

GDT ECH with linear Slab Profiles

cold electrons, 50ev ions

Open Additional files:

Get dispersion routines by evaluating Disper.nb

Get plotting and printing routines by evaluating PlotPack.nb

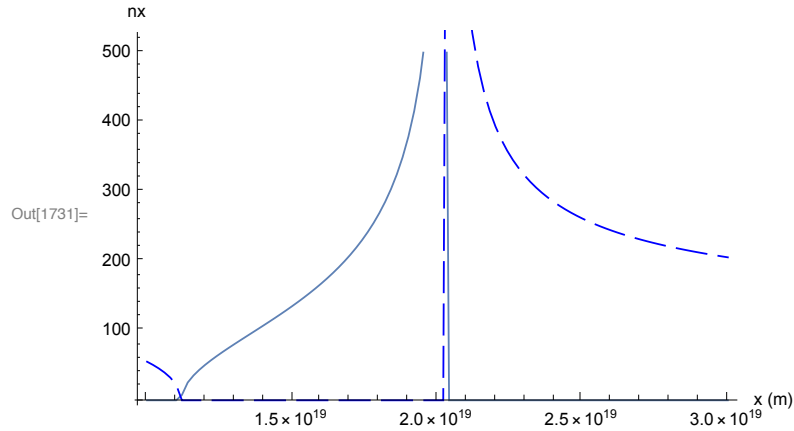
Set Parameters by opening a Parameter Window

Note: Slab profile models defined in initialization cells at the bottom of this notebook.

First Do Cold Plasma

Plot Real and Imaginary parts of n_x from 2nd order warm plasma dispersion relation (i.e. $E_{||} \equiv 0$)

```
In[1729]:= nPerpCold[x_] := Module[{ne, b, x0}, x0 = x;  
    ne = nprof[x0];  
    b = bprof[x0];  
    ColdDis0[freq, ne, b, nz, etaList]  
  
    nt = Table[{x, nPerpCold[x]}, {x, xmin, xmax,  $\frac{x_{\max} - x_{\min}}{nPoints - 1}$ }]  
  
    ComplexListPlot[nt, "x (m)", "nx"]  
    paramPrint[{dataSet, xProfileMin, xProfileMax,  
        nXmin, nXmax, BXmin, BXmax, freq, nz, etaList, xmin, xmax}];
```



```
dataSet=GDT Kinetic Alfven 50 ev
```

```
xProfileMin=1. × 1019
```

```
xProfileMax=3. × 1019
```

```
nXmin=1. × 1019
```

```
nXmax=3. × 1019
```

```
BXmin=1.2
```

```
BXmax=1.2
```

```
freq=7.5
```

```
nz=127.324
```

```
etaList={0., 1., 0., 0., 0.}
```

```
xmin=1. × 1019
```

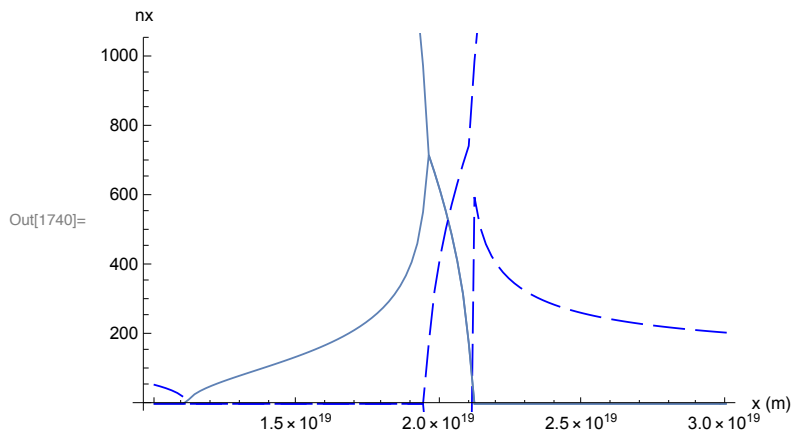
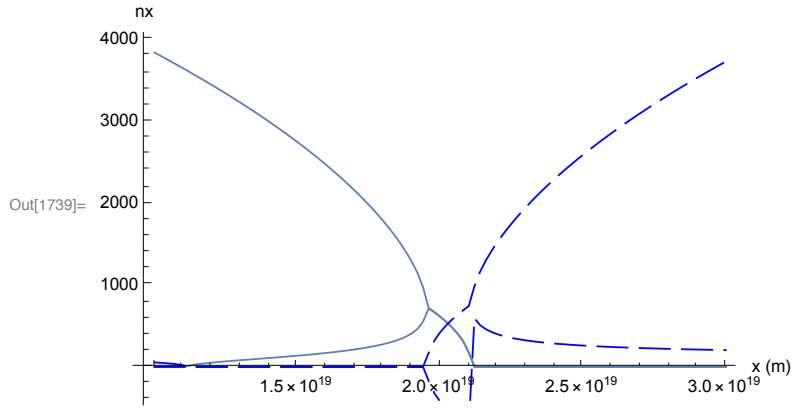
```
xmax=3. × 1019
```

Plot Real and Imaginary parts of nx from 4nd order cold plasma dispersion relation (i.e. fast and slow)

```
In[1733]:= nPerp2FS[x_] := Module[{ne, b, x0}, x0 = x;
  ne = nprof[x0];
  b = bprof[x0];
  ColdDis2FS[freq, ne, b, nz, etaList]]

nt2FS = Table[Flatten[{x, nPerp2FS[x]}], {x, xmin, xmax,  $\frac{x_{\max} - x_{\min}}{nPoints - 1}$ }]];

nF = Transpose[{Transpose[nt2FS][[1]], Transpose[nt2FS][[2]]}];
nS = Transpose[{Transpose[nt2FS][[1]], Transpose[nt2FS][[3]]}];
g1 = ComplexListPlot[nF, "x (m)", "nx"];
g2 = ComplexListPlot[nS, "x (m)", "nx"];
Show[{g1, g2}, PlotRange → All]
Show[{g1, g2}, PlotRange → {0., 1000.}]
paramPrint[{dataSet, xProfileMin, xProfileMax,
  nXmin, nXmax, BXmin, BXmax, freq, nz, etaList, xmin, xmax}];
```



```
dataSet=GDT Kinetic Alfven 50 ev
```

```
xProfileMin= $1. \times 10^{19}$ 
```

```
xProfileMax= $3. \times 10^{19}$ 
```

```
nXmin= $1. \times 10^{19}$ 
```

```
nXmax= $3. \times 10^{19}$ 
```

```
BXmin=1.2
```

```
BXmax=1.2
```

```
freq=7.5
```

```
nz=127.324
```

```
etaList={0., 1., 0., 0., 0.}
```

```
xmin= $1. \times 10^{19}$ 
```

```
xmin= $1. \times 10^{19}$ 
```

Now Warm Plasma Stuff

Plot Real and Imaginary parts of nx from 6th order warm plasma dispersion relation (expanded to 2nd order in $k_{\perp} \rho$)

```
In[1742]:= nPerpWarm6[x_] := Module[{ne, te, b, x0, TL},
      x0 = x;
      ne = nprof[x0];
      b = bprof[x0];
      TL = tprof[x0] * TList;
      WarmDis6[freq, ne, b, nz, etaList, TL]]
```

Compare to cold plasma roots

```
In[1743]:= nPerp2FS[xmin]
nPerpWarm6[xmin]

Out[1743]= {0. + 54.7543 i, 3838.1}

Out[1744]= {1.52288 × 10-9 + 50.1853 i, 3825.44 - 2529.23 i, 3825.44 + 2529.23 i,
      -1.52288 × 10-9 - 50.1853 i, -3825.44 + 2529.23 i, -3825.44 - 2529.23 i}
```

Gets a root near the cutoff fast wave branch, but slow wave is way off. Check if small Γ expansion is valid.

```
In[1745]:= xpoint = 1.0 * 10^19;
      TL = tprof[xpoint] * TList;
      roots6 = nPerpWarm6[xpoint];
      Warmgamma[freq, BXmin, roots6[[1]], TL]
      Warmgamma[freq, BXmin, roots6[[2]], TL]
      Warmgamma[freq, BXmin, roots6[[3]], TL]

Out[1748]= {0. + 0. i, 0. + 0. i, -0.0000899484 + 5.45898 × 10-15 i, 0. + 0. i, 0. + 0. i, 0. + 0. i}

Out[1749]= {0. + 0. i, 0. + 0. i, 0.294178 - 0.691097 i, 0. + 0. i, 0. + 0. i, 0. + 0. i}

Out[1750]= {0. + 0. i, 0. + 0. i, 0.294178 + 0.691097 i, 0. + 0. i, 0. + 0. i, 0. + 0. i}
```

Small $k_x \rho$ expansion not valid for 2 large roots.

```
In[1751]:= nPerp2FS[1.8 * 10^19]
nPerpWarm6[1.8 * 10^19]

Out[1751]= {276.548, 1784.18}

Out[1752]= {300.421 + 1.02095 × 10-8 i, 2424.99 - 1850.53 i, 2424.99 + 1850.53 i,
      -300.421 - 1.02095 × 10-8 i, -2424.99 + 1850.53 i, -2424.99 - 1850.53 i}
```

Here we get 3 propagating roots. Check if small Γ expansion is valid.

```

In[1753]:= xpoint = 1.8 * 10^19;
           TL = tprof[xpoint] * TList;
           roots6 = nPerpWarm6[xpoint];
           Warmgamma[freq, BXmin, roots6[[1]], TL]
           Warmgamma[freq, BXmin, roots6[[2]], TL]
           Warmgamma[freq, BXmin, roots6[[3]], TL]

Out[1756]= {0. + 0. i, 0. + 0. i, 0.00322329 + 2.1908 × 10-13 i, 0. + 0. i, 0. + 0. i, 0. + 0. i}

Out[1757]= {0. + 0. i, 0. + 0. i, 0.0877185 - 0.320536 i, 0. + 0. i, 0. + 0. i, 0. + 0. i}

Out[1758]= {0. + 0. i, 0. + 0. i, 0.0877185 + 0.320536 i, 0. + 0. i, 0. + 0. i, 0. + 0. i}

           Not valid for the large root, and I worry about the slow wave root too.

In[1759]:= nPerp2FS[1.9 * 10^19]
           nPerpWarm6[1.9 * 10^19]

Out[1759]= {407.791, 1288.35}

Out[1760]= {459.141 + 2.97858 × 10-8 i, 2018.57 - 1547.85 i, 2018.57 + 1547.85 i,
           -459.141 - 2.97858 × 10-8 i, -2018.57 + 1547.85 i, -2018.57 - 1547.85 i}

In[1761]:= nPerp2FS[2.0 * 10^19]
           nPerpWarm6[2.0 * 10^19]

Out[1761]= {612.814 - 423.427 i, 612.814 + 423.427 i}

Out[1762]= {708.224 - 1084.76 i, 708.224 + 1084.76 i, 1866.67 + 2.24612 × 10-7 i,
           -708.224 + 1084.76 i, -708.224 - 1084.76 i, -1866.67 - 2.24612 × 10-7 i}

In[1838]:= nPerp2FS[2.1 * 10^19]
           nPerpWarm6[2.1 * 10^19]

Out[1838]= {175.467 - 742.538 i, 175.467 + 742.538 i}

Out[1839]= {8.28196 × 10-8 - 2518.38 i, 2.46314 × 10-8 + 483.329 i, 2697.35 + 5.79755 × 10-8 i,
           -8.28196 × 10-8 + 2518.38 i, -2.46314 × 10-8 - 483.329 i, -2697.35 - 5.79755 × 10-8 i}

```

Here we get 1 propagating root. Check if small Γ expansion is valid.

```

In[1842]:= xpoint = 2.1 * 10^19;
           TL = tprof[xpoint] * TList;
           roots6 = nPerpWarm6[xpoint];
           Warmgamma[freq, BXmin, roots6[[1]], TL]
           Warmgamma[freq, BXmin, roots6[[2]], TL]
           Warmgamma[freq, BXmin, roots6[[3]], TL]

Out[1845]= {0. + 0. i, 0. + 0. i, -0.226507 - 1.48979 × 10-11 i, 0. + 0. i, 0. + 0. i, 0. + 0. i}

Out[1846]= {0. + 0. i, 0. + 0. i, -0.00834307 + 8.5036 × 10-13 i, 0. + 0. i, 0. + 0. i, 0. + 0. i}

Out[1847]= {0. + 0. i, 0. + 0. i, 0.259845 + 1.117 × 10-11 i, 0. + 0. i, 0. + 0. i, 0. + 0. i}

```

Propagating root has $\Gamma \sim 0.26$, expansion questionable

```

In[1818]:= nPerp2FS[3.0 * 10^19]
           nPerpWarm6[3.0 * 10^19]

Out[1818]= {0. + 205.063 i, 0. + 3728.56 i}

Out[1819]= {1.56244 × 10-8 - 5301.1 i, 7.17984 × 10-10 + 202.855 i, 3991.03 + 9.86508 × 10-9 i,
           -1.56244 × 10-8 + 5301.1 i, -7.17984 × 10-10 - 202.855 i, -3991.03 - 9.86508 × 10-9 i}

```

Plot kx profile for 6 th order system

```

In[1765]:= nxwarm = Table[Flatten[{x, nPerpWarm6[x]}], {x, xmin, xmax,  $\frac{x_{\max} - x_{\min}}{nPoints - 1}$ }}];

           roots = rootSort[nxwarm];
           rootsRe = Table[Flatten[{roots[[i]][[1]],
           Table[Re[roots[[i]][[j]]], {j, 2, Length[roots[[i]]}]}], {i, Length[roots]}];

           rootsIm = Table[Flatten[{roots[[i]][[1]],
           Table[Im[roots[[i]][[j]]], {j, 2, Length[roots[[i]]}]}], {i, Length[roots]}];

           g6 = ComplexVectorListPlot[roots, "x (m)", "nx"];
           paramPrint[{dataSet, xProfileMin, xProfileMax, nXmin, nXmax, BXmin,
           BXmax, freq, nz, etaList, TeXmin, TeXmax, TList, modelList, xmin, xmax}];
           Show[g6, PlotRange → All]
           Show[g6, PlotRange → {-2000., 2000.}]
           ComplexVectorListPlot[rootsRe, "x", "Re[kx]", PlotRange → {-2000., 2000.}]
           ComplexVectorListPlot[rootsIm, "x", "Im[kx]", PlotRange → {-2000., 2000.}]

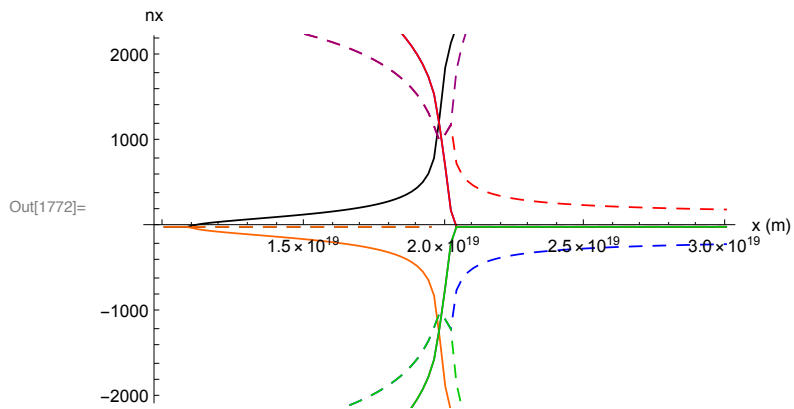
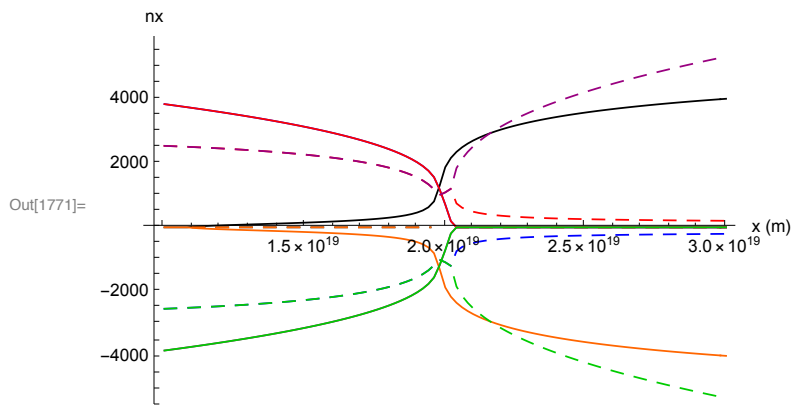
           dataSet=GDT Kinetic Alfven 50 ev
           xProfileMin=1. × 1019
           xProfileMax=3. × 1019
           nXmin=1. × 1019

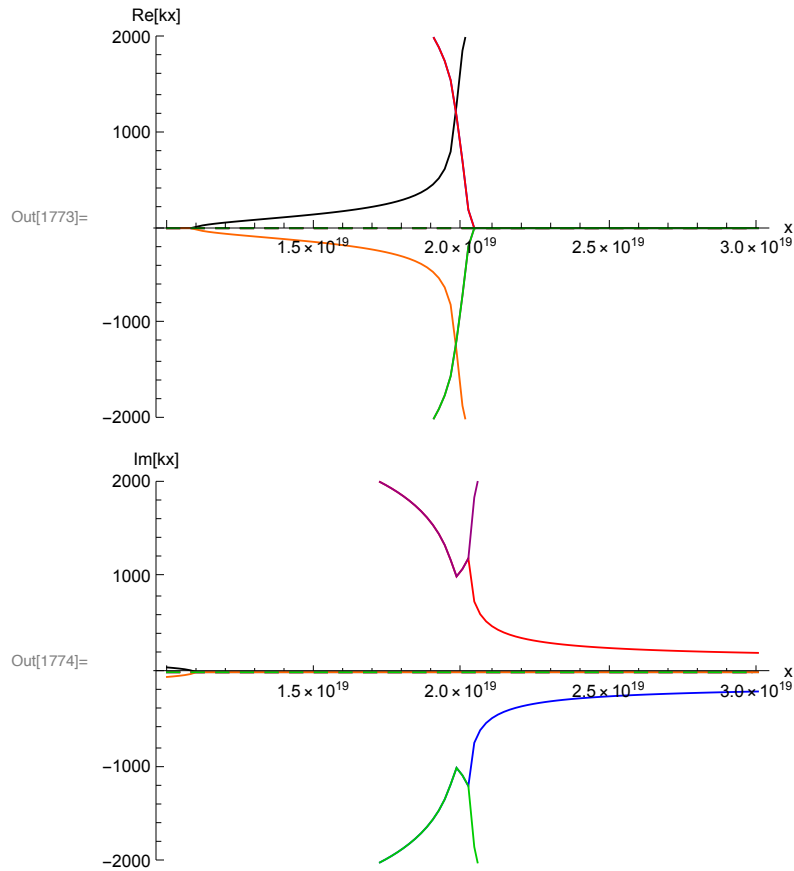
```

```

nXmax=3. × 1019
BXmin=1.2
BXmax=1.2
freq=7.5
nz=127.324
etaList={0., 1., 0., 0., 0.}
TeXmin=0.1
TeXmax=0.1
TList={0., 0., 1., 0., 0., 0.}
modelList={2, 2, 2, 2, 2, 2}
xmin=1. × 1019
xmax=3. × 1019

```





There finally is something that looks like Stix's KAW picture. BUT electrons are cold and $k_x \rho$ expansion is questionable.

Now Try with Hot Plasma Dispersion using root finder

```
In[1775]:= nPerpHot[x_, nxGuess_] := Module[{ne, b, x0, TL, nx0},
  x0 = x;
  nx0 = nxGuess;
  ne = nprof[x0];
  b = bprof[x0];
  TL = tprof[x0] * TLlist;
  rootRule = FindRoot[DisFuncGeneral[freq, ne, b, nz, nx, etaList, TL,
    nminList, nmaxList, modelList], {nx, nx0}, MaxIterations -> 30];
  nx /. rootRule]
```

First try root finding on warm plasma dispersion rel (model = 1)

```
In[1776]:= modelList = Table[1, {i, 1, 6}];
```


Compare to cold plasma and 6th order system roots

```
In[1777]:= xpoint = 1.0 * 10^19;
           TL = tprof[xpoint] * TList;
           roots6 = nPerpWarm6[xpoint]
           nPerpHot[xpoint, roots6[[1]]]
           nPerpHot[xpoint, roots6[[2]]]
           nPerpHot[xpoint, roots6[[1]]]
           nPerpHot[xpoint, roots6[[2]]]
           nPerpHot[xpoint, roots6[[3]]]

Out[1779]= {1.52288 × 10-9 + 50.1853 i, 3825.44 - 2529.23 i, 3825.44 + 2529.23 i,
           -1.52288 × 10-9 - 50.1853 i, -3825.44 + 2529.23 i, -3825.44 - 2529.23 i}

Out[1780]= 1.52288 × 10-9 + 50.1853 i
Out[1781]= 3825.44 - 2529.23 i
Out[1782]= 1.52288 × 10-9 + 50.1853 i
Out[1783]= 3825.44 - 2529.23 i
Out[1784]= 3825.44 + 2529.23 i
```

Solutions from FindRoot (nPerpHot) agree with solutions from NSolve (nPerpWarm). But still not valid.

Look closer to resonance

```
In[1785]:= xpoint = 1.8 * 10^19;
           TL = tprof[xpoint] * TList;
           roots6 = nPerpWarm6[xpoint]
           nPerpHot[xpoint, roots6[[1]]]
           nPerpHot[xpoint, roots6[[2]]]
           nPerpHot[xpoint, roots6[[1]]]
           nPerpHot[xpoint, roots6[[2]]]
           nPerpHot[xpoint, roots6[[3]]]

Out[1787]= {300.421 + 1.02095 × 10-8 i, 2424.99 - 1850.53 i, 2424.99 + 1850.53 i,
           -300.421 - 1.02095 × 10-8 i, -2424.99 + 1850.53 i, -2424.99 - 1850.53 i}

Out[1788]= 300.421 + 1.02095 × 10-8 i
Out[1789]= 2424.99 - 1850.53 i
Out[1790]= 300.421 + 1.02095 × 10-8 i
Out[1791]= 2424.99 - 1850.53 i
Out[1792]= 2424.99 + 1850.53 i
```

Plot nx profile from root finder but still with 6th order system (model = 1)

```

In[1793]:= modellList = Table[1, {i, 1, 6}]; (* Set model *)

In[1794]:= nxhot[iRoot_] := Module[{iRoot0, nxWarm, rootsWarm, nxH, x0, ne, b, t, TL},

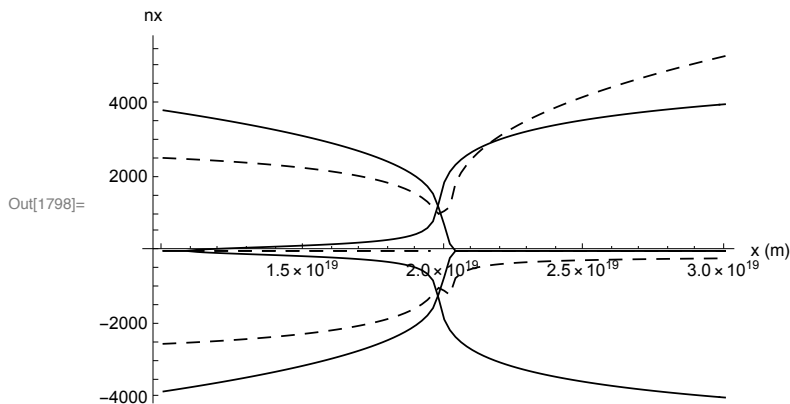
  nxWarm = Table[Flatten[{x, nPerpWarm6[x]}], {x, xmin, xmax,  $\frac{x_{\max} - x_{\min}}{nPoints - 1}$ }}];

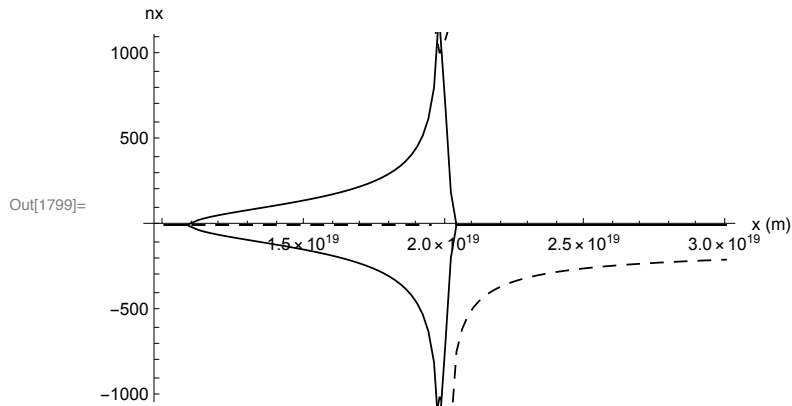
  iRoot0 = iRoot;
  rootsWarm = rootSort[nxWarm];
  nxH = Table[0., {i, 1, nPoints}];
  Do[
    ( $x0 = xmin + (i - 1) \frac{x_{\max} - x_{\min}}{nPoints - 1}$ ;
    nxGuess = rootsWarm[[i]][[iRoot0 + 1]];
    (*Print["x0 = ", x0, " nxGuess= ", nxGuess];*)
    nxH[[i]] = {x0, nPerpHot[x0, nxGuess]}]; {i, 1, nPoints}];

  nxH];

g7 = ComplexVectorListPlot[nxhot[1], "x (m)", "nx"];
g8 = ComplexVectorListPlot[nxhot[2], "x (m)", "nx"];
g9 = ComplexVectorListPlot[nxhot[3], "x (m)", "nx"];
g10 = ComplexVectorListPlot[nxhot[4], "x (m)", "nx"];
g11 = ComplexVectorListPlot[nxhot[5], "x (m)", "nx"];
g12 = ComplexVectorListPlot[nxhot[6], "x (m)", "nx"];
Show[{g7, g8, g10, g11}, PlotRange → All]
Show[{g7, g8, g10, g11}, PlotRange → {-1000., 1000.}]
paramPrint[{dataSet, xProfileMin, xProfileMax, nXmin, nXmax, BXmin,
  BXmax, freq, nz, etaList, TeXmin, TeXmax, TList, modellList, xmin, xmax}];

```





```
dataSet=GDT Kinetic Alfven 50 ev
```

```
xProfileMin=1. × 1019
```

```
xProfileMax=3. × 1019
```

```
nXmin=1. × 1019
```

```
nXmax=3. × 1019
```

```
BXmin=1.2
```

```
BXmax=1.2
```

```
freq=7.5
```

```
nz=127.324
```

```
etaList={0., 1., 0., 0., 0.}
```

```
TeXmin=0.1
```

```
TeXmax=0.1
```

```
TList={0., 0., 1., 0., 0., 0.}
```

```
modelList={1, 1, 1, 1, 1, 1}
```

```
xmin=1. × 1019
```

```
xmax=3. × 1019
```

Now try with hot plasma (model = 2) for all species. Can I find the warm plasma roots with the full dispersion relation?

■ Change model to 2

```
In[1801]:= modelList = Table[2, {i, 1, 6}];
```

```
In[1802]:= xPoint = xmin;
```

```
nPerp2FS[xPoint]
```

```
guesses = nPerpWarm6[xPoint] (* Warm plasma roots at x=0 *)
```

```
Out[1803]= {0. + 54.7543 i, 3838.1}
```

```
Out[1804]= {1.52288 × 10-9 + 50.1853 i, 3825.44 - 2529.23 i, 3825.44 + 2529.23 i,  
-1.52288 × 10-9 - 50.1853 i, -3825.44 + 2529.23 i, -3825.44 - 2529.23 i}
```

```
In[1805]:= {nPerpHot[xPoint, guesses[[1]]],
            nPerpHot[xPoint, guesses[[2]]],
            nPerpHot[xPoint, guesses[[3]]],
            nPerpHot[xPoint, guesses[[4]]],
            nPerpHot[xPoint, guesses[[5]]],
            nPerpHot[xPoint, guesses[[6]]]}
paramPrint[{dataSet, xProfileMin, xProfileMax, nXmin, nXmax, BXmin, BXmax, freq,
            nz, etaList, TeXmin, TeXmax, TList, modelList, nminList, nmaxList, xmin, xmax}];
```

```
Out[1805]= {1.52288 × 10-9 + 50.1853 i, 4476.34 - 3958.2 i, 4476.34 + 3958.2 i,
            -1.52288 × 10-9 - 50.1853 i, -4476.34 + 3958.2 i, -4476.34 - 3958.2 i}
```

```
dataSet=GDT Kinetic Alfven 50 ev
```

```
xProfileMin=1. × 1019
```

```
xProfileMax=3. × 1019
```

```
nXmin=1. × 1019
```

```
nXmax=3. × 1019
```

```
BXmin=1.2
```

```
BXmax=1.2
```

```
freq=7.5
```

```
nz=127.324
```

```
etaList={0., 1., 0., 0., 0.}
```

```
TeXmin=0.1
```

```
TeXmax=0.1
```

```
TList={0., 0., 1., 0., 0., 0.}
```

```
modelList={2, 2, 2, 2, 2, 2}
```

```
nminList={-1, -2, -2, -2, -2, -2}
```

```
nmaxList={1, 2, 2, 2, 2, 2}
```

```
xmin=1. × 1019
```

```
xmax=3. × 1019
```

Hot plasma root finder gets fast root but slow root is way different from 6th order, i.e. not damped. Not surprising since $k_x \rho$ expansion not valid Try initializing with cold plasma roots

```
In[1807]:= nPerp2FS[xPoint]
            {nPerpHot[xPoint, nPerp2FS[xPoint][[1]]], nPerpHot[xPoint, nPerp2FS[xPoint][[2]]]}
```

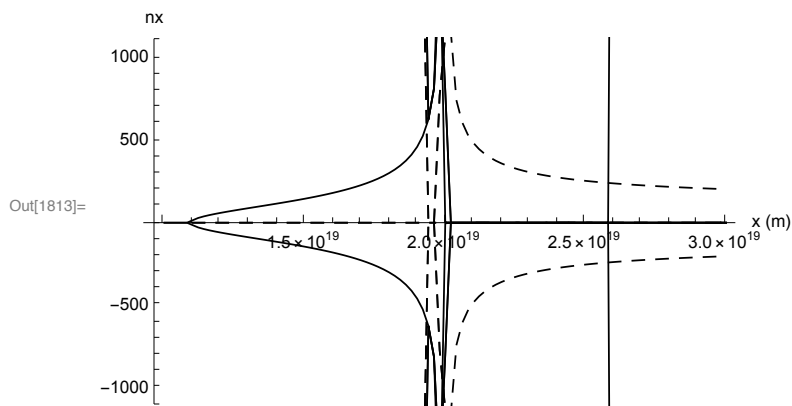
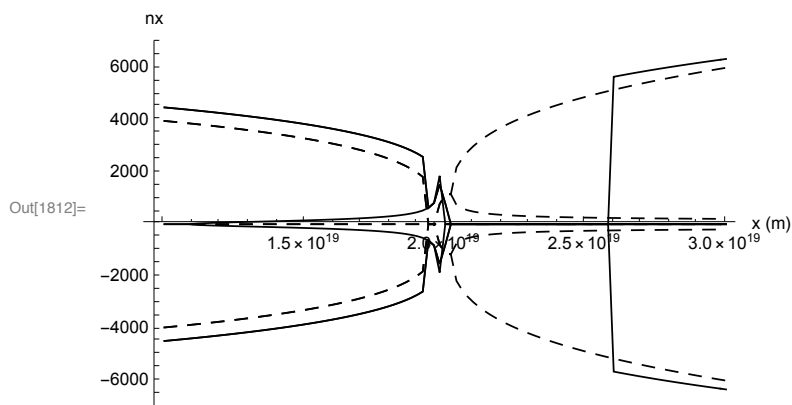
```
Out[1807]= {0. + 54.7543 i, 3838.1}
```

```
Out[1808]= {1.52288 × 10-9 + 50.1853 i, 0.0114009 + 3.46098 × 10-8 i}
```

Same as before.

Plot hot plasma vs x

```
In[1809]:= g7 = ComplexVectorListPlot[nxhot[1], "x (m)", "nx"];
g8 = ComplexVectorListPlot[nxhot[2], "x (m)", "nx"];
g9 = ComplexVectorListPlot[nxhot[3], "x (m)", "nx"];
g10 = ComplexVectorListPlot[nxhot[4], "x (m)", "nx"];
g11 = ComplexVectorListPlot[nxhot[5], "x (m)", "nx"];
g12 = ComplexVectorListPlot[nxhot[6], "x (m)", "nx"];
Show[{g7, g8, g9, g10, g11, g12}, PlotRange → All]
Show[{g7, g8, g9, g10, g11, g12}, PlotRange → {-1000., 1000.}]
paramPrint[{dataSet, xProfileMin, xProfileMax, nXmin, nXmax, BXmin, BXmax, freq, nz,
etaList, TeXmin, TeXmax, TList, modelList, nminList, nmaxList, xmin, xmax}];
```



dataSet=GDT Kinetic Alfven 50 ev

xProfileMin= $1. \times 10^{19}$

xProfileMax= $3. \times 10^{19}$

nXmin= $1. \times 10^{19}$

nXmax= $3. \times 10^{19}$

BXmin=1.2

BXmax=1.2

freq=7.5

```

nz=127.324
etaList={0., 1., 0., 0., 0.}
TeXmin=0.1
TeXmax=0.1
TList={0., 0., 1., 0., 0., 0.}
modelList={2, 2, 2, 2, 2, 2}
nminList={-1, -2, -2, -2, -2, -2}
nmaxList={1, 2, 2, 2, 2, 2}
xmin= $1. \times 10^{19}$ 
xmax= $3. \times 10^{19}$ 

```

No sign of any kinetic Alfven wave. Really looks about the same as with warm electrons

Initialization

Magnetic field,Density and Temperature Profiles

In[1815]:=

```

bprof[x_] := If[Abs[(BXmax - BXmin) / BXmax] > 10-6,
  BXmin + (x - xProfileMin) / (xProfileMax - xProfileMin) (BXmax - BXmin), BXmin];

```

In[1816]:=

```

nprof[x_] := If[Abs[(nXmax - nXmin) / nXmax] > 10-6,
  nXmin + (x - xProfileMin) / (xProfileMax - xProfileMin) (nXmax - nXmin), nXmin];

```

In[1817]:=

```

tprof[x_] := If[Abs[(TeXmax - TeXmin) / TeXmax] > 10-6,
  TeXmin + (x - xProfileMin) / (xProfileMax - xProfileMin) (TeXmax - TeXmin), TeXmin];

```