Agenda

- Duality Review
  - Motivation example
  - Weak/Strong duality, duality for LPs
  - Farkas Lemma
- Taking the dual example
- Dual of l norm example

Logistics

- HW 5 out, due Friday March 25 at 9pm
- HW 6 due Friday April 1 at 9pm
- Midterm 2 on Tuesday April 12

Duality motivation / obtaining lower bounds

- We want to get a lower bound for

$$\begin{aligned} \min_{x} \quad & x_1 + 3x_2 \\ \text{s.t.} \quad & x_1 + x_2 \geq 2 \\ & x_2 \geq 1 \\ & x_1 - x_2 \geq 3 \end{aligned}$$

Farkas Lemma

Given A and b, one of the following 2 statements true

1. $\exists x$ s.t. $Ax = b$, $x \geq 0$
2. $\exists y$ s.t. $A^T y \geq 0$, $b^T y < 0$

Dual of std LP

$$\min_x \; c^T x \quad \text{s.t. } Ax = b,\ x \geq 0$$

$$\max_y \; -b^T y \quad \text{s.t. } A^T y + c \geq 0$$

Inequality form

$$\min_x \; c^T x \quad \text{s.t. } Ax \leq b$$

$$\max_y \; -b^T y \quad \text{s.t. } A^T y + c = 0,\ y \geq 0$$

Example: find the dual

one way: pattern matching

$$\min_x \; 3x_1 + 4x_2 \quad \text{s.t. } x_1 + x_2 \geq 5,\ 2x_1 + x_2 \geq 6,\ x \geq 0$$

Other way: derive the dual

$$\min_x \; c^T x \quad \text{s.t. } Ax \geq b$$

Dual of l norm problem

$$\min_x \; c^T x \quad \text{s.t.} \quad \|Ax+b\|_1 \le 1$$

a. formulate this LP inequality form

b. derive the dual LP and show that it's equivalent to

$$\max_z \; b^T z - \|z\|_\infty \quad \text{s.t.} \quad A^T z + c = 0$$

c. give a direct argument that whenever x is primal feasible and z is dual feasible, $c^T x \geq b^T z - \|z\|_\infty$

$$\min_x \; c^T x \quad \text{s.t.} \quad \|Ax+b\|_1 \leq 1$$

$$\max_z \; b^T z - \|z\|_\infty \quad \text{s.t.} \quad A^T z + c = 0$$