MATH 665: TOPICS IN QUANTUM ALGEBRA

FALL 2024 SYLLABUS

We discuss the relationship between representations of linear groups over finite and p-adic fields, a part of Lie theory, and isotopy invariants of knot and links, a part of geometric topology. The bridge is the theory of Hecke algebras and their cocenters.

Instructor	Minh-Tâm Trinh (minh-tam.trinh@yale.edu)
Time	TTh 2:30–3:45 PM
Place	17 Hillhouse Ave, Room 03 (basement) (**NEW**)
Webpage	https://mqtrinh.github.io/math/teaching/yale/math-665/

In place of a textbook, I will typeset course notes and post them to the webpage as we go along. See also the bibliography at the end of this syllabus.

(Tentative) Schedule

8/29	Introduction	
9/3–9/5	1. Reductive Groups and Hecke Algebras	Set 0 (*due* 10/10) Set 1 (*due* 9/19)
9/10–9/12		
9/17–9/19		
9/24–9/26	2. Quantum Link Invariants	Set 2 (*due* 10/10)
10/1–10/3		
10/8–10/10		
10/15–10/17	*October Recess*	
10/22–10/24	3. Hecke Categories	Set 3 (*due* 11/7)
10/29–10/31		
11/5–11/7	4. Symmetric Functions and Hall Algebras	
11/12–11/14		
11/19–11/21	5. Current Topics	Set 4 (*due* 12/5)
11/26–11/28	*November Recess*	
12/3–12/5		

Logistics

Emails. If you need to email me about the course, please put "MATH 665" in the email subject. That helps me keep everything organized. You may address me as "Minh-Tam" or as "Dr. Trinh".

Problem Sets. You should write in complete sentences. The grammar does not have to be perfect. You do not have to rewrite the problem statements.

Attendance. There is no attendance grade. If you get sick, please don't come to class! Stay at home, take care of yourself, and notify me early.

Office Hours. By appointment.

References

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