Precept 2

Agenda

- Ballistics example
- Inverse of matrix w/ LU
- More plotting

Logistics

- Change in rotation of preceptors - I'll be here
  2 out of every 3 weeks
  - Hao will take the 3rd week
- HW1 out, due Friday Feb 12 at 9pm
- HW2 out Thursday, due Friday Feb 19 at 9pm

Ballistics Example

Overview

- a projectile moving in 2-dim space
- sample position and velocity at times $\tau = 0, h, 2h, \ldots$
- $p_t \in \mathbb{R}^2$ is the position at time $\tau = th$
- $v_t \in \mathbb{R}^2$ is the velocity at time $\tau = th$
- $f_t \in \mathbb{R}^2$ is the total force on projectile at time $\tau = th$
- $x_t = \begin{pmatrix} p_t \\ v_t \end{pmatrix}$ is the projectile state at time $\tau = th$

Force model

- $f_t = mg - \eta v_t$
- $\eta \in \mathbb{R}$ is the drag coefficient
- $g = \begin{pmatrix} 0 \\ -9.8 \end{pmatrix}$ is gravity

Dynamics

- approximate velocity as constant over time interval $th \le \tau \le (t+1)h$

$$p_{t+1} = p_t + h v_t$$

- approximating force as constant over the time interval

$$v_{t+1} = v_t + \left(\frac{h}{m}\right) f_t$$

$$= \left(1 - \frac{h\eta}{m}\right) v_t + hg$$

Now write this more compactly as $x_{t+1} = Ax_t + b$

$$A = \begin{pmatrix} 1 & 0 & h & 0 \\ 0 & 1 & 0 & h \\ 0 & 0 & 1-\frac{h\eta}{m} & 0 \\ 0 & 0 & 0 & 1-\frac{h\eta}{m} \end{pmatrix}, \quad b = \begin{pmatrix} 0 \\ 0 \\ hg_1 \\ hg_2 \end{pmatrix}$$

Propagating the state through time

$$x_1 = Ax_0 + b$$

$$x_2 = Ax_1 + b = A(Ax_0 + b) + b = A^2 x_0 + Ab + b$$

$$\vdots$$

not transpose

$$x_T = A^T x_0 + (A^{T-1} + \dots + A + I)b$$

Targeting Problem

Given

- initial position p_0
- parameters h, m, η
- flighttime Th
- desired final position (target) p_T

Goal

- find the initial velocity

Final State

$$x_T = A^T x_0 + (A^{T-1} + \dots + A + I)b$$

$$= F x_0 + j$$

where $F = A^T$, $j = (A^{T-1} + \dots + A + I)b$

- $F \in \mathbb{R}^{4,4}$, $j \in \mathbb{R}^4$

$$F = \begin{pmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{pmatrix}$$

each subblock is a 2×2 matrix

$$j = \begin{pmatrix} j_1 \\ j_2 \end{pmatrix} \quad j_i \in \mathbb{R}^2$$

Final position

$$x_T = \begin{pmatrix} p_T \\ v_T \end{pmatrix} = \begin{pmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{pmatrix} \begin{pmatrix} p_0 \\ v_0 \end{pmatrix} + j_1$$

$$\Rightarrow \underbrace{p_T = C v_0 + d}_{\text{solve this linear system to find } v_0} \quad \text{where } C = F_{12},\ d = F_{11} p_0 + j_1$$

Robust ballistics

- Suppose we have uncertainty in the drag coefficient
- uncertainty modeled as K scenarios
  - each scenario has its own
    - $A^{(j)}, b^{(j)}$
    - $C^{(j)}, d^{(j)}$

Robust targetting

$$\min_{v_0} \frac{1}{K} \sum_{j=1}^{K} \| C^{(j)} v_0 + d^{(j)} - p_T \|^2$$