PARAMETER SENSITIVITY ANALYSIS

ERT 474/574

Open-Source Hydro Data Analytics

Nov 19th 2025

University at Buffalo The State University of New York

Final Project

Timeline:

- FAIR Principles discussion (Dec 1)
- Nov 24: progress update (each group is expected to finish setting up VIC and mizuRoute models, and conducting initial evaluation against the USGS time series)
  - 5 min for each group
- Dec 8: Final presentation (30%)
  - 15 min for each group
- Dec 15: Final report (60%)
  - 10-page report for each group (including reference)

Paper reading for Week 15

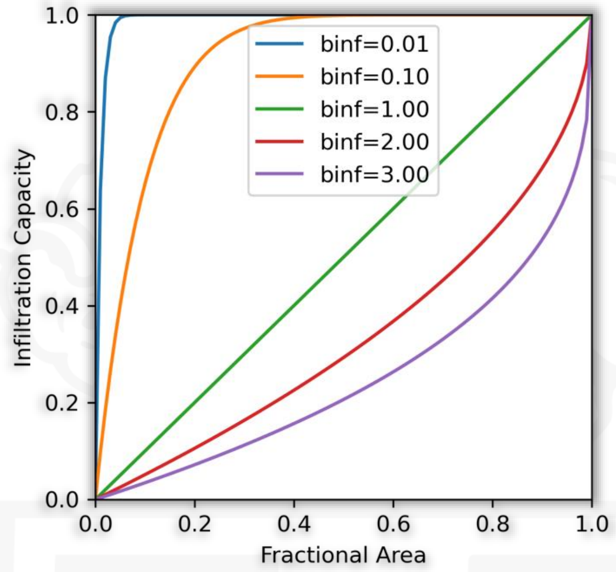
- FAIR Principles discussion (Dec 1)
- CARE Principles discussion (Dec 3)

Outline

- What is a parameter?
- What is parameter sensitivity?
- Why important in hydrology?
- Types of sensitivity analysis
- Steps in sensitivity analysis
- Expectations for the final project

What is a parameter?

- In hydrologic models, a parameter is a variable that defines the characteristics of a grid cell, such as soil properties, vegetation, or model configurations.



- Examples of parameters in VIC models

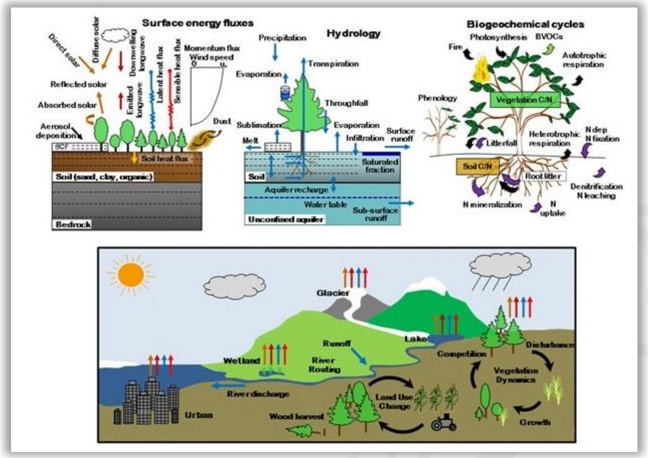
Variable Name	Dimension	Units	Type	Number of Values	Description
infilt	[lat, lon]	N/A	double	1	Variable infiltration curve parameter (binfilt)
Ds	[lat, lon]	fraction	double	1	Fraction of Dsmax where non-linear baseflow begins
Dsmax	[lat, lon]	mm/day	double	1	Maximum velocity of baseflow
Ws	[lat, lon]	fraction	double	1	Fraction of maximum soil moisture where non-linear baseflow occurs

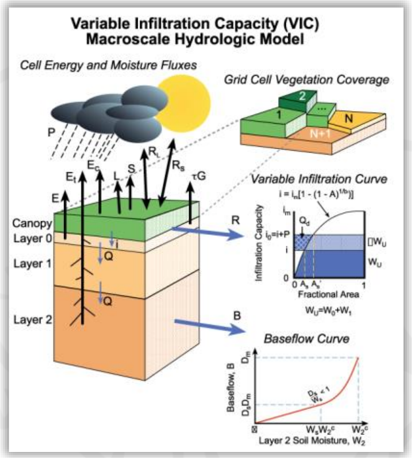
CLM Diagram

VIC Diagram

Process representations and their parameters vary across different models

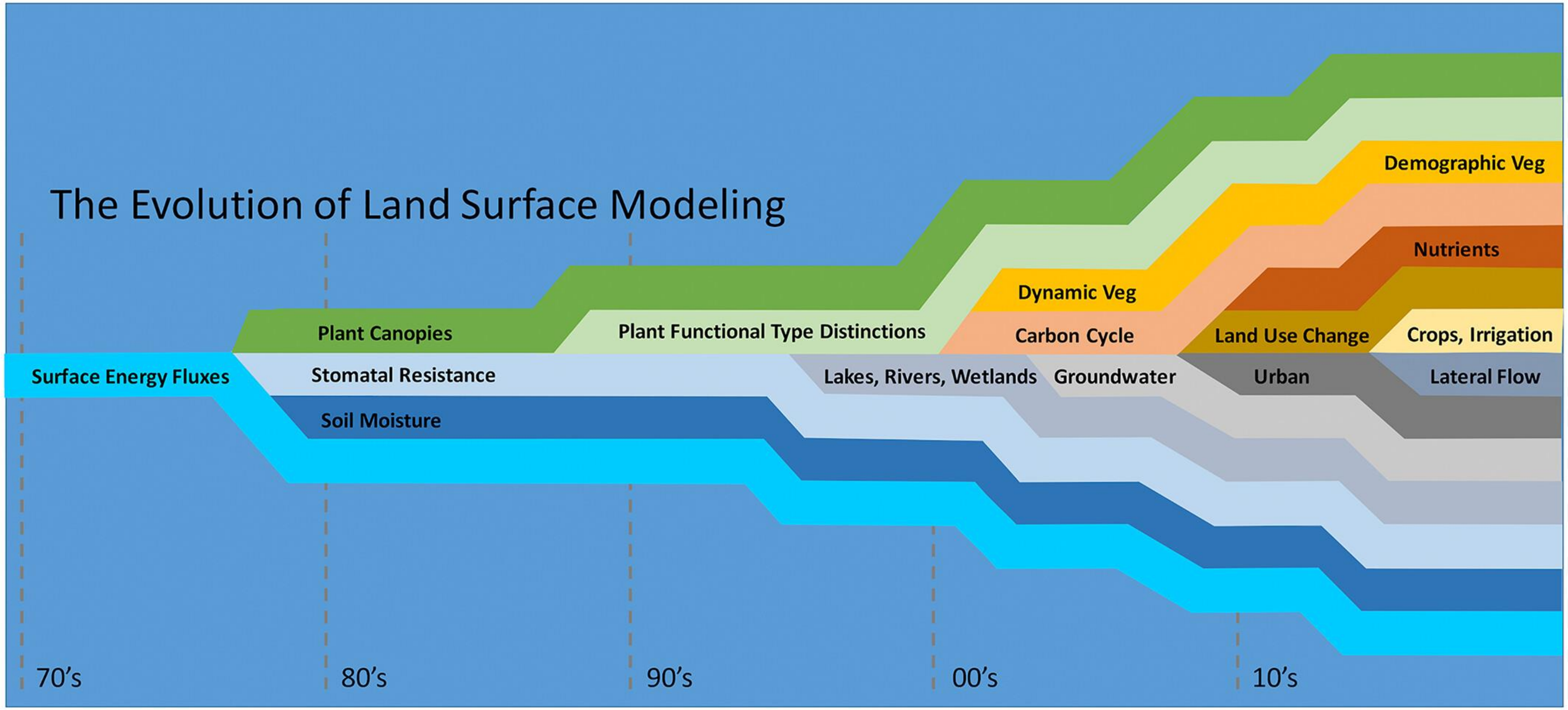
- Different philosophies were applied when developing different models





Aspect	CLM	VIC
Goal	Earth system integration, process realism	Hydrologic prediction, water balance
Parameter treatment	Global constants (one value for global application) + surface datasets (spatially distributed dataset)	Spatially heterogenous, calibrated locally
Calibration Focus	Limited (often avoided for ESM consistency)	Essential for regional streamflow accuracy

There are many, many parameters in the model!



Out of hundreds of parameters, which parameters should we calibrate??

Fisher, R. A. & C. D. Koven (2020), *JAMES*

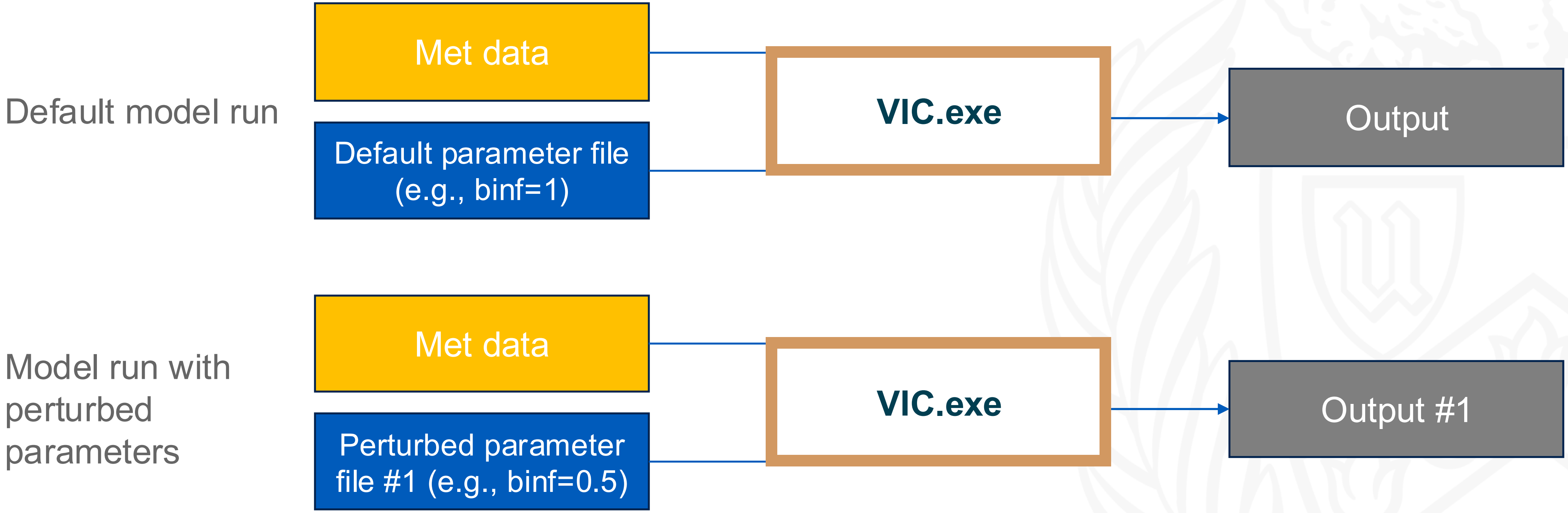
Why important in hydrology?

- Helps identify which parameters need careful calibration.
- Reduces uncertainty by focusing on influential parameters.
- Improves model efficiency and reliability.

What is parameter sensitivity?

- Parameter sensitivity refers to how much a model's output changes in response to variations in a specific parameter.
  - It measures the influence or importance of a parameter on the model behavior.

How do we perform a sensitivity analysis?



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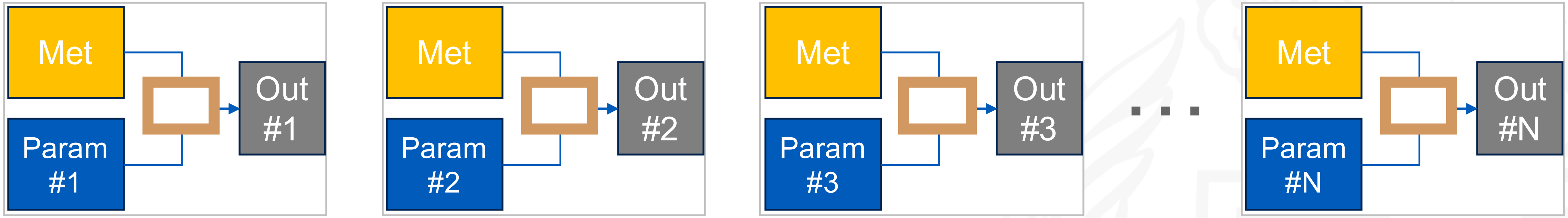

Therefore, before running sensitivity analysis, we need to define the parameter space and how parameters will be varied or sampled!

Processes are interconnected

One parameter can modify multiple processes!
When the soil parameters are modified, it might not only affect the infiltration and runoff but also soil moisture or even evapotranspiration.

Types of sensitivity analyses

Local Sensitivity Analysis

- Vary one parameter at a time around a baseline (One-At-A-Time, or OAAT)

Global Sensitivity Analysis

- Vary multiple parameters simultaneously across ranges.
- Methods:
  - The Sobol′ method
  - The Sum-of-Trees (SOT) model
  - The Multivariate Adaptive Regression Splines (MARS) technique
  - The Delta Test (DT)

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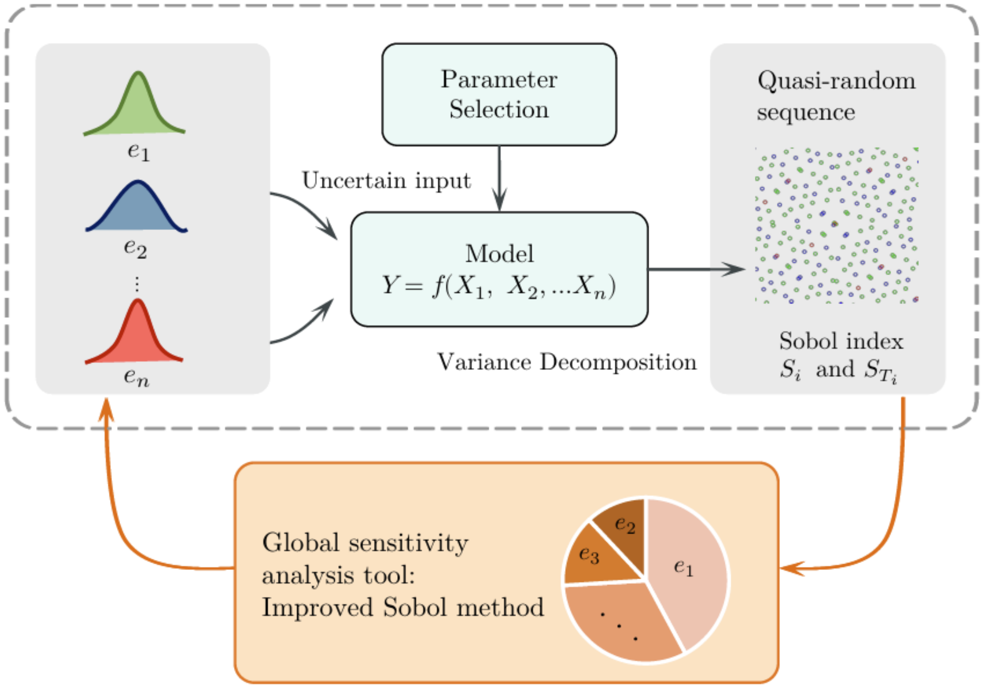
One-At-A-Time (OAAT)

- OAAT means we vary **one parameter at a time** while keeping all others fixed, and observe how the model output changes. This helps us understand the direct effect of each parameter individually.
- Pros: Simple, quick
- Cons: Ignores interactions

Experiment ID	B_{inf}	Ds,max	Ds
Default	1	50	0.5
Exp 1	0.5	50	0.5
Exp 2	2	50	0.5
Exp 3	1	25	0.5
Exp 4	1	100	0.5
Exp 5	1	50	0.25
Exp 6	1	50	0.75

Sobol's Method

- A global sensitivity analysis method that decomposes the variance of a model output into contributions from each input parameter and their interactions.
- It's widely used in environmental modeling, hydrology, and climate science because it helps identify which parameters most influence uncertainty in predictions.



Credit: https://www.researchsquare.com/article/rs-1558167/v1

Sobol's Method

- Uses decomposition of the model output function $y = f(x)$ into summands of variance using combinations of input parameters in increasing dimensionality.
- **Main Idea**: decomposition of the function $f(x)$ into summands of increasing dimensionality is given by:

$$f(x_1 \ldots x_k) = f_0 + \sum_{i=1}^{k} f_i(x_i) + \sum_{1 \le i < j \le k} f_{i,j}(x_i, x_j) + \cdots + f_{1,\ldots,k}(x_1, \ldots, x_k)$$

- Therefore, we can decompose the variance of the function Y as follows

$$\mathbb{V}(Y) = \sum_{i=1}^{k} \mathbb{V}_i(Y) + \sum_{1 \le i < j \le k} \mathbb{V}_{i,j}(Y) + \cdots + \mathbb{V}_{1,\ldots,k}(Y)$$

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Introducing conditional variance

$$\mathbb{V}_i(Y) = \mathbb{V}[\mathbb{E}(Y|x_i)] \qquad \mathbb{V}_{ij}(Y) = \mathbb{V}[\mathbb{E}(Y|x_i x_j)] - \mathbb{V}_i(Y) - \mathbb{V}_j(Y)$$

Sobol's indices are expressed as

$$S_i = \frac{\mathbb{V}_i(Y)}{\mathbb{V}[Y]} \qquad S_{ij} = \frac{\mathbb{V}_{ij}(Y)}{\mathbb{V}[Y]}$$

S_i corresponds to the first-order term which apprises the contribution of the i-th parameter, while S_{ij} corresponds to the second-order term which informs about the contribution of interactions between the i-th and the j-th parameters. These equations can be generalized to compute higher order terms; however, they are expensive to compute and their interpretation is complex. This is why only first-order indices are provided.

Sobol's Method

- First Order Effects
  - The first-order effect, or the main effect, of a single input parameter is the effect of a single parameter on the model output variance.
- Total Effects
  - The total effects include the first-order effects and all the interactions involving that parameter. The total effects of the response are shown in the Figures below

$$S_i = \frac{\mathbb{V}_i(Y)}{\mathbb{V}[Y]}$$

$$S_{T_i} = S_i + \sum_j S_{ij} + \sum_{j,k} S_{ijk} = 1 - \frac{\mathbb{V}[\mathbb{E}(Y|x_{\sim i})]}{\mathbb{V}[Y]}$$

First order indices sum to at most 1, while total order indices sum to at least 1. If there are no interactions, then first and total order indices are equal, and both first and total order indices sum to 1.

Sobol’s Method

- Sensitivity index--the fractional contribution to the output variance due to uncertainties in the inputs.
- Example: if there are only three input parameters, A, B, and C, then the total effects of a parameter A, for

$$TS(A)=S(A)+S(AB)+S(AC)+S(ABC)$$

- where TS(i) is the total sensitivity index of parameter i, and S(A) denotes the first order sensitivity index for parameter A, S(A_j) denote the second-order sensitivity index for the parameter A and j (for j(≠A)), i.e. the interaction between parameters A and j(≠A), and so on.

Steps for sensitivity analysis

- Step 1: Define model (VIC) and output variables (e.g., streamflow, ET).
- Step 2: Select parameters and ranges (physical plausibility).
- Step 3: Choose sensitivity method (local vs global).
- Step 4: Run simulations (Monte Carlo or Latin Hypercube Sampling for global).
- Step 5: Compute sensitivity metrics (e.g., Sobol indices, correlation).
- Step 6: Interpret results (rank parameters by influence).

Expectations in final project

- Let's practice an OAAT sensitivity experiment!
- Based on your model performance, pick one parameter
  - Such as: Binf, Ws, Wsmax, Ds, Dsmax…
- For the selected parameter, please apply multiplicative factors,
  - For example, if we decide to perturb Binf, we can multiply the parameters by factors ranging from [0.8, 1.2] for all Binf across the basin.
- Check the impacts of parameter perturbations to 1) Water Balance terms, including surface runoff, baseflow, ET, etc., 2) Energy flux terms, such as latent heat fluxes and sensible heat fluxes.
- Based on the sensitivity analysis, based on its corresponding impacts on runoff, manually pick one optimal value that might lead to the best flow performance! And check whether it indeed improves the flow performance!

Questions?