Agenda

- Review of Simplex method
- Optimality condition question
- Degeneracy and cycling
- Example

Logistics

- HW 5 due Friday March 25 9pm
- Midterm 2 Thursday April 12
- Midterm 1 grades out

Standard form polyhedra

Standard form LP?

What is P?

Feasible direction

a vector d is a feasible direction at point $x \in P$

$\Leftrightarrow$

$\exists \theta > 0$ s.t. $x + \theta d \in P$

Claim: $Ad = 0$

Basic feasible solution

- A basis: e.g. $B = \{1, 4, 5\}$
- A basis matrix $A_B = [A_{B(1)}, \ldots, A_{B(m)}]$
- What does x satisfy?

- When is x a basic feasible solution?

- basic direction d (also feasible)

Cost Improvement

$$\underbrace{c^T(x+\theta d)}_{\text{new cost}} - \underbrace{c^T x}_{\text{old cost}} = \overbrace{\theta\, c^T d}^{\text{cost improvement}}$$

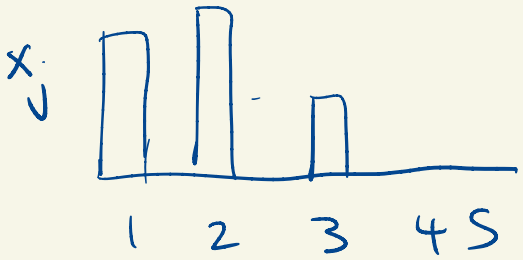
$c^T d$ is the reduced cost associated with this d

$$\bar{c}_j = c^T d = \sum_{i=1}^{2} c_i d_i = c_j + c_B^T d_B = c_j - c_B^T A_B^{-1} A_j$$

Example: $m=3$, $n=5$

$B = \{1, 2, 3\}$

want to add 4 to the basis



$A_B x_B = b$

$$(A_1\ A_2\ A_3)\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = b$$

- if $\bar{c} \geq 0$ then?
- if some $\bar{c}_j < 0$ then?

- How far can we step after finding d?

- if $d \geq 0$ in this case, then?

Optimality conditions

(a) a feasible solution x is optimal $\Leftrightarrow c^T d \geq 0$ for any feasible direction at x

$\Rightarrow$

$\Leftarrow$

(b) a feasible solution x is the unique optimal solution if and only if $c^T d > 0$ for every nonzero feasible direction d at x