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Two kinases, one substrate, simplified model

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> with(LinearAlgebra) :
> interface(rtablesize = 40) :
>
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Consider the system with two kinases (K, G), both allosteric, and a simple substrate.
The reactions are as follows (the label of the reaction is indicated in the arrows).

KR + S --1-> KRS --2-> KR + Sp
KT + S --3-> KTS --4-> KT + Sp
KR --5-> KT
KRS <-6-- KTS

GR + S --7-> GRS --8-> GR + Sp
GT + S --9-> GTS --10-> GT + Sp
GR --11-> GT
GRS <-12-- GTS

Sp --13-> S

The system has been further simplified such that the binding reaction
KR+S -> KRS
is not reversible and the allosteric part is only one way.
It might not be realistic, but due to some mathematical theorems, if we can show that this system has 5 steady states (3 stable), then the full system will as well.
To start with it makes sense to make the simplification since the equations become simpler.

Species
{S (1) ,Sp (2) ,KR (3) ,KT (4), KRS (5),KTS (6), GR (7) ,GT (8), GRS (9),GTS (10)}

There are a total of 13 reactions and 10 species.

I construct the mass-action ODE system by defining the stoichiometric matrix A, and the vector of rates "krates".
Then, the ODE system is

dx/dt = A.krates

and hence steady states are given as A.krates = 0.

Stoichiometric matrix:

I enter first the reactions as rows and then transpose the matrix

```
> A := Matrix(13, 10) :
> A[1, 1] := -1 : A[1, 3] := -1 : A[1, 5] := 1 :
> A[2, 3] := 1 : A[2, 2] := 1 : A[2, 5] := -1 :
> A[3, 1] := -1 : A[3, 4] := -1 : A[3, 6] := 1 :
> A[4, 4] := 1 : A[4, 2] := 1 : A[4, 6] := -1 :
> A[5, 3] := -1 : A[5, 4] := 1 :
> A[6, 5] := 1 : A[6, 6] := -1 :
> A[13, 2] := -1 : A[13, 1] := 1 :
> A[7, 1] := -1 : A[7, 7] := -1 : A[7, 9] := 1 :
> A[8, 7] := 1 : A[8, 2] := 1 : A[8, 9] := -1 :
> A[9, 1] := -1 : A[9, 8] := -1 : A[9, 10] := 1 :
> A[10, 8] := 1 : A[10, 2] := 1 : A[10, 10] := -1 :
> A[11, 7] := -1 : A[11, 8] := 1 :
> A[12, 9] := 1 : A[12, 10] := -1 :
> A := Transpose(A) :
```

Vector of rates:

here x_i is the concentration of the i-th species

> $krates := Vector\left(\left[k_1 \cdot x_3 \cdot x_1, k_2 \cdot x_5, k_3 \cdot x_4 \cdot x_1, k_4 \cdot x_6, k_5 \cdot x_3, k_6 \cdot x_6, k_7 \cdot x_7 \cdot x_1, k_8 \cdot x_9, k_9 \cdot x_8 \cdot x_1, k_{10} \cdot x_{10}, k_{11} \cdot x_7, k_{12} \cdot x_{10}, k_{13} \cdot x_2\right]\right)$

(1.1)

$$krates := \begin{bmatrix} k_1 x_3 x_1 \\ k_2 x_5 \\ k_3 x_4 x_1 \\ k_4 x_6 \\ k_5 x_3 \\ k_6 x_6 \\ k_7 x_7 x_1 \\ k_8 x_9 \\ k_9 x_8 x_1 \\ k_{10} x_{10} \\ k_{11} x_7 \\ k_{12} x_{10} \\ k_{13} x_2 \end{bmatrix} \tag{1.1}$$

Steady state equations:

> $eqs := A.krates$

$$eqs := \begin{bmatrix} -k_1 x_1 x_3 - k_3 x_1 x_4 - k_7 x_1 x_7 - k_9 x_1 x_8 + k_{13} x_2 \\ k_2 x_5 + k_4 x_6 + k_8 x_9 + k_{10} x_{10} - k_{13} x_2 \\ -k_1 x_1 x_3 + k_2 x_5 - k_5 x_3 \\ -k_3 x_1 x_4 + k_4 x_6 + k_5 x_3 \\ k_1 x_1 x_3 - k_2 x_5 + k_6 x_6 \\ k_3 x_1 x_4 - k_4 x_6 - k_6 x_6 \\ -k_7 x_1 x_7 + k_8 x_9 - k_{11} x_7 \\ -k_9 x_1 x_8 + k_{10} x_{10} + k_{11} x_7 \\ k_7 x_1 x_7 - k_8 x_9 + k_{12} x_{10} \\ k_9 x_1 x_8 - k_{10} x_{10} - k_{12} x_{10} \end{bmatrix} \tag{1.2}$$

Conservation laws:

This system has total amount of kinase and total amount of substrate conserved:

> $F := ReducedRowEchelonForm(Transpose(Matrix([op(NullSpace(Transpose(A)))])))$

$$F := \begin{bmatrix} 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix} \quad \textbf{(1.3)}$$

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the conservation laws are:

> $cons := [x_1 + x_2 + x_5 + x_6 - T_1, x_3 + x_4 + x_5 + x_6 - T_2, x_7 + x_8 + x_9 + x_{10} - T_3]:$

Therefore, the steady states constrained by the conservation laws are solutions to myeqs=0 (because there are two conservation laws, two of the equations in eqs can be disregarded).

> $myeqs2 := [eqs[2], eqs[4], eqs[5], eqs[6], eqs[8], eqs[9], eqs[10], x_1 + x_2 + x_5 + x_6 - T_1, x_3 + x_4 + x_5 + x_6 - T_2, x_7 + x_8 + x_9 + x_{10} - T_3]$

$$myeqs2 := [k_2 x_5 + k_4 x_6 + k_8 x_9 + k_{10} x_{10} - k_{13} x_2, -k_3 x_1 x_4 + k_4 x_6 + k_5 x_3, k_1 x_1 x_3 - k_2 x_5 + k_6 x_6, k_3 x_1 x_4 - k_4 x_6 - k_6 x_6, -k_9 x_1 x_8 + k_{10} x_{10} + k_{11} x_7, k_7 x_1 x_7 - k_8 x_9 + k_{12} x_{10}, k_9 x_1 x_8 - k_{10} x_{10} - k_{12} x_{10}, x_1 + x_2 + x_5 + x_6 - T_1, x_3 + x_4 + x_5 + x_6 - T_2, x_7 + x_8 + x_9 + x_{10} - T_3] \quad \textbf{(1.4)}$$

We parameterise the steady states as functions of x1 and x3, using the four steady state equations:
When x1 and x3 are positive, then so are the rest.

> $sol1 := solve([eqs[4], eqs[5], eqs[6], cons[2]], [x_3, x_4, x_5, x_6])$

$$sol1 := \left[\left[x_3 = \frac{T_2 x_1 k_6 k_3 k_2}{k_1 k_3 k_6 x_1^2 + k_2 k_3 k_5 x_1 + k_2 k_3 k_6 x_1 + k_3 k_5 k_6 x_1 + k_2 k_4 k_5 + k_2 k_5 k_6}, x_4 = \frac{(k_6 + k_4) k_5 k_2 T_2}{k_1 k_3 k_6 x_1^2 + k_2 k_3 k_5 x_1 + k_2 k_3 k_6 x_1 + k_3 k_5 k_6 x_1 + k_2 k_4 k_5 + k_2 k_5 k_6}, x_5 = \frac{x_1 k_3 T_2 k_6 (k_1 x_1 + k_5)}{k_1 k_3 k_6 x_1^2 + k_2 k_3 k_5 x_1 + k_2 k_3 k_6 x_1 + k_3 k_5 k_6 x_1 + k_2 k_4 k_5 + k_2 k_5 k_6}, x_6 = \frac{x_1 k_5 k_3 k_2 T_2}{k_1 k_3 k_6 x_1^2 + k_2 k_3 k_5 x_1 + k_2 k_3 k_6 x_1 + k_3 k_5 k_6 x_1 + k_2 k_4 k_5 + k_2 k_5 k_6}\right]\right] \quad \textbf{(1.5)}$$

> $sol2 := solve\left(\left[eqs[8], eqs[9], eqs[10], cons[3]\right], \left[x_7, x_8, x_9, x_{10}\right]\right)$

$$sol2 := \left[\left[x_7 = \frac{T_3\, k_{12}\, x_1\, k_9\, k_8}{k_7\, k_9\, k_{12}\, x_1^2 + k_8\, k_9\, k_{11}\, x_1 + k_8\, k_9\, k_{12}\, x_1 + k_9\, k_{11}\, k_{12}\, x_1 + k_8\, k_{10}\, k_{11} + k_8\, k_{11}\, k_{12}}, x_8 = \frac{\left(k_{12} + k_{10}\right) k_{11}\, k_8\, T_3}{k_7\, k_9\, k_{12}\, x_1^2 + k_8\, k_9\, k_{11}\, x_1 + k_8\, k_9\, k_{12}\, x_1 + k_9\, k_{11}\, k_{12}\, x_1 + k_8\, k_{10}\, k_{11} + k_8\, k_{11}\, k_{12}}, x_9 = \frac{x_1\, k_9\, T_3\, k_{12}\, \left(k_7\, x_1 + k_{11}\right)}{k_7\, k_9\, k_{12}\, x_1^2 + k_8\, k_9\, k_{11}\, x_1 + k_8\, k_9\, k_{12}\, x_1 + k_9\, k_{11}\, k_{12}\, x_1 + k_8\, k_{10}\, k_{11} + k_8\, k_{11}\, k_{12}}, x_{10} = \frac{k_{11}\, x_1\, k_9\, k_8\, T_3}{k_7\, k_9\, k_{12}\, x_1^2 + k_8\, k_9\, k_{11}\, x_1 + k_8\, k_9\, k_{12}\, x_1 + k_9\, k_{11}\, k_{12}\, x_1 + k_8\, k_{10}\, k_{11} + k_8\, k_{11}\, k_{12}}\right]\right] \quad \textbf{(1.6)}$$

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> $sol3 := solve\left(eqs[2], \left\{x_2\right\}\right)$

$$sol3 := \left\{x_2 = \frac{k_2\, x_5 + k_4\, x_6 + k_8\, x_9 + k_{10}\, x_{10}}{k_{13}}\right\} \quad \textbf{(1.7)}$$

> $sol4 := simplify(subs(sol2[1], sol1[1], \{sol3[1]\}))$

$$sol4 := \Big\{x_2 = \Big(x_1\, \Big(T_2\, k_1\, k_2\, k_3\, k_6\, k_7\, k_9\, k_{12}\, x_1^3 + T_3\, k_1\, k_3\, k_6\, k_7\, k_8\, k_9\, k_{12}\, x_1^3 \quad \textbf{(1.8)}$$
$$+ T_2\, k_1\, k_2\, k_3\, k_6\, k_8\, k_9\, k_{11}\, x_1^2 + T_2\, k_1\, k_2\, k_3\, k_6\, k_8\, k_9\, k_{12}\, x_1^2 + T_2\, k_1\, k_2\, k_3\, k_6\, k_9\, k_{11}\, k_{12}\, x_1^2$$
$$+ T_2\, k_2\, k_3\, k_4\, k_5\, k_7\, k_9\, k_{12}\, x_1^2 + T_2\, k_2\, k_3\, k_5\, k_6\, k_7\, k_9\, k_{12}\, x_1^2 + T_3\, k_1\, k_3\, k_6\, k_8\, k_9\, k_{10}\, k_{11}\, x_1^2$$
$$+ T_3\, k_1\, k_3\, k_6\, k_8\, k_9\, k_{11}\, k_{12}\, x_1^2 + T_3\, k_2\, k_3\, k_5\, k_7\, k_8\, k_9\, k_{12}\, x_1^2 + T_3\, k_2\, k_3\, k_6\, k_7\, k_8\, k_9\, k_{12}\, x_1^2$$
$$+ T_3\, k_3\, k_5\, k_6\, k_7\, k_8\, k_9\, k_{12}\, x_1^2 + T_2\, k_1\, k_2\, k_3\, k_6\, k_8\, k_{10}\, k_{11}\, x_1 + T_2\, k_1\, k_2\, k_3\, k_6\, k_8\, k_{11}\, k_{12}\, x_1$$
$$+ T_2\, k_2\, k_3\, k_4\, k_5\, k_8\, k_9\, k_{11}\, x_1 + T_2\, k_2\, k_3\, k_4\, k_5\, k_8\, k_9\, k_{12}\, x_1 + T_2\, k_2\, k_3\, k_4\, k_5\, k_9\, k_{11}\, k_{12}\, x_1$$
$$+ T_2\, k_2\, k_3\, k_5\, k_6\, k_8\, k_9\, k_{11}\, x_1 + T_2\, k_2\, k_3\, k_5\, k_6\, k_8\, k_9\, k_{12}\, x_1 + T_2\, k_2\, k_3\, k_5\, k_6\, k_9\, k_{11}\, k_{12}\, x_1$$

$$+ T_3 k_2 k_3 k_5 k_8 k_9 k_{10} k_{11} x_1 + T_3 k_2 k_3 k_5 k_8 k_9 k_{11} k_{12} x_1 + T_3 k_2 k_3 k_6 k_8 k_9 k_{10} k_{11} x_1$$
$$+ T_3 k_2 k_3 k_6 k_8 k_9 k_{11} k_{12} x_1 + T_3 k_2 k_4 k_5 k_7 k_8 k_9 k_{12} x_1 + T_3 k_2 k_5 k_6 k_7 k_8 k_9 k_{12} x_1$$
$$+ T_3 k_3 k_5 k_6 k_8 k_9 k_{10} k_{11} x_1 + T_3 k_3 k_5 k_6 k_8 k_9 k_{11} k_{12} x_1 + T_2 k_2 k_3 k_4 k_5 k_8 k_{10} k_{11}$$
$$+ T_2 k_2 k_3 k_4 k_5 k_8 k_{11} k_{12} + T_2 k_2 k_3 k_5 k_6 k_8 k_{10} k_{11} + T_2 k_2 k_3 k_5 k_6 k_8 k_{11} k_{12}$$
$$+ T_3 k_2 k_4 k_5 k_8 k_9 k_{10} k_{11} + T_3 k_2 k_4 k_5 k_8 k_9 k_{11} k_{12} + T_3 k_2 k_5 k_6 k_8 k_9 k_{10} k_{11}$$
$$+ T_3 k_2 k_5 k_6 k_8 k_9 k_{11} k_{12} \big) \big) \Big/ \Big(\big(k_1 k_3 k_6 x_1^2 + k_2 k_3 k_5 x_1 + k_2 k_3 k_6 x_1 + k_3 k_5 k_6 x_1$$
$$+ k_2 k_4 k_5 + k_2 k_5 k_6 \big) \big(k_7 k_9 k_{12} x_1^2 + k_8 k_9 k_{11} x_1 + k_8 k_9 k_{12} x_1 + k_9 k_{11} k_{12} x_1$$
$$+ k_8 k_{10} k_{11} + k_8 k_{11} k_{12} \big) k_{13} \Big) \Big\}$$

> *collect*(*numer*(*simplify*(*subs*(*sol2*[1], *sol1*[1], *sol4*, *cons*[1]))), x_1)

$$k_1 k_3 k_6 k_7 k_9 k_{12} k_{13} x_1^5 + \big(-T_1 k_1 k_3 k_6 k_7 k_9 k_{12} k_{13} + T_2 k_1 k_2 k_3 k_6 k_7 k_9 k_{12} \tag{1.9}$$
$$+ T_2 k_1 k_3 k_6 k_7 k_9 k_{12} k_{13} + T_3 k_1 k_3 k_6 k_7 k_8 k_9 k_{12} + k_1 k_3 k_6 k_8 k_9 k_{11} k_{13}$$
$$+ k_1 k_3 k_6 k_8 k_9 k_{12} k_{13} + k_1 k_3 k_6 k_9 k_{11} k_{12} k_{13} + k_2 k_3 k_5 k_7 k_9 k_{12} k_{13}$$
$$+ k_2 k_3 k_6 k_7 k_9 k_{12} k_{13} + k_3 k_5 k_6 k_7 k_9 k_{12} k_{13} \big) x_1^4 + \big(-T_1 k_1 k_3 k_6 k_8 k_9 k_{11} k_{13}$$
$$- T_1 k_1 k_3 k_6 k_8 k_9 k_{12} k_{13} - T_1 k_1 k_3 k_6 k_9 k_{11} k_{12} k_{13} - T_1 k_2 k_3 k_5 k_7 k_9 k_{12} k_{13}$$
$$- T_1 k_2 k_3 k_6 k_7 k_9 k_{12} k_{13} - T_1 k_3 k_5 k_6 k_7 k_9 k_{12} k_{13} + T_2 k_1 k_2 k_3 k_6 k_8 k_9 k_{11}$$
$$+ T_2 k_1 k_2 k_3 k_6 k_8 k_9 k_{12} + T_2 k_1 k_2 k_3 k_6 k_9 k_{11} k_{12} + T_2 k_1 k_3 k_6 k_8 k_9 k_{11} k_{13}$$
$$+ T_2 k_1 k_3 k_6 k_8 k_9 k_{12} k_{13} + T_2 k_1 k_3 k_6 k_9 k_{11} k_{12} k_{13} + T_2 k_2 k_3 k_4 k_5 k_7 k_9 k_{12}$$
$$+ T_2 k_2 k_3 k_5 k_6 k_7 k_9 k_{12} + T_2 k_2 k_3 k_5 k_7 k_9 k_{12} k_{13} + T_2 k_3 k_5 k_6 k_7 k_9 k_{12} k_{13}$$
$$+ T_3 k_1 k_3 k_6 k_8 k_9 k_{10} k_{11} + T_3 k_1 k_3 k_6 k_8 k_9 k_{11} k_{12} + T_3 k_2 k_3 k_5 k_7 k_8 k_9 k_{12}$$
$$+ T_3 k_2 k_3 k_6 k_7 k_8 k_9 k_{12} + T_3 k_3 k_5 k_6 k_7 k_8 k_9 k_{12} + k_1 k_3 k_6 k_8 k_{10} k_{11} k_{13}$$
$$+ k_1 k_3 k_6 k_8 k_{11} k_{12} k_{13} + k_2 k_3 k_5 k_8 k_9 k_{11} k_{13} + k_2 k_3 k_5 k_8 k_9 k_{12} k_{13}$$
$$+ k_2 k_3 k_5 k_9 k_{11} k_{12} k_{13} + k_2 k_3 k_6 k_8 k_9 k_{11} k_{13} + k_2 k_3 k_6 k_8 k_9 k_{12} k_{13}$$
$$+ k_2 k_3 k_6 k_9 k_{11} k_{12} k_{13} + k_2 k_4 k_5 k_7 k_9 k_{12} k_{13} + k_2 k_5 k_6 k_7 k_9 k_{12} k_{13}$$
$$+ k_3 k_5 k_6 k_8 k_9 k_{11} k_{13} + k_3 k_5 k_6 k_8 k_9 k_{12} k_{13} + k_3 k_5 k_6 k_9 k_{11} k_{12} k_{13} \big) x_1^3 + \big($$
$$-T_1 k_1 k_3 k_6 k_8 k_{10} k_{11} k_{13} - T_1 k_1 k_3 k_6 k_8 k_{11} k_{12} k_{13} - T_1 k_2 k_3 k_5 k_8 k_9 k_{11} k_{13}$$
$$- T_1 k_2 k_3 k_5 k_8 k_9 k_{12} k_{13} - T_1 k_2 k_3 k_5 k_9 k_{11} k_{12} k_{13} - T_1 k_2 k_3 k_6 k_8 k_9 k_{11} k_{13}$$
$$- T_1 k_2 k_3 k_6 k_8 k_9 k_{12} k_{13} - T_1 k_2 k_3 k_6 k_9 k_{11} k_{12} k_{13} - T_1 k_2 k_4 k_5 k_7 k_9 k_{12} k_{13}$$
$$- T_1 k_2 k_5 k_6 k_7 k_9 k_{12} k_{13} - T_1 k_3 k_5 k_6 k_8 k_9 k_{11} k_{13} - T_1 k_3 k_5 k_6 k_8 k_9 k_{12} k_{13}$$
$$- T_1 k_3 k_5 k_6 k_9 k_{11} k_{12} k_{13} + T_2 k_1 k_2 k_3 k_6 k_8 k_{10} k_{11} + T_2 k_1 k_2 k_3 k_6 k_8 k_{11} k_{12}$$

$$+ T_2 k_1 k_3 k_6 k_8 k_{10} k_{11} k_{13} + T_2 k_1 k_3 k_6 k_8 k_{11} k_{12} k_{13} + T_2 k_2 k_3 k_4 k_5 k_8 k_9 k_{11}$$
$$+ T_2 k_2 k_3 k_4 k_5 k_8 k_9 k_{12} + T_2 k_2 k_3 k_4 k_5 k_9 k_{11} k_{12} + T_2 k_2 k_3 k_5 k_6 k_8 k_9 k_{11}$$
$$+ T_2 k_2 k_3 k_5 k_6 k_8 k_9 k_{12} + T_2 k_2 k_3 k_5 k_6 k_9 k_{11} k_{12} + T_2 k_2 k_3 k_5 k_8 k_9 k_{11} k_{13}$$
$$+ T_2 k_2 k_3 k_5 k_8 k_9 k_{12} k_{13} + T_2 k_2 k_3 k_5 k_9 k_{11} k_{12} k_{13} + T_2 k_3 k_5 k_6 k_8 k_9 k_{11} k_{13}$$
$$+ T_2 k_3 k_5 k_6 k_8 k_9 k_{12} k_{13} + T_2 k_3 k_5 k_6 k_9 k_{11} k_{12} k_{13} + T_3 k_2 k_3 k_5 k_8 k_9 k_{10} k_{11}$$
$$+ T_3 k_2 k_3 k_5 k_8 k_9 k_{11} k_{12} + T_3 k_2 k_3 k_6 k_8 k_9 k_{10} k_{11} + T_3 k_2 k_3 k_6 k_8 k_9 k_{11} k_{12}$$
$$+ T_3 k_2 k_4 k_5 k_7 k_8 k_9 k_{12} + T_3 k_2 k_5 k_6 k_7 k_8 k_9 k_{12} + T_3 k_3 k_5 k_6 k_8 k_9 k_{10} k_{11}$$
$$+ T_3 k_3 k_5 k_6 k_8 k_9 k_{11} k_{12} + k_2 k_3 k_5 k_8 k_{10} k_{11} k_{13} + k_2 k_3 k_5 k_8 k_{11} k_{12} k_{13}$$
$$+ k_2 k_3 k_6 k_8 k_{10} k_{11} k_{13} + k_2 k_3 k_6 k_8 k_{11} k_{12} k_{13} + k_2 k_4 k_5 k_8 k_9 k_{11} k_{13}$$
$$+ k_2 k_4 k_5 k_8 k_9 k_{12} k_{13} + k_2 k_4 k_5 k_9 k_{11} k_{12} k_{13} + k_2 k_5 k_6 k_8 k_9 k_{11} k_{13}$$
$$+ k_2 k_5 k_6 k_8 k_9 k_{12} k_{13} + k_2 k_5 k_6 k_9 k_{11} k_{12} k_{13} + k_3 k_5 k_6 k_8 k_{10} k_{11} k_{13}$$
$$+ k_3 k_5 k_6 k_8 k_{11} k_{12} k_{13}\big) x_1^2 + \big(-T_1 k_2 k_3 k_5 k_8 k_{10} k_{11} k_{13} - T_1 k_2 k_3 k_5 k_8 k_{11} k_{12} k_{13}$$
$$- T_1 k_2 k_3 k_6 k_8 k_{10} k_{11} k_{13} - T_1 k_2 k_3 k_6 k_8 k_{11} k_{12} k_{13} - T_1 k_2 k_4 k_5 k_8 k_9 k_{11} k_{13}$$
$$- T_1 k_2 k_4 k_5 k_8 k_9 k_{12} k_{13} - T_1 k_2 k_4 k_5 k_9 k_{11} k_{12} k_{13} - T_1 k_2 k_5 k_6 k_8 k_9 k_{11} k_{13}$$
$$- T_1 k_2 k_5 k_6 k_8 k_9 k_{12} k_{13} - T_1 k_2 k_5 k_6 k_9 k_{11} k_{12} k_{13} - T_1 k_3 k_5 k_6 k_8 k_{10} k_{11} k_{13}$$
$$- T_1 k_3 k_5 k_6 k_8 k_{11} k_{12} k_{13} + T_2 k_2 k_3 k_4 k_5 k_8 k_{10} k_{11} + T_2 k_2 k_3 k_4 k_5 k_8 k_{11} k_{12}$$
$$+ T_2 k_2 k_3 k_5 k_6 k_8 k_{10} k_{11} + T_2 k_2 k_3 k_5 k_6 k_8 k_{11} k_{12} + T_2 k_2 k_3 k_5 k_8 k_{10} k_{11} k_{13}$$
$$+ T_2 k_2 k_3 k_5 k_8 k_{11} k_{12} k_{13} + T_2 k_3 k_5 k_6 k_8 k_{10} k_{11} k_{13} + T_2 k_3 k_5 k_6 k_8 k_{11} k_{12} k_{13}$$
$$+ T_3 k_2 k_4 k_5 k_8 k_9 k_{10} k_{11} + T_3 k_2 k_4 k_5 k_8 k_9 k_{11} k_{12} + T_3 k_2 k_5 k_6 k_8 k_9 k_{10} k_{11}$$
$$+ T_3 k_2 k_5 k_6 k_8 k_9 k_{11} k_{12} + k_2 k_4 k_5 k_8 k_{10} k_{11} k_{13} + k_2 k_4 k_5 k_8 k_{11} k_{12} k_{13}$$
$$+ k_2 k_5 k_6 k_8 k_{10} k_{11} k_{13} + k_2 k_5 k_6 k_8 k_{11} k_{12} k_{13}\big) x_1 - T_1 k_2 k_4 k_5 k_8 k_{10} k_{11} k_{13}$$
$$- T_1 k_2 k_4 k_5 k_8 k_{11} k_{12} k_{13} - T_1 k_2 k_5 k_6 k_8 k_{10} k_{11} k_{13} - T_1 k_2 k_5 k_6 k_8 k_{11} k_{12} k_{13}$$

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This is a degree 5 polynomial which presumably admits 5 positive real roots. Any real root of this polynomial leads to a steady state for the fixed rate constants and total amounts. The values of the other variables at steady states are found by plugging the value of x1 (the root of the polynomial) into the expressions in sol1, sol2, and sol3 above.

A necessary condition for 5 positive roots is that the signs of the coefficient of the polynomial (in x1) alternate.
This is a pre-check when you do the sampling: you need to impose the coefficient of x1^4 to be negative , the coefficient of x1^3 to be positive, the coefficient of x1^2 to be negative and the coefficient of x1 to be positive. The coefficients of x1^5 and the independent term always have the right sign.