Small-bore imager drive filter

Eli Mattingly

July 2021

1 Introduction

A persistent challenge in the design and construction of the small-bore imaging platform is effectively filtering and optimally powering the drive coil, and then filtering the received signal. This document will summarize the design of these system, in detail. The goal of the filter is to allow for the coil to be powered with a maximally pure waveform as well as being optimally thermally stable.

2 Drive Filter

2.1 Optimal impedance matching

The drive coil is an inductor with an impedance (ignoring parasitics) of $Z_L = j\omega L_D + R_D$, where L_D and R_D and are the drive coil's inductance ($130\mu H$) and series resistance($400m\Omega$) respectively, as seen in fig. 2. The amplifier (AE Techron 7548) is a voltage source that for maximum power delivery will drive a 4Ω load. This will be modeled as a source impedance of 4Ω (fig. 1) going forward. If we were to directly power the drive coil at 25kHz with the amplifier, the reactive impedance of the drive coil would dominate:

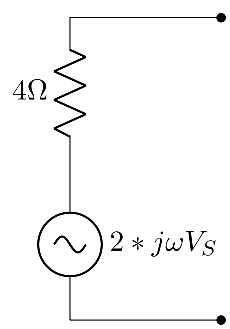


Figure 1: The equivalent model for the amplifier. The source resistance is not physical but rather is included to ensure the load is matched to a proper source impedance. In order to ensure the voltage is seen at the load is $j\omega V_s$ and compensate for the artificial resistor, the source voltage is doubled. See appendix A for more detail.

$$Z_D = \sqrt{(2 * \pi * 25kHz * 130\mu H)^2 + (4\Omega + 0.4\Omega)^2} = 20.9\Omega \tag{1}$$

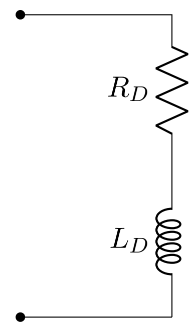


Figure 2: The model for the drive coil. Winding capacitances are assumed to be negligible.

Resulting in only a small amount of current being delivered to the coil:

$$I_D = \frac{2V_S}{20.9\Omega} \rightarrow 95\text{mA / Volt} \tag{2}$$

Where the 20.9Ω is the source impedance in series with the load. This reactance can be easily nulled with a series capacitor, such that at the drive frequency $\omega L_D = \frac{1}{\omega C_D}$, so:

$$C_D = \frac{1}{\omega^2 * L_D} = \frac{1}{(2 * pi * 25000Hz)^2 * 130\mu H} = 312nF \tag{3}$$

With that capacitor in place the effective circuit is now simply a $400m\Omega$ resistor, but the amplifiers internal current limits would prevent it from maximum power delivery. This is illustrated by the 4Ω source resistance, which effectively divides the voltage seen at the load. Now the drive coil would receive:

$$I_D = \frac{2V_S}{4.4\Omega} = 450\text{mA / Volt} \tag{4}$$

Which is much better, but there is still substantial room to improve both in terms of matching and filtering. The issue currently is the load is not impedance matched to the source. Ideally, for maximum power delivery, the load would present itself as 4 Ohms to the source.

To accomplish this, we can simply add an impedance matching stage as seen in figure 3. Defining the "matching factor (m)" as the relative ratios of source and load impedance:

$$m = \frac{R_S}{R_L} = \frac{4\Omega}{0.4\Omega} = 10 \tag{5}$$

And:

$$Q = \sqrt{m-1} = 3 \tag{6}$$

For matching to occur:

$$X_L = Q * R_L = 1.2\Omega \tag{7}$$

Which, for resonance at the drive frequency requires the capacitor's reactance to equal:

$$\frac{1}{\omega C_1} = X_L * (1 + Q^{-2}) \rightarrow C_1 = \frac{1}{\omega^2 * L_1 * (10/9)} = 4.77\mu F \tag{8}$$

Implementing this results in approximately 800mA per volt. This is the nearly the theoretical maximum with this source impedance, because if we consider conservation of energy, the power

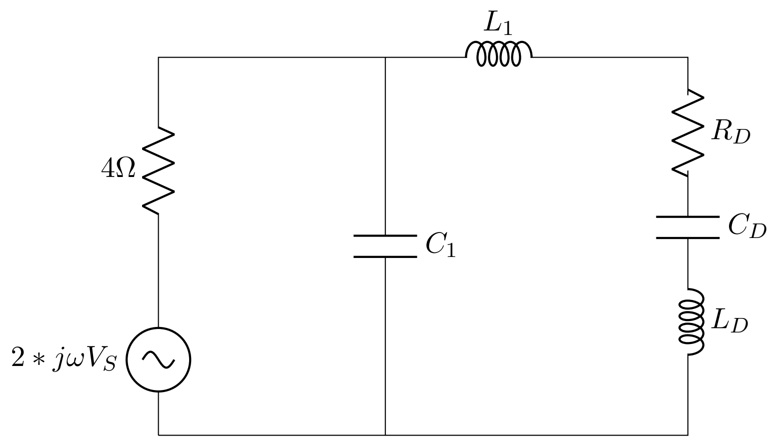


Figure 3: The drive coil circuit with a single impedance matching stage added (L_1, C_1). In order to form an effective impedance match, $L_1 = 7.6\mu H, C_1 = 4.7\mu F, C_D = 312nF$. This is "Design C-1" in Table 1. L_1 can be removed if C_D and C_1 are picked to be $C_1 = 4.77\mu F, C_D = 331nF$. This version (with no L_1) is "Design C-2".

dissipated by the source $P_S = V^2/R$ must equal the power dissipated in the load (if it is perfectly matched and the other elements are ideal). So:

$$P_S = V^2/R = P_D = I_D^2 * R_D \rightarrow I_D = \sqrt{\frac{1}{R_S * R_D}} = 800\text{mA / Volt} \tag{9}$$

In the case where C_D is picked properly, we can do the full impedance matching without needing an additional inductor. To do so, we first observe that a parallel capacitor to L_D, C_D, R_D will only add positive imaginary components to the admittance (also called the "susceptance", "B") when looking from the source. This means the real part of the admittance (called the "conductance", G) of the series components, L_D, C_D, R_D must equal $\frac{1}{R_S}$. For this to occur, the following must hold:

$$X_{Series} = +QR_D \tag{10}$$

Where the Q is the same as previously defined. The necessary series capacitor is straightforward to calculate:

$$X_{Series} = X_C + X_L \tag{11}$$

$$X_C = -(X_L - QR_D) = 1.2 - 20.4\Omega = -19.2\Omega = \frac{-1}{\omega C_D} \tag{12}$$

$$C_D = \frac{1}{\omega X_C} = 331nF \tag{13}$$

The equivalent impedance of the series drive coil with capacitor is now $R_D + jQR_D$. To remove the imaginary component, we first convert the impedance(Z) to an admittance(Y) given $Z = 1/Y$. Now, looking from the source, the admittance looks like:

$$Y = \frac{1}{R_D + jQR_D} \tag{14}$$

Multiplying the numerator and denominator by the complex conjugate gives:

$$Y = \frac{R_D - jQR_D}{(R_D + jQR_D)(R_D - jQR_D)} = \frac{R_D - jQR_D}{(R_D^2 + (QR_D)^2)} = \frac{R_D}{R_D^2(1+Q^2)} - \frac{jQR_D}{R_D^2(1+Q^2)} \tag{15}$$

Utilizing Eqns. 5,6 gives:

$$Y = \frac{R_D}{R_D^2(R_s/R_D)} - \frac{jQR_D}{R_D^2(R_s/R_D)} = \frac{1}{R_s} - \frac{jQ}{R_s} \tag{16}$$

Therefore, the susceptance of the shunt capacitor must equal:

$$B_C = +\frac{Q}{R_S} = \omega C \tag{17}$$

Which results in a capacitor of:

$$C = +\frac{Q}{\omega R_S} = 4.77\mu F \tag{18}$$

With that implemented, the equivalent input impedance at the drive frequency is equal to:

$$Z = \frac{1}{Y} = R_S \tag{19}$$

A third alternative design would be to impedance match to the drive coil's reactive and resistive load without adding a capacitor and cancel the reactance with a parallel C to the drive and a series L before that as in figure 4, and this results in $L_2 = 410\mu H, C_2 = 410nF$. While the inductor value is very large, the current flowing through is roughly 3x less than the drive coil, so 9x less power/heat (probably 2x resistance, though so overall more like 5x less power than the drive coil). This design is less favorable due to needing to wind another inductor, which can become quite large.

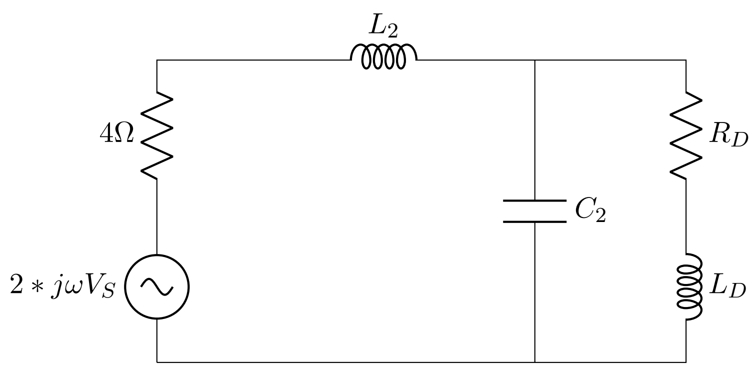


Figure 4: The drive coil circuit with an alternative impedance matching stage added (L_2, C_2). Note there is no series capacitor to the drive coil.For matching $L_2 = 409\mu H, C_1 = 410nF$. This has the advantage of very low required capacitance. This is "Design D" in Table 1

Regardless of the design specifics, the Q of the filter is dependent on the matching ratio (Eqn. 6). Increasing the filter Q has multiple effects: First, the losses in a filter with higher impedance will be much lower, and second, the way the system drifts with temperature will change–this is discussed later, though. To the first note, we observe for conservation of power (and ignoring filter losses):

$$I_S^2 R_S = I_D^2 R_D \tag{20}$$

$$(\frac{I_S}{I_D})^2 = \frac{R_D}{R_S} \tag{21}$$

So, as the source impedance increases, the current decreases. For an impedance matched filter, the same logic holds, where high impedance filters have smaller currents flowing. The characteristic impedance of a filter is defined as:

$$Z_0 = \sqrt{\frac{L}{C}} \to Z_0^2 C = L \tag{22}$$

and in an ideal resonant circuit energy flows between the inductor and capacitor such that

$$1/2 * LI^2 = 1/2 * CV^2 \tag{23}$$

Plugging in the characteristic impedance and cancelling like terms:

$$Z_0^2 I^2 = V^2 \to Z_0 = V/I \tag{24}$$

If we consider the losses are proportional to the current squared, then the impedance of the filter is inversely proportional to the losses, squared. This is a simplification of course, as capacitors dissipate heat and resistive components are a function of inductance, but it can still be useful to guide the design.

This suggests a filter matched to a higher impedance would be less lossy at the cost of potentially higher voltages and bigger capacitors and inductors. At higher impedances, the amplifier may lose efficiency, though. Taking AE Techron 7224 as an example, the max power it can transfer at 8 Ohms is 900W, but at 16 Ohms it can still power over 700W. The AE Techron 7548, at 4 Ohms can supply 3300W, but at 16 Ohms it can only supply 1100W. Given the resistance of the drive coil is $400m\Omega$ and we want 25A, that means we only need the amplifier to supply 250W (plus any filter losses).

2.2 Filtering

The next challenge is the low F3/F1 attenuation, which can be solved by adding a LC band pass filter with a minimally small pass-band around the drive frequency. So that means making a T network where the series sections are series resonant L-C sections and the shunt sections are parallel L-C sections such that at resonance the series sections have no reactance and minimal resistance, and the shunt sections appear to have practically infinite impedance. With this arrangement, the filter does not alter the impedance matching.

3 Transformer Design

Transformers can be used on both the Tx and Rx systems for galvanic isolation, balancing loads, common-mode noise rejection among other reasons. While they present an attractive option for those reasons, the design must be carefully considered to minimize risk of harmonic distortion, and added noise. In particular, the major risks (in this context) are the core becoming saturated and adding harmonic distortion, heating, or (for Rx) adding resistive noise.

For any transformer, the operation is fundamentally the same: current in the N_p primary windings induce an alternating flux, Φ_C in the core that energizes the secondary N_s windings with $\frac{N_s}{N_p}$ volts and an inverse ratio of current (ideally). The ratio of turns therefore is the governing factor for the voltages/currents, but the absolute number of turns in each, in conjunction with geometric/composition parameters, determines the "magnetizing inductance", L_M, the leakage inductance, L_L, as well as the flux density in the core, B_c.

First, looking at the inductances, there are two major values to consider. The "magnetizing" inductance, and the second is the "leakage" inductance. Simply, the magnetizing inductance is the integrated flux in the core that couples to the secondary per ampere, and the leakage is the integrated remaining flux per ampere. There is a magnetizing inductance for both the primary and secondary.

Next, we must consider the flux density within the core to determine if it will saturate. This was derived in (among other sources) *Magnetic Circuits and Transformers, 1943* (Pg. 167-8), and is repeated here for completion. Consider the situation where a transformer (negligible wire losses) has a voltage v applied to the primary side, which must be equivalent to $N\frac{d\phi}{dt}$, where N is the primary-side turn count. ϕ can be written as:

$$\phi = \phi_{max} sin(\omega t) \tag{25}$$

differentiating gives:

$$v = N\frac{d\phi}{dt} = \omega N \phi_{max} cos(\omega t) \tag{26}$$

Substituting $2\pi f$ for ω and solving for ϕ_{max}, and recognizing the cosine goes to a maximum of 1:

$$\phi_{max} = \frac{v}{2\pi f N} \tag{27}$$

It is worth noting that in many texts it is written as:

$$v_{eff} = \frac{2\pi f N \phi_{max}}{\sqrt{2}} \approx 4.44 f N \phi_{max} \tag{28}$$

But this assumes the RMS value. In order to determine saturation or not, it is useful to put it in terms of the flux density, which is a simple transformation as $BA = \phi$, so:

$$B_{max} = \frac{v}{2\pi f N A} \tag{29}$$

This expression is in terms of the voltage at the primary side, but does not include secondary-side effects. This can be explain intuitively, if we assume a near-ideal 1:1 transformer. If a load is applied to the secondary side and draws X amps, the primary side current will increase an equal amount, so the flux within the core remains essentially the same.

3.0.1 Equivalent circuit

As mentioned, the transformer has two inductances each, for the primary and secondary side: the "L_M", the magnetizing inductance, and "L_l" the leakage inductance. With these four parameters, an equivalent circuit can be formed as in figure 5. It is common to refer all of the impedances to either the primary or secondary side. For this discussion, the secondary side will be more simple to refer the impedances to.

In the case that the magnetizing inductive impedance is substantially higher than the effective resistive load (at the drive frequency), this term can be ignored. This results in the equivalent circuit simply being a series inductor with a value equal to the sum of the leakages referred to the secondary side. This can be easily tuned out by adding a series capacitor such that it resonates at the drive frequency.

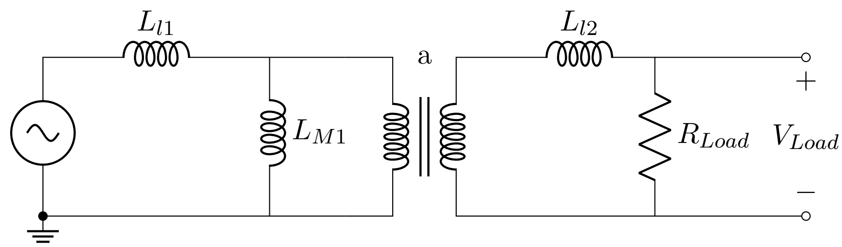


Figure 5: Circuit model for a generalized transformer with both secondary and primary-side leakages as well as the magnetizing inductance on the primary side. The transformer in the center is an ideal transformer with a turns ratio equivalent to the square root of the magnetizing inductance ratio ($\sqrt{\frac{L_{M1}}{L_{M2}}} = a$). This model assumes negligible capacitive effects, and core losses.

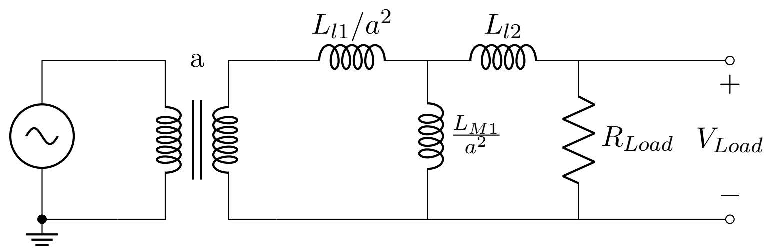
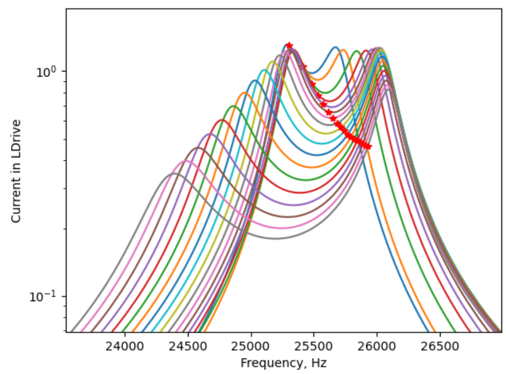


Figure 6: Circuit model for a generalized transformer with both secondary and primary-side leakages as well as the magnetizing inductance on the primary side. The transformer in the center is an ideal transformer with a turns ratio equivalent to the square root of the magnetizing inductance ratio ($\sqrt{\frac{L_{M1}}{L_{M2}}} = a$).

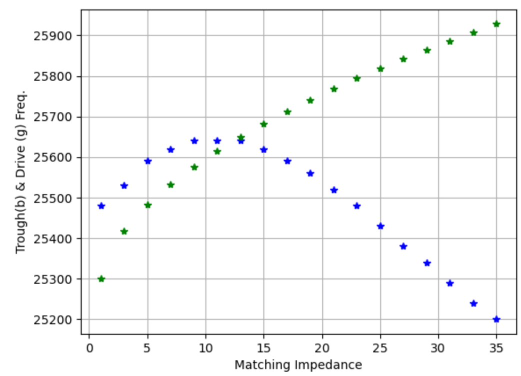
4 Implementation

The discussion has been mostly theory-based so far, which in a general sense is useful to guide a design, but reality tends to fight back on the implementation front. A number of new effects must be considered. Firstly, the amplifier is actually a fairly low impedance source ($< 10m\Omega$) not the four or so Ohms that was assumed earlier. This may result in some unexpected behavior in non-drive frequencies, because now the amplifier can supply substantially more current to low-impedance loads (whereas before the simulated source resistance attenuated it).

The transfer function (voltage input to drive current) ends up having a peak on either side of the drive frequency, which may come as a surprise. The relative location of these peaks depend on which impedance the load is matched to, and a number of different impedances are plotted in figure 7a. When plotting the total efficiency (power out/power in), the drive frequency is indeed the absolute maximum.



(a) The transfer function from the V_{in} to I_{Drive} for many different matching impedances. Red stars are the drive frequencies



(b) The "trough" location and the operating drive frequency at different matched impedances

Figure 7: Impedance matching to different levels (changing matching ratio) causes the relative position of the drive frequency and the "trough", which is the local minima, to be varied.

Appendices

A Modeling voltage sources

Throughout this document I model the amplifier as a voltage source in series with a resistor that is equivalent to the amplifier's maximum output voltage divided by the maximum current:

$$R_{Src} = \frac{V_{Max}}{I_{Max}} \tag{30}$$

This is an approximation that can be used to help design the filter, but this is not a physical effect. As shown in figure 10, it is evident that the match is far from perfect, but it does indeed match exactly at the maximum power point, and roughly matches the off-peak characteristics in that it monotonically attenuates them. If this series resistor, the maximum power would be at zero ohms load as $P = V^2/R$.

B Phasor view of impedance matching

A second way to understand the impedance matching is by looking at the phasors of the impedance and admittance depicted in figure 11. From this prespective the steps are more clear: first, looking at the admittance, the goal is to move the green admittance such that the real part is aligned with the target. This is done by adding series capacitance. Next, adding parallel capacitance adds positive, imaginary admittance, so that parallel capacitor is picked to cancel out all of the reactance.

Design	I_D per V_S	SPICE Source Impedance	F3/F1	Filer losses
A	95mA/V	$20.9\Omega, 100°$	-9.4dB	-
B	450mA/V	$4.4\Omega, 0°$	-21dB	-
C-1	800mA/V	$8.2\Omega, -2°$	-46dB	30 W
C-2	800mA/V	$7.9\Omega, 2°$	-46dB	-
D	800mA/V	$7.96\Omega, 2°$	-58dB	25.4 W
E	700mA/V	$6.58\Omega, 22°$	-80dB	46 W
F	800mA/V	$8.0\Omega, -3°$	-81dB	43.5 W
F-2	800mA/V	$7.94\Omega, -3°$	-79dB	28 W

Table 1: Tabulated parameters for each of the different filter versions. The SPICE source impedance is in series with the 4 Ω source impedance. For the losses, capacitors are assumed to be ideal, as are wires, and resistance is taken to be approximately $200m\Omega per\sqrt{100\mu H}$, so for example, a $25\mu H$ would have $100m\Omega$ resistance

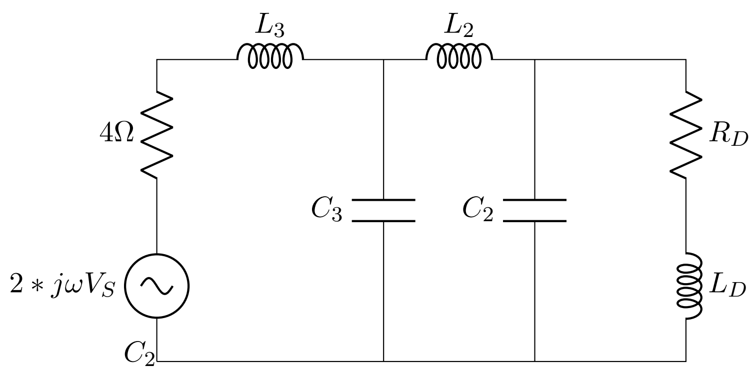


Figure 8: The drive coil circuit with a single impedance matching stage added (L_2, C_2) as well as a filter (L_3, C_3) to attenuate harmonic distortions that originate from the amplifier. This is "Design E" in Table 1

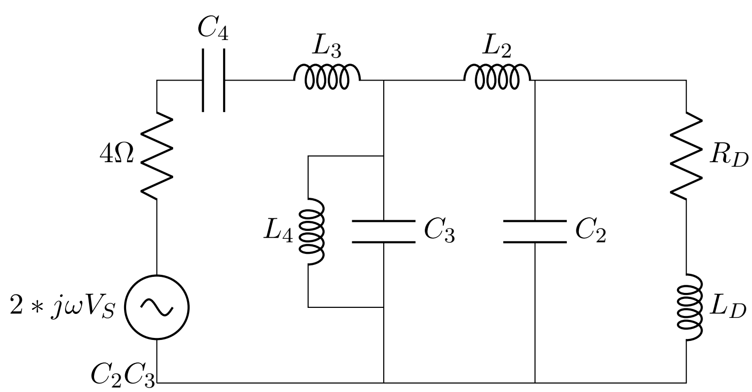


Figure 9: The drive coil circuit with a single impedance matching stage added $(L_2, C_2.)$ as well as a filter $(L_3, C_3.)$ which has been tuned such that it is resonant at the drive frequency $(L_4, C_4.)$. This is "Design F" in Table 1

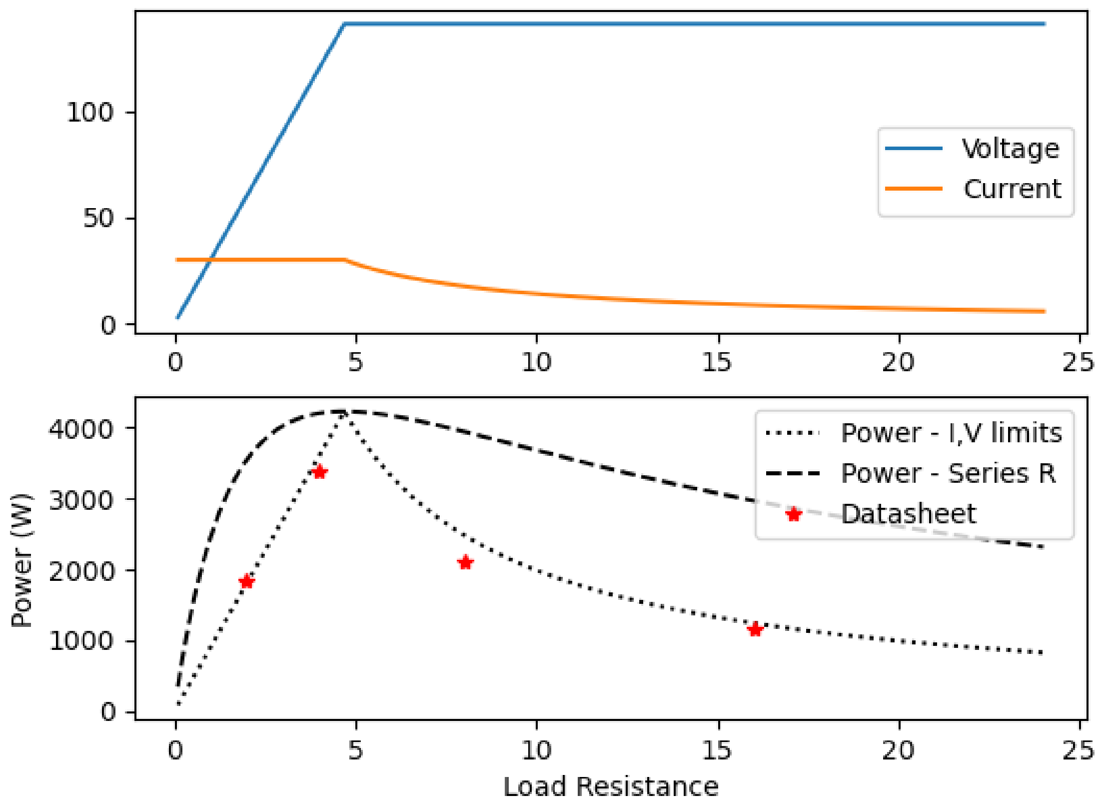
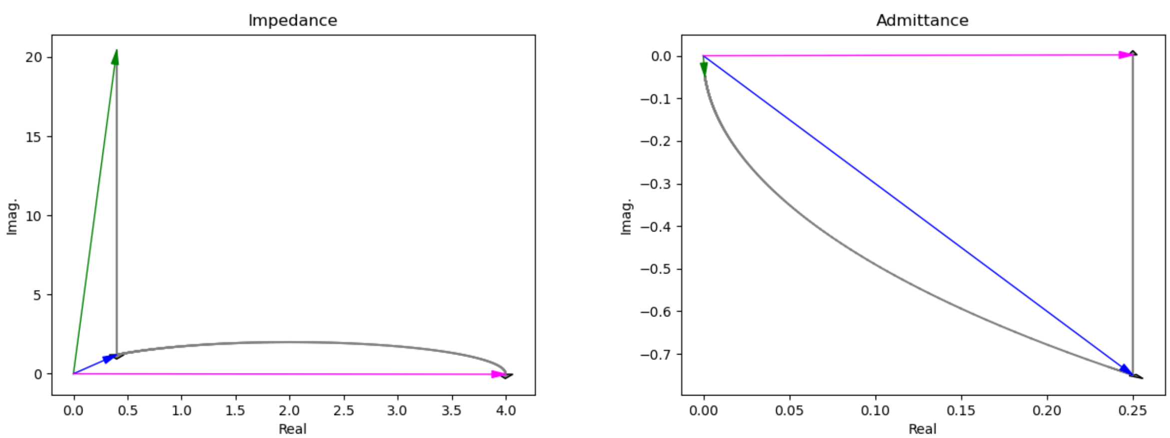


Figure 10: **Top:** The actual currents and voltages that one would expect from an amplifier given the maximum current and voltages of the AE Techron 7548 (RMS, steady state). **Bottom:** The dotted line is the power as a function of load impedance that corresponds to the top I and V. The dashed line is the approximation that utilizes the source having twice the voltage with a series resistance as calculated in Eqn.30. The four red stars are the datapoints found in the AE Techron 7548 datasheet.



(a) The impedance phasors

(b) The admittance phasors

Figure 11: Impedance matching from a graphical point of view. The green represents the original impedance , the blue is the drive coil with the 331nF series capacitor, and the magenta is the final, impedance matched system (4.77μF capacitor added in parallel as in design "C-2".