RIVER ROUTING

ERT 474/574

Open-Source Hydro Data Analytics

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Why do we need a routing model??

Saint-Venant Equation

1-D conservation equation for continuity

$$\frac{\partial q}{\partial x} + \frac{\partial A}{\partial t} = 0$$

1-D conservation equation for momentum

$$\underbrace{\frac{\partial v}{\partial t}}_{a} + \underbrace{v\frac{\partial v}{\partial x}}_{b} + \underbrace{g\frac{\partial y}{\partial x}}_{c} - \underbrace{g(S_0}_{d} - \underbrace{S_f)}_{e} = 0$$

- (a) is the local acceleration term
- (b) is the convective acceleration term
- (c) is the pressure gradient term
- (d) is the gravity term
- (e) is the friction term.

Simplification of Saint-Venant Equation in IRF

- IRF
  - **IRF** uses a **diffusive wave approximation**, which retains pressure gradient and dispersion, but neglects inertia.

$$\cancel{\frac{\partial v}{\partial t}} + v\cancel{\frac{\partial v}{\partial x}} + g\frac{\partial y}{\partial x} - g(S_0 - S_f) = 0$$

$$Continuity\ Equation$$

$$Manning's\ Equation$$

$$\frac{\partial q}{\partial t} = D\frac{\partial^2 q}{\partial x^2} - C\frac{\partial q}{\partial x}$$

It is important to note that both diffusivity (D) and wave speed (C) are constant. This is a linear advection-diffusion equation, which can be solved using Green's functions to derive the Impulse Response Function.

Think of it as a **response kernel**—it tells you how a unit input of runoff at a grid cell spreads out over time as it travels downstream. It's derived analytically and used in a convolution framework.

Simplification of Saint-Venant Equation in KWT

- KWT
  - **KWT** uses a **kinematic wave approximation**, which neglects both inertia and pressure gradient, assuming flow is driven purely by gravity and friction.

Models the **movement of a wave of water** down a slope, assuming the flow is driven purely by gravity and friction. It's more physically detailed but less flexible for large-scale or linear routing.

$$\cancel{\frac{\partial v}{\partial t}} + v\cancel{\frac{\partial v}{\partial x}} + g\cancel{\frac{\partial y}{\partial x}} - g(S_0 - S_f) = 0$$

$$\boxed{\begin{gathered} S_0 = S_f \\ Continuity\ Equation \\ Manning's\ Equation \end{gathered}}$$

$$\boxed{C = (\alpha + 1) \cdot (w)^{\frac{-\alpha}{\alpha+1}} \cdot \left(\frac{k}{n}\sqrt{S_0}\right)^{\frac{1}{\alpha+1}} \cdot q^{\frac{\alpha}{\alpha+1}}}$$

It is important to note that the calculated wave speed (C) is NOT constant. The wave speed is a function of channel width, Manning's coefficient, channel slope, and discharge.

IRF versus KWT

Feature	Impulse Response Function (IRF)	Kinematic Wave Tracking (KWT)
Approach	Linear convolution using precomputed response functions	Solves simplified Saint-Venant equations

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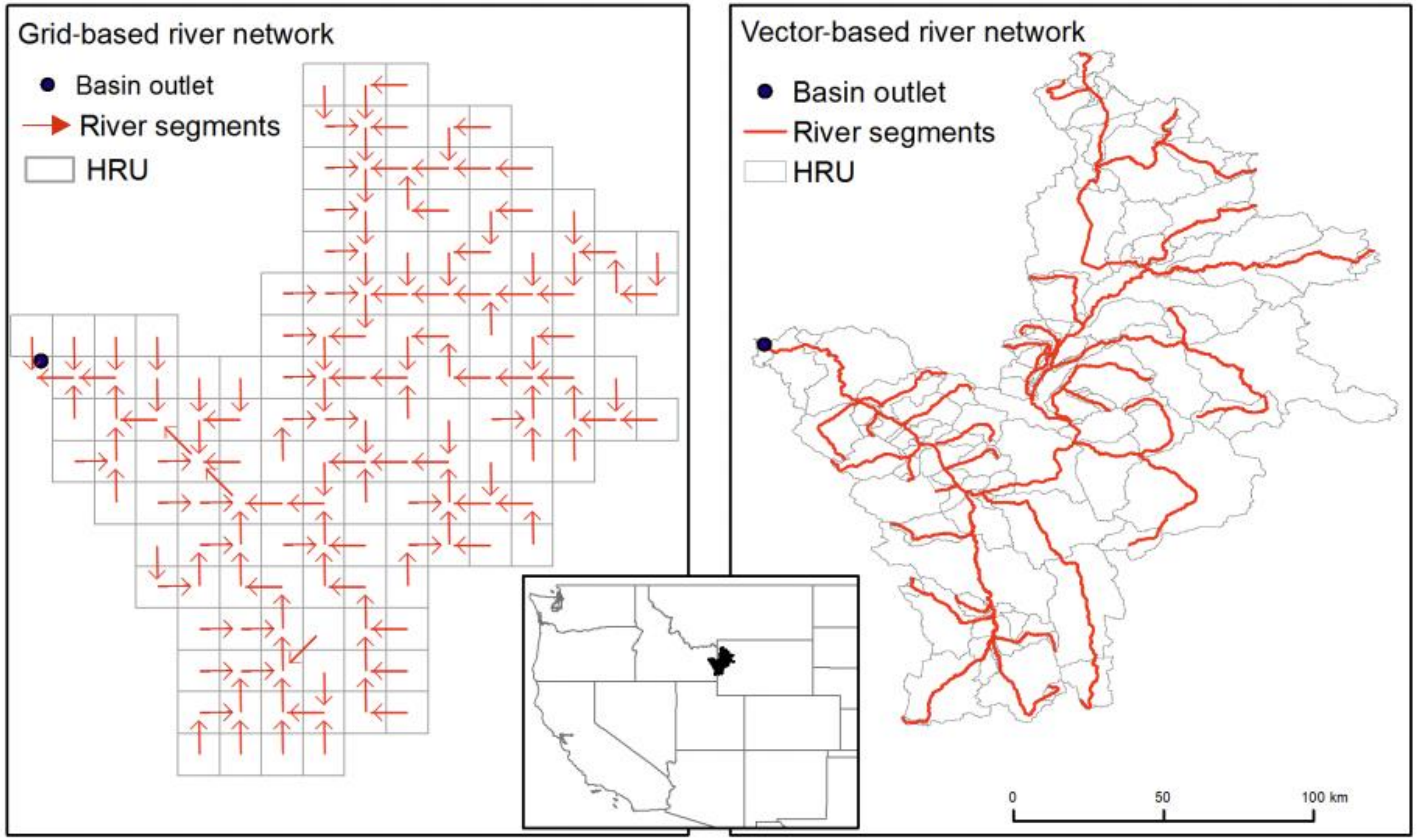
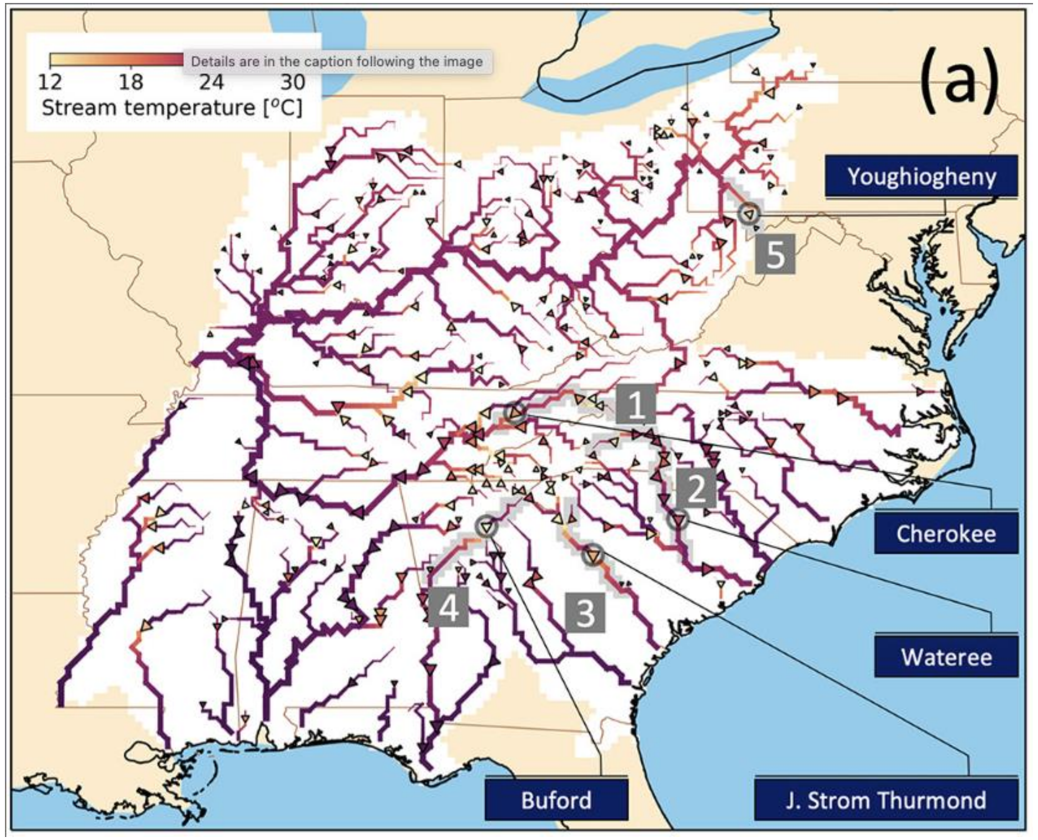
Data Requirements	Flow direction, velocity, and diffusion maps	Channel geometry, slope, roughness, and inflow hydrographs

Grid-based v.s. vector-based river networks



A vector-based river network for a large river basin



The left figure shows the mean summer river temperature for major river basins in the Southeastern United States.

What are the pros/cons of a gridded river network?

Grid-based *versus* vector-based river networks

Feature	Gridded River Network	Vector-Based River Network
Implementation Difficulty	Easier to implement; Naturally following by hydrologic modeling configured in grid cells	More complex; requires detailed preprocessing

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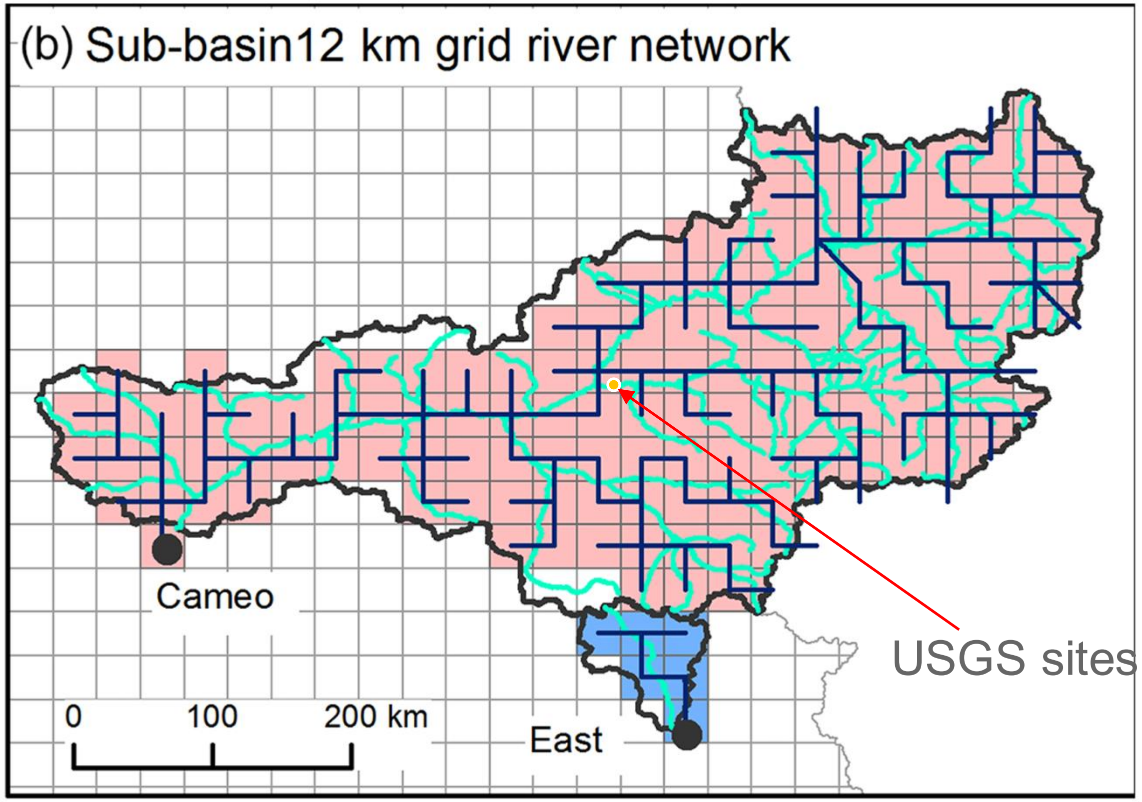
Model Evaluation	The observation sites that will be used for evaluation might not be located in the nearest grid cell.	It is very easy to find the corresponding HUCs for the observation sites.

Example: if there is a USGS site at the red dot, which grid cells of streamflow should we use to evaluate?



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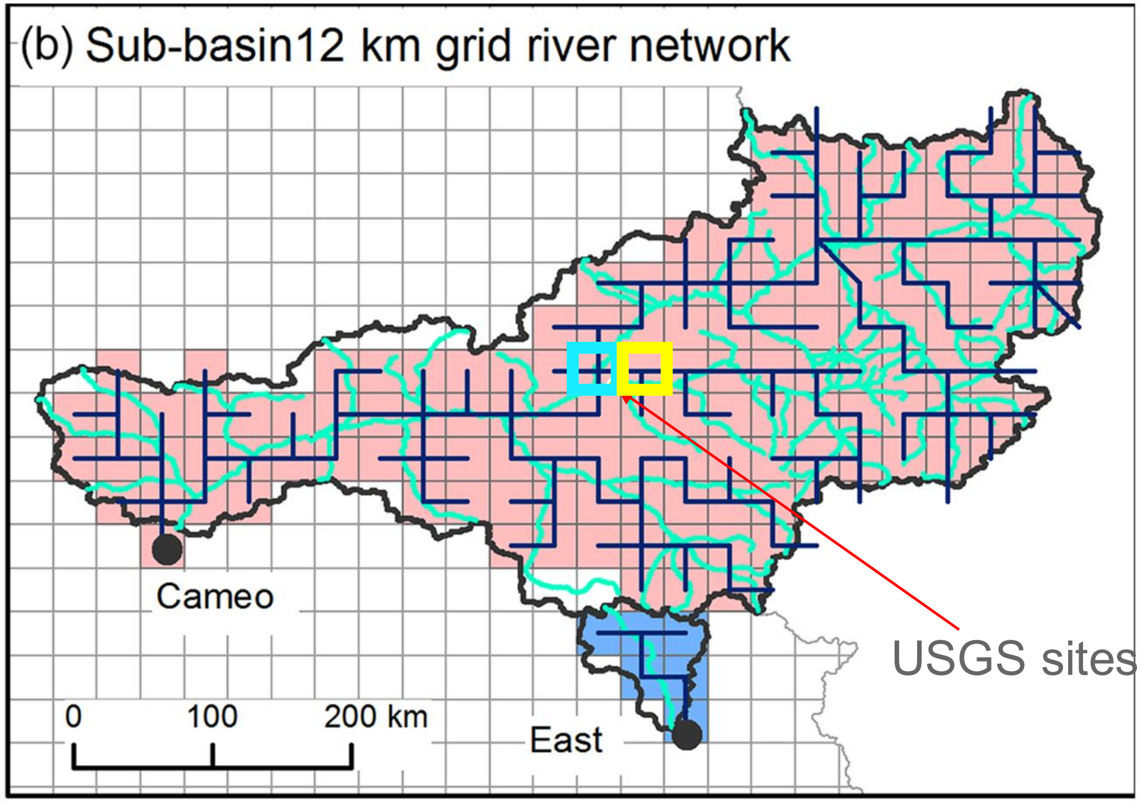

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Why?

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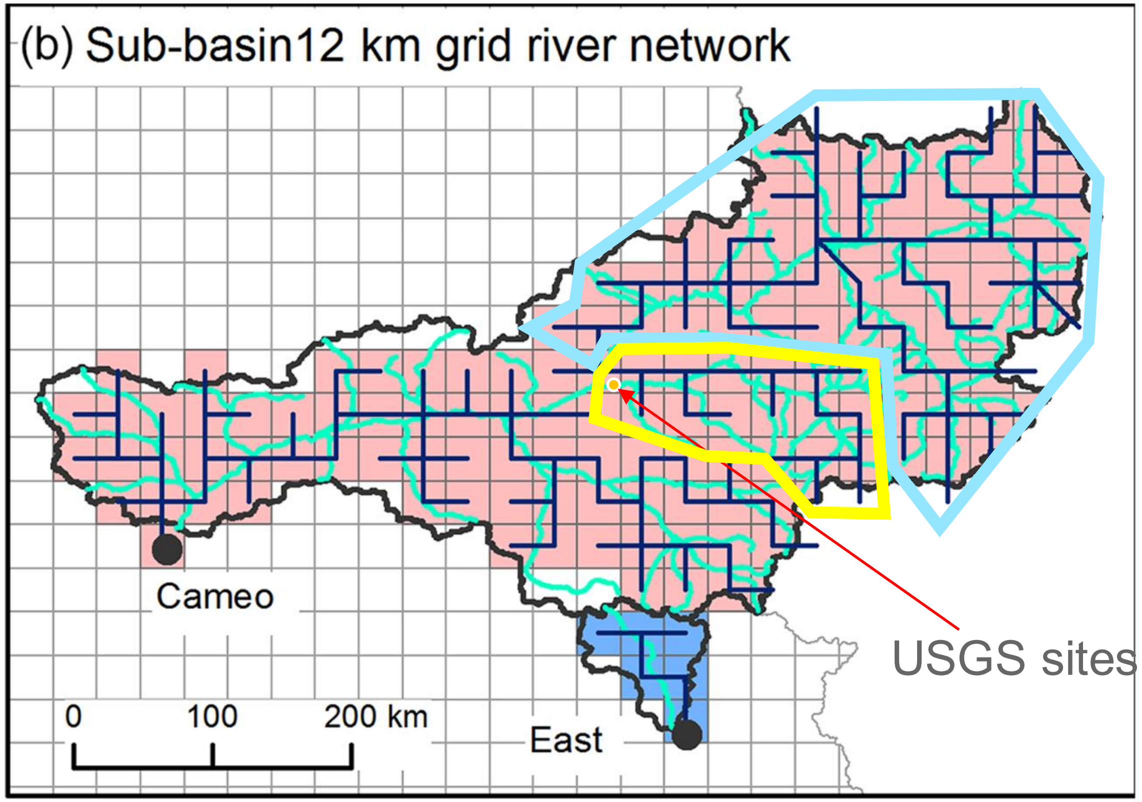

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Why? The river flow in the blue grid also confluences the streamflow from the upper tributary (highlighted in blue), while the USGS site was located upstream of the confluence point, so its confluence area should be the boundary highlighted in yellow.

mizuRoute – routing model

- Why name it mizuRoute?
  - ***mizu*** (水) means water in Japanese
  - The developer's last name is ***mizu***kami.
- Why mizuRoute?
  - It can be applied to a user-defined catchment-based river network (both vector-based and grid-based).
  - We can add Lakes into the river network, allowing the model to simulate discharge and volume in rivers and lakes (natural lakes or reservoirs).

What input files do we need to run mizuRoute for a vector-based river network?

Runoff data
Model output from hydrologic models

River network and topology file
River segment connection, channel slope, segment length, etc.

Parameter file
Such as C(wave velocity) and D (Diffusivity) in IRF

Mapping file
It is used to map the runoff from the grid-based network to each hydrologic response unit (HRU)

Questions?