Agenda

- Flop Count
- Least Squares
- A bit of coding

Logistics

- HW2 out due Friday 2/11 9pm
- HW3 released Thursday, due Friday 2/18 9pm
- Ed Forum

Flop Count

$A, B \in \mathbb{R}^{m,n}$, $C \in \mathbb{R}^{n,p}$, $x \in \mathbb{R}^n$, $\alpha \in \mathbb{R}$

matrix operations	flop count
αA	mn
$A+B$	mn
$x^T x$	$2n-1$
Ax	$(2n-1)m$
AC	$(2n-1)mp$
A^TA	$\sim mnp$

$$x^Tx = \sum_{i=1}^{n} x_i^2$$

n multiplications

n-1 additions

$$Ax = \begin{pmatrix} - a_1^T - \\ \vdots \\ - a_m^T - \end{pmatrix} x = \begin{pmatrix} a_1^T x \\ \vdots \\ a_m^T x \end{pmatrix} \begin{array}{l} \leftarrow 2n-1 \\ \vdots \\ \leftarrow 2n-1 \end{array}$$

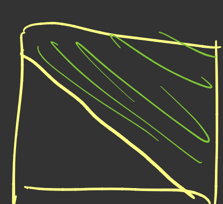
So $m(2n-1)$

$$AC = A\begin{pmatrix} | & & | \\ c_1 & \cdots & c_p \\ | & & | \end{pmatrix} = \begin{pmatrix} | & & | \\ Ac_1 & \cdots & Ac_p \\ | & & | \end{pmatrix}$$

$$\underset{m(2n-1)}{\uparrow} \quad \cdots \quad \underset{m(2n-1)}{\uparrow}$$

total: $mp(2n-1)$

A^TA symmetric, so only need



about $\frac{1}{2}$ of entries, so about mnp

Least Squares $\quad \min_x \|Ax-b\|_2^2 \quad$ (A is Tall)

- Derive the normal equations

$$\|Ax-b\|_2^2 = (Ax-b)^T(Ax-b)$$
$$= x^TA^TAx - 2b^TAx + b^Tb$$

$$\nabla_x \|Ax-b\|_2^2 = 2A^TAx - 2A^Tb = 0$$

set to zero for first order condition

Normal Equations $\longrightarrow$ $A^TAx^* = A^Tb$

Gram matrix A^TA

1. Show that A^TA symmetric
2. Show that $A^TA \succ 0$ if columns of A linearly independent

1. $(AB)^T = B^TA^T$

So $(A^TA)^T = A^T(A^T)^T = A^TA$

2. Recall $H \succeq 0 \Leftrightarrow x^THx \geq 0 \ \forall x \neq 0$

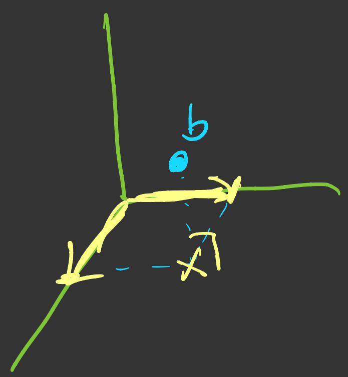
$$x^TA^TAx = \|Ax\|_2^2 \geq 0$$

$\|Ax\|_2^2 > 0 \ \forall x \neq 0$ since cols of A are lin. ind.

Least Squares interpretation

$$A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix} \qquad b = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$$\min_x \|Ax - b\|_2^2$$



Least Squares Orthogonality Principle

$$r = Ax^* - b$$

$Ax \perp r$ for any x i.e. $(Ax)^T r = 0 \quad \forall x$

$$A^T A x^* = A^T b$$

$$A^T(Ax^* - b) = 0 = A^T r$$

$$(Ax)^T r = x^T A r = 0 \quad \checkmark$$