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Some stuff on conversion to Kinetic Alfven Wave

Condition for wave to be weakly damped, i.e. parallel cold

$$\xi_0 = V_{phase}/V_{th} = \frac{\omega}{k_z V_{th}} = \frac{\omega}{c k_z}\frac{c}{V_{th}} = \frac{1}{n_z}\frac{c}{V_{th}} \qquad (1.1)$$

For ξ_0 somewhere between 2 & 3, $Z(\xi_0)$ is essentially asymptotic. Also $\xi_{\pm 1}$, $\xi_{\pm 2}$ are similar size, so pick some $\xi_{crit} \sim 3$. Then

$$\frac{1}{n_z}\frac{c}{V_{th}} > \xi_{crit} \implies n_z < \frac{1}{\xi_{crit}}\frac{V_{th}}{c} \qquad (1.2)$$

or

$$n_z^2 < \frac{1}{\xi_{crit}^2}\frac{c^2}{V_{th}^2} = \frac{1}{\xi_{crit}^2}\frac{mc^2}{2T} \qquad (1.3)$$

So for electrons $mc^2 = 511$ kev, Need

$$n_z^2 < \frac{1}{\xi_{crit}^2}\frac{256\text{ kev}}{T_e(\text{kev})} \qquad (1.4)$$

So for example take $\xi_{crit} = \sqrt{5}$, $Z(\xi_{crit}) = -.5 + .01i$. We could call that cold. Then for weak Landau damping need

$$n_z^2 < \frac{50}{T_e(\text{kev})} \qquad (1.5)$$

Condition for Alfven Resonance

$$S = 1 - \Sigma \frac{\omega_{pi}^2}{\omega^2 - \Omega_i^2} = n_z^2 \qquad (1.6)$$

For $\omega \approx \Omega_i$ this is

$$S = 1 - \frac{\omega_{pe}^2}{\omega^2 - \Omega_e^2} - \frac{\omega_{pi}^2}{\omega^2 - \Omega_i^2} \simeq 1 + \frac{\omega_{pe}^2}{\Omega_e^2} - \frac{\omega_{pi}^2}{\omega^2 - \Omega_i^2} \approx \frac{\omega_{pi}^2}{\Omega_i^2 - \omega^2} \qquad (1.7)$$

resonance condition: $\omega_{pi}^2 = (\Omega_i^2 - \omega^2)\, n_z^2$ or (1.8)

$$\omega_{pi}^2 = \Omega_i^2 (1 - \omega^2/\Omega_i^2)\, n_z^2 \;;\; \frac{m_e}{m_i}\omega_{pe}^2 = \frac{m_e^2}{m_i^2}\Omega_e^2 (1 - \omega^2/\Omega_i^2)\, n_z^2$$

$$\omega_{pe}^2 = \frac{m_e}{m_i}\Omega_e^2 (1 - \omega^2/\Omega_i^2)\, n_z^2 \qquad (1.9)$$

Using (NRL Formulary) $f_{pe} = 8.98 \times 10^3 \sqrt{n_e(cm^{-3})} = 8.98 \sqrt{n_e(m^{-3})}$

$f_{ce} = 2.8 \times 10^{10}\, B(\text{Tesla})$ Eq (1.9) becomes

$$8.64\; n_e(m^{-3}) = 7.84 \times 10^{20}\, B(\text{Tesla})^2 \frac{m_e}{m_i}\left(1 - \frac{\omega^2}{\Omega_i^2}\right) n_z^2 \qquad (2.1)$$

$$n_e^{res} = 9.7 \times 10^{18}\, B(\text{Tesla})^2 \frac{m_e}{m_i}\left(1 - \frac{\omega^2}{\Omega_i^2}\right) n_z^2 \qquad (2.2)$$

So for D ion $m_e/m_i = 1/3670$ and using (1.5) for n_z^2 for weak damping

$$n_e^{res} = 1.3 \times 10^{17} \frac{B^2}{T}\left(1 - \frac{\omega^2}{\Omega_i^2}\right), \quad B(\text{Tesla}),\ T(\text{kev}) \qquad (2.3)$$

Looking at standard case 3: $B = 2.1$ T, $f = 15$ MHz, ~~$f_{...}$~~
$T_e = .01$ kev to 1 kev

$$n_e^{res} = 7.2 \times 10^{16} \frac{1}{T(\text{kev})} \qquad (2.4)$$

This is a very low density.

Check (2.2) against case 1

Stix's Dispersion Rel (Sect 13-8)

Look at Stix Eq(39), k^4 term

$$A = \frac{S}{P} + \frac{\beta_i}{2}\left(\frac{\omega^2}{\Omega_i^2-\omega^2} - \frac{\omega^2}{4\Omega_i^2-\omega^2}\right) \quad (3.1)$$

$$S = \frac{\omega_{pi}^2}{\Omega_i^2-\omega^2} \qquad P = 1 - \frac{\omega_{pi}^2}{\omega^2} - \frac{\omega_{pe}^2}{\omega^2} \simeq -\frac{\omega_{pe}^2}{\omega^2} \quad (3.2)$$

$$S/P = -\frac{\omega^2}{\omega_{pe}^2}\frac{\omega_{pi}^2}{\Omega_i^2-\omega^2} = -\frac{m_e}{m_i}\frac{\omega^2}{\Omega_i^2-\omega^2} \quad (3.3)$$

$$A = \frac{\omega^2}{\Omega_i^2-\omega^2}\left[-\frac{m_e}{m_i} + \frac{\beta_i}{2}\left[1 - \frac{\Omega_i^2-\omega^2}{4\Omega_i^2-\omega^2}\right]\right] \quad (3.4)$$

Depends on n_e only through β_i

$$A = \frac{\omega^2}{\Omega_i^2-\omega^2}\left[\frac{3\beta_i}{8}\frac{1}{1-\omega^2/4\Omega_i^2} - \frac{m_e}{m_i}\right] \quad (3.5)$$

Which agrees w. Stix criterion Eq(40).

Look at Stix dispersion rel. Eq(39)

$$A n_x^4 + B n_x^2 + C = A n_x^4 - (S-n_z^2)\, n_x^2 + (R-n_z^2)(L-n_z^2) = 0 \quad (3.6)$$

At resonance $B=0$, $L-n_z^2 > 0$ (above left hand cutoff), $R-n_z^2 < 0$ (below right hand cutoff)

So $(R-n_z^2)(L-n_z^2) < 0$ so there should be real n_z^2 roots when $A > 0$, I have never seen such roots!

Can relate conversion condition (3.5) to resonance condition and weak damping condition (1.9) & (1.3).

$$\beta_i = \frac{8\pi n_i T_i}{B^2} = \frac{4\pi n_i e^2}{m_i}\frac{2T_i}{m_i c^2}\frac{m_i^2 c^2}{e^2 B^2} = \frac{\omega_{pi}^2}{\Omega_i^2}\frac{2T_i}{m_i c^2} = \frac{\omega_{pi}^2}{\Omega_i^2}\frac{2T_e}{m_e c^2}\frac{m_e}{m_i}\frac{T_i}{T_e} \quad (3.7)$$

Using (1.8) for resonance condition

$$\frac{\omega_{pe}^{2\,res}}{\Omega_i^2} = \left(1 - \frac{\omega^2}{\Omega_i^2}\right) n_z^2 \qquad (4.1)$$

Using (1.3) weak damping condition

$$n_z^2 < \frac{1}{\xi_{crit}^2} \frac{m_e c^2}{2T_e} \qquad \text{so} \qquad (4.2)$$

$$\frac{\omega_{pe}^{2\,res}}{\Omega_i^2} < \left(1 - \frac{\omega^2}{\Omega_i^2}\right) \frac{1}{\xi_{crit}^2} \frac{m_e c^2}{2T_e} \qquad (4.3)$$

Using this in (3.7)

$$\beta_i^{res} < \frac{1}{\xi_{crit}^2} \left(1 - \frac{\omega^2}{\Omega_i^2}\right) \frac{m_e c^2}{2T_e} \frac{2T_e}{m_e c^2} \frac{m_e}{m_i} \frac{T_i}{T_e} = \frac{1}{\xi_{crit}^2}\left(1 - \frac{\omega^2}{\Omega_i^2}\right) \frac{m_e}{m_i} \frac{T_i}{T_e} \qquad (4.4)$$

And using (4.4) in the conversion condition (3.5), $A > 0$ requires

$$\frac{1}{\xi_{crit}^2} \frac{3}{8} \frac{1 - \omega^2/\Omega_i^2}{1 - \omega^2/4\Omega_i^2} \frac{T_i}{T_e} > 1 \qquad \text{or} \qquad (4.5)$$

$$T_i > \frac{8}{3} \xi_{crit}^2 \frac{1 - \omega^2/4\Omega_i^2}{1 - \omega^2/\Omega_i^2} T_e \qquad (4.6)$$

For $\xi_{crit} \sim 3$ this going to be really hard to satisfy.

The only way around this I can see would be if for some reason the waves were weakly damped despinte $v_{th} \sim v_{phase}$. Maybe because of polarization. Still with $\xi_{crit} \sim 1$, (4.6) would be hard to get.

Forgetting about the weak damping condition, take $T_e = 0$. Use (4.1) in (3.7) to get basic conversion condition.

$$\beta_i^{res} = \frac{2T_i}{m_i c^2}\left(1 - \frac{\omega^2}{\Omega_i^2}\right) n_z^2 = \frac{m_e}{m_i} \frac{2T_i}{m_e c^2}\left(1 - \frac{\omega^2}{\Omega_i^2}\right) n_z^2 \qquad (4.7)$$

$$T_i^{res} > \frac{4}{3} \frac{1 - \omega^2/\Omega_i^2}{1 - \omega^2/4\Omega_i^2} \frac{m_e c^2}{n_z^2} \qquad (4.8)$$

So need $T_i \gtrsim 511\ \text{keV}/n_z^2$