Session 10
Causal Inference

Tao Jiang

Motivating Examples

- “My headache went away because I took an aspirin.”
- “She got a good job last year because she went to college.”
- “She has long hair because she is a girl.”

The goal is to draw causal inference on the effect of “treatment”:

  - would like to be able to say that such an effect is attributable, or “caused by” treatment

Key notions

- X = 1 if treatment, = 0 if control
- Z vector of pre-exposure covariates
- Y observed outcome

- "My headache went away because I took an aspirin."
- "She got a good job last year because she went to college."
- "She has long hair because she is a girl."

Definition of Causal Effects

- "My headache went away because I took an aspirin."

Unit	Not Observable			Known	
	Potential Outcomes		Causal Effect	Actual Treatment	Observed Outcome
	Y(Aspirin)	Y(No Aspirin)			
You	No Headache	Headache	Improvement due to Aspirin	Aspirin	No Headache

1. Headache gone only with aspirin:
   Y(Aspirin) = No Headache, Y(No Aspirin) = Headache
2. No effect of aspirin, with a headache in both cases:
   Y(Aspirin) = Headache, Y(No Aspirin) = Headache
3. No effect of aspirin, with the headache gone in both cases:
   Y(Aspirin) = No Headache, Y(No Aspirin) = No Headache
4. Headache gone only without aspirin:
   Y(Aspirin) = Headache, Y(No Aspirin) = No Headache

Model

- X = 1 if treatment, = 0 if control (observed, not assigned)
- Z vector of pre-exposure covariates
- Y observed outcome

- Observed data sets are i.i.d. copies (Yi, Zi, Xi) for each subject i = 1, ..., n
- Based on the data, we estimate the average causal treatment effect.

Counterfactual Model

- Counterfactuals: Each subject has potential outcomes (Y0, Y1)
  - Y0 outcome the subject would have if they received control
  - Y1 outcome the subject would have if they received treatment

- Average causal treatment effect:
  - The probability distribution of Y0 represents how outcomes in the population would turn out if everyone received control , with mean E(Y0) (= P(Y0 = 1) for binary outcome)
  - The probability distribution of Y1 represents this if everyone received treatment , with mean E(Y1) (= P (Y1 = 1) for binary outcome)
  - Thus, the average causal treatment effect is

$$\Delta = \mu_1 - \mu_0 = E(Y_1) - E(Y_0)$$

Counterfactual Model

$$\Delta = \mu_1 - \mu_0 = E(Y_1) - E(Y_0)$$

- Counterfactuals: Each subject has potential outcomes (Y0, Y1)
  - Y0 outcome the subject would have if they received control
  - Y1 outcome the subject would have if they received treatment

- However, we do not observe (Y0, Y1) for all n subjects; instead we only observe

Y = Y1*X + Y0*(1-X)

- Question:how do we estimate E(Y1) and E(Y0)?

Unconfounded Assumption

- Counterfactuals: Each subject has potential outcomes (Y0, Y1)
  - Y0 outcome the subject would have if they received control
  - Y1 outcome the subject would have if they received treatment

- The key assumption we make is that Y1, Y0 are independent of X given Z.
  - i.e. Given a subject, there is no association between exposure X and potential outcome (Y0,Y1)
  - E.g. My headache would go away if I took an aspirin, and my headache would not go away if I did not take an aspirin. The potential outcome is fixed regardless I took an aspirin when I had a headache today.
  - E.g. She would get a good job if she went to college, and she would not get a good job if she did not go to college. The fact would not change regardless she went to college eventually.