RIVER ROUTING

ERT 474/574

Open-Source Hydro Data Analytics

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Why do we need a routing model??

What is the output from a hydrologic model?

Water balance equation

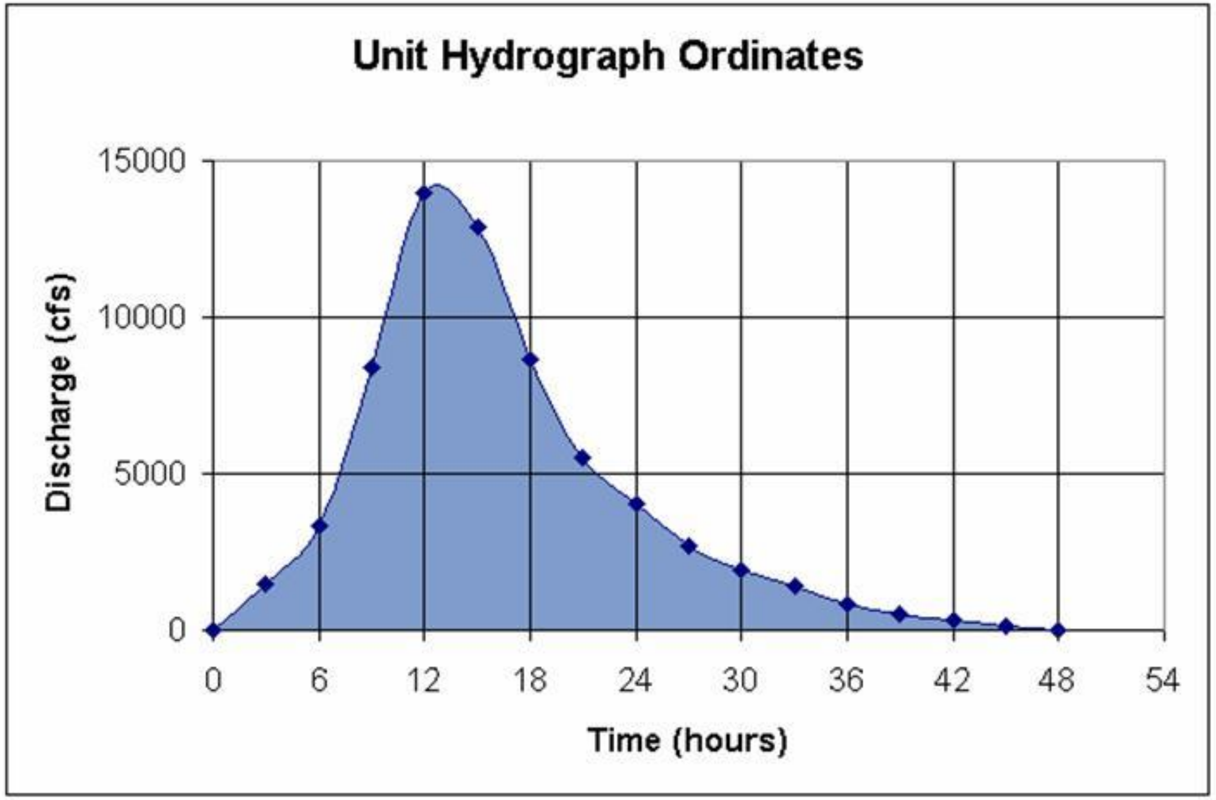
$$\underbrace{P}_{\text{Input}} = \underbrace{ET + R + \Delta S}_{\text{Output}}$$

We have already calculated the runoff in the hydrologic model, but measuring it is very challenging!

As runoff eventually will contribute to the streamflow in the river, we will need to take a step further and leverage a ***Routing Model*** to convert runoff into streamflow so that we can evaluate the accuracy of the numerical model.

Routing Scheme #1: Impulse Response Function (IRF)

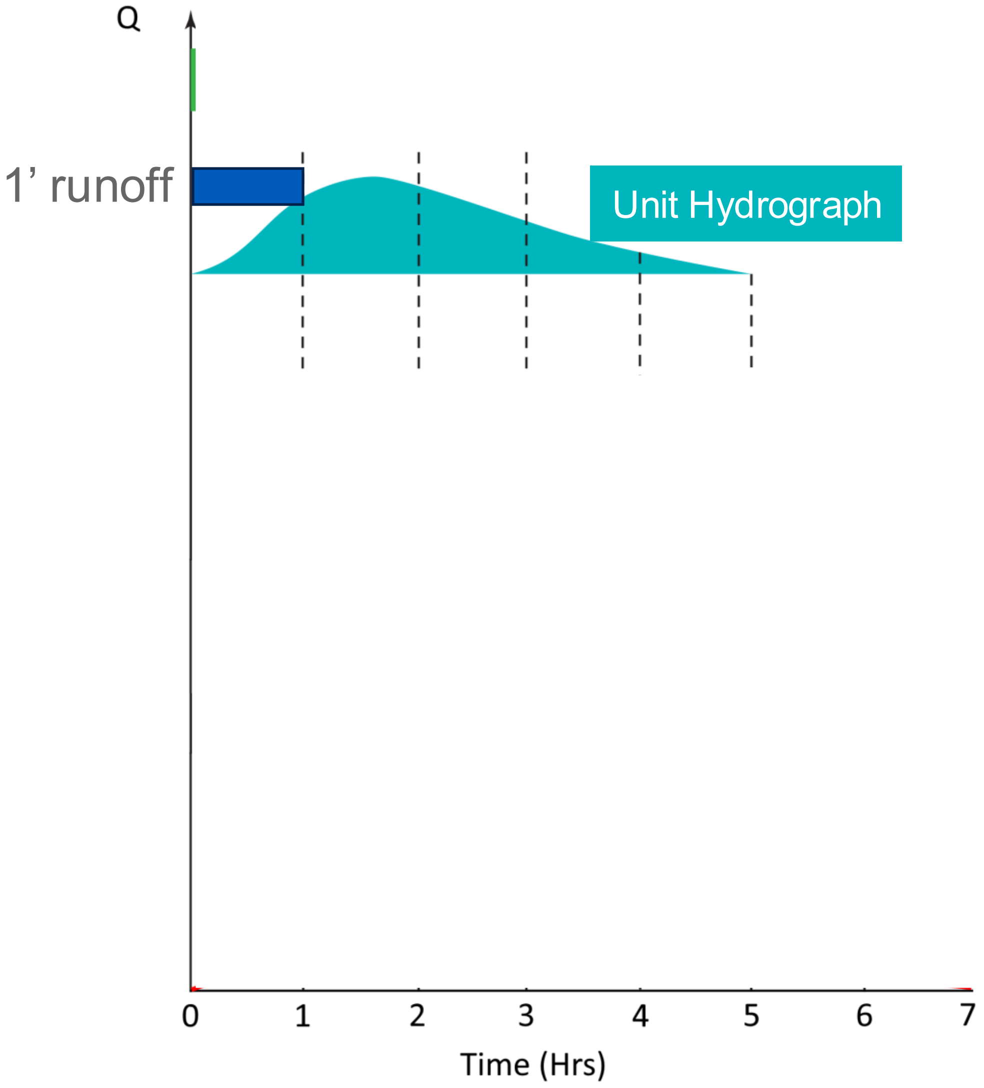
- IFR is used by the Lohmann Routing Scheme
  - IRF is called as the Unit Hydrograph (UH) by hydrologists
- What is a unit hydrograph?
  - A unit hydrograph represents the hydrograph from direct or surface runoff from one inch of runoff over the entire basin in a given unit of time.



Unit Hydrograph for Runoff

Unit hydrograph theory converts runoff values for a basin into flow or discharge at the basin outlet.

This method is used in the default routing model for VIC, called ***RVIC***.



https://training.weather.gov/nwstc/Hydrology/HYDRO/BHModule/BH-unit5.HTML

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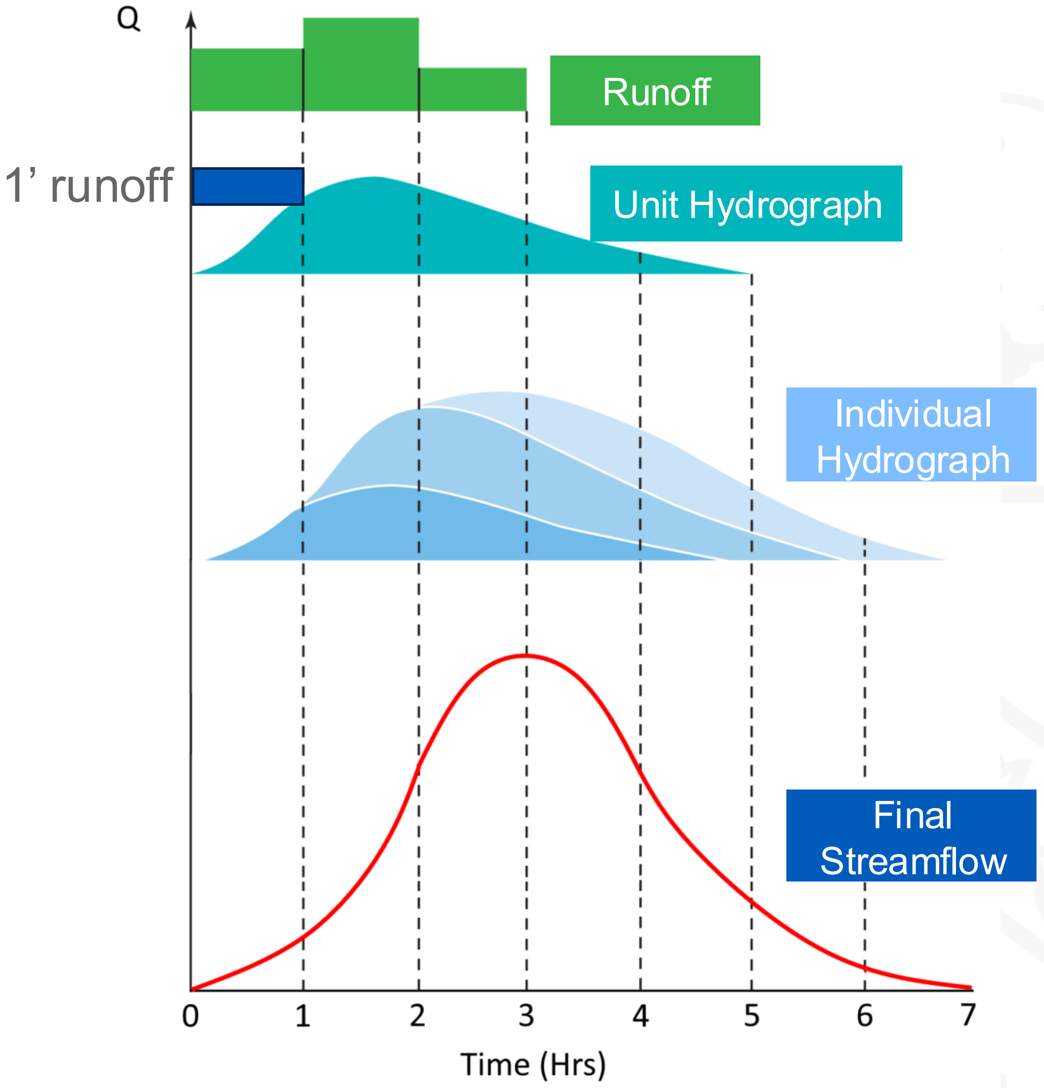

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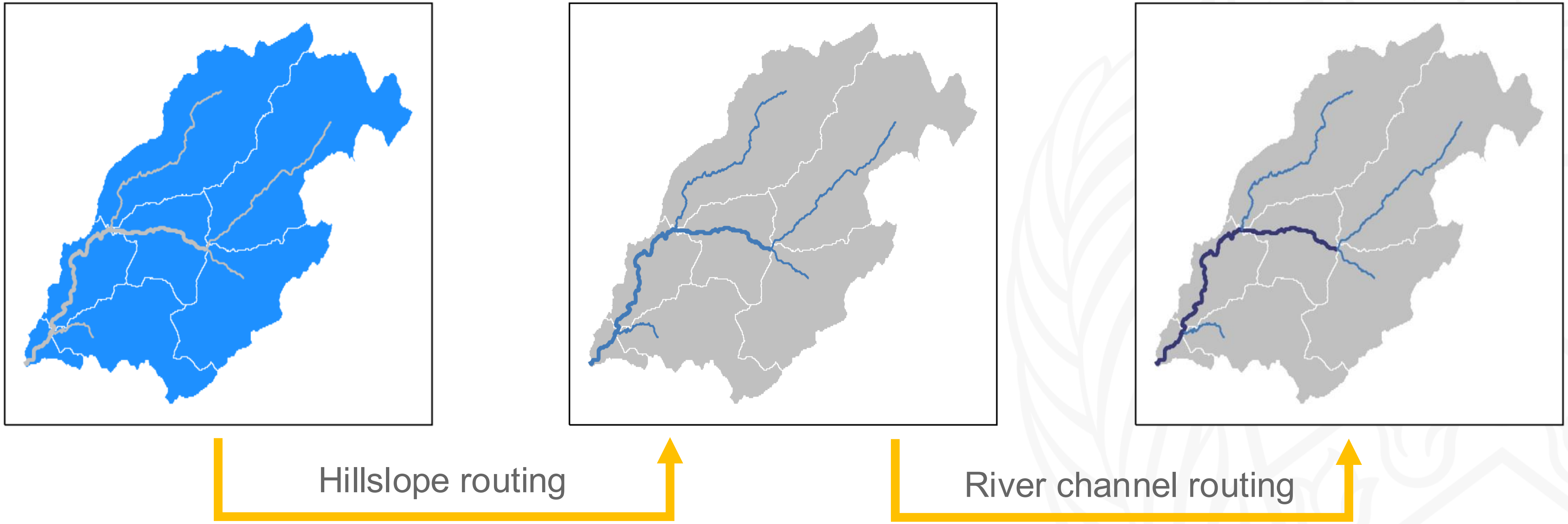
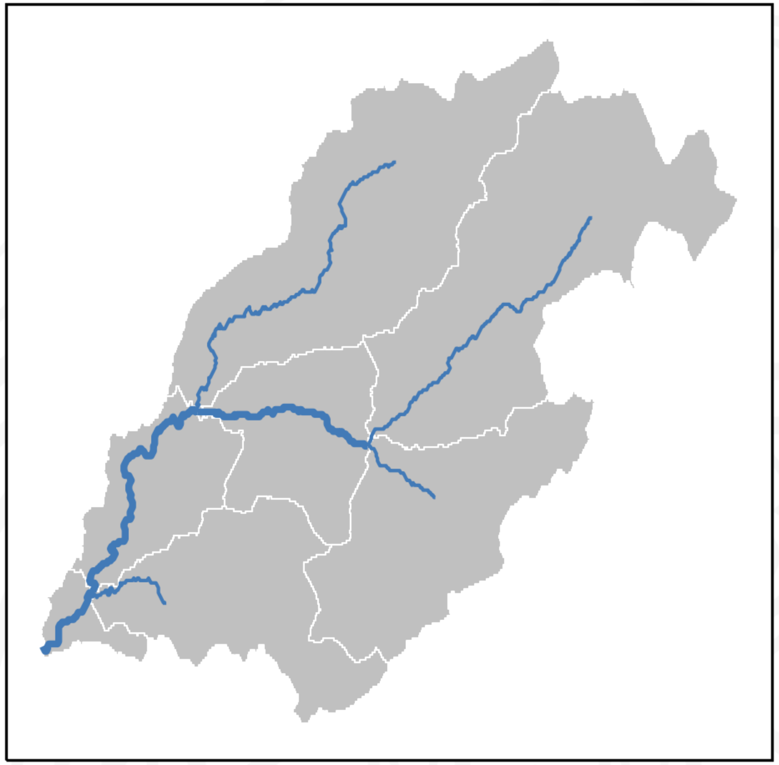
RVIC model separate the routing into two-steps



Hillslope routing is performed with a gamma-distribution-based unit-hydrograph to transport runoff from a hillslope to a catchment outlet

What are the assumptions for IRF?

- **Time-Invariant Impulse Response Functions (IRFs)**
  The routing behavior from each grid cell to a downstream point is characterized by an IRF, which is assumed to be linear and time-invariant. The response to runoff inputs does not change over time, which can be precomputed.
- **Neglect of Nonlinear Channel Processes**
  The model does not account for complex channel hydraulics, backwater effects, or floodplain interactions. It is best suited for large-scale applications where such details are less critical.



Each pair of the upstream and downstream HRUs has one unit hydrograph

Routing Scheme #2: Kinematic wave tracking (KWT)

- **KWT method** computes a wave speed or a celerity for the runoff (or discharge) that enters an individual stream segment from the corresponding HRU at each time step using kinematic approximation
  - The runoff, represented as a particle, is propagated through the river network based on a travel time (the celerity divided by the segment length). Note that the wave celerity differs from the flow velocity, as the wave typically moves faster than water mass
  - The wave celerity [LT^{-1}] is a function of channel width [L], Manning's coefficient [–], channel slope [unitless], and discharge [L^3T^{-1}].

Saint-Venant Equation

1-D conservation equation for continuity

$$\frac{\partial q}{\partial x} + \frac{\partial A}{\partial t} = 0$$

1-D conservation equation for momentum

$$\underbrace{\frac{\partial v}{\partial t}}_{a} + \underbrace{v\frac{\partial v}{\partial x}}_{b} + \underbrace{g\frac{\partial y}{\partial x}}_{c} - \underbrace{g(S_0}_{d} \underbrace{- S_f)}_{e} = 0$$

- (a) is the local acceleration term
- (b) is the convective acceleration term
- (c) is the pressure gradient term
- (d) is the gravity term
- (e) is the friction term.

Simplification of Saint-Venant Equation in IRF

- IRF
  - **IRF** uses a **diffusive wave approximation**, which retains pressure gradient and dispersion, but neglects inertia.

Think of it as a **response kernel**—it tells you how a unit input of runoff at a grid cell spreads out over time as it travels downstream. It's derived analytically and used in a convolution framework.

$$\cancel{\frac{\partial v}{\partial t}} + v\cancel{\frac{\partial v}{\partial x}} + g\frac{\partial y}{\partial x} - g(S_0 - S_f) = 0$$

$$Continuity\ Equation$$

$$Manning's\ Equation$$

$$\frac{\partial q}{\partial t} = D\frac{\partial^2 q}{\partial x^2} - C\frac{\partial q}{\partial x}$$

It is important to note that both diffusivity (D) and wave speed (C) are constant. This is a linear advection-diffusion equation, which can be solved using Green's functions to derive the Impulse Response Function.

Simplification of Saint-Venant Equation in KWT

- KWT
  - **KWT** uses a **kinematic wave approximation**, which neglects both inertia and pressure gradient, assuming flow is driven purely by gravity and friction.

$$\cancel{\frac{\partial v}{\partial t}} + \cancel{v\frac{\partial v}{\partial x}} + \cancel{g\frac{\partial y}{\partial x}} - g(S_0 - S_f) = 0$$

$$\boxed{\begin{array}{c} S_0 = S_f \\ Continuity\ Equation \\ Manning's\ Equation \end{array}} \rightarrow \boxed{C = (\alpha+1)\cdot(w)^{\frac{-\alpha}{\alpha+1}} \cdot \left(\frac{k}{n}\sqrt{S_0}\right)^{\frac{1}{\alpha+1}} \cdot q^{\frac{\alpha}{\alpha+1}}}$$

Models the **movement of a wave of water** down a slope, assuming the flow is driven purely by gravity and friction. It's more physically detailed but less flexible for large-scale or linear routing.

It is important to note that the calculated wave speed (C) is NOT constant. The wave speed is a function of channel width, Manning's coefficient, channel slope, and discharge.

IRF versus KWT

Feature	Impulse Response Function (IRF)	Kinematic Wave Tracking (KWT)
Approach	Linear convolution using precomputed response functions	Solves simplified Saint-Venant equations

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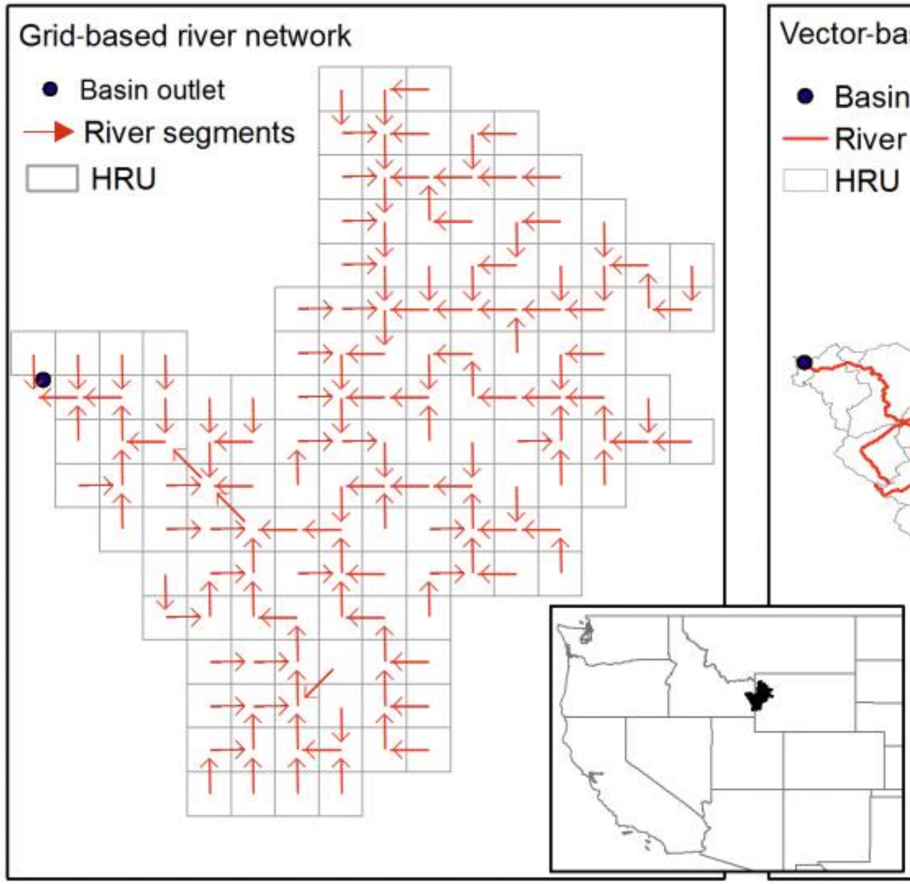
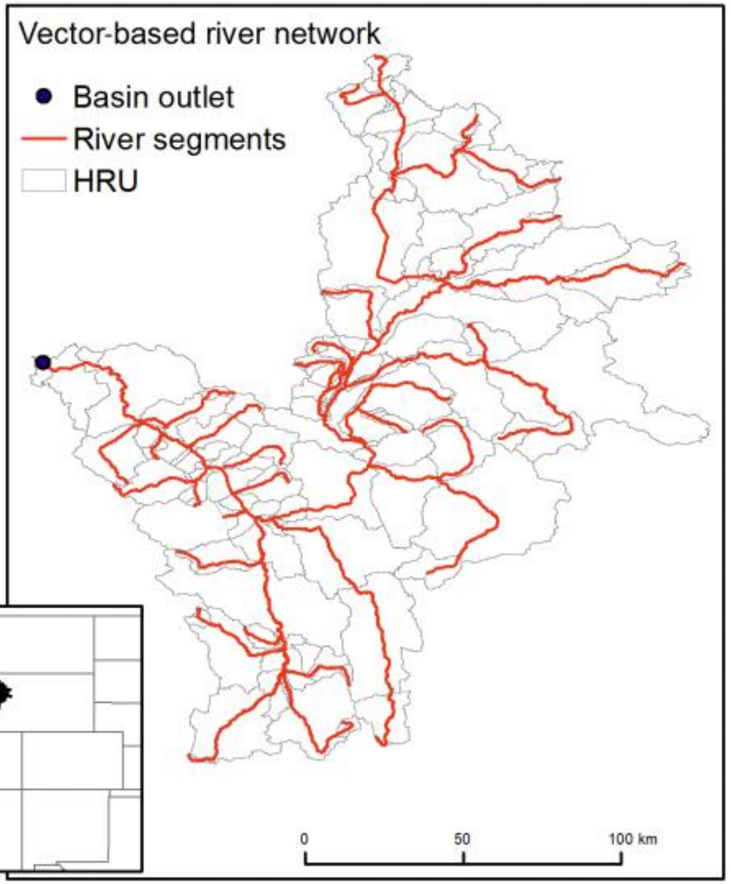
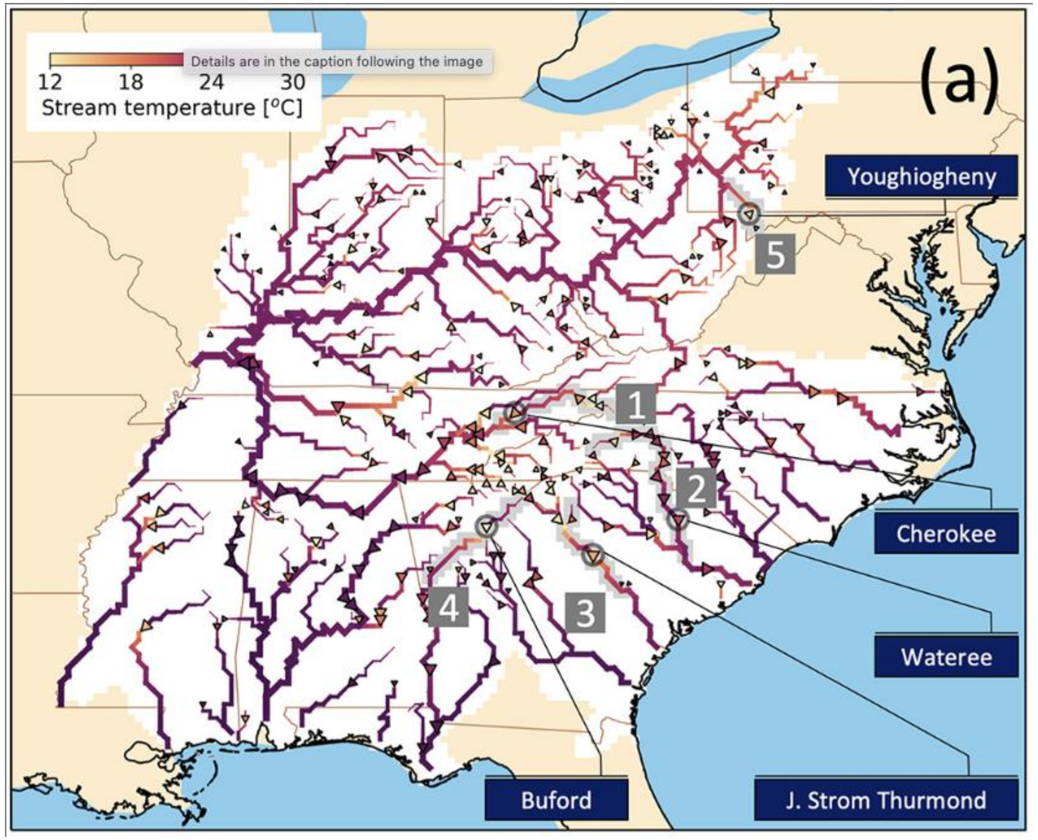
Data Requirements	Flow direction, velocity, and diffusion maps	Channel geometry, slope, roughness, and inflow hydrographs

Grid-based v.s. vector-based river networks





A vector-based river network for a large river basin



The left figure shows the mean summer river temperature for major river basins in the Southeastern United States.

What are the pros/cons of a gridded river network?

Grid-based *versus* vector-based river networks

Feature	Gridded River Network	Vector-Based River Network
Implementation Difficulty	Easier to implement; Naturally following by hydrologic modeling configured in grid cells	More complex; requires detailed preprocessing

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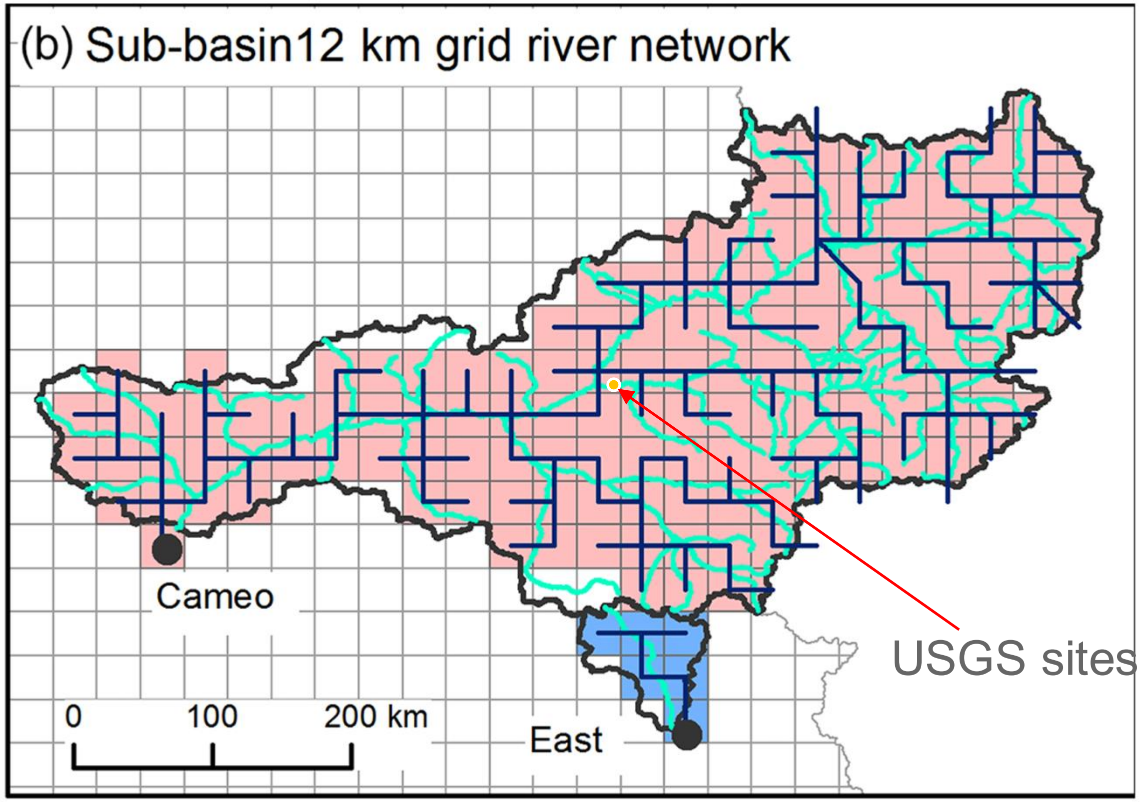
Routing Accuracy	May misrepresent flow paths, especially in flat or complex terrain	Preserves true river geometry and connectivity
Model Evaluation	The observation sites that will be used for evaluation might not be located in the nearest grid cell.	It is very easy to find the corresponding HUCs for the observation sites.

Example: if there is a USGS site at the red dot, which grid cells of streamflow should we use to evaluate?



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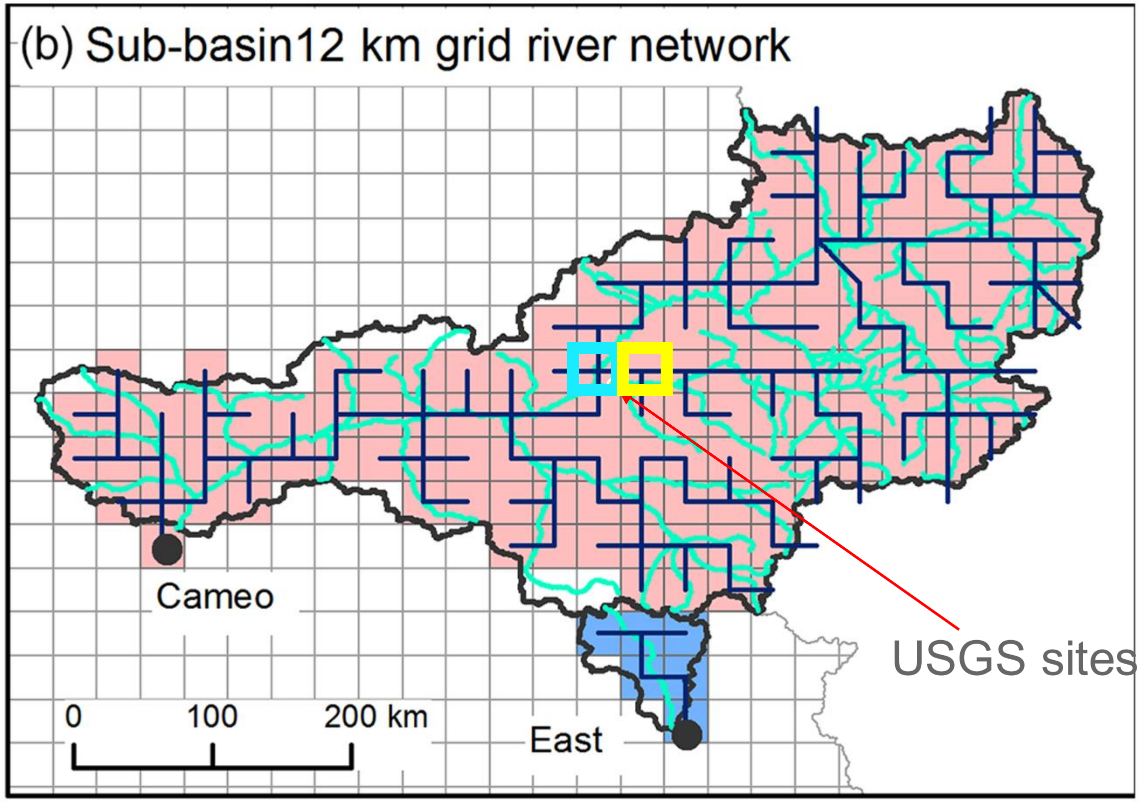

Even though the USGS site is located in the grid cell with the blue boundary, we should use the yellow grid instead.

Why? The river flow in the blue grid also confluences the streamflow from the upper tributary, while the USGS site was located upstream of the confluence point.

Questions?