THE LEADER IN ELECTRONIC CHARTS

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TITOLO */TITLE*

MERCATOR/GEOGRAPHIC COORDINATES CONVERSION EQUATIONS

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REVISIONE */REVISION HISTORY*

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List of Introduced Changes

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1	08 Sep 1998	Added chapter 4: Navionics Datum	M.Tanzi	
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1. Purpose

This document specifies the set of equations used to convert geographical coordinates from/to Mercator coordinates.

2. Reference Documents

N/A

3. Mercator Projection

Since the Navionics database is viewed on a flat screen, the graphics must be a representation of the mercator projection of what is actually on the surface of the earth.

A mercator projection is obtained by wrapping a sheet of paper around the circumference of the earth, tangent to the equator, and then projecting points on the earth's surface onto the sheet by constructing a projection line from the center of the earth, through the point to where it intersects the flat sheet.

A physical vertical distance on the earth's surface very close to the equator, is very slightly larger on the mercator projected drawing.

As the latitude increases towards either pole, the same distance on the earth's surface becomes increasingly larger on the projected image. Thus, if one draws 1º latitude lines on the flat projected image, they will be spaced closer together at the equator than they will at higher northern or southern latitudes. The actual distance traversed on the globe itself is the same, but the physical distance on the projected image is larger.

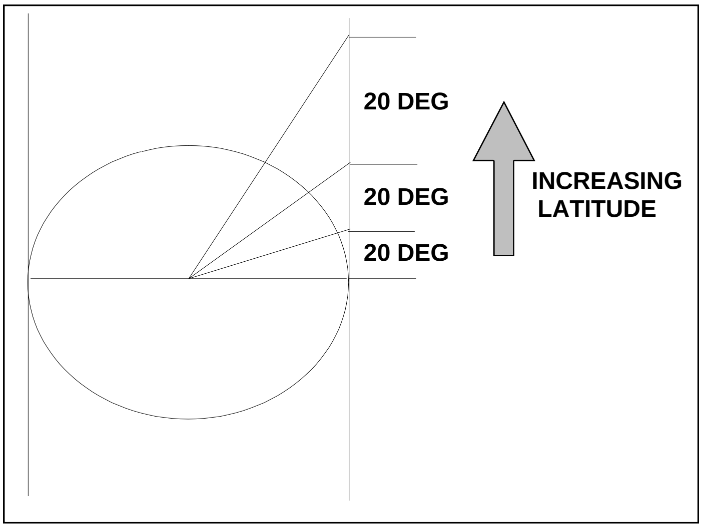


Figure 9 - Geometry of a Mercator Projection

The Navionics database is kept in mercator projected format. It is a linear transformation that may be edited using conventional drawing tools, but requires that the inverse transformation be performed every time a geographic position needs to be obtained.

The data stored in Navionics cartridges is also kept in this same projection, except it is has been converted to pixels based on a 512 X 512 pixel image area. The image is actually linear in terms of Mercator units, but it is highly non-linear in terms of Geographic units.

Whenever a geographic position must be obtained from the image, a transformation must be performed to locate the proper pixel position on the screen. The inverse is also true whenever a selected position on the image must be returned to the user in geographic coordinates. Figure 1 shows a diagram of the projection technique used for forming a mercator projected image.

Conversely, there is no problem in obtaining the projection position of any meridian since the earth is essentially circular at the equator.

There are actually two sets of formulae in use at Navionics. One is used in the plotter products, and is an approximation of the more complete version used by the cartography department in order to maintain the accuracy of the raw master database down to 1 Mercator meter of accuracy.

The difference between the 2 sets of equations amounts to about 6 meters on the earth's surface at the highest latitude for which the equations are valid (about 75º). While this accuracy is fine for plotters, it is not sufficient to maintain the accuracy of the database in its raw form.

4. Datum

By definition a Datum is an Ellipsoid with its tangent point. In other words a Datum is an Ellipsoid opportunely positioned around the Earth.

The Navionics Datum has the ellipsoid below defined:

Equatorial Radius	a = 6378388 meters.
Polar Radius	b = 6356912 meters.
Flattening Ratio	(a-b)/a = 1/297.0
Eccentricity	$\sqrt{\frac{a^2 - b^2}{a^2}}$ = 0.081991889977

This Ellipsoid is identical to **Hayford 1909** (same as International 1924).

The position of the Ellipsoid is often given as vector component deltas to WGS-84, in this case Navionics Ellipsoid and WGS-84 Elliposid have same position (deltas are 0).

As example is given a procedure to configure a Navionics Datum in AutoCAD MAP. The procedure is composed basically of two steps:

1) Create Navionics Datum
2) Create Navionics Category

1) Create Navionics Datum:
- From menu "Map->Map Tools" choose "Define Global Coordinate System"
- Press "Datums" button
- Insert "NAV84" as Datum Code
- Insert "WGS84 per Navionics" as Description
- Choose "International 1924" as Ellipsoid
- Fill with "0" next records
- Press "Add" button
- Press "OK"
- Press "OK"

2) Create Navionics Category
- From menu "Map->Map Tools" choose "Define Global Coordinate System"
- Press "New Catergory" button
- Insert "Navionics" then press "OK"
- Press "New" button
- Insert "Nav84" as CS Code
- Insert the Description "Coordinate Navionics"
- Choose "Traditional Mercator Projection" in the Projection list box
- Choose "WGS84 per Navionics" in the Datum list box
- Press "OK"
- Press "OK"

5. Complete Equation Set

This set of formulae is used by the following software:

- AutoCAD I/O routines before Jun 95
- CreMap DOS version

Mercator Longitude:

$$M\lambda = \lambda * \frac{\pi}{180} * 6378388.0$$

where:
λ = geographic longitude in degrees;
Mλ = Mercator longitude in Mercator meters.

Mercator Latitude:

$$M\varphi = 6378388.0 * \ln\left\{\tan\left[\left(\frac{\pi}{4} + \frac{(\varphi * \pi / 180)}{2}\right) * \left(\frac{1 - \mathrm{k} * \sin(\varphi * \pi / 180)}{1 + \mathrm{k} * \sin(\varphi * \pi / 180)}\right)^{\frac{\mathrm{k}}{2}}\right]\right\}$$

$$\mathrm{k} = \sqrt{6.722670022 * 10^{-3}}$$

where:
φ = geographic longitude in degrees;
Mφ = Mercator longitude in Mercator meters.

Geographic Longitude:

$$\lambda = \frac{M\lambda}{6378388.0} * \frac{180}{\pi}$$

Geographic Latitude:

$$\alpha = \frac{180}{\pi} * \mathrm{atan}\left\{1.00676425 * \tan\left[2 * \mathrm{atan}\left(\exp\left(\frac{|M\varphi|}{6378388.0}\right)\right) - \frac{\pi}{2}\right]\right\}$$

$$\gamma = \frac{1}{3600} * \left[0.1925 * \sin\left(\frac{\pi}{180} * 4 * \alpha\right)\right]$$

$$\varepsilon = 10^{-8} * \left[20.77777778 * \alpha - 0.288580246 * (\alpha)^2\right]$$

$$\varphi = \mathrm{sign}(M\varphi) * (\alpha + \gamma + \varepsilon)$$

The geographic latitude equation comes from the approximate one, since the inverted form cannot be derived from the complete equation. In order to counterbalance the error, an appropriate correction factor is added.

The error between the direct and the inverse equations is less than $1*10^{-6}$, and is shown in the following diagrams:

$step := 1$ plot density

$i := -1 \cdot step \cdot 80 .. step \cdot 80$ plot range

$g(i) := \frac{i}{step}$ latitude (degree

$f(i) := \frac{\pi}{180} \cdot g(i)$ latitude (radians

$k := \sqrt{6.722670022 \cdot 10^{-3}}$ $sign(x) := if\left(x = 0, 1, \frac{x}{|x|}\right)$

$$U(i) := 6378388 \ln\left[\tan\left(\frac{\pi}{4} + \frac{f(i)}{2}\right) \cdot \left(\frac{1 - k \cdot \sin(f(i))}{1 + k \cdot \sin(f(i))}\right)^{\frac{k}{2}}\right]$$

Complete equation [3]

$$Ru(i) := \frac{180}{\pi} \cdot atan\left(1.00676425 \tan\left(2 \cdot atan\left(\exp\left(\frac{|U(i)|}{6378388}\right)\right) - \frac{\pi}{2}\right)\right)$$

$Run(i) := sign(U(i)) \cdot Ru(i)$ Inverse approximate eq. applied to (3) [4]

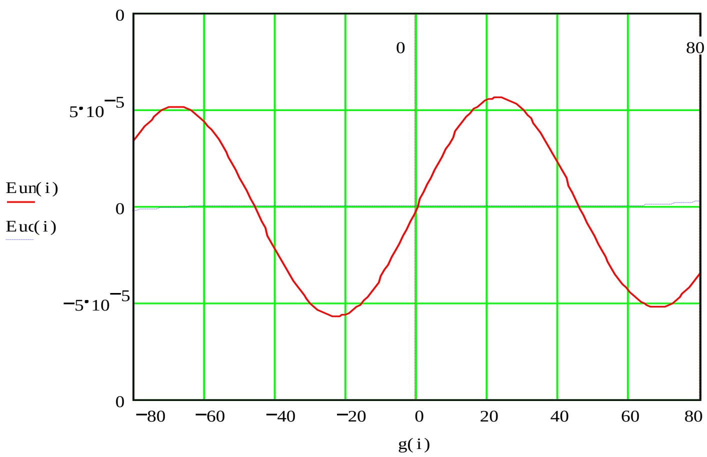
$Eun(i) := g(i) - Run(i)$ Error of direct/inverse conversion using (3) and (4)

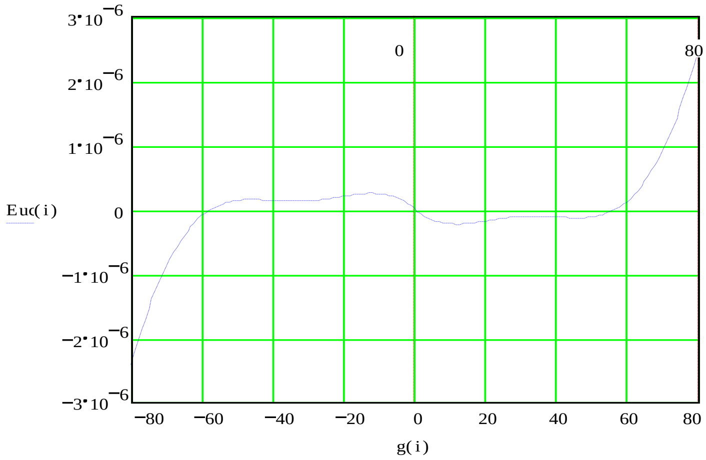
$gamma(i) := \frac{1}{3600} \cdot \left[0.1925 \sin\left[\frac{\pi}{180} \cdot 4 \cdot (Ru(i))\right]\right]$ Correction factor [5]

$eps(i) := 10^{-8} \cdot \left[20.7777777 (Ru(i)) - 0.28858024 (Ru(i)^2\right]$ Correction factor [6]

$Ruc(i) := sign(U(i)) \cdot (Ru(i) + gamma(i) + eps(i))$ Same as (4) corrected with (5) and (6)

$Euc(i) := g(i) - Ruc(i)$ Error of direct/inverse conversion using (3) and (7)





6. Approximate Equation Set

This set of formulae is used by the following software:

- AutoCAD routines after Jun 95
- BAS library (all versions)
- BAS-based software: NamEdit, IOGDB, MCM, CreMap/CreCart Windows version (aka MapMaker/CartMaker)
- NavDisp-based software: NavDisp, SedEdit, NavCat
- NEC V25-based plotter software
- Geocore library
- WinGeocore library
- Geocore Plus library

Mercator Longitude:

$$M\lambda = \lambda * \frac{\pi}{180} * 6378388.0$$

where:
λ = geographic longitude in degrees;
Mλ = Mercator longitude in Mercator meters.

Mercator Latitude:

$$M\varphi = 6378388.0 * \ln\left\{\tan\left[\frac{1}{2} * \operatorname{atan}\left(\frac{\tan(\varphi * \pi / 180)}{1.00676425}\right) + \frac{\pi}{4}\right]\right\}$$

where:
φ = geographic longitude in degrees;
Mφ = Mercator longitude in Mercator meters.

Geographic Longitude:

$$\lambda = \frac{M\lambda}{6378388.0} * \frac{180}{\pi}$$

Geographic Latitude:

$$\varphi = \frac{180}{\pi} * \operatorname{atan}\left\{1.00676425 * \tan\left[2 * \operatorname{atan}\left(\exp\left(\frac{M\varphi}{6378388.0}\right)\right) - \frac{\pi}{2}\right]\right\}$$

It is suggested that the functions for latitude conversion can conveniently be tested after they are coded by constructing a table of Mercator latitudes, converting them back into Geographic latitudes and checking to see that the converted Geographic latitudes are within $1x10^{-13}$ degrees of the original latitudes (less than $3.6x10^{-10}$ sec).

The error between the direct and the inverse equations is less than $2.5*10^{-14}$, and is shown in the following diagrams:

$step := 1$ plot density

$i := -1 \cdot step \cdot 80 .. step \cdot 80$ plot range

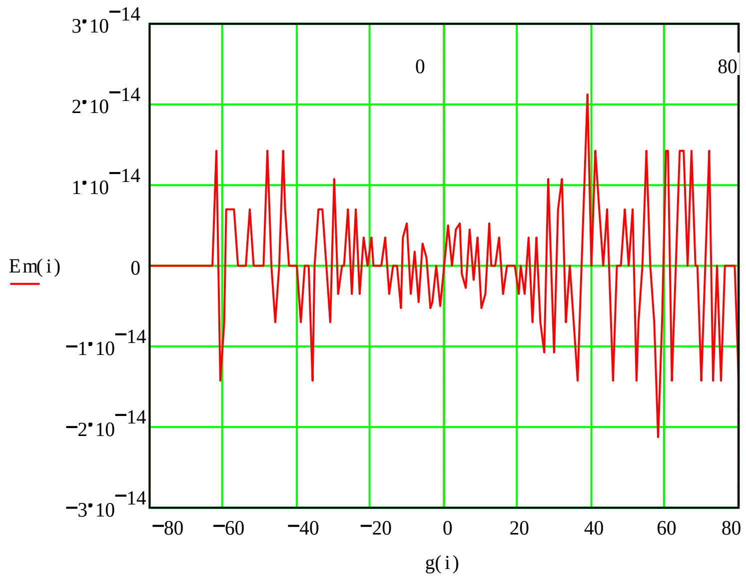
$g(i) := \dfrac{i}{step}$ latitude (degrees)

$f(i) := \dfrac{\pi}{180} \cdot g(i)$ latitude (radians)

$$M(i) := 6378388 \ln\left(\left(\tan\left(0.5 \operatorname{atan}\left(\frac{\tan(f(i))}{1.006764225}\right) + \frac{\pi}{4}\right)\right)\right)$$ Direct approximate equati [1]

$$Rr(i) := \frac{180}{\pi} \cdot \operatorname{atan}\left(1.006764225 \tan\left(2 \cdot \operatorname{atan}\left(\exp\left(\frac{M(i)}{6378388}\right)\right) - \frac{\pi}{2}\right)\right)$$ Inverse approximate eq. applied to (1)

$Er(i) := g(i) - Rr(i)$ Error of direct/inverse conversion using (1) and (2)



7. Approximate vs. Complete Equations Comparison

The following diagrams show the error introduced by the approximate equation with respect to the complete one.

As shown in the diagram above, the maximum is less than 17 Mercator meters, which means approximatively less than 6 meters on earth's surface.

$step := 1$ plot density

$i := -1 \cdot step \cdot 80 .. step \cdot 80$ plot range

$g(i) := \frac{i}{step}$ latitude (degrees)

$f(i) := g(i) \cdot \frac{\pi}{180}$ latitude (radians)

$k := \sqrt{6.722670022 \cdot 10^{-3}}$

$$U(i) := 6378388 \ln\left[\tan\left(\frac{\pi}{4} + \frac{f(i)}{2}\right) \cdot \left(\frac{1 - k \cdot \sin(f(i))}{1 + k \cdot \sin(f(i))}\right)^{\frac{k}{2}}\right]$$ Complete equation

$$M(i) := 6378388 \ln\left(\left(\tan\left(0.5 \operatorname{atan}\left(\frac{\tan(f(i))}{1.00676425}\right) + \frac{\pi}{4}\right)\right)\right)$$ Approximate equatio

$D(i) := (U(i) - M(i))$ Error in Mercator meters

$H(i) := D(i) \cdot \cos(f(i))$ Approximate error in meters on earth's surface