


Agenda

- Optimality Conditions
- Sensitivity Analysis
 - new variable
 - new constraint
 - global/local
 - example
- Farkas Lemma

Logistics

- HW 6 due Fri 4/1
- HW 7 due Fri 4/8
- Midterm 2 4/12

Optimality Conditions

Primal and dual solns are optimal



What is maintained for all iterations?

Primal feas, Dual feas, Duality gap

Primal Simplex

Dual Simplex

Proof for duality gap = 0 for primal simplex

Sensitivity Analysis: new variable

$$\begin{array}{ll} \min_x & c^T x \\ \text{s.t.} & Ax = b \\ & x \geq 0 \end{array} \quad (x^*, y^*) \text{ opt}$$

Add new var :

$$\begin{array}{ll} \min_{x, x_{n+1}} & c^T x + c_{n+1} x_{n+1} \\ \text{s.t.} & Ax + A_{n+1} x_{n+1} = b \\ & x, x_{n+1} \geq 0 \end{array}$$

Sensitivity Analysis: new constraint

$$\begin{array}{ll}\min_x & c^T x \\ \text{s.t.} & Ax = b \\ & a_{m+1}^T x = b_{m+1} \\ & x \geq 0\end{array}$$

Global/Local Sensitivity Analysis

Consider the problem

$$\begin{aligned} \min_x \quad & -5x_1 - x_2 + 12x_3 \\ \text{s.t.} \quad & 3x_1 + 2x_2 + x_3 = 10 \\ & 5x_1 + 3x_2 + x_4 = 16 \\ & x \geq 0 \end{aligned}$$

$$\text{Opt soln: } \bar{x} = \begin{pmatrix} 2 \\ 2 \\ 0 \\ 0 \end{pmatrix}$$

- Suppose we change a_{11} from 3 to $3 + \delta$
- Keep x_1, x_2 as basic variables, let $B(\delta)$ be the corresponding basis matrix
- 1. Compute $B(\delta)^{-1}b$. For which values of δ is $B(\delta)$ a feasible basis?

b. Compute $C_B^T B(\delta)^{-1}$. For which values of δ is $B(\delta)$ an optimal basis?

Another Farkas Lemma

Prove that exactly 1 of the following 2 statements holds

(1) $\exists x$ s.t. $Ax \leq b$

(2) $\exists y$ s.t. $A^T y = 0, b^T y < 0, y \geq 0$