# Bistable motif: 2kinase I substrate

## Finding the condition of multistationarity

We consider the following reactions:

```
\begin{array}{l} \texttt{K} + \texttt{S} \to \texttt{KS} \to \texttt{K} + \texttt{S}_p \\ \texttt{K}_p + \texttt{S} \to \texttt{K}_p \ \texttt{S} \to \texttt{K}_p + \texttt{S}_p \\ \texttt{K} \to \texttt{K}_p \\ \texttt{KS} \leftarrow \texttt{K}_p \ \texttt{S} \\ \texttt{E} + \texttt{S} \to \texttt{ES} \to \texttt{E} + \texttt{S}_p \\ \texttt{E}_p + \texttt{S} \to \texttt{E}_p \ \texttt{S} \to \texttt{E}_p + \texttt{S}_p \\ \texttt{E} \to \texttt{E}_p \\ \texttt{ES} \leftarrow \texttt{E}_p \ \texttt{S} \\ \texttt{S}_p \to \texttt{S} \end{array} The species of the system are:
```

 $\{\,\texttt{S}\,,\,\,\texttt{S}_{\texttt{p}}\,,\,\,\texttt{K}\,,\,\,\texttt{K}_{\texttt{p}}\,,\,\,\,\texttt{KS}\,,\,\,\,\texttt{K}_{\texttt{p}}\,\,\texttt{S}\,,\,\,\,\texttt{E}\,,\,\,\,\texttt{E}_{\texttt{p}}\,,\,\,\,\,\texttt{ES}\,,\,\,\,\texttt{E}_{\texttt{p}}\,\,\texttt{S}\,\}$ 

In total, there are 13 reations and 10 species.

We firstly construct the ordinary differential equations based on mass-action kinetics. Then compute the determinant of Jacobian, using the solution at critical point (steady state) to calculate the determinant. The (necessary) condition for multistationarity is to make determinant equal to zero (non-zero determinant implys injectivity).

```
A = Table[0, {13}, {10}];
A[[1]][[1]] = -1; A[[1]][[3]] = -1; A[[1]][[5]] = 1;
A[[2]][[3]] = 1; A[[2]][[2]] = 1; A[[2]][[5]] = -1;
A[[3]][[1]] = -1; A[[3]][[4]] = -1; A[[3]][[6]] = 1;
A[[4]][[4]] = 1; A[[4]][[2]] = 1; A[[4]][[6]] = -1;
A[[5]][[3]] = -1;
A[[5]][[4]] = 1;
A[[6]][[5]] = 1;
A[[6]][[6]] = -1;
A[[13]][[2]] = -1; A[[13]][[1]] = 1;
A[[7]][[1]] = -1; A[[7]][[7]] = -1; A[[7]][[9]] = 1;
A[[8]][[7]] = 1; A[[8]][[2]] = 1; A[[8]][[9]] = -1;
A[[9]][[1]] = -1; A[[9]][[8]] = -1; A[[9]][[10]] = 1;
A[[10]][[8]] = 1; A[[10]][[2]] = 1; A[[10]][[10]] = -1;
A[[11]][[7]] = -1; A[[11]][[8]] = 1;
A[[12]][[9]] = 1;
A[[12]][[10]] = -1;
stoiM = Transpose[A]
\{\{-1, 0, -1, 0, 0, 0, -1, 0, -1, 0, 0, 0, 1\}, \{0, 1, 0, 1, 0, 0, 0, 1, 0, 1, 0, 0, -1\},
 \{-1,\,1,\,0,\,0,\,-1,\,0,\,0,\,0,\,0,\,0,\,0,\,0,\,0\}\,,\,\{0,\,0,\,-1,\,1,\,1,\,0,\,0,\,0,\,0,\,0,\,0,\,0,\,0\}\,,
 \{1, -1, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 1, -1, 0, -1, 0, 0, 0, 0, 0, 0, 0\},
 \{0, 0, 0, 0, 0, 0, -1, 1, 0, 0, -1, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0, -1, 1, 1, 0, 0\},
 \{0, 0, 0, 0, 0, 0, 1, -1, 0, 0, 0, 1, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0, 1, -1, 0, -1, 0\}\}
```

```
ks = \{k_1 \times x_3 \times x_1, k_2 \times x_5, k_3 \times x_4 \times x_1, k_4 \times x_6, k_5 \times x_3, k_6 \times x_6,
         k_7 \times x_7 \times x_1, k_8 \times x_9, k_9 \times x_8 * x_1, k_{10} * x_{10}, k_{11} * x_7, k_{12} \times x_{10}, k_{13} * x_2
 \{k_1 x_1 x_3, k_2 x_5, k_3 x_1 x_4, k_4 x_6, k_5 x_3, k_6 x_6,
   k_7 x_1 x_7, k_8 x_9, k_9 x_1 x_8, k_{10} x_{10}, k_{11} x_7, k_{12} x_{10}, k_{13} x_2
 ssEqns = stoiM.ks
 \{k_{13} x_2 - k_1 x_1 x_3 - k_3 x_1 x_4 - k_7 x_1 x_7 - k_9 x_1 x_8,
     -\,k_{13}\;x_{2}\,+\,k_{2}\;x_{5}\,+\,k_{4}\;x_{6}\,+\,k_{8}\;x_{9}\,+\,k_{10}\;x_{10}\,\text{,}\;\;-\,k_{5}\;x_{3}\,-\,k_{1}\;x_{1}\;x_{3}\,+\,k_{2}\;x_{5}\,\text{,}\;\;k_{5}\;x_{3}\,-\,k_{3}\;x_{1}\;x_{4}\,+\,k_{4}\;x_{6}\,\text{,}\;\;k_{5}\;x_{1}\,+\,k_{10}\;x_{10}\,,\;\;-\,k_{10}\;x_{10}\,,\;\;-\,k_{10}\;x_{10}\,,\;\;-\,k_{10}\;x_{10}\,,\;\;-\,k_{10}\;x_{10}\,,\;\;-\,k_{10}\;x_{10}\,,\;\;-\,k_{10}\;x_{10}\,,\;\;-\,k_{10}\;x_{10}\,,\;\;-\,k_{10}\;x_{10}\,,\;\;-\,k_{10}\;x_{10}\,,\;\;-\,k_{10}\;x_{10}\,,\;\;-\,k_{10}\;x_{10}\,,\;\;-\,k_{10}\;x_{10}\,,\;\;-\,k_{10}\;x_{10}\,,\;\;-\,k_{10}\;x_{10}\,,\;\;-\,k_{10}\;x_{10}\,,\;\;-\,k_{10}\;x_{10}\,,\;\;-\,k_{10}\;x_{10}\,,\;\;-\,k_{10}\;x_{10}\,,\;\;-\,k_{10}\;x_{10}\,,\;\;-\,k_{10}\;x_{10}\,,\;\;-\,k_{10}\;x_{10}\,,\;\;-\,k_{10}\;x_{10}\,,\;\;-\,k_{10}\;x_{10}\,,\;\;-\,k_{10}\;x_{10}\,,\;\;-\,k_{10}\;x_{10}\,,\;\;-\,k_{10}\;x_{10}\,,\;\;-\,k_{10}\;x_{10}\,,\;\;-\,k_{10}\;x_{10}\,,\;\;-\,k_{10}\;x_{10}\,,\;\;-\,k_{10}\;x_{10}\,,\;\;-\,k_{10}\;x_{10}\,,\;\;-\,k_{10}\;x_{10}\,,\;\;-\,k_{10}\;x_{10}\,,\;\;-\,k_{10}\;x_{10}\,,\;\;-\,k_{10}\;x_{10}\,,\;\;-\,k_{10}\;x_{10}\,,\;\;-\,k_{10}\;x_{10}\,,\;\;-\,k_{10}\;x_{10}\,,\;\;-\,k_{10}\;x_{10}\,,\;\;-\,k_{10}\;x_{10}\,,\;\;-\,k_{10}\;x_{10}\,,\;\;-\,k_{10}\;x_{10}\,,\;\;-\,k_{10}\;x_{10}\,,\;\;-\,k_{10}\;x_{10}\,,\;\;-\,k_{10}\;x_{10}\,,\;\;-\,k_{10}\;x_{10}\,,\;\;-\,k_{10}\;x_{10}\,,\;\;-\,k_{10}\;x_{10}\,,\;\;-\,k_{10}\;x_{10}\,,\;\;-\,k_{10}\;x_{10}\,,\;\;-\,k_{10}\;x_{10}\,,\;\;-\,k_{10}\;x_{10}\,,\;\;-\,k_{10}\;x_{10}\,,\;\;-\,k_{10}\;x_{10}\,,\;\;-\,k_{10}\;x_{10}\,,\;\;-\,k_{10}\;x_{10}\,,\;\;-\,k_{10}\;x_{10}\,,\;\;-\,k_{10}\;x_{10}\,,\;\;-\,k_{10}\;x_{10}\,,\;\;-\,k_{10}\;x_{10}\,,\;\;-\,k_{10}\;x_{10}\,,\;\;-\,k_{10}\;x_{10}\,,\;\;-\,k_{10}\;x_{10}\,,\;\;-\,k_{10}\;x_{10}\,,\;\;-\,k_{10}\;x_{10}\,,\;\;-\,k_{10}\;x_{10}\,,\;\;-\,k_{10}\;x_{10}\,,\;\;-\,k_{10}\;x_{10}\,,\;\;-\,k_{10}\;x_{10}\,,\;\;-\,k_{10}\;x_{10}\,,\;\;-\,k_{10}\;x_{10}\,,\;\;-\,k_{10}\;x_{10}\,,\;-\,k_{10}\;x_{10}\,,\;-\,k_{10}\;x_{10}\,,\;-\,k_{10}\;x_{10}\,,\;-\,k_{10}\;x_{10}\,,\;-\,k_{10}\;x_{10}\,,\;-\,k_{10}\;x_{10}\,,\;-\,k_{10}\;x_{10}\,,\;-\,k_{10}\;x_{10}\,,\;-\,k_{10}\;x_{10}\,,\;-\,k_{10}\;x_{10}\,,\;-\,k_{10}\;x_{10}\,,\;-\,k_{10}\;x_{10}\,,\;-\,k_{10}\;x_{10}\,,\;-\,k_{10}\;x_{10}\,,\;-\,k_{10}\;x_{10}\,,\;-\,k_{10}\;x_{10}\,,\;-\,k_{10}\;x_{10}\,,\;-\,k_{10}\;x_{10}\,,\;-\,k_{10}\;x_{10}\,,\;-\,k_{10}\;x_{10}\,,\;-\,k_{10}\;x_{10}\,,\;-\,k_{10}\;x_{10}\,,\;-\,k_{10}\;x_{10}\,,\;-\,k_{10}\;x_{10}\,,\;-\,k_{10}\;x_{10}\,,\;-\,k_{10}\;x_
     k_1 x_1 x_3 - k_2 x_5 + k_6 x_6, k_3 x_1 x_4 - k_4 x_6 - k_6 x_6, -k_{11} x_7 - k_7 x_1 x_7 + k_8 x_9,
     k_{11} x_7 - k_9 x_1 x_8 + k_{10} x_{10}, k_7 x_1 x_7 - k_8 x_9 + k_{12} x_{10}, k_9 x_1 x_8 - k_{10} x_{10} - k_{12} x_{10}}
mC = RowReduce[NullSpace[A]]
 \{\{1, 1, 0, 0, 1, 1, 0, 0, 1, 1\},\
     \{0, 0, 1, 1, 1, 1, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 1, 1, 1, 1\}\}
 cons = \{x_1 + x_2 + x_5 + x_6 + x_9 + x_{10} - T_1, x_3 + x_4 + x_5 + x_6 - T_2, x_7 + x_8 + x_9 + x_{10} - T_3\};
 subsEqns = {ssEqns[[2]], ssEqns[[4]], ssEqns[[5]], ssEqns[[6]],
         ssEqns[[8]], ssEqns[[9]], cons[[1]], cons[[2]], cons[[3]]}
 \{-k_{13} x_2 + k_2 x_5 + k_4 x_6 + k_8 x_9 + k_{10} x_{10}, k_5 x_3 - k_3 x_1 x_4 + k_4 x_6, k_1 x_1 x_3 - k_2 x_5 + k_6 x_6, k_1 x_1 x_2 - k_2 x_5 + k_6 x_6, k_1 x_1 x_2 - k_2 x_5 + k_6 x_6, k_1 x_1 x_2 - k_2 x_5 + k_6 x_6, k_1 x_1 x_2 - k_2 x_5 + k_6 x_6, k_1 x_1 x_2 - k_2 x_5 + k_6 x_6, k_1 x_1 x_2 - k_2 x_5 + k_6 x_6, k_1 x_1 x_2 - k_2 x_5 + k_6 x_6, k_1 x_1 x_2 - k_2 x_5 - k_2
    k_3 x_1 x_4 - k_4 x_6 - k_6 x_6, k_{11} x_7 - k_9 x_1 x_8 + k_{10} x_{10}, k_7 x_1 x_7 - k_8 x_9 + k_{12} x_{10},
     -T_{1}+x_{1}+x_{2}+x_{5}+x_{6}+x_{9}+x_{10}\text{, }-T_{2}+x_{3}+x_{4}+x_{5}+x_{6}\text{, }-T_{3}+x_{7}+x_{8}+x_{9}+x_{10}\text{ }\}
 sol1 =
     Solve[\{ssEqns[[4]], ssEqns[[5]], ssEqns[[6]], cons[[2]]\} == 0, \{x_3, x_4, x_5, x_6\}]
                                                                                                                        k_2 k_3 k_6 T_2 x_1
                           k_2 \ k_4 \ k_5 \ + \ k_2 \ k_5 \ k_6 \ + \ k_2 \ k_3 \ k_5 \ x_1 \ + \ k_2 \ k_3 \ k_6 \ x_1 \ + \ k_3 \ k_5 \ k_6 \ x_1 \ + \ k_1 \ k_3 \ k_6 \ x_1^2
                                                                                                                 k_2 k_5 (k_4 + k_6) T_2
                            k_2 k_4 k_5 + k_2 k_5 k_6 + k_2 k_3 k_5 x_1 + k_2 k_3 k_6 x_1 + k_3 k_5 k_6 x_1 + k_1 k_3 k_6 x_1^2
                                                                                                     k_3 T_2 (k_5 k_6 x_1 + k_1 k_6 x_1^2)
                            k_2 k_4 k_5 + k_2 k_5 k_6 + k_2 k_3 k_5 x_1 + k_2 k_3 k_6 x_1 + k_3 k_5 k_6 x_1 + k_1 k_3 k_6 x_1^2
                                                                                                                      \mathbf{k_2} \ \mathbf{k_3} \ \mathbf{k_5} \ \mathbf{T_2} \ \mathbf{x_1}
                           sol2 =
     Solve[{ssEqns[[8]], ssEqns[[9]], ssEqns[[10]], cons[[3]]} == 0, \{x_7, x_8, x_9, x_{10}\}]
                                                                                                                                      k<sub>8</sub> k<sub>9</sub> k<sub>12</sub> T<sub>3</sub> x<sub>1</sub>
                           k_8 \ k_{10} \ k_{11} + k_8 \ k_{11} \ k_{12} + k_8 \ k_9 \ k_{11} \ x_1 + k_8 \ k_9 \ k_{12} \ x_1 + k_9 \ k_{11} \ k_{12} \ x_1 + k_7 \ k_9 \ k_{12} \ x_1^2
                                                                                                                           k_8 \ k_{11} \ (k_{10} + k_{12}) \ T_3
                            k_{8} k_{10} k_{11} + k_{8} k_{11} k_{12} + k_{8} k_{9} k_{11} x_{1} + k_{8} k_{9} k_{12} x_{1} + k_{9} k_{11} k_{12} x_{1} + k_{7} k_{9} k_{12} x_{1}^{2}
                                                                                                             k_9 T_3 (k_{11} k_{12} x_1 + k_7 k_{12} x_1^2)
                            k_8 \ k_{10} \ k_{11} + k_8 \ k_{11} \ k_{12} + k_8 \ k_9 \ k_{11} \ x_1 + k_8 \ k_9 \ k_{12} \ x_1 + k_9 \ k_{11} \ k_{12} \ x_1 + k_7 \ k_9 \ k_{12} \ x_1^2
                                                                                                                                         k_8 k_9 k_{11} T_3 x_1
                              \frac{}{k_8 \ k_{10} \ k_{11} + k_8 \ k_{11} \ k_{12} + k_8 \ k_9 \ k_{11} \ x_1 + k_8 \ k_9 \ k_{12} \ x_1 + k_9 \ k_{11} \ k_{12} \ x_1 + k_7 \ k_9 \ k_{12} \ x_1^2} \right\} \Big\}
 sol3 = x_2 /. Solve[{ssEqns[[2]] = 0}, {x_2}]
      k_2\ x_5\ +\ k_4\ x_6\ +\ k_8\ x_9\ +\ k_{10}\ x_{10}
```

### $sol4 = Solve[{x_2 = sol3[[1]]} /. Join[sol1[[1]], sol2[[1]]], {x_2}]$

$$\begin{split} \Big\{ \Big\{ x_2 \to \frac{1}{k_{13}} \left( \frac{k_2 \, k_3 \, k_4 \, k_5 \, T_2 \, x_1}{k_2 \, k_4 \, k_5 + k_2 \, k_5 \, k_6 + k_2 \, k_3 \, k_5 \, x_1 + k_2 \, k_3 \, k_6 \, x_1 + k_3 \, k_5 \, k_6 \, x_1 + k_1 \, k_3 \, k_6 \, x_1^2} + \\ \frac{k_2 \, k_3 \, T_2 \, \left( k_5 \, k_6 \, x_1 + k_1 \, k_6 \, x_1^2 \right)}{k_2 \, k_4 \, k_5 + k_2 \, k_5 \, k_6 + k_2 \, k_3 \, k_5 \, x_1 + k_2 \, k_3 \, k_6 \, x_1 + k_3 \, k_5 \, k_6 \, x_1 + k_1 \, k_3 \, k_6 \, x_1^2} + \\ \frac{k_8 \, k_9 \, k_{10} \, k_{11} \, T_3 \, x_1}{k_8 \, k_{10} \, k_{11} + k_8 \, k_{11} \, k_{12} + k_8 \, k_9 \, k_{11} \, x_1 + k_8 \, k_9 \, k_{12} \, x_1 + k_9 \, k_{11} \, k_{12} \, x_1 + k_7 \, k_9 \, k_{12} \, x_1^2} + \\ \frac{k_8 \, k_9 \, T_3 \, \left( k_{11} \, k_{12} \, x_1 + k_7 \, k_{12} \, x_1^2 \right)}{k_8 \, k_{10} \, k_{11} + k_8 \, k_{11} \, k_{12} + k_8 \, k_9 \, k_{11} \, x_1 + k_8 \, k_9 \, k_{12} \, x_1 + k_9 \, k_{11} \, k_{12} \, x_1 + k_7 \, k_9 \, k_{12} \, x_1^2} \Big\} \Big\} \Big\} \end{split}$$

#### term =

$$Full Simplify[x_1 + x_2 + x_5 + x_6 + x_9 + x_{10} - T_1 /. \ Join[sol1[[1]], \ sol2[[1]], \ sol4[[1]]]]$$

$$-T_{1} + \frac{1}{k_{13}}x_{1} \left( \frac{k_{3} \ T_{2} \ (k_{5} \ (k_{6} \ k_{13} + k_{2} \ (k_{4} + k_{6} + k_{13}) \ ) \ + k_{1} \ k_{6} \ (k_{2} + k_{13}) \ x_{1})}{k_{3} \ k_{6} \ x_{1} \ (k_{5} + k_{1} \ x_{1}) \ + k_{2} \ (k_{5} \ (k_{4} + k_{6}) \ + k_{3} \ (k_{5} + k_{6}) \ x_{1})} \ + \\ (k_{9} \ k_{12} \ k_{13} \ (T_{3} + x_{1}) \ (k_{11} + k_{7} \ x_{1}) \ + \\ k_{8} \ (k_{11} \ ((k_{10} + k_{12}) \ k_{13} + k_{9} \ (k_{10} + k_{12} + k_{13}) \ T_{3}) \ + k_{9} \ ((k_{11} + k_{12}) \ k_{13} + k_{7} \ k_{12} \ T_{3}) \ x_{1})) \ / \\ (k_{9} \ k_{12} \ x_{1} \ (k_{11} + k_{7} \ x_{1}) \ + k_{8} \ (k_{11} \ (k_{10} + k_{12}) \ + k_{9} \ (k_{11} + k_{12}) \ x_{1})) \ )$$

#### polynomial = Collect[Numerator[Together[term]], x1]

```
-\;k_2\;k_4\;k_5\;k_8\;k_{10}\;k_{11}\;k_{13}\;T_1\;-\;k_2\;k_5\;k_6\;k_8\;k_{10}\;k_{11}\;k_{13}\;T_1\;-\;
                      k_2 k_4 k_5 k_8 k_{11} k_{12} k_{13} T_1 - k_2 k_5 k_6 k_8 k_{11} k_{12} k_{13} T_1 +
                                (k_2 \ k_4 \ k_5 \ k_8 \ k_{10} \ k_{11} \ k_{13} + k_2 \ k_5 \ k_6 \ k_8 \ k_{10} \ k_{11} \ k_{13} + k_2 \ k_4 \ k_5 \ k_8 \ k_{11} \ k_{12} \ k_{13} + k_2 \ k_4 \ k_5 \ k_8 \ k_{11} \ k_{12} \ k_{13} + k_8 \ k_{10} \ k_{11} \ k_{12} \ k_{13} + k_8 \ k_{10} \ 
                                                                                                      k_2 \; k_5 \; k_6 \; k_8 \; k_{11} \; k_{12} \; k_{13} \; - \; k_2 \; k_4 \; k_5 \; k_8 \; k_9 \; k_{11} \; k_{13} \; T_1 \; - \; k_2 \; k_5 \; k_6 \; k_8 \; k_9 \; k_{11} \; k_{13} \; T_1 \; - \; k_2 \; k_5 \; k_6 \; k_8 \; k_9 \; k_{11} \; k_{13} \; T_1 \; - \; k_2 \; k_5 \; k_6 \; k_8 \; k_9 \; k_{11} \; k_{13} \; T_1 \; - \; k_2 \; k_5 \; k_6 \; k_8 \; k_9 \; k_{11} \; k_{13} \; T_1 \; - \; k_2 \; k_5 \; k_6 \; k_8 \; k_9 \; k_{11} \; k_{13} \; T_1 \; - \; k_2 \; k_5 \; k_6 \; k_8 \; k_9 \; k_{11} \; k_{13} \; T_1 \; - \; k_2 \; k_5 \; k_6 \; k_8 \; k_9 \; k_{11} \; k_{13} \; T_1 \; - \; k_2 \; k_5 \; k_6 \; k_8 \; k_9 \; k_{11} \; k_{13} \; T_1 \; - \; k_2 \; k_5 \; k_6 \; k_8 \; k_9 \; k_{11} \; k_{13} \; T_1 \; - \; k_2 \; k_5 \; k_6 \; k_8 \; k_9 \; k_{11} \; k_{13} \; T_1 \; - \; k_2 \; k_5 \; k_6 \; k_8 \; k_9 \; k_{11} \; k_{13} \; T_1 \; - \; k_2 \; k_5 \; k_6 \; k_8 \; k_9 \; k_{11} \; k_{13} \; T_1 \; - \; k_2 \; k_5 \; k_6 \; k_8 \; k_9 \; k_{11} \; k_{13} \; T_1 \; - \; k_2 \; k_5 \; k_6 \; k_8 \; k_9 \; k_{11} \; k_{13} \; T_1 \; - \; k_2 \; k_5 \; k_8 \; k_9 \; k_{11} \; k_{13} \; T_1 \; - \; k_2 \; k_5 \; k_8 \; k_9 \; k_{11} \; k_{13} \; T_1 \; - \; k_2 \; k_5 \; k_8 \; k_9 \; k_{11} \; k_{13} \; T_1 \; - \; k_2 \; k_5 \; k_8 \; k_9 \; k_{11} \; k_{13} \; T_1 \; - \; k_2 \; k_5 \; k_8 \; k_9 \; k_{11} \; k_{13} \; T_1 \; - \; k_2 \; k_5 \; k_8 \; k_9 \; k_{11} \; k_{13} \; T_1 \; - \; k_2 \; k_5 \; k_8 \; k_9 \; k_{11} \; k_{13} \; T_1 \; - \; k_2 \; k_5 \; k_8 \; k_9 \; k_{11} \; k_{13} \; T_1 \; - \; k_2 \; k_5 \; k_8 \; k_9 \; k_{11} \; k_{13} \; T_1 \; - \; k_2 \; k_5 \; k_8 \; k_9 \; k_{11} \; k_{13} \; T_1 \; - \; k_2 \; k_5 \; k_8 \; k_9 \; k_{11} \; k_{13} \; T_1 \; - \; k_2 \; k_5 \; k_8 \; k_9 \; k_{11} \; k_{12} \; T_1 \; - \; k_2 \; k_5 \; k_8 \; k_9 \; k_{11} \; k_{12} \; T_1 \; - \; k_2 \; k_5 \; k_8 \; k_9 \; k_{11} \; k_{12} \; T_1 \; - \; k_2 \; k_5 \; k_8 \; k_9 \; k_{11} \; k_{12} \; T_1 \; - \; k_2 \; k_5 \; k_8 \; k_9 \; k_{11} \; k_{12} \; T_1 \; - \; k_2 \; k_5 \; k_8 \; k_9 \; k_{11} \; k_{12} \; T_1 \; - \; k_2 \; k_5 \; k_8 \; k_9 \; k_{11} \; k_{12} \; T_1 \; - \; k_2 \; k_9 \; k_9 \; k_{12} \; k_{13} \; T_1 \; - \; k_2 \; k_9 \; k_9 \; k_{12} \; K_1 \; - \; k_9 \; k_9 \; k_{11} \; k_{12} \; T_1 \; - \; k_9 \; 
                                                                                                      k_2 \ k_3 \ k_5 \ k_8 \ k_{10} \ k_{11} \ k_{13} \ T_1 - k_2 \ k_3 \ k_6 \ k_8 \ k_{10} \ k_{11} \ k_{13} \ T_1 - k_3 \ k_5 \ k_6 \ k_8 \ k_{10} \ k_{11} \ k_{13} \ T_1 - k_{10} \ k_{11} \ k_{12} \ K_1 - k_{11} \ k_{12} \ K_1 - k_{12} \ K_1 - k_{13} \ K_1 - k
                                                                                                      k_2\ k_4\ k_5\ k_8\ k_9\ k_{12}\ k_{13}\ T_1\ -\ k_2\ k_5\ k_6\ k_8\ k_9\ k_{12}\ k_{13}\ T_1\ -\ k_2\ k_3\ k_5\ k_8\ k_{11}\ k_{12}\ k_{13}\ T_1\ -\ k_2\ k_3\ k_5\ k_8\ k_{11}\ k_{12}\ k_{13}\ T_1\ -\ k_2\ k_{13}\ k_{12}\ k_{13}\ k_{14}\ k_{15}\ k_{15
                                                                                                      k_2 \; k_3 \; k_6 \; k_8 \; k_{11} \; k_{12} \; k_{13} \; T_1 \; - \; k_3 \; k_5 \; k_6 \; k_8 \; k_{11} \; k_{12} \; k_{13} \; T_1 \; - \; k_2 \; k_4 \; k_5 \; k_9 \; k_{11} \; k_{12} \; k_{13} \; T_1 \; - \; k_1 \; k_2 \; k_1 \; k
                                                                                                      k_2 \ k_5 \ k_6 \ k_9 \ k_{11} \ k_{12} \ k_{13} \ T_1 + k_2 \ k_3 \ k_4 \ k_5 \ k_8 \ k_{10} \ k_{11} \ T_2 + k_2 \ k_3 \ k_5 \ k_6 \ k_8 \ k_{10} \ k_{11} \ T_2 + k_8 \ k_8 \ k_{10} \ k_{11} \ k_{12} \ k_{13} \ k_{10} \ k_{11} \ k_{12} \ k_{13} \ k_{14} \ k_{15} \ k_{10} \ k_{1
                                                                                                      k_2 \ k_3 \ k_4 \ k_5 \ k_8 \ k_{11} \ k_{12} \ T_2 + k_2 \ k_3 \ k_5 \ k_6 \ k_8 \ k_{11} \ k_{12} \ T_2 + k_2 \ k_3 \ k_5 \ k_8 \ k_{10} \ k_{11} \ k_{13} \ T_2 +
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                                                                                                      k_2 \ k_4 \ k_5 \ k_8 \ k_9 \ k_{10} \ k_{11} \ T_3 + k_2 \ k_5 \ k_6 \ k_8 \ k_9 \ k_{10} \ k_{11} \ T_3 + k_2 \ k_4 \ k_5 \ k_8 \ k_9 \ k_{11} \ k_{12} \ T_3 + k_1 \ k_2 \ k_4 \ k_5 \ k_8 \ k_9 \ k_{11} \ k_{12} \ k_{12} \ k_{13} \ k_{14} \ k_{15} \ k_{15} \ k_{15} \ k_{16} \ k
                                                                                                      k_2 \ k_5 \ k_6 \ k_8 \ k_9 \ k_{11} \ k_{12} \ T_3 + k_2 \ k_4 \ k_5 \ k_8 \ k_9 \ k_{11} \ k_{13} \ T_3 + k_2 \ k_5 \ k_6 \ k_8 \ k_9 \ k_{11} \ k_{13} \ T_3 + k_8 \ k_9 \ k_{11} \ k_{13} 
                                                                                                      k_2 \ k_4 \ k_5 \ k_9 \ k_{11} \ k_{12} \ k_{13} \ T_3 \ + \ k_2 \ k_5 \ k_6 \ k_9 \ k_{11} \ k_{12} \ k_{13} \ T_3 \ ) \ x_1 \ +
                                   (k_2 \ k_4 \ k_5 \ k_8 \ k_9 \ k_{11} \ k_{13} + k_2 \ k_5 \ k_6 \ k_8 \ k_9 \ k_{11} \ k_{13} + k_2 \ k_3 \ k_5 \ k_8 \ k_{10} \ k_{11} \ k_{13} + k_{13} \ k_{13} \ k_{14} \ k_{15} \ k_{15} \ k_{10} \ k_{11} \ k_{12} + k_{13} \ k_{14} \ k_{15} \ k_{15} \ k_{10} \ k_{11} \ k_{12} \ k_{13} \ k_{14} \ k_{15} \ k_{15} \ k_{10} \ k_{11} \ k_{12} \ k_{13} \ k_{15} \ k_{15} \ k_{10} \ k_{10} \ k_{11} \ k_{12} \ k_{13} \ k_{15} \ k_{15}
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                                                                                                      k_2\ k_5\ k_6\ k_8\ k_9\ k_{12}\ k_{13}\ +\ k_2\ k_3\ k_5\ k_8\ k_{11}\ k_{12}\ k_{13}\ +\ k_2\ k_3\ k_6\ k_8\ k_{11}\ k_{12}\ k_{13}\ +
                                                                                                      k_3 \ k_5 \ k_6 \ k_8 \ k_{11} \ k_{12} \ k_{13} \ + \ k_2 \ k_4 \ k_5 \ k_9 \ k_{11} \ k_{12} \ k_{13} \ + \ k_2 \ k_5 \ k_6 \ k_9 \ k_{11} \ k_{12} \ k_{13} \ -
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                                                                                                      k_2 \; k_3 \; k_5 \; k_8 \; k_9 \; k_{12} \; k_{13} \; T_1 \; - \; k_2 \; k_3 \; k_6 \; k_8 \; k_9 \; k_{12} \; k_{13} \; T_1 \; - \; k_3 \; k_5 \; k_6 \; k_8 \; k_9 \; k_{12} \; k_{13} \; T_1 \; - \; k_1 \; k_2 \; k_3 \; k_1 \; k_1 \; k_2 \; k_3 \; k_1 \; k_2 \; k_3 \; k_1 \; k_2 \; k_3 \; k_1 \; k_2 \; k_3 \; k_1 \; k_1 \; k_2 \; k_3 \; k_1 \; k_1 \; k_2 \; k_3 \; k_1 \; k_2 \; k_3 \; k_1 \; k_1 \; k_2 \; k_3 \; k_1 \; k_1 \; k_2 \; k_3 \; k_1 \; k_2 \; k_3 \; k_1 \; k_1 \; k_2 \; k_3 \; k_1 \; k_1 \; k_2 \; k_3 \; k_1 \; k_2 \; k_3 \; k_1 \; k_1 \; k_1 \; k_2 \; k_3 \; k_1 \; k_1 \; k_2 \; k_3 \; k_1 \; k_1 \; k_2 \; k_1 \; k_1 \; k_2 \; k_3 \; k_1 \; k_1 \; k_1 \; k_2 \; k_1 \; k_1 \; k_2 \; k_3 \; k_1 \; k_2 \; k_1 \; k_1 \; k_1 \; k_2 \; k_3 \; k_1 \; k_2 \; k_1 \; k_2 \; k_1 \; k_1 \; k_1 \; k_2 \; k_1 \; k_2 \; k_1 \; k_2 \; k_1 \; k_1 \; k_2 \; k_1 \; k_2 \; k_1 \; k_2 \; k_1 \; k_1 \; k_2 \; k_1 
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                                                                                                      k_1 \; k_2 \; k_3 \; k_6 \; k_8 \; k_{10} \; k_{11} \; T_2 \; + \; k_2 \; k_3 \; k_4 \; k_5 \; k_8 \; k_9 \; k_{12} \; T_2 \; + \; k_2 \; k_3 \; k_5 \; k_6 \; k_8 \; k_9 \; k_{12} \; T_2 \; + \; k_8 \; k_9 \; k_{10} \; K_1 \; K_2 \; K_3 \; k_9 \; k_{10} \; K_1 \; K_2 \; K_3 \; k_9 \; k_{10} \; K_1 \; K_2 \; K_3 \; k_9 \; k_{10} \; K_1 \; K_2 \; K_3 \; K_1 \; K_2 \; K_3 \; K_1 \; K_2 \; K_3 \; K_2 \; K_3 \; K_3 \; K_4 \; K_1 \; K_2 \; K_3 \; K_1 \; K_2 \; K_3 \; K_2 \; K_3 \; K_3 \; K_4 \; K_1 \; K_2 \; K_3 \; K_2 \; K_3 \; K_3 \; K_4 \; K_1 \; K_2 \; K_3 \; K_1 \; K_2 \; K_3 \; K_2 \; K_3 \; K_3 \; K_4 \; K_1 \; K_2 \; K_3 \; K_1 \; K_2 \; K_2 \; K_3 \; K_1 \; K_2 \; K_1 \; K_2 \; K_3 \; K_1 \; K_2 \; K_1 \; K_1 \; K_2 \; K_3 \; K_1 \; K_2 \; K_1 \; K_2 \; K_1 \; K_2 \; K_1 \; K_1 \; K_2 \; K_2 \; K_1 \; K_2 \; K_1 \; K_2 \; K_2 \; K_3 \; K_1 \; K_2 \; K_1 \; K_2 \; K_1 \; K_2 \; K_2 \; K_3 \; K_1 \; K_2 \; K_1 \; K_2 \; K_2 \; K_2 \; K_1 \; K_2 \; K_2 \; K_1 \; K_2 \; K_2 \; K_1 \; K_2 \; K_2 \; K_3 \; K_1 \; K_2 \; K_1 \; K_2 \; K_2 \; K_2 \; K_3 \; K_1 \; K_2 \; K_2 \; K_3 \; K_1 \; K_2 \; K_2 \; K_3 \; K_1 \; K_2 \; K_1 \; K_2 \; K_1 \; K_2 \; K_2 \; K_1 \; K_2 \; K_2 \; K_3 \; K_1 \; K_2 \; K_2 \; K_1 \; K_2 \; K_2 \; K_3 \; K_1 \; K_2 \; K_1 \; K_2 \; K_2 \; K_3 \; K_1 \; K_2 \; K_1 \; K_2 \; K_2 \; K_1 \; K_2 \; K_2 \; K_3 \; K_1 \; K_2 \; K_1 \; K_2 \; K_2 \; K_3 \; K_1 \; K_2 \; K_2 \; K_1 \; K_2 \; K_2 \; K_1 \; K_2 \; K_1 \; K_2 \; K_1 \; K_2 \; K_2 \; K_2 \; K_1 \; K_2 \; K_2 \; K_2 \; K_1 \; K_2 \; K_2 
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                                                                                                      k_2 \; k_3 \; k_6 \; k_8 \; k_9 \; k_{10} \; k_{11} \; T_3 + k_3 \; k_5 \; k_6 \; k_8 \; k_9 \; k_{10} \; k_{11} \; T_3 + k_2 \; k_4 \; k_5 \; k_7 \; k_8 \; k_9 \; k_{12} \; T_3 + k_8 \; k_9 \; k_{10} \; k_{11} \; k_9 \; k_{10} \; k_{11} \; k_9 \; k_{10} \; k_{11} \; k_{11} \; k_{11} \; k_{12} \; k_{13} \; k_{14} \; k_{15} \; k_{15} \; k_{16} \; k_{16} \; k_{18} \; k_{19} \; k_{10} \; k_{11} \; k_{10} \; k_{11} \; k_{10} \; k_{11} \; k_{11} \; k_{12} \; k_{13} \; k_{14} \; k_{15} \; k_{15} \; k_{16} \; k_{18} \; k_{19} \; k_{10} \; k_{11} \; k_{10} \; k_{11} \; k_{10} \; k_{11} \; k_{10} \; k_{11} \; k_{10} 
                                                                                                      k_2 \ k_5 \ k_6 \ k_7 \ k_8 \ k_9 \ k_{12} \ T_3 + k_2 \ k_3 \ k_5 \ k_8 \ k_9 \ k_{11} \ k_{12} \ T_3 + k_2 \ k_3 \ k_6 \ k_8 \ k_9 \ k_{11} \ k_{12} \ T_3 + k_8 \ k_9 \ k_{11} \ k_{12} \ T_3 + k_8 \ k_9 \ k_{11} \ k_{12} \ k_{12} \ k_{13} \ k_{14} \ k_{15} \ k_{15} \ k_{16} \ k_{17} \ k_{18} \ k
                                                                                                      k_3 \; k_5 \; k_6 \; k_8 \; k_9 \; k_{11} \; k_{12} \; T_3 \; + \; k_2 \; k_3 \; k_5 \; k_8 \; k_9 \; k_{11} \; k_{13} \; T_3 \; + \; k_2 \; k_3 \; k_6 \; k_8 \; k_9 \; k_{11} \; k_{13} \; T_3 \; + \; k_2 \; k_3 \; k_6 \; k_8 \; k_9 \; k_{11} \; k_{13} \; T_3 \; + \; k_2 \; k_3 \; k_6 \; k_8 \; k_9 \; k_{11} \; k_{12} \; T_3 \; + \; k_2 \; k_3 \; k_6 \; k_8 \; k_9 \; k_{11} \; k_{12} \; T_3 \; + \; k_2 \; k_3 \; k_6 \; k_8 \; k_9 \; k_{11} \; k_{12} \; K_3 \; k_8 \; k_9 \; k_{11} \; k_{12} \; K_3 \; k_8 \; k_9 \; k_{11} \; k_{12} \; K_3 \; k_8 \; k_9 \; k_{11} \; k_{12} \; K_3 \; k_8 \; k_9 \; k_{11} \; k_{12} \; K_3 \; k_8 \; k_9 \; k_{11} \; k_{12} \; K_3 \; k_8 \; k_9 \; k_{11} \; k_{12} \; K_3 \; k_8 \; k_9 \; k_{11} \; k_{12} \; K_3 \; k_8 \; k_9 \; k_{11} \; k_{12} \; K_3 \; k_8 \; k_9 \; k_{11} \; k_{12} \; K_3 \; k_8 \; k_9 \; k_{11} \; k_{12} \; K_3 \; k_8 \; k_9 \; k_{11} \; k_{12} \; K_3 \; k_8 \; k_9 \; k_{11} \; k_{12} \; K_3 \; k_8 \; k_9 \; k_{11} \; k_{12} \; K_3 \; k_8 \; k_9 \; k_{11} \; k_{12} \; K_3 \; k_8 \; k_9 \; k_{11} \; k_{12} \; K_3 \; k_8 \; k_9 \; k_{11} \; k_{12} \; K_3 \; k_8 \; k_9 \; k_{11} \; k_{12} \; K_3 \; k_8 \; k_9 \; k_{11} \; k_{12} \; K_3 \; k_8 \; k_9 \; k_{11} \; k_{12} \; K_3 \; k_9 \; k_{12} \; k_{13} \; K_9 \; k_9 \; k_{11} \; k_{12} \; K_9 \; k_{12} \; k_{13} \; k_{13} \; K_9 \; k_{13} \; k_{14} \; k_{14} \; k_{15} \; K_9 \; k_{15} \; k_{1
                                                                                                      k_3 \; k_5 \; k_6 \; k_8 \; k_9 \; k_{11} \; k_{13} \; T_3 \; + \; k_2 \; k_4 \; k_5 \; k_7 \; k_9 \; k_{12} \; k_{13} \; T_3 \; + \; k_2 \; k_5 \; k_6 \; k_7 \; k_9 \; k_{12} \; k_{13} \; T_3 \; + \; k_2 \; k_5 \; k_6 \; k_7 \; k_9 \; k_{12} \; k_{13} \; T_3 \; + \; k_2 \; k_5 \; k_6 \; k_7 \; k_9 \; k_{12} \; k_{13} \; T_3 \; + \; k_2 \; k_5 \; k_6 \; k_7 \; k_9 \; k_{12} \; k_{13} \; T_3 \; + \; k_2 \; k_5 \; k_6 \; k_7 \; k_9 \; k_{12} \; k_{13} \; K_3 \; k_9 \; k_{14} \; k_{15} \; K_4 \; k_9 \; k_{15} \; k_{1
                                                                                                      k_2 k_3 k_5 k_9 k_{11} k_{12} k_{13} T_3 + k_2 k_3 k_6 k_9 k_{11} k_{12} k_{13} T_3 + k_3 k_5 k_6 k_9 k_{11} k_{12} k_{13} T_3) x_1^2 +
                                   (k_2 \ k_3 \ k_5 \ k_8 \ k_9 \ k_{11} \ k_{13} + k_2 \ k_3 \ k_6 \ k_8 \ k_9 \ k_{11} \ k_{13} + k_3 \ k_5 \ k_6 \ k_8 \ k_9 \ k_{11} \ k_{13} + k_1 \ k_3 \ k_6 \ k_8 \ k_{10} \ k_{11} \ k_{13} + k_1 \ k_2 \ k_3 \ k_6 \ k_8 \ k_{10} \ k_{11} \ k_{13} + k_1 \ k_2 \ k_1 \ k_{10} \ k_{11} \ k_{12} + k_2 \ k_1 \ k_2 \ 
                                                                                                      k_2 \; k_4 \; k_5 \; k_7 \; k_9 \; k_{12} \; k_{13} \; + \; k_2 \; k_5 \; k_6 \; k_7 \; k_9 \; k_{12} \; k_{13} \; + \; k_2 \; k_3 \; k_5 \; k_8 \; k_9 \; k_{12} \; k_{13} \; + \; k_2 \; k_3 \; k_6 \; k_8 \; k_9 \; k_{12} \; k_{13} \; + \; k_2 \; k_3 \; k_6 \; k_8 \; k_9 \; k_{12} \; k_{13} \; + \; k_2 \; k_3 \; k_6 \; k_8 \; k_9 \; k_{12} \; k_{13} \; + \; k_2 \; k_3 \; k_6 \; k_8 \; k_9 \; k_{12} \; k_{13} \; + \; k_2 \; k_3 \; k_6 \; k_8 \; k_9 \; k_{12} \; k_{13} \; + \; k_2 \; k_3 \; k_6 \; k_8 \; k_9 \; k_{12} \; k_{13} \; + \; k_2 \; k_3 \; k_6 \; k_8 \; k_9 \; k_{12} \; k_{13} \; + \; k_2 \; k_3 \; k_6 \; k_8 \; k_9 \; k_{12} \; k_{13} \; + \; k_2 \; k_3 \; k_6 \; k_8 \; k_9 \; k_{12} \; k_{13} \; + \; k_2 \; k_3 \; k_6 \; k_8 \; k_9 \; k_{12} \; k_{13} \; + \; k_2 \; k_3 \; k_6 \; k_8 \; k_9 \; k_{12} \; k_{13} \; + \; k_2 \; k_3 \; k_6 \; k_8 \; k_9 \; k_{12} \; k_{13} \; + \; k_2 \; k_3 \; k_6 \; k_8 \; k_9 \; k_{12} \; k_{13} \; + \; k_2 \; k_3 \; k_6 \; k_8 \; k_9 \; k_{12} \; k_{13} \; + \; k_2 \; k_3 \; k_6 \; k_8 \; k_9 \; k_{12} \; k_{13} \; + \; k_2 \; k_3 \; k_6 \; k_8 \; k_9 \; k_{12} \; k_{13} \; + \; k_2 \; k_3 \; k_6 \; k_8 \; k_9 \; k_{12} \; k_{13} \; + \; k_2 \; k_3 \; k_6 \; k_8 \; k_9 \; k_{12} \; k_{13} \; + \; k_2 \; k_3 \; k_8 \; k_9 \; k_{12} \; k_{13} \; + \; k_3 \; k_8 \; k_9 \; k_{12} \; k_{13} \; + \; k_3 \; k_8 \; k_9 \; k_{12} \; k_{13} \; + \; k_3 \; k_8 \; k_9 \; k_{12} \; k_{13} \; + \; k_3 \; k_8 \; k_9 \; k_{13} \; k_{13} \; + \; k_3 \; k_8 \; k_9 \; k_{12} \; k_{13} \; + \; k_3 \; k_8 \; k_9 \; k_{12} \; k_{13} \; + \; k_3 \; k_9 \; k_{12} \; k_{13} \; + \; k_3 \; k_9 \; k_{12} \; k_{13} \; + \; k_3 \; k_9 \; k_{12} \; k_{13} \; + \; k_3 \; k_9 \; k_{12} \; k_{13} \; + \; k_3 \; k_9 \; k_{12} \; k_{13} \; + \; k_3 \; k_9 \; k_{12} \; k_{13} \; + \; k_3 \; k_9 \; k_{12} \; k_{13} \; + \; k_3 \; k_9 \; k_{12} \; k_{13} \; + \; k_9 \; k_{13} \; k_{13} \; + \; k_9 \; k_{12} \; k_{13} \; + \; k_9 \; k_{13} \; k_{13} \; + \; k_9 \; k_{12} \; k_{13} \; + \; k_9 \; k_{13} \; k_{13} \; + \; k_9 \; k_{14} \; k_{13} \; + \; k_9 \; k_{14} \; k_{14} \; + \; k_9 \; k_{14} \; k_{14} \; + \; k_
                                                                                                      k_3 \; k_5 \; k_6 \; k_8 \; k_9 \; k_{12} \; k_{13} \; + \; k_1 \; k_3 \; k_6 \; k_8 \; k_{11} \; k_{12} \; k_{13} \; + \; k_2 \; k_3 \; k_5 \; k_9 \; k_{11} \; k_{12} \; k_{13} \; + \; k_2 \; k_3 \; k_6 \; k_9 \; k_{11} \; k_{12} \; k_{13} \; + \; k_1 \; k_2 \; k_3 \; k_6 \; k_9 \; k_{11} \; k_{12} \; k_{13} \; + \; k_2 \; k_3 \; k_6 \; k_9 \; k_{11} \; k_{12} \; k_{13} \; + \; k_1 \; k
                                                                                                      k_3 \; k_5 \; k_6 \; k_9 \; k_{11} \; k_{12} \; k_{13} \; - \; k_1 \; k_3 \; k_6 \; k_8 \; k_9 \; k_{11} \; k_{13} \; T_1 \; - \; k_2 \; k_3 \; k_5 \; k_7 \; k_9 \; k_{12} \; k_{13} \; T_1 \; - \; k_8 \; k_8 \; k_9 \; k_{11} \; k_{12} \; k_{13} \; k_{14} \; k_{15} \; k_{16} \; k_{18} \; k_{18} \; k_{19} \; k_{11} \; k_{11} \; k_{12} \; k_{13} \; k_{16} \; k_{18} \; k_{19} \; k_{11} \; k_{11} \; k_{11} \; k_{12} \; k_{13} \; k_{14} \; k_{11} \; k_{11} \; k_{12} \; k_{13} \; k_{14} \; k_{15} \; k_{18} \; k_{18} \; k_{19} \; k_{11} \; k
                                                                                                      k_2 \ k_3 \ k_6 \ k_7 \ k_9 \ k_{12} \ k_{13} \ T_1 - k_3 \ k_5 \ k_6 \ k_7 \ k_9 \ k_{12} \ k_{13} \ T_1 - k_1 \ k_3 \ k_6 \ k_8 \ k_9 \ k_{12} \ k_{13} \ T_1 - k_1 \ k_2 \ k_2 \ k_1 \ k_2 \ k_1 \ k_2 \ k_1 \ k_2 \ k_1 \ k_2 \ k_2 \ k_2 \ k_1 \ k_2 
                                                                                                      k_1 \ k_3 \ k_6 \ k_9 \ k_{11} \ k_{12} \ k_{13} \ T_1 + k_1 \ k_2 \ k_3 \ k_6 \ k_8 \ k_9 \ k_{11} \ T_2 + k_2 \ k_3 \ k_4 \ k_5 \ k_7 \ k_9 \ k_{12} \ T_2 + k_8 \ k_8 \ k_9 \ k_{11} \ k_8 \ k_9 \ k_{12} \ k_{12} \ k_{12} \ k_{13} \ k_{14} \ k_{15} \ k_{15} \ k_{16} \ k_{16} \ k_{18} \ k
                                                                                                      k_2 k_3 k_5 k_6 k_7 k_9 k_{12} T_2 + k_1 k_2 k_3 k_6 k_8 k_9 k_{12} T_2 + k_1 k_2 k_3 k_6 k_9 k_{11} k_{12} T_2 +
                                                                                                      k_1 \ k_3 \ k_6 \ k_8 \ k_9 \ k_{11} \ k_{13} \ T_2 + k_2 \ k_3 \ k_5 \ k_7 \ k_9 \ k_{12} \ k_{13} \ T_2 + k_3 \ k_5 \ k_6 \ k_7 \ k_9 \ k_{12} \ k_{13} \ T_2 + k_7 \ k_9 \ k_{12} \ k_{13} \ k_{13} \ k_{13} \ k_{14} \ k_{15} 
                                                                                                      k_1 \; k_3 \; k_6 \; k_8 \; k_9 \; k_{12} \; k_{13} \; T_2 + k_1 \; k_3 \; k_6 \; k_9 \; k_{11} \; k_{12} \; k_{13} \; T_2 + k_1 \; k_3 \; k_6 \; k_8 \; k_9 \; k_{10} \; k_{11} \; T_3 + k_1 \; k_1 \; k_2 \; k_1 \; k_3 \; k_1 \; k_2 \; k_2 \; k_1 \; k_2 \; k_2 \; k_1 \; k_2 \; k_1 \; k_2 \; k_1 \; k_2 \; k_1 \; k_2 \; k_2 \; k_1 \; k_2 \; k_2 \; k_1 \; k_2 \; k_2 \; k_2 \; k_1 \; k_2 \; k_2 \; k_2 \; k_1 \; k_2 \; k_1 \; k_2 \; k
                                                                                                      k_2 \ k_3 \ k_5 \ k_7 \ k_8 \ k_9 \ k_{12} \ T_3 \ + \ k_2 \ k_3 \ k_6 \ k_7 \ k_8 \ k_9 \ k_{12} \ T_3 \ + \ k_3 \ k_5 \ k_6 \ k_7 \ k_8 \ k_9 \ k_{12} \ T_3 \ +
                                                                                                      k_1 \ k_3 \ k_6 \ k_8 \ k_9 \ k_{11} \ k_{12} \ T_3 + k_1 \ k_3 \ k_6 \ k_8 \ k_9 \ k_{11} \ k_{13} \ T_3 + k_2 \ k_3 \ k_5 \ k_7 \ k_9 \ k_{12} \ k_{13} \ T_3 + k_1 \ k_2 \ k_3 \ k_5 \ k_7 \ k_9 \ k_{12} \ k_{13} \ T_3 + k_1 \ k_2 \ k_3 \ k_5 \ k_7 \ k_9 \ k_{12} \ k_{13} \ T_3 + k_2 \ k_3 \ k_5 \ k_7 \ k_9 \ k_{12} \ k_{13} \ T_3 + k_1 \ k_2 \ k_3 \ k_5 \ k_7 \ k_9 \ k_{12} \ k_{13} \ k_7 \ k_9 \ k_{14} \ k_{15} \ k_7 \ k_9 \ k_{15} \ k_{15} \ k_7 \ k_9 \ k_{15} 
                                                                                                      k_2 k_3 k_6 k_7 k_9 k_{12} k_{13} T_3 + k_3 k_5 k_6 k_7 k_9 k_{12} k_{13} T_3 + k_1 k_3 k_6 k_9 k_{11} k_{12} k_{13} T_3) x_1^3 + k_1 k_2 k_3 k_6 k_9 k_{12} k_{13} k_{13} k_{14} k_{15} k_{1
                                (\ k_1\ k_3\ k_6\ k_8\ k_9\ k_{11}\ k_{13}\ +\ k_2\ k_3\ k_5\ k_7\ k_9\ k_{12}\ k_{13}\ +\ k_2\ k_3\ k_6\ k_7\ k_9\ k_{12}\ k_{13}\ +
                                                                                                      k_3 \ k_5 \ k_6 \ k_7 \ k_9 \ k_{12} \ k_{13} + k_1 \ k_3 \ k_6 \ k_8 \ k_9 \ k_{12} \ k_{13} + k_1 \ k_3 \ k_6 \ k_9 \ k_{11} \ k_{12} \ k_{13} -
                                                                                                      k_1 \ k_3 \ k_6 \ k_7 \ k_9 \ k_{12} \ k_{13} \ T_1 \ + \ k_1 \ k_2 \ k_3 \ k_6 \ k_7 \ k_9 \ k_{12} \ T_2 \ + \ k_1 \ k_3 \ k_6 \ k_7 \ k_9 \ k_{12} \ T_2 \ +
                                                                                                      k_1 k_3 k_6 k_7 k_8 k_9 k_{12} T_3 + k_1 k_3 k_6 k_7 k_9 k_{12} k_{13} T_3) x_1^4 + k_1 k_3 k_6 k_7 k_9 k_{12} k_{13} x_1^5
```

This is a degree 5 polynomial which presumably admits 5 positive real roots. Any real root of this polynomial leads to a steady state for the fixed rate constants and total amounts. The values of the other variables at steady states are found by plugging the value of x1 (the root of the polynomial) into the expressions in sol1, sol2, and sol3 above.

A necessary condition for 5 positive roots is that the signs of the coefficient of the polynomial (in x1) alternate.

This is a pre-check when you do the sampling: you need to impose the coefficient of x1<sup>4</sup> to be negative, the coefficient of x1<sup>3</sup> to be positive, the coefficient of x1<sup>2</sup> to be negative and the coefficient of x1 to be positive. The coefficients of x1<sup>5</sup> and the independent term always have the right sign.

## Sampling the parameters to make the term has 5 roots

```
ClearAll["Global *"];
pol = -k_2 k_4 k_5 k_8 k_{10} k_{11} k_{13} T_1 -
                        k_2 k_5 k_6 k_8 k_{10} k_{11} k_{13} T_1 - k_2 k_4 k_5 k_8 k_{11} k_{12} k_{13} T_1 - k_2 k_5 k_6 k_8 k_{11} k_{12} k_{13} T_1 +
                          (k_2 \ k_4 \ k_5 \ k_8 \ k_{10} \ k_{11} \ k_{13} + k_2 \ k_5 \ k_6 \ k_8 \ k_{10} \ k_{11} \ k_{13} + k_2 \ k_4 \ k_5 \ k_8 \ k_{11} \ k_{12} \ k_{13} +
                                                k_2 k_5 k_6 k_8 k_{11} k_{12} k_{13} - k_2 k_4 k_5 k_8 k_9 k_{11} k_{13} T_1 - k_2 k_5 k_6 k_8 k_9 k_{11} k_{13} T_1 -
                                                \mathbf{k_2} \ \mathbf{k_3} \ \mathbf{k_5} \ \mathbf{k_8} \ \mathbf{k_{10}} \ \mathbf{k_{11}} \ \mathbf{k_{13}} \ \mathbf{T_1} - \mathbf{k_2} \ \mathbf{k_3} \ \mathbf{k_6} \ \mathbf{k_8} \ \mathbf{k_{10}} \ \mathbf{k_{11}} \ \mathbf{k_{13}} \ \mathbf{T_1} - \mathbf{k_3} \ \mathbf{k_5} \ \mathbf{k_6} \ \mathbf{k_8} \ \mathbf{k_{10}} \ \mathbf{k_{11}} \ \mathbf{k_{13}} \ \mathbf{T_1} - \mathbf{k_{10}} \ \mathbf{k_{10}} \ \mathbf{k_{11}} \ \mathbf{k_{13}} \ \mathbf{T_1} - \mathbf{k_{10}} \ \mathbf{k_{10}} \ \mathbf{k_{10}} \ \mathbf{k_{11}} \ \mathbf{k_{13}} \ \mathbf{T_1} - \mathbf{k_{10}} \ \mathbf{k_{10
                                                k_2 k_4 k_5 k_8 k_9 k_{12} k_{13} T_1 - k_2 k_5 k_6 k_8 k_9 k_{12} k_{13} T_1 - k_2 k_3 k_5 k_8 k_{11} k_{12} k_{13} T_1 -
                                                k_2 k_3 k_6 k_8 k_{11} k_{12} k_{13} T_1 - k_3 k_5 k_6 k_8 k_{11} k_{12} k_{13} T_1 - k_2 k_4 k_5 k_9 k_{11} k_{12} k_{13} T_1 -
                                                k_2 k_5 k_6 k_9 k_{11} k_{12} k_{13} T_1 + k_2 k_3 k_4 k_5 k_8 k_{10} k_{11} T_2 + k_2 k_3 k_5 k_6 k_8 k_{10} k_{11} T_2 +
                                                k_2 k_3 k_4 k_5 k_8 k_{11} k_{12} \mathbf{T}_2 + k_2 k_3 k_5 k_6 k_8 k_{11} k_{12} \mathbf{T}_2 + k_2 k_3 k_5 k_8 k_{10} k_{11} k_{13} \mathbf{T}_2 +
                                                k_3 \ k_5 \ k_6 \ k_8 \ k_{10} \ k_{11} \ k_{13} \ T_2 + k_2 \ k_3 \ k_5 \ k_8 \ k_{11} \ k_{12} \ k_{13} \ T_2 + k_3 \ k_5 \ k_6 \ k_8 \ k_{11} \ k_{12} \ k_{13} \ T_2 + k_3 \ k_5 \ k_6 \ k_8 \ k_{11} \ k_{12} \ k_{13} \ T_2 + k_3 \ k_5 \ k_6 \ k_8 \ k_{11} \ k_{12} \ k_{13} \ T_2 + k_3 \ k_5 \ k_6 \ k_8 \ k_{11} \ k_{12} \ k_{13} \ T_2 + k_3 \ k_5 \ k_6 \ k_8 \ k_{11} \ k_{12} \ k_{13} \ k_{12} \ k_{13} \ k_{12} \ k_{13} \ k_{13} \ k_{14} \ k_{15} \ k_{1
                                                k_2 k_4 k_5 k_8 k_9 k_{10} k_{11} T_3 + k_2 k_5 k_6 k_8 k_9 k_{10} k_{11} T_3 + k_2 k_4 k_5 k_8 k_9 k_{11} k_{12} T_3 +
                                                k_2 k_5 k_6 k_8 k_9 k_{11} k_{12} T_3 + k_2 k_4 k_5 k_8 k_9 k_{11} k_{13} T_3 + k_2 k_5 k_6 k_8 k_9 k_{11} k_{13} T_3 +
                                                k_2 k_4 k_5 k_9 k_{11} k_{12} k_{13} T_3 + k_2 k_5 k_6 k_9 k_{11} k_{12} k_{13} T_3) x_1 +
                          (k_2 k_4 k_5 k_8 k_9 k_{11} k_{13} + k_2 k_5 k_6 k_8 k_9 k_{11} k_{13} + k_2 k_3 k_5 k_8 k_{10} k_{11} k_{13} +
                                                k_2 k_3 k_6 k_8 k_{10} k_{11} k_{13} + k_3 k_5 k_6 k_8 k_{10} k_{11} k_{13} + k_2 k_4 k_5 k_8 k_9 k_{12} k_{13} +
                                                k_2 k_5 k_6 k_8 k_9 k_{12} k_{13} + k_2 k_3 k_5 k_8 k_{11} k_{12} k_{13} + k_2 k_3 k_6 k_8 k_{11} k_{12} k_{13} +
                                                k_3 k_5 k_6 k_8 k_{11} k_{12} k_{13} + k_2 k_4 k_5 k_9 k_{11} k_{12} k_{13} + k_2 k_5 k_6 k_9 k_{11} k_{12} k_{13} -
                                                k_2 k_3 k_5 k_8 k_9 k_{11} k_{13} T_1 - k_2 k_3 k_6 k_8 k_9 k_{11} k_{13} T_1 - k_3 k_5 k_6 k_8 k_9 k_{11} k_{13} T_1 -
                                                \mathbf{k}_1 \ \mathbf{k}_3 \ \mathbf{k}_6 \ \mathbf{k}_8 \ \mathbf{k}_{10} \ \mathbf{k}_{11} \ \mathbf{k}_{13} \ \mathbf{T}_1 - \mathbf{k}_2 \ \mathbf{k}_4 \ \mathbf{k}_5 \ \mathbf{k}_7 \ \mathbf{k}_9 \ \mathbf{k}_{12} \ \mathbf{k}_{13} \ \mathbf{T}_1 - \mathbf{k}_2 \ \mathbf{k}_5 \ \mathbf{k}_6 \ \mathbf{k}_7 \ \mathbf{k}_9 \ \mathbf{k}_{12} \ \mathbf{T}_1 - \mathbf{k}_{13} \ \mathbf{T}_1 - \mathbf{k}_2 \ \mathbf{k}_{13} \ \mathbf{K}_1 \ \mathbf{K}_1 \ \mathbf{K}_2 \ \mathbf{K}_1 \ \mathbf{K}_2 \ \mathbf{K}_2 \ \mathbf{K}_3 \ \mathbf{K}_1 \ \mathbf{K}_2 \ \mathbf{K}_2 \ \mathbf{K}_3 \ \mathbf{K}_4 \ \mathbf{K}_5 \ \mathbf{K}_7 \ \mathbf{K}_9 \ \mathbf{K}_{12} \ \mathbf{K}_{13} \ \mathbf{K}_{13} \ \mathbf{K}_{13} \ \mathbf{K}_{14} \ \mathbf{K}_{15} 
                                                k_2 k_3 k_5 k_8 k_9 k_{12} k_{13} T_1 - k_2 k_3 k_6 k_8 k_9 k_{12} k_{13} T_1 - k_3 k_5 k_6 k_8 k_9 k_{12} k_{13} T_1 -
                                                k_1 \ k_3 \ k_6 \ k_8 \ k_{11} \ k_{12} \ k_{13} \ T_1 - k_2 \ k_3 \ k_5 \ k_9 \ k_{11} \ k_{12} \ k_{13} \ T_1 - k_2 \ k_3 \ k_6 \ k_9 \ k_{11} \ k_{12} \ k_{13} \ T_1 -
                                                k_3 \ k_5 \ k_6 \ k_9 \ k_{11} \ k_{12} \ k_{13} \ T_1 + k_2 \ k_3 \ k_4 \ k_5 \ k_8 \ k_9 \ k_{11} \ T_2 + k_2 \ k_3 \ k_5 \ k_6 \ k_8 \ k_9 \ k_{11} \ T_2 + k_8 \ k_9 \ k_{11} \ k_{12} \ k_{13} \ k_{14} \ k_{15} \ k
                                                k_1 \ k_2 \ k_3 \ k_6 \ k_8 \ k_{10} \ k_{11} \ T_2 + k_2 \ k_3 \ k_4 \ k_5 \ k_8 \ k_9 \ k_{12} \ T_2 + k_2 \ k_3 \ k_5 \ k_6 \ k_8 \ k_9 \ k_{12} \ T_2 + k_8 \ k_9 \ k_{12} \ k_9 \ k_{12} \ k_{12} \ k_{12} \ k_{12} \ k_{13} \ k_{14} \ k_{15} \ k_{16} 
                                                k_1 \ k_2 \ k_3 \ k_6 \ k_8 \ k_{11} \ k_{12} \ T_2 + k_2 \ k_3 \ k_4 \ k_5 \ k_9 \ k_{11} \ k_{12} \ T_2 + k_2 \ k_3 \ k_5 \ k_6 \ k_9 \ k_{11} \ k_{12} \ T_2 +
                                                k_2 k_3 k_5 k_8 k_9 k_{11} k_{13} T_2 + k_3 k_5 k_6 k_8 k_9 k_{11} k_{13} T_2 + k_1 k_3 k_6 k_8 k_{10} k_{11} k_{13} T_2 +
                                                k_2 k_3 k_5 k_8 k_9 k_{12} k_{13} T_2 + k_3 k_5 k_6 k_8 k_9 k_{12} k_{13} T_2 + k_1 k_3 k_6 k_8 k_{11} k_{12} k_{13} T_2 +
                                                k_2 k_3 k_5 k_9 k_{11} k_{12} k_{13} T_2 + k_3 k_5 k_6 k_9 k_{11} k_{12} k_{13} T_2 + k_2 k_3 k_5 k_8 k_9 k_{10} k_{11} T_3 +
                                                k_2 k_3 k_6 k_8 k_9 k_{10} k_{11} T_3 + k_3 k_5 k_6 k_8 k_9 k_{10} k_{11} T_3 + k_2 k_4 k_5 k_7 k_8 k_9 k_{12} T_3 +
                                                k_2 k_5 k_6 k_7 k_8 k_9 k_{12} T_3 + k_2 k_3 k_5 k_8 k_9 k_{11} k_{12} T_3 + k_2 k_3 k_6 k_8 k_9 k_{11} k_{12} T_3 +
                                                k_3 k_5 k_6 k_8 k_9 k_{11} k_{12} \mathbf{T}_3 + k_2 k_3 k_5 k_8 k_9 k_{11} k_{13} \mathbf{T}_3 + k_2 k_3 k_6 k_8 k_9 k_{11} k_{13} \mathbf{T}_3 +
                                                k_3 k_5 k_6 k_8 k_9 k_{11} k_{13} T_3 + k_2 k_4 k_5 k_7 k_9 k_{12} k_{13} T_3 + k_2 k_5 k_6 k_7 k_9 k_{12} k_{13} T_3 +
                                                k_2 k_3 k_5 k_9 k_{11} k_{12} k_{13} T_3 + k_2 k_3 k_6 k_9 k_{11} k_{12} k_{13} T_3 + k_3 k_5 k_6 k_9 k_{11} k_{12} k_{13} T_3) x_1^2 +
                          (k_2 k_3 k_5 k_8 k_9 k_{11} k_{13} + k_2 k_3 k_6 k_8 k_9 k_{11} k_{13} + k_3 k_5 k_6 k_8 k_9 k_{11} k_{13} +
                                                k_1 k_3 k_6 k_8 k_{10} k_{11} k_{13} + k_2 k_4 k_5 k_7 k_9 k_{12} k_{13} + k_2 k_5 k_6 k_7 k_9 k_{12} k_{13} +
                                                k_2 k_3 k_5 k_8 k_9 k_{12} k_{13} + k_2 k_3 k_6 k_8 k_9 k_{12} k_{13} + k_3 k_5 k_6 k_8 k_9 k_{12} k_{13} +
                                                k_1 \ k_3 \ k_6 \ k_8 \ k_{11} \ k_{12} \ k_{13} + k_2 \ k_3 \ k_5 \ k_9 \ k_{11} \ k_{12} \ k_{13} + k_2 \ k_3 \ k_6 \ k_9 \ k_{11} \ k_{12} \ k_{13} + k_{12} \ k_{13} + k_{12} \ k_{13} + k_{13} \ k_{14} \ k_{15} \ k_{15} + k_{15} \ k_{15} \ k_{15} \ k_{15} \ k_{15} \ k_{15} + k_{15} \ k
                                                k_3 k_5 k_6 k_9 k_{11} k_{12} k_{13} - k_1 k_3 k_6 k_8 k_9 k_{11} k_{13} T_1 - k_2 k_3 k_5 k_7 k_9 k_{12} k_{13} T_1 -
                                                k_2 k_3 k_6 k_7 k_9 k_{12} k_{13} T_1 - k_3 k_5 k_6 k_7 k_9 k_{12} k_{13} T_1 - k_1 k_3 k_6 k_8 k_9 k_{12} k_{13} T_1 -
                                                k_1 k_3 k_6 k_9 k_{11} k_{12} k_{13} T_1 + k_1 k_2 k_3 k_6 k_8 k_9 k_{11} T_2 + k_2 k_3 k_4 k_5 k_7 k_9 k_{12} T_2 +
                                                k_2 k_3 k_5 k_6 k_7 k_9 k_{12} \mathbf{T}_2 + k_1 k_2 k_3 k_6 k_8 k_9 k_{12} \mathbf{T}_2 + k_1 k_2 k_3 k_6 k_9 k_{11} k_{12} \mathbf{T}_2 +
                                                k_1 \ k_3 \ k_6 \ k_8 \ k_9 \ k_{11} \ k_{13} \ T_2 + k_2 \ k_3 \ k_5 \ k_7 \ k_9 \ k_{12} \ k_{13} \ T_2 + k_3 \ k_5 \ k_6 \ k_7 \ k_9 \ k_{12} \ k_{13} \ T_2 +
                                                k_1 \ k_3 \ k_6 \ k_8 \ k_9 \ k_{12} \ k_{13} \ T_2 + k_1 \ k_3 \ k_6 \ k_9 \ k_{11} \ k_{12} \ k_{13} \ T_2 + k_1 \ k_3 \ k_6 \ k_8 \ k_9 \ k_{10} \ k_{11} \ T_3 +
                                                k_2 k_3 k_5 k_7 k_8 k_9 k_{12} \mathbf{T}_3 + k_2 k_3 k_6 k_7 k_8 k_9 k_{12} \mathbf{T}_3 + k_3 k_5 k_6 k_7 k_8 k_9 k_{12} \mathbf{T}_3 +
                                                k_1 k_3 k_6 k_8 k_9 k_{11} k_{12} T_3 + k_1 k_3 k_6 k_8 k_9 k_{11} k_{13} T_3 + k_2 k_3 k_5 k_7 k_9 k_{12} k_{13} T_3 +
                                                k_2 k_3 k_6 k_7 k_9 k_{12} k_{13} T_3 + k_3 k_5 k_6 k_7 k_9 k_{12} k_{13} T_3 + k_1 k_3 k_6 k_9 k_{11} k_{12} k_{13} T_3) x_1^3 + k_1 k_2 k_1 x_2 k_
                          (k_1 k_3 k_6 k_8 k_9 k_{11} k_{13} + k_2 k_3 k_5 k_7 k_9 k_{12} k_{13} + k_2 k_3 k_6 k_7 k_9 k_{12} k_{13} +
                                                k_3 k_5 k_6 k_7 k_9 k_{12} k_{13} + k_1 k_3 k_6 k_8 k_9 k_{12} k_{13} + k_1 k_3 k_6 k_9 k_{11} k_{12} k_{13} -
                                                k_1 k_3 k_6 k_7 k_9 k_{12} k_{13} T_1 + k_1 k_2 k_3 k_6 k_7 k_9 k_{12} T_2 + k_1 k_3 k_6 k_7 k_9 k_{12} k_{13} T_2 +
                                                k_1 k_3 k_6 k_7 k_8 k_9 k_{12} T_3 + k_1 k_3 k_6 k_7 k_9 k_{12} k_{13} T_3) x_1^4 + k_1 k_3 k_6 k_7 k_9 k_{12} k_{13} x_1^5;
```

#### Coeffs

### Sampling

```
multistableParSets = {};
multistablePolSets = {};
multistableSolSets = {};
bistableParSets = {};
bistablePolSets = {};
bistableSolSets = {};
biCount = 0;
multiCount = 0;
termCount = 0;
Timing[
    Do [ {
                pars = Exp[-RandomVariate[
                                  ExponentialDistribution[Log[2] / (-Log[0.001])], 13]] * 1000;
                tots = Exp[-RandomVariate[ExponentialDistribution[
                                      Log[2] / (-Log[0.001])], 3]] * 1000;
                 (*pars=Exp[RandomReal[{Log[0.001],Log[1000.]},13]];*)
                 (*tots=Exp[RandomReal[{Log[0.001],Log[10.]},3]];*)
                subs = \{k_1 \rightarrow pars[[1]], k_2 \rightarrow pars[[2]], k_3 \rightarrow pars[[3]], k_4 \rightarrow pars[[4]], k_4 \rightarrow pars[[4]], k_4 \rightarrow pars[[4]], k_8 \rightarrow pars[[4]], k_9 \rightarrow pars[[4
                       k_5 \rightarrow pars[[5]], k_6 \rightarrow pars[[6]], k_7 \rightarrow pars[[7]], k_8 \rightarrow pars[[8]],
                       k_9 \rightarrow \texttt{pars} \texttt{[[9]], } k_{10} \rightarrow \texttt{pars} \texttt{[[10]], } k_{11} \rightarrow \texttt{pars} \texttt{[[11]], } k_{12} \rightarrow \texttt{pars} \texttt{[[12]], }
                       k_{13} \rightarrow pars[[13]], T_1 \rightarrow tots[[1]], T_2 \rightarrow tots[[2]], T_3 \rightarrow tots[[3]];
                 (*term4=coeff4/.subs;
                term3=coeff3/.subs;
                term2=coeff2/.subs;
                term1=coeff1/.subs;
                If[term4<0&&term3>0&&term2<0&&term1>0,{*)
                solution = NSolve[\{pol = 0 \&\& x_1 > 0\} /. subs, x_1, Reals];
                 (*termCount++;*)
                If[Length[Flatten[solution]] > 1, {
                       AppendTo[bistableParSets, Flatten[Join[pars, tots]]];
                       AppendTo[bistablePolSets, pol /. subs];
                       AppendTo[bistableSolSets, Flatten[solution]];
                       biCount++;
                       If [Length[Flatten[solution]] > 3, {
                               AppendTo[multistableParSets, Flatten[Join[pars, tots]]];
                               AppendTo[multistablePolSets, pol /. subs];
                               AppendTo[multistableSolSets, Flatten[solution]];
                               multiCount++;
                           }];
                    }];
                 (*}];*)
            }, {i, 1000000}];
]
{2294.45, Null}
Length[bistableParSets]
Length [multistableParSets]
```

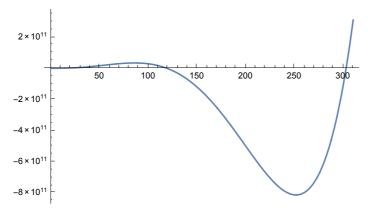
InputForm[bistableParSets[[1;; 50]]]

#### bistableSolSets[[1;; 50]]

```
\{ \{ x_1 \rightarrow 0.00275821, x_1 \rightarrow 4.47861, x_1 \rightarrow 14.0472 \}, 
   \{x_1 \rightarrow 0.000281256, x_1 \rightarrow 0.204154, x_1 \rightarrow 2.98431\}
   \{\,x_1 	o 0.0358597\,,\; x_1 	o 0.143274\,,\; x_1 	o 782.206\,\} ,
   \left\{ \, \mathbf{x}_1 
ightarrow 5.7227 	imes 10^{-7} \, , \, \, \mathbf{x}_1 
ightarrow 0.0000834245 \, , \, \, \mathbf{x}_1 
ightarrow 725.452 \, 
ight\} ,
   \left\{\mathbf{x}_{1} 
ightarrow 1.20477 	imes 10^{-6} , \mathbf{x}_{1} 
ightarrow 0.00321882 , \mathbf{x}_{1} 
ightarrow 61.7904 
ight\} ,
   \{x_1 \rightarrow 0.0263379, x_1 \rightarrow 29.1998, x_1 \rightarrow 102.667\}
    \left\{ \mathbf{x}_{1} 
ightarrow 2.23995 	imes 10^{-6} , \mathbf{x}_{1} 
ightarrow 0.390836 , \mathbf{x}_{1} 
ightarrow 4.48494 
ight\} ,
   \left\{ \, \mathbf{x}_1 
ightarrow 	extbf{1.08418} 	imes 	extbf{10}^{-14} 	extbf{,} \, \, \mathbf{x}_1 
ightarrow 	extbf{0.00668897} 	extbf{,} \, \, \mathbf{x}_1 
ightarrow 	extbf{260.404} 
ight\} 	extbf{,}
    \left\{ \mathbf{x}_{1} 
ightarrow 	extstyle{7.70987} 	imes 10^{-6} , \mathbf{x}_{1} 
ightarrow 22.668 , \mathbf{x}_{1} 
ightarrow 171.73 
ight\} ,
   \{x_1 \rightarrow 0.625486, x_1 \rightarrow 2.44407, x_1 \rightarrow 165.798\}
   \{\,x_1 \to 0\,\text{.}\,0000224073\,\text{,}\ x_1 \to 0\,\text{.}\,0000701277\,\text{,}\ x_1 \to 0\,\text{.}\,723713\,\}\,\text{,}
   \left\{\, x_1 
ightarrow 5.97223 	imes 10^{-8}\, 	ext{, } x_1 
ightarrow 0.00214513\, 	ext{, } x_1 
ightarrow 781.638 \, 
ight\} ,
   \left\{\mathbf{x}_1 
ightarrow 7.25378 	imes 10^{-8} , \mathbf{x}_1 
ightarrow 0.0000446211 , \mathbf{x}_1 
ightarrow 0.00175757 
ight\} ,
   \{x_1 \rightarrow 9.00434, x_1 \rightarrow 44.2176, x_1 \rightarrow 67.7837\}
   \{x_1 \rightarrow 0.00642932, x_1 \rightarrow 1.21019, x_1 \rightarrow 19.4252\}
   \{\,x_1 \to \text{0.217426}\,\text{,}\ x_1 \to \text{102.641}\,\text{,}\ x_1 \to \text{562.132}\,\}\,\text{,}
   \{x_1 \rightarrow 0.000118194, x_1 \rightarrow 0.872202, x_1 \rightarrow 266.415\}
   \{x_1 	o 0.00419153 , x_1 	o 0.172909 , x_1 	o 89.9307\} ,
   \{x_1 \rightarrow 3.15635, x_1 \rightarrow 4.5011, x_1 \rightarrow 39.0505\}
   \left\{ \, \mathbf{x}_1 
ightarrow \mathbf{4.66659} 	imes 10^{-6} , \, \mathbf{x}_1 
ightarrow \mathbf{0.00320223} , \, \mathbf{x}_1 
ightarrow \mathbf{408.028} \, 
ight\} ,
   \{x_1 \rightarrow 0.0924295, x_1 \rightarrow 0.37375, x_1 \rightarrow 250.431\}
   \left\{ \, \mathbf{x}_1 
ightarrow 4.52792 	imes 10^{-6} , \, \mathbf{x}_1 
ightarrow 0.00128723 , \, \mathbf{x}_1 
ightarrow 0.0329867 \, 
ight\} ,
   \{x_1 \rightarrow 0.0938524, x_1 \rightarrow 0.552778, x_1 \rightarrow 688.816\}
   \left\{ \, \mathbf{x}_1 
ightarrow 	extbf{2.66262} 	imes 10^{-7} \, 	extbf{,} \, \, \mathbf{x}_1 
ightarrow 	extbf{0.191586} \, 	extbf{,} \, \, \mathbf{x}_1 
ightarrow 17.1547 \, 
ight\} \, 	extbf{,}
   \{x_1 \rightarrow 0.0330878, x_1 \rightarrow 0.174425, x_1 \rightarrow 375.352\}
   \{\,x_1 \to \text{0.000788716}\,\text{,}\,\,x_1 \to \text{0.00627802}\,\text{,}\,\,x_1 \to \text{0.79084}\,\}\,\text{,}
   \{\,x_1 \to \text{0.0153617}\,\text{,}\ x_1 \to \text{0.061438}\,\text{,}\ x_1 \to \text{95.079}\,\}\,\text{,}
   \left\{ \, \mathbf{x}_1 
ightarrow 7 \, 	extbf{.}\, 34605 	imes 10^{-6} \, 	extbf{,} \, \, \mathbf{x}_1 
ightarrow 0 \, 	extbf{.}\, 000156223 \, 	extbf{,} \, \, \mathbf{x}_1 
ightarrow 0 \, 	extbf{.}\, 02106 \, 
ight\} \, 	extbf{,}
   \{\,x_1 \,\rightarrow\, 2\, \ldotp\, 92156\, \text{,}\ x_1 \,\rightarrow\, 4\, \ldotp\, 01718\, \text{,}\ x_1 \,\rightarrow\, 208\, \ldotp\, 318\, \} ,
   \{x_1 \rightarrow 0.107459, x_1 \rightarrow 4.09316, x_1 \rightarrow 433.066\}
   \{x_1 \rightarrow 0.0000198778, x_1 \rightarrow 0.0216719, x_1 \rightarrow 18.5379\},
  \{x_1 \rightarrow \textbf{0.00831671}, x_1 \rightarrow \textbf{1.83148}, x_1 \rightarrow \textbf{61.9437}\}
  \{x_1 \rightarrow 0.0252228, x_1 \rightarrow 2.35283, x_1 \rightarrow 181.456\}
   \{\,x_1 \to \text{0.00356962}\,\text{,}\ x_1 \to \text{0.147133}\,\text{,}\ x_1 \to 176\text{.}507\,\}\,\text{,}
   \{x_1 \rightarrow 0.000181545, x_1 \rightarrow 0.226576, x_1 \rightarrow 0.815798\},
   \{x_1 \rightarrow 0.0178888, x_1 \rightarrow 58.368, x_1 \rightarrow 431.63\}
   \{x_1 \rightarrow 2.99332, x_1 \rightarrow 65.5889, x_1 \rightarrow 131.292\}
   \{x_1 \rightarrow 0.000817703, x_1 \rightarrow 0.0253721, x_1 \rightarrow 0.67715\}
   \{\,x_1 \to \text{0.13928,}\ x_1 \to \text{15.6635,}\ x_1 \to \text{75.2222}\,\} ,
   \{\,x_1 \rightarrow 5 \mathinner{\ldotp\ldotp} 01776\,\text{,}\ x_1 \rightarrow 35 \mathinner{\ldotp\ldotp} 6684\,\text{,}\ x_1 \rightarrow 184 \mathinner{\ldotp\ldotp} 249\,\}\,\text{,}
   \left\{ \mathbf{x}_1 
ightarrow 1.6696 	imes 10^{-6} , \mathbf{x}_1 
ightarrow 0.00527399 , \mathbf{x}_1 
ightarrow 0.0431478 
ight\} ,
   \{\,x_1 \to 0.118118\,\text{, } x_1 \to 0.712658\,\text{, } x_1 \to 15.9451\,\}\,\text{,}
   \{\,x_1 \to \text{0.000273715}\,\text{,}\ x_1 \to \text{6.02954}\,\text{,}\ x_1 \to \text{470.976}\,\}\,\text{,}
   \{\,x_1 \,\to\, 0\,\text{.}\,0700232\,\text{,}\ x_1 \,\to\, 4\,\text{.}\,22994\,\text{,}\ x_1 \,\to\, 621\,\text{.}\,588\,\}\,\text{,}
   \{\,x_1 \to \text{0.0157948}\,\text{,}\ x_1 \to \text{7.99914}\,\text{,}\ x_1 \to \text{884.458}\,\}\,\text{,}
   \{x_1 \rightarrow 0.000121473, x_1 \rightarrow 0.287795, x_1 \rightarrow 227.411\},
   \{x_1 \rightarrow 0.0000983331, x_1 \rightarrow 0.903753, x_1 \rightarrow 103.688\}
   \{\,x_1 \, \to \, 0\, \ldotp \, 0103199\, \hbox{, } x_1 \, \to \, 0\, \ldotp \, 804873\, \hbox{, } x_1 \, \to \, 40\, \ldotp \, 9821\, \} ,
   \{x_1 \rightarrow 1.31798 \times 10^{-7}, x_1 \rightarrow 0.0119621, x_1 \rightarrow 6.38003\}
   \{x_1 \rightarrow 0.0705423, x_1 \rightarrow 59.1696, x_1 \rightarrow 338.86\}
Solve[bistablePolSets[[14]] = 0, x_1]
\{\{x_1 \rightarrow -0.384066 - 10.0926 \,\dot{\mathbb{1}}\}, \{x_1 \rightarrow -0.384066 + 10.0926 \,\dot{\mathbb{1}}\}, \}
```

 $\{\,x_1 \rightarrow \text{0.0105899}\,\}$  ,  $\,\{\,x_1 \rightarrow 118.909\,\}$  ,  $\,\{\,x_1 \rightarrow 302.746\,\}\,\}$ 

### ${\tt Plot[bistablePolSets[[14]], \{x_1, \, 0, \, 310\}, \, PlotRange \rightarrow Full]}$



## Test