Agenda

- Optimality Conditions
- Sensitivity Analysis
  - new variable
  - new constraint
  - global/local
  - example
- Farkas Lemma

Logistics

- HW 6 due Fri 4/1
- HW 7 due Fri 4/8
- Midterm 2 4/12

Optimality Conditions

Primal and dual solns are optimal

$\Updownarrow$

What is maintained for all iterations?

	Primal feas.	Dual feas.	Duality gap
Primal Simplex			
Dual Simplex			

Proof for duality gap $= 0$ for primal simplex

Sensitivity Analysis: new variable

$$\begin{aligned} \min_{x} \quad & c^T x \\ \text{s.t.} \quad & Ax = b \\ & x \geq 0 \end{aligned} \qquad (x^*, y^*) \text{ opt}$$

Add new var:

$$\begin{aligned} \min_{x, x_{n+1}} \quad & c^T x + c_{n+1} x_{n+1} \\ \text{s.t.} \quad & Ax + A_{n+1} x_{n+1} = b \\ & x, x_{n+1} \geq 0 \end{aligned}$$

Sensitivity Analysis: new constraint

$$\begin{aligned} \min_{x} \quad & c^T x \\ \text{s.t.} \quad & Ax = b \\ & a_{m+1}^T x = b_{m+1} \\ & x \geq 0 \end{aligned}$$

Global / Local Sensitivity Analysis

Consider the problem

$$\min_x \; -5x_1 - x_2 + 12x_3$$
$$\text{s.t. } 3x_1 + 2x_2 + x_3 = 10$$
$$5x_1 + 3x_2 + x_4 = 16$$
$$x \geq 0$$

Opt soln: $\bar{x} = \begin{pmatrix} 2 \\ 2 \\ 0 \\ 0 \end{pmatrix}$

- Suppose we change a_{11} from 3 to $3 + \delta$
- Keep x_1, x_2 as basic variables, let $B(\delta)$ be the corresponding basis matrix

a. Compute $B(\delta)^{-1} b$. For which values of δ is $B(\delta)$ a feasible basis?

b. Compute $c_B^T B(\delta)^{-1}$. For which values of δ is $B(\delta)$ an optimal basis?

Another Farkas Lemma

Prove that exactly 1 of the following 2 statements holds

(1) $\exists\, x$ s.t. $Rx > 0$

(2) $\exists\, y$ s.t. $R^T y = 0$, $y \geq 0$, $y \neq 0$