A Behavioral Heterogeneous Agent New Keynesian Model

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Abstract

We develop a New Keynesian model with household heterogeneity and bounded rationality in the form of cognitive discounting. The model can account for recent empirical findings on the transmission mechanisms and effectiveness of monetary and fiscal policy. In particular, the interaction of household heterogeneity and bounded rationality generates amplification of monetary and fiscal policy through indirect general equilibrium effects while simultaneously ruling out the forward guidance puzzle and remaining stable at the effective lower bound. When abstracting from either household heterogeneity or bounded rationality the model fails to do so. Our framework nests a broad range of existing models, none of which can be consistent with all these empirical facts simultaneously. Thus, our model overcomes a major shortcoming of these existing models. We use our model to revisit the policy implications of inflationary supply shocks and show that central banks have to increase interest rates more strongly than in the rational model to fully stabilize inflation. While fully stabilizing inflation keeps output at potential, higher real interest rates lead to a substantial increase in inequality and government debt. Our model thus indicates a more pronounced trade-off between aggregate efficiency and price stability on the one hand, and distributional consequences and fiscal sustainability on the other hand.

Keywords: Heterogeneous Households, Behavioral Macroeconomics, Monetary Policy, Forward Guidance, Fiscal Policy, Lower Bound, Inflation, Macroeconomic Stabilization

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1 Introduction

Recent empirical evidence sheds new light on the transmission mechanisms and the effectiveness of monetary and fiscal policy. Monetary policy is transmitted to household consumption to a large extent through indirect general equilibrium effects that tend to amplify the effectiveness of contemporaneous monetary policy. Announcements of future monetary policy changes, however, have relatively weak effects on current economic activity. Despite these weak effects of forward guidance, advanced economies have not experienced large instabilities during long spells at the binding effective lower bound. In addition, government spending has been found to increase private consumption.[1] Accounting for all these facts within one framework, however, turns out to be challenging for existing macroeconomic workhorse models.

In this paper, we propose a new framework that can account for all these facts *simultaneously*: the behavioral heterogeneous agent New Keynesian model—or *behavioral HANK model*, for short. The model features a standard New Keynesian core, but we allow for household heterogeneity and bounded rationality in the form of cognitive discounting. Key to our results is that due to the interaction of household heterogeneity and bounded rationality, aggregate demand responds less than one-for-one to expected future output whereas general equilibrium channels amplify the effects of current policy changes on aggregate demand. Both model ingredients are necessary for these patterns of aggregate demand. If we abstract from either bounded rationality or household heterogeneity, the model cannot be consistent with all these facts simultaneously. Indeed, the behavioral HANK model nests a broad range of existing models, none of which can be consistent with all the empirical facts.

We develop our framework using two complementary approaches: first, we rely on a limited-heterogeneity setup which enables us to derive all results in closed form and thus, provides a clear understanding of the role of bounded rationality, household heterogeneity and their interaction. In the second approach, we replace the limited-heterogeneity setup with a standard incomplete markets model. First, we show that all our results carry over to this quantitative behavioral HANK model. Second, we use the quantitative behavioral HANK model to revisit the monetary policy implications of inflationary supply shocks and show that, compared to rational HANK models, our model indicates a more pronounced trade-off between price stability on the one side and inequality and fiscal sustainability on

[1]See, e.g., Ampudia et al. (2018), Slacalek et al. (2020) Samarina and Nguyen (2019) and Holm et al. (2021) for the empirical relevance of indirect channels in the transmission of monetary policy, Galí et al. (2007), Perotti (2007) or Dupor et al. (2021) for empirical evidence on the positive consumption response to fiscal spending, and see, for example, Del Negro et al. (2015), D'Acunto et al. (2020), Miescu (2022) and Roth et al. (2021) for empirical evidence on the (in-)effectiveness of forward guidance and Debortoli et al. (2020) and Cochrane (2018) on the stability at the lower bound.

the other side.

To arrive at the analytically-tractable version of our model, we assume that there are two groups of households.[2] One group of households is "unconstrained", in the sense that they participate in financial markets so that they are on their Euler equation. Households in the other group consume all their disposable income in each period. These "hand-to-mouth" households exhibit high marginal propensities to consume (MPCs) and their income fluctuates more strongly with aggregate income. In addition, each household faces an idiosyncratic risk of switching from one type to the other such that unconstrained households have a precautionary savings motive with respect to becoming hand-to-mouth.

We introduce bounded rationality in the form of cognitive discounting. Households anchor their expectations about future macroeconomic variables to the steady state and cognitively discount expected future deviations from it, as introduced in a representative agent setup by Gabaix (2020). As a result, average expectations underreact to news, as we show to be the case empirically across all income groups.[3]

We can then summarize our tractable model in just three equations. The key novelty is the behavioral HANK IS equation which together with monetary policy characterizes the aggregate demand block of the economy. In contrast to the textbook Representative Agent New Keynesian (RANK) IS equation, the behavioral HANK IS equation features both discounting of future output and amplification of contemporaneous policy changes.

The reason for the latter is that hand-to-mouth households benefit disproportionately more from an increase in current output. Given their high MPCs, they spend a large fraction out of their extra income which further amplifies the increase in output. As a consequence, contemporaneous monetary policy that increases current output is amplified through indirect, general equilibrium effects. A decomposition into direct and indirect effects shows that indeed the major share of the monetary policy transmission works through indirect effects. In addition, after an increase in fiscal spending, private consumption increases even in the benchmark case of constant real interest rates.

For an expected future increase in output, these channels imply a relaxation of households' precautionary-savings motive as households expect to disproportionately benefit from the

[2]Models with a similar household structure are often referred to as TANK (Two Agent New Keynesian) models with type switching or as THANK (Tractable HANK) models (Bilbiie (2021)). To have only one name for our framework throughout the paper (for the tractable and the quantitative model), we simply refer to our our framework as behavioral HANK model.

[3]We show how to microfound cognitive discounting as a noisy-signal extraction problem of otherwise rational agents. Angeletos and Lian (2022) show how other forms of bounded rationality or lack of common knowledge can lead to observationally-equivalent outcomes. For further evidence on the underreaction of aggregate expectations to news, see, for example, Coibion and Gorodnichenko (2015), Bordalo et al. (2020) and Angeletos et al. (2021).

increase in output in the hand-to-mouth state. Under rational expectations, this would lead to compounding in the IS equation, i.e., a higher sensitivity towards expected future output (see Werning (2015), Acharya and Dogra (2020), and Bilbiie (2021)). Yet, in the behavioral HANK model, if output is expected to increase, households cognitively discount both the expected cause of the increase in output as well as its implication for their precautionary savings motive leading, in sum, to discounting of future changes in output in the IS equation.

As a result, announced policies that increase output in the future, such as announced future interest rate decreases, are less effective in stimulating current output. Furthermore, the effectiveness on today's output decreases with the horizon of the announcement. Hence, the model does not suffer from the forward guidance puzzle.[4] In addition, the behavioral HANK model remains stable even when the effective lower bound (ELB) is expected to bind for a prolonged period of time. This is in contrast to the rational model, in which output implodes. The behavioral HANK model remains determinate even in the limiting case of an ever-binding ELB, as the model features equilibrium determinacy under an interest-rate peg.

The fact that the behavioral HANK model can generate amplification through indirect effects and resolve the forward-guidance puzzle simultaneously is in stark contrast to its rational counterpart. The rational model generates either amplification or resolves the forward guidance puzzle but not both at the same time (see Werning (2015) and Bilbiie (2021)). The behavioral HANK model, on the other hand, overcomes this *Catch-22*.

To show that the results of the tractable version of our model are not driven by the limited heterogeneity assumptions, we then extend our analysis and build on a standard incomplete markets setup. In particular, ex-ante identical households face uninsurable idiosyncratic risk, incomplete markets and borrowing constraints. Households self-insure against their idiosyncratic risk by accumulating liquid assets which are now in positive net supply. As in the tractable version, households with higher MPCs tend to be more exposed to the business cycle. Households are fully rational in the stationary equilibrium, i.e., in the presence of idiosyncratic risk but absent aggregate shocks. Yet, after the realization of an aggregate shock, households anchor their expectations about future macroeconomic variables to the stationary equilibrium, but they cognitively discount expected future deviations from it.

We first show that all the results from the tractable version of the model carry over to the quantitative model. Second, the quantitative behavioral HANK model allows us to consider heterogeneous degrees of bounded rationality. We document that in the data households with

[4] The forward guidance puzzle describes the paradoxical finding in many models that announced future interest-rate changes are at least as effective in stimulating current output than contemporaneous interest-rate changes (Del Negro et al. (2015), McKay et al. (2016)).

higher income tend to deviate somewhat less from rational expectations than households with lower income. Incorporating heterogeneous degrees of bounded rationality along these lines, we then show that our results remain robust.

Having established that the behavioral HANK model can account for recent empirical findings on the transmission channels and effectiveness of monetary and fiscal policy, we use the model to revisit the implications of inflationary supply shocks for stabilization policy. Many advanced economies have recently experienced a dramatic surge in inflation and at least part of this is attributed to disruptions in production, such as supply-chain "bottlenecks" (see di Giovanni et al. (2022)). We model these supply disruptions as a negative productivity shock and examine how monetary policy has to be implemented after such a shock in order to fully stabilize inflation.

In both the rational and the behavioral HANK model, output falls by the same amount as potential output when monetary policy fully stabilizes inflation. Yet, the nominal interest rate hike is more than twice as strong in the behavioral HANK model than in the rational HANK model. The reason is that agents expect interest rates to remain elevated for some time due to the persistence of the shock and the higher expected interest rates help to stabilize current inflation. These effects are especially pronounced in rational HANK models—a corollary of the forward guidance puzzle in these models. As the behavioral HANK does not suffer from the forward guidance puzzle, these effects are less powerful and, hence, the monetary authority needs to increase the interest rates more forcefully. Higher interest rates lead to an increase in government debt and consumption inequality, with both increasing by about three times as much in the behavioral HANK model. In contrast, under a more dovish monetary policy regime, inflation and the output gap increase by more in the behavioral HANK model. Due to the positive output gap, inequality now decreases and it does so by more in the behavioral HANK model. We show that these patterns are even more evident when initial government debt levels are high as they tend to be "post Covid" and we find similar implications for monetary and fiscal policy when considering cost-push shocks. In sum, the behavioral HANK model indicates a more pronounced trade-off between aggregate efficiency and price stability on one hand, and distributional consequences and fiscal sustainability on the other hand.

We close by extending our tractable framework along three dimensions to highlight how the interaction of bounded rationality and household heterogeneity helps to match additional empirical facts. First, we allow for positive savings and analytically show that the behavioral HANK model matches empirical estimates of intertemporal MPCs (iMPCs)—a key statistic in HANK models for monetary and fiscal policy analysis (Auclert et al. (2018), Wolf (2021), Kaplan and Violante (2020)).

Second, we allow for sticky wages and show how the interplay of household heterogeneity and bounded rationality leads to hump-shaped responses of macroeconomic variables in response to aggregate shocks, as documented empirically (see, e.g., Auclert et al. (2020)). Furthermore, we find that the households' expectations initially underreact followed by a delayed overshooting. Thus, the model matches recent findings from survey expectations data (see Angeletos et al. (2021) and Adam et al. (2022)). Neither the rational HANK model nor the representative-agent model (behavioral or rational) matches these two features.

Third, we show that if agents anchor their beliefs to *past observations* of the respective variable instead of the respective steady state values, the model is observationally equivalent with models featuring incomplete information and learning (see Angeletos and Huo (2021) and Gallegos (2021)). This model features myopia and anchoring in the aggregate IS equation and can also produce hump-shaped responses and is consistent with expectations from survey data.

Related Literature. The literature treats the empirical facts laid out in the Introduction mostly independent from each other. The HANK and TANK literature—both with quantitative and analytical models—has highlighted the transmission of monetary policy through indirect, general equilibrium effects (Kaplan et al. (2018), Auclert (2019), Auclert et al. (2020), Bilbiie (2020), Luetticke (2021)), fiscal multipliers (Auclert et al. (2018), Galí et al. (2007), Ferriere and Navarro (2018), Hagedorn et al. (2019b), Bayer et al. (2020a), Bayer et al. (2020b)), and potential resolutions of the forward guidance puzzle (McKay et al. (2016), McKay et al. (2017), Hagedorn et al. (2019a)).

Werning (2015) and Bilbiie (2021) combine the themes of policy amplification and forward guidance puzzle in HANK and establish a trade-off inherent in models with household heterogeneity: if HANK models amplify contemporaneous monetary policy (and fiscal policy) through redistribution towards high MPC households, they also dampen precautionary savings desires after a forward guidance shock which aggravates the forward guidance puzzle.[5] One of our contributions is to show how our behavioral HANK model overcomes this Catch-22 (Bilbiie (2021)).[6]

[5] Acharya and Dogra (2020) construct a pseudo-RANK model, in which they isolate and highlight the role of precautionary savings dynamics to explain the solution or aggravation of the forward guidance puzzle.

[6] Bilbiie (2021) provides two theoretical possibilities of how to sidestep the Catch-22. The first possibility is a pure risk channel which can, in theory, break the comovement of income risk and inequality. Yet to do so, it requires a calibration which seems highly at odds with the data. A second possibility is to drastically narrow down the policy space: in a world in which monetary policy is described by Wicksellian price level targeting or fiscal policy follows a nominal bond rule, there would be no Catch-22. Hagedorn et al. (2019a) use a similar description of fiscal policy to solve the forward guidance puzzle in a quantitative HANK model, in which contemporaneous monetary policy is amplified. Similarly, Kaplan et al. (2016) show that in their quantitative HANK model in Kaplan et al. (2018), there is no Forward Guidance puzzle, conditional on specific fiscal policy responses to a monetary policy shock. In contrast, in our model, there is no Catch-22

Other papers relax the assumption of full-information rational expectations (FIRE) to weaken the effectiveness of future monetary policies, thereby resolving the forward guidance puzzle (Wiederholt (2015), Angeletos and Lian (2018), Andrade et al. (2019), Gabaix (2020), and Pfäuti (2021). We complement these papers by introducing household heterogeneity in terms of income, MPCs, and exposure to the business cycle. This way, our model also generates amplification of contemporaneous monetary and fiscal policy through indirect GE channels. In terms of policy implications, our model highlights the additional side-effects of an inflation-stabilizing monetary policy on inequality.

Farhi and Werning (2019) show that the combination of incomplete markets and level k-thinking can resolve the forward guidance puzzle. We employ cognitive discounting instead of level k-thinking and we do not only focus on the forward guidance puzzle but show that our behavioral HANK model can combine the resolution of the forward guidance puzzle with indirect, general-equilibrium amplification of monetary and fiscal policy. Other papers that share the combination of household heterogeneity and some deviation from FIRE but do not share our focus include Broer et al. (2021a), Angeletos and Huo (2021), Laibson et al. (2021), Gallegos (2021), and Bonciani and Oh (2022). Our extended model with sticky wages can be seen as a tractable complementary to Auclert et al. (2020) who introduce sticky information in HANK to generate humps-shaped responses of macroeconomic aggregates to aggregate shocks. In contrast to all these papers, we offer analytical insights into how household heterogeneity and bounded rationality matter for policy analysis, and how the interaction of these two ingredients is key to reconcile the model with recent empirical facts outlined above. Additionally, we are the first ones to study the monetary and fiscal policy implications of inflationary-supply shocks in a model of household heterogeneity and bounded rationality.

Outline. The rest of the paper is structured as follows. We present our tractable behavioral HANK model in Section 2 and our main analytical results in Section 3. In Section 4, we develop the quantitative behavioral HANK model and show that the results from the tractable model carry over to the quantitative model. We use the quantitative model to study the policy implications of an inflationary supply-side shock in Section 5. We discuss three extensions of the behavioral HANK model in Section 6 and Section 7 concludes.

2 A Behavioral HANK Model

In this section, we present our tractable New Keynesian model featuring household heterogeneity and bounded rationality (BR). For now, we focus on a limited-heterogeneity setup which is typical in the analytical HANK literature to ensure closed-form solutions (e.g.,

independently of the exact specification of monetary and fiscal policy.

McKay et al. (2017), Bilbiie (2021)). We turn to a full-blown incomplete markets setup in Section 4 to show that none of our results are driven by our simplifying assumptions in this section.

2.1 Structure of the Model

Households. The economy is populated by a unit mass of households, indexed by $i \in [0,1]$. Households obtain utility from (non-durable) consumption, C_t^i, and dis-utility from working N_t^i. Households discount future utility at rate $\beta \in [0,1]$. We assume a standard CRRA utility function

$$\mathcal{U}(C_t^i, N_t^i) \equiv \begin{cases} \frac{(C_t^i)^{1-\gamma}}{1-\gamma} - \frac{(N_t^i)^{1+\varphi}}{1+\varphi}, & \text{if } \gamma \neq 1, \\ \log\left(C_t^i\right) - \frac{(N_t^i)^{1+\varphi}}{1+\varphi}, & \text{if } \gamma = 1, \end{cases} \tag{1}$$

where φ denotes the inverse Frisch elasticity and γ the relative risk aversion.

Households can save in government bonds B_{t+1}^i, paying nominal interest i_t, and they can acquire shares ι_t of intermediate monopolistic firms, introduced later. Households face an exogenous borrowing constraint which we set to zero. Households participate in financial markets infrequently. When they do participate, they can freely trade bonds and shares and receive the intermediate firm profits, D_t. Otherwise, they simply receive the payoff from their previously acquired bonds. For now, asset-market participation is exogenous and can be interpreted, for example, as a shock to the household's taste or patience. We denote households participating in financial markets by U as, in equilibrium, they will be Unconstrained in the sense that they are on their Euler equation. We denote the non-participants by H as they neither save nor borrow and are thus, $H$and-to-mouth. An unconstrained household remains unconstrained with probability s and becomes hand-to-mouth with probability $1-s$. Hand-to-mouth households remain hand-to-mouth with probability h and switch to being unconstrained with probability $1-h$. In what follows, we focus on stationary equilibria where $\lambda \equiv \frac{1-s}{2-s-h}$ denotes the constant share of hand-to-mouth households.

Households belong to a family whose intertemporal welfare is maximized by its utilitarian family head. The head can only provide insurance within types but not across types, i.e., the head pools all the resources within types. Thus, in equilibrium every U household will consume and work the same amount and every H household will consume and work the same amount but the H households' consumption and labor supply is not necessarily the same as those of U households. When households switch from being unconstrained to being hand-to-mouth, they only keep their government bonds. Shares, in contrast, cannot be used to self-insure.

We allow for the possibility that the family head is boundedly rational in the way we describe in detail in Section 2.3.[7] The program of the family head is

$$V\left(B_t^U, \iota_t\right) = \max_{\{C_t^U, C_t^H, B_{t+1}^U, N_t^U, N_t^H, \iota_{t+1}\}} \left[(1-\lambda)\mathcal{U}\left(C_t^U, N_t^U\right) + \lambda\mathcal{U}\left(C_t^H, N_t^H\right)\right] + \beta\mathbb{E}_t^{BR} V\left(B_{t+1}^U, \iota_{t+1}\right)$$

subject to the flow budget constraints of unconstrained households

$$C_t^U + B_{t+1}^U + v_t\iota_{t+1} = W_t N_t^U + \iota_t(v_t + \tilde{D}_t) + s\frac{1+i_{t-1}}{1+\pi_t}B_t^U + T_t^U, \tag{2}$$

and the hand-to-mouth households

$$C_t^H = W_t N_t^H + T_t^H + (1-s)\frac{1+i_{t-1}}{1+\pi_t}\frac{1-\lambda}{\lambda}B_t^U, \tag{3}$$

as well as the borrowing constraint $B_{t+1}^U \geq 0$, where W_t is the real wage, v_t is the stock price, and T_t^i are transfers to type-i households. As we will detail below, we assume that these transfers are financed by a proportional tax on profits, τ^D, such that they entail a redistribution from U households (who receive the profits) to H households. The family head takes these transfers as given. $\tilde{D}_t$ denotes the after-tax profits of the intermediate firms. The budget constraints reflect our assumption that households keep their acquired government bonds when switching their type as well as the assumption of full-insurance within type, as the bonds are equally shared within types.

The optimality conditions are given by the Euler equation of unconstrained households

$$\frac{\partial\mathcal{U}\left(C_t^U, N_t^U\right)}{\partial C_t^U} \geq \beta\mathbb{E}_t^{BR}\left[R_t\left(s\frac{\partial\mathcal{U}\left(C_{t+1}^U, N_{t+1}^U\right)}{\partial C_{t+1}^U} + (1-s)\frac{\partial\mathcal{U}\left(C_{t+1}^H, N_{t+1}^H\right)}{\partial C_{t+1}^H}\right)\right], \tag{4}$$

where $R_t \equiv \frac{1+i_t}{1+\pi_{t+1}}$ denotes today's real interest rate, and the respective labor-leisure equations of both types:

$$-\frac{\partial\mathcal{U}\left(C_t^i, N_t^i\right)}{\partial N_t^i} = W_t\frac{\partial\mathcal{U}\left(C_t^i, N_t^i\right)}{\partial C_t^i}.$$

Importantly, the Euler equation of the unconstrained households features a self-insurance motive as unconstrained households demand bonds to self-insure their idiosyncratic risk of becoming hand-to-mouth. The optimality condition for shares, given by $\frac{\partial\mathcal{U}\left(C_t^U, N_t^U\right)}{\partial C_t^U} \geq \beta\mathbb{E}_t^{BR}\left[\frac{v_{t+1}+\tilde{D}_{t+1}}{v_t}\frac{\partial\mathcal{U}\left(C_{t+1}^U, N_{t+1}^U\right)}{\partial C_{t+1}^U}\right]$, only prices the bonds residually as shares cannot be used to self-insure.

We follow the tradition of analytical HANK models and assume a zero liquidity equilibrium (i.e., bond supply B_t^G is equal to zero for all t) to keep our model tractable (Krusell et al. (2011), McKay et al. (2017), Ravn and Sterk (2017), and Bilbiie (2021)).

[7] We show in Appendix A.9 how the family head's expectation can be understood as an average expectation over all households' expectations within the family where each household receives a noisy signal about the future state.

Firms. We assume a standard New Keynesian firm side with sticky prices. All households consume the same aggregate basket of individual goods, $j \in [0,1]$, $C_t = \left(\int_0^1 C_t(j)^{\frac{\epsilon-1}{\epsilon}} dj\right)^{\frac{\epsilon}{\epsilon-1}}$, where $\epsilon > 1$ is the elasticity of substitution between the individual goods. Each firm faces demand $C_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon} C_t$, where $P_t(j)/P_t$ denotes the individual price relative to the aggregate price index, $P_t^{1-\epsilon} = \int_0^1 P_t(j)^{1-\epsilon} dj$, and produces with the linear technology $Y_t(j) = N_t(j)$. The real marginal cost is given by W_t. We assume that the government pays a constant subsidy τ^S on revenues to induce marginal cost pricing in the steady state. The subsidy is financed by a lump-sum tax on firms T_t^F. Hence, the profit function is $D_t(j) = (1+\tau^S)[P_t(j)/P_t]Y_t(j) - W_t N_t(j) - T_t^F$. Total profits are then $D_t = Y_t - W_t N_t$ and are zero in steady state. Zero steady-state profits imply full insurance in steady state, as households only differ (potentially) in their profit income, i.e., in steady state we have $C^H = C^U = C$. In the log-linear dynamics around this steady state, profits vary inversely with the real wage, $\widehat{d}_t = -\widehat{w}_t$, where variables with a hat on top denote log-deviations from steady state. We allow for steady state inequality in Appendix D and show that our results are not driven by this assumption and are in fact barely affected even by substantial inequality in the steady state.

Government. Fiscal policy induces the optimal steady state subsidy financed by lump-sum taxation of firms and taxes profits at rate τ^D and rebates these taxes as a transfer to H households, such that

$$T_t^H = \frac{\tau^D}{\lambda} D_t.$$

As will become clear later the level of τ^D allows us to vary the exposure of H households to the business cycle through a redistribution channel and thus, the cyclicality of inequality. That said, we can also abstract from these transfers, set $\tau^D = 0$, and all our results are qualitatively unchanged. We set $T_t^U = 0$ and we abstract from government spending for now, but introduce it in Section 3 to study fiscal multipliers.

In most of the analysis, we assume that monetary policy follows a standard (log-linearized) Taylor rule

$$\widehat{i}_t = \phi \pi_t + \epsilon_t^{MP}, \tag{5}$$

with ϵ_t^{MP} being a monetary policy shock (Appendix A discusses more general Taylor rules).

Market Clearing. Market clearing requires that the goods market clears $Y_t = C_t = \lambda C_t^H + (1-\lambda) C_t^U$ and the labor market clears $N_t = \lambda N_t^H + (1-\lambda) N_t^U$. Bond market clearing implies $B_{t+1}^U = 0$, at all t.

2.2 Log-Linearized Model

We now focus on the log-linearized dynamics around the full-insurance, zero-liquidity steady state. First, we can write consumption of the hand-to-mouth households as

$$\widehat{c}_t^H = \chi \widehat{y}_t, \tag{6}$$

with

$$\chi \equiv 1 + \varphi\left(1 - \frac{\tau^D}{\lambda}\right) \tag{7}$$

measuring the cyclicality of the H household's consumption (see appendix A.1). Auclert (2019) and Patterson (2019) document that households with higher MPCs tend to be more exposed to aggregate income fluctuations, which is the case when $\chi > 1$. For given φ, this requires $\tau^D < \lambda$.

Why does $\tau^D < \lambda$ imply that the consumption of hand-to-mouth households moves more than one-to-one with aggregate output? If output increases, firms increase their labor demand, leading to an increase in wages. Due to the assumption of sticky prices and flexible wages, profits in the New Keynesian model decrease. In the representative agent model, the representative agent both incurs the increase in wages and the decrease in profits coming from firms. With household heterogeneity, this is not necessarily the case. If the hand-to-mouth households receive less of the profits than their share in the population ($\tau^D < \lambda$) the increase in the real wage is fully transmitted to their income whereas the decrease in profits is not. Thus, H households increase their consumption more than aggregate output increases. The unconstrained households whose profit share is disproportionally large, on the other hand, work more to make up for the income loss due to lower profit income. It is thus mainly the unconstrained households who produce the additional output (see Bilbiie (2021) for an extensive discussion of this).

Combining equation (6) with the goods market clearing condition yields

$$\widehat{c}_t^U = \frac{1 - \lambda\chi}{1 - \lambda} \widehat{y}_t, \tag{8}$$

which implies that consumption inequality is given by:

$$\widehat{c}_t^U - \widehat{c}_t^H = \frac{1 - \chi}{1 - \lambda} \widehat{y}_t. \tag{9}$$

Thus, if $\chi > 1$, inequality is countercyclical as it varies negatively with total output, i.e., inequality increases in recessions and decreases in booms. In line with the empirical evidence on the covariance between MPCs and business-cycle exposure the data also points towards $\chi > 1$ when looking at the cyclicality of inequality, conditional on monetary policy: Coibion et al. (2017), Mumtaz and Theophilopoulou (2017), Ampudia et al. (2018) and Samarina and Nguyen (2019) all provide evidence of countercyclical inequality conditional on monetary policy shocks.

The second key equilibrium equation is the log-linearized bond Euler equation of U households:

$$\widehat{c}_t^U = s\mathbb{E}_t^{BR}\left[\widehat{c}_{t+1}^U\right] + (1-s)\mathbb{E}_t^{BR}\left[\widehat{c}_{t+1}^H\right] - \frac{1}{\gamma}\left(\widehat{i}_t - \mathbb{E}_t^{BR}\pi_{t+1}\right). \tag{10}$$

For the case without type-switching, i.e., for $s = 1$, equation (10) boils down to a standard Euler equation. For $s \in [0, 1)$, however, the agent takes into account that she might switch her type and self-insures against becoming hand-to-mouth next period. How strongly this precautionary-saving motive affects the household's consumption will depend on the household's degree of bounded rationality. We will, following the assumption in Gabaix (2020), often focus on the case in which households are rational with respect to the real rate, i.e., we replace $\mathbb{E}_t^{BR}\pi_{t+1}$ with $\mathbb{E}_t\pi_{t+1}$ in equation (10). We show in Appendix D that our results go through with boundedly-rational real-rate expectations.

Supply Side. For simplicity and to get a clear understanding of the mechanisms driving our results, we focus on a static Phillips curve in Section 3:

$$\pi_t = \kappa\widehat{y}_t, \tag{11}$$

where $\kappa \geq 0$ captures the slope of the Phillips curve. Such a static Phillips curve arises if we assume that firms are either completely myopic or if they face Rotemberg-style price adjustment costs relative to yesterday's market average price index, instead of their own price (see Bilbiie (2021)). In Appendix D we show that a forward-looking Phillips Curve (rational or behavioral) does not qualitatively affect our results.

2.3 Bounded Rationality

We follow Gabaix (2020) and model bounded rationality in the form of cognitive discounting.[8] Let X_t be a random variable (or vector of variables) and let us define X_t^d as some default value the agent may have in mind and let $\tilde{X}_{t+1} \equiv X_{t+1} - X_t^d$ denote the deviation from this default value.[9] The behavioral agent's expectation about X_{t+1} is then defined as

$$\mathbb{E}_t^{BR}\left[X_{t+1}\right] = \mathbb{E}_t^{BR}\left[X_t^d + \tilde{X}_{t+1}\right] \equiv X_t^d + \bar{m}\mathbb{E}_t\left[\tilde{X}_{t+1}\right], \tag{12}$$

[8] While Gabaix (2020) embeds bounded rationality in a NK model the basic idea of behavioral inattention (or sparsity) has been proposed by Gabaix earlier already (see Gabaix (2014, 2016)) and a handbook treatment of behavioral inattention is given in Gabaix (2019). Benchimol and Bounader (2019) and Bonciani and Oh (2021) study optimal monetary policy in a RANK and TANK model, respectively, with this kind of behavioral frictions.

[9] Gabaix (2020) focuses on the case in which X_t denotes the state of the economy. He shows (Lemma 1 in Gabaix (2020)) that this form of cognitive discounting also applies to all other variables. We, on the other hand, directly apply cognitive discounting to all variables. Given Lemma 1 in Gabaix (2020), our results would be unchanged, but our more direct method simplifies some of the derivations, especially in Section 6.3. Appendix A.8 derives our results following the approach in Gabaix (2020).

where $\mathbb{E}_t[\cdot]$ is the rational expectations operator and $\bar{m} \in [0, 1]$ is the behavioral parameter capturing the degree of rationality. A higher $\bar{m}$ denotes a smaller deviation from rational expectations and rational expectations are captured by $\bar{m} = 1$. Intuitively, the behavioral agent anchors her expectations to the default value and cognitively discounts expected future deviations from this default value. For now, we focus on the steady state as the default value but relax this assumption in Section 6.3.

While we present a way how to microfound $\bar{m}$ in Appendix A.9, note, that the exact microfoundation or underlying behavioral friction is not crucial for the rest of our analysis. Angeletos and Lian (2022) show how other forms of bounded rationality or lack of common knowledge lead to observationally-equivalent expectations.

Log-linearizing equation (12) around the steady state yields

$$\mathbb{E}_t^{BR}[\widehat{x}_{t+1}] = (1 - \bar{m})\widehat{x}_t^d + \bar{m}\mathbb{E}_t[\widehat{x}_{t+1}] \tag{13}$$

and when X_t^d is the steady state value, we obtain $\mathbb{E}_t^{BR}[\widehat{x}_{t+1}] = \bar{m}\mathbb{E}_t[\widehat{x}_{t+1}]$. In Appendix E.1, we estimate $\bar{m}$ for different household groups based on their income and in Appendix B, we discuss other empirical estimates of $\bar{m}$ and how we can map recent evidence in Coibion and Gorodnichenko (2015) and Angeletos et al. (2021) to $\bar{m}$. As a benchmark, we follow Gabaix (2020) and set $\bar{m}$ to 0.85, which is a rather conservative deviation from rational expectations, given that the empirical evidence points towards a $\bar{m}$ between 0.6 and 0.85.

Discussion of assumptions. Throughout this section, we have imposed several assumptions that allow us in the following section to analytically characterize our main results as well as to generate analytical insights into how household heterogeneity and bounded rationality interact. In particular, we assume full insurance within types, exogenous type switching, a zero-liquidity equilibrium, no inequality in the steady state and a static Phillips Curve. We relax all these assumptions in Section 4 and show that our results presented in the following do not depend on these assumptions.

3 Results

In this section, we derive the three-equation representation of the tractable behavioral HANK model and show that the model is consistent with the discussed empirical facts. We also show that the model nests a wide spectrum of existing models—none of which can account for all the empirical facts simultaneously.

3.1 The Three-Equation Representation

The behavioral HANK model can be summarized by three equations: a Phillips curve, representing the aggregate supply side captured by equation (11), and a rule for monetary policy (equation (5)), which together with the *behavioral HANK IS equation* determines aggregate demand. To obtain the behavioral HANK IS equation, we combine the hand-to-mouth households' consumption (6) with the consumption of unconstrained households (8) and their consumption Euler equation (10) (see appendix A for all the derivations).

Proposition 1. *The behavioral HANK IS equation is given by*

$$\widehat{y}_t = \psi_f \mathbb{E}_t \widehat{y}_{t+1} - \psi_c \frac{1}{\gamma} \left(\widehat{i}_t - \mathbb{E}_t \pi_{t+1} \right), \tag{14}$$

where

$$\psi_f \equiv \bar{m}\delta = \bar{m} \left[1 + (\chi - 1) \frac{1-s}{1-\lambda\chi} \right] \quad \textit{and} \quad \psi_c \equiv \frac{1-\lambda}{1-\lambda\chi}.$$

Compared to RANK, two new coefficients show up: ψ_c and ψ_f. ψ_c governs the sensitivity of today's output with respect to the contemporaneous real interest rate. ψ_c is shaped by household heterogeneity, in particular by the share of H households λ and their business-cycle exposure χ. As the H households are more exposed to the business cycle ($\chi > 1$), $\psi_c > 1$ which makes current output more sensitive to changes in the contemporaneous real interest rate due to general equilibrium forces, as we show later.

The second new coefficient in the behavioral HANK IS equation (14), ψ_f, captures the sensitivity of today's output with respect to changes in expected future output. ψ_f is shaped by household heterogeneity and the behavioral friction as it depends on the cyclicality of income risk *and* the degree of bounded rationality of households as well as the interaction of these two. Given countercyclical income inequality, income risk is also countercyclical which manifests itself in $\delta > 1$. Countercyclical risk induces compounding in the Euler equation and, thus, competes with the empirically observed underreaction of aggregate expectations ($\bar{m} < 1$) which induces discounting in the Euler equation. We see in the following sections that even for a small degree of bounded rationality—much smaller than the empirics suggest—the discounting through bounded rationality dominates the compounding through countercyclical income risk. Hence, in the behavioral HANK model it holds that $\psi_f < 1$ which makes the economy less sensitive to expectations and news about the future which is key to resolve the forward guidance puzzle as well as to obtain a determinate, locally unique equilibrium.

Equation (14) nests a wide range of existing IS equations: the IS equation in the standard rational-expectations RANK model by setting $\psi_f = \psi_c = 1$, RANK models deviating from

FIRE by $\delta = \psi_c = 1$, TANK models by setting $\bar{m} = \psi_f = 1$, and rational HANK models by $\bar{m} = 1$.

Baseline Calibration. We set the parameters close to the calibration in Bilbiie (2020) and Bilbiie (2021) which is set in order to replicate several findings on monetary and fiscal policy coming from more quantitative HANK models. We set τ^D such that $\chi = 1.5$ which implies that H agents' income is relatively more sensitive to aggregate fluctuations, in line with empirical findings in Auclert (2019) and Patterson (2019). We set the share of H agents to one third, $\lambda = 0.33$, and the probability of an U household to become hand-to-mouth next period to 5.4%, i.e., $s = 0.946$ (this corresponds to $s = 0.8$ in annual terms). We focus on log utility, $\gamma = 1$, set $\beta = 0.99$, and the slope of the Phillips Curve to $\kappa = 0.02$. The cognitive discounting parameter, $\bar{m}$ is set to 0.85, as explained in Section 2.3. Details on the calibration and a discussion of the robustness of our findings for different calibrations are presented in Appendix B. Note, that even when we vary certain parameters, we keep $\lambda < \chi^{-1}$.

3.2 Monetary Policy

We now show how the behavioral HANK model generates amplification of contemporaneous monetary policy through indirect effects while resolving the forward guidance puzzle at the same time. Additionally, we discuss determinacy conditions and show that the model remains stable at the effective lower bound.

To derive these results, it is sometimes convenient to combine the IS equation (14) with the static Phillips Curve (11) and the Taylor rule (5) so that we can represent the model in a single first-order difference equation:

$$\widehat{y}_t = \frac{\psi_f + \psi_c \frac{\kappa}{\gamma}}{1 + \psi_c \phi \frac{\kappa}{\gamma}} \mathbb{E}_t \widehat{y}_{t+1} - \frac{\psi_c \frac{1}{\gamma}}{1 + \psi_c \phi \frac{\kappa}{\gamma}} \varepsilon_t^{MP}. \tag{15}$$

General Equilibrium Amplification and Forward Guidance. We start by showing how the behavioral HANK model generates general equilibrium amplification of current monetary policy, while simultaneously ruling out the forward guidance puzzle. The forward guidance puzzle states that announcements about future changes in the interest rate affect output today as strong (or even stronger) than contemporaneous changes in the interest rate.[10] Such strong effects of future interest rate changes, however, seem puzzling and are not supported by the data (Del Negro et al. (2015), Miescu (2022), Roth et al. (2021)).

[10] Detailed analyses of the forward guidance puzzle in RANK are provided by McKay et al. (2016) and Del Negro et al. (2015).

Let us now consider two different monetary policy experiments: (i) a contemporaneous monetary policy shock, i.e., a surprise decrease in the interest rate today, and (ii) a forward guidance shock, i.e., a news shock today about a decrease in the interest rate k periods in the future. In both cases, we focus on *i.i.d.* shocks and $\phi = 0$, as in Bilbiie (2021).[11]

Proposition 2. *In the behavioral HANK model, there is amplification of contemporaneous monetary policy relative to RANK if and only if*

$$\chi > 1, \tag{16}$$

and the forward guidance puzzle is ruled out if

$$\bar{m}\delta + \frac{1}{\gamma}\frac{1-\lambda}{1-\lambda\chi}\kappa < 1. \tag{17}$$

The behavioral HANK model generates amplification of contemporaneous monetary policy with respect to RANK whenever $\chi > 1$, that is, when high-MPC households' consumption is relatively more sensitive to aggregate income fluctuations. As discussed in Section 2.2, this is the case empirically. With $\chi > 1$ the income of H agents moves more than one for one with aggregate output mainly due to an increase in the real wage.

After a decrease in the interest rate, wages increase and profits decline. As H agents receive a relatively smaller share of profits but fully benefit from the increase in wages, they experience an increase in their overall income, which they consume immediately, thus, increasing the initial effect on total output. The unconstrained households, on the other hand, experience a decline in their income due to the fall in their profit income. To make up for this, they supply more labor and hence, produce the extra output. As a result, $\psi_c > 1$ and the increase in output is amplified through general equilibrium effects, mainly due to the response of hand-to-mouth households to their increase in labor income. To see the importance of GE or indirect effects, the following Lemma disentangles the direct and indirect effects.

Lemma 1. *The consumption function in the behavioral HANK model is given by*

$$\widehat{c}_t = [1 - \beta(1-\lambda\chi)]\,\widehat{y}_t - \frac{(1-\lambda)\beta}{\gamma}\widehat{r}_t + \beta\bar{m}\delta(1-\lambda\chi)\mathbb{E}_t\widehat{c}_{t+1}. \tag{18}$$

Let ρ denote the exogenous persistence and define the indirect effects as the change in total consumption due to the change in total income but for fixed real rates. The share of indirect

[11] If we instead impose $\phi > 0$, contemporaneous amplification in the following proposition is not affected but the condition to rule out the forward guidance puzzle is further relaxed. Similarly, assuming completely fixed prices ($\kappa = 0$), as for example in Farhi and Werning (2019), or modelling forward guidance as changes in the *real* interest rate, as for example in McKay et al. (2016), would also leave the amplification condition unaltered but relaxes the condition to rule out the forward guidance puzzle.

effects, Ξ^{GE}, out of the total effect is then given by

$$\Xi^{GE} = \frac{1-\beta(1-\lambda\chi)}{1-\beta\bar{m}\delta\rho(1-\lambda\chi)}.$$

Given our calibration and assuming an AR(1) monetary policy shock with a persistence of 0.8, indirect effects account for about 78%, consistent with larger quantitative models as for example in Kaplan et al. (2018). For comparison, the representative agent model generates an indirect share of

$$\Xi^{GE} = \frac{1-\beta}{1-\beta\bar{m}\rho},$$

thus, about 3% in the behavioral RANK model and 5% in the rational RANK model.

Note, that in the case of an i.i.d. shock the behavioral friction leaves the relative importance of direct vs. indirect effects—i.e., amplification of contemporaneous monetary policy—unaltered, as amplification of a contemporaneous i.i.d. shock is solely determined by the static redistribution towards the high MPC households. It is through these indirect general equilibrium effects that monetary policy gets amplified as the H households do not directly respond to interest rate changes because they do not participate in asset markets.

Turning to forward guidance, note, that the forward guidance puzzle is ruled out if the term $\frac{\psi_f+\psi_c\frac{\kappa}{\gamma}}{1+\psi_c\phi\frac{\kappa}{\gamma}}$ in front of $\mathbb{E}_t\widehat{y}_{t+1}$ in the first-order difference equation (15) is smaller than 1. Given that we consider $\phi=0$, this boils down to the condition stated in Proposition 2.

What determines whether condition (17) holds or not? First, note that as in the discussion of contemporaneous monetary policy, it is still the case that with $\chi > 1$ the income of H agents moves more than one for one with aggregate income. In this case, unconstrained households who self-insure against becoming hand-to-mouth in the future want less insurance when they expect a decrease in the interest rate since if they become hand-to-mouth they would benefit more from the increase in aggregate income. Hence, after a forward guidance shock, unconstrained households decrease their precautionary savings which compounds the increase in output today ($\delta > 1$). Yet, as households are boundedly rational, they cognitively discount these effects taking place in the future. Importantly, unconstrained households cognitively discount both the future increase in output as well as the general equilibrium implications for their precautionary savings, thereby decreasing the effects of the forward guidance shock on today's consumption.

This last part clearly illustrates the main interaction of bounded rationality and household heterogeneity that enables the behavioral HANK model to resolve the forward guidance puzzle while simultaneously generating amplification through indirect effects. Households fully understand their idiosyncratic risk of switching their type as well as the implications of switching type in case there are no aggregate shocks, i.e., in the steady state. If the monetary authority makes an unexpected announcement about its future policy, however, behavioral households do not fully incorporate the effects of this policy on their own income risk and thus, their precautionary savings. Numerically, already a small underreaction of the behavioral households is enough to resolve the forward guidance puzzle. Given our calibration there is no forward guidance puzzle in the behavioral HANK model as long as $\bar{m} < 0.92$ which is above the upper bounds for empirical estimates (see Section 2.3).

We now compare the behavioral HANK model to its rational counterpart to show how the behavioral HANK model overcomes a major shortcoming inherent in the rational HANK model – the *Catch-22* (Bilbiie (2021), see also Werning (2015)). The Catch-22 describes the trade-off that the rational HANK model can either generate amplification of contemporaneous monetary policy *or* solve the forward guidance puzzle. To see this, note that with $\bar{m} = 1$ the forward guidance puzzle is resolved when

$$\delta + \frac{1}{\gamma}\frac{1-\lambda}{1-\lambda\chi}\kappa < 1$$

which requires $\chi < 1$, as otherwise $\delta > 1$. Assuming $\chi < 1$, however, leads to *dampening* of contemporaneous monetary policy instead of amplification. We graphically illustrate the Catch-22 of the rational model and its resolution in the behavioral HANK model in Appendix C. Note that also rational TANK models (thus, turning off type switching) or the behavioral RANK model would not deliver amplification and resolve the forward guidance puzzle simultaneously. TANK models would face the same issues as the rational RANK model in the sense that they cannot solve the forward guidance puzzle while bounded rationality in a RANK model does not deliver initial amplification.

A direct consequence of Proposition 2 is that in the behavioral HANK model, highly persistent monetary policy shocks have smaller effects on contemporaneous output than in RANK whereas less persistent shocks have relatively larger effects in the behavioral HANK model. The reason is that persistent shocks also work through a forward guidance channel which is dampened in the behavioral HANK model. As the persistence of the shocks approaches unity, an exogenous increase in the nominal interest rate becomes expansionary in the rational model. The behavioral HANK model, on the other hand, does not suffer from these paradoxical model predictions. We elaborate these points in more detail in Appendix D.2. A similar result applies to fiscal spending shocks, as we discuss below.

Determinacy in Behavioral HANK. According to the Taylor principle, monetary policy needs to respond sufficiently strongly to inflation in order to guarantee a determinate equilibrium. In the rational RANK model the Taylor principle is given by $\phi > 1$, where ϕ is the inflation-response coefficient in the Taylor rule (5). We now derive a similar determinacy condition in the behavioral HANK model and show that both household heterogeneity and bounded rationality affect this condition. The following proposition provides the behavioral HANK Taylor principle.[12]

Proposition 3. *The behavioral HANK model has a determinate, locally unique equilibrium if and only if:*

$$\phi > \phi^* = 1 + \frac{\bar{m}\delta - 1}{\frac{\kappa}{\gamma}\frac{1-\lambda}{1-\lambda\chi}}. \tag{19}$$

We obtain Proposition 3 directly from the difference equation (15). For determinacy, we need that the coefficient in front of $\mathbb{E}_t\widehat{y}_{t+1}$ is smaller than 1 (the eigenvalues associated with any exogenous variables are assumed to be $\rho < 1$, which is stable). Solving this condition for ϕ yields Propositon 3. Appendix A.4 outlines the details and extends the result to more general Taylor rules.

To understand the condition in Proposition 3, consider first $\bar{m} = 1$ and, thus, focus solely on the role of household heterogeneity. With $\chi > 1$, it follows that $\phi^* > 1$ and, hence, the threshold is higher than the RANK Taylor principle states. This insufficiency of the Taylor principle in the rational HANK model has been shown by Bilbiie (2021) and in a similar way by Ravn and Sterk (2021) and Acharya and Dogra (2020). As a future aggregate sunspot increases the income of households in state H disproportionately, unconstrained households cut back on precautionary savings today which further increases output today. This calls for a stronger response of the central bank to not let the sunspot become self-fulfilling.

On the other hand, bounded rationality $\bar{m} < 1$ relaxes the condition as unconstrained households now cognitively discount both the future aggregate sunspot as well as its implications for their idiosyncratic risk. A smaller response of the central bank is needed in order to prevent the sunspot to become self-fulfilling. Given our calibration the cutoff value for $\bar{m}$ to restore the RANK Taylor principle in the behavioral HANK model is 0.949. What is more, given our baseline choice of $\bar{m} = 0.85$, we obtain $\phi^* = -2.9$. Thus, in the behavioral HANK model it is not necessary that monetary policy responds to inflation at all as the economy features a stable unique equilibrium even under an interest rate peg. In this sense the behavioral HANK model overcomes the famous result in Sargent and Wallace (1975) who have shown that an interest rate peg leads to equilibrium indeterminacy.

Stability at the Effective Lower Bound. Related to the determinacy issues under a peg the traditional New Keynesian model struggles to explain how the economy can remain stable when the effective lower bound (ELB) on nominal interest rates is binding for an extended period of time, as observed in many advanced economies over recent decades (see, e.g., Debortoli et al. (2020) and Cochrane (2018)). If the ELB binds for a sufficiently long

[12] We focus on local determinacy and bounded equilibria.

time, RANK predicts unreasonably large recessions and, in the limit case in which the ELB binds forever, even indeterminacy.[13] Similar to the forward guidance puzzle, this is even more severe in rational HANK models.

We now show that the behavioral HANK model resolves these issues. To this end, let us add a *natural rate shock* (i.e., a demand shock) $\widehat{r}_t^n$ to the IS equation:

$$\widehat{y}_t = \psi_f \mathbb{E}_t \widehat{y}_{t+1} - \psi_c \left(\widehat{i}_t - \mathbb{E}_t \pi_{t+1} - \widehat{r}_t^n \right).$$

We assume that in period t the natural rate decreases to a value $\widetilde{r}^n$ that is sufficiently negative such that the natural rate in levels is below the ELB. The natural rate stays at $\widetilde{r}^n$ for $k \geq 0$ periods and after k periods the economy returns immediately back to steady state. Agents correctly anticipate the length of the binding ELB. Iterating the IS equation forward, it follows that output in period t is given by

$$\widehat{y}_t = -\frac{1}{\gamma} \psi_c \underbrace{\left(\widehat{i}_{ELB} - \widetilde{r}^n \right)}_{>0} \sum_{j=0}^{k} \left(\psi_f + \frac{\kappa}{\gamma} \psi_c \right)^j, \tag{20}$$

where the term $\left(\widehat{i}_{ELB} - \widetilde{r}^n \right) > 0$ captures the shortfall of the policy response due to the binding ELB. Under rational expectations, we have that $\delta > 1$ and $\psi_f > 1$, meaning that output implodes as $k \to \infty$. The same is true in the rational RANK model which is captured by $\psi_f = \psi_c = 1$. In the behavioral HANK model, however, this is not the case. As long as $\psi_f + \frac{\kappa}{\gamma} \psi_c < 1$ the output response in t is bounded even as $k \to \infty$. It follows that $\bar{m} < 0.92$ is enough to rule out unboundedly-severe recessions at the ELB even if the ELB is expected to persist forever. We graphically illustrate in Appendix C that the behavioral HANK model remains stable also for long spells of the ELB in which output in the rational models collapses.

3.3 Fiscal Policy

We now show that the sufficient statistic for amplification of contemporaneous monetary policy is also a sufficient statistic to generate positive consumption multipliers of fiscal policy under constant real rates, as estimated empirically. Dupor et al. (2021) and Galí et al. (2007), for example, provide empirical evidence for positive effects of government spending on private consumption. Furthermore, Nakamura and Steinsson (2014), Ramey (2019) and Chodorow-Reich (2019) document fiscal multipliers above 1, which through the lens of our model is

[13] The intuition is directly related to our discussion about determinacy under a peg: a forever binding ELB basically implies that the Taylor coefficient is equal to zero and, thus, the nominal rate is pegged at the lower bound, thereby violating the Taylor principle. Note, that this statement also extends to models featuring more elaborate monetary policy rules including Taylor rules responding to output or also the Wicksellian price-level targeting rule, as they all collapse to a constant nominal rate in a world of an ever-binding ELB.

equivalent to saying that consumption responds positively to government spending.

To characterize fiscal multipliers, we assume government spending g_t to follow an AR(1) with persistence $\rho_g \geq 0$, and to be 0 in steady state. The government taxes all agents uniformly to finance g_t.

The behavioral HANK IS equation with government spending is given by:

$$\widehat{c}_t = \psi_f \mathbb{E}_t \widehat{c}_{t+1} - \psi_c \frac{1}{\gamma}\left(\widehat{i}_t - \mathbb{E}_t \pi_{t+1}\right) + \zeta\left[\frac{\lambda(\chi-1)}{1-\lambda\chi}\left(g_t - \bar{m}\mathbb{E}_t g_{t+1}\right) + (\psi_f - \bar{m})\mathbb{E}_t g_{t+1}\right],$$

where $\zeta \equiv \frac{\varphi}{\gamma\left(1+\frac{\varphi}{\gamma}\right)}$ (see appendix A.5). The static Phillips Curve in this setting is given by $\pi_t = \kappa c_t + \kappa\zeta g_t$. The following Proposition characterizes the fiscal multiplier in the behavioral HANK model.

Proposition 4. *The fiscal multiplier in the behavioral HANK model is given by*

$$\frac{\partial \widehat{c}_t}{\partial g_t} = \frac{1}{1-\nu\rho_g}\frac{\zeta}{1+\frac{1}{\gamma}\psi_c\phi\kappa}\left[\frac{\chi-1}{1-\lambda\chi}\left[\lambda(1-\bar{m}\rho_g) + \bar{m}\rho_g(1-s)\right] - \kappa\frac{1}{\gamma}\psi_c\left(\phi - \rho_g\right)\right],$$

where $\nu \equiv \frac{\psi_f + \kappa\frac{1}{\gamma}\psi_c}{1+\frac{1}{\gamma}\psi_c\phi\kappa}$.

To make the argument as clear as possible, we assume prices to be fully rigid, $\kappa = 0$ which, given our Taylor rule, implies that the real interest rate is held constant after the government spending shock. This is a useful benchmark as in this case the consumption response in RANK is 0 (see Bilbiie (2011) and Woodford (2011)).[14]

From Proposition 4, we derive the constant-real-rate multiplier in the behavioral HANK model:

$$\frac{\partial \widehat{c}_t}{\partial g_t} = \frac{1}{1-\nu\rho_g}\zeta\left[\frac{\chi-1}{1-\lambda\chi}\left[\lambda(1-\bar{m}\rho_g) + \bar{m}\rho_g(1-s)\right]\right].$$

As $\chi > 1$ the fiscal multiplier is bounded from below by 0 irrespective of the persistence ρ_g. In other words, the constant-real-rate multiplier in the behavioral HANK model is strictly positive. With $\chi > 1$ the high MPC households benefit disproportionately more from the extra income out of the increase in government spending which increases the fiscal multiplier through a Keynesian type argument. We graphically illustrate the fiscal multiplier in the behavioral HANK model and its dependence on bounded rationality in Appendix C.

The behavioral HANK model does not rely on a specific financing type to achieve positive consumption responses to fiscal spending. This is in contrast to the behavioral RANK model in Gabaix (2020). In the behavioral RANK model, bounded rationality can also increase the multiplier but only if the government delays taxing the agents to finance the contemporaneous spending as boundedly-rational agents will then discount the future increases in taxes. In

[14] Auclert et al. (2018) also use a constant real interest rate case to show that their HANK model can generate (output) fiscal multipliers larger than one.

HANK models, on the other hand, the fiscal multiplier can in principle be larger than one with $\chi \leq 1$ if the hand-to-mouth households pay relatively less of the fiscal spending's cost than unconstrained households (see Bilbiie (2020) or Ferriere and Navarro (2018)). Both of these channels would also push up the multiplier in the behavioral HANK model, yet it does not depend on any of these two to achieve fiscal multipliers larger than 0.

A corollary of Propositon 4 is that with persistent government spending, $\rho_g > 0$, and with $\chi > 1$, more bounded rationality, i.e., a lower $\bar{m}$, leads to a lower fiscal multiplier.[15] Bounded rationality decreases the fiscal multiplier as boundedly-rational agents discount the fact that an increase in government spending today has a positive effect on future spending as well. In the case of an i.i.d. spending shock the fiscal multiplier is independent of $\bar{m}$. Furthermore, the fiscal multiplier is bounded from above in the behavioral HANK model as $\nu\rho_g < 1$ even for highly persistent shocks. In the rational model, on the other hand, this is not the case. The fiscal multiplier approaches infinity as $\nu\rho_g \to 1$, which can occur because in the rational HANK model $\nu > 1$. As $\nu\rho_g > 1$ the multiplier even becomes negative. The behavioral HANK model, on the other hand, rules out these undesirable model implications.

Comparison to Nested Models. The behavioral HANK model nests three classes of models in the literature: the representative-agent rational expectations (RANK) model for $\lambda = 0$ and $\bar{m} = 1$ (see Galí (2015), Woodford (2003)), representative agent models without FIRE for $\lambda = 0$ and $\bar{m} \in (0, 1)$ as, for example, in Gabaix (2019), Angeletos and Lian (2018), and Woodford (2019); and TANK and tractable HANK models as e.g. in Bilbiie (2008), Bilbiie (2021), McKay et al. (2017), or Debortoli and Galí (2018) for $\bar{m} = 1$. In contrast to these classes of models, the behavioral HANK model combines the indirect general equilibrium amplification of monetary and fiscal policy with a resolution of the forward guidance puzzle and stability at the ELB. In representative agent models monetary policy mainly works through direct intertemporal substitution channels and cannot have $\psi_c \neq 1$, rational HANK models on the other hand do not feature $\psi_f < 1$ and $\psi_c > 1$ simultaneously as discussed in Section 3.2 and in Bilbiie (2021). Hence, if we abstract from either bounded rationality or household heterogeneity (or both), the models fails in accounting for all the documented empirical facts about the transmission mechanisms and effectiveness of monetary and fiscal policy.

[15] We focus on the case in which $\nu\rho_g < 1$, which holds in the behavioral HANK model even for $\rho_g = 1$, and we assume $1 - s - \lambda < 0$, which holds under all reasonable parameterizations.

4 A Quantitative Behavioral HANK Model

In this section, we develop a *quantitative* behavioral heterogeneous agent New Keynesian model and show that the main insights of our tractable three-equation model do not depend on the simplifying assumptions we have imposed to keep the model tractable.

We replace the household setup described in Section 2 by an incomplete markets setup as in Bewley (1986), Huggett (1993), and Aiyagari (1994) which is standard in the quantitative HANK literature. There is a continuum of ex-ante identical households all subject to idiosyncratic productivity risk, incomplete markets, and exogenous borrowing constraints. Households self-insure against their idiosyncratic risk by accumulating government bonds. Bonds are now in positive net supply as the fiscal authority issues real government debt, B^G. To finance its interest payments, the fiscal authority collects tax payments from households. Given these assumptions, households differ ex-post in their current productivity level, e, and their wealth B. The households' utility function is the same as in the tractable model (equation (1)).

Household i faces the budget constraint

$$C_{i,t} + \frac{B_{i,t+1}}{R_t} = B_{i,t} + W_t e_{i,t} N_{i,t} + D_t d(e) - \tau_t(e)$$

and the borrowing constraint $B_{i,t+1} \geq \underline{B}$, where $\underline{B}$ denotes an exogenous borrowing limit. Households pay taxes conditional on their productivity, $\tau_t(e)$, and, in particular, we assume that only the most productive households pay taxes, as in McKay et al. (2016). Households receive a share of the dividends, $D_t d(e)$ also conditional on their productivity. Similar to the setup in the tractable model, we assume that the households with higher productivity receive a larger share of the dividends than households with a lower productivity. As dividends are countercyclical in the model, this assumption makes sure that households with higher MPCs (which is highly correlated with lower productivity states) tend to be more exposed to the business cycle, in line with the tractable model and the empirical evidence (Auclert (2019), Patterson (2019)). This is different from McKay et al. (2016) who assume that every household receives the same share of the dividends which leads to procyclical inequality.

We introduce bounded rationality in the same way as in our tractable model. Households are fully rational with respect to their idiosyncratic risk, but they cognitively discount the expected deviations of future aggregates (including prices such as wages) from their respective values in the stationary equilibrium. As a household's individual consumption depends on these aggregates, she cognitively discounts expected future deviations of her marginal utility in each state from its stationary equilibrium counterpart.

Hence, the Euler equation of household i is given by

$$\begin{aligned} C_{i,t}^{-\gamma} &\geq \beta R_t \mathbb{E}_t^{BR}\left[C_{i,t+1}^{-\gamma}\right] \\ &= \beta R_t \mathbb{E}_t^{BR}\left[C_i^{-\gamma} + \left(C_{i,t+1}^{-\gamma} - C_i^{-\gamma}\right)\right] \\ &= \beta R_t \left[C_i^{-\gamma} + \bar{m}\mathbb{E}_t\left(C_{i,t+1}^{-\gamma} - C_i^{-\gamma}\right)\right], \end{aligned} \tag{21}$$

where $C_i^{-\gamma}$ denotes the marginal utility of household i (which depends on the household's individual states B and e) in the stationary equilibrium, i.e., when all aggregate variables and prices are constant. The Euler equation (21) holds with equality for non-constrained households, while it holds with strict inequality for households whose borrowing constraint binds. The labor-leisure condition is identical to the one in the tractable model and holds for every household. With rational expectations ($\bar{m} = 1$), the model collapses to a standard one-asset HANK model, similar to McKay et al. (2016) or Debortoli and Galí (2018). We relegate the details and the calibration to Appendix E.

4.1 Monetary Policy

We now consider the same two monetary policy experiments as in the tractable model. First, how does the economy respond to an i.i.d. expansionary monetary policy shock compared to RANK and second, how do these effects change as the shock is announced today to take place at k periods in the future? In particular, we assume that the monetary authority announces in period 0 to decrease the nominal interest rate by 10 basis points in period k and keeps the nominal rate at its steady state value in all other periods. Following Farhi and Werning (2019), we focus on the case with fully rigid prices such that the change in the nominal rate translates one for one to changes in the real rate and is thus also consistent with the exercise in McKay et al. (2016). In addition, we also follow Farhi and Werning (2019) and McKay et al. (2016) in assuming that the government debt level remains constant, $B_t^G = \bar{B}^G$.

Figure 1 shows on the vertical axis the response of output in period 0, Y_0, to an announced real rate change implemented in period k (horizontal axis). The white horizontal line represents the response in the rational RANK model.[16] The constant response in RANK is a consequence of the assumption that forward guidance is implemented through changes in the real rate.

The blue-dashed line shows the results for the quantitative behavioral HANK model. We see that contemporaneous monetary policy has stronger effects than in RANK. The intuition is the same as in the tractable model: as households with higher MPCs tend to be more exposed to the business cycle, monetary policy is amplified through indirect general

[16]Note that for an easier interpretation, we normalized the y-axis by dividing through the response in the rational RANK model which is 0.05% after a shock of 10 basis points.

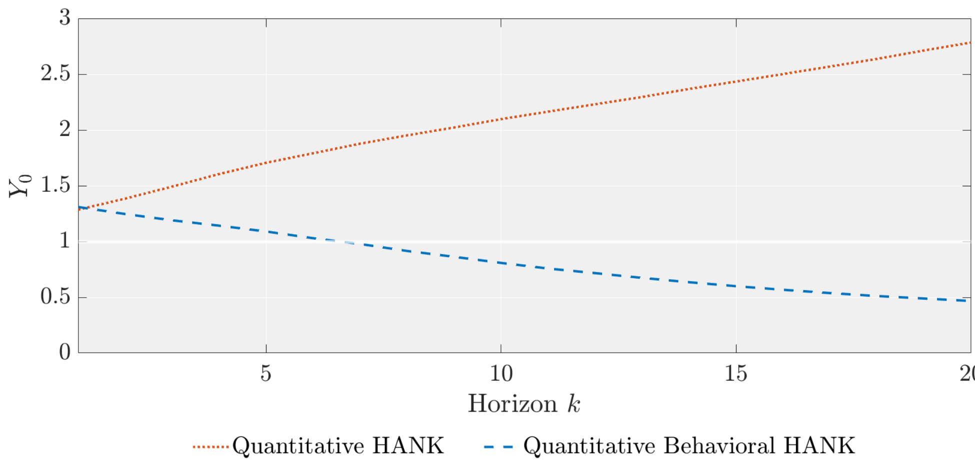

Figure 1: Monetary Policy in the Quantitative Model

Note: This figure shows the response of total output in period 0 to anticipated i.i.d. monetary policy shocks occurring at different horizons k, relative to the response in the RANK model under rational expectations (normalized to 1).

equilibrium effects. Turning again to an AR(1)-process with a persistence of 0.8, we find that indirect effects account for 73% of the total effect in the quantitative behavioral HANK, consistent with other HANK models (for example as in Kaplan et al. (2018)) as well as our tractable behavioral HANK model. At the same time, the behavioral HANK model does not suffer from the forward guidance puzzle, as shown by the decline in the blue-dashed line. Interest rate changes announced to take place in the future have relatively weaker effects on contemporaneous output and the effects decrease with the horizon.

The orange-dotted line shows that this is not the case in the rational HANK model. Contemporaneous monetary policy is as strong as in the behavioral model, but with rational expectations the amplification through indirect effects extends intertemporally and results in an aggravation of the forward guidance puzzle. Indeed, we see that the farther away the announced interest rate change takes place, the stronger the response of output today. A change that is announced to take place in twenty quarters leads to a response of today's output that is more than twice as strong as to a contemporaneous monetary policy shock.[17]

Stability at the ELB and positive fiscal multipliers. The quantitative behavioral HANK model is also consistent with the other empirical facts laid out in the introduction—stability at the effective lower bound as well as positive fiscal multipliers on consumption. To show the first, we employ a transitory shock to the discount factor which pushes the economy to the ELB for twelve periods, in the behavioral and the rational model. After

[17]We discuss how the rational HANK model can resolve the forward-guidance puzzle but then cannot generate amplification of conventional monetary policy in Appendix E.

that the shock jumps back to its steady state value. Consistent with the tractable model, the recession in the rational model is substantially more severe. While output drops only by 5.8% in the behavioral model, it drops by 9.8% in the rational model (see Appendix E for details).

Turning to fiscal policy, we also confirm that the quantitative behavioral HANK model generates positive consumption multipliers under a constant real rate. To this end, we run the same exercise—a temporary increase in government consumption financed by lump-sum transfers—as in Section 3.3. To such a fiscal policy shock, private consumption increases independent of the persistence of the fiscal shock (see Appendix E for details). Overall, we conclude that our main insights of the tractable behavioral HANK model carry over to the quantitative behavioral HANK model.

4.2 Heterogeneous Cognitive Discounting

So far, we have assumed that all households exhibit the same degree of rationality. In reality, however, there might be heterogeneity with respect to the degree of cognitive discounting. Indeed, as we show in Appendix E.1, while cognitive discounting is found across all income groups, the data suggests that higher income households deviate somewhat less from rational expectations.

To capture the positive correlation between households' income and the degree of rationality, we assume that a household's rationality is a function of her productivity level e: $\bar{m}(e = e_1) = 0.8$, $\bar{m}(e = e_2) = 0.85$ and $\bar{m}(e = e_3) = 0.9$. This parameterization serves three purposes: first, in line with the data, the lowest-productivity households exhibit the largest deviation from rational expectations and the degree of rationality increases monotonically with productivity. Second, the average degree of bounded rationality remains 0.85 such that we can isolate the effect of heterogeneous degrees of bounded rationality. And third, this is a very conservative parameterization—both in terms of the degree of heterogeneity and in the level of rationality—compared to the results in the data which points more towards lower level of rationality across all households and less dispersion. Appendix E.1 discusses alternative calibrations.

Figure 2 compares the model with heterogeneous degrees of bounded rationality (black-dashed-dotted line) to our baseline quantitative behavioral HANK model (blue-dashed line) for the same monetary policy experiments as above. The effect of a contemporaneous monetary policy shock is practically identical across the two scenarios consistent with the insight that amplification of a contemporaneous monetary policy shock is barely affected by the degree of rationality. At longer horizons, however, monetary policy is more effective in the economy in which households differ in their degrees of rationality.

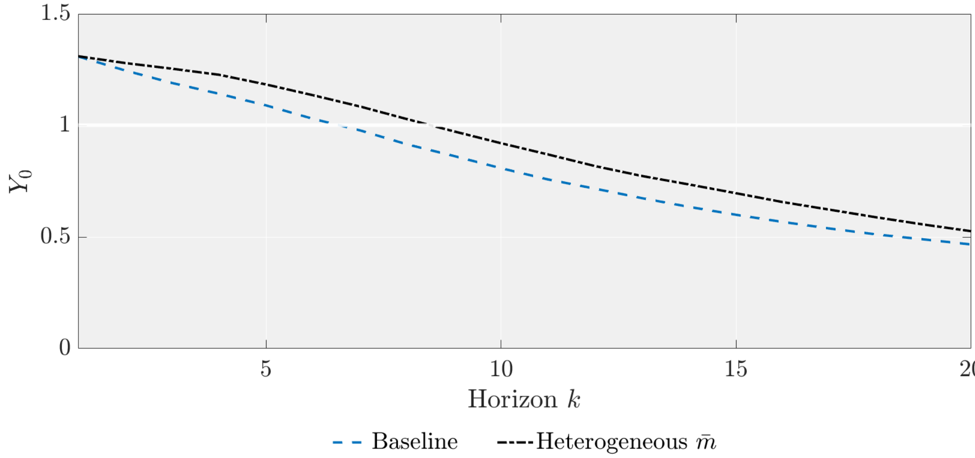

Figure 2: Heterogeneous $\bar{m}$ and Monetary Policy

Note: This figure shows the response of total output in period 0 to anticipated i.i.d. monetary policy shocks occurring at different horizons k for the baseline calibration with $\bar{m} = 0.85$ for all households (blue-dashed line) and for the model in which households differ in their levels of cognitive discounting (black-dashed-dotted line).

There are two competing effects: first, high productivity households are now more rational such that they react stronger to announced future changes in the interest rate compared to the baseline which increases the effectiveness of forward guidance. Second, low productivity households are less rational which tends to dampen the effectiveness of forward guidance. Yet, a large share of low productivity households are at their borrowing constraint and, thus, do not directly react to future changes in the interest rate anyway while most of the high productivity households are unconstrained. Hence, the first effect dominates and forward guidance is more effective compared to the baseline model. Overall, however, the differences across the two calibrations are rather small.

5 Policy Implications: Inflationary Supply Shocks

Having established that our quantitative behavioral HANK model is consistent with recent facts about the transmission and effectiveness of monetary and fiscal policy, we now use it to revisit the policy implications of inflationary supply shocks. Many advanced economies have recently experienced a dramatic surge in inflation and at least part of this is attributed to disruptions in production, such as supply-chain "bottlenecks" (see, e.g., di Giovanni et al. (2022)). We model these disruptions as a negative total factor productivity shock and analyze how monetary policy has to be implemented after such a shock in order to stabilize inflation.

Production of intermediate-goods firm j is given by $Y_t(j) = A_t N_t(j)$, where A_t is total factor productivity following an AR(1)-process, $A_t = (1-\rho_A)\bar{A} + \rho_A A_{t-1} + \varepsilon_t^A$, and ε_t^A is an i.i.d. shock, $\bar{A}$ the steady-state level of TFP and ρ_A the persistence of the shock process which we set to $\rho_A = 0.9$. Each firm can adjust its price with probability 0.15 in a given quarter and we assume that firms have rational expectations to fully focus on the role of bounded rationality on the household side.

Government debt is time-varying and total tax payments, T_t, follow a standard debt feedback rule, $T_t - \bar{T} = \vartheta \frac{B_{t+1} - \bar{B}}{\bar{Y}}$, where we set $\vartheta = 0.05$. We consider two different monetary policy regimes: in the first one, monetary policy follows a strict inflation-targeting rule and implements a zero inflation rate in all periods. In the second one, monetary policy follows a more dovish Taylor rule.

The size of the shock is such that output in the model with fully-flexible prices, complete markets and rational expectations—what we from now on call *potential output*—decreases by 1% in terms of deviations from its steady state. We normalize the leisure parameter in the complete markets, flexible price model such that it has the same steady state output as our behavioral HANK model. The *output gap* is then defined as the difference between the actual output and potential output divided by steady state output.

Figure 3 shows the impulse-response functions of output, the output gap, inflation, nominal interest rates, government debt and the consumption Gini index as a measure of inequality after the negative supply shock when monetary policy fully stabilizes inflation. The blue-dashed lines show the responses in the behavioral HANK model and the orange-dotted lines in the rational HANK model. The output responses are almost indistinguishable across the two models and practically identical to the fall in potential output such that the output gap is essentially zero.

Yet, the reaction of monetary policy differs across the two models. The nominal interest rate in the behavioral HANK model increases twice as much on impact and also more persistently than in the rational HANK model. The reason is that behavioral households cognitively discount the future interest rate hikes that they expect due to the persistence of the shock. Hence, these expected future rate hikes are less effective for stabilizing inflation today. Thus, to induce zero inflation in every period, monetary policy needs to increase interest rates by more than in the rational HANK model, in which the expected future interest rate hikes are very powerful. As this line of reasoning applies in every period, the interest rate hike is also more persistent than under rational expectations.

Raising interest rates increases the cost of debt for the government which it finances in the short run by issuing additional debt. The bottom-middle panel in Figure 3 shows that government debt in the behavioral model increases by more than twice as much as in the rational model. On top of the stronger increase in government debt and interest rates, consumption inequality increases more strongly in the behavioral model compared

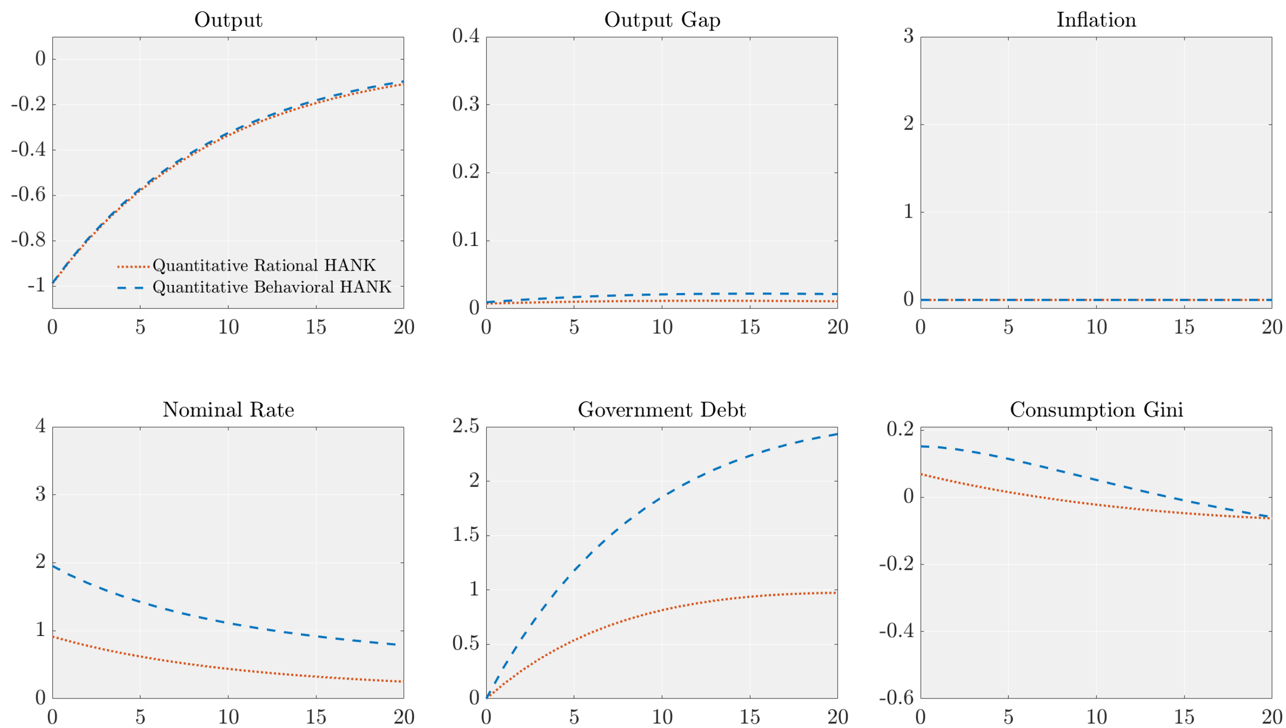

Figure 3: Inflationary supply shock: strict inflation-targeting

Note: This figure shows the impulse responses after a TFP shock that decreases potential output by 1% in the inflation-stabilizing monetary policy regime. Output and the output gap are shown as percentage deviations from steady state output, nominal interest rate and inflation as annualized percentage points and government debt level as percentage point deviations in the debt-per annual GDP level. The lower-right figure shows the change in the consumption Gini index as a percentage deviation from the stationary equilibrium.

to the rational model. The reason is that along the wealth distribution, increases in the real interest rate redistribute to wealthier households and, hence, households who already have a higher consumption level. As the increases in the real interest rate are higher in the behavioral HANK model, these redistribution effects are more pronounced. Note that since monetary policy fully stabilizes inflation and the output gap after a productivity shock, profits and wage income fall by the same relative amount, such that also each household's labor and dividend income falls by the same amount. Hence, the redistribution channels present in Sections 3 and 4 after policy shocks are muted here.

In Appendix F, we show that both the increase in consumption inequality and the implications for fiscal policy are even more evident when initial debt levels are high, especially in the behavioral HANK model.

What if monetary policy does not fully stabilize inflation but rather follows a standard Taylor rule? In particular, we assume that the nominal interest rate responds to inflation and we set the respective coefficient to 1.5. Figure 4 shows that in this case, inflation, the nominal interest rate and the output gap increase by more in the behavioral HANK model compared to the rational model. The government debt level also increases by more than in the rational HANK model, but both increase by less than when monetary policy fully stabilizes inflation. Consumption inequality is now decreasing instead of increasing both in the rational as well as in the behavioral HANK model and it decreases even more in the behavioral HANK model.

The overheating of the economy—reflected in the positive output gap and increase in inflation (which are more pronounced in the behavioral HANK model)—increases wages and decreases profits relative to the inflation stabilizing regime in the same way as expansionary policy shocks in Sections 3 and 4 do. This redistributes towards lower income households which decreases consumption inequality. While the higher interest rates still redistribute towards high-asset households which tends to increase inequality, this channel is now dominated by the former.

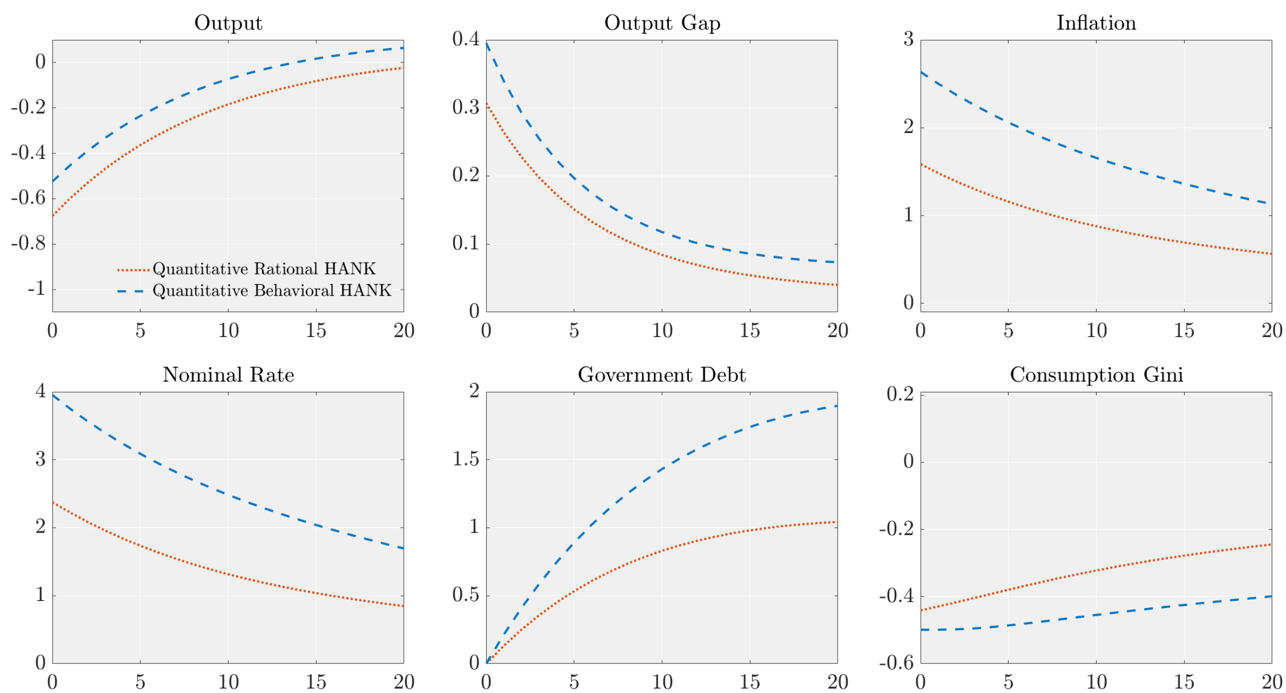

Figure 4: Inflationary supply shock: Taylor rule

Note: This figure shows the impulse responses after a productivity shock for the case that monetary policy follows a Taylor rule. Output and the output gap are shown as percentage deviations from steady state output, nominal interest rate and inflation as annualized percentage points and government debt level as percentage point deviations in the debt-per annual GDP level. The lower-right figure shows the change in the consumption Gini index as a percentage deviation from the stationary equilibrium.

Comparing the case of full inflation stabilizing monetary policy with the more dovish Taylor rule shows that while monetary policy can fully stabilize inflation and keep output (practically) at potential, this is more costly in terms of inequality and government debt in the behavioral HANK model. Dávila and Schaab (2022) show that—unless the central bank only cares about aggregate efficiency—these distributional consequences change the conduct of *optimal* monetary policy in HANK models. Our results indicate that these trade-offs are even more pronounced in the behavioral HANK model.

Cost-push shocks. So far, we have focused on the inflationary pressure coming from negative TFP shocks. We show in Appendix F that if the inflationary pressure comes from a cost-push shock instead, the monetary and fiscal implications are very similar: the central bank needs to raise interest rates much more strongly in the behavioral HANK model than in the rational HANK model to fully stabilize inflation. This pushes up the government debt level which increases more strongly in the behavioral HANK model. If monetary policy instead follows a Taylor rule, again inflation, the output gap and government debt increase by more in the behavioral HANK model than in the rational HANK model.

6 Model Extensions

We now extend our baseline tractable model along three dimensions to show how the interaction of household heterogeneity and bounded rationality helps to match further empirical facts. First, we allow for positive savings which enables us to show that the behavioral HANK model matches the empirical estimates of the iMPCs and how they depend on bounded rationality, heterogeneity and the interaction of the two. Second, we allow for sticky wages and show how the interplay of sticky wages, household heterogeneity and bounded rationality leads to hump-shaped responses of macroeconomic variables in response to aggregate shocks, as well as forecast-error dynamics consistent with recent findings from survey data. Third, we derive an equivalence result between HANK models with bounded rationality and HANK models with incomplete information and learning.

6.1 Intertemporal MPCs

The HANK literature shows that intertemporal marginal propensities to consume are a key statistic for conducting policy analysis (see, e.g., Auclert et al. (2018), Auclert et al. (2020), and Kaplan and Violante (2020)).[18] We follow the tractable HANK literature and define the aggregate iMPCs in the behavioral HANK model as the partial derivative of aggregate consumption at time k, $\widehat{c}_k$, with respect to aggregate disposable income, $\widetilde{y}_0$, keeping everything else fixed (see Bilbiie (2021), Cantore and Freund (2021)). The following Proposition characterizes the iMPCs in the behavioral HANK model (see Appendix G).

[18]See, e.g., Lian (2021) or Boutros (2022) for MPC analyses in models deviating from FIRE.

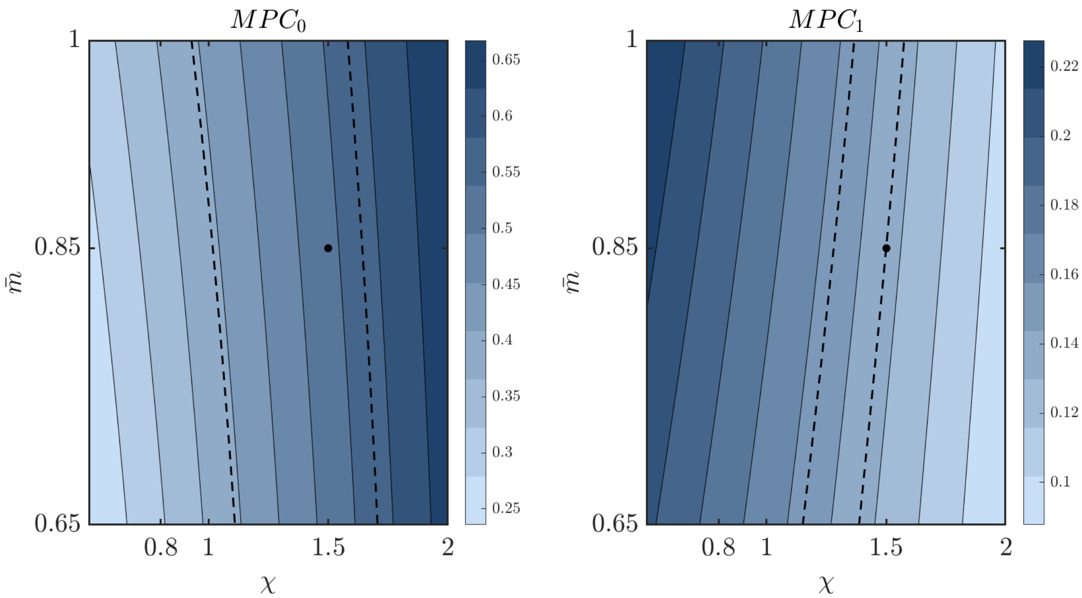

Figure 5: Intertemporal MPCs, Bounded Rationality and Household Heterogeneity

Note: This figure shows the aggregate intertemporal MPCs, i.e., the aggregate consumption response in year 0 (left) and year 1 (right) to a change in aggregate disposable income in year 0 for different χ (x-axis) and $\bar{m}$ (y-axis). The dashed lines show the range of empirically-estimated iMPCs and the black dot shows the model estimate given our baseline calibration. Darker colors represent higher MPCs, see the colorbars on the right side of the figures.

Proposition 5. *The intertemporal MPCs in the behavioral HANK model, i.e., the aggregate consumption response in period k to a one-time change in aggregate disposable income in period 0, are given by*

$$MPC_0 \equiv \frac{d\widehat{c}_0}{d\widetilde{y}_0} = 1 - \frac{1-\lambda\chi}{s\bar{m}}\mu_2^{-1}$$

$$MPC_k \equiv \frac{d\widehat{c}_k}{d\widetilde{y}_0} = \frac{1-\lambda\chi}{s\bar{m}}\mu_2^{-1}\left(\beta^{-1}-\mu_1\right)\mu_1^{k-1}, \quad \text{for } k>0,$$

where the parameters μ_1 and μ_2 depend on the underlying parameters, including $\bar{m}$ and χ and are explicitly spelled out in Appendix G.

We calibrate the model annually as the empirical evidence on the iMPCs is annual (see Fagereng et al. (2021) and Auclert et al. (2018)). We set $s = 0.8$ and $\beta = 0.95$, and keep the rest of the calibration as above. Figure 5 graphically depicts how the interplay of bounded rationality $\bar{m}$ and household heterogeneity χ determines the size of the aggregate iMPCs. The left panel depicts the aggregate MPCs within the first year (in period 0) and the right panel the aggregate MPCs within the second year (in period 1). Darker colors represent higher MPCs. First, note that with our baseline calibration—$\chi = 1.5$ and $\bar{m} = 0.85$ as shown by the black dots—the behavioral HANK model generates iMPCs within the first year of 0.56 and within the second year of 0.14. These values lie within the estimated bounds for the iMPCs in the data (Auclert et al. (2018)) which are between $0.42 - 0.6$ for the first and $0.14 - 0.16$ for the second year (see dashed lines). Away from our baseline calibration, an increase in χ increases the MPCs in the first year but decreases them in the second year. In contrast, an increase in $\bar{m}$ increases the aggregate MPC in the first year and in the second year.

Let us first turn to the role of χ for the iMPCs. Recall, the higher χ the more sensitive is the income of the H households to a change in aggregate income. Thus, with higher χ, H households gain weight in relative terms for the aggregate iMPCs while unconstrained households loose weight in relative terms. This pushes up the aggregate MPC within the first year as the H households spend all of their income windfall, but pushes down the aggregate MPC within the second year as households that were hand-to-mouth in the period of the income windfall have a MPC of 0 in the second year.

Bounded rationality, captured by $\bar{m}$, affects only the MPCs of the initially-unconstrained households as these are the only households who intertemporarily optimize. Their Euler equation dictates that the decrease in today's marginal utility of consumption—due to the increase in consumption—is equalized by a decrease in tomorrow's expected marginal utility. For behavioral households, however, the decrease in tomorrow's marginal utility needs to be more substantial as they cognitively discount it. Hence, behavioral households save relatively more out of the income windfall. This pushes down the aggregate MPCs in $t = 0$. The same is true for the aggregate MPC in $t = 1$, in which there are now two opposing forces at work: on the one hand, unconstrained households again cognitively discount the expectations about the future decrease in their marginal utility which depresses their consumption. On the other hand, unconstrained households have accumulated more wealth from period $t = 0$ which tends to increase consumption. Given our calibration, in $t = 1$ the former dominates. Figure 19 in Appendix G shows that, beginning in $k = 3$, the latter effect starts to dominate. For a higher idiosyncratic risk of becoming hand-to-mouth, i.e., an increase in the transition probability $1 - s$, the aggregate MPC is already higher in $t = 1$ for lower $\bar{m}$. The reason is that a smaller fraction of initially-unconstrained households remains unconstrained which pushes consumption upwards in $k = 1$ (see Figure 20 in Appendix G).

The effects of a change in $\bar{m}$ are more pronounced at lower levels of χ. This follows directly from our discussion about the role of χ and $\bar{m}$: the lower χ, the higher is the relative importance of unconstrained households for the aggregate iMPCs and, in turn, the stronger is the effect of $\bar{m}$ on the aggregate iMPCs. These interaction effects are quite substantial: at $\chi = 1.5$, a decrease of $\bar{m}$ from 1 to 0.65 decreases the MPC_0 by 7% and the MPC_1 by more than 12%.

6.2 Sticky Wages

Recent HANK models have relaxed the assumption of fully-flexible wages and rather assume wages to be sticky, bringing these models closer to the data (see, e.g., Auclert et al. (2020) or Broer et al. (2020)). To introduce sticky wages, we follow Colciago (2011) and assume a centralized labor market in which a labor union allocates the hours of households to firms and makes sure that U and H households work the same amount. The labor union faces the typical Calvo (1983) friction, such that it can re-optimize the wage within a given period only with a certain probability, giving rise to a wage Phillips Curve. We assume that the labor union sets wages based on rational expectations to focus on the effects of bounded rationality solely on the household side.

The wage Phillips curve is given by

$$\pi_t^w = \beta \mathbb{E}_t \pi_{t+1}^w + \kappa_w \widehat{\mu}_t^w,$$

where π_t^w denotes wage inflation, κ_w the slope of the wage Phillips curve and $\widehat{\mu}_t^w$ is a time-varying wage markup, given by $\widehat{\mu}_t^w = \gamma \widehat{c}_t + \varphi \widehat{n}_t - \widehat{w}_t$. We set $\kappa_w = 0.075$ as in Bilbiie et al. (2021).

We follow Auclert et al. (2020) and introduce interest-rate smoothing in the Taylor rule: $\widehat{i}_t = \rho_i \widehat{i}_{t-1} + (1-\rho_i)\phi\pi_t + \varepsilon_t^{MP}$ with $\rho_i = 0.89$ and $\phi = 1.5$ as estimated by Auclert et al. (2020) and assume the shocks ε_t^{MP} to be completely transitory. Similar to the wage setters, we assume price-setting firm managers to be fully rational, giving rise to the standard New Keynesian Phillips curve

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa_\pi \widehat{mc}_t,$$

where $\widehat{mc}_t$ denotes the time-varying price markup. The rest of the model is as above. We relegate the details and the parameterization to Appendix H.

Hump-shaped responses to monetary policy shocks. Figure 6 shows the impulse-response functions of output, real wages and consumption of the two household types to a monetary policy shock for the behavioral HANK model with sticky wages (blue-dashed lines). Importantly, the figure shows that the output response to a monetary policy shock is hump-shaped in the behavioral HANK model but neither in its rational counterpart (orange-dotted lines) nor in its representative-agent counterpart (black-solid lines show the rational RANK results and the black-dashed-dotted lines the results for the behavioral RANK model).

Note, that the introduction of wage rigidity leads to a hump-shaped response in real wages, which is the case in all four models. Since wages determine the H households' income in the rational and the behavioral HANK, their consumption also follows a hump-shape (see lower right figure). Crucial for the overall response, however, is not only the response of H

households but also the response of unconstrained households.

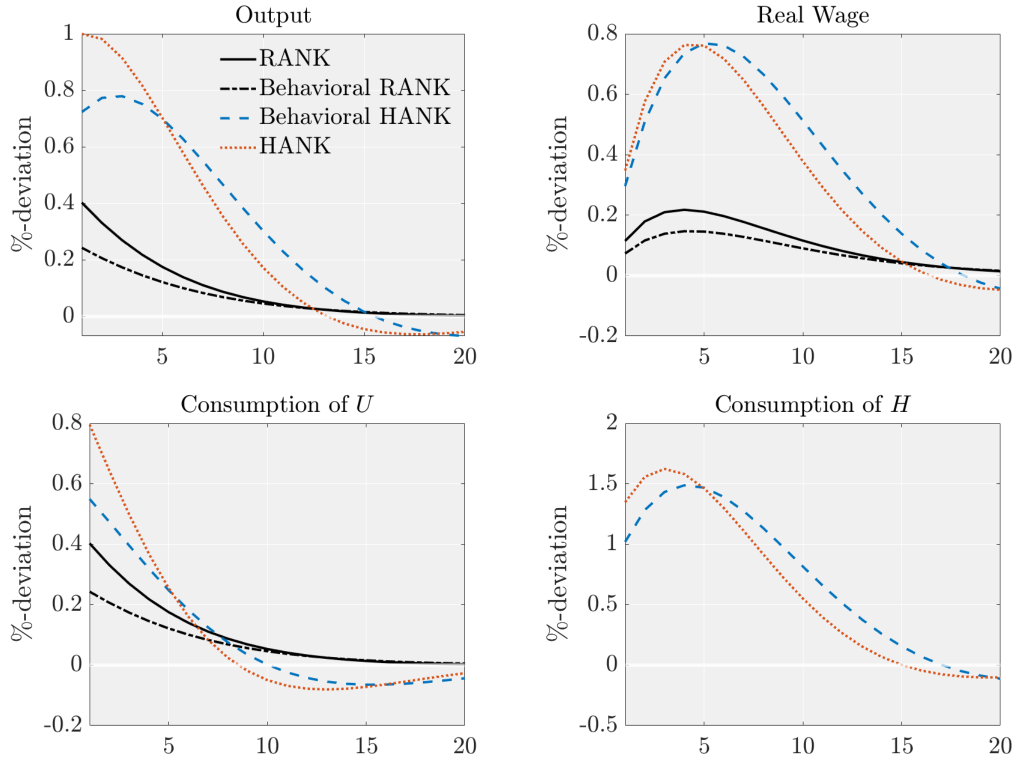

Figure 6: Monetary Policy Shock

Note: This figure shows the impulse-response functions of output, real wages and consumption of the two household types to a monetary policy shock in the behavioral HANK model, the rational and the behavioral RANK model and the rational HANK model with sticky wages. The shock size is normalized such that output in the rational model increases by 1pp on impact.

Under rational expectations, unconstrained households perfectly understand how the consumption of H agents will respond in the future and what this implies for their idiosyncratic risk induced by type switching. In particular, they understand already on impact that their self-insurance motive will be relaxed for some periods. Thus, unconstrained households immediately cut back on precautionary savings and, thus, their consumption responds strongly on impact. Under bounded rationality, however, unconstrained households cognitively discount the future and thus, underreact to the expected increase in wages and, thus, the relaxation of their idiosyncratic risk. Hence, on impact, they do not cut back on precautionary savings as strong as a rational household would. Going forward, they learn that their self-insurance motive is still (or even more) relaxed. As a consequence, their consumption decreases more slowly inducing a flatter consumption profile compared to a rational unconstrained household. It is the combination of the flatter consumption profile of unconstrained households and the hump-shaped consumption profile of the hand-to-mouth that generates the hump-shaped response of consumption in the aggregate.

The model with a representative (behavioral) agent does not generate the hump-shaped response. The reason is that without hand-to-mouth agents, the wage profile does not translate into hump-shaped consumption of (a sub population of) households to begin with. It is thus indeed the *interaction* of household heterogeneity and bounded rationality that produces these hump-shaped responses.

Auclert et al. (2020) argue that many macroeconomic models fail to generate the *micro jumps and macro humps* that we observe in the data, i.e., iMPCs that respond strongly on impact and hump-shaped responses of macroeconomic variables to aggregate shocks. Our results on iMPCs in Section 6.1 as well as the results presented in Figure 6 show how the behavioral HANK model offers a tractable analogue to the full-blown HANK model presented in Auclert et al. (2020).

Forecast-errors dynamics. We now show that the sticky-wage behavioral HANK model generates dynamic forecast errors as observed in survey data. In particular, households' expectations initially underreact followed by delayed overreaction as recently documented empirically in Angeletos et al. (2021) for unemployment and inflation and in Adam et al. (2022) for housing prices.[19] Consistent with the empirical exercise in Angeletos et al. (2021), we focus on three-quarter ahead forecasts. For a variable $\widehat{x}$, the three-period ahead forecast error is defined as

$$FE^{\widehat{x}}_{t+h+3|t+h} \equiv \widehat{x}_{t+h+3} - \bar{m}^3 \mathbb{E}_{t+h}\left[\widehat{x}_{t+h+3}\right],$$

such that a positive forecast error means the forecast was lower than the actual outcome.

Figure 7 shows the forecast errors of output, the real wage and consumption of the two household types starting in the first period in which the expectations start to change which in this case corresponds to the fourth period after the shock. For completeness, the orange-dashed lines at zero show that under rational expectations, i.e., $\bar{m} = 1$, forecast errors are equal to 0. In the behavioral HANK model, however, this is not the case. In fact, forecast errors are positive in the first few quarters after the shock, illustrating the underreaction of the agents' expectations to the shock.

After about 10-15 quarters, however, forecast errors turn negative. Put differently, the behavioral agents' expectations show patterns of delayed overreaction. In contrast to Angeletos et al. (2021) or Adam et al. (2022), the behavioral HANK model with sticky wages generates these dynamic patterns of forecast errors even though the behavioral agents' expectations

[19] In fact, Angeletos et al. (2021) argue that looking at the dynamics of forecast errors in response to structural shocks is more informative than other tests of FIRE. The dynamic responses reconcile seemingly conflicting evidence on underreaction (as in Coibion and Gorodnichenko (2015)) and overreaction (as in Adam et al. (2017) or Kohlhas and Walther (2021)).

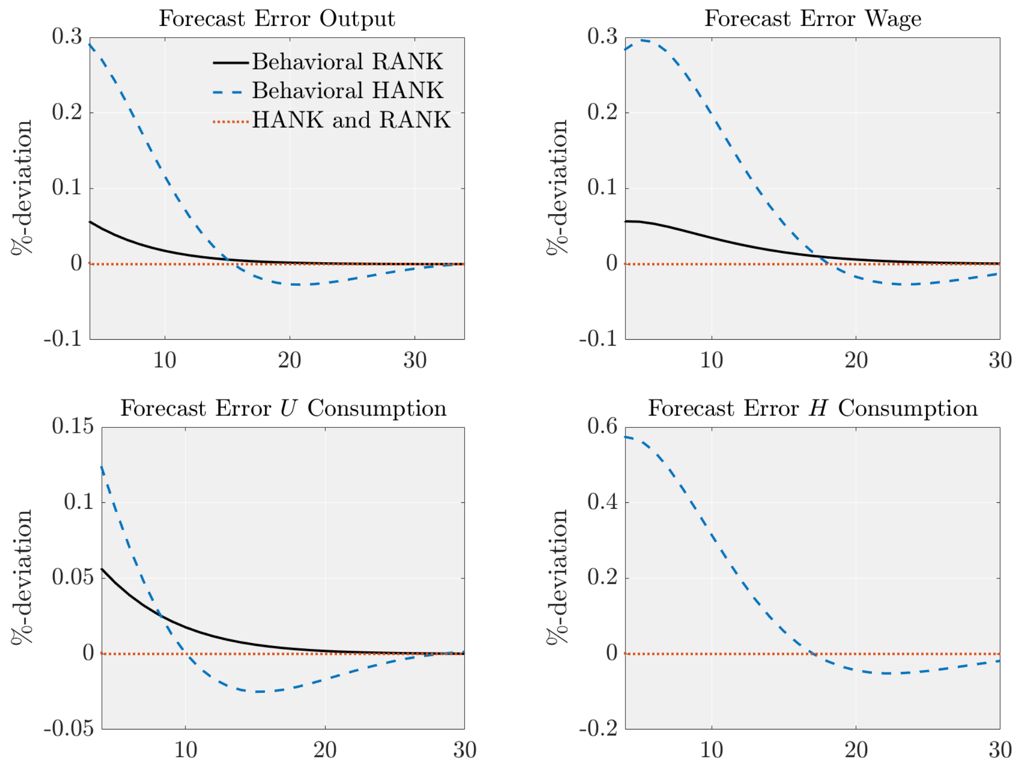

Figure 7: Forecast Error Dynamics

Note: This figure shows the forecast error dynamics of output, the real wage, consumption of unconstrained households and of hand-to-mouth households after an expansionary monetary policy shock.

are purely forward looking.

Where does the delayed overshooting come from? As figure 6 shows, output falls below its steady-state level after some periods in the HANK models. The reason is that with sticky wages, wages increase very persistently. In HANK, this makes the consumption of the H households very persistent which, ceteris paribus, makes the increase in aggregate demand more persistent. Monetary policy reacts to this by increasing the nominal interest rate more strongly and more persistently. Due to inertia in the Taylor rule, however, the interest rate stays high even as aggregate demand returns to its steady state level, generating a mild recession after about 15 quarters (consistent with larger HANK models, see, for example, Auclert et al. (2020)). The behavioral agents then not only underestimate the boom after the monetary policy shock in the short-run, but also underestimate the mild recession in the medium-run, which causes the delayed overshooting in their expectations.

Note that the behavioral RANK model (black solid lines) does not generate these delayed overreactions. Only when allowing for both—household heterogeneity and bounded rationality—the model is able to generate hump-shaped responses of macroeconomic aggregates and forecast error dynamics that are consistent with recent evidence from household survey expectations.

6.3 Bounded Rationality and Incomplete Information with Learning: An Equivalence Result

In this section, we derive an equivalence result of heterogeneous-household models featuring bounded rationality and those featuring incomplete information with learning. In particular, we show how a change in the default value in the behavioral setup leads to an observationally equivalent IS equation as in models with incomplete information and learning (see Angeletos and Huo (2021) and Gallegos (2021)). To this end, we now assume that behavioral agents anchor their expectations to their *last observation* instead of the steady state values which induces a backward-looking component in the expectations as well as in the IS equation:

Proposition 6. *Set the boundedly-rational agents' default value to the variable's past value* $X_t^d = X_{t-1}$. *In this case, the boundedly-rational agent's expectations of* X_{t+1} *becomes*

$$\mathbb{E}_t^{BR}\left[X_{t+1}\right] = (1-\bar{m})X_{t-1} + \bar{m}\mathbb{E}_t\left[X_{t+1}\right] \tag{22}$$

and the behavioral HANK IS equation is then given by

$$\widehat{y}_t = \psi_f \mathbb{E}_t \widehat{y}_{t+1} - \psi_c \frac{1}{\gamma}\left(\widehat{i}_t - \mathbb{E}_t \pi_{t+1}\right) + (1-\bar{m})\delta \widehat{y}_{t-1}. \tag{23}$$

Proposition 6 shows that the change in the agents' default value does not change the existing behavioral and heterogeneity coefficients ψ_f and ψ_c. Yet, anchoring to past realizations introduces an additional backward-looking term in the IS equation, similar to models relying on habit persistence. The IS equation thus features *myopia* and *anchoring* as in Angeletos and Huo (2021) and Gallegos (2021) who derive an IS equation with the same reduced form. Their setup, however, is based on incomplete-information and learning. We complement their findings by showing how we can generate the equivalent outcome based on a *behavioral* relaxation of FIRE.

7 Conclusion

We develop a new framework for business-cycle and policy analysis: the behavioral HANK model. The model accounts for recent empirical findings on the transmission channels and effectiveness of monetary and fiscal policy. To arrive at this framework, we introduce bounded rationality in the form of cognitive discounting and household heterogeneity into a sticky price model. We show that the interaction of these two ingredients enables the model to be reconciled with the data. Both features are thus crucial to arrive at our results. We present a tractable version of our model that relies on a limited heterogeneity setup which allows us to derive our results in closed-form. The model also nests a wide range of existing models—none of which can account for all the empirical patterns. We then relax the limited heterogeneity setup and show that the results from the tractable model carry over to a behavioral HANK model with full heterogeneity including a setup with heterogeneous degrees of bounded rationality. The behavioral HANK model predicts that central banks that want to stabilize inflation after an inflationary supply shock need to hike the nominal interest rate much more strongly and more persistently than under rational expectations. Hiking interest rates, however, leads to a more pronounced increase in public debt and inequality, especially when initial debt levels are already high.

Given its consistency with empirical facts about the transmission of monetary policy and fiscal policy, the behavioral HANK model provides a natural laboratory for both business-cycle and policy analysis. Our framework can also easily be extended along many dimensions, some of which have been done in the paper, whereas others are left for future work.

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Appendix

A Model Details and Derivations

A.1 Derivation of χ

In Section 2, we stated that

$$\widehat{c}_t^H = \chi \widehat{y}_t, \tag{24}$$

where $\chi \equiv 1+\varphi\left(1-\frac{\tau^D}{\lambda}\right)$ is *the* crucial statistic coming from the limited heterogeneity setup. We now show how we arrive at equation (24) from the H-households' budget constraint, optimality conditions and market clearing.

The labor-leisure condition of the H households is given by

$$(N_t^H)^{\varphi} = W_t (C_t^H)^{-\gamma}, \tag{25}$$

and similarly for the U households. As we focus on the steady state with no inequality, we have that in steady state $C = C^H = C^U$ and $N = N^U = N^H$ and market clearing and the production function imply $Y = C = N$, which we normalize to 1.

Thus, log-linearizing the labor-leisure conditions yields

$$\begin{aligned}
\varphi \widehat{n}_t^H &= \widehat{w}_t - \gamma \widehat{c}_t^H \\
\varphi \widehat{n}_t^U &= \widehat{w}_t - \gamma \widehat{c}_t^U.
\end{aligned}$$

Since both households work for the same wage, we obtain

$$\varphi \widehat{n}_t^H + \gamma \widehat{c}_t^H = \varphi \widehat{n}_t^U + \gamma \widehat{c}_t^U \tag{26}$$

Log-linearizing the market clearing conditions yields

$$\begin{aligned}
\widehat{n}_t &= \lambda \widehat{n}_t^H + (1-\lambda)\widehat{n}_t^U \\
\widehat{c}_t &= \lambda \widehat{c}_t^H + (1-\lambda)\widehat{c}_t^U,
\end{aligned}$$

which can be re-arranged as (using $\widehat{y}_t = \widehat{c}_t = \widehat{n}_t$)

$$\begin{aligned}
\widehat{n}_t^U &= \frac{1}{1-\lambda}\left(\widehat{y}_t - \lambda \widehat{n}_t^H\right) \\
\widehat{c}_t^U &= \frac{1}{1-\lambda}\left(\widehat{y}_t - \lambda \widehat{c}_t^H\right).
\end{aligned}$$

Replacing $\widehat{n}_t^U$ and $\widehat{c}_t^U$ in equation (26) then gives

$$\varphi \widehat{n}_t^H + \gamma \widehat{c}_t^H = (\varphi + \gamma)\widehat{y}_t. \tag{27}$$

The budget constraint of H households (accounting for the fact that bond holdings are zero in equilibrium) is given by

$$C_t^H = W_t N_t^H + \frac{\tau^D}{\lambda} D_t, \tag{28}$$

where we replaced T_t^H with $\frac{\tau^D}{\lambda} D_t$. In log-linearized terms, we get

$$\widehat{c}_t^H = \widehat{w}_t + \widehat{n}_t^H + \frac{\tau^D}{\lambda}\widehat{d}_t, \tag{29}$$

and using that $\widehat{w}_t = -\widehat{d}_t = \varphi \widehat{n}_t^H + \gamma \widehat{c}_t^H$, we get

$$\widehat{c}_t^H = \left(\varphi \widehat{n}_t^H + \gamma \widehat{c}_t^H\right)\left(1 - \frac{\tau^D}{\lambda}\right) + \widehat{n}_t^H. \tag{30}$$

Using (27) to solve for $\widehat{n}_t^H$ and plugging it into (30) yields

$$\widehat{c}_t^H = \widehat{c}_t^H \gamma \left(1 - \frac{\tau^D}{\lambda}\right) + \chi \left(\frac{\varphi + \gamma}{\varphi}\widehat{y}_t - \frac{\gamma}{\varphi}\widehat{c}_t^H\right).$$

Grouping terms, we obtain

$$\widehat{c}_t^H = \chi \widehat{y}_t,$$

with $\chi \equiv 1 + \varphi\left(1 - \frac{\tau^D}{\lambda}\right)$, as stated above.

A.2 Derivation of Proposition 1.

Combining equations (6) and (8) with the bounded-rationality setup in equation (13) for $\widehat{x}_t^d = 0$ as X_t^d is given by the steady state, we have

$$\begin{aligned}
\mathbb{E}_t^{BR}\left[\widehat{c}_{t+1}^H\right] &= \bar{m}\mathbb{E}_t\left[\widehat{c}_{t+1}^H\right] = \bar{m}\chi\mathbb{E}_t\left[\widehat{y}_{t+1}\right] \\
\mathbb{E}_t^{BR}\left[\widehat{c}_{t+1}^U\right] &= \bar{m}\mathbb{E}_t\left[\widehat{c}_{t+1}^U\right] = \bar{m}\frac{1-\lambda\chi}{1-\lambda}\mathbb{E}_t\left[\widehat{y}_{t+1}\right].
\end{aligned}$$

Plugging these two equations as well as equation (8) into the Euler equation of unconstrained households (10) yields

$$\frac{1-\lambda\chi}{1-\lambda}\widehat{y}_t = s\bar{m}\frac{1-\lambda\chi}{1-\lambda}\mathbb{E}_t\left[\widehat{y}_{t+1}\right] + (1-s)\bar{m}\chi\mathbb{E}_t\left[\widehat{y}_{t+1}\right] - \frac{1}{\gamma}\left(\widehat{i}_t - \mathbb{E}_t\pi_{t+1}\right).$$

Combining the $\mathbb{E}_t\left[\widehat{y}_{t+1}\right]$ terms and dividing by $\frac{1-\lambda\chi}{1-\lambda}$ yields the following coefficient in front of $\mathbb{E}_t\left[\widehat{y}_{t+1}\right]$:

$$\begin{aligned}
\psi_f &\equiv \bar{m}\left[s + (1-s)\chi\frac{1-\lambda}{1-\lambda\chi}\right] \\
&= \bar{m}\left[1 - 1 + s + (1-s)\chi\frac{1-\lambda}{1-\lambda\chi}\right] \\
&= \bar{m}\left[1 - \frac{1-\lambda\chi}{1-\lambda\chi} + s + (1-s)\chi\frac{1-\lambda}{1-\lambda\chi}\right] \\
&= \bar{m}\left[1 - \frac{1-\lambda\chi}{1-\lambda\chi} + \frac{(1-\lambda\chi)s}{1-\lambda\chi} + (1-s)\chi\frac{1-\lambda}{1-\lambda\chi}\right] \\
&= \bar{m}\left[1 + (\chi - 1)\frac{1-s}{1-\lambda\chi}\right].
\end{aligned}$$

Defining $\psi_c \equiv \frac{1-\lambda}{1-\lambda\chi}$ yields the behavioral HANK IS equation in Proposition 1:

$$\widehat{y}_t = \psi_f \mathbb{E}_t \widehat{y}_{t+1} - \psi_c \frac{1}{\gamma}\left(\widehat{i}_t - \mathbb{E}_t \pi_{t+1}\right).$$

A.3 Derivation of Proposition 2.

The first part comes from the fact that amplification is obtained when

$$\frac{1-\lambda}{1-\lambda\chi} > 1,$$

which requires $\chi > 1$.

For the second part, recall how we define the forward guidance experiment (following Bilbiie (2021)). We assume a Taylor coefficient of 0, i.e., $\phi = 0$, such that the nominal interest rate is given by $\widehat{i}_t = \varepsilon_t^{MP}$. Replacing inflation using the Phillips curve (11), i.e., $\pi_t = \kappa \widehat{y}_t$, we can re-write the behavioral HANK IS equation from Proposition 1 as

$$\begin{aligned}\widehat{y}_t &= \psi_f \mathbb{E}_t \widehat{y}_{t+1} - \psi_c \frac{1}{\gamma}\left(\varepsilon_t^{MP} - \kappa \mathbb{E}_t \widehat{y}_{t+1}\right) \\ &= \left(\psi_f + \psi_c \frac{1}{\gamma}\kappa\right) \mathbb{E}_t \widehat{y}_{t+1} - \psi_c \frac{1}{\gamma} \varepsilon_t^{MP}\end{aligned}$$

The forward guidance puzzle is ruled out if and only if

$$\left(\psi_f + \psi_c \frac{1}{\gamma}\kappa\right) < 1,$$

which is the same as the condition stated in Proposition 2:

$$\bar{m}\delta + \frac{1}{\gamma}\frac{1-\lambda}{1-\lambda\chi}\kappa < 1.$$

Solving this for $\bar{m}$ yields

$$\bar{m} < \frac{1 - \frac{1-\lambda}{\gamma(1-\lambda\chi)}\kappa}{\delta},$$

which completes Proposition 2.

A.4 Derivation of Proposition 3.

Replacing $\widehat{i}_t$ by $\phi\pi_t = \phi\kappa\widehat{y}_t$ and $\mathbb{E}_t \pi_{t+1} = \kappa \mathbb{E}_t \widehat{y}_{t+1}$ in the IS equation (14), we get

$$\widehat{y}_t = \psi_f \mathbb{E}_t \widehat{y}_{t+1} - \psi_c \frac{1}{\gamma}\left(\phi\kappa\widehat{y}_t - \kappa \mathbb{E}_t \widehat{y}_{t+1}\right),$$

which can be re-written as

$$\widehat{y}_t \left(1 + \psi_c \frac{1}{\gamma}\phi\kappa\right) = \mathbb{E}_t \widehat{y}_{t+1}\left(\psi_f + \psi_c \frac{1}{\gamma}\kappa\right).$$

Dividing by $\left(1+\psi_c\frac{1}{\gamma}\phi\kappa\right)$ and plugging in for ψ_f and ψ_c yields

$$\widehat{y}_t = \frac{\bar{m}\delta + \frac{(1-\lambda)\kappa}{\gamma(1-\lambda\chi)}}{1+\kappa\phi\frac{1}{\gamma}\frac{(1-\lambda)}{1-\lambda\chi}}\mathbb{E}_t\widehat{y}_{t+1}.$$

To obtain determinacy, the term in front of $\mathbb{E}_t\widehat{y}_{t+1}$ has to be smaller than 1. Solving this for ϕ yields

$$\phi > \phi^* = 1 + \frac{\bar{m}\delta - 1}{\frac{\kappa}{\gamma}\frac{1-\lambda}{1-\lambda\chi}}, \tag{31}$$

which is the condition in Proposition 3. This illustrates how bounded rationality raises the likelihood that the Taylor principle ($\phi^* = 1$) is sufficient for determinacy, as the Taylor principle can only hold if

$$\bar{m}\delta \leq 1.$$

In the rational model, this boils down to $\delta \leq 1$. However, the Taylor principle can be sufficient under bounded rationality, i.e., $\bar{m} < 1$, even when $\delta > 1$, thus, even when allowing for amplification. Note that we could also express condition (31) as

$$\phi > \phi^* = 1 + \frac{\psi_f - 1}{\frac{\kappa}{\gamma}\psi_c}.$$

Proposition 3 can be extended to allow for Taylor rules of the form

$$\widehat{i}_t = \phi_\pi \pi_t + \phi_y \widehat{y}_t$$

and in which the behavioral agents do not have rational expectations about the real interest rate but rather perceive the real interest rate to be equal to

$$\widehat{r}_t^{BR} \equiv \widehat{i}_t - \bar{m}^r \mathbb{E}_t \pi_{t+1},$$

where $\bar{m}^r$ can be equal to $\bar{m}$ or can potentially differ from it (if it equals 1, we are back to the case in which the behavioral agent is rational with respect to real interest rates).

Combining the static Phillips Curve with the generalized Taylor rule and the behavioral HANK IS equation, it follows that

$$\widehat{y}_t = \frac{\omega_f + \frac{\kappa}{\gamma}\omega_c\bar{m}^r}{1+\frac{\omega_c}{\gamma}\left(\kappa\phi_\pi + \phi_y\right)}\mathbb{E}_t\widehat{y}_{t+1}. \tag{32}$$

From equation (32), it follows that we need

$$\phi_\pi > \bar{m}^r - \phi_y + \frac{\omega_f - 1}{\omega_c\frac{\kappa}{\gamma}} = \bar{m}^r - \phi_y + \frac{\bar{m}\delta - 1}{\frac{1-\lambda}{1-\chi\lambda}\frac{\kappa}{\gamma}} \tag{33}$$

for the model to feature a determinate, locally unique equilibrium. Condition (33) shows that both, $\bar{m}^r < 1$ and $\phi_y > 0$, weaken the condition in Proposition 3. Put differently, bounded rationality with respect to the real rate or a Taylor rule that responds to changes in output, both relax the condition on ϕ_π to yield determinacy.

A.5 IS Curve with Government Spending

Since government spending is financed by uniform taxes, $\tau_t^H = \tau_t^U = G_t$, household H's net income is:

$$\widehat{c}_t^H = \widehat{w}_t + \widehat{n}_t^H + \frac{\tau^D}{\lambda}\widehat{d}_t - g_t, \tag{34}$$

where $g_t = log(G_t/Y)$.

We first derive households H consumption as a function of total income $\widehat{y}_t$. The good markets clearing condition is now

$$\widehat{y}_t = \lambda\widehat{c}_t^H + (1-\lambda)\widehat{c}_t^U + g_t. \tag{35}$$

Plugging this and the labor market clearing condition into (26), yields:

$$\varphi\widehat{n}_t^H + \gamma\widehat{c}_t^H = (\varphi+\gamma)\widehat{y}_t - \gamma g_t. \tag{36}$$

Replacing wages and the dividends in the households' budget constraint yields:

$$\widehat{c}_t^H = \left(\varphi\widehat{n}_t^H + \gamma\widehat{c}_t^H\right)\left(1 - \frac{\tau^D}{\lambda}\right) + \widehat{n}_t^H - g_t. \tag{37}$$

and using (36) yields:

$$\widehat{c}_t^H = \left(\varphi\widehat{n}_t^H + \gamma\widehat{c}_t^H\right)\left(1 - \frac{\tau^D}{\lambda}\right) + \widehat{n}_t^H - g_t. \tag{38}$$

Finally, consumption of H is given by:

$$\widehat{c}_t^H = \chi\widehat{y}_t - \left[\frac{\chi-1}{1+\frac{\varphi}{\gamma}} + 1\right] g_t \tag{39}$$

which is

$$\widehat{c}_t^H = \chi\left(\widehat{y}_t - g_t\right) + \left[\frac{\chi-1}{1+\frac{\gamma}{\varphi}}\right] g_t. \tag{40}$$

The consumption of unconstrained households is then given by (using the market clearing condition):

$$\widehat{c}_t^U = \frac{1-\lambda\chi}{1-\lambda}\left(\widehat{y}_t - g_t\right) - \frac{\lambda}{1-\lambda}\frac{\chi-1}{1+\frac{\gamma}{\varphi}} g_t. \tag{41}$$

The IS curve in terms of aggregate consumption is then obtained by plugging the consumption of the hand-to-mouth and of unconstrained households into the Euler equation of unconstrained households and using $\widehat{c}_t = (\widehat{y}_t - g_t)$.

$$\begin{aligned} \frac{1-\lambda\chi}{1-\lambda}\widehat{c}_t - \frac{\lambda}{1-\lambda}\frac{\chi-1}{1+\frac{\gamma}{\varphi}} g_t = s\mathbb{E}_t^{BR}&\left[\frac{1-\lambda\chi}{1-\lambda}\widehat{c}_{t+1} - \frac{\lambda}{1-\lambda}\frac{\chi-1}{1+\frac{\gamma}{\varphi}} g_{t+1}\right] \\ &+(1-s)\mathbb{E}_t^{BR}\left[\chi\widehat{c}_{t+1} + \left[\frac{\chi-1}{1+\frac{\gamma}{\varphi}}\right] g_{t+1}\right] - \frac{1}{\gamma}\mathbb{E}_t(\widehat{i}_t - \pi_{t+1}), \end{aligned}$$

which can be re-written as (using similar derivations as in Appendix A.2)

$$\begin{aligned}
\widehat{c}_t &= \psi_f \mathbb{E}_t \widehat{c}_{t+1} - \frac{1}{\gamma}\psi_c \mathbb{E}_t(\widehat{i}_t - \pi_{t+1}) + \frac{\lambda}{1+\frac{\gamma}{\varphi}} \frac{\chi - 1}{1-\lambda\chi} g_t \\
&\quad - \left[s\frac{\lambda}{1-\lambda\chi}\frac{\chi-1}{1+\frac{\gamma}{\varphi}} + (1-s)\frac{\chi-1}{1+\frac{\gamma}{\varphi}}\frac{1-\lambda}{1-\chi\lambda} \right] \mathbb{E}_t^{BR} g_{t+1} \\
&= \psi_f \mathbb{E}_t \widehat{c}_{t+1} - \frac{1}{\gamma}\psi_c \mathbb{E}_t(\widehat{i}_t - \pi_{t+1}) + \zeta \left[\frac{\lambda(\chi-1)}{1-\lambda\chi}(g_t - \bar{m}\mathbb{E}_t g_{t+1}) + (\delta - 1)\bar{m}\mathbb{E}_t g_{t+1} \right]
\end{aligned}$$

with $\zeta = \frac{1}{1+\frac{\gamma}{\varphi}}$. Replacing the expectations and taking the derivative with respect to g_t yields the consumption multiplier.

A.6 Derivation of Lemma 1

Let us first state a few auxiliary results that will prove helpful later. First, in log-linearized terms, the stochastic discount factor is given by

$$\frac{1}{\gamma}\mathbb{E}_t^{BR} \widehat{q}_{t,t+1}^U = \widehat{c}_t^U - s\bar{m}\mathbb{E}_t \widehat{c}_{t+1}^U - (1-s)\bar{m}\mathbb{E}_t \widehat{c}_{t+1}^H$$

and for i periods ahead:

$$\frac{1}{\gamma}\mathbb{E}_t^{BR} \widehat{q}_{t,t+i}^U = \widehat{c}_t^U - s\bar{m}^i\mathbb{E}_t \widehat{c}_{t+i}^U - (1-s)\bar{m}^i\mathbb{E}_t \widehat{c}_{t+i}^H.$$

Furthermore, we have:

$$\begin{aligned}
\frac{1}{\gamma}\mathbb{E}_t^{BR} \widehat{q}_{t+1,t+2}^U &= \mathbb{E}_t^{BR}\left[\widehat{c}_{t+1}^U - s\widehat{c}_{t+2}^U - (1-s)\widehat{c}_{t+2}^H\right] \\
&= \bar{m}\mathbb{E}_t \widehat{c}_{t+1}^U - s\bar{m}^2\mathbb{E}_t \widehat{c}_{t+2}^U - (1-s)\bar{m}^2\mathbb{E}_t \widehat{c}_{t+2}^H
\end{aligned}$$

and the stochastic discount factor has the property

$$\mathbb{E}_t^{BR}\left[\widehat{q}_{t,t+i}^U\right] = \mathbb{E}_t^{BR}\left[\widehat{q}_{t,t+1}^U + \widehat{q}_{t+1,t+2}^U + ... + \widehat{q}_{t+i-1,t+i}^U\right].$$

Using these results, $\mathbb{E}_t^{BR}\left[\widehat{q}_{t,t+i}^U\right]$ can be written as

$$\begin{aligned}
\frac{1}{\gamma}\mathbb{E}_t^{BR} \widehat{q}_{t,t+i}^U = \widehat{c}_t^U &+ (1-s)\bar{m}\mathbb{E}_t\left[\widehat{c}_{t+1}^U - \widehat{c}_{t+1}^H\right] \\
&+ (1-s)\bar{m}^2\mathbb{E}_t\left[\widehat{c}_{t+2}^U - \widehat{c}_{t+2}^H\right] + ...+ \\
&+ (1-s)\bar{m}^i\mathbb{E}_t\left[\widehat{c}_{t+i}^U - \widehat{c}_{t+1}^H\right] - \bar{m}^i\mathbb{E}_t \widehat{c}_{t+i}^U,
\end{aligned}$$

or put differently

$$\frac{1}{\gamma}\mathbb{E}_t^{BR} \widehat{q}_{t,t+i}^U + \bar{m}^i\mathbb{E}_t \widehat{c}_{t+i}^U = \widehat{c}_t^U + (1-s)\mathbb{E}_t \sum_{k=1}^{i} \bar{m}^k \left(\widehat{c}_{t+k}^U - \widehat{c}_{t+k}^H\right). \tag{42}$$

The (linearized) budget constraint can be written as

$$\mathbb{E}_t^{BR}\sum_{i=0}^{\infty}\beta^i\left(\frac{1}{\gamma}\widehat{q}_{t,t+i}^U+\widehat{c}_{t+i}^U\right)=\mathbb{E}_t^{BR}\sum_{i=0}^{\infty}\beta^i\left(\frac{1}{\gamma}\widehat{q}_{t,t+i}^U+\widehat{y}_{t+i}^U\right)$$

$$\Leftrightarrow \mathbb{E}_t^{BR}\sum_{i=0}^{\infty}\beta^i\left(\frac{1}{\gamma}\widehat{q}_{t,t+i}^U\right)+\mathbb{E}_t\sum_{i=0}^{\infty}(\beta\bar{m})^i\,\widehat{c}_{t+i}^U=\mathbb{E}_t^{BR}\sum_{i=0}^{\infty}\beta^i\left(\frac{1}{\gamma}\widehat{q}_{t,t+i}^U\right)+\mathbb{E}_t\sum_{i=0}^{\infty}(\beta\bar{m})^i\,\widehat{y}_{t+i}^U.$$

Now, focus on the left-hand side and notice that the sum $\mathbb{E}_t\sum_{i=0}^{\infty}(\beta\bar{m})^i\,\widehat{c}_{t+i}^U$ cancels with the $\bar{m}^i\mathbb{E}_t\widehat{c}_{t+i}^U$ terms in equation (42) when summing them up. The left-hand side of the budget constraint can thus be written as

$$\begin{aligned}
&\mathbb{E}_t\sum_{i=0}^{\infty}\beta^i\left(\widehat{c}_t^U+(1-s)\sum_{k=1}^{i}\bar{m}^k\left(\widehat{c}_{t+k}^U-\widehat{c}_{t+k}^H\right)\right)\\
&=\frac{1}{1-\beta}\widehat{c}_t^U+(1-s)\mathbb{E}_t\sum_{i=0}^{\infty}\beta^i\sum_{k=1}^{i}\bar{m}^k\left(\widehat{c}_{t+k}^U-\widehat{c}_{t+k}^H\right)\\
&=\frac{1}{1-\beta}\widehat{c}_t^U+\frac{1-s}{1-\beta}\mathbb{E}_t\sum_{i=1}^{\infty}(\beta\bar{m})^i\left(\widehat{c}_{t+i}^U-\widehat{c}_{t+i}^H\right).
\end{aligned}$$

Note, from the Euler equation of the unconstrained households, we obtain the real interest rate

$$\begin{aligned}
-\frac{1}{\gamma}\widehat{r}_t&=\widehat{c}_t^U-s\mathbb{E}_t^{BR}\widehat{c}_{t+1}^U-(1-s)\mathbb{E}_t^{BR}\widehat{c}_{t+1}^H\\
&=\frac{1}{\gamma}\mathbb{E}_t^{BR}\widehat{q}_{t,t+1}^U,
\end{aligned}$$

and similarly,

$$-\frac{1}{\gamma}\bar{m}^i\mathbb{E}_t\widehat{r}_{t+i}=\frac{1}{\gamma}\mathbb{E}_t^{BR}\widehat{q}_{t+i,t+i+1}^U,$$

where $\widehat{r}_t$ is the (linearized) real interest rate.

Combining these results, we see that

$$\mathbb{E}_t^{BR}\sum_{i=0}^{\infty}\beta^i\frac{1}{\gamma}\widehat{q}_{t,t+i}^U=-\frac{1}{1-\beta}\frac{1}{\gamma}\beta\mathbb{E}_t\sum_{i=0}^{\infty}(\beta\bar{m})^i\,\widehat{r}_{t+i}.$$

Plugging this into the right-hand side of the budget constraint and multiplying both sides by $1-\beta$ yields

$$\begin{aligned}
\widehat{c}_t^U=&-\frac{1}{\gamma}\beta\widehat{r}_t+(1-\beta)\widehat{y}_t^U-(1-s)\mathbb{E}_t\sum_{i=1}^{\infty}(\beta\bar{m})^i\left(\widehat{c}_{t+i}^U-\widehat{c}_{t+i}^H\right)\\
&-\frac{1}{\gamma}\beta\mathbb{E}_t\sum_{i=1}^{\infty}(\beta\bar{m})^i\,\widehat{r}_{t+i}+(1-\beta)\mathbb{E}_t\sum_{i=1}^{\infty}(\beta\bar{m})^i\,\widehat{y}_{t+i}^U,
\end{aligned}$$

or written recursively

$$\widehat{c}_t^U = -\frac{1}{\gamma}\beta\widehat{r}_t + (1-\beta)\widehat{y}_t^U + \beta\bar{m}s\mathbb{E}_t\widehat{c}_{t+1}^U + \beta\bar{m}(1-s)\mathbb{E}_t\widehat{c}_{t+1}^H.$$

Now, aggregating, i.e., multiplying the expression for $\widehat{c}_t^U$ by $(1-\lambda)$, adding $\lambda\widehat{c}_t^H$ and using $\widehat{c}_t^H = \chi\widehat{y}_t$ as well as $\widehat{y}_t^U = \frac{1-\lambda\chi}{1-\lambda}\widehat{y}_t$, yields the consumption function

$$\widehat{c}_t = [1-\beta(1-\lambda\chi)]\,\widehat{y}_t - \frac{(1-\lambda)\beta}{\gamma}\widehat{r}_t + \beta\bar{m}\delta(1-\lambda\chi)\mathbb{E}_t\widehat{c}_{t+1}, \tag{43}$$

as stated in the main text.

To obtain the share of indirect effects, note that the model does not feature any endogenous state variables and hence, endogenous variables inherit the persistence of the exogenous variables, ρ. Thus, $\mathbb{E}_t\widehat{c}_{t+1} = \rho\widehat{c}_t$. Plugging this into the consumption function (43), we get

$$\widehat{c}_t = \frac{1-\beta(1-\lambda\chi)}{1-\beta\bar{m}\delta\rho(1-\lambda\chi)}\widehat{y}_t - \frac{(1-\lambda)\beta}{\gamma(1-\beta\bar{m}\delta\rho(1-\lambda\chi))}\widehat{r}_t.$$

The term in front of $\widehat{y}_t$ is the share of indirect effects.

A.7 Derivation of Proposition 6

To prove Proposition 6, we start from the Euler equation (10). Plugging in for $\widehat{c}_t^U$, $\widehat{c}_{t+1}^U$ and $\widehat{c}_{t+1}^H$ from equations (6) and (8), we get

$$\widehat{y}_t = s\mathbb{E}_t^{BR}\left[\widehat{y}_{t+1}\right] + (1-s)\frac{1-\lambda}{1-\lambda\chi}\mathbb{E}_t^{BR}\left[\widehat{y}_{t+1}\right] - \psi_c\left(\widehat{i}_t - \mathbb{E}_t\pi_{t+1}\right),$$

which can be re-written as

$$\widehat{y}_t = \delta\mathbb{E}_t^{BR}\left[\widehat{y}_{t+1}\right] - \psi_c\left(\widehat{i}_t - \mathbb{E}_t\pi_{t+1}\right).$$

Now, using the expectations setup from Proposition 6, we get $\delta\mathbb{E}_t^{BR}\left[\widehat{y}_{t+1}\right] = (1-\bar{m})\delta\widehat{y}_{t-1} + \bar{m}\delta\mathbb{E}_t\left[\widehat{y}_{t+1}\right]$ which proves Proposition 6.

A.8 Cognitive Discounting of the State Vector

In Section 2, we assume that cognitive discounting applies to all variables, which differs slightly from the assumption in Gabaix (2020) who assumes that cognitive discounting applies to the *state* of the economy (exogenous shocks as well as announced monetary and fiscal policies). He then proves (Lemma 1 in Gabaix (2020)) how cognitive discounting applies as a result (instead of as an assumption) to all future variables, including future consumption choices. For completeness, we show in this section how our results are unaffected when following the approach in Gabaix (2020).

Let X_t denote the (de-meaned) state vector which evolves as

$$X_{t+1} = G^X\left(X_t, \varepsilon_{t+1}\right), \tag{44}$$

where G^X denotes the transition function of X in equilibrium and ε are zero-mean innovations. Linearizing equation (44) yields

$$X_{t+1} = \Gamma X_t + \varepsilon_{t+1}, \tag{45}$$

where ε_{t+1} might have been renormalized. The assumption in Gabaix (2020) is that the behavioral agent perceives the state vector to follow

$$X_{t+1} = \bar{m} G^X(X_t, \varepsilon_{t+1}), \tag{46}$$

or in linearized terms

$$X_{t+1} = \bar{m}\left(\Gamma X_t + \varepsilon_{t+1}\right). \tag{47}$$

The expectation of the boundedly-rational agent of X_{t+1} is thus $\mathbb{E}_t^{BR}\left[X_{t+1}\right] = \bar{m}\mathbb{E}_t\left[X_{t+1}\right] = \bar{m}\Gamma X_t$. Iterating forward, it follows that $\mathbb{E}_t^{BR}\left[X_{t+k}\right] = \bar{m}^k\mathbb{E}_t\left[X_{t+k}\right] = \bar{m}^k\Gamma^k X_t$.

Now, consider any variable $z(X_t)$ with $z(0) = 0$ (e.g., demeaned consumption of unconstrained households $C^U(X_t)$). Linearizing $z(X)$, we obtain $z(X) = b_X^z X$ for some b_X^z and thus

$$\begin{aligned}
\mathbb{E}_t^{BR}\left[z(X_{t+k})\right] &= \mathbb{E}_t^{BR}\left[b_X^z X_{t+k}\right] \\
&= b_X^z \mathbb{E}_t^{BR}\left[X_{t+k}\right] \\
&= b_X^z \bar{m}^k \mathbb{E}_t\left[X_{t+k}\right] \\
&= \bar{m}^k \mathbb{E}_t\left[b_X^z X_{t+k}\right] \\
&= \bar{m}^k \mathbb{E}_t\left[z(X_{t+k})\right].
\end{aligned}$$

For example, expected consumption of unconstrained households tomorrow (in linearized terms) is given by

$$\mathbb{E}_t^{BR}\left[\widehat{c}^U(X_{t+1})\right] = \bar{m}\mathbb{E}_t\left[\widehat{c}^U(X_{t+1})\right], \tag{48}$$

which we denote in the main text as

$$\mathbb{E}_t^{BR}\left[\widehat{c}_{t+1}^U\right] = \bar{m}\mathbb{E}_t\left[\widehat{c}_{t+1}^U\right]. \tag{49}$$

Now, take the linearized Euler equation (10) of unconstrained households:

$$\widehat{c}_t^U = s\mathbb{E}_t^{BR}\left[\widehat{c}_{t+1}^U\right] + (1-s)\mathbb{E}_t^{BR}\left[\widehat{c}_{t+1}^H\right] - \frac{1}{\gamma}\widehat{r}_t, \tag{50}$$

where $\widehat{r}_t \equiv \widehat{i}_t - \mathbb{E}_t\pi_{t+1}$.

Using the notation in Gabaix (2020), we can write the Euler equation as

$$\widehat{c}^U(X_t) = s\mathbb{E}_t^{BR}\left[\widehat{c}^U(X_{t+1})\right] + (1-s)\mathbb{E}_t^{BR}\left[\widehat{c}^H(X_{t+1})\right] - \frac{1}{\gamma}\widehat{r}(X_t). \tag{51}$$

Now, applying the results above, we obtain

$$\widehat{c}^U(X_t) = s\bar{m}\mathbb{E}_t\left[\widehat{c}^U(X_{t+1})\right] + (1-s)\bar{m}\mathbb{E}_t\left[\widehat{c}^H(X_{t+1})\right] - \frac{1}{\gamma}\widehat{r}(X_t), \tag{52}$$

which after writing $\widehat{c}^U(X_t)$, $\widehat{c}^U(X_{t+1})$ and $\widehat{c}^H(X_{t+1})$ in terms of total output yields exactly the behavioral HANK IS equation in Proposition 1.

A.9 Microfounding $\bar{m}$

Gabaix (2020) shows how to microfound $\bar{m}$ from a noisy signal extraction problem in the case of a representative agent. Following these lines, we show how this signal-extraction problem generates a setup in which the family head behaves as if she was boundedly rational.

The (linearized) law of motion of the state variable, X_t, is given by $X_{t+1} = \Gamma X_t + \varepsilon_{t+1}$ (a similar reasoning extends to the non-linearized case), where X has been demeaned. Now assume that every agent j within the family of unconstrained households (the expectations of the hand-to-mouth agents are irrelevant) receives a noisy signal of X_{t+1}, S^j_{t+1}, given by

$$S^j_{t+1} = \begin{cases} X_{t+1} & \text{with probability } p \\ X'_{t+1} & \text{with probability } 1-p \end{cases}$$

where X'_{t+1} is an i.i.d. draw from the unconditional distribution of X_{t+1}, which has an unconditional mean of zero. In words, with probability p the agent j receives perfectly precise information and with probability $1-p$ agent j receives a signal realization that is completely uninformative. A fully-informed rational agent would have $p = 1$.

The conditional mean of X_{t+1}, given the signal S^j_{t+1}, is given by

$$X^e_{t+1} \equiv \mathbb{E}\left[X_{t+1} | S_{t+1} = s^j_{t+1}\right] = p \cdot s^j_{t+1}.$$ [20]

The intuition is that the signal distribution is such that the agent either receives a perfectly precise signal or a completely uninformative signal. As the perfectly-precise signal arrives with probability p and the unconditional mean is zero, it follows that the agent puts a weight p on the signal.

[20] To see this, note that the joint distribution of (X_{t+1}, S^j_{t+1}) is

$$f(x_{t+1}, s^j_{t+1}) = pg(s^j_{t+1})\delta_{s^j_{t+1}}(x_{t+1}) + (1-p)g(s^j_{t+1})g(x_{t+1}),$$

where $g(X_{t+1})$ denotes the distribution of X_{t+1} and δ is the Dirac function. Given that the unconditional mean of X_{t+1} is 0, i.e., $\int x_{t+1}g(x_{t+1})dx_{t+1} = 0$, it follows that

$$\begin{aligned}
\mathbb{E}_t\left[X_{t+1} | S^j_{t+1} = s^j_{t+1}\right] &= \frac{\int x_{t+1} f(x_{t+1}, s^j_{t+1}) dx_{t+1}}{\int f(x_{t+1}, s^j_{t+1}) dx_{t+1}} \\
&= \frac{pg(s^j_{t+1})s^j_{t+1} + (1-p)g(s^j_{t+1}) \int x_{t+1} g(x_{t+1}) dx_{t+1}}{g(s^j_{t+1})} \\
&= ps^j_{t+1}.
\end{aligned}$$

Furthermore, we have

$$\mathbb{E}\left[S_{t+1}|X_{t+1}\right] = pX_{t+1} + (1-p)\mathbb{E}\left[X'_{t+1}\right] = pX_{t+1}.$$

So, it follows that the *average* expectation of X_{t+1} within the family is given by

$$\begin{aligned}\mathbb{E}\left[X^e_{t+1}(S_{t+1})|X_{t+1}\right] &= \mathbb{E}\left[p \cdot S_{t+1}|X_{t+1}\right] \\ &= p \cdot \mathbb{E}\left[S_{t+1}|X_{t+1}\right] \\ &= p^2 X_{t+1}.\end{aligned}$$

Defining $\bar{m} \equiv p^2$ and since $X_{t+1} = \Gamma X_t + \varepsilon_{t+1}$, we have that the family head perceives the law of motion of X to equal

$$X_{t+1} = \bar{m}\left(\Gamma X_t + \varepsilon_{t+1}\right), \tag{53}$$

as imposed in equation (47). The boundedly-rational expectation of X_{t+1} is then given by

$$\mathbb{E}^{BR}_t\left[X_{t+1}\right] = \bar{m}\mathbb{E}_t\left[X_{t+1}\right].$$

B Calibration

Our baseline calibration is summarized in Table 1. The values for γ and κ are directly taken from Bilbiie (2021, 2020) and are quite standard in the literature. Gabaix (2020), however, sets $\kappa = 0.11$ and $\gamma = 5$. Even though these coefficients differ quite substantially from our baseline calibration, note that our results would barely be affected by this. To see this, note that *amplification* is only determined by λ and χ, both independent of κ and γ. The determinacy condition on the other hand depends on both, κ and γ, but what ultimately matters is the fraction $\frac{\kappa}{\gamma}$ (see Proposition 3). As κ and γ are both approximately five times larger in Gabaix (2020) compared to Bilbiie (2021) and our baseline calibration, the fraction is approximately the same and thus, the determinacy region under an interest-rate peg remains unchanged.

Table 1: Baseline Calibration

Parameter	Description	Value
γ	Risk Aversion	1
κ	Slope of NKPC	0.02
χ	Business-Cycle Exposure of H	1.5
λ	Share of H	0.33
s	Type-Switching Probability	$0.8^{1/4}$
β	Time Discount Factor	0.99
$\bar{m}$	Cognitive Discounting Parameter	0.85

The household heterogeneity parameters, χ, λ and s are also standard in the analytical HANK literature (see Bilbiie (2020)). The most important assumption for our qualitative results in Section 3 is $\chi > 1$, which is consistent with the data. Patterson (2019) provides empirical evidence for the countercyclicality of inequality. Coibion et al. (2017), Mumtaz and Theophilopoulou (2017), Ampudia et al. (2018) and Samarina and Nguyen (2019) provide evidence of countercyclical inequality conditional on monetary policy shocks. Almgren et al. (2019) show that output in countries with higher shares of hand-to-mouth households responds more strongly to monetary policy shocks which, through the lens of the model, calls for $\chi > 1$.

For figure 5, i.e., to compute the iMPCs we choose a yearly calibration with $s = 0.8$ and $\beta = 0.95$ (this calibration is close to the iMPC exercise in Bilbiie (2021) but while he fixes χ to match the empirically-observed iMPCs, we vary χ together with $\bar{m}$ to examine their joint effects on iMPCs).

The Cognitive Discounting Parameter $\bar{m}$. The cognitive discounting parameter $\bar{m}$ is set to 0.85, as in Gabaix (2020) and Benchimol and Bounader (2019). Fuhrer and Rudebusch

(2004), for example, estimate an IS equation and find that $\psi_f \approx 0.65$, which together with $\delta > 1$, would imply a $\bar{m}$ much lower than 0.85 and especially our determinacy results would be even stronger under such a calibration.

Another way to calibrate $\bar{m}$ (as pointed out in Gabaix (2020)) is to interpret the estimates in Coibion and Gorodnichenko (2015) through the "cognitive-discounting lens". They regress forecast errors on forecast revisions

$$x_{t+h} - F_t x_{t+h} = c + b^{CG} \left(F_t x_{t+h} - F_{t-1} x_{t+h} \right) + u_t,$$

where $F_t x_{t+h}$ denotes the forecast at time t of variable x, h periods ahead. Focusing on inflation, they find that $b^{CG} > 0$ in consensus forecasts, pointing to *underreaction* (similar results are, for example, found in Angeletos et al. (2021) and Adam et al. (2022) for other variables).

In the model, the law of motion of x is $x_{t+1} = \Gamma \left(x_t + \varepsilon_{t+1} \right)$ whereas the behavioral agents perceive it to be $x_{t+1} = \bar{m}\Gamma \left(x_t + \varepsilon_{t+1} \right)$. It follows that $F_t x_{t+h} = \left(\bar{m}\Gamma \right)^h x_t$ and thus, forecast revisions are equal to

$$\begin{aligned} F_t x_{t+h} - F_{t-1} x_{t+h} &= \left(\bar{m}\Gamma \right)^h x_t - \left(\bar{m}\Gamma \right)^{h+1} x_{t-1} \\ &= \left(\bar{m}\Gamma \right)^h \Gamma(1-\bar{m}) x_{t-1} + \left(\bar{m}\Gamma \right)^h \varepsilon_t. \end{aligned}$$

The forecast error is given by

$$x_{t+h} - F_t x_{t+h} = \Gamma^h (1 - \bar{m}^h) \Gamma x_{t-1} + \Gamma^h (1 - \bar{m}^h) \varepsilon_t + \sum_{j=0}^{h-1} \Gamma^j \varepsilon_{t+h-j},$$

where $\sum_{j=0}^{h-1} \Gamma^j \varepsilon_{t+h-j}$ is the rational expectations forecast error. Gabaix (2020) shows that b^{CG} is bounded below $b^{CG} \geq \frac{1-\bar{m}^h}{\bar{m}^h}$, showing that $\bar{m} < 1$ yields $b^{CG} > 0$, as found empirically. When replacing the weak inequality with an equality, we get

$$\bar{m}^h = \frac{1}{1 + b^{CG}}.$$

Most recently, Angeletos et al. (2021) estimate b^{CG} (focusing on a horizon $h = 3$) to lie between $b^{CG} \in [0.74, 0.81]$ for unemployment forecasts and $b^{CG} \in [0.3, 1.53]$ for inflation, depending on the considered period (see their Table 1). These estimates imply $\bar{m} \in [0.82, 0.83]$ for unemployment and $\bar{m} \in [0.73, 0.92]$ for inflation, and are thus close to our preferred value of 0.85. Note, however, that these estimates pertain to professional forecasters and should therefore be seen as upper bounds on $\bar{m}$. We provide direct evidence on $\bar{m}$ for households (of different income groups) in Appendix E.1. We find that households are less rational than professional forecasters.

C Figures to Section 3

C.1 Resolving the Catch-22

We graphically illustrate the Catch-22 of the rational model and the resolution of it in the behavioral HANK model in Figure 8. The figure shows on the vertical axis the response of contemporaneous output relative to the initial response in the RANK model with rational expectations for anticipated i.i.d. monetary policy shocks occurring at different times k on the horizontal axis.[21]

The orange-dotted line represents the baseline calibration of the rational HANK model. We see that this model is able to generate contemporaneous amplification of monetary policy shocks, that is, an output response that is relatively stronger than in RANK. Put differently the GE effects amplify the effects of monetary policy shocks. Yet, at the same time, it exacerbates the forward guidance puzzle as shocks occurring in the future have even stronger effects on today's output than contemporaneous shocks.

The black-dashed-dotted line shows how the forward guidance puzzle can be resolved by allowing for $\chi < 1$. Yet, this comes at the cost that the model is unable to generate amplification of contemporaneous monetary policy shocks. Recent empirical findings, however, document that GE effects indeed amplify monetary policy changes (Auclert (2019)).

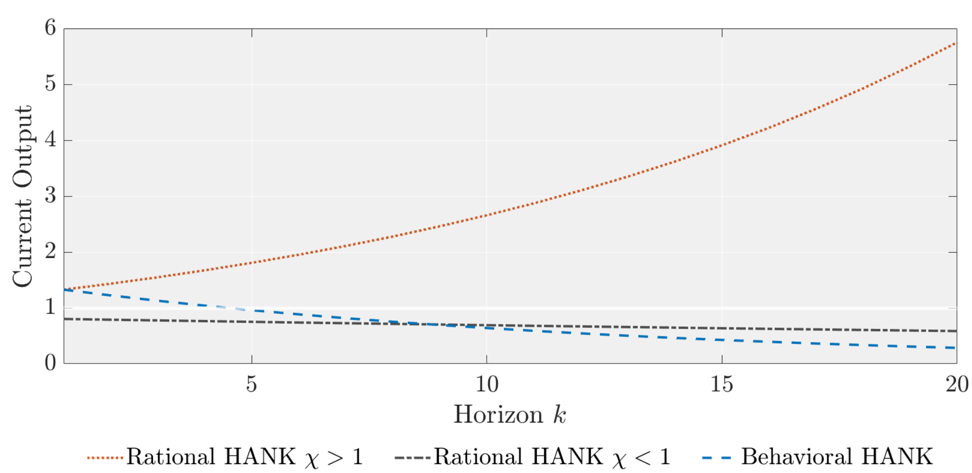


Figure 8: Resolving the Catch-22

Note: This figure shows the response of total output in period 0 to anticipated i.i.d. monetary policy shocks occurring at different horizons k (horizontal axis), relative to the initial response in the RANK model under rational expectations (equal to 1).

[21] Under fully-rigid prices (i.e., $\kappa = 0$) the RANK model would deliver a constant response for all k. The same is true for two-agent NK models (TANK), i.e., tractable HANK models without type switching. Whether the constant response would lie above or below its RANK counterpart depends on $\chi \lesseqgtr 1$ in the same way the initial response depends on $\chi \lesseqgtr 1$.

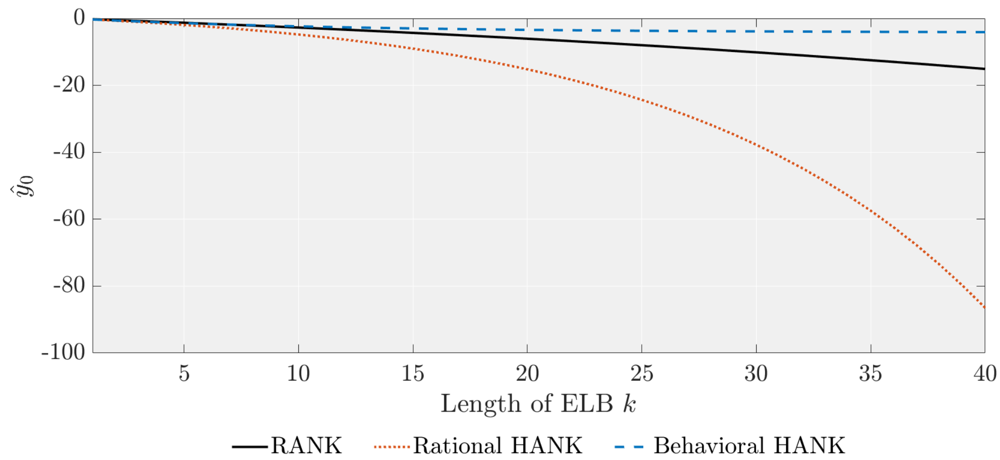


Figure 9: The Effective Lower Bound Problem

Note: This figure shows the contemporaneous output response for different lengths of a binding ELB k (horizontal axis) and compares the responses across different models.

The blue-dashed line shows that the behavioral HANK model, on the other hand, generates both: amplification of contemporaneous monetary policy and a resolution of the forward guidance puzzle, both consistent with the empirical facts.

C.2 Stability at the Effective Lower Bound

We illustrate the stability of the behavioral HANK model at the lower bound graphically in Figure 9. The figure shows the output response in RANK, the rational HANK and the behavioral HANK to different lengths of a binding ELB (depicted on the horizontal axis). The shortcoming of monetary policy due to the ELB, i.e., the gap $\left(\widehat{i}_{ELB} - \widetilde{r}^n\right) > 0$, is set to a relatively small value of 0.25% (1% annually), and we set $\bar{m} = 0.85$. Figure 9 shows the implosion of output in the rational RANK (back-solid line) and even more so in the rational HANK model (orange-dotted line): an ELB that is expected to bind for 40 quarters would decrease today's output in the rational RANK by 15% and in the rational HANK model by 85%. On the other hand—and consistent with recent experiences in advanced economies—output in the behavioral HANK model remains quite stable and drops by a mere 4%, as illustrated by the blue-dashed line.

C.3 Fiscal Multipliers

Figure 10 illustrates the effect of bounded rationality on the fiscal multiplier by plotting the fiscal multiplier in the behavioral HANK model for varying degrees of $\bar{m}$ (blue-solid line) and compares it to the multiplier in the rational HANK model and RANK. For this exercise, we set the persistence parameter to an intermediate value $\rho_g = 0.6$. It shows that the fiscal multiplier decreases with decreasing $\bar{m}$. Yet, even for the extreme case of $\bar{m} = 0$, in which households fully discount all future increases in government spending the fiscal multiplier is still substantially above zero even though it is somewhat weaker than under rational expectations. In fact, the behavioral HANK model generates consumption responses to fiscal spending that are quantitatively close to the empirical estimates in Dupor et al. (2021) who estimate the non-durable consumption response to lie between 0.2 and 0.29. Note, that we did not target this moment.

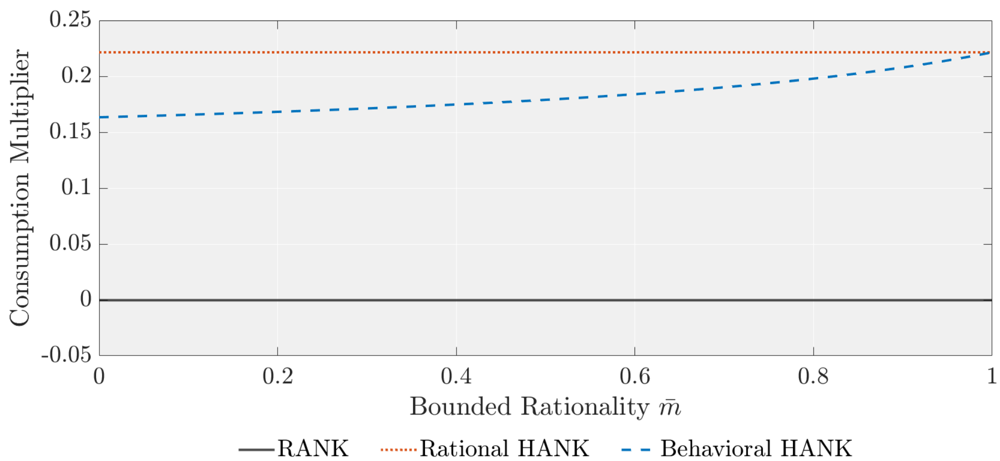


Figure 10: Consumption Response to Government Spending

Note: This figure shows the consumption multipliers (the consumption response to government spending) for different degrees of bounded rationality (blue-dashed line). The orange-dotted line plots the multiplier in the rational version of the model and the black-solid line shows the zero-multiplier in the RANK model.

D Further Extensions

D.1 Allowing for Steady State Inequality

So far, we have assumed that there is no steady state inequality, i.e., $C^H = C^U$. In the following, we relax this assumption and denote steady state inequality by $\Omega \equiv \frac{C^U}{C^H}$. Recall the Euler equation of unconstrained households

$$\left(C_t^U\right)^{-\gamma} = \beta R_t \mathbb{E}_t^{BR}\left[s\left(C_t^U\right)^{-\gamma} + (1-s)\left(C_t^H\right)^{-\gamma}\right],$$

from which we can derive the steady state real rate

$$R = \frac{1}{\beta(s+(1-s)\Omega^\gamma)}.$$

Log-linearizing the Euler equation yields

$$\widehat{c}_t^U = \beta R\bar{m}\left[s\mathbb{E}_t\widehat{c}_{t+1}^U + (1-s)\Omega^\gamma\mathbb{E}_t\widehat{c}_{t+1}^H\right] - \frac{1}{\gamma}\left(\widehat{i}_t - \mathbb{E}_t\pi_{t+1}\right).$$

Combining this with the consumption functions and the steady state real rate yields the IS equation

$$\widehat{y}_t = \bar{m}\tilde{\delta}\mathbb{E}_t\widehat{y}_{t+1} - \frac{1}{\gamma}\frac{1-\lambda}{1-\lambda\chi}\left(\widehat{i}_t - \mathbb{E}_t\pi_{t+1}\right), \tag{54}$$

with

$$\tilde{\delta} \equiv 1 + (\chi - 1)\frac{(1-s)\Omega^\gamma}{s+(1-s)\Omega^\gamma}\frac{1}{1-\lambda\chi}.$$

From a qualitative perspective, the whole analysis in the paper could be carried out with $\tilde{\delta}$ instead of δ. Quantitatively the differences are small as well. For example, if we set $\Omega = 1.5$, we get $\tilde{\delta} = 1.074$ instead of $\delta = 1.054$. Thus, we need $\bar{m} < 0.91$ instead of $\bar{m} < 0.92$ for determinacy under a peg.

D.2 Persistent Monetary Policy Shocks

In the main text in Section 3, we illustrated the resolution of the Catch-22 by considering i.i.d. monetary policy shocks (following Bilbiie (2021)). The behavioral HANK model delivers initial amplification of these monetary shocks but the effects decrease with the horizon of the shock, i.e., the behavioral HANK model resolves the forward guidance puzzle. Another way to see this is by considering persistent shocks.

Figure 11 illustrates this. The figure shows the response of output in period t to a shock in period t for different degrees of persistence (x-axis). The black-solid line shows the output response in RANK and the blue-dashed line in the behavioral HANK. The forward guidance puzzle in RANK manifests itself in the sense that highly persistent shocks have stronger effects in RANK than in the behavioral HANK. Persistent shocks are basically a form of forward guidance and thus, with high enough persistence in the shocks, the RANK model predicts stronger effects than the behavioral HANK model.

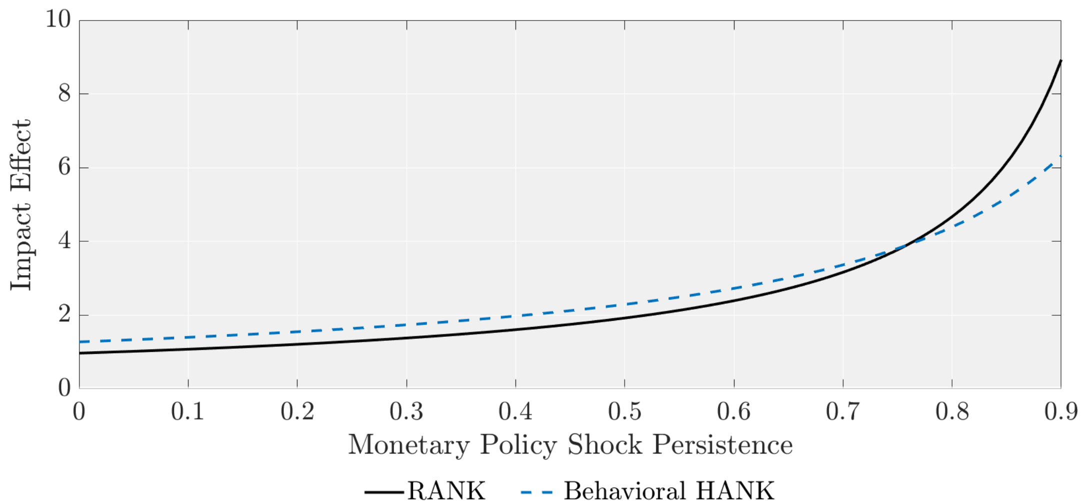


Figure 11: Initial Output Response for Varying Degrees of the Persistence

Note: This figure shows the initial output response to monetary policy shocks with different degrees of persistence.

As the persistence of the monetary policy shock approaches unity, the rational model leads to the paradoxical finding that an exogenous increase in the nominal interest rate leads to an expansion in output. To see this, note that we can write output as

$$\widehat{y}_t = -\frac{\frac{\psi_c}{\gamma}}{1 + \frac{\psi_c}{\gamma}\phi\kappa - \left(\psi_f + \psi_c \frac{\kappa}{\gamma}\right)\rho}\varepsilon_t^{MP}. \tag{55}$$

Given our baseline calibration and a Taylor coefficient of $\phi = 1.5$, the rational model would produce these paradoxical findings for $\rho > 0.97$. The behavioral HANK model, on the other hand, does not suffer from this as the denominator is always positive, even when $\phi = 0$ and $\rho = 1$.

D.3 Forward-Looking NKPC and Real Interest Rates

In the main part of the paper, we made the assumption that agents are rational with respect to real interest rates (as in Gabaix (2020)) and assumed a static Phillips Curve (as in Bilbiie (2021)). We now show that the results are barely affected when considering a forward-looking New Keynesian Phillips Curve (NKPC) and that agents are also boundedly rational with respect to real rates. Gabaix (2020) derives the NKPC under bounded rationality and shows that it takes the form:

$$\pi_t = \beta M^f \mathbb{E}_t \pi_{t+1} + \kappa \widehat{y}_t,$$

with

$$M^f \equiv \bar{m} \left(\theta + \frac{1 - \beta\theta}{1 - \beta\theta\bar{m}} (1 - \theta) \right),$$

where $1 - \theta$ captures the Calvo probability of price adjustment.

Taking everything together (including the bounded rationality with respect to real interest rates), the model can be summarized by the following three equations:

$$\begin{aligned}
\widehat{y}_t &= \psi_f \mathbb{E}_t \widehat{y}_{t+1} - \psi_c \frac{1}{\gamma} \left(\widehat{i}_t - \bar{m} \mathbb{E}_t \pi_{t+1} \right) \\
\pi_t &= \beta M^f \mathbb{E}_t \pi_{t+1} + \kappa \widehat{y}_t \\
\widehat{i}_t &= \phi \pi_t.
\end{aligned}$$

Plugging the Taylor rule into the IS equation, we can write everything in matrix form:

$$\begin{pmatrix} \mathbb{E}_t \pi_{t+1} \\ \mathbb{E}_t \widehat{y}_{t+1} \end{pmatrix} = \underbrace{\begin{pmatrix} \frac{1}{\beta M^f} & -\frac{\kappa}{\beta M^f} \\ \frac{\psi_c}{\gamma \psi_f} \left(\phi - \frac{\bar{m}}{\beta M^f} \right) & \frac{1}{\psi_f} \left(1 + \frac{\psi_c \bar{m} \kappa}{\gamma \beta M^f} \right) \end{pmatrix}}_{\equiv A} \begin{pmatrix} \pi_t \\ \widehat{y}_t \end{pmatrix}. \tag{56}$$

For determinacy, we need

$$det(A) > 1; \quad det(A) - tr(A) > -1; \quad det(A) + tr(A) > -1.$$

The last condition is always satisfied. The first two conditions are satisfied if and only if

$$\phi > \max \left\{ \frac{\beta \delta M^f \bar{m} - 1}{\frac{\kappa}{\gamma} \frac{1-\lambda}{1-\lambda\chi}}, \bar{m} + \frac{(\delta \bar{m} - 1)(1 - \beta M^f)}{\frac{\kappa}{\gamma} \frac{1-\lambda}{1-\lambda\chi}} \right\}.$$

In the case of a static Phillips curve but bounded rationality with respect to the real rate, the second condition is the crucial one. To capture the static Phillips curve, we can simply set $M^f = 0$. We can see that bounded rationality with respect to the real rate relaxes the determinacy condition whereas a forward-looking NKPC tightens it. But even in the case of a forward-looking NKPC (rational or behavioral), cognitive discounting relaxes the determinacy condition and thus, all our results from the static Phillips curve are qualitatively unchanged.

E Quantitative Behavioral HANK Model

Table 2 shows how we calibrate the quantitative model introduced in Section 4. The calibration closely follows the parameterization in McKay et al. (2016). As in McKay et al. (2016), we assume that high productivity households pay all the taxes. The main difference to their calibration is that they assume that every household receives an equal share of the dividends whereas we assume that the high productivity households receive 80% of the dividend payments, while the middle productivity households receive 20% of it. The low productivity households do not receive any dividend payments. We choose this calibration such that the contemporaneous amplification in the quantitative HANK model matches the one from the tractable model, outlined in Section 2. Note that this dividend distribution is in line with empirical findings in Kuhn et al. (2020).

Table 2: Baseline Calibration Of Quantitative HANK Model

Parameter	Description	Value
R	Steady State Real Rate (annualized)	2%
γ	Risk aversion	2
φ	Inverse of Frisch elasticity	2
μ	Markup	1.2
θ	Calvo Price Stickiness	0
ρ_e	Autocorrelation of idiosyncratic risk	0.966
σ_e^2	Variance of idiosyncratic risk	0.0384
$\tau(e)$	Tax shares	[0, 0, 1]
$d(e)$	Dividend shares	$[0, \frac{0.2}{0.5}, \frac{0.8}{0.25}]$
$\frac{B^G}{4Y}$	Total wealth	0.625

Resolving the forward guidance puzzle in a rational quantitative HANK model. McKay et al. (2016) resolve the forward guidance puzzle by assuming that every household receives an equal share of the dividends, leading to pro-cyclical inequality. Thus, the low-productivity households—who also exhibit larger MPCs on average than households with higher productivities—are less exposed to monetary policy. Therefore, the effectiveness of monetary policy is dampened overall, leading to a resolution of the forward guidance puzzle but also to a dampening of contemporaneous shocks, as shown in Figure 12.

Stability at the ELB. Figure 13 shows the output and nominal interest rate response after a shock to the discount factor in the quantitative behavioral HANK model and in its rational counterpart. In particular, the discount factor jumps on impact by 0.8% for 12 quarters before it returns to steady state.

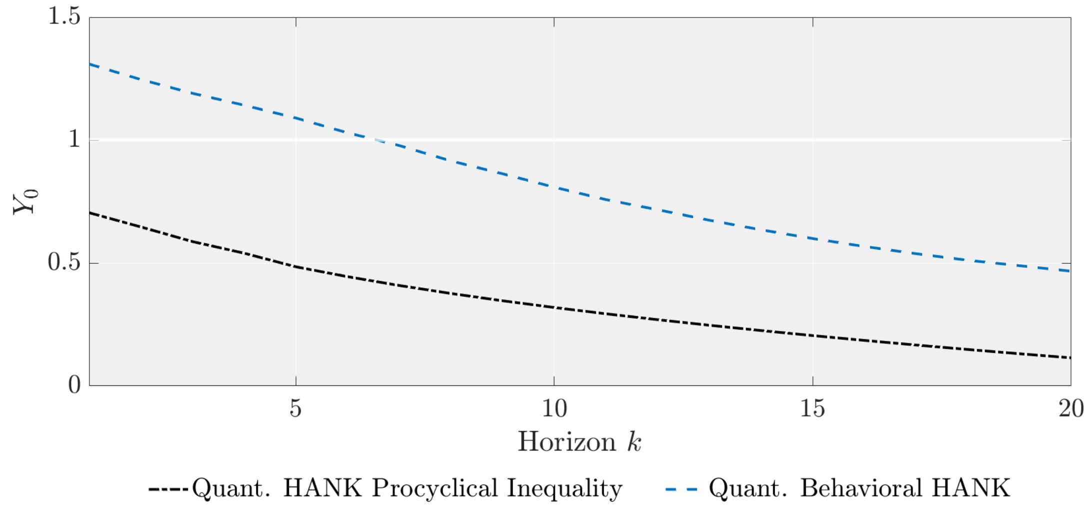
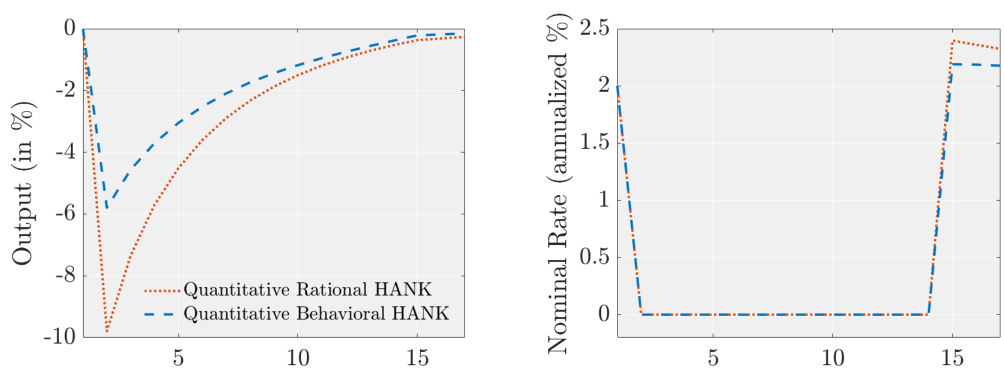


Figure 12: Resolving the Forward Guidance Puzzle in HANK

Note: This figure shows the response of total output in period 0 to anticipated i.i.d. monetary policy shocks occurring at different horizons k, relative to the response in the RANK model under rational expectations (equal to 1).

Figure 13: ELB recession in the quantitative behavioral HANK model



Note: This figure shows the impulse responses of total output and of the nominal interest rate after a discount factor shock that brings the economy to the ELB for 12 quarters.

Fiscal Multiplier. To verify that the quantitative behavioral HANK model generates positive consumption multiplier under a constant real rate, we redo the experiments in Section 3.3: the government exogeneously increases government consumption (which is assumed to be zero in steady state) which follows an AR(1)-process. The increase in government consumption is immediately financed by lump-sum taxes. Figure 14 shows the impact multiplier on consumption for various degrees of persistence, ρ_G. It shows that while the multiplier increases in persistence, it is bounded from below by zero. In other words, also the quantitative behavioral HANK model generates positive consumption multipliers, such that, the result from our tractable model carries over to the behavioral HANK model.

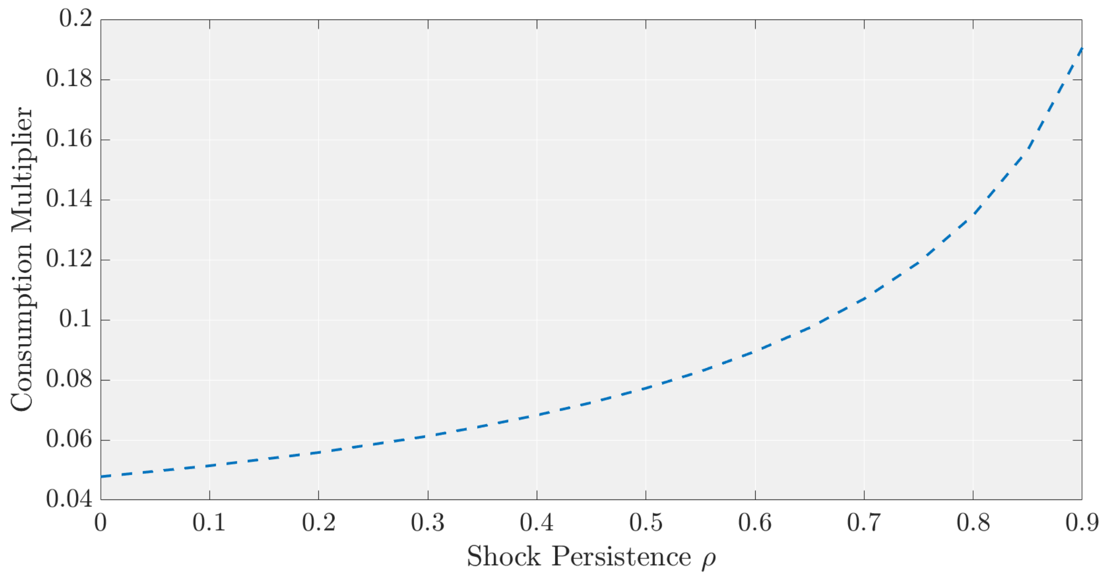


Figure 14: Consumption multiplier in the quantitative behavioral HANK

Note: This figure shows the impact consumption multiplier after an exogenous increase in government consumption which is financed by lump-sum taxes for various degrees of persistence.

E.1 Heterogeneous $\bar{m}$

To test for heterogeneity in the degree of cognitive discounting, we follow Coibion and Gorodnichenko (2015) and regress forecast errors on forecast revisions as follows

$$x_{t+4} - \mathbb{E}_t^{e,BR} x_{t+4} = c^e + b^{e,CG} \left(\mathbb{E}_t^{e,BR} x_{t+4} - \mathbb{E}_{t-1}^{e,BR} x_{t+4} \right) + \epsilon_t^e, \tag{57}$$

to estimate $b^{e,CG}$ for different groups of households, indexed by e. As shown in Appendix B, $b^{e,CG} > 0$ is consistent with underreaction and the corresponding cognitive discounting parameter is approximately given by

$$\bar{m}^e = \left(\frac{1}{1 + b^{e,CG}} \right)^{1/4}. \tag{58}$$

Ideally, we would use actual data and expectations data about future marginal utilities of consumption which, however, are not available. Instead, we focus on expectations about future unemployment. The Survey of Consumers from the University of Michigan provides 1-year ahead unemployment expectations and we use the unemployment rate from the FRED database as our measure of actual unemployment. Consistent with the model, we split the households into three groups based on their income. The bottom and top income groups each contain the 25% households with the lowest and highest income, respectively, and the remaining 50% are assigned to the middle income group.

The Michigan Survey asks households whether they expect unemployment to increase, decrease or to remain about the same over the next twelve months. We follow Carlson and Parkin (1975), Mankiw (2000) and Bhandari et al. (2019) to translate these categorical unemployment expectation into numerical expectations.

Focus on group $e \in \{L, M, H\}$ and let $q_t^{e,D}$, $q_t^{e,S}$ and $q_t^{e,U}$ denote the shares within income group e reported at time t that think unemployment will go down, stay roughly the same, or go up over the next year, respectively. We assume that these shares are drawn from a cross-sectional distribution of responses that are normally distributed according to $\mathcal{N}\left(\mu_t^e, (\sigma_t^e)^2\right)$ and a threshold a such that when a household expects unemployment to remain within the range $[-a, a]$ over the next year, she responds that unemployment will remaine "about the same". We thus have

$$q_t^{e,D} = \Phi\left(\frac{-a - \mu_t^e}{\sigma_t^e}\right) \qquad q_t^{e,U} = 1 - \Phi\left(\frac{a - \mu_t^e}{\sigma_t^e}\right),$$

which after some rearranging yields

$$\sigma_t^e = \frac{2a}{\Phi^{-1}\left(1 - q_t^{e,U}\right) - \Phi^{-1}\left(q_t^{e,D}\right)}$$
$$\mu_t^e = a - \sigma_t^e \Phi^{-1}\left(1 - q_t^{e,U}\right).$$

This leaves us with one degree of freedom, namely a. We make two assumptions. First, a is independent of the income group. The second assumption is that we set $a = 0.5$ which means that if a household expects the change in unemployment to be less than half a percentage point (in absolute terms), she reports that she expects unemployment to be about the same as it is at the time of the survey. We discuss different a later on.

As the question in the survey is about the expected change in unemployment, we add the actual unemployment rate at the time of the survey to μ_t^e to construct a time-series of unemployment expectations, as in Bhandari et al. (2019). That said, we will also report the case of expected unemployment *changes*.

Given the so-constructed expectations, we can compute forecast revisions as

$$\mu_t^e - \mu_{t-1}^e$$

and four-quarter-ahead forecast errors using the actual unemployment rate u_t obtained from FRED as

$$u_{t+4} - \mu_t^e. \tag{59}$$

For the case of expected unemployment changes, we replace u_{t+4} with $(u_{t+4} - u_t)$ in equation (59).

Following Coibion and Gorodnichenko (2015), we then regress forecast errors on forecast revisions

$$u_{t+4} - \mu_t^e = c^e + b^{e,CG}\left(\mu_t^e - \mu_{t-1}^e\right) + \epsilon_t^e, \tag{60}$$

to estimate $b^{e,CG}$ for each income group e. Note, however, that the expectations in the forecast revisions are about unemployment at different points in time. To account for this, we instrument forecast revisions by the *main business cycle shock* obtained from Angeletos et al. (2020).

Table 3: Regression Results of Equation (57)

	IV Regression			OLS		
	Bottom 25%	Middle 50%	Top 25%	Bottom 25%	Middle 50%	Top 25%
$\widehat{b}^{e,CG}$	0.85	0.75	0.63	1.22	1.10	0.90
s.e.	(0.471)	(0.453)	(0.401)	(0.264)	(0.282)	(0.247)
F-stat.	24.76	18.74	17.86	-	-	-
N	152	152	152	157	157	157

Note: This table provides the estimated $\widehat{b}^{e,CG}$ from regression (57) for different income groups. The first three columns show the results when the right-hand side in equation (57) is instrumented using the *main business cycle shock* from Angeletos et al. (2020) and the last three columns using OLS. Standard errors are robust with respect to heteroskedasticity and are reported in parentheses. The row "F-stat." reports the first-stage F-statistic for the IV regressions.

Table 3 shows the results. The first three columns report the estimated $b^{e,CG}$ from the IV regressions and the last three columns the same coefficients estimated via OLS. Standard errors are robust with respect to heteroskedasticity and are reported in parentheses. The row "F-stat." reports the first-stage F-statistic for the IV regressions. We see that in all cases $\widehat{b}^{e,CG}$ is positive, suggesting that households of all income groups tend to underreact, consistent with our assumption of $\bar{m} < 1$.

Using equation (58) we obtain $\bar{m}^e$ equal to 0.86, 0.87 and 0.88 for the bottom 25%, the middle 50% and the top 25%, respectively for the estimates from the IV regressions and 0.82, 0.83 and 0.85 for the OLS estimates. When estimating $\bar{m}^e$ using expected unemployment *changes* instead of the level, the estimated $\bar{m}^e$ equal 0.57, 0.59 and 0.64 for the IV regressions and 0.77, 0.80 and 0.86 for the OLS regressions.

There are two main take-aways from this empirical exercise: first, it further confirms that $\bar{m} = 0.85$ is a reasonable (but rather conservative) deviation from rational expectations. Second, the data suggests that there is heterogeneity in the degree of rationality conditional on households income. In particular, households with higher income tend to exhibit higher degrees of rationality.[22]

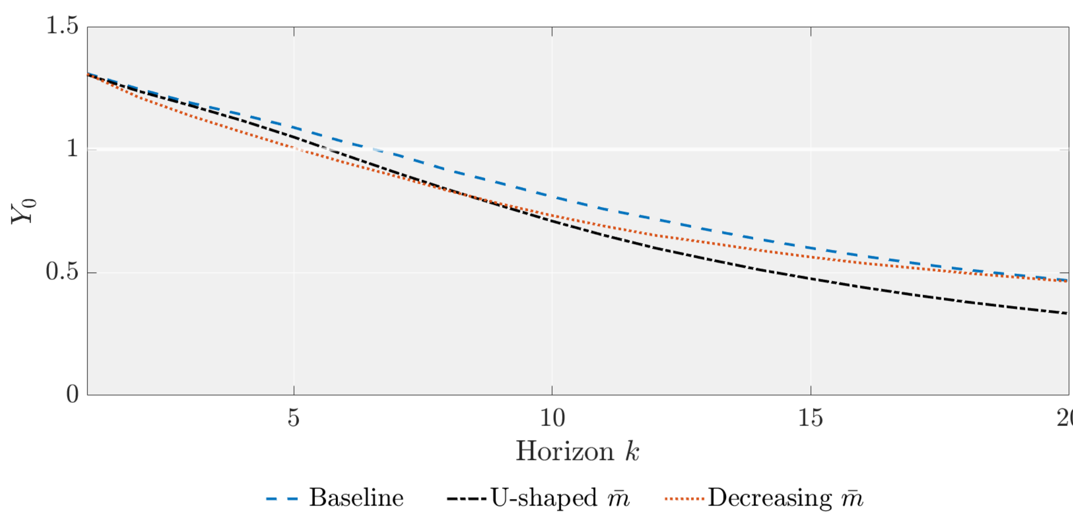
[22]This is consistent with other empirical findings on heterogeneous deviations from FIRE. Broer et al.

If we consider inflation expectations instead of unemployment expectations, we obtain estimated cognitive discounting parameters of 0.70, 0.75 and 0.78 for the bottom 25%, the middle 50% and the top 25%, respectively. Thus, somewhat lower than for unemployment and the differences across income groups are larger. In particular, higher-income households tend to be more attentive (they discount less) than lower-income households. The differences, however, are overall rather small.

E.1.1 Heterogeneous $\bar{m}$: Alternative Scenarios

Empirically, we document that richer households tend to deviate somehwat less from rational expectations than poorer households. Broer et al. (2021a) find that the relation between income and forecast accuracy is non-monotonic. In particular, they find that relatively rich and poor households tend to make smaller forecast errors than households with medium level income. To mirror this, we set $\bar{m} = 0.9$ for the high- and low-productivity households and $\bar{m} = 0.8$ for the medium-productivity households. Given that 50% of households fall into the medium category, this calibration again features an average $\bar{m}$ of 0.85. The black-dashed-dotted line in Figure 15 shows the results when re-running the monetary policy experiments outlined in Section 4.

Figure 15: Heterogeneous $\bar{m}$ and Monetary Policy



Note: This figure shows the response of total output in period 0 to anticipated i.i.d. monetary policy shocks occurring at different horizons k for the baseline calibration with $\bar{m} = 0.85$ for all households (blue-dashed line), for the model in which low- and high productivity households have $\bar{m} = 0.9$ and medium-level productivity households have $\bar{m} = 0.9$ (black-dashed-dotted line), and the model with $\bar{m} = 0.9$, $\bar{m} = 0.85$, and $\bar{m} = 0.8$ for low- medium- and high-productivity households, respectively (orange-dotted line).

(2021a), for example, document that wealthier households tend to have more accurate beliefs, as measured by forecast errors.

Overall, the results are quite similar to the baseline calibration. Forward guidance is somewhat weaker, which is driven by the lower $\bar{m}$ of the medium-productivity households. These households are usually unconstrained and thus, respond to forward guidance directly. Since they account for half of all households their lower $\bar{m}$ outweighs the higher $\bar{m}$ of high-productivity households, even though these are even less likely to be constrained. But only 25% of all households are high-productivity households whereas 50% are medium-productivity households.

The orange-dotted line shows the result for the case in which low-productivity households are closest to rational expectations, i.e., when their $\bar{m}$ is set to 0.9 and the high-productivity households have a $\bar{m}$ of 0.8. We see that compared to the baseline calibration, the effectiveness of monetary policy drops faster with the horizon as we increase the horizon.

F Additional Results and Figures to Section 5

F.1 Productivity Shock with High Initial Debt Levels

Figure 16 shows the impulse responses after a negative productivity shock when monetary policy fully stabilizes inflation when the initial debt level is 90% of annual GDP instead of 62.5%.

Compared to Figure 3, we see that government debt increases more strongly when initial debt levels are higher. The increase in the real interest rate is more costly for the government which is financed by an additional increase in debt. Consumption inequality also increases more than at lower levels of government debt, even tough the differences are rather small.

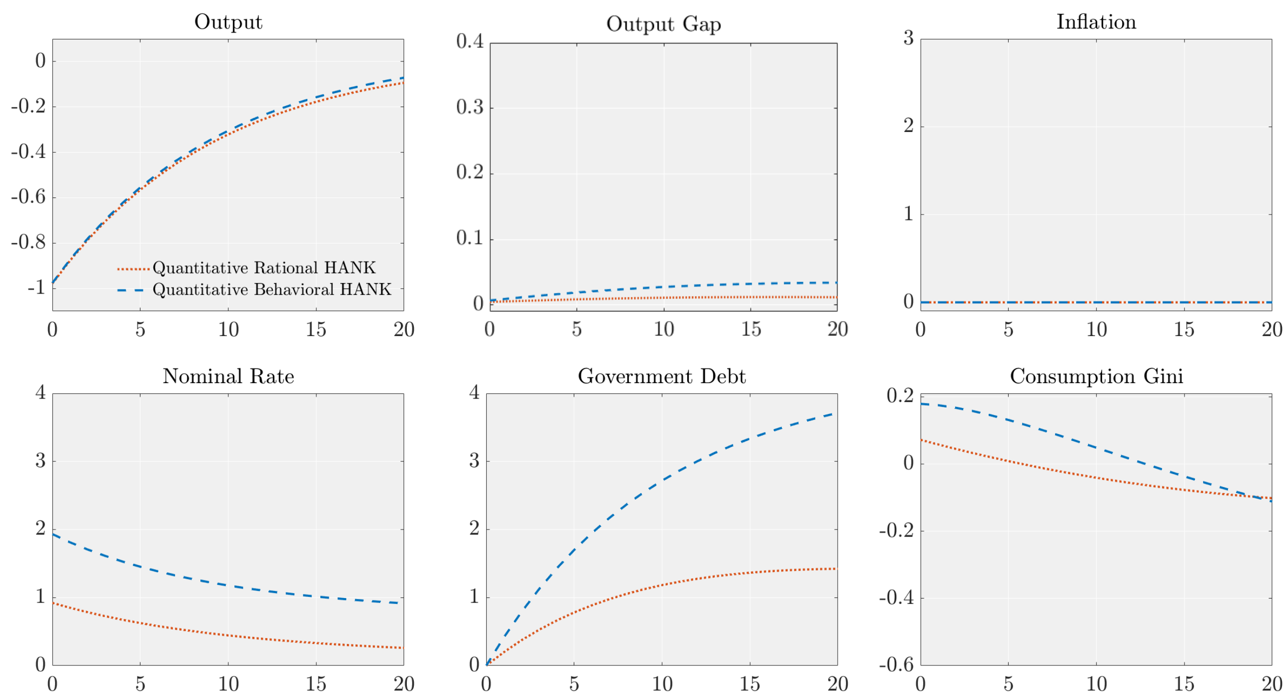


Figure 16: Inflationary productivity shock with high debt: strict inflation targeting

Note: This figure shows the impulse responses after a TFP shock that decreases potential output by 1% in the inflation-stabilizing monetary policy regime when the initial debt level is 90% instead of 62.5% of annual GDP. Output and the output gap are shown as percentage deviations from steady state output, nominal interest rate and inflation as annualized percentage points and government debt level as percentage point deviations in the debt-per annual GDP level. The lower-right figure shows the change in the consumption Gini index as a percentage deviation from the stationary equilibrium.

F.1.1 Cost-Push Shocks

We now show that the fiscal and monetary implications are very similar for an inflationary cost-push shock. To introduce cost-push shocks, we assume that the desired mark-up of firms, μ_t follows an AR(1)-process, $\mu_t = (1 - \rho_\mu)\bar{\mu} + \rho_\mu \mu_{t-1} + \varepsilon_t^\mu$, where ε_t^μ is an i.i.d. shock, $\bar{\mu}$ the steady-state level of the desired markup and ρ_μ the persistence of the shock process which we set to $\rho_\mu = 0.9$. The rest of the model is as in Section 5.

Figure 17 shows the impulse-response functions of output, the output gap, inflation, nominal interest rates, government debt and the consumption Gini index as a measure of consumption inequality following an inflationary cost-push shock. The blue-dashed lines show the responses in the behavioral HANK model and the orange-dotted lines in the rational HANK model. In both cases, monetary policy fully stabilizes inflation by assumption. Output drops in both cases, with the responses being practically identical across the two models. Again, the output gap is almost closed in both models. The required response of the nominal interest rate, however, differs substantially across the two models, as was the case after a negative productivity shock, discussed in Section 5. In the behavioral HANK the monetary authority increases the nominal rate much more strongly and more persistently. The reason for this strong response is that households cognitively discount future (expected) interest rate hikes making them less effective for stabilizing inflation today. Thus, in order to achieve the same stabilization outcome in every period, the interest rate needs to increase by more.

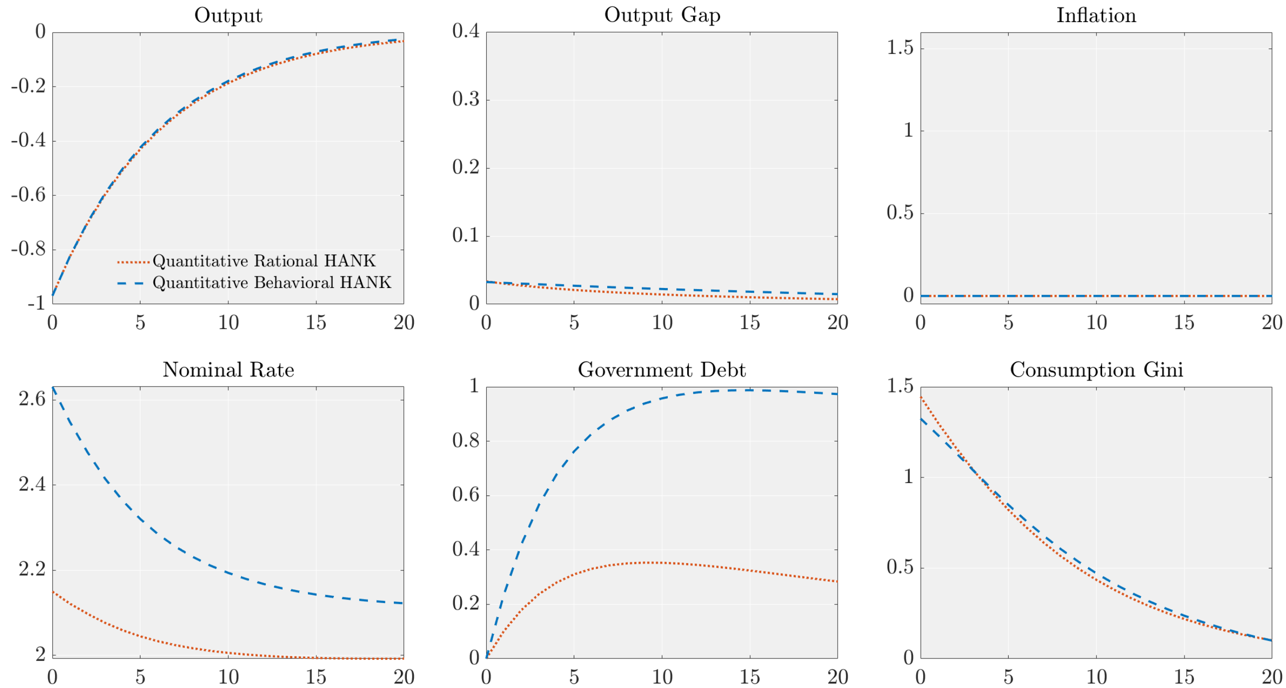


Figure 17: Inflationary cost-push shock: strict inflation targeting

Note: This figure shows the impulse responses after a cost-push shock that decreases potential output by 1% in the inflation-stabilizing monetary policy regime. Output and the output gap are shown as percentage deviations from steady state output, nominal interest rate and inflation as annualized percentage points and government debt level as percentage point deviations in the debt-per annual GDP level. The lower-right figure shows the change in the consumption Gini index as a percentage deviation from the stationary equilibrium.

Increasing the interest rates increases the cost of debt for the government which it finances in the short run by issuing more debt. The middle panel on the bottom line in Figure 17 shows that government debt in the behavioral model increases more than three times as much as in the rational model. Furthermore, consumption inequality increases in both models, somewhat stronger in the rational model. There are two channels: first and most important, the cost-push shock increases dividends and decreases wages which redistributes from low to high productivity households thereby pushing up consumption inequality. Second, the increase in the real interest rate redistributes towards high wealth households but it is the high productivity households who eventually pay the tax burden. This slightly decreases the consumption of high productivity households and increases the consumption of middle productivity households who hold some assets but do not face tax increases. Thus, the second channel slightly dampens the increase in inequality and, as real interest rates increase by more, this channel is stronger in the behavioral HANK model.

Figure 18 shows the impulse-response functions of output, the output gap, inflation, nominal interest rates, government debt (as a share of annual GDP) and consumption inequality for the same cost-push shock but for the case in which monetary policy follows a simple Taylor rule with a response coefficient of 1.5.

As in the case where monetary policy fully stabilizes inflation, the nominal interest rate increases more strongly in the behavioral HANK model than in its rational version. The difference across the two models, however, is somewhat smaller compared to the case in which inflation is completely stable. Inflation, however, increases more strongly in the behavioral model and also government debt increases more substantially.

Consumption inequality increases less strongly than with fully stabilizing inflation. The overheating economy—reflected in the positive output gap and increase in inflation—increases wages and decreases profits (relative to the inflation stabilizing regime) in the same way as expansionary policy shocks in Sections 3 and 4 do, thereby redistributing towards lower income households which dampens the increase in consumption inequality.

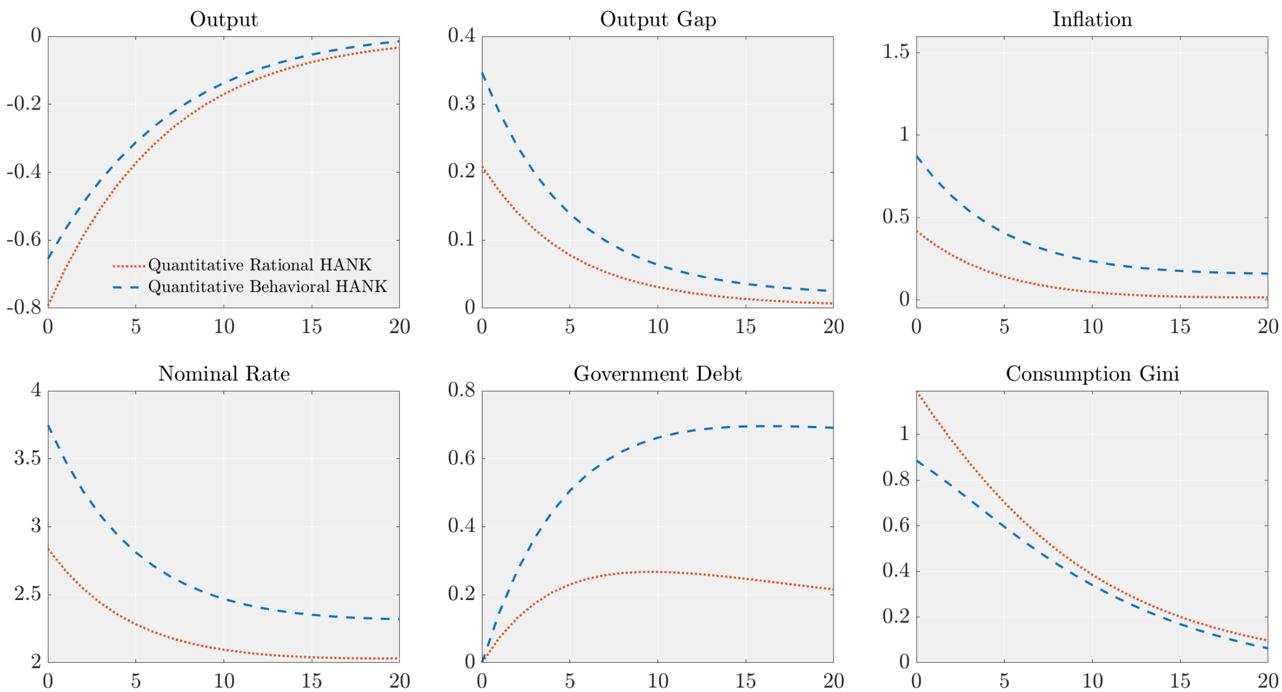


Figure 18: Inflationary cost-push shock: Taylor rule

Note: This figure shows the impulse responses after a cost-push shock that decreases potential output by 1% in the Taylor rule monetary policy regime. Output and the output gap are shown as percentage deviations from steady state output, nominal interest rate and inflation as annualized percentage points and government debt level as percentage point deviations in the debt-per annual GDP level. The lower-right figure shows the change in the consumption Gini index as a percentage deviation from the stationary equilibrium.

G Details on Intertemporal MPCs

In this section, we derive the iMPCs discussed in Section 6.1. Defining Y_t^j as type j's disposable income, we can write the households' budget constraints as

$$C_t^H = Y_t^H + \frac{1-s}{\lambda} R_t B_t$$
$$C_t^U + \frac{1}{1-\lambda} B_{t+1} = Y_t^U + \frac{s}{1-\lambda} R_t B_t,$$

where R_t denotes the real interest rate and B_t real bonds. Log-linearizing the two budget constraints around the zero-liquidity steady state and $R = \beta^{-1}$ yields

$$\widehat{c}_t^H = \widehat{y}_t^H + \frac{1-s}{\lambda}\beta^{-1} b_t \tag{61}$$
$$\widehat{c}_t^U + \frac{1}{1-\lambda} b_{t+1} = \widehat{y}_t^U + \frac{s}{1-\lambda}\beta^{-1} b_t, \tag{62}$$

where b_t denotes real bonds in shares of steady state output. Aggregating (61) and (62) delivers

$$\widehat{c}_t = \tilde{y}_t + \beta^{-1} b_t - b_{t+1}, \tag{63}$$

where $\tilde{y}_t$ denotes aggregate disposable income.

By plugging equations (61) and (62) into the Euler equation of unconstrained households (10), we can derive the dynamics of liquid assets b_t (ignoring changes in the real rate as this is a partial equilibrium exercise):

$$\mathbb{E}_t b_{t+2} - b_{t+1}\left[\frac{1}{s\bar{m}} + \beta^{-1}s + \frac{(1-s)^2\beta^{-1}(1-\lambda)}{s\lambda}\right] + \frac{\beta^{-1}}{\bar{m}} b_t =$$
$$(1-\lambda)\mathbb{E}_t\widehat{y}_{t+1}^U + \frac{1-s}{s}(1-\lambda)\mathbb{E}_t\widehat{y}_{t+1}^H - \frac{1-\lambda}{s\bar{m}}\widehat{y}_t^U. \tag{64}$$

Note that a change in total disposable income by one changes the hand-to-mouth households' disposable income by χ and the disposable income of unconstrained households by $\frac{1-\lambda\chi}{1-\lambda}$.

Let us denote the right-hand side of equation (64) by $-\mathbb{E}_t\widehat{z}_t$. Factorizing the left-hand side and letting F denote the forward-operator, it follows that

$$(F-\mu_1)(F-\mu_2)\mathbb{E}_t b_t = -\mathbb{E}_t\widehat{z}_t, \tag{65}$$

where μ_1 and μ_2 denote the roots of the characteristic equation

$$\mathbb{E}_t b_{t+2} - \phi_1 b_{t+1} - \phi_2 b_t = 0, \tag{66}$$

where

$$\phi_1 \equiv \left[\frac{1}{s\bar{m}} + \beta^{-1}s + \frac{(1-s)^2\beta^{-1}(1-\lambda)}{s\lambda}\right] \tag{67}$$

and

$$\phi_2 \equiv -\frac{\beta^{-1}}{\bar{m}}. \tag{68}$$

Thus, the roots are given by

$$\mu_{1,2} = \frac{\phi_1 \pm \sqrt{\phi_1^2 + 4\phi_2}}{2}. \tag{69}$$

It follows that

$$\begin{aligned} b_{t+1} &= \mu_1 b_t - (F - \mu_2)^{-1} \mathbb{E}_t \widehat{z}_t \\ &= \mu_1 b_t + \frac{\mu_2^{-1}}{1 - F\mu_2^{-1}} \mathbb{E}_t \widehat{z}_t. \end{aligned}$$

Note that $\mathbb{E}_t \widehat{z}_t$ can be written as $\frac{1-\lambda\chi}{s}\left(\delta \mathbb{E}_t \widehat{y}_{t+1} - \frac{1}{\bar{m}}\widehat{y}_t\right)$. Without loss of generality, we let $\mu_2 > \mu_1$ and we have $\mu_2 > 1$. We have $(1 - F\mu_2^{-1})^{-1} = \sum_{l=0}^{\infty} \mu_2^{-l} F^l$. Thus, we end up with

$$b_{t+1} = \mu_1 b_t + \frac{1 - \lambda\chi}{s} \sum_{l=0}^{\infty} \mu_2^{-(l+1)} \mathbb{E}_t \left(\frac{1}{\bar{m}} \widehat{y}_{t+l} - \delta \widehat{y}_{t+1+l} \right). \tag{70}$$

Plugging this in equation (63) and taking derivatives with respect to $\widehat{y}_{t+k}$ yields Proposition 5.

iMPCs for more than two periods. Figure 19 plots the MPCs for the year of the income windfall as well as the five consecutive years for different degrees of rationality. As discussed in section 6.1, under our benchmark calibration, the rational model predicts somewhat larger initial MPCs as behavioral, unconstrained households save relatively more. Over time, however, the MPCs in the behavioral model lie above their rational counterparts due to the fact that more and more of the initial unconstrained households become hand-to-mouth and start consuming their (higher) savings. As Figure 20 shows, the probability of type switching, $1 - s$, matters for when exactly the behavioral model starts to generate larger MPCs compared to the rational model.

iMPCs and the Role of Idiosyncratic Risk. In Figure 20, we plot he MPCs in the year of the income windfall (left panel) and the first year after the windfall (right panel) for a relatively high idiosyncratic risk of $1 - s = 0.5$. The high probability of becoming hand-to-mouth flips the role of $\bar{m}$ for the MPC_1 compared to our baseline calibration as discussed in Section 6.1. The reason being that the behavioral, unconstrained households save a relatively large amount of the received income windfall in period 0 as they cognitively discount the decrease in their future marginal utility. Thus, they end up with relatively more disposable income in year 1. Now, given the relatively high probability of type switching, there are many unconstrained households who end up being hand-to-mouth in year 1 after the income windfall. As they are hand-to-mouth, they consume their previously-accumulated savings which increases the MPC_1. The more behavioral unconstrained households are, i.e., the lower $\bar{m}$ is, the more pronounced this effect and hence, a lower $\bar{m}$ increases the MPC_1 in the case of a relatively high $1 - s$.

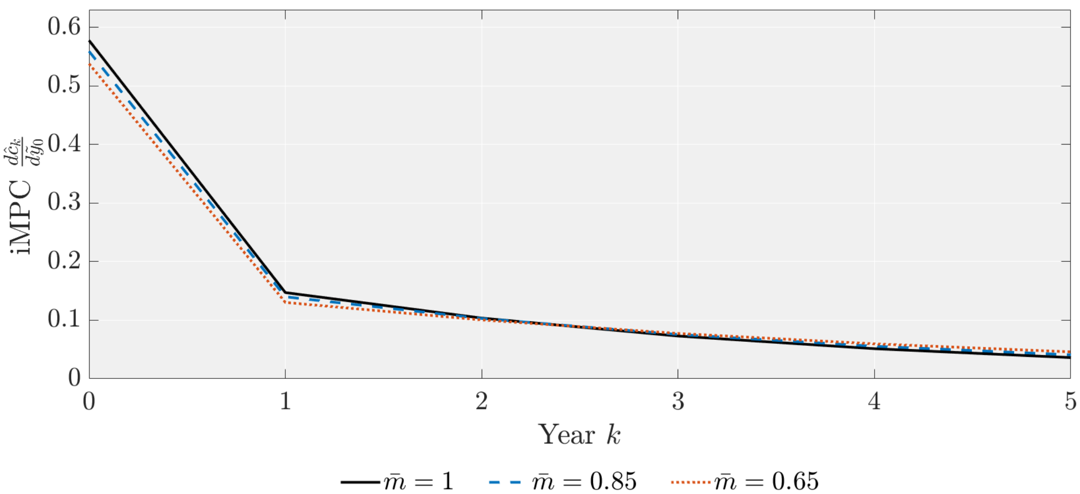


Figure 19: Intertemporal MPCs

Note: This figure shows the aggregate intertemporal MPCs, i.e., the aggregate consumption response in year k to a change in aggregate disposable income in year 0 for different $\bar{m}$.

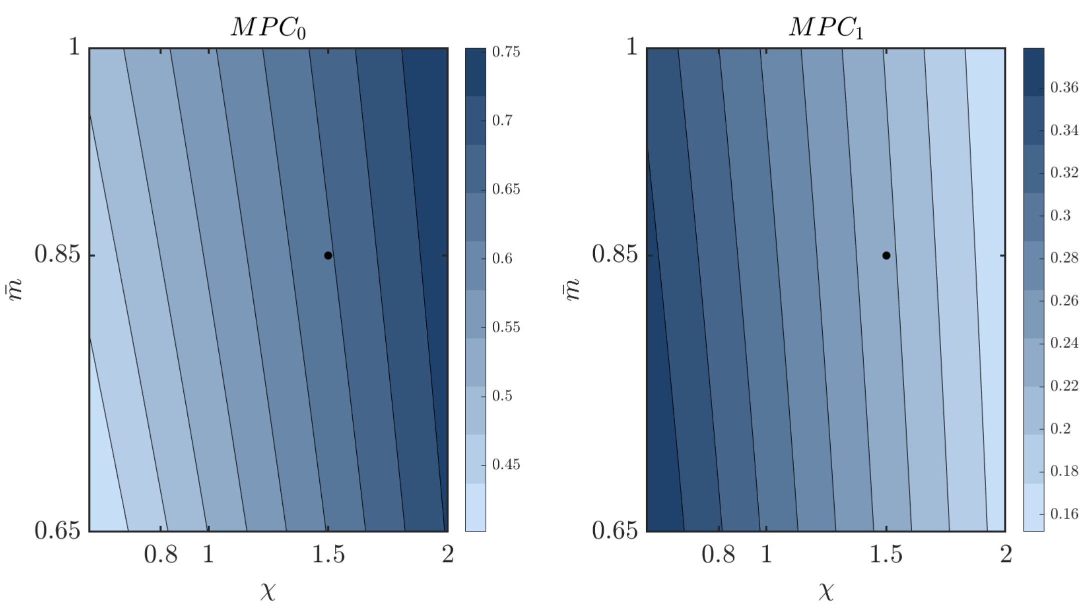


Figure 20: Intertemporal MPCs, Bounded Rationality and Household Heterogeneity

Note: This figure shows the aggregate intertemporal MPCs, i.e., the aggregate consumption response in year 0 (left) and year 1 (right) to a change in aggregate disposable income in year 0 for a transition probability $1 - s = 0.5$.

H Sticky Wages

In this section, we provide details on the sticky-wage extension presented in Section 6.2 as well as the calibration used to produce Figures 6 and 7. The way we introduce sticky wages follows Colciago (2011) and recently adopted by Bilbiie et al. (2021).[23]

In the household block, the only difference to our benchmark model is that we assume that there is a labor union pooling labor and setting wages on behalf of households. This leads to a condition similar to the labor-leisure conditions in Section 2. But instead of individual conditions, the condition is the same for every household:

$$\varphi\widehat{n}_t = \widehat{w}_t - \gamma\widehat{c}_t,$$

and $\widehat{n}_t = \widehat{n}_t^U = \widehat{n}_t^H$.

The labor union, however, is subject to wage rigidities. The nominal wage can only be re-optimized with a constant probability, which leads to a time-varying wage markup

$$\widehat{\mu}_t^w = \varphi\widehat{n}_t - \widehat{w}_t + \gamma\widehat{c}_t,$$

and a wage Phillips Curve

$$\pi_t^w = \beta\mathbb{E}_t\pi_{t+1}^w + \kappa_w\widehat{\mu}_t^w.$$

Wage inflation is given by

$$\pi_t^w = \widehat{w}_t - \widehat{w}_{t-1} + \pi_t.$$

The firm side is exactly the same as in the main text but we focus on the case with rational firms, which gives rise to a standard Phillips Curve:

$$\pi_t = \beta\mathbb{E}_t\pi_{t+1} + \kappa_\pi\widehat{mc}_t,$$

where $\widehat{mc}_t$ is a time-varying price markup. Table 4 summarizes all equilibrium equations.

The calibration of this extended model is presented in Table 5. The parameters γ, φ, s, β and $\bar{m}$ are as in our baseline calibration. The parameters of the Taylor rule, ρ_i and ϕ, are set as estimated in Auclert et al. (2020).

The slope of the wage Phillips curve, κ_w, is set as in Bilbiie et al. (2021) and we focus on the *no-redistribution* case $\tau^D = 0$. Note, that this leads to impact responses of consumption of the two household types that are very close to the ones in our baseline model: $\widehat{c}_t^H$ increases by about 1.42, whereas output increases by 1. The baseline calibration of $\chi = 1.5$ would predict that in the model without sticky wages, $\widehat{c}_t^H$ increases by 1.5 when output increases by 1. We focus on a relatively stable inflation and set κ_π to 0.01.

The only parameter that we change with respect to our baseline calibration is λ which we

[23] See also Erceg et al. (2000). Broer et al. (2020) and Broer et al. (2021b) discuss the role of sticky wages in (rational) TANK models for the analysis of monetary and fiscal policy, respectively.

Table 4: Sticky Wages, Equilibrium Equations

Name	Equation
Wage Markup	$\widehat{\mu}_t^w = \gamma \widehat{c}_t + \varphi \widehat{n}_t - \widehat{w}_t$
Wage Phillips Curve	$\pi_t^w = \beta \mathbb{E}_t \pi_{t+1}^w + \kappa_w \widehat{\mu}_t^w$
Wage Inflation	$\pi_t^w = \widehat{w}_t - \widehat{w}_{t-1} + \pi_t$
Bond Euler	$\widehat{c}_t^U = s\bar{m}\mathbb{E}_t \widehat{c}_{t+1}^U + (1-s)\bar{m}\mathbb{E}_t \widehat{c}_{t+1}^H - \frac{1}{\gamma}(\widehat{i}_t - \mathbb{E}_t \pi_{t+1})$
H Budget Constraint	$\widehat{c}_t^H = \widehat{w}_t + \widehat{n}_t + \widehat{t}_t^H$
H Transfer	$\widehat{t}_t^H = \frac{\tau^D}{\lambda} D_t$
Profits	$\widehat{d}_t = \widehat{y}_t - (\widehat{w}_t + \widehat{n}_t)$
Labor Demand	$\widehat{w}_t = \widehat{mc}_t + \widehat{y}_t - \widehat{n}_t$
Phillips Curve	$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa_\pi \widehat{mc}_t$
Production	$\widehat{y}_t = \widehat{n}_t$
Consumption	$\widehat{c}_t = \lambda \widehat{c}_t^H + (1-\lambda)\widehat{c}_t^U$
Resource Constraint	$\widehat{y}_t = \widehat{c}_t$
Taylor Rule	$\widehat{i}_t = \rho_i \widehat{i}_{t-1} + (1-\rho_i)\phi \pi_t + \varepsilon_t^{MP}$

Table 5: Sticky Wage Model Calibration

Parameter	γ	κ_π	λ	s	φ	τ^D	κ_w	β	ρ_i	ϕ
Value	1	0.01	0.37	$0.8^{1/4}$	1	0	0.075	0.99	0.89	1.5

set to 0.37 instead of 0.33. A value of 0.37 is still in the range of often used values (see, for example Bilbiie (2020)). We increase λ somewhat compared to our baseline calibration in order to increase the role of hand-to-mouth households in the response to monetary policy shocks and thus, allows the model to generate the pronounced hump-shaped responses. Setting $\lambda = 0.33$ still produces hump-shaped responses but those are somewhat less pronounced.