Agenda

- Branch & Bound Overview
- Disjunctive constraints problem
- Production/Distribution problem

Logistics

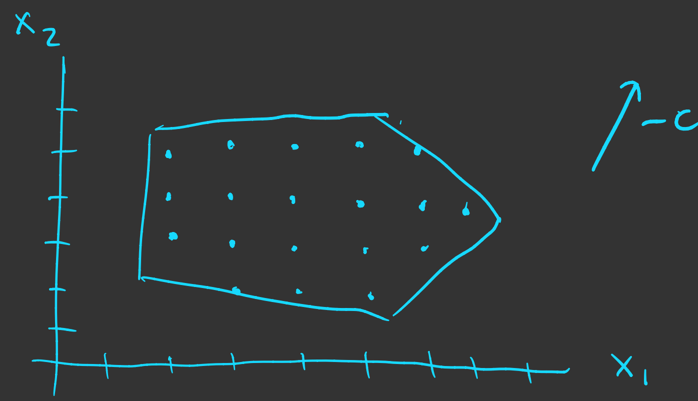
- HW8 out due Friday April 30 9pm
- This is the last precept
- Final project
  - more details to be announced
  - coding project
  - 24 hours to Complete
  - Out May 6
  - Deadline May 14

Graduate School

- Reach out to me if you're interested/ have questions

Mixed integer program

$$p^{IP} = \begin{array}{ll} \min_x & c^T x \\ \text{s.t.} & Ax \le b \\ & x_i \in \mathbb{Z}, \ i \in \mathcal{I} \end{array}$$



Relaxation

$$p^{rel} = \begin{array}{ll} \min_x & c^T x \\ \text{s.t.} & Ax \le b \end{array}$$

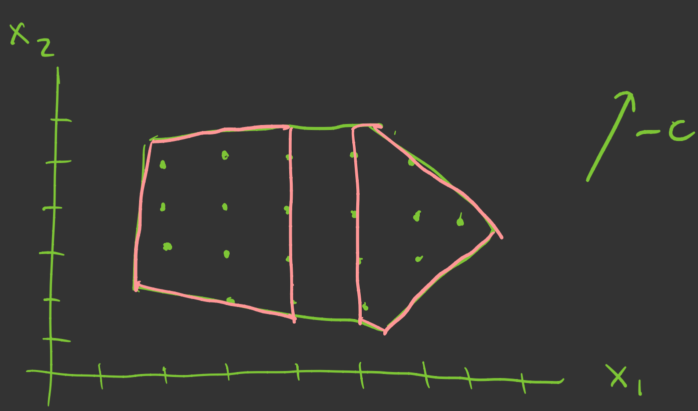
$p^{rel} \le p^{IP}$ way to achieve LBs

If x^{*rel} is integer then $p^{rel} = p^{IP}$,

– this happens if conv $P = \{Ax \le b\}$

Branch & Bound

- main idea: divide & conquer



Partition into smaller sets

Solve subproblems

$$\Phi(S^j) = \begin{array}{ll} \min_x & c^T x \\ \text{s.t.} & x \in S^j \end{array}$$

$$\Phi_{LB}(S^j) \le \Phi(S^j) \le \Phi_{UB}(S^j)$$ ← any feas. point

relaxation →

Branch & Bound Algorithm

$$\min_x \; c^T x \quad \text{s.t. } x \in P_{IP} \qquad P_{IP} = \{ x \mid Ax \le b,\; x_i \in \mathbb{Z} \;\; i \in \mathcal{I} \}$$

Iterations

1. Branch: create/refine the partition of P_{IP} and get S^j
2. Bound:
   - Compute lower and upper bounds

     $$L_j = \Phi_{LB}(S^j) \qquad U_j = \Phi_{UB}(S^j) \quad \forall j$$

   - Update global bounds on $c^T x^*$

     $$L = \min_j L_j \qquad U = \min_j U_j$$

3. If $U - L \le \epsilon$ break

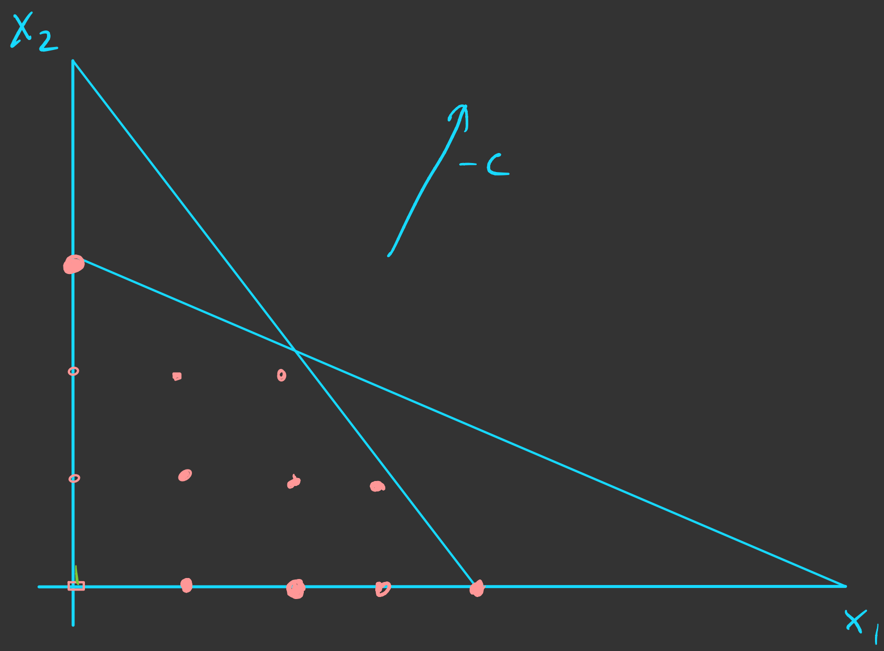
- Assume U is nonincreasing, L nondecreasing
- Pruning: S^j is active if $L_j \le \min_j U_j$

  else: it is inactive and we can prune it

Branch & Bound example

$$\begin{aligned} \min_{x} \quad & -2x_1 - 3x_2 \\ \text{s.t.} \quad & \tfrac{2}{9}x_1 + \tfrac{1}{4}x_2 \le 1 \\ & \tfrac{1}{7}x_1 + \tfrac{1}{3}x_2 \le 1 \\ & x \ge 0 \\ & x \in \mathbb{Z} \end{aligned}$$

$$x^{*IP} = (2,2)$$



Disjunctive constraints

Suppose we are given m constraints $a_i^T x \geq b_i$ $i = 1, \ldots, m$
Model the requirement that at least k of them are satisfied

Recall from lecture

goal: either $a^T x \leq b$ or $d^T x \leq f$ is valid

$$a^T x \leq b + yM$$
$$d^T x \leq f + (1-y)M$$
$$y \in \{0,1\}$$
$$x \in \mathbb{R}$$

Production / Distribution problem

A company produces a set of K products at I plants and ships these products to J market zones $(k=1,\dots,K)$ $(i=1,\dots,I)$ $(j=1,\dots,J)$

v_{ik}: cost of producing 1 unit of product k at plant i

c_{ijk}: cost of shipping 1 unit of product k from plant i to zone j

f_{ik}: fixed cost of producing product k at plant i

M_{ik}: maximal quantity of product k produced at plant i

m_{ik}: minimal quantity of product k that can be produced at plant i if plant i introduces a nonzero quantity

q_{ik}: capacity of plant i to produce 1 unit of product k

Q_i: capacity of plant i

d_{jk}: demand for product k at market zone j

(a) formulate minimizing the total cost as an integer program

vars: $x_{ik} \in \mathbb{Z}^{I,K}$: units of product k produced at plant i

$y_{ijk} \in \mathbb{Z}^{I,J,K}$: units of product k shipped from plant i to market zone j

$z_{ik} \in \{0,1\}^{I,K}$: binary variable indicating if product k produced at plant i

(b) No plant can produce more than K_1 products

(c) Every product can be produced in at most I_1 plants

(d) For a particular product k_0, plant 3 must produce it if neither plant 1 or plant 2 produces it

(e) Each market zone must be sourced by exactly one plant for all products