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**Second Theory Test**

**Task 1**

**Problem**: Characterize equation **R**;write (and explain) polynomial (of the 3rd degree) approximation for the IVP: ; define (and explain) the order of the polynomial approximation in some neighborhood of 0.

**Solution:**

Firstly, it is nonlinear Bernoulli differential equation with degree=2 that matches pattern from lecture notes (topic I): **R .** In our case k = 2**.**

To write polynomial approximation for the IVP, we will use series approach (as in lecture slides of topic IV)

Solution in the form of Maclaurin series:

For polynomial (of the 3rd degree as it given in statements) approximation, we should find , , and . our initial value from statements.

, then . (move to the right)

Then we differentiate both sides of , we get the following: . So:

Again differentiate both sides: , then: . For point x=0:

Hence, by Maclaurin series formula that was mentioned above:

So, 3rd degree approximation:

**But*:*** In my equation there is a discontinuity point. Using analytical solution of IVP we get:

, which has discontinuity point at , that

lies in . So, approximation is valid only up to this point.

To define the order of the polynomial approximation in some neighborhood of 0, Taylor’s theorem with Peano’s remainder should be used:

where represents a function g(x) with g(x)/ → 0 as x → 0

Taking as 0 and as y(x), y(x) can be written as:

So, the order of the polynomial approximation of y(x) in some neighborhood of 0 is 3.

**Answer:**  for x in [0, 0.5] that are up to point of discontinuity, y R. For bigger values of x, approximation is not valid. The order of the polynomial approximation of y(x) in some neighborhood of 0 is 3.

**Task 2**

**Problem**: where y: [0, +) → **R .** Characterize it, define what is stability of the equation at equilibrium point (0,0) and prove stability

Firstly, it is a second-order linear homogeneous ordinary differential equation.

**Proof:**

Let be equal to y(x) and

and From the initial equation, , = = .

So we have: , .

Consider the system:

, where  = (, f() = (,)

Using Tutorial 7 notes, topic V: Assume that solution exists for all, then define the stability of the equation at the equilibrium point (0,0):

**“The solution is called stable by Lyapunov**

**if ∀ > 0 ∃δ > 0, such that ∀t >**

**|| (t) − || <  *(In our problem: || (t) - (0,0) || = || (t) || < ),* (\*)**

**if || () − || < δ (*In our problem:* || () || < δ). ”**

Now, consider the system = f(), f(0) = 0, where = (, f() = (,)

We will use Second Lyapunov’s stability theorem from Tutorial 7 slides to prove that the equation at equilibrium point (0,0) is stable:

**“If there exist the Lyapunov function for the system**

**= f(), f(0) = 0, (\*\*)**

**then the zero equilibrium is stable.”**

Definition of the Lyapunov function from the Tutorial 7 slides:

**“A continuously differentiable function L(x) is called Lyapunov function for the system**

**= f(x), f (0) = 0,**

**at the equilibrium x = 0 if:**

**➧ L ∈ (), L(x): → R; (\*\*\*)**

**➧ L(x) = 0, x = 0;**

**➧ L(x) > 0, x 0;**

**➧ ∃ > 0 such that (x(t)) ≤ 0, as ∀||x|| < .”**

Let us define and consider new function: L() = , where = (, L: .

**➧L is continuously differentiable, L():** ,

**➧**L(0, 0) = ,

**➧Since , if , then L() > 0,**

**➧**

**So, () for all , then ∃ > 0 : () ≤ 0, as ∀|||| <**

Using this properties, L(x) actually is a Lyapunov function for system  **(\*)***,* because of the definition from Tutorial that was mentioned above **(\*\*)**.

**Answer**: So, by the theorem from Tutorial  **(\*\*\*)**, **because there exists a Lyapunov function for the system, then the initially given equation is stable at equilibrium point (0,0)**.