Main Exercise 1. Given $i = 1, \ldots, n-1$ let

$$\pi_i : [x_1 : \cdots : x_n] \longmapsto \frac{1}{x_n^{1+q+\cdots+q^{n-i}}} \Delta(x_n, x_{n-1}, \ldots, x_i).$$

(a) Show that for all i, the map

$$[x_1 : \cdots : x_n] \longmapsto (\pi_1, \ldots, \pi_i, [x_{i+1} : \cdots : x_n])$$

induces a surjective morphism $\phi_i : \mathbf{X}_n \twoheadrightarrow (\mathbb{G}_m)^i \times \mathbf{X}_{n-i}$.

(b) Let $\mathbf{V}$ be the unipotent radical of $\mathbf{P}_I$ with $I = \{s_2, \ldots, s_{n-1}\}$. Show that ϕ_1 induces a bijective morphism

$$V\backslash\mathbf{X}_n \xrightarrow{\sim} \mathbb{G}_m \times \mathbf{X}_{n-1}.$$

(c) Generalize this to ϕ_i for all $i \geqslant 2$.