction, mentioned above and documented in Angeletos et al. (2020). Thus, my model of inflation expectations matches these empirical findings.

Figure 13: Impulse Response Functions of Forecast Errors



Note: This figure shows the impulse-response functions of inflation forecast errors after a three-standard deviation positive (left) and negative (right) natural rate shock.

F.6 No Random Walk

A potential concern with the results stated in Section 3, in particular the *inflation-attention trap* in Figure 3, is that these findings are driven by the random walk assumption in the belief process of the agents. Relaxing the random-walk assumption requires to take a stand on the perceived average inflation. In this case, where I solve the model around the zero inflation steady state, this is quite innocuous. But later on, when I focus on Ramsey optimal policy, this cannot be done anymore without distorting the results, in the sense that agents might have a mean bias.

Figure 14 shows the same impulse response functions as reported in Figure 3 for the case of $\rho_\pi = 0.95$ and an average inflation of 0. We see a similar pattern, even though somewhat less pronounced. Inflation is persistently lower under limited attention due to slowly-adjusting inflation expectations. Expectations are updated even more sluggishly when $\rho_\pi < 1$. Further, this also dampens the initial response in inflation expectations, and thus, of inflation itself. Therefore, the attention trap is somewhat mitigated and the economy escapes the lower bound faster than with $\rho_\pi = 1$. Nevertheless, the nominal interest rate is low for longer due to the slow recovery of inflation.

Optimal Policy with a Bias in Inflation Expectations In the main analysis, I have assumed that agents believe that inflation follows a random walk. Under this assumption, inflation expectations and inflation coincide on average. In the following, I relax this assumption and assume that the perceived persistence parameter is less than 1, $\rho_\pi < 1$. As discussed earlier, this yields the following inflation-expectations formation

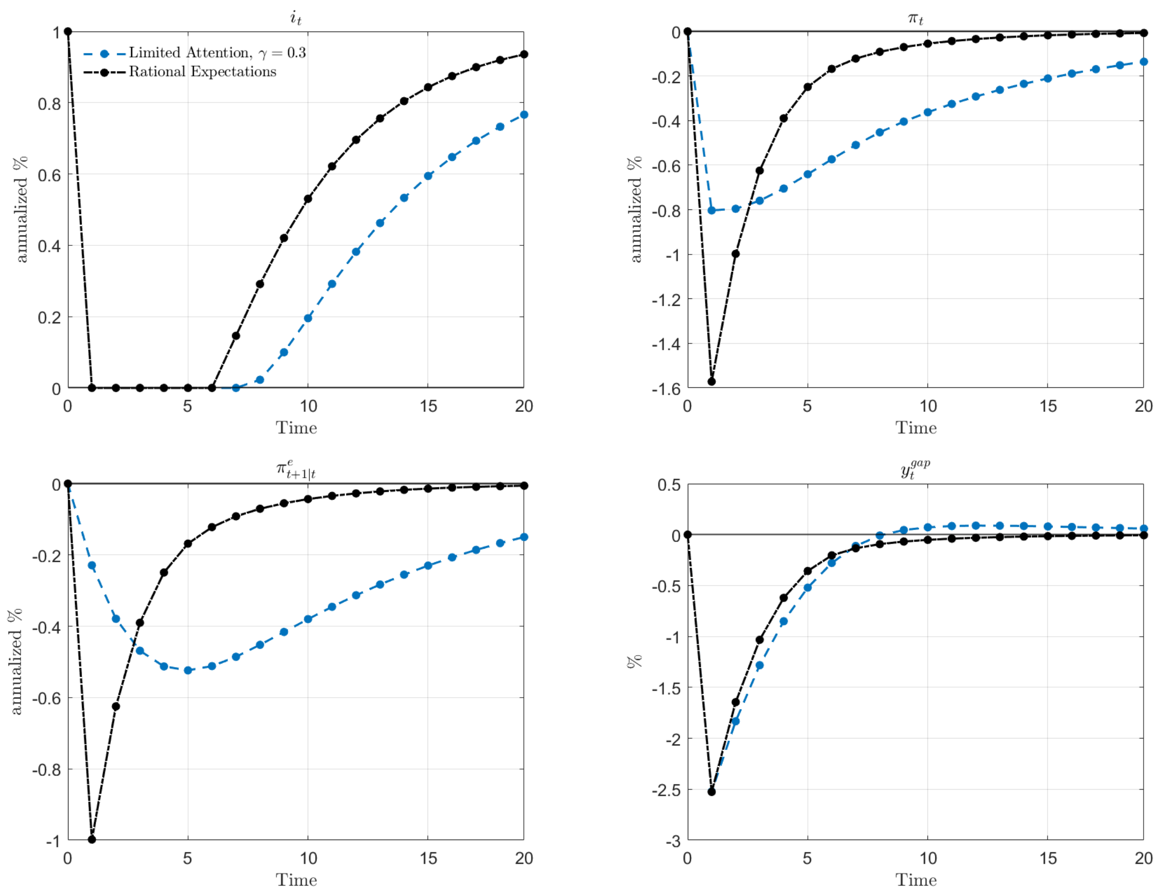
$$\pi^e_{t+1|t} = (1-\rho_\pi)\bar{\pi} + \rho_\pi \pi^e_{t|t-1} + \rho_\pi \gamma \left(\pi_t - \pi^e_{t|t-1}\right),$$

where $\bar{\pi}$ captures the long-run expectations of the agent. I set $\rho_\pi = 0.95$ and compare economies with different $\bar{\pi}$, namely $\bar{\pi} \in \{0\%, 2\%, 4\%\}$ (annualized).

Figure 15 shows the optimal inflation target (left panel) and welfare (22) (right panel) under Ramsey optimal policy for different levels of attention and different mean beliefs, $\bar{\pi}$. The blue-dashed lines show the results for the case with $\rho_\pi = 1$ (which is the baseline case discussed above), the gray-dashed-dotted lines show the results for $\rho_\pi = 0.95$ and $\bar{\pi} = 0\%$, the black-solid lines for $\rho_\pi = 0.95$ and $\bar{\pi} = 2\%$, and the red-dotted lines for $\rho_\pi = 0.95$ and $\bar{\pi} = 4\%$.

We see that introducing a mean bias in general leads to an increase in the optimal inflation target and additional welfare losses, independent of $\bar{\pi}$. This mainly comes from the fact that ρ_π is now below 1, which dampens the degree of updating captured by γ. Thus, once the

Figure 14: Impulse Response Functions to a Negative Natural Rate Shock



Note: This figure shows the impulse-response functions of the nominal interest rate (upper-right panel), inflation (upper-left panel), inflation expectations (lower-right) and the output gap (lower-left) to a negative natural rate shock of three standard deviations. The blue-dashed lines show the case for the limited-attention model and the black-dashed-dotted lines for the rational expectations model. Everything is in terms of percentage deviations from the respective steady state levels, expect the nominal rate is in levels.

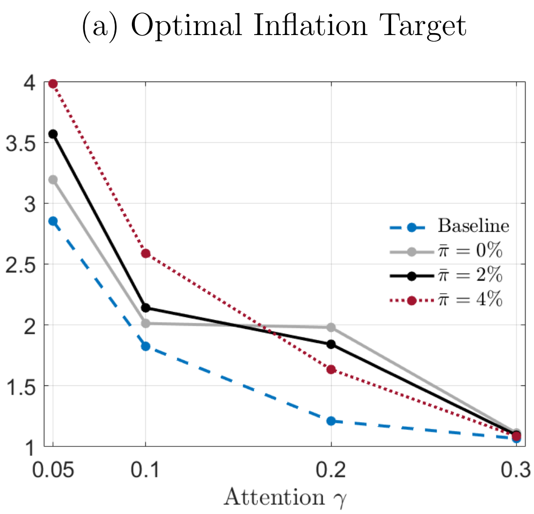
economy gets stuck at the ELB and the policymaker tries to decrease real rates by increasing inflation expectations, actual inflation needs to increase more strongly. Therefore, a lower ρ_π can exacerbate attention traps when they occur.

Interestingly, the relationship between the optimal target and $\bar{\pi}$ is non-monotonic in the level of attention. While, for example, at $\gamma = 0.2$, the optimal target is highest at $\bar{\pi} = 0\%$, it is highest at $\bar{\pi} = 4\%$ when $\gamma = 0.05$. To understand this, we can write the unconditional average inflation expectations as

$$\pi^e = \frac{(1-\rho_\pi)\bar{\pi} + \rho_\pi \gamma \pi}{1 - \rho_\pi(1-\gamma)}.$$

The following Lemma sheds light on how $\bar{\pi}$ matters for average inflation expectations and

Figure 15: Mean Bias, Optimal Inflation Target and Welfare



Notes: This figure shows the average inflation rate under Ramsey optimal policy (left panel) and welfare (22) (right panel) under Ramsey optimal policy for different levels of attention and different mean beliefs, $\bar{\pi}$. The blue-dashed lines show the results for the case with $\rho_\pi = 1$ (which is the baseline case), the gray-dashed-dotted lines show the results for $\rho_\pi = 0.95$ and $\bar{\pi} = 0\%$, the black-solid lines for $\rho_\pi = 0.95$ and $\bar{\pi} = 2\%$, and the red-dotted lines for $\rho_\pi = 0.95$ and $\bar{\pi} = 4\%$.

how this depends on the level of attention, γ.

Lemma 6 *For the case $\rho_\pi = 1$, average inflation expectations move one-for-one with average inflation, independent of γ:*

$$\pi^e = \pi.$$

For the case $0 < \rho_\pi < 1$, average inflation expectations move less than one-for-one with average inflation

$$0 < \frac{\partial \pi^e}{\partial \pi} = \frac{\rho_\pi \gamma}{1 - \rho_\pi(1-\gamma)} < 1,$$

and the strength of this dependency increases with γ

$$\frac{\partial^2 \pi^e}{\partial \pi \partial \gamma} > 0.$$

Average inflation expectations move less than one-for-one with $\bar{\pi}$

$$0 < \frac{\partial \pi^e}{\partial \bar{\pi}} = \frac{(1-\rho_\pi)}{1 - \rho_\pi(1-\gamma)} < 1,$$

and the strength of this dependency decreases with γ

$$\frac{\partial^2 \pi^e}{\partial \bar{\pi} \partial \gamma} < 0.$$

So, as attention falls, there are several opposing forces at work. On the one hand, the effect of $\bar{\pi}$ on average inflation expectations becomes stronger and thus, also exerts more pressure on actual inflation via the Phillips Curve. On the other hand, increasing the inflation target—average inflation—has a smaller effect on average inflation expectations at low levels of attention. Thus, to increase inflation expectations in this case, the inflation target needs to increase more strongly, which is of course costly. Comparing the optimal inflation targets in Figure 15, we see that at low levels of attention the first effect dominates. If $\bar{\pi}$ is relatively high, the inflation target is high.

F.7 The Recursified Ramsey Problem and Solution Algorithm

To solve the Ramsey problem numerically, I first recursify the Lagrangian as proposed in Marcet and Marimon (2019), which has been applied in similar problems (see, e.g., Adam and Billi (2006) or Adam et al. (2020)).. Let $\mu_{1,t}$ and $\mu_{2,t}$ denote the Lagrange multipliers for constraints (24) and (25), respectively. Instead of defining an additional multiplier for the constraint (26), I use this equation to substitute for $\pi^e_{t+1|t}$. The Lagrange multiplier $\mu_{1,t}$ will be multiplied with y^{gap}_{t+1}, a choice variable in period $t+1$. I rename the corresponding multiplier as ζ_{t+1}, with the transition equation $\zeta_{t+1} = \mu_{1,t}$. The recursified problem is then given by

$$\begin{aligned} V\left(\zeta_t, u_t, r^n_t, \pi^e_{t|t-1}\right) = \min_{\{\mu_{1,t},\mu_{2,t}\}} \max_{\{y^{gap}_t,\pi_t,i_t\}} \Big\{ & h\left(y^{gap}_t, \pi_t, i_t, \mu_{1,t}, \mu_{2,t}, \zeta_t, u_t, r^n_t\right) \\ & + \beta E_t \left[V\left(\zeta_{t+1}, u_{t+1}, r^n_{t+1}, \pi^e_{t+1|t}\right)\right] \Big\}, \end{aligned} \tag{43}$$

where the one-period return function $h(\cdot)$ is given by

$$\begin{aligned} h\left(y^{gap}_t, \pi_t, i_t, \mu_{1,t}, \mu_{2,t}, \zeta_t, u_t, r^n_t, \pi^e_{t|t-1}\right) = & -\pi_t^2 - \chi\left(y^{gap}_t\right)^2 \\ & + \mu_{1,t}\left[\pi_t - \kappa y^{gap}_t - \beta\left((1-\gamma)\pi^e_{t|t-1} + \gamma\pi_t\right) - u_t\right] \\ & + \mu_{2,t}\left\{y^{gap}_t + \varphi\left[i_t - \left((1-\gamma)\pi^e_{t|t-1} + \gamma\pi_t\right) - r^n_t\right]\right\} \\ & - \zeta_t \frac{1}{\beta} y^{gap}_t, \end{aligned}$$

and the optimization is subject to the ELB constraint on the nominal interest rate, the law of motion $\zeta_{t+1} = \mu_{1,t}$, the shock processes for u_t and r^n_t, and the initial value of ζ is set to 0.

I use the collocation method to approximate the value function v that solves the Bellman equation and then obtain the optimal policy functions (for details, see Miranda and Fackler (2004) whose toolbox is used to solve the model). Adam and Billi (2006) and Adam et al. (2020), for example, solve similar problems using the same approach. The following exposition follows closely the one in Appendix A.3 of Adam and Billi (2006).

In a first step, I discretize the state space $S = (\zeta_t, u_t, r^n_t, \pi^e_{t|t-1}) \subset \mathbb{R}^4$ into a set of N collocation nodes $\mathcal{N} = \{s_n | n = 1, ..., N\}$, where $s_n \in S$. I then interpolate the value function over the collocation notes by choosing basis coefficients c_n such that

$$V(s_n) = \sum_{n=1,\dots,N} c_n \theta(s_n) \tag{44}$$

at each $s_n \in \mathcal{N}$, and where $\theta(\cdot)$ is a four-dimensional cubic spline function. Equation (44) approximates the left-hand side of (43).

To approximate the right-hand side of (43), I need to approximate $EV(t(s_n, x_1, x_2, \epsilon))$, where $t(\cdot)$ is the state-transition function, $x_1 = (\mu_1, \mu_2)$ and $x_2 = (y^{gap}, \pi, i)$ are the vectors of control variables, and $\epsilon = (\varepsilon_u, \varepsilon_{r^n})$ are the shock innovations. Given the normality assumption for the shock innovations, the expected value function is approximated by Gaussian-Hermite quadrature, for which the shocks are discretized into a set of quadrature nodes ϵ_m and associated probability weights ω_m for $m = 1, ..., M$.

Putting this together, the right-hand side of (43) can be approximated as

$$\inf_{x_1} \sup_{x_2} \left\{ h(s_n, x_1, x_2) + \beta \sum_m \sum_n \omega_m c_n \theta_j(t(s_n, x_1, x_2, \epsilon_m)) \right\} \tag{45}$$

at each node $s_n \in \mathcal{N}$.

Taking the lower-bound constraint into account, the optimization problem (45) can be solved using Newton methods, delivering $RHS_c(\cdot)$, and policy functions x_{1c} and x_{2c} at the collocation nodes. RHS_c can then be approximated by a new set of basis coefficients c'_n such that

$$RHS_c(s_n) = \sum_n c'_n \theta(s_n) \tag{46}$$

at each node.

Equations (44)-(46) define the iteration

$$c \rightarrow \Phi(c), \tag{47}$$

where c is the initial vector of basis coefficients and $\Phi(c)$ the one of c'. The algorithm solves for the fixed point $c^* = \Phi(c^*)$ as follows:

1. Choose N and M, and collocation and quadrature nodes. Guess c^0.

2. Iterate on (47) and update c^k to c^{k+1}.

3. Stop if $|c^{k+1} - c^k|_{max} < \tau$, where $\tau > 0$ is the tolerance level and $|\cdot|_{max}$ denotes the maximum absolute norm. Otherwise, repeat step 2.