

# Inflation – who cares?

## Monetary Policy in Times of Low Attention

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### Abstract

I propose an approach to quantify *attention to inflation* in the data and show that the decrease in the volatility and persistence of U.S. inflation after the Great Inflation period was accompanied by a decline in the public's attention to inflation. This decline in attention has important implications for monetary policy as it renders managing inflation expectations more difficult. I augment the standard New Keynesian model with limited attention to inflation and a lower-bound constraint on nominal interest rates and show that low attention can lead to *inflation-attention traps*: prolonged periods of a binding lower bound and low inflation due to slowly-adjusting inflation expectations. I derive the Ramsey optimal policy in this framework and find that it is optimal to increase the average inflation rate as attention declines. Accounting for the lower bound fundamentally changes the normative implications of declining attention. While lower attention raises welfare absent the lower-bound constraint, it decreases welfare when accounting for the lower bound.

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# 1 Introduction

Managing inflation expectations is an important instrument for monetary policy. It offers a powerful substitute to conventional tools if the nominal interest rate is constrained by the lower bound. By making promises about the future conduct of its policy, the monetary authority can shape inflation expectations, and thus, steer the real interest rate even when the nominal rate is constrained. At least this is how it works in theory.<sup>1</sup> But while traditional analyses assume that agents have full-information rational expectations, recent empirical evidence suggests that the general public is usually poorly informed about and inattentive to monetary policy and inflation.<sup>2</sup> What do these low levels of attention imply for the conduct of monetary policy? And how has the public’s attention to inflation changed over the last fifty years?

To tackle these questions, I first propose an approach to quantify *attention to inflation* in the data. The approach is based on a model of optimal attention choice subject to information acquisition costs. The result is a law of motion for inflation expectations in which attention governs how strongly agents update their expectations following an inflation surprise. The optimal degree of attention depends positively on how volatile and persistent inflation is.

Using micro survey data of professional forecasters and consumers in the U.S., I then use this approach to estimate attention to inflation in the data and show that attention was very low in the years just before the Covid-19 crisis. In the 1970s and 1980s, on the other hand, attention to inflation was substantially higher. Consistent with the underlying model, times of higher inflation volatility and persistence are characterized by higher attention to inflation.

How does the decline in attention matter for monetary policy? To answer this question, I augment the standard New Keynesian model with an effective lower bound (ELB) on the nominal interest rate and with inflation expectations that are characterized by limited attention. Incorporating limited attention considerably affects the conduct of monetary policy. On the one hand, lower attention has a stabilizing effect on inflation expectations and actual inflation, resembling more anchored (short-run) expectations. I show analytically how a decline in attention mutes the reaction of inflation to shocks and to fluctuations in

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<sup>1</sup>For optimal monetary policy under full-information rational expectations, see e.g., [Clarida et al. \(1999\)](#), [Woodford \(2003\)](#) or [Gali \(2015\)](#). See [Eggertsson and Woodford \(2003\)](#), [Adam and Billi \(2006\)](#) or [Coibion et al. \(2012\)](#) for early analyses including the lower bound on nominal interest rates.

<sup>2</sup>Most recently, [Candia et al. \(2021\)](#) and [Coibion et al. \(2020\)](#) show that U.S. firms as well as households are usually poorly informed about and quite inattentive to monetary policy. [Coibion and Gorodnichenko \(2012, 2015a\)](#), for example, show that models of limited attention more closely align with empirical patterns of inflation expectations, compared to models with full-information rational expectations. In line with limited attention, [D’Acunto et al. \(2020\)](#) show that forward guidance is quite ineffective in stimulating inflation expectations.

the output gap due to a muted response of inflation expectations. Put differently, a decline in attention resembles a flatter Phillips Curve as empirically documented, e.g., in [Del Negro et al. \(2020\)](#). On the other hand, however, lower attention renders managing expectations more difficult, which is particularly relevant if the nominal interest rate is constrained by the lower bound.<sup>3</sup>

Indeed, lower attention to inflation weakens the effectiveness of make-up policies such as forward guidance. On top of that, a downward shift of the lower bound—i.e., going from a zero lower bound to a negative lower bound—is less stimulating if agents’ attention is low. A promising alternative to make-up policies is to increase the inflation target. A higher average inflation rate increases the average nominal rate, thus, provides the policymaker with additional space when faced with an adverse shock such that the lower bound binds less frequently. On top of that, even if the lower bound becomes binding after an adverse shock, the real rate is lower under a higher inflation target due to the slow adjustment of inflation expectations.

Under sub-optimal policy, limited attention can lead to substantially longer periods at the ELB. Even though lower levels of attention attenuate the initial response of inflation expectations to a given shock, the decline in expectations becomes more persistent which hinders actual inflation from recovering quickly. Due to the persistently-low inflation, the monetary authority keeps the interest rate at the ELB for longer. I refer to these periods of long spells at the ELB and persistent declines in inflation and inflation expectations as *inflation-attention traps*. The response of the output gap, on the other hand, is very similar to the one under rational expectations. Thus, low attention offers a potential explanation for why inflation was relatively stable during the Great Recession but was persistently low during the subsequent recovery, seemingly disconnected from output (as documented in [Del Negro et al. \(2020\)](#)).

I then derive the Ramsey optimal policy in the presence of limited attention and the ELB. As attention declines, monetary policy becomes less powerful in managing inflation expectations, which is critical at the lower bound. To mitigate this loss of control and to prevent attention traps from happening, the optimal inflation target—the average inflation rate under Ramsey optimal policy—substantially increases as attention falls. Increasing the optimal inflation target comes as a precautionary response to the ineffectiveness of forward guidance and other make-up policies under limited attention. The higher inflation target provides additional space when cutting the interest rate following adverse shocks, and there-

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<sup>3</sup>[Coibion et al. \(2019\)](#) and [Coibion et al. \(2020\)](#) show that managing expectations by the central bank is indeed a difficult task and the effects of monetary policy are much smaller than in most theoretical models. [D’Acunto et al. \(2020\)](#) find small effects of forward guidance on inflation expectations and durable consumption.

fore helps to prevent long spells at the ELB.<sup>4</sup> Given the estimates of attention just before the Covid-19 crisis, the optimal inflation target under limited attention is roughly 2-3 percentage points higher than under rational expectations.

Another instrument to combat the drawbacks of limited attention are negative interest rate policies. Allowing for negative interest rates up to  $-0.5\%$  (annualized) lowers the necessary increase in the optimal inflation target. As attention declines, however, the effectiveness of negative interest rate policies decreases and the optimal increase in the inflation target is close to the one without negative rates.

The paper concludes by shedding light on the normative implications of lower attention while accounting for the lower-bound constraint. Absent the ELB constraint, lower attention is welfare improving through its stabilization effects on actual inflation (similar to the findings in [Paciello and Wiederholt \(2014\)](#)). When taking the ELB into account, however, welfare decreases as attention declines due to the increase in the average *level* of inflation. This level effect dominates the stabilization benefits. Thus, accounting for the ELB and its interactions with low levels of attention drastically changes the welfare implications of declining attention.

**Related Literature** The empirical part of this paper is related to recent findings in [Jørgensen and Lansing \(2021\)](#) who show that inflation expectations have become more anchored over the last decades.<sup>5</sup> My measure of attention is inversely related to their definition of anchoring, but attention is concerned with short-run expectations whereas anchoring usually refers to the stabilization of long-run expectations. I complement their empirical analysis along several dimensions which I detail more closely in the empirical Section 2. Overall, my paper aligns well with their empirical findings and offers new insights in how stabilized expectations matter when nominal interest rates are constrained by a lower-bound constraint and what this implies for forward guidance and optimal monetary policy.

My limited-attention model of inflation expectations is closely related to the general information choice problem in [Mackowiak et al. \(2020\)](#). In contrast to their model, and the rational inattention literature more generally, agents in my model have a perceived law of

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<sup>4</sup>Thus, the reason for a strictly positive inflation target is different from earlier papers that also considered an occasionally-binding ELB (e.g., [Adam and Billi \(2006\)](#), [Adam et al. \(2020\)](#)). In these papers, the higher average inflation rate arises due to promises the policymaker makes *at the lower bound*. In the present paper, on the other hand, the higher inflation rate arises due to considerations *before the lower bound* binds, *foreseeing what will happen* at the lower bound.

<sup>5</sup>Similarly, [Gáti \(2020\)](#) documents that anchoring of long-run inflation expectations is time varying and has substantially increased recently.

motion of inflation that can potentially differ from the actual law of motion.<sup>6</sup> The reduced-form of the model that I bring to the data is close to the one in [Vellekoop and Wiederholt \(2019\)](#). In contrast to their paper, I focus on how attention changed over the last fifty years and show that attention tends to be higher in times of volatile and persistent inflation.

[Ball et al. \(2005\)](#), [Adam \(2007\)](#), [Paciello and Wiederholt \(2014\)](#) and [Gáti \(2020\)](#) characterize optimal monetary policy in models with different forms of limited attention. I contribute to this literature by allowing for an occasionally-binding lower-bound constraint. I show that accounting for the lower bound can lead to *qualitatively* different welfare implications due to changes in the optimal *level* of inflation. [Wiederholt \(2015\)](#) and [Gabaix \(2020\)](#) examine how information rigidities and inattention matter at the zero lower bound. [Angeletos and Lian \(2018\)](#) study the implications of relaxing the common knowledge assumption for forward guidance and show that the effects of forward guidance are attenuated in such a setting. My paper complements these three papers by studying the Ramsey optimal policy in a fully stochastic setup and focuses on the implications for the optimal inflation target. To the best of my knowledge, the present paper is the first to study the trade off of lower attention in a fully stochastic model with an occasionally binding lower-bound constraint and to characterize the Ramsey optimal monetary policy in such a setting.

**Road Map** The rest of the paper is structured as follows. The empirical strategy to quantify attention, the description of the data and the empirical results are presented in Section 2. In Section 3, I show how limited attention renders make-up policies less effective and how it can lead to inflation-attention traps, before I then study optimal policy in Section 4. Section 5 concludes.

## 2 Quantifying Attention

In this section, I show how I derive an expectations-formation process under limited attention that provides a straightforward approach to measure attention to inflation empirically. The model is an application of [Mackowiak et al. \(2020\)](#), who study a general problem of optimal information acquisition. I relegate all the details and derivations to appendix A.

The main difference to [Mackowiak et al. \(2020\)](#) is that agents in my model do not exactly know the underlying process of inflation but have a simplified view of how inflation

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<sup>6</sup>[Mackowiak et al. \(2020\)](#) provide a recent overview of this literature, which was inspired by the seminal paper [Sims \(2003\)](#). For further developments in this literature, see, among others, [Mackowiak and Wiederholt \(2009\)](#), [Paciello and Wiederholt \(2014\)](#), [Maćkowiak et al. \(2018\)](#), [Afrouzi and Yang \(2021\)](#), and see [Gabaix \(2019\)](#) for an overview of *behavioral* inattention.

evolves.<sup>7</sup> In particular, the agent believes that (demeaned) inflation tomorrow,  $\pi'$ , depends on (demeaned) inflation today,  $\pi$ , as follows

$$\pi' = \rho_\pi \pi + \nu,$$

where  $\rho_\pi \in [0, 1]$  denotes the perceived persistence of inflation and  $\nu \sim i.i.N.(0, \sigma_\nu^2)$ . This assumption is supported by empirical evidence (see e.g., [Faust and Wright \(2013\)](#) or [Canova \(2007\)](#)).<sup>8</sup> Note, that the perceived volatility and persistence need not be the same as their actual counterparts, consistent with the empirical evidence on inflation expectations (see, e.g., Table 5 in the Appendix). The agent wants to minimize her expected forecast error but inflation in the current period is unobservable and acquiring and processing information is costly. The agent thus faces a trade off how attentive she wants to be. I follow the literature on rational inattention and assume that the loss arising from making mistakes in her forecasts is quadratic with a scaling factor  $r$ , and the cost of information acquisition and processing is linear in mutual information with a scaling parameter  $\lambda$ .

In this setup and with a normal prior, the optimal signal takes the form

$$s = \pi + \varepsilon,$$

with  $\varepsilon \sim i.i.N.(0, \sigma_\varepsilon^2)$  (see [Matějka and McKay \(2015\)](#)).

The optimal forecast is given by  $\pi^e = \rho_\pi E[\pi|s]$ , and Bayesian updating implies

$$\pi^e = \rho_\pi (1 - \gamma) \hat{\pi} + \rho_\pi \gamma s, \tag{1}$$

where  $\gamma = 1 - \frac{\sigma_{\pi|s}^2}{\sigma_\pi^2} \in [0, 1]$  measures how much attention the agent pays to inflation, and  $\hat{\pi}$  denotes the prior mean of  $\pi$ .

Solving for the optimal  $\gamma$  and writing the cost of information relative to the stakes,  $\tilde{\lambda} \equiv \frac{\lambda}{r}$ , yields the *optimal* level of attention, summarized in the following Lemma.

**Lemma 1** *The optimal level of attention is given by*

$$\gamma = \max \left( 0, 1 - \frac{\tilde{\lambda}}{2\rho_\pi^2 \sigma_\pi^2} \right), \tag{2}$$

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<sup>7</sup>In the monetary model, later on, I will disentangle the effects from this potentially misperceived law of motion from the assumption of costly attention and show that most of the results are driven by limited attention and not the misperceived law of motion (see section 4.4).

<sup>8</sup>[Fulton and Hubrich \(2021\)](#) show that simple models such as AR(1) models are hard to beat when forecasting inflation in real time.

which shows that the optimal level of attention is

- (i) decreasing in the relative cost of information acquisition,  $\tilde{\lambda} \equiv \frac{\lambda}{r}$ ,
- (ii) increasing in inflation volatility,  $\sigma_\pi$ , and
- (iii) increasing in inflation persistence,  $\rho_\pi$ .

From Lemma 1, we see that aside from the relative information cost,  $\tilde{\lambda}$ , the persistence,  $\rho_\pi$ , and the volatility of inflation,  $\sigma_\pi$ , are crucial drivers of attention. The model predicts a positive relationship between attention and  $\sigma_\pi$ , as well as between attention and  $\rho_\pi$ . In the following, I will first estimate attention  $\gamma$ , assess how it has changed over time and then test whether there is indeed evidence for these positive relations.

## 2.1 Bringing the Model to the Data

To estimate attention in the data, I extend the law of motion of inflation expectations, equation (1), to a dynamic setup. The agent believes that inflation  $\pi$  follows

$$\pi_t = (1 - \rho_\pi)\bar{\pi} + \rho_\pi\pi_{t-1} + \nu_t,$$

where  $\bar{\pi}$  is the agent's long-run belief about inflation and  $\rho_\pi$  is the perceived persistence of inflation. I assume that the error term  $\nu_t$  is normally distributed with mean zero and variance  $\sigma_\nu^2$ .

The agent receives a signal about inflation of the form

$$s_{it} = \pi_t + \varepsilon_{it},$$

where the noise  $\varepsilon_{it}$  is assumed to be normally distributed with variance  $\sigma_\varepsilon^2$ .

Given these assumptions, it follows from the (steady state) Kalman filter that optimal updating is given by

$$\pi_{t+1|t,i}^e = (1 - \rho_\pi)\bar{\pi} + \rho_\pi\pi_{t|t-1,i}^e + \rho_\pi\gamma(\pi_t - \pi_{t|t-1,i}^e) + u_{i,t}. \quad (3)$$

From equation (3), we observe that lower attention, i.e., a lower  $\gamma$ , implies that the agent updates her expectations to a given forecast error,  $(\pi_t - \pi_{t|t-1,i}^e)$ , less strongly. Lower attention is reflected in more noisy signals, and more noise means the agent trusts her received signals less and thus, puts less weight on these signals. Hence, her expectations remain more strongly anchored at her prior beliefs.

In the estimation of equation (3), I allow for individual-specific intercepts. This can either reflect a mean bias in the perceived inflation rate,  $\bar{\pi}_i \neq \bar{\pi}$ , or that the agent believes her signals are biased on average, as in [Vellekoop and Wiederholt \(2019\)](#).

## 2.2 Data

I focus on the Survey of Professional Forecasters (SPF) from the Federal Reserve Bank of Philadelphia, as well as the Survey of Consumers from the University of Michigan (SoC). In the Appendix, I show that the findings extend to other data sets as well. For the SPF, I consider individual and aggregate forecasts. The main focus is on expectations about the quarter-on-quarter percentage change in the GDP deflator, which is available since 1969. I drop forecasters for which I have less than eight observations. As a robustness check, I will show that the results are robust to using expectations about the consumer price index, CPI. This data series, however, is only available since 1979.

While the SPF provides data on expectations about the next quarter, the SoC only provides one-year-ahead expectations. Therefore, I will compare them to the actual year-on-year changes in the CPI.<sup>9</sup> A drawback of the SoC is that it includes the same survey respondents at most twice. Therefore, estimating how individuals update their expectations over time is infeasible. Instead, I consider average (and median) expectations. Additionally, I estimate attention using the Survey of Consumer Expectations from the Federal Reserve Bank of New York (SCE). The SCE, launched in 2013, has a panel structure and thus allows me to estimate attention using individual-consumer data, at least for the period after 2013. Data on actual inflation comes from the FRED database from the Federal Reserve Bank of St. Louis. Appendix B provides summary statistics and plots the discussed time series.

As discussed in the literature review above, [Jørgensen and Lansing \(2021\)](#) estimate how strongly anchored inflation expectations in the U.S. are and how this changed over the last fifty years. I extend their empirical strategy in several dimensions. First, I allow the persistence of perceived inflation to change over time and do not restrict it to follow a random walk. Second, I show that not only aggregate professional forecasters' expectations have become more anchored, but also consider individual-specific expectations, as well as consumers' inflation expectations. Third, I do not impose the structure of the New Keynesian Phillips Curve on the data but directly estimate attention simply based on the proposed law of motion for inflation expectations. That said, we will see that my results are consistent with the results in [Jørgensen and Lansing \(2021\)](#).

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<sup>9</sup>The question in the SoC is not explicitly about the CPI but about “prices”. Using GDP deflator inflation instead of CPI inflation barely affects the results.



## 2.3 Estimation Results

Before estimating attention, I rewrite the updating equation (3) as

$$\pi_{t+1|t,i}^e = \beta_i + \beta_1 \pi_{t|t-1,i}^e + \beta_2 (\pi_t - \pi_{t|t-1,i}^e) + u_{i,t}, \quad (4)$$

where  $\beta_i = (1 - \rho_\pi)\bar{\pi}_i$ ,  $\beta_1 = \rho_\pi$  and  $\frac{\beta_2}{\beta_1} = \gamma$ . For the SPF, where I can use individual forecasts, I estimate (4) using a forecaster-fixed-effects regression. Since the dependent variable shows up with a lag on the right-hand side, however, the exogeneity assumption is violated. Therefore, I apply the estimator proposed by [Blundell and Bond \(1998\)](#) (BB for short) and I use all available lags of the dependent variable as instruments.<sup>10</sup> All reported standard errors are robust with respect to heteroskedasticity and serial correlation. As an alternative to the BB estimator, I also estimate (4) using pooled OLS. For the Survey of Consumers I cannot include individual fixed effects as I use average and median expectations and thus, I apply the Newey-West estimator using four lags ([Newey and West \(1987\)](#)).

To examine how attention changed over time, I run regression (4) for the period before and after 1990, separately. The results are robust to different split points (see Appendix D). Later on, I will estimate (4) using rolling-windows of ten years each.

Table 1: Regression Results of Equation (4)

	Survey of Professional Forecasters				Survey of Consumers			
	Blundell Bond		Pooled OLS		Averages		Median	
	< 1990	≥ 1990	< 1990	≥ 1990	< 1990	≥ 1990	< 1990	≥ 1990
$\hat{\gamma}$	0.70	0.41	0.44	0.22	0.75	0.31	0.43	0.24
s.e.	(0.1005)	(0.0522)	(0.0397)	(0.0290)	(0.1574)	(0.0881)	(0.0970)	(0.0601)
$N$	2235	3566	2235	3566	84	120	47	120

Note: This table shows the results from regression (4) for professional forecasters (SPF) as well as for consumers. For the SPF, I use the [Blundell and Bond \(1998\)](#) (BB) estimator (first two columns), as well as pooled OLS (columns 3-4). For the Survey of Consumer, I consider average expectations (columns 5-6) and median expectations (columns 7-8). The standard errors, reported in parentheses, are robust with respect to heteroskedasticity and serial correlation.

Table 1 shows the results. We see that the estimated attention parameter  $\gamma$  is substantially lower after 1990 compared to the period before 1990.<sup>11</sup> This is true for professional forecasters and for consumers, and as I show in Appendix D, also for other datasets. The point estimates after 1990 are basically half of what they were before the 1990s.

<sup>10</sup>Appendix D shows that the results are robust to using fewer lags.

<sup>11</sup>I test the validity of the instruments in the Blundell-Bond estimation by testing for autocorrelation of order one and two in the first-differenced error terms. The respective  $p$ -values are 0.000 (order 1) and 0.973 (order 2) for the period before 1990 and 0.000 (order 1) and 0.737 (order 2) for the period after 1990. This indicates that the instruments used in the estimation are valid.

The decline in attention is even more pronounced when focusing on the most recent decade. To show this, I run regression (4) for the period between 2010 and 2020. Additionally, I also use data from the New York Fed Survey of Consumer Expectations, starting in 2013. The advantage of this survey compared to the Michigan Survey is that it surveys the same consumers up to twelve times in a row, providing a much larger sample size. Table 2 shows the results. We see that overall, attention declined substantially compared to earlier periods and is between 0.04 and 0.17 during this period of low and stable inflation. Furthermore, the results from the SCE lie in the same ballpark as the ones from the Michigan Survey, which indicates that using average (or median) consumer expectations does not fundamentally affect the results. In fact, the estimated attention parameter for the average expectations from the Michigan Survey is 0.04 when restricting the sample to 2013-2020, which is exactly the same as the estimate obtained from the New York Fed Survey.

Table 2: Attention since 2010

	Survey of Professional Forecasters		Michigan Survey		NY Fed Survey
	Blundell Bond	Pooled OLS	Averages	Median	Pooled OLS
$\hat{\gamma}$	0.17	0.07	0.12	0.09	0.04
s.e.	(0.0729)	(0.0333)	(0.0658)	(0.0616)	(0.0316)
$N$	1322	1322	40	40	74229

Note: This table shows the results from regression (4) for the period between 2010 and 2020 for professional forecasters (SPF) as well as for consumers. For the SPF, I use the [Blundell and Bond \(1998\)](#) (BB) estimator (first column), as well as pooled OLS (column 2). For the Survey of Consumer, I consider average expectations (column 3) and median expectations (columns 4). The standard errors, reported in parentheses, are robust with respect to heteroskedasticity and serial correlation. Additionally, column 5 shows the results for consumer inflation expectations from the New York Fed Survey of Consumer Expectations.

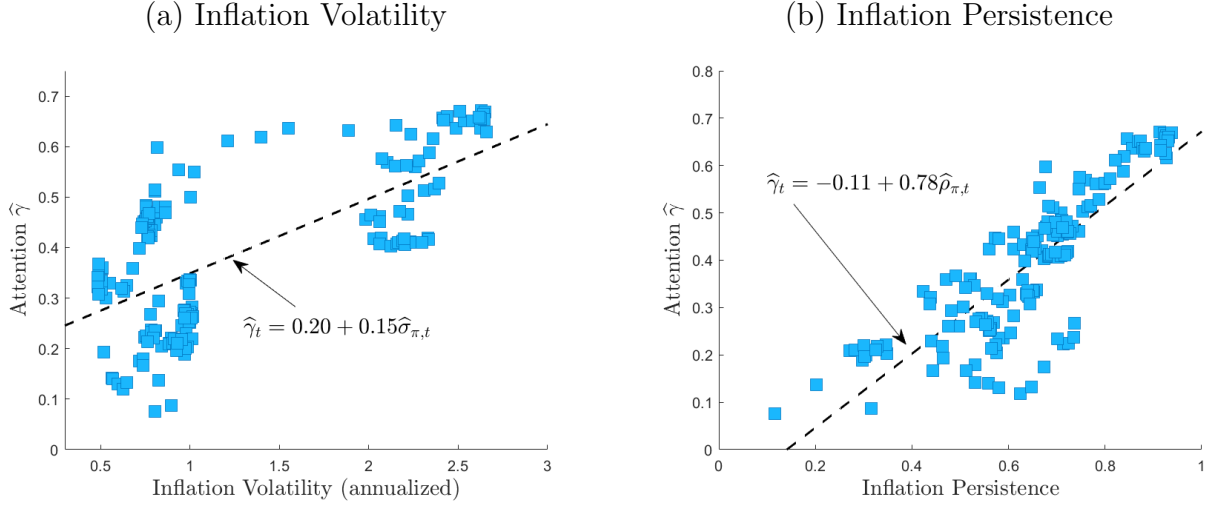
### 2.3.1 When is Attention High?

Lemma 1 states that two key drivers of attention are inflation volatility and inflation persistence. To examine these relationships empirically, I estimate regression (4), using a rolling-window approach in which every window is 10 years long, and estimate one attention parameter for every window,  $\hat{\gamma}_t$ , as well as the period-specific inflation volatility,  $\hat{\sigma}_{\pi,t}$  and the persistence parameter,  $\hat{\rho}_{\pi,t}$ . In particular, I use the window-specific standard deviation of inflation as my measure of  $\hat{\sigma}_{\pi,t}$  and the first-order autocorrelation of inflation for  $\hat{\rho}_{\pi,t}$ . Appendix D shows that the following results also hold for different window lengths or when using the standard deviation and persistence of *expected* inflation for  $\hat{\sigma}_{\pi,t}$  and  $\hat{\rho}_{\pi,t}$ .

Figure 1 summarizes the results graphically. The left scatterplot shows the inflation volatility on the horizontal axis and the estimated attention parameter,  $\hat{\gamma}$ , on the vertical

axis. The attention parameters shown in the figure are the ones for individual professional forecasters, obtained via pooled OLS. The right panel shows the relationship between attention and inflation persistence (on the horizontal axis). In both cases, we see that there is a clear positive relationship, just as the limited-attention model predicts.

Figure 1: Attention, Inflation Volatility and Inflation Persistence



Notes: The right panel shows the relationship between inflation volatility and the estimated attention parameter,  $\hat{\gamma}$ . The left panel shows the relationship between the persistence of inflation and the estimated attention parameter. Both panels report the results for individual professional forecasters where the attention parameter was estimated via pooled OLS.

To check if these findings are statistically significant, I regress attention on inflation volatility or on inflation persistence as follows

$$\hat{\gamma}_t = \alpha_1 + \beta\hat{\sigma}_{\pi,t} + u_t \quad (5)$$

$$\hat{\gamma}_t = \alpha_2 + \zeta\hat{\rho}_{\pi,t} + v_t. \quad (6)$$

Table 3: Attention, Inflation Volatility and Inflation Persistence

Estimator	Survey of Professional Forecasters		Michigan Survey
	Blundell-Bond	Pooled OLS	OLS
$\hat{\beta}$	0.13***	0.15***	0.09***
s.e.	(0.0155)	(0.0105)	(0.0272)
$\hat{\zeta}$	0.71***	0.78***	0.56***
s.e.	(0.0568)	(0.0349)	(0.0714)
$N$	165	165	163

Note: This table shows the results of regression (5) and (6). Standard errors are robust with respect to heteroskedasticity. \*\*\* :  $p$ -value < 0.01, \*\* :  $p$ -value < 0.05, \* :  $p$ -value < 0.1.

Table 3 reports the results. Standard errors are robust with respect to heteroskedasticity. We see that the observed patterns in Figure 1 are indeed statistically significant. Attention to inflation is positively correlated with inflation volatility and inflation persistence, as Lemma 1 predicts. This is true for professional forecasters as well as for consumers. Overall, the magnitudes of the estimates indicate that the results are somewhat stronger for professional forecasters. Appendix D shows that these results hold when regressing attention on inflation volatility and persistence jointly. Given the decline in inflation volatility and inflation persistence over the last fifty years (see Table 5 in Appendix B), the positive correlation with attention supports the findings in Table 1: attention declined as the volatility and persistence of inflation declined.<sup>12</sup>

In Appendix C, I document a decline in news coverage of inflation in popular news papers as well as in books, thus, providing additional, complementary evidence on the decline in attention after the 1980s. In Appendix D, I show that the presented results are robust to using different data sources, different sample splits, different specifications of the BB estimator, allowing for time-fixed effects to account, for example, for varying trend inflation, as well as constructing a quasi panel of consumers, based on their income.

### 3 Monetary Policy Implications of Limited Attention

How does the decline in attention to inflation affect the conduct of monetary policy? To answer this question, I augment the standard New Keynesian model with inflation expectations that are characterized by limited attention, and a lower-bound constraint on the nominal interest rate. To do this, I replace the inflation expectations under full-information rational expectations (FIRE) with their limited-attention counterpart derived in the previous section.<sup>13</sup> This way of incorporating limited attention in the model allows me to study the Ramsey optimal policy in the presence of an occasionally-binding lower bound on the

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<sup>12</sup>Benati (2008) documents a decline in inflation persistence in advanced economies, especially for countries that introduced inflation targeting regimes.

<sup>13</sup>One way to *micro-found* this would be similar to Section V in Angeletos and La'o (2010), where, for example, firms have two managers that do not communicate with each other. In my setting, this would require that one manager sets the firm's price and takes its employment decision for some *given inflation expectation* and the other manager provides this inflation forecast, according to the steps outlined in Section 2, without taking the other manager's decisions into account (and similarly for the representative household). An approach similar to mine, namely replacing the expectations after the derivation of the equilibrium conditions, is taken in Jørgensen and Lansing (2021) or parts of the *learning* literature (see, for example, Evans and Honkapohja (2003) or Milani (2007) for early contributions to the monetary policy literature).

policy rate.<sup>14</sup> Apart from the formation of inflation expectations, I build on the standard New Keynesian model without capital, with rigid prices in the spirit of [Calvo \(1983\)](#) and [Yun \(1996\)](#) and with a lower bound on the nominal interest rate. The government pays a subsidy to intermediate-goods producers to eliminate steady state distortions arising from market power.

The linearized model yields an aggregate supply equation, the so-called *New Keynesian Phillips Curve*, and an aggregate Euler (or IS) equation:

$$\pi_t = \beta \pi_{t+1|t}^e + \kappa y_t^{gap} + u_t, \quad (7)$$

$$y_t^{gap} = E_t y_{t+1}^{gap} - \varphi (i_t - \pi_{t+1|t}^e - r_t^n), \quad (8)$$

where  $\kappa$  measures the sensitivity of aggregate inflation to changes in the output gap,  $y_t^{gap}$ ,  $\beta \in (0, 1)$  denotes the time discount factor of the representative household, and  $u_t$  are cost-push shocks, following an AR(1) process with persistence  $\rho_u \in [0, 1]$  and innovations  $\varepsilon^u \sim i.i.N.(0, \sigma_u^2)$ . The output gap is the log deviation of output from its efficient counterpart that would prevail under flexible prices. Altogether, equation (7) summarizes the aggregate supply side of the economy. Equation (8), together with monetary policy, determines aggregate demand in this model. Here,  $\varphi > 0$  measures the real rate elasticity of output,  $i_t$  is the nominal interest rate which is set by the monetary authority, and  $r_t^n$  is the natural interest rate. The natural interest rate is the real rate that prevails in the economy with fully flexible prices and is exogenous. It follows an AR(1) process with persistence  $\rho_r \in [0, 1]$  and innovations  $\varepsilon^r \sim i.i.N.(0, \sigma_r^2)$ , independent of  $\varepsilon^u$ . The nominal interest rate and the natural rate are both expressed in absolute deviations of their respective steady state values,  $\bar{i}$  and  $\bar{r}^n$ , with  $\bar{i} = \bar{r}^n$ , as I linearize the model around the zero-inflation steady state.  $E_t$  denotes the full-information rational expectations operator.

Inflation expectations are characterized by limited attention and are given by

$$\pi_{t+1|t}^e = (1 - \rho_\pi) \bar{\pi} + \rho_\pi \pi_{t|t-1}^e + \rho_\pi \gamma (\pi_t - \pi_{t|t-1}^e), \quad (9)$$

where the notation is the same as in Section 2. Given the representative agent assumption, I abstract from noise shocks in (9). The law of motion of inflation expectations, equation (9), is driven by two main assumptions. First, the perceived law of motion follows an AR(1) process and second, paying attention is costly. In Section 4.4, I will disentangle the two

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<sup>14</sup>See, e.g., [Nimark \(2008\)](#), [Mackowiak and Wiederholt \(2009\)](#), [Angeletos and Huo \(2021\)](#), [Gáti \(2020\)](#), [Afrouzi and Yang \(2021\)](#), and [Gallegos \(2021\)](#), for papers that derive the New Keynesian model (or certain blocks of it) under some form of non-FIRE assumptions without this ad-hoc assumption. These papers, however, do not account for the occasionally binding ELB.

and show that it is mainly the second assumption that drives the implications for optimal monetary policy. For the most part, I will focus on  $\rho_\pi = 1$  in which case average inflation expectations align with actual average inflation and long-run beliefs  $\bar{\pi}$  are irrelevant. This belief formation process is empirically plausible, in the sense that it is consistent with recent empirical findings, documented in [Angeletos et al. \(2020\)](#): after a shock, expectations initially underreact, followed by a delayed overreaction (see Appendix [G.2](#)).

As is standard in the rational inattention literature, I assume that the attention parameter  $\gamma$  is constant.<sup>15</sup> The usual assumption to obtain this is that in period  $t = 0$  the agent chooses her level of attention and then obtains all future signals at this point. This leaves conditional second moments time-invariant and thus, the optimal level of attention constant. I will, however, compare economies with different levels of attention.

Table 4: Model Parameterization

Parameter	Value	Source/Target
<i>Preferences and technology</i>		
$\beta$	0.9975	Average natural rate of 1%
$\varphi$	1	<a href="#">Adam and Billi (2006)</a>
$\kappa$	0.057	<a href="#">Adam and Billi (2006)</a>
<i>Exogenous shock processes</i>		
$\rho_r$	0.8	<a href="#">Adam and Billi (2006)</a>
$\sigma_r$	0.2940%	<a href="#">Adam and Billi (2006)</a>
$\rho_u$	0	<a href="#">Adam and Billi (2006)</a>
$\sigma_u$	0.154%	<a href="#">Adam and Billi (2006)</a>

**Calibration** I calibrate the model to quarterly frequency. I assume an annualized steady state natural rate of 1%. The rest of the calibration is taken from [Adam and Billi \(2006\)](#). Table [4](#) summarizes the calibration.

**Attention and the Phillips Curve** Inflation expectations are a crucial driver of actual inflation through the supply side of the economy, and thus, changing levels of attention affect the Phillips Curve. The following Proposition summarizes these effects.

**Proposition 1** *The New Keynesian Phillips Curve under limited attention is given by*

$$\pi_t = \frac{\beta(1-\gamma)}{1-\beta\gamma} \pi_{t|t-1}^e + \frac{\kappa}{1-\beta\gamma} y_t^{gap} + \frac{1}{1-\beta\gamma} u_t.$$

*A decrease in attention  $\gamma$  leads to a weaker reaction of inflation*

<sup>15</sup>See, e.g., [Mackowiak and Wiederholt \(2009\)](#), [Maćkowiak et al. \(2018\)](#); [Mackowiak et al. \(2020\)](#).

- (i) to changes in the output gap, i.e., it resembles a flatter Phillips Curve,
- (ii) to cost-push shocks,  $u_t$ , when keeping  $y_t^{gap}$  constant, and
- (iii) to changes in prior beliefs,  $\pi_{t|t-1}^e$ , when keeping  $y_t^{gap}$  constant.

**Proof.** See Appendix F. ■

Proposition 1 captures the stabilizing effects of lower attention. As attention declines, firms' inflation expectations react less to changes in actual inflation. Through the Phillips Curve, this muted reaction of expectations in turn stabilizes inflation itself. What cannot be seen from Proposition 1, but what will be crucial in the subsequent analysis, is that lower attention not only affects the initial response of inflation and inflation expectations but the dynamics as well. Changes in inflation and inflation expectations become more persistent at low levels of attention, even though the initial response is muted.

Furthermore, declining attention also matters for the effectiveness of monetary policy, as Lemma 2 shows.

**Lemma 2** *Inflation under limited attention can be written as*

$$\begin{aligned}\pi_t = & \frac{1}{1 - \gamma(\beta + \kappa\varphi)} u_t + \frac{\kappa\varphi}{1 - \gamma(\beta + \kappa\varphi)} r_t^n \\ & + \frac{\kappa}{1 - \gamma(\beta + \kappa\varphi)} E_t y_{t+1}^{gap} - \frac{\kappa\varphi}{1 - \gamma(\beta + \kappa\varphi)} i_t \\ & + \frac{\beta + \varphi\kappa}{1 - \gamma(\beta + \kappa\varphi)} (1 - \gamma) \pi_{t|t-1}^e.\end{aligned}$$

Under the assumption  $1 - \gamma(\beta + \kappa\varphi) > 0$ , which is the case with my calibration for all estimated values of  $\gamma$ , Lemma 2 shows that lower attention mutes the reaction of inflation to shocks,  $u_t$  and  $r_t^n$ . Thus, lower attention has a stabilizing effect on actual inflation. Lower attention, however, also weakens the inflation response to changes in current nominal interest rates,  $i_t$ , as well as expected future output gaps,  $E_t y_{t+1}^{gap}$ . The last point already hints at the fact that a given promise about the future outlook of the economy, for example an announced *make-up policy* after a binding ELB, becomes less effective in stimulating the current level of inflation. The role of attention for the effects of higher prior beliefs on inflation is ambiguous. This is mainly the case because, on the one hand, lower attention implies that agents put more weight on their prior beliefs. On the other hand, lower attention stabilizes inflation, thus, weakening the effects of prior beliefs. In the following, I explore these different implications of declining attention in more detail.

### 3.1 Forward Guidance and Attention

To see how lower attention weakens the effectiveness of forward guidance, consider the following stylized experiment.<sup>16</sup> The economy is hit by a negative natural rate shock in period  $t = 0$  that pushes the nominal interest rate to the effective lower bound, i.e.,  $r_0^n < 0$  and  $i_0 = -\underline{i}$ . In  $t = 1$ , the natural rate returns to its steady state value and stays there indefinitely,  $r_t^n = 0$  for all  $t \geq 1$ . From period  $t = 2$  onwards, the output gap, and the real rate are back at their steady states,  $y_t^{gap} = 0$  and  $i_t - \pi_{t+1|t}^e = 0$  for all  $t \geq 2$ .

To model forward guidance, the real rate is assumed to be below the natural rate in  $t = 1$ . To make it comparable across different degrees of attention, I impose that

$$r_1 \equiv i_1 - \pi_{2|1}^e < 0$$

is the same for all  $\gamma$  and known in advance.<sup>17</sup> Hence, forward guidance here means to announce a certain value for the real rate. In Appendix E, I discuss the implications of forward guidance via the nominal rate. In the following, I assume that  $(-\underline{i} - r_0^n + r_1)$  is negative, which means that the announced policy, captured by  $r_1 < 0$ , *makes up* for the binding lower bound in  $t = 0$ , captured by  $-\underline{i} - r_0^n > 0$ .

Given the real rate  $r_1$  and the fact that  $y_2^{gap} = 0$ , the Euler equation in  $t = 1$  determines the output gap in period 1 as

$$y_1^{gap} = -\varphi(r_1) > 0. \quad (10)$$

Equation (10) captures the *make-up policy*: by keeping the real rate below the natural rate, output is (expected to be) above potential after the lower-bound constraint stops to be binding.

In  $t = 0$ , the ELB binds and the natural rate is negative. Thus, the Euler equation in  $t = 0$  yields

$$y_0^{gap} = \underbrace{-\varphi(r_1)}_{=E_0 y_1^{gap}} - \varphi(-\underline{i} - \pi_{1|0}^e - r_0^n).$$

Substituting the law of motion for inflation expectations

$$\pi_{1|0}^e = (1 - \gamma)\pi_{0|-1}^e + \gamma\pi_0,$$

into the Phillips Curve

$$\pi_0 = \frac{\beta}{1 - \beta\gamma}(1 - \gamma)\pi_{0|-1}^e + \frac{\kappa}{1 - \beta\gamma}y_0^{gap}$$

<sup>16</sup>For clarity, I focus on the case  $\rho_\pi = 1$ . Appendix E discusses the general case with  $\rho_\pi < 1$ .

<sup>17</sup>This is different to Angeletos and Lian (2018), where private agents are uncertain about future policies.



yields an expression for inflation expectations:

$$\pi_{1|0}^e = \frac{1-\gamma}{1-\beta\gamma}\pi_{0|-1}^e + \frac{\kappa\gamma}{1-\beta\gamma}y_0^{gap}.$$

Putting everything together, we arrive at the following result.

**Proposition 2** *The output gap in the period when the shock hits,  $t = 0$ , is given by*

$$y_0^{gap} = -\frac{\varphi(1-\beta\gamma)}{1-\gamma(\beta+\varphi\kappa)}[-\underline{i} - r_0^n + r_1] + \frac{\varphi(1-\gamma)}{1-\gamma(\beta+\varphi\kappa)}\pi_{0|-1}^e \quad (11)$$

*and inflation in  $t = 0$  is given by*

$$\pi_0 = -\frac{\kappa\varphi}{1-\gamma(\beta+\varphi\kappa)}[-\underline{i} - r_0^n + r_1] + (1-\gamma)\left[\frac{\beta}{1-\beta\gamma} + \frac{\varphi}{1-\gamma(\beta+\varphi\kappa)}\right]\pi_{0|-1}^e. \quad (12)$$

Proposition 2 captures the effectiveness of forward guidance on the output gap and inflation in the period when the shock hits. Assuming  $(1-\gamma(\beta+\varphi\kappa))$  is positive makes sure that forward guidance has a stimulating effect on output and inflation in  $t = 0$ . Proposition 2 captures several channels how a change in attention affects the economy's response to forward guidance, which I discuss in the following two corollaries.

**Corollary 1** *Lower attention weakens*

- (i) *the negative effects of the shock,*
- (ii) *the positive effects of forward guidance,*
- (iii) *the positive effects of a decrease in the lower bound  $-\underline{i}$*

*on the output gap and inflation.*

Corollary 1 follows from the fact that the terms  $\frac{\varphi(1-\beta\gamma)}{1-\gamma(\beta+\varphi\kappa)}$  and  $\frac{\kappa\varphi}{1-\gamma(\beta+\varphi\kappa)}$  in front of  $[-\underline{i} - r_0^n + r_1]$  are both increasing in  $\gamma$ . Points (i) and (ii) capture the main trade off of lower attention. While lower attention has a stabilizing effect via more anchored inflation expectations (point (i)), it renders forward guidance less effective (point (ii)). The reason why forward guidance becomes less effective as attention declines is because inflation expectations increase less in response to the announced policy, and thus, the real rate remains higher. Point (iii) illustrates an additional drawback of lower attention. A reduction of the effective lower bound,  $-\underline{i}$ , is less stimulating if agents in the economy are less attentive. Thus, going from a zero lower bound to a lower bound in negative territory, as conducted in several advanced economies over the last ten years, becomes less effective in terms of stimulating output and

inflation if the public is inattentive. Away from the lower bound, point (iii) implies that the effectiveness of conventional monetary policy via the nominal interest rate becomes less effective as attention declines.

How attention matters for the transmission of prior inflation expectations on the output gap and inflation is ambiguous, as the following Corollary shows.

**Corollary 2** *Lower attention*

- (i) *weakens the positive effect of higher prior inflation beliefs,  $\pi_{0|-1}^e$ , on the output gap if and only if,*

$$(\beta + \varphi\kappa) > 1, \quad (13)$$

- (ii) *weakens the positive effect of higher prior inflation beliefs on inflation if and only if*

$$\frac{\beta(\beta - 1)}{(1 - \beta\gamma)^2} + \frac{\varphi((\beta + \varphi\kappa) - 1)}{(1 - \gamma(\beta + \varphi\kappa))^2} > 0. \quad (14)$$

Overall, the role of attention for the effects of higher prior beliefs on output and inflation is ambiguous. This is mainly the case because, on the one hand, lower attention implies that agents put more weight on their prior beliefs. On the other hand, as discussed previously, lower attention leads to more stable inflation overall, thus, weakening the effects of prior beliefs. This can also be seen in the discussion of the Phillips Curve, see Proposition 1.

Given the calibration in Table 4, conditions (13) and (14) both hold for all  $\gamma < 0.99$ . The effects of changes in  $\gamma$ , however, are numerically small. Thus, an increase in the average inflation rate—which increases average prior beliefs—is a promising monetary instrument to combat the loss of control via forward guidance as attention declines. By *ex-ante* increasing the average inflation rate, the policymaker not only supports higher inflation expectations and thus, lower real rates for a given nominal rate, but also gains additional policy space through the increase in the average nominal rate. Higher average inflation, however, is also costly. In the analysis of optimal policy, later on, I will explore this trade off and characterize the optimal inflation target for different levels of attention.

In Appendix E, I discuss the general case with  $\rho_\pi < 1$  and show that a similar logic applies when forward guidance is conducted via the *nominal* instead of the real interest rate. Furthermore, I discuss the case in which firms and households may be heterogeneous in terms of their attention. Given this stylized example on the effectiveness of forward guidance and its dependence on the public's attention, I now show in a numerical example an additional drawback of lower attention, namely that the economy can get stuck in an *inflation-attention trap*.

### 3.2 Inflation-Attention Traps

To close the model from Section 3, I assume for now that, away from the lower bound, the monetary authority sets the nominal interest rate according to a Taylor rule

$$\tilde{i}_t = \rho_i \tilde{i}_{t-1} + (1 - \rho_i) (\phi_\pi \pi_t + \phi_y y_t^{gap}), \quad (15)$$

where  $\rho_i \in [0, 1)$  captures persistence in the interest rate,  $\phi_\pi > 1$  and  $\phi_y \geq 0$  denote the reaction coefficients to inflation and the output gap, respectively. The actual interest rate,  $i_t$ , however is constrained by the lower bound

$$i_t = \max\{\tilde{i}_t, -\bar{i}\}, \quad (16)$$

where I set the lower bound (in levels) to zero.

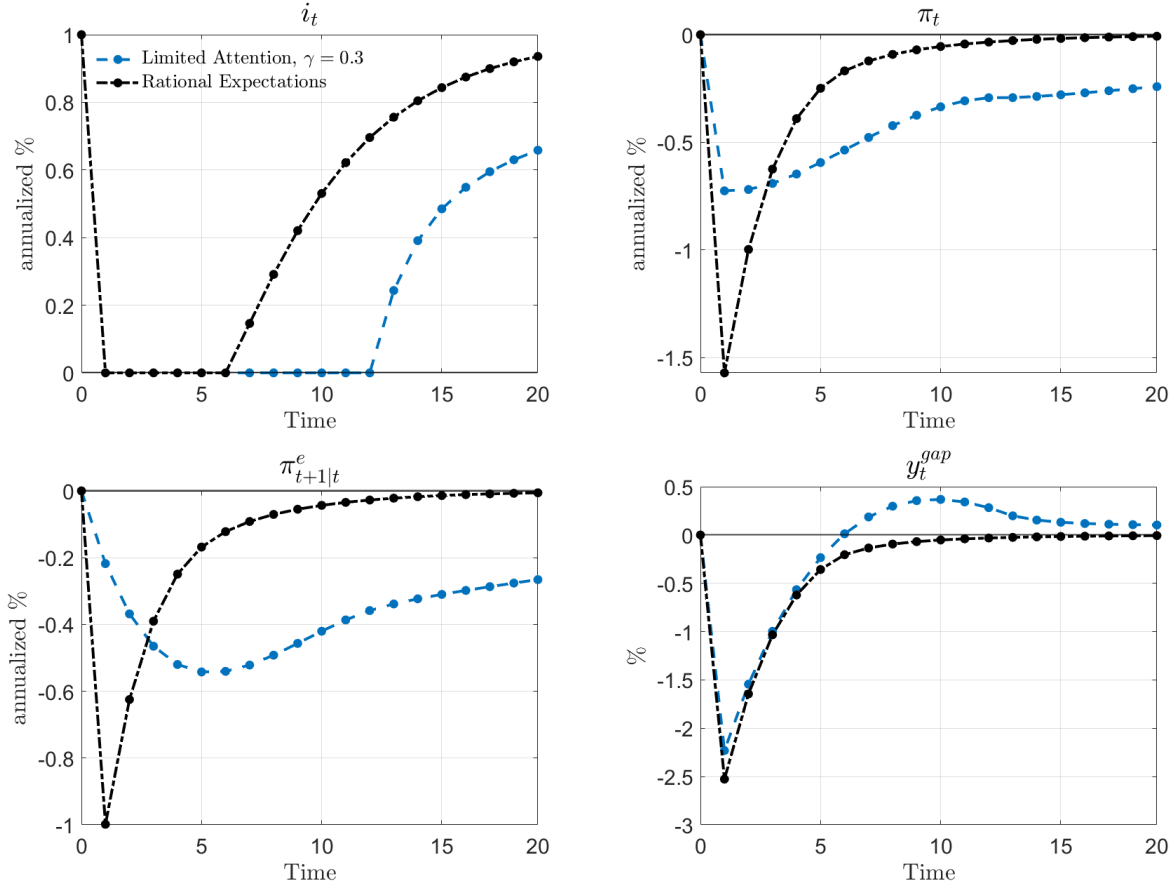
I set the persistence parameter of the nominal interest rate to 0.7, and the reaction coefficients  $\phi_\pi = 2$  and  $\phi_y = 0.5$ , as in [Andrade et al. \(2019\)](#). In Appendix G.1, I show that the exact specification of the Taylor rule is inconsequential for the following results.

What happens if a negative natural rate shock hits the economy and pushes the nominal rate to the lower bound? To answer this, Figure 2 plots the impulse response functions of the model's main variables to a negative natural rate shock of three standard deviations. The black-dashed-dotted lines are the IRFs in the model under FIRE and the blue-dashed lines are the ones under limited attention for the case  $\gamma = 0.3$ .

In both cases, the shock is large enough to push the economy to the lower bound. While the reaction of the output gap is very similar in both economies, the responses of inflation and inflation expectations are strikingly different. Initially, the muted response of inflation expectations to the adverse shock is reflected in a smaller downturn of inflation itself under limited attention. This captures the stabilizing effects that come with lower attention. The sluggish adjustment of inflation expectations in the following, however, leads to a very persistent undershooting of inflation. Even five years after the shock, inflation and inflation expectations are still substantially below their steady state levels of zero. The result is a prolonged period of a binding lower bound. While the economy under rational expectations escapes the ELB six periods after the shock, the economy under limited attention is stuck for twice as long. This is what I label *inflation-attention trap*. A side-effect of these traps is that the long ELB period leads to an output boom. As discussed earlier, this (expected) output boom in the future, however, has rather small effects on the economy today if people are inattentive.

Overall, limited attention to inflation offers a possible explanation for why several ad-

Figure 2: Impulse Response Functions to a Negative Natural Rate Shock



Note: This figure shows the impulse-response functions of the nominal interest rate (upper-left panel), inflation (upper-right panel), inflation expectations (lower-left) and the output gap (lower-right) to a negative natural rate shock of three standard deviations. The blue-dashed lines show the case for the limited-attention model and the black-dashed-dotted lines for the rational expectations model. Everything is in terms of percentage deviations from the respective steady state levels, except the nominal rate is in levels.

vanced economies were stuck at the ELB after the financial crisis, as well as inflation that undershot the central banks' inflation targets, even though the initial decrease was muted and output recovered quite strongly after declining severely initially (Del Negro et al. (2020)). In other words, the limited-attention model can explain the *missing deflation puzzle* as well as the *missing inflation puzzle* (Coibion and Gorodnichenko (2015b), Constancio (2015)).

## 4 Optimal Monetary Policy

To understand how monetary policy should optimally deal with declining attention, I now derive the Ramsey optimal monetary policy in this economy. I focus on the case of  $\rho_\pi = 1$ , in which average inflation expectations coincide with the actual inflation average. Appendix

G.3 reports the results when relaxing this assumption.

The policymaker's objective is to maximize the representative household's utility, taking the household's and firms' optimal behavior, including their attention choice, as given. Thus, the policymaker cannot exploit the private agent's lack of information. Nevertheless, the policymaker can affect inflation expectations by influencing inflation itself and he can set the average inflation expectations by setting the average inflation rate.

The policymaker is paternalistic in the sense of [Benigno and Paciello \(2014\)](#) and evaluates the household's utility under rational expectations. A second-order approximation to the household's utility function yields the policymaker's objective

$$-\frac{1}{2}E_0 \sum_{t=0}^{\infty} \beta^t \left[ \pi_t^2 + \chi (y_t^{gap})^2 \right], \quad (17)$$

where  $\chi$  is the relative weight of the output gap. Following [Adam and Billi \(2006\)](#), I set  $\chi = 0.007$ . In the following, I refer to (17) as *welfare*.

In sum, the optimal policy problem is given by

$$\max_{\pi_t, y_t^{gap}, i_t} -\frac{1}{2}E_0 \sum_{t=0}^{\infty} \beta^t \left[ \pi_t^2 + \chi (y_t^{gap})^2 \right] \quad (18)$$

subject to

$$\pi_t = \beta \pi_{t+1|t}^e + \kappa y_t^{gap} + u_t \quad (19)$$

$$y_t^{gap} = E_t y_{t+1}^{gap} - \varphi (i_t - \pi_{t+1|t}^e - r_t^n) \quad (20)$$

$$\pi_{t+1|t}^e = \pi_{t|t-1}^e + \gamma (\pi_t - \pi_{t|t-1}^e) \quad (21)$$

$$u_t = \rho_u u_{t-1} + \varepsilon_t^u \quad (22)$$

$$r_t^n = \rho_r r_{t-1}^n + \varepsilon_t^r \quad (23)$$

$$i_t \geq -\bar{i}, \quad (24)$$

with  $\varepsilon_t^u \sim i.i.N. (0, \sigma_u^2)$  and  $\varepsilon_t^r \sim i.i.N. (0, \sigma_r^2)$  and (24) is the lower-bound constraint.<sup>18</sup> All variables are in percent deviations from their respective steady state, except the nominal interest rate and the natural rate which are in absolute deviations.

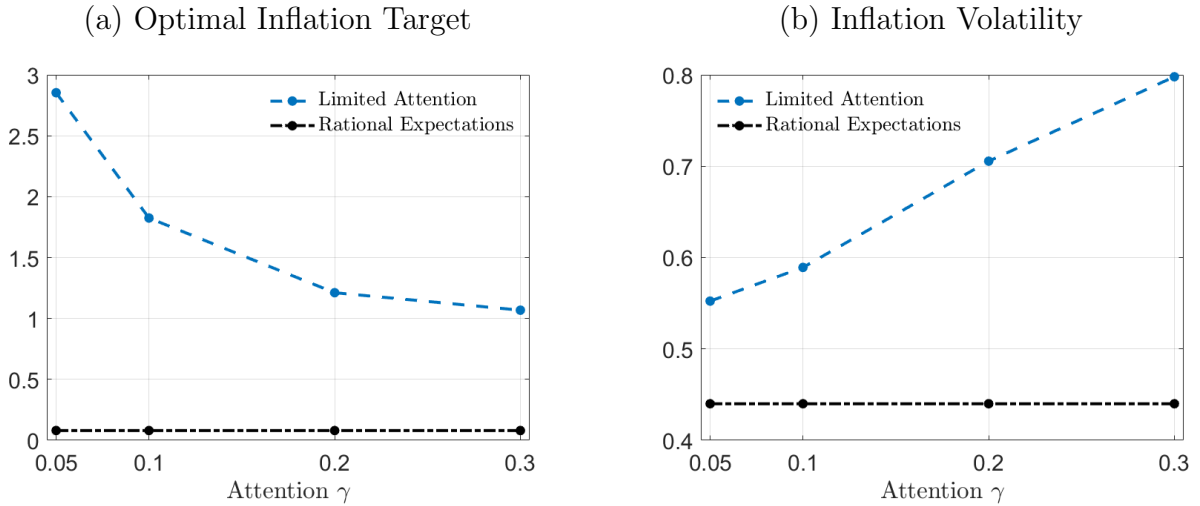
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<sup>18</sup>I solve this numerically by recursifying the constrained optimization problem, as in [Marcet and Marimon \(2019\)](#).

## 4.1 The Optimal Inflation Target

What do low levels of attention imply for inflation volatility and the optimal inflation target? For this, I solve the Ramsey problem for different levels of attention, namely  $\gamma \in \{0.05, 0.1, 0.2, 0.3\}$ . An attention parameter of 0.3 is close to the estimates for consumers' attention after 1990, and the lower levels of 0.05 and 0.1 are close to the ones observed since 2010.

Figure 3: Optimal Inflation and Inflation Volatility



Notes: This figure shows the average inflation rate under Ramsey optimal policy (left panel) and inflation volatility for different attention levels. The blue-dashed lines show the results for the model under limited attention, and the black-dashed-dotted lines for the rational-expectations model.

Figure 3 shows the results. The average inflation rate under Ramsey optimal policy—the optimal inflation target—is plotted in the left panel and the inflation volatility in the right panel. The blue-dashed lines show the results for the model under limited attention, and the black-dashed-dotted lines for the rational-expectations model. We see that the optimal inflation target increases substantially as attention declines. At current estimates of attention,  $\gamma \in \{0.05, 0.1\}$ , the inflation target is about 2-3 percentage points higher than under rational expectations due to the discussed ineffectiveness of forward-guidance policies. By increasing the average level of inflation the average nominal interest rate increases and thus makes it less likely that the ELB becomes binding. Indeed, the frequency of a binding ELB decreases substantially (not shown). For  $\gamma = 0.3$ , the ELB is binding 22% of the time, whereas this value shrinks to 1.4% for an attention level of 0.05.

While lower attention renders forward guidance, make-up policies and other policies that work (partly) through inflation expectations less effective, lower attention also stabilizes inflation, as can be seen from the right panel in Figure 3. First, because lower attention

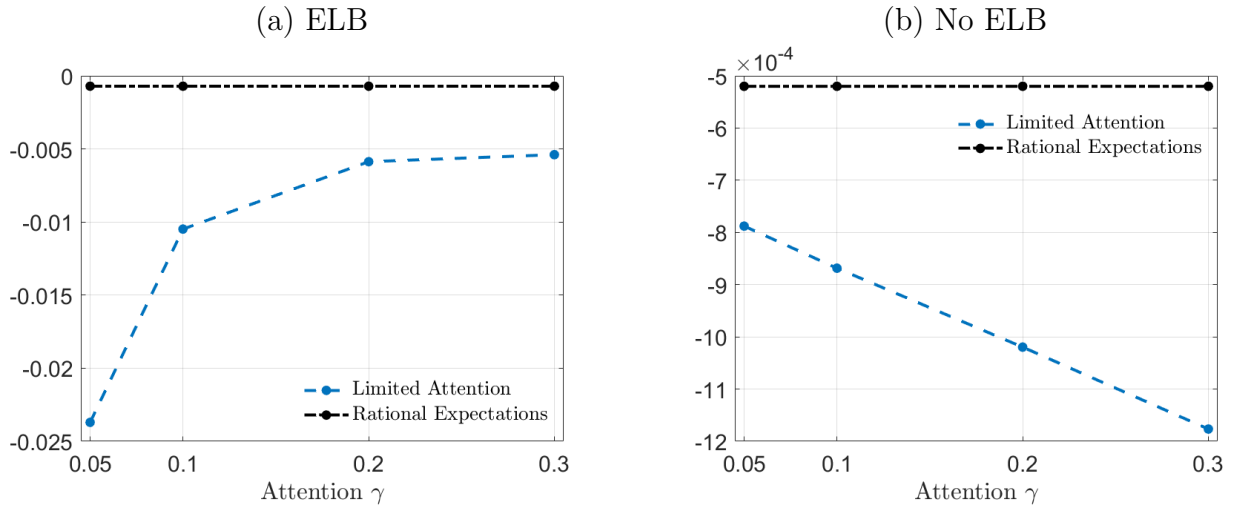
reduces the inflation response to shocks and output (see Proposition 1). Second, the lower ELB frequency further stabilizes the economy. Thus, lower attention to inflation can help stabilizing actual inflation and reduces the number of binding-ELB periods. The lower inflation volatility at lower levels of attention in fact justifies these low attention levels, as optimal attention depends positively on inflation volatility (see Section A). This low volatility, however, requires an increase in the inflation target, which is costly. Thus, it is not clear *a priori* whether lower attention leads to welfare gains or not.

## 4.2 Welfare

What are the effects of declining attention on overall welfare? Welfare is given by equation (17) and from the previous discussion, we know that lower attention poses a trade off. On the one hand, inflation volatility decreases and the ELB binds less frequently, when attention is low. This raises welfare. On the other hand, lower attention complicates managing inflation expectations and thus, the optimal average *level* of inflation increases, which is costly. Which effect dominates?

Panel (a) in Figure 4 shows that the cost of the level effect outweighs the stabilization benefits. As attention falls, welfare decreases. This is especially pronounced at low levels of attention, where the optimal inflation target increases substantially (see Figure 3).

Figure 4: Welfare and Attention



Notes: This figure shows welfare (17) under Ramsey optimal policy for different levels of attention. The left panel shows the results for the case with an occasionally-binding ELB, and the right panel without an ELB. The blue-dashed lines show the results for the model under limited attention, and the black-dashed-dotted lines for the rational-expectations model.

Absent the lower-bound constraint, the complications in managing inflation expectations

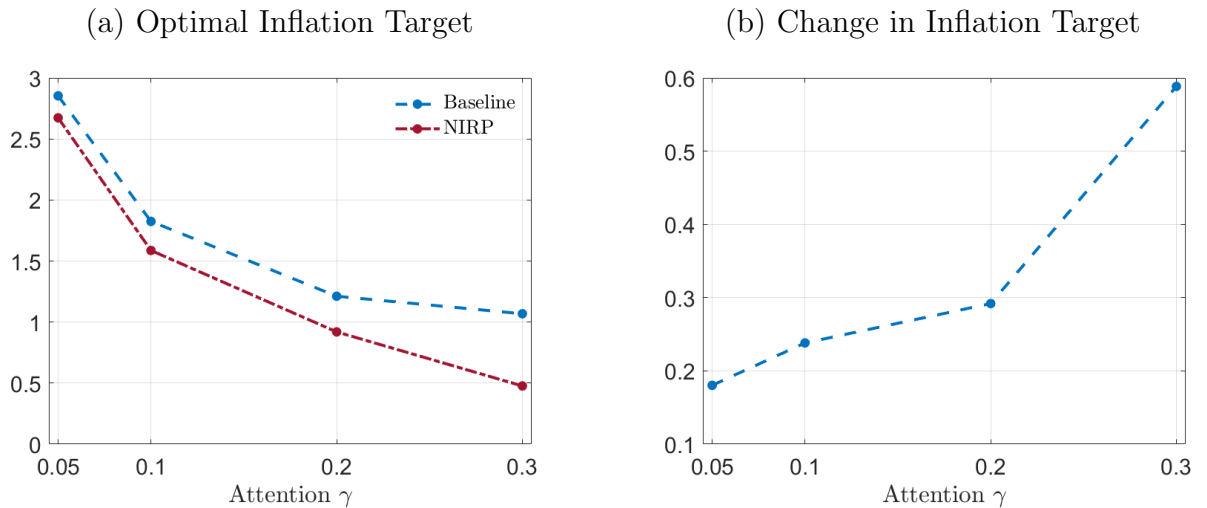
due to limited attention are much less pronounced since managing expectations is particularly important at the lower bound. In fact, lower attention is welfare improving in the case without an ELB. Panel (b) in Figure 4 shows this graphically. The stabilization benefits that arise from lower attention—which is reflected in more anchored expectations—lead to an increase in welfare.

These findings show that accounting for the ELB is crucial for making a normative statement about costs and benefits of stabilizing inflation expectations. The ELB highlights the drawbacks that arise from the stabilization of expectations due to the fall in attention, as the management of expectations becomes particularly relevant when the ELB binds.

### 4.3 Negative Interest Rate Policies

In recent years, several central banks in advanced economies have implemented negative interest rate policies (NIRP).<sup>19</sup> Could negative rates limit the negative consequences of declining attention? In order to answer this question, I solve the same Ramsey optimal policy problem as above, but set the effective lower bound to  $-0.5\%$  (annualized).

Figure 5: Negative Interest Rate Policies and Attention



Notes: The left panel shows the average inflation rate under optimal policy for different degrees of attention  $\gamma$ . The blue-dashed lines show the results for the benchmark model where the lower bound is at 0, and the red-dashed-dotted lines show the results when allowing for negative interest rates up to  $-0.5\%$  (annualized). The right panel shows the difference in the optimal inflation targets, defined as  $\pi^{*,ZLB} - \pi^{*,NIRP}$ , where  $\pi^*$  denotes the optimal inflation target and the superscripts *ZLB* and *NIRP* denote the two cases where the ELB is at 0% or  $-0.5\%$ , respectively.

Figure 5 reports the outcomes. Panel (a) shows the optimal inflation target (red-dashed-

<sup>19</sup>See [Brandão-Marques et al. \(2021\)](#) for a recent survey on negative interest rate policies and its effectiveness.



dotted line) and compares it to the case with an ELB at 0 (blue-dashed line). We see that the additional policy space due to the negative lower bound indeed calls for a lower inflation target. As the discussion in the analytical section 3 foreshadowed, however, the decline in attention also weakens the effectiveness of NIRP. We see this by observing that the optimal inflation target under NIRP gets closer to the one without negative rates as attention declines. To see these gaps clearly, panel (b) shows the difference in the optimal inflation targets, defined as  $\pi^{*,ZLB} - \pi^{*,NIRP}$ , where  $\pi^*$  denotes the optimal inflation target and the superscripts *ZLB* and *NIRP* denote the two cases where the ELB is at 0% or -0.5%, respectively. Overall, allowing for negative policy rates can help limiting the drawbacks of low attention but these policies itself become less effective as attention declines.

## 4.4 Full Attention

There are two main assumptions underlying the law of motion of inflation expectations (9). First, the assumption that agents perceive inflation to follow an AR(1) process. Second, that paying attention to inflation is costly and thus, their attention is limited. To disentangle these two effects, Figure 14 in Appendix G.4 shows the implications of shutting down the second channel, i.e., if we set  $\gamma = 1$ . Put differently, how much of the results in previous sections is solely due to the misperception of the law of motion of inflation? Panel (a) shows that the optimal average inflation rate is practically identical to the one under rational expectations. Thus, as argued above, it is really the lack of attention that pushes up the optimal inflation rate, whereas the assumption of a misperceived law of motion is rather innocuous from this perspective.

Panel (b) in Figure 14 reports the inflation volatility under Ramsey optimal policy for different levels of attention, including the full attention case,  $\gamma = 1$ . Inflation is substantially more volatile than under rational expectations. Thus, while the misperception of the law of motion of inflation is rather inconsequential for the optimal inflation target, it predicts a higher inflation volatility. Indeed, there is a trade-off. On the one hand, higher attention increases the response of inflation to shocks, as discussed in Lemma 2. On the other hand, if agents are fully attentive, inflation also reacts more strongly to changes in expected future output. Thus, to achieve a certain effect on today's inflation rate, the policymaker is required to make smaller promises about its policies in the future which in turn stabilizes inflation and output already today. While the first channel dominates at lower levels of  $\gamma$ , the second effect pushes inflation volatility down at higher levels of  $\gamma$ .

Finally, panel (c) in Figure 14 shows the welfare implications of the misperceived law of motion. Welfare is slightly more negative when agents have a misperceived law of motion

of inflation compared to fully rational agents. Given the results in panels (a) and (b), we see that these additional welfare losses are mainly due to increased inflation volatility rather than its level.

## 5 Conclusion

With the stabilization of inflation in advanced economies since the Great Inflation period, inflation has become less important in people’s everyday lives. In this paper, I quantify this using a limited-attention model of inflation expectations. In line with this model, I show that attention to inflation decreased together with inflation volatility and inflation persistence since the 1970s. Especially in the period between 2010 and 2020, the general public’s attention to inflation was close to zero.

For monetary policy the decline in attention was desirable at first, since lower attention stabilizes inflation expectations and hence, stabilizes actual inflation. With the outbreak of the Great Recession and nominal rates at their lower bound, however, managing inflation expectations became a central tool for monetary policy. But managing inflation expectations is difficult when people are inattentive.

In this paper, I study how lower attention matters for monetary policy. After an adverse shock that pushes the nominal rate to the lower bound, inflation expectations and inflation remain persistently low and thus, the lower bound binds for a prolonged period. To mitigate these drawbacks that arise from low attention, the optimal policy response is a substantial increase in the inflation target. This increases the average nominal rate and thus, binding ELB periods become less likely. The costs of this increase in inflation, however, outweigh the stabilization benefits of lower attention. Lower attention, therefore, decreases welfare if we account for the lower bound. This stands in stark contrast to the case without an ELB in which case lower attention leads to welfare gains through the stabilization of inflation expectations and inflation. My paper thus shows that accounting for the ELB is crucial when assessing the role of the public’s attention to inflation.

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# Appendix

## A A Limited-Attention Model of Inflation Expectations

In this section, I derive the expectations-formation process under limited attention sketched in Section 2. The agent believes that (demeaned) inflation tomorrow,  $\pi'$ , depends on (demeaned) inflation today,  $\pi$ , as follows

$$\pi' = \rho_\pi \pi + \nu,$$

where  $\rho_\pi \in [0, 1]$  denotes the perceived persistence of inflation and  $\nu \sim i.i.N.(0, \sigma_\nu^2)$ . Inflation in the current period is unobservable, so before forming an expectation about future inflation, the agent needs to form an expectation about today's inflation. I denote this nowcast  $\tilde{\pi}$ , and the resulting forecast about next period's inflation  $\pi^e = \rho_\pi \tilde{\pi}$ . Given her beliefs, the full-information forecast  $\pi^{e*}$  is

$$\pi^{e*} \equiv \rho_\pi \pi.$$

But since  $\pi$  is not perfectly observable, the actual forecast will deviate from the full-information forecast. Deviating, however, is costly, as this causes the agent to make mistakes in her decisions.

The agent's choice is not only about how to form her expectations given certain information, but about how to choose this information optimally, while taking into account how this will later affect her forecast. That is, she chooses the form of the signal  $s$  she receives about current inflation. Since acquiring information is costly, it cannot be optimal to acquire different signals that lead to an identical forecast. Due to this one-to-one relation of signal and forecast, we can directly work with the joint distribution of  $\pi^e$  and  $\pi$ ,  $f(\pi^e, \pi)$ , instead of working with the signal.

Let  $U(\pi^e, \pi)$  denote the negative of the loss that is incurred when the agent's forecast deviates from the forecast under full information, and  $C(f)$  the cost of information. Then, the agent's problem is given by

$$\begin{aligned} & \max_f \int U(\pi^e, \pi) f(\pi^e, \pi) d\pi d\pi^e - C(f) \\ & \text{subject to } \int f(\pi^e, \pi) d\pi^e = g(\pi), \text{ for all } \pi, \end{aligned} \tag{25}$$



where  $g(\pi)$  is the agent's prior, which is assumed to be Gaussian;  $\pi \sim N(\hat{\pi}, \sigma_\pi^2)$ .  $C(\cdot)$  is the cost function that captures how costly information acquisition is. It is linear in *mutual information*  $I(\pi; \pi^e)$ , i.e., the expected reduction in entropy of  $\pi$  due to knowledge of  $\pi^e$ :

$$C(f) = \lambda I(\pi; \pi^e) = \lambda (H(\pi) - E[H(\pi|\pi^e)]),$$

where  $H(x) = -\int f(x) \log(f(x)) dx$  is the entropy of  $x$  and  $\lambda$  is a parameter that measures the cost of information.

The objective function  $U(\cdot)$  is assumed to be quadratic:

$$U(\pi^e, \pi) = -r (\rho_\pi \pi - \pi^e)^2,$$

where  $r$  measures the stakes of making a mistake.<sup>20,21</sup>

In this setup, Gaussian signals are optimal (and in fact the unique solution, see [Matějka and McKay \(2015\)](#)). The optimal signal thus has the form

$$s = \pi + \varepsilon,$$

with  $\varepsilon \sim i.i.N.(0, \sigma_\varepsilon^2)$ .<sup>22</sup> The problem (25) now reads

$$\max_{\sigma_{\pi|s}^2 \leq \sigma_\pi^2} E_\pi [E_s [-r \rho_\pi^2 (\pi - E[\pi|s])^2]] - \lambda I(\pi; \pi^e) = \max_{\sigma_{\pi|s}^2 \leq \sigma_\pi^2} \left( -r \rho_\pi^2 \sigma_{\pi|s}^2 - \frac{\lambda}{2} \log \frac{\sigma_\pi^2}{\sigma_{\pi|s}^2} \right). \quad (26)$$

The optimal forecast is given by  $\pi^e = \rho_\pi E[\pi|s]$ , and Bayesian updating implies

$$\pi^e = \rho_\pi (1 - \gamma) \hat{\pi} + \rho_\pi \gamma s, \quad (27)$$

where  $\gamma = 1 - \frac{\sigma_{\pi|s}^2}{\sigma_\pi^2} \in [0, 1]$  measures how much attention the agent pays to inflation, and  $\hat{\pi}$  denotes the prior mean of  $\pi$ .

---

<sup>20</sup>A quadratic loss function is usually derived from a second-order approximation of the household's utility function or the firm's profit function (see, e.g., [Mackowiak and Wiederholt \(2009\)](#)).

<sup>21</sup>These stakes (or also the information cost parameter  $\lambda$ ) can be interpreted as a way to incorporate other variables to which the agent might pay attention. For example, a household might not only want to forecast inflation but also her own income stream going forward. In this case, a smaller  $r$  could capture an increase in her idiosyncratic income volatility. Thus, paying attention to inflation is relatively less beneficial, as the relative importance of her idiosyncratic income increases. Such an interpretation also explains why professional forecasters might not be fully informed about inflation, given that they usually forecast a whole array of variables.

<sup>22</sup>In this case, the entropy becomes  $H(x) = \frac{1}{2} \log(2\pi e \sigma_x^2)$ , where  $\sigma_x^2$  is the variance of  $x$ . Note, that here  $\pi$  denotes the number "pi" and not inflation.

An equivalent way of writing  $\gamma$  is

$$\gamma = \frac{\sigma_\pi^2}{\sigma_\pi^2 + \sigma_\varepsilon^2}. \quad (28)$$

Now, since the agent *chooses* the level of attention, we can re-formulate (26) as

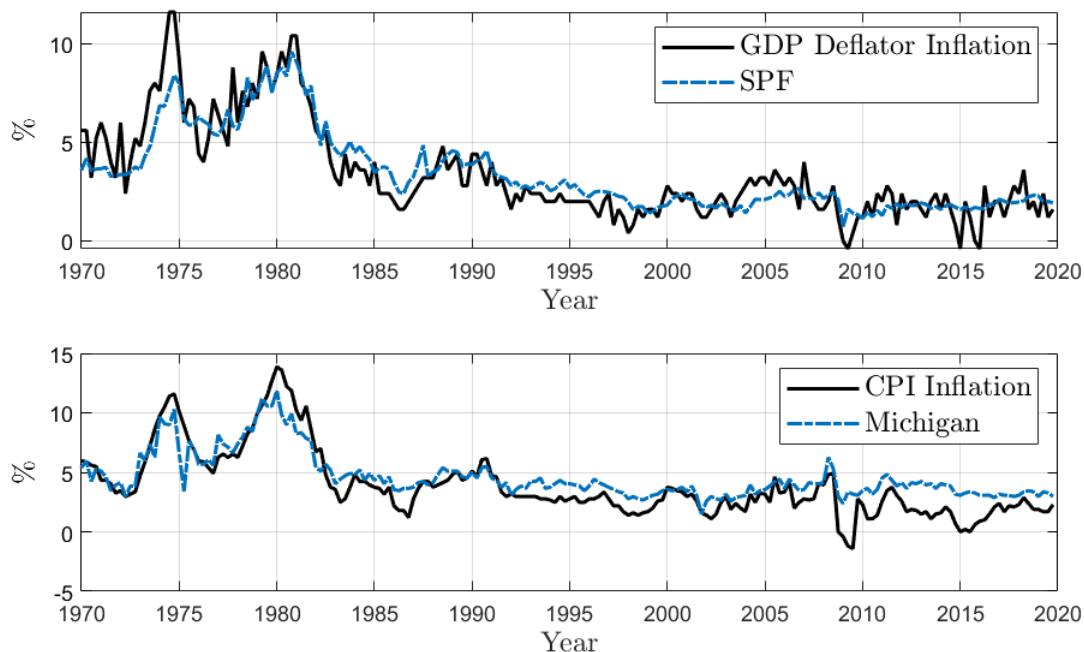
$$\max_{\gamma \in [0,1]} \left( -r\rho_\pi^2(1-\gamma)\sigma_\pi^2 - \frac{\lambda}{2} \log \frac{1}{1-\gamma} \right). \quad (29)$$

Writing the cost of information relative to the stakes,  $\tilde{\lambda} \equiv \frac{\lambda}{r}$ , and solving the optimization problem (29) yields the *optimal* level of attention, presented in Lemma 1.

## B Data and Summary Statistics

Figure 6 shows the main time series that are used in the empirical analyses of Section 2. Apart from the apparent decrease in the level and volatility of inflation as well as inflation expectations, we see that expectations became more and more detached from actual inflation. First, consumer expectations seem to be biased on average in the most recent decades, as can be seen in the lower panel. While these expectations closely tracked inflation in the 70s and 80s, this is not the case anymore.<sup>23</sup> Second, professional forecasters' expectations seem to perform quite well on average. In the last twenty years, however, they barely react to actual changes in inflation anymore. Overall, these observations suggest that attention decreased in the last decades.

Figure 6: Inflation and Inflation Expectations



Note: This figure shows the raw time series of inflation, as well as survey expectations about future inflation. Everything is in annualized percentages.

Table 5 shows the summary statistics, for the period before and after the 1990s, separately. For professional forecasters, the perceived persistence is higher than the actual one. This is especially the case when the actual persistence is relatively low, as was the case after 1990. Afrouzi et al. (2020) document a similar finding in an experimental setting. This might point towards lower attention since the 1990s. Note, that in the empirical analysis I account for changes in the perceived persistence.

<sup>23</sup>In the empirical analysis I account for this mean bias by including an intercept in the regressions.

Table 5: Summary Statistics

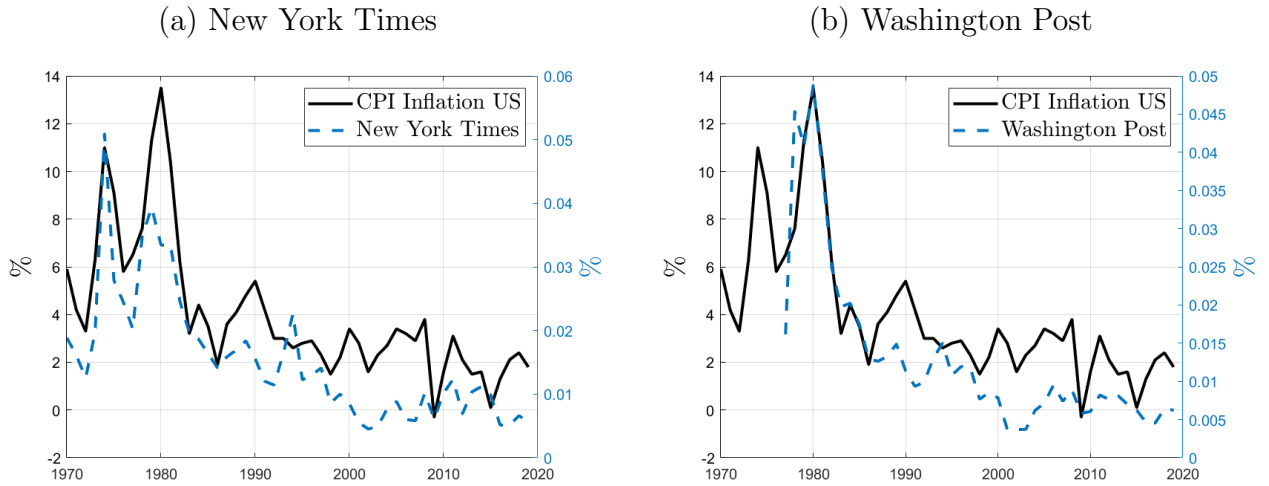
	GDP Deflator Inflation		SPF Expectations	
	1968-1990	1990-2020	1968-1990	1990-2020
Mean (%)	5.44	2.00	5.18	2.15
Std. Dev. (%)	2.43	0.90	1.87	0.63
Persistence	0.84	0.55	0.93	0.92
	CPI Inflation		Consumer Expectations	
	1968-1990	1990-2020	1968-1990	1990-2020
Mean (%)	6.09	2.42	6.00	3.62
Std. Dev. (%)	3.00	1.26	2.17	0.68
Persistence	0.96	0.77	0.85	0.70

Note: This table shows the summary statistics of the data. The upper panel shows the statistics for the quarter-on-quarter GDP deflator inflation (left) and the corresponding inflation expectations from the Survey of Professional Forecasters (right). The lower panel shows the year-on-year CPI inflation (left) and the corresponding inflation expectations from the Survey of Consumers from the University of Michigan. All data are annualized.

## C News and Book Coverage of Inflation

In this section, I provide complementary evidence to the one presented in Section 2 based on news coverage. Figure 7 shows the relative frequency of the word “inflation” among all words in two major U.S. newspapers (blue-dashed lines), the New York Times (left panel) and the Washington Post (right panel), together with the annual U.S. CPI inflation (black-solid lines). It is evident that news coverage is higher in times of high and volatile inflation as was the case during the 1970s and early 1980s. Moreover, the figure suggests that the public’s attention to inflation—proxied here by news coverage—has not always been as low as in recent years, but declined over time.

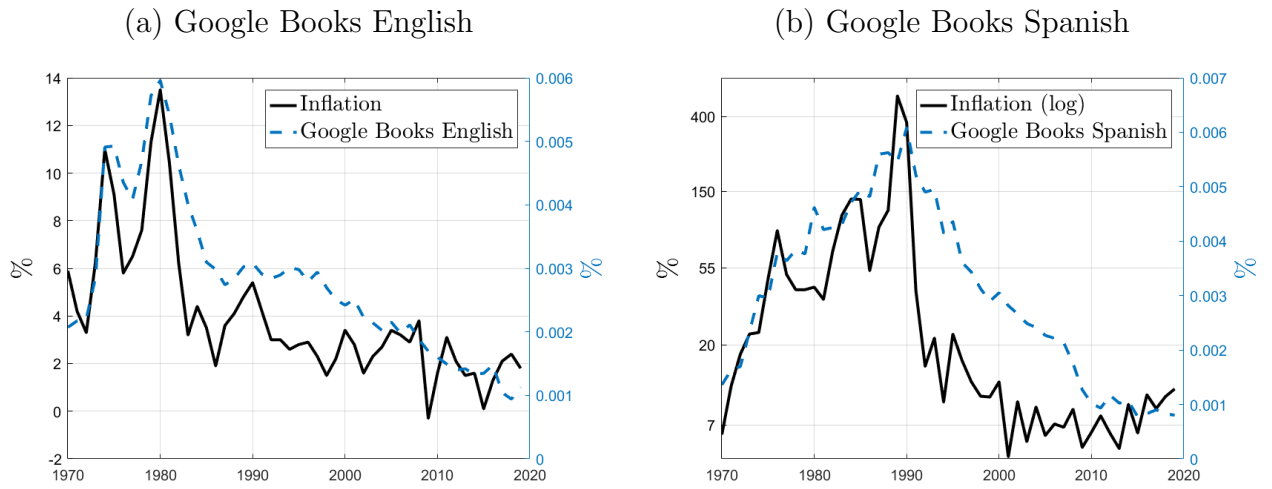
Figure 7: News Coverage of Inflation



Notes: This figure shows the relative frequency (blue dashed lines, right axis) of the word “inflation” in the New York Times (left) and the Washington Post (right). The black solid line shows annual U.S. CPI inflation (left axis).

In Figure 8, we see that a similar picture emerges when looking at the coverage of “inflation” in books, according to *Google Books Ngram Viewer*. In the left panel, we see that “inflation” is covered more frequently in English books written in times of high inflation. But this is not simply a U.S. phenomenon. To see this, I show the same statistic for books written in Spanish for the word “inflación”. To contrast this with inflation, the black solid line shows the average inflation (in logs) of the four largest Spanish-speaking countries, weighted by their 2020 population size. These are Argentina, Colombia, Mexico and Spain. Again, we observe that attention to inflation—measured by book coverage—is higher in times of high and volatile inflation.

Figure 8: Book Coverage of Inflation



Notes: The blue dashed lines show the frequency of the words “inflation” and “inflación”, respectively, in English and Spanish books, according to Google Books Ngram Viewer. The black solid line shows the corresponding inflation rates.

## D Robustness

In this section, I show that the empirical results are robust along several dimensions.

### Additional Data Sources

In Table 6, I show how attention changed over time for different data sources. The first two columns show the results for the Greenbook forecasts, columns 3-4 for the Livingston Survey, and columns 5-6 and 7-8 are for CPI forecasts from the SPF instead of forecasts about the GDP deflator. As in the main text, I use two different estimators. First, the Blundell-Bond estimator (columns 5-6) and pooled OLS (columns 7-8). All standard errors are robust with respect to heteroskedasticity and serial correlation. We see that the main finding of lower attention in inflation expectations in the period after 1990 compared to the period before is robust to these changes in the data source and/or exact variable.

Table 6: Regression Results of Equation (4)

	Greenbook		Livingston		SPF CPI BB		SPF CPI OLS	
	< 1990	≥ 1990	< 1990	≥ 1990	< 1990	≥ 1990	< 1990	≥ 1990
$\hat{\gamma}$	0.39	0.24	0.28	0.17	0.36	0.23	0.17	0.13
s.e.	(0.0851)	(0.0715)	(0.0554)	(0.0624)	(0.1444)	(0.0328)	(0.0409)	(0.0142)
$N$	84	100	83	61	550	3,577	550	3,577

Note: This table shows the results from regression (4) for different data sources. The standard errors, reported in parentheses, are robust with respect to heteroskedasticity and serial correlation.

## Different Sample Splits

Table 1 in the main body of the paper shows that attention to inflation declined by focusing on a sample split in 1990. To show that this is robust to the exact split point, Tables 7 and 8 show that the result holds when splitting the sample in 1985 or 1995, respectively. In fact, the decline in attention is even somewhat more pronounced when splitting the sample in 1985. This is in line with the theoretical prediction of the limited-attention model. Namely, the period between 1985 and 1990 was a period of relatively low and stable inflation compared to the period pre 1985 (see Figure 6), and thus, a period in which the model would predict a relatively low level of attention.

Table 7: Regression Results of Equation (4), pre 1985 vs. post 1985

	Survey of Professional Forecasters				Survey of Consumers			
	Blundell Bond		Pooled OLS		Averages		Median	
	< 1985	≥ 1985	< 1985	≥ 1985	< 1985	≥ 1985	< 1985	≥ 1985
$\hat{\gamma}$	0.75	0.37	0.45	0.25	0.77	0.31	0.50	0.26
s.e.	(0.1247)	(0.0399)	(0.0403)	(0.0338)	(0.1688)	(0.0811)	(0.0955)	(0.0561)
$N$	1914	3887	1914	3887	64	140	27	140

Note: This table shows the results from regression (4) for professional forecasters (SPF) as well as for consumers. For the SPF, I use the [Blundell and Bond \(1998\)](#) (BB) estimator (first two columns), as well as pooled OLS (columns 3-4). For the Survey of Consumer, I consider average expectations (columns 5-6) and median expectations (columns 7-8). The standard errors, reported in parentheses, are robust with respect to heteroskedasticity and serial correlation.

Table 8: Regression Results of Equation (4), pre 1995 vs. post 1995

	Survey of Professional Forecasters				Survey of Consumers			
	Blundell Bond		Pooled OLS		Averages		Median	
	< 1995	≥ 1995	< 1995	≥ 1995	< 1995	≥ 1995	< 1995	≥ 1995
$\hat{\gamma}$	0.70	0.41	0.44	0.21	0.72	0.27	0.43	0.22
s.e.	(0.0907)	(0.0654)	(0.0379)	(0.0344)	(0.1473)	(0.0962)	(0.0819)	(0.0654)
$N$	2708	3093	2708	3093	104	100	67	100

Note: This table shows the results from regression (4) for professional forecasters (SPF) as well as for consumers. For the SPF, I use the [Blundell and Bond \(1998\)](#) (BB) estimator (first two columns), as well as pooled OLS (columns 3-4). For the Survey of Consumer, I consider average expectations (columns 5-6) and median expectations (columns 7-8). The standard errors, reported in parentheses, are robust with respect to heteroskedasticity and serial correlation.



## Different Specifications of the BB Estimator

In the baseline estimation, reported in Table 1, I included all potential lags for the Blundell-Bond estimation. To show that the results are robust to this specification, I show in Table 9 that for maximum lag lengths of 20 and 10 periods, the estimated attention parameter  $\hat{\gamma}$  is in all cases higher before 1990 compared to the period after 1990.

Table 9: Different Maximum Lag Lengths

	All Lags		20 Lags		10 Lags	
	< 1990	$\geq$ 1990	< 1990	$\geq$ 1990	< 1990	$\geq$ 1990
$\hat{\gamma}$	0.70	0.41	0.74	0.51	0.84	0.69
s.e.	(0.1005)	(0.0522)	(0.1086)	(0.0632)	(0.1247)	(0.1127)
$N$	2235	3566	2235	3566	2235	3566

Note: This table shows the results from regression (4) for different numbers of lags included in the BB estimation. The standard errors, reported in parentheses, are robust with respect to heteroskedasticity and serial correlation.

## Time Fixed Effects

To account for potential changes in trend inflation, I include time-fixed effects in regression (4). To do so, recall that (4) is given by

$$\pi_{t+1|t,i}^e = \beta_i + \beta_1 \pi_{t|t-1,i}^e + \beta_2 (\pi_t - \pi_{t|t-1,i}^e) + u_{i,t}. \quad (30)$$

To include time fixed effects, I first compute a period-specific persistence parameter,  $\rho_\pi$ . Note, that in (30),  $\beta_1$  measures this persistence. Therefore, I subtract  $\hat{\rho}_\pi \pi_{t|t-1,i}^e$  from both sides and then to directly estimate  $\gamma$ , I further divide both sides by  $\hat{\rho}_\pi$ :

$$\frac{\pi_{t+1|t,i}^e - \hat{\rho}_\pi \pi_{t|t-1,i}^e}{\hat{\rho}_\pi} = \delta_i + d_t + \gamma (\pi_t - \pi_{t|t-1,i}^e) + v_{i,t}, \quad (31)$$

where  $d_t$  captures time-fixed effects,  $\delta_i = \frac{\beta_i}{\hat{\rho}_\pi}$  and  $v_{i,t} = \frac{u_{i,t}}{\hat{\rho}_\pi}$ . I do this transformation for the period before and after 1990 separately. Note, that this transformation also deals with the endogeneity problem explained in Section 2.

The estimated attention levels are 0.75 (s.e. 0.0327) for the period before 1990 and 0.61 (s.e. 0.0295) after 1990 if I use the first-order autocorrelation of expected inflation as my measure of  $\rho_\pi$ . If I use the estimate of  $\beta_1$  from equation (31) as my measure of  $\rho_\pi$ , the estimated attention before the 1990s is 0.68 (s.e. 0.0252) and the one after the 1990s is 0.46 (s.e. 0.0242). Thus, we see that the decrease in attention is robust to controlling for time-fixed effects, even though the decline is somewhat muted.

When using the first-order autocorrelation of expected inflation as my measure of  $\rho_\pi$ , estimating equation (5) in this way, delivers a point estimate of 0.06 (s.e. 0.0111) that is statistically significant on all conventional significance levels. The estimate for  $\zeta$  in regression (6) is 0.23 (s.e. 0.0220), statistically significant on all conventional significance levels. When using  $\hat{\beta}_1$  from (31) as the measure of  $\rho_\pi$ , the point estimate of  $\beta$  in equation (5) is 0.06 (s.e. 0.0074) and the estimate of  $\zeta$  in (6) is 0.29 (s.e. 0.0306), both statistically significant on all conventional levels of significance. Thus, the positive relationships between attention and volatility, as well as between attention and inflation persistence, are robust to controlling for time fixed effects.

## Professional Forecasters in the Aggregate

When estimating attention of professional forecasters' average expectations instead of individual ones, we obtain a value of 0.24 (s.e. 0.0481) for the period before 1990 and of 0.09 (s.e. 0.0353) after 1990. Consistent with the main results, attention substantially decreased in recent decades and is about half after 1990 compared to before.

Estimating regression (5) on aggregate SPF data delivers a coefficient of 0.15 ( $p$ -value of 0.000) and the estimate of  $\zeta$  in regression (6) is 0.69 ( $p$ -value of 0.000). Thus, the results reported in the main text are robust.

## Joint Regressions

Instead of running regressions (5) and (6) separately, I estimate

$$\hat{\gamma}_t = \alpha + \beta \hat{\sigma}_{\pi,t} + \zeta \hat{\rho}_{\pi,t} + u_t. \quad (32)$$

Table 10 shows that the results are robust to this change in specification.

Table 10: Attention, Inflation Volatility and Inflation Persistence

Estimator	Survey of Professional Forecasters		Michigan Survey
	Blundell-Bond	Pooled OLS	OLS
$\hat{\beta}$	0.04***	0.05***	0.06***
s.e.	(0.0153)	(0.0128)	(0.0150)
$\hat{\zeta}$	0.59***	0.65***	0.31***
s.e.	(0.0597)	(0.0499)	(0.0772)
$N$	165	165	163

Note: This table shows the results of regression (32). Standard errors are robust with respect to heteroskedasticity. \*\*\* :  $p$ -value < 0.01, \*\* :  $p$ -value < 0.05, \* :  $p$ -value < 0.1.

## Quasi-Panel of Consumers

The Survey of Consumers does not follow consumers over time. Therefore, I could not allow for individual-specific fixed effects but rather consider average and/or median inflation expectations. I now group the survey respondents into four groups, based on their income. The SoC provides data on this starting in the last quarter of 1979.

Table 11 shows the results. The first two columns report the results for the split point in 1990, and the third and fourth column for the split point in 1995. We see that the estimated attention levels using this quasi panel are similar to the ones obtained using average expectations (Table 1).

Table 11: Regression Results of Equation (4), Quasi-Panel

	Survey of Consumers			
	< 1990	≥ 1990	< 1995	≥ 1995
$\hat{\gamma}$	0.77	0.33	0.70	0.29
s.e.	(0.0933)	(0.0263)	(0.1078)	(0.0289)
$N$	160	480	240	400

Note: This table shows the results from regression (4), estimated using the [Blundell and Bond \(1998\)](#) estimator, for consumers grouped into four groups, based on their income. The standard errors, reported in parentheses, are robust with respect to heteroskedasticity and serial correlation.

Table 12 shows the results of regressions (5) and (6) (first column), as well as of the joint regression (32), using this quasi panel of consumers. We see that the results are robust and that there is indeed a significantly positive relation between attention and inflation volatility, as well as between attention and inflation persistence.

Table 12: Attention, Inflation Volatility and Inflation Persistence

Estimator	Survey of Consumers	
	Separate	Joint
$\hat{\beta}$	0.13***	0.13***
s.e.	(0.0106)	(0.0126)
$\hat{\zeta}$	0.20***	0.12***
s.e.	(0.0787)	(0.0620)
$N$	121	121

Note: This table shows the results of regressions (5), (6) (first column) and (32) (second column) using a quasi panel of consumers. The attention parameters have been estimated using the BB-estimator. Standard errors are robust with respect to heteroskedasticity. \*\*\* :  $p$ -value < 0.01, \*\* :  $p$ -value < 0.05, \* :  $p$ -value < 0.1.

## Volatility and Persistence of Inflation Expectations

Table 13 shows the results of regressions (5) and (6) using the volatility and persistence of inflation expectations instead of actual inflation as independent variables. Standard errors are robust with respect to heteroskedasticity.

Table 13: Attention, Inflation Volatility and Inflation Persistence

Estimator	Survey of Professional Forecasters		Michigan Survey
	Blundell-Bond	Pooled OLS	OLS
$\hat{\beta}$	0.14***	0.16***	0.13***
s.e.	(0.0153)	(0.0098)	(0.0172)
$\hat{\zeta}$	1.06***	1.21***	0.23***
s.e.	(0.1272)	(0.0838)	(0.0685)
$N$	165	165	163

Note: This table shows the results of regressions 5 and 6 using the volatility and persistence of inflation expectations instead of actual inflation as dependent variables. Standard errors are robust with respect to heteroskedasticity. \*\*\* :  $p$ -value < 0.01, \*\* :  $p$ -value < 0.05, \* :  $p$ -value < 0.1.

## Window Length

As predicted by the underlying model of optimal information acquisition, I showed that there is indeed a positive relationship between attention to inflation and inflation volatility, as well as between attention and inflation persistence. In the baseline specification, I relied on a rolling-window approach in which every window was 10 years. Tables 14 and 15 show that these results are robust to using different window lengths, namely 5 and 15 years.

Table 14: Attention, Inflation Volatility and Inflation Persistence

Estimator	Survey of Professional Forecasters		Michigan Survey
	Blundell-Bond	Pooled OLS	OLS
$\hat{\beta}$	-0.01	0.06***	0.13***
s.e.	(0.1643)	(0.0185)	(0.0411)
$\hat{\zeta}$	0.73	0.44***	0.40***
s.e.	(0.6731)	(0.0551)	(0.1547)
$N$	185	185	183

Note: This table shows the results of regression (32) using windows of 5 years each. Standard errors are robust with respect to heteroskedasticity. \*\*\* :  $p$ -value < 0.01, \*\* :  $p$ -value < 0.05, \* :  $p$ -value < 0.1.

Table 15: Attention, Inflation Volatility and Inflation Persistence

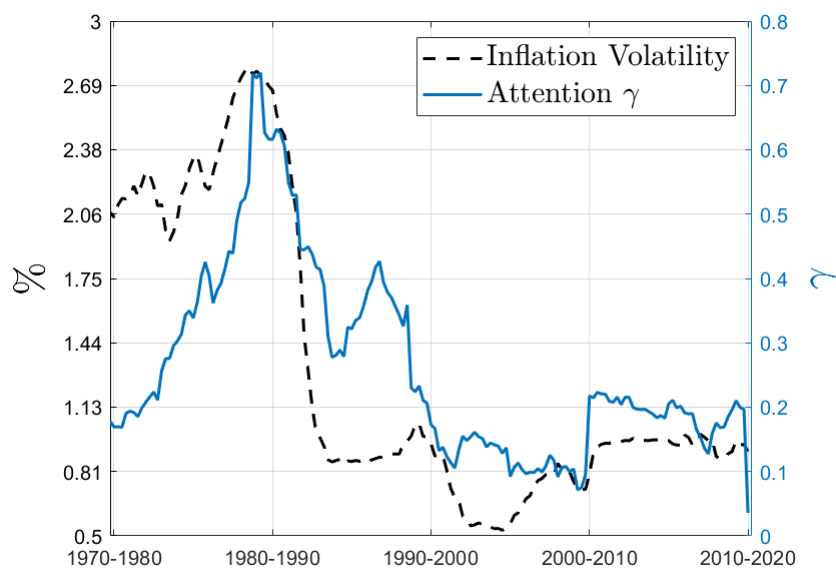
Estimator	Survey of Professional Forecasters		Michigan Survey
	Blundell-Bond	Pooled OLS	OLS
$\hat{\beta}$	0.01	0.01	0.07***
s.e.	(0.0124)	(0.0115)	(0.0136)
$\hat{\zeta}$	0.90***	1.00***	0.43***
s.e.	(0.0603)	(0.0552)	(0.0706)
$N$	145	145	143

Note: This table shows the results of regression (32) using windows of 15 years each. Standard errors are robust with respect to heteroskedasticity. \*\*\* :  $p$ -value < 0.01, \*\* :  $p$ -value < 0.05, \* :  $p$ -value < 0.1.

## Attention over Time

Figure 9 shows the estimated attention levels,  $\gamma$ , (black-solid line) from the SPF consensus forecasts, together with the volatility of GDP deflator inflation (blue-dashed lines). We clearly see the aforementioned decrease in attention over time, as well as the positive correlation of attention and inflation volatility.

Figure 9: Attention and Inflation Volatility over Time



Notes: This figure shows the estimated attention levels,  $\gamma$ , (black-solid line) from the SPF consensus forecasts, together with the volatility of GDP deflator inflation (blue-dashed lines).

## E More on Forward Guidance and Attention

In Section 3, I discussed how lower attention weakens the effectiveness of forward guidance. For the sake of clarity, I focused on the case  $\rho_\pi = 1$ . In this section, I now show that all the results go through when relaxing this assumption and also discuss how forward guidance via the nominal (instead of the real) interest rate changes the results and I also allow for attention heterogeneity across firms and households. We consider the same stylized experiment but now the law of motion for inflation expectations is given by

$$\pi_{1|0}^e = (1 - \rho_\pi)\bar{\pi} + \rho_\pi(1 - \gamma)\pi_{0|-1}^e + \rho_\pi\gamma\pi_0,$$

which can be substituted into the Phillips Curve:

$$\pi_0 = \frac{\beta}{1 - \beta\rho_\pi\gamma} ((1 - \rho_\pi)\bar{\pi} + \rho_\pi(1 - \gamma)\pi_{0|-1}^e) + \frac{\kappa}{1 - \beta\rho_\pi\gamma} y_0^{gap}.$$

Thus, inflation expectations are given by

$$\pi_{1|0}^e = \frac{1 - \rho_\pi}{1 - \beta\rho_\pi\gamma} \bar{\pi} + \frac{\rho_\pi(1 - \gamma)}{1 - \beta\rho_\pi\gamma} \pi_{0|-1}^e + \frac{\kappa\rho_\pi\gamma}{1 - \beta\rho_\pi\gamma} y_0^{gap}.$$

Putting everything together, we arrive at the following Proposition.

**Proposition 3** *The output gap in the period when the shock hits,  $t = 0$ , is given by*

$$\begin{aligned} y_0^{gap} = & -\frac{\varphi(1 - \beta\rho_\pi\gamma)}{1 - \rho_\pi\gamma(\beta + \varphi\kappa)} [-\underline{i} - r_0^n + r_1] \\ & + \frac{\varphi}{1 - \rho_\pi\gamma(\beta + \varphi\kappa)} [(1 - \rho_\pi)\bar{\pi} + \rho_\pi(1 - \gamma)\pi_{0|-1}^e] \end{aligned}$$

and inflation in  $t = 0$  is given by

$$\begin{aligned} \pi_0 = & -\frac{\kappa\varphi}{1 - \rho_\pi\gamma(\beta + \varphi\kappa)} [-\underline{i} - r_0^n + r_1] + (1 - \rho_\pi) \left[ \frac{\beta}{1 - \beta\rho_\pi\gamma} + \frac{\varphi}{1 - \rho_\pi\gamma(\beta + \varphi\kappa)} \right] \bar{\pi} \\ & + \rho_\pi(1 - \gamma) \left[ \frac{\beta}{1 - \beta\rho_\pi\gamma} + \frac{\varphi}{1 - \rho_\pi\gamma(\beta + \varphi\kappa)} \right] \pi_{0|-1}^e. \end{aligned} \tag{33}$$

Proposition 3 captures the effectiveness of forward guidance on the output gap and inflation in the period when the shock hits. The assumption that  $(1 - \rho_\pi\gamma(\beta + \varphi\kappa))$  is positive, makes sure that forward guidance, i.e, a lower  $r_1$  has a stimulating effect on output and inflation in  $t = 0$ . Proposition 3 captures several channels how a change in attention affects the economy's response to forward guidance, which I now collect in a series of corollaries.



**Corollary 3** *Lower attention*

- (i) *weakens the negative effect of the shock on impact,*
- (ii) *weakens the effects of forward guidance on the output gap and inflation,*
- (iii) *weakens the stimulative effects of a decrease in the lower bound  $-\underline{i}$ .*

Corollary 3 follows from the fact that the terms  $\frac{\varphi(1-\beta\rho_\pi\gamma)}{1-\rho_\pi\gamma(\beta+\varphi\kappa)}$  and  $\frac{\kappa\varphi}{1-\rho_\pi\gamma(\beta+\varphi\kappa)}$  in front of  $[-\underline{i} - r_0^n + r_1]$  are both increasing in  $\gamma$ . Points (i) and (ii) capture the main trade off of lower attention. While lower attention has a stabilizing effect via more anchored inflation expectations (point (i)), it renders forward guidance less effective (point (ii)). The reason why forward guidance becomes less effective as attention declines is because inflation expectations increase less in response to the announced policy, and thus, the real rate remains higher. Point (iii) illustrates an additional drawback of lower attention. A reduction of the effective lower bound,  $-\underline{i}$ , is less stimulating if agents in the economy are less attentive. Thus, going from a zero lower bound to a lower bound in negative territory, as conducted in several advanced economies over the last ten years, becomes less effective in terms of stimulating output and inflation if the public is inattentive. Note, that a decrease in the perceived inflation persistence,  $\rho_\pi$ , has the exact same implications as a decrease in  $\gamma$ .

The next corollary discusses how changes in attention affect the role of long-run inflation beliefs on the output gap and inflation.

**Corollary 4** *Lower attention weakens the positive effects of higher long-run inflation beliefs  $\bar{\pi}$  on output and inflation,*

Corollary 4 says that higher long-run beliefs have a positive effect on inflation and the output gap, but lower attention weakens these effects. However, as long as  $\gamma(\beta + \varphi\kappa) < 1$ , a higher  $\rho_\pi$  mutes the effects of  $\bar{\pi}$  on the output gap. Since this condition is usually satisfied and because  $\rho_\pi$  is in general close to 1, the role of high long-run inflation beliefs is quite weak. In the limit case  $\rho_\pi \rightarrow 1$ , long-run beliefs become irrelevant.

How attention matters for the transmission of prior inflation expectations on the output gap and inflation is ambiguous, as the following Corollary shows.

**Corollary 5** *Lower attention*

- (i) *weakens the positive effect of higher prior inflation beliefs,  $\pi_{0|-1}^e$ , on the output gap if and only if,*

$$\rho_\pi(\beta + \varphi\kappa) > 1, \quad (34)$$

(ii) weakens the positive effect of higher prior inflation beliefs on inflation if and only if

$$\frac{\rho_\pi \beta (\rho_\pi \beta - 1)}{(1 - \beta \rho_\pi \gamma)^2} + \frac{\rho_\pi \varphi (\rho_\pi (\beta + \varphi \kappa) - 1)}{(1 - \rho_\pi \gamma (\beta + \varphi \kappa))^2} > 0. \quad (35)$$

Overall, the role of attention for the effects of higher prior beliefs on output and inflation is ambiguous. This is mainly the case because, on the one hand, lower attention implies that agents put more weight on their prior beliefs. On the other hand, as discussed previously, lower attention leads to more stable inflation overall, thus, weakening the effects of prior beliefs. This can also be seen in the discussion of the Phillips Curve, see Proposition 1.

Given the calibration in Table 4, conditions (34) and (35) both hold. The effects of changes in  $\gamma$ , however, are numerically small. Thus, an increase in the average inflation rate—which also increases average prior beliefs—is a promising monetary instrument to combat the loss of control via forward guidance as attention declines. By *ex-ante* increasing the average inflation rate, the policymaker not only supports higher inflation expectations and thus, lower real rates for a given nominal rate, but also gains additional policy space through the increase in the average nominal rate. Higher average inflation, however, is also costly. In the analysis of optimal policy, later on, I will explore this trade off and characterize the optimal inflation target for different levels of attention.

**Forward Guidance via Nominal Interest Rates** So far, forward guidance was characterized as a promise to keep the *real* rate low. Now, assume that forward guidance is conducted via promising lower *nominal* rates instead. Thus,  $i_1$  will be fixed across different  $\gamma$ . For simplicity, I focus on the case with  $\rho_\pi = 1$  and  $\pi_{0|-1}^e = 0$ . It follows from the Euler equation in  $t = 1$  that

$$y_1^{gap} = -\varphi (i_1 - (1 - \gamma) \gamma \pi_0 - \gamma \pi_1).$$

The Phillips Curve in  $t = 1$  yields

$$\pi_1 = \frac{(1 - \gamma) \gamma}{1 - \beta \gamma} \pi_0 + \frac{\kappa}{1 - \beta \gamma} y_1^{gap},$$

so that we get an expression for  $y_1^{gap}$  in terms of  $\pi_0$ :

$$y_1^{gap} = -\frac{\varphi (1 - \beta \gamma)}{1 - \gamma (\beta + \varphi \kappa)} i_1 + \varphi (1 - \gamma) \gamma \frac{1 + \gamma (1 - \beta)}{1 - \gamma (\beta + \varphi \kappa)} \pi_0. \quad (36)$$

Given  $\pi_{1|0}^e = \gamma\pi_0$ , the Phillips Curve in  $t = 0$  yields

$$\pi_0 = \frac{\kappa}{1 - \beta\gamma} y_0^{gap},$$

and hence,  $\pi_{1|0}^e = \frac{\kappa\gamma}{1 - \beta\gamma} y_0^{gap}$ . Plugging this into the Euler equation in  $t = 0$  gives

$$y_0^{gap} = \mathbb{E}_0 y_1^{gap} - \varphi \left( -\underline{i} - \frac{\kappa\gamma}{1 - \beta\gamma} y_0^{gap} - r_0^n \right).$$

Solving for  $y_0^{gap}$  leads to the following Lemma.

**Lemma 3** *Forward guidance via the nominal interest rate yields the following output gap*

$$y_0^{gap} = A_1 \left[ -\frac{\varphi(1 - \beta\gamma)}{1 - \gamma(\beta + \varphi\kappa)} i_1 - \varphi(-\underline{i} - r_0^n) \right], \quad (37)$$

and inflation

$$\pi_0 = \frac{\kappa}{1 - \beta\gamma} A_1 \left[ -\frac{\varphi(1 - \beta\gamma)}{1 - \gamma(\beta + \varphi\kappa)} i_1 - \varphi(-\underline{i} - r_0^n) \right], \quad (38)$$

where

$$A_1 \equiv \frac{1}{1 - \varphi(1 - \gamma)\gamma \frac{1 + \gamma(1 - \beta)}{1 - \gamma(\beta + \varphi\kappa)} \frac{\kappa}{1 - \beta\gamma} - \frac{\varphi\kappa\gamma}{1 - \beta\gamma}}. \quad (39)$$

Given the calibration in Table 4,  $A_1$  is positive and increasing in  $\gamma$ . Thus, promising lower future nominal interest rates can indeed stimulate the economy. But similar to the case in which the policy maker commits to a certain future *real* rate, forward guidance becomes less effective when agents are less attentive. In fact, all three results from Corollary 3 go through.

Recall equation (36):

$$y_1^{gap} = -\frac{\varphi(1 - \beta\gamma)}{1 - \gamma(\beta + \varphi\kappa)} i_1 + \varphi(1 - \gamma)\gamma \frac{1 + \gamma(1 - \beta)}{1 - \gamma(\beta + \varphi\kappa)} \pi_0.$$

Note, that the first term becomes less negative as  $\gamma$  declines. Given the calibration in Table 4, also the second term decreases as attention declines. Thus, for a given  $\pi_0$ , a particular  $i_1$  has weaker effects on the output gap in  $t = 1$  at lower levels of attention. Since lower attention also weakens the positive effects of forward guidance on  $\pi_0$ , the output gap (and inflation) stay lower also in  $t = 1$ .

Since inflation in  $t = 0$  and  $t = 1$  is lower at smaller values of  $\gamma$ , also  $\pi_{2|1}^e$  will be lower and thus, for a given nominal rate  $i_1$ , the *real* rate,  $r_1 \equiv i_1 - \pi_{2|1}^e$ , will be higher. Hence, to achieve a certain forward guidance in terms of the real interest rate, the promise in terms of the nominal rate needs to be larger when firms and households are inattentive. Combining

this with the findings on the effectiveness of forward guidance via the *real* rate (Proposition 3) shows how lower attention renders forward guidance less powerful *even though* the promise in terms of the *nominal* rate is stronger.

## E.1 Heterogeneous Attention

So far, I assumed that firms and households are equally attentive. But what if firms and households differ in their attention to inflation? Let us denote firms' attention by  $\gamma_F$  and households' attention by  $\gamma_H$  with  $\gamma_F \neq \gamma_H$ . For clarity, I focus on the case with  $\rho_\pi = 1$  and  $\pi_{0|-1}^{e,j} = 0$  for  $j \in \{F, H\}$ .

**Lemma 4** *With heterogeneous attention to inflation, the output gap in  $t = 0$  is given by*

$$y_0^{gap} = \frac{-\varphi(1 - \beta\gamma_F)}{1 - \beta\gamma_F - \kappa\varphi\gamma_H} [-\underline{i} + r_1 - r_0^n], \quad (40)$$

and inflation by

$$\pi_0 = \frac{-\varphi\kappa}{1 - \beta\gamma_F - \kappa\varphi\gamma_H} [-\underline{i} + r_1 - r_0^n], \quad (41)$$

where  $r_1 \equiv i_1 - \pi_{2|1}^{e,H}$  is the real rate given the households' expectations.

Lemma 4 shows that a similar result as in Corollary 3 holds under heterogeneous attention levels.

**Corollary 6** *Lower attention of either firms or households*

- (i) *weakens the negative effect of the shock on the output gap and inflation on impact,*
- (ii) *weakens the effects of forward guidance on the output gap and inflation,*
- (iii) *weakens the stimulative effects of a decrease in the lower bound  $-\underline{i}$  on the output gap and inflation.*

The parts concerning the output gap in Corollary 6 follow because the term in front of the brackets in equation (40) becomes more negative as either of  $\{\gamma_F, \gamma_H\}$  increases:

$$\begin{aligned} \frac{\partial \left[ \frac{-\varphi(1 - \beta\gamma_F)}{1 - \beta\gamma_F - \kappa\varphi\gamma_H} \right]}{\partial \gamma_F} &= -\frac{\beta\kappa\varphi^2\gamma_H}{(1 - \beta\gamma_F - \kappa\varphi\gamma_H)^2} < 0 \\ \frac{\partial \left[ \frac{-\varphi(1 - \beta\gamma_F)}{1 - \beta\gamma_F - \kappa\varphi\gamma_H} \right]}{\partial \gamma_H} &= -\frac{\varphi^2(1 - \beta\gamma_F)}{(1 - \beta\gamma_F - \kappa\varphi\gamma_H)^2} < 0, \end{aligned}$$

and the parts concerning inflation because the term  $\frac{-\varphi\kappa}{1-\beta\gamma_F-\kappa\varphi\gamma_H}$  in equation (41) becomes more negative as either of  $\{\gamma_F, \gamma_H\}$  increases, too.

Thus, if either firms or households (or both) become less attentive, forward guidance becomes less effective. In fact, the two degrees of attention reinforce each other, as the following Corollary shows.

**Corollary 7** *Lower levels of households' attention to inflation weaken the effectiveness of forward guidance, especially when firms' attention to inflation is low, and vice-versa.*

To see this, note that

$$\begin{aligned}\frac{\partial^2 \left[ -\frac{\varphi\kappa}{1-\beta\gamma_F-\kappa\varphi\gamma_H} \right]}{\partial\gamma_F\partial\gamma_H} &= \frac{-2\varphi^2\kappa^2\beta}{(1-\beta\gamma_F-\kappa\varphi\gamma_H)^3} < 0, \\ \frac{\partial^2 \left[ \frac{-\varphi(1-\beta\gamma_F)}{1-\beta\gamma_F-\kappa\varphi\gamma_H} \right]}{\partial\gamma_F\partial\gamma_H} &= \frac{-\beta\kappa\varphi^2 [1-\beta\gamma_F+\kappa\varphi\gamma_H]}{(1-\beta\gamma_F-\kappa\varphi\gamma_H)^3} < 0.\end{aligned}$$

## F Proof of Proposition 1

**Proof.** The New Keynesian Phillips Curve is given by

$$\pi_t = \beta \pi_{t+1|t}^e + \kappa y_t^{gap} + u_t.$$

Substituting

$$\pi_{t+1|t}^e = \pi_{t|t-1}^e + \gamma (\pi_t - \pi_{t|t-1}^e)$$

for  $\pi_{t+1|t}^e$  yields

$$\begin{aligned} \pi_t &= \beta (\pi_{t|t-1}^e + \gamma (\pi_t - \pi_{t|t-1}^e)) + \kappa y_t^{gap} + u_t \\ \Leftrightarrow \pi_t (1 - \beta\gamma) &= \beta \pi_{t|t-1}^e (1 - \gamma) + \kappa y_t^{gap} + u_t \\ \Leftrightarrow \pi_t &= \frac{\beta \pi_{t|t-1}^e (1 - \gamma) + \kappa y_t^{gap} + u_t}{(1 - \beta\gamma)} \\ \Leftrightarrow \pi_t &= \frac{\beta (1 - \gamma)}{(1 - \beta\gamma)} \pi_{t|t-1}^e + \frac{\kappa}{(1 - \beta\gamma)} y_t^{gap} + \frac{u_t}{(1 - \beta\gamma)}. \end{aligned}$$

Now, taking derivatives with respect to  $y_t^{gap}$ ,  $u_t$ , and  $\pi_{t|t-1}^e$ , respectively, yields the results (i), (ii), and (iii). ■

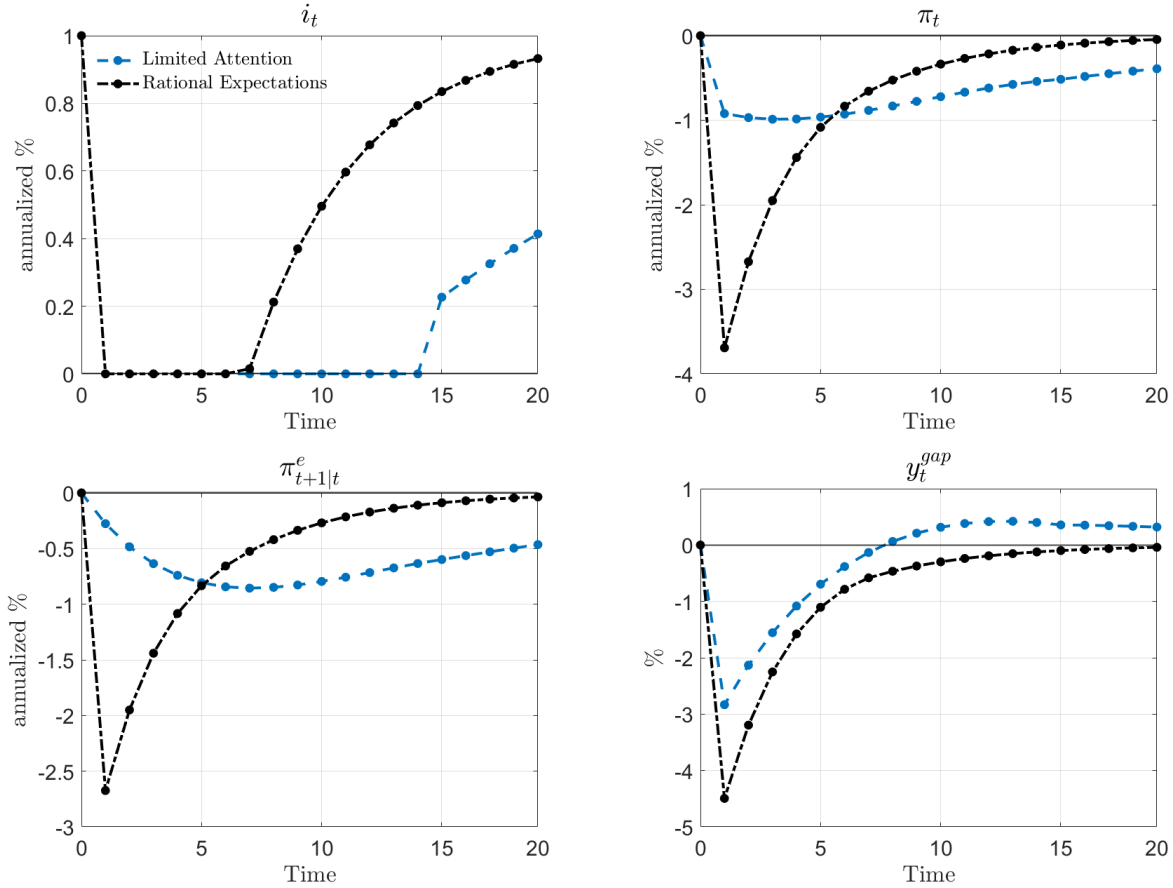
## G Additional Model Insights

### G.1 Different Taylor Rule

To show that the exact specification of the Taylor rule is not essential for the occurrence of inflation-attention traps, Figure 10 shows the impulse-response functions of the nominal interest rate, inflation, inflation expectations and the output gap for the model in which the Taylor rule absent the ELB is given by

$$i_t = 1.5\pi_t. \quad (42)$$

Figure 10: IRFs to Natural Rate Shock for Taylor rule (42)

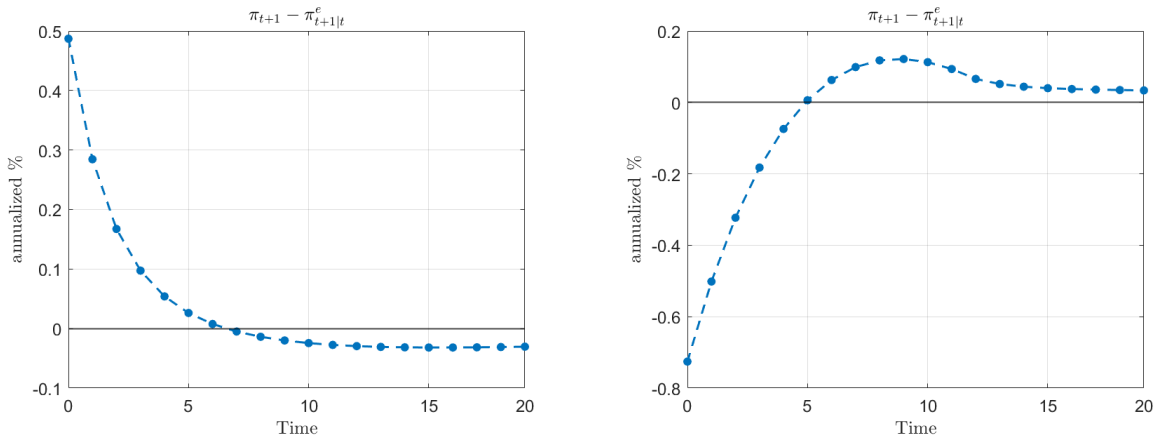


Note: This figure shows the impulse-response functions of the nominal interest rate (upper-left panel), inflation (upper-right panel), inflation expectations (lower-left) and the output gap (lower-right) to a negative natural rate shock of three standard deviations. The blue-dashed lines show the case for the limited-attention model and the black-dashed-dotted lines for the rational expectations model. Everything is in terms of percentage deviations from the respective steady state levels, except the nominal rate is in levels.

## G.2 Forecast Errors

[Angeletos et al. \(2020\)](#) propose a new test of models that deviate from FIRE. Namely, that expectations should initially underreact but overshoot eventually. A straightforward way to test this is to look at the model-implied impulse response functions of the forecast error,  $\pi_{t+1} - \pi_{t+1|t}^e$ , to an exogenous shock. Figure 11 shows these IRFs. The left panel shows the IRF of the forecast error after a positive natural rate shock and the right panel shows the corresponding IRF to a negative natural rate shock. In both cases, we see an underreaction in expectations, which manifests itself in a positive forecast error after a shock that increases the forecasted variable, and vice-versa following a negative shock. After about 5-6 periods, the forecast error response, however, flips sign. This is exactly the eventual overreaction, mentioned above and documented in [Angeletos et al. \(2020\)](#). Thus, my model of inflation expectations matches these empirical findings.

Figure 11: Impulse Response Functions of Forecast Errors



Note: This figure shows the impulse-response functions of inflation forecast errors after a three-standard deviation positive (left) and negative (right) natural rate shock.



### G.3 No Random Walk

A potential concern with the results stated in Section 3, in particular the *inflation-attention trap* in Figure 2, is that these findings are driven by the random walk assumption in the belief process of the agents. Relaxing the random-walk assumption requires to take a stand on the perceived average inflation. In this case, where I solve the model around the zero inflation steady state, this is quite innocuous. But later on, when I focus on Ramsey optimal policy, this cannot be done anymore without distorting the results, in the sense that agents might have a mean bias.

Figure 12 shows the same impulse response functions as reported in Figure 2 for the case of  $\rho_\pi = 0.95$  and an average inflation of 0. We see a similar pattern, even though somewhat less pronounced. Inflation is persistently lower under limited attention due to slowly-adjusting inflation expectations. Expectations are updated even more sluggishly when  $\rho_\pi < 1$ . Further, this also dampens the initial response in inflation expectations, and thus, of inflation itself. Therefore, the attention trap is somewhat mitigated and the economy escapes the lower bound faster than with  $\rho_\pi = 1$ . Nevertheless, the nominal interest rate is low for longer due to the slow recovery of inflation.

**Optimal Policy with a Bias in Inflation Expectations** In the main analysis, I have assumed that agents believe that inflation follows a random walk. Under this assumption, inflation expectations and inflation coincide on average. In the following, I relax this assumption and assume that the perceived persistence parameter is less than 1,  $\rho_\pi < 1$ . As discussed earlier, this yields the following inflation-expectations formation

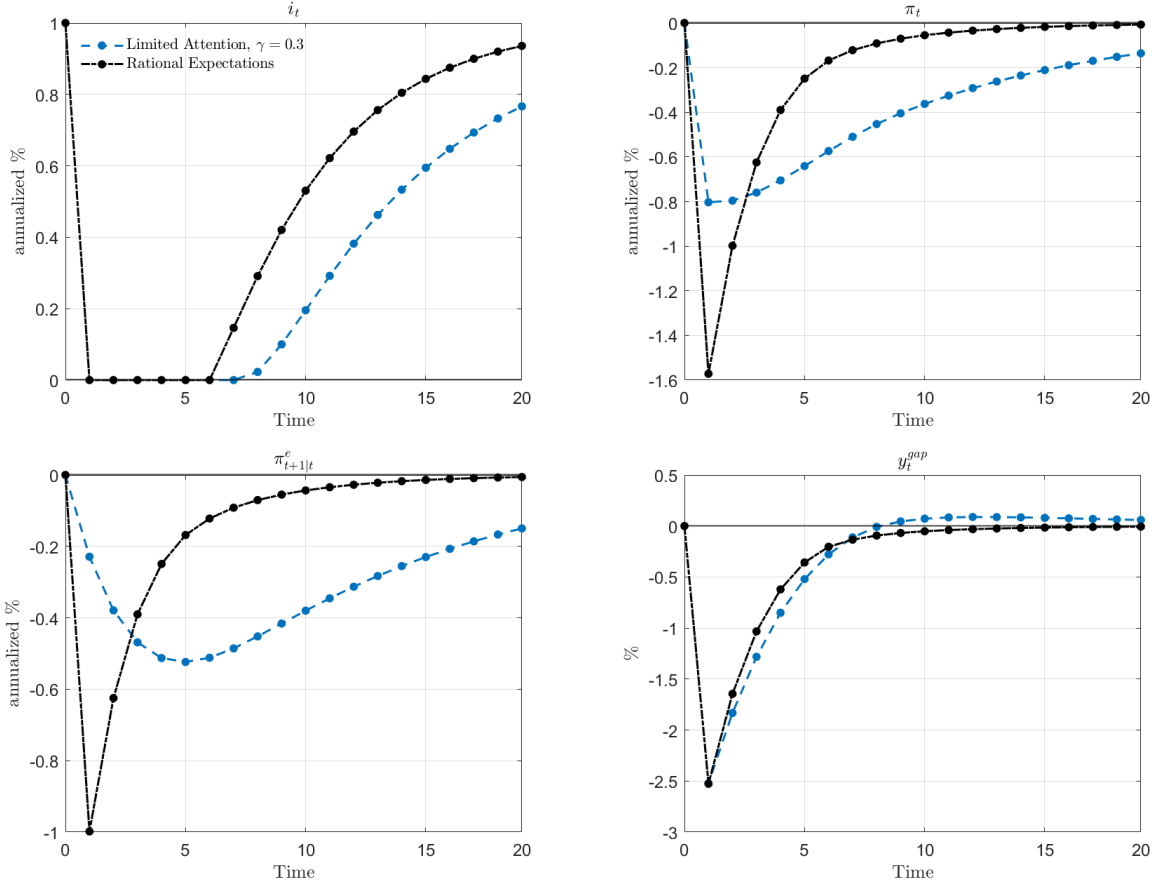
$$\pi_{t+1|t}^e = (1 - \rho_\pi)\bar{\pi} + \rho_\pi\pi_{t|t-1}^e + \rho_\pi\gamma(\pi_t - \pi_{t|t-1}^e),$$

where  $\bar{\pi}$  captures the long-run expectations of the agent. I set  $\rho_\pi = 0.95$  and compare economies with different  $\bar{\pi}$ , namely  $\bar{\pi} \in \{0\%, 2\%, 4\%\}$  (annualized).

Figure 13 shows the optimal inflation target (left panel) and welfare (17) (right panel) under Ramsey optimal policy for different levels of attention and different mean beliefs,  $\bar{\pi}$ . The blue-dashed lines show the results for the case with  $\rho_\pi = 1$  (which is the baseline case discussed above), the gray-dashed-dotted lines show the results for  $\rho_\pi = 0.95$  and  $\bar{\pi} = 0\%$ , the black-solid lines for  $\rho_\pi = 0.95$  and  $\bar{\pi} = 2\%$ , and the red-dotted lines for  $\rho_\pi = 0.95$  and  $\bar{\pi} = 4\%$ .

We see that introducing a mean bias in general leads to an increase in the optimal inflation target and additional welfare losses, independent of  $\bar{\pi}$ . This mainly comes from the fact that  $\rho_\pi$  is now below 1, which dampens the degree of updating captured by  $\gamma$ . Thus, once the

Figure 12: Impulse Response Functions to a Negative Natural Rate Shock



Note: This figure shows the impulse-response functions of the nominal interest rate (upper-right panel), inflation (upper-left panel), inflation expectations (lower-right) and the output gap (lower-left) to a negative natural rate shock of three standard deviations. The blue-dashed lines show the case for the limited-attention model and the black-dashed-dotted lines for the rational expectations model. Everything is in terms of percentage deviations from the respective steady state levels, except the nominal rate is in levels.

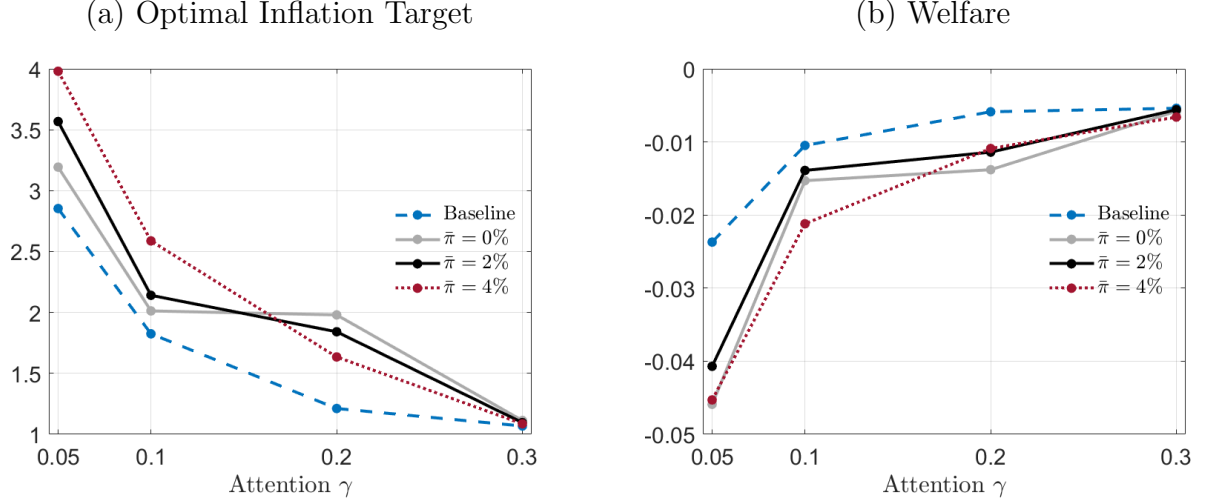
economy gets stuck at the ELB and the policymaker tries to decrease real rates by increasing inflation expectations, actual inflation needs to increase more strongly. Therefore, a lower  $\rho_\pi$  can exacerbate attention traps when they occur.

Interestingly, the relationship between the optimal target and  $\bar{\pi}$  is non-monotonic in the level of attention. While, for example, at  $\gamma = 0.2$ , the optimal target is highest at  $\bar{\pi} = 0\%$ , it is highest at  $\bar{\pi} = 4\%$  when  $\gamma = 0.05$ . To understand this, we can write the unconditional average inflation expectations as

$$\pi^e = \frac{(1 - \rho_\pi)\bar{\pi} + \rho_\pi\gamma\pi}{1 - \rho_\pi(1 - \gamma)}.$$

The following Lemma sheds light on how  $\bar{\pi}$  matters for average inflation expectations and

Figure 13: Mean Bias, Optimal Inflation Target and Welfare



Notes: This figure shows the average inflation rate under Ramsey optimal policy (left panel) and welfare (17) (right panel) under Ramsey optimal policy for different levels of attention and different mean beliefs,  $\bar{\pi}$ . The blue-dashed lines show the results for the case with  $\rho_\pi = 1$  (which is the baseline case), the gray-dashed-dotted lines show the results for  $\rho_\pi = 0.95$  and  $\bar{\pi} = 0\%$ , the black-solid lines for  $\rho_\pi = 0.95$  and  $\bar{\pi} = 2\%$ , and the red-dotted lines for  $\rho_\pi = 0.95$  and  $\bar{\pi} = 4\%$ .

how this depends on the level of attention,  $\gamma$ .

**Lemma 5** *For the case  $\rho_\pi = 1$ , average inflation expectations move one-for-one with average inflation, independent of  $\gamma$ :*

$$\pi^e = \pi.$$

*For the case  $0 < \rho_\pi < 1$ , average inflation expectations move less than one-for-one with average inflation*

$$0 < \frac{\partial \pi^e}{\partial \pi} = \frac{\rho_\pi \gamma}{1 - \rho_\pi(1 - \gamma)} < 1,$$

*and the strength of this dependency increases with  $\gamma$*

$$\frac{\partial^2 \pi^e}{\partial \pi \partial \gamma} > 0.$$

*Average inflation expectations move less than one-for-one with  $\bar{\pi}$*

$$0 < \frac{\partial \pi^e}{\partial \bar{\pi}} = \frac{(1 - \rho_\pi)}{1 - \rho_\pi(1 - \gamma)} < 1,$$

*and the strength of this dependency decreases with  $\gamma$*

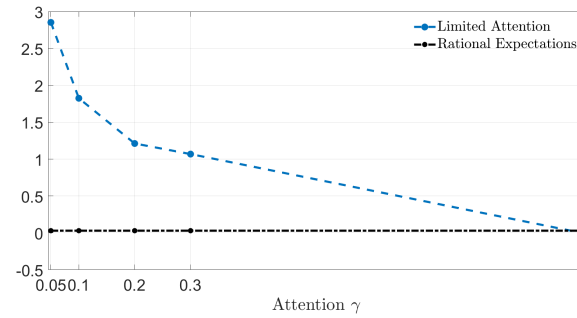
$$\frac{\partial^2 \pi^e}{\partial \bar{\pi} \partial \gamma} < 0.$$

So, as attention falls, there are several opposing forces at work. On the one hand, the effect of  $\bar{\pi}$  on average inflation expectations becomes stronger and thus, also exerts more pressure on actual inflation via the Phillips Curve. On the other hand, increasing the inflation target—average inflation—has a smaller effect on average inflation expectations at low levels of attention. Thus, to increase inflation expectations in this case, the inflation target needs to increase more strongly, which is of course costly. Comparing the optimal inflation targets in Figure 13, we see that at low levels of attention the first effect dominates. If  $\bar{\pi}$  is relatively high, the inflation target is high.

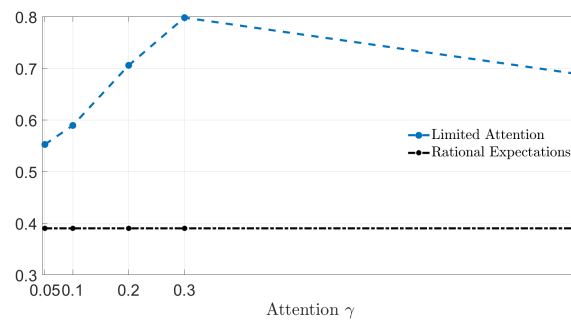
## G.4 Figures to Section 4.4

Figure 14: Full Attention,  $\gamma = 1$

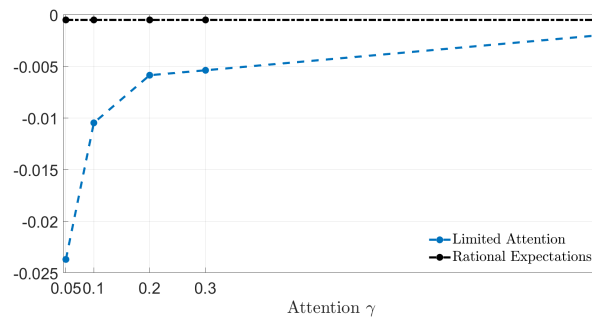
(a) Optimal Inflation Target



(b) Inflation Volatility



(c) Welfare



Notes: This figure shows the optimal inflation target (panel (a)), inflation volatility (panel (b)) and welfare (panel (c)) under Ramsey optimal policy for different levels of attention, including full attention, i.e.,  $\gamma = 1$  and compares it to the full-information rational expectations counterparts (black-dashed-dotted lines).