```
> with(LinearAlgebra) :
> interface(rtablesize = 80) :
>
```

This is analysis for minimal system extend 10:

We consider the following biochemical reaction network

K + SR <-> K_SR -> K + SRp
K + ST <-> K_ST -> K + STp
P + SRp <-> P_SRp -> P + SR
P + STp <-> P_STp -> P + ST
SR <-> ST
SRp <-> STp
K_SR <-> K_ST
P_SRp <-> P_STp

The species for this reaction networks are

{SR (1), SRp (2), ST (3), STp (4), K (5), P (6), K_SR (7), K_ST (8), P_SRp (9), P_STp (10)}

There are total 20 reactions and 10 species.

Using the same method, we construct the mass-action ODE system by defining the stoichiometric matrix A, and the vector of rates k_{rs}.

$$\frac{dx}{dt} = A \cdot k_{rs}$$

The steady states is given by $A \cdot k_{rs} = 0$. Now we construct the stoichiomatric matrix:

```
> A := Matrix(20, 10) :
> A[1, 1] := -1 : A[1, 5] := -1 : A[1, 7] := 1 : A[2] := -A[1] :
> A[3, 2] := 1 : A[3, 5] := 1 : A[3, 7] := -1 :
> A[4, 3] := -1 : A[4, 5] := -1 : A[4, 8] := 1 : A[5] := -A[4] :
> A[6, 4] := 1 : A[6, 5] := 1 : A[6, 8] := -1 :
> A[7, 2] := -1 : A[7, 6] := -1 : A[7, 9] := 1 : A[8] := -A[7] :
> A[9, 1] := 1 : A[9, 6] := 1 : A[9, 9] := -1 :
> A[10, 3] := -1 : A[10, 6] := -1 : A[10, 10] := 1 : A[11] := -A[10] :
> A[12, 3] := 1 : A[12, 6] := 1 : A[12, 10] := -1 :
> A[13, 1] := -1 : A[13, 3] := 1 : A[14] := -A[13] :
> A[15, 2] := -1 : A[15, 4] := 1 : A[16] := -A[15] :
> A[17, 7] := -1 : A[17, 8] := 1 : A[18] := -A[17] :
> A[19, 9] := -1 : A[19, 10] := 1 : A[20] := -A[19] :
> A := Transpose(A)
```

A := **(1)**

$[[-1, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, -1, 1, 0, 0, 0, 0, 0, 0],$
$[0, 0, 1, 0, 0, 0, -1, 1, 0, 0, 0, 0, 0, 0, -1, 1, 0, 0, 0, 0],$
$[0, 0, 0, -1, 1, 0, 0, 0, 0, -1, 1, 1, 1, -1, 0, 0, 0, 0, 0, 0],$
$[0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1, -1, 0, 0, 0, 0],$
$[-1, 1, 1, -1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0],$
$[0, 0, 0, 0, 0, 0, -1, 1, 1, -1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0],$
$[1, -1, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 1, 0, 0],$
$[0, 0, 0, 1, -1, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, -1, 0, 0],$
$[0, 0, 0, 0, 0, 0, 1, -1, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 1],$
$[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, -1, -1, 0, 0, 0, 0, 0, 0, 1, -1]]$

>

> $ks := Vector\left(\left[k_1 \cdot x_5 \cdot x_1, k_2 \cdot x_7, k_3 \cdot x_7, k_4 \cdot x_3 \cdot x_5, k_5 \cdot x_8, k_6 \cdot x_8, k_7 \cdot x_2 \cdot x_6, k_8 \cdot x_9, k_9 \cdot x_9, k_{10} \cdot x_4 \cdot x_6, k_{11} \cdot x_{10}, k_{12} \cdot x_{10}, k_{13} \cdot x_1, k_{14} \cdot x_3, k_{15} \cdot x_2, k_{16} \cdot x_4, k_{17} \cdot x_7, k_{18} \cdot x_8, k_{19} \cdot x_9, k_{20} \cdot x_{10}\right]\right)$

(2)

$$ks := \begin{bmatrix} k_1\, x_5\, x_1 \\ k_2\, x_7 \\ k_3\, x_7 \\ k_4\, x_3\, x_5 \\ k_5\, x_8 \\ k_6\, x_8 \\ k_7\, x_2\, x_6 \\ k_8\, x_9 \\ k_9\, x_9 \\ k_{10}\, x_4\, x_6 \\ k_{11}\, x_{10} \\ k_{12}\, x_{10} \\ k_{13}\, x_1 \\ k_{14}\, x_3 \\ k_{15}\, x_2 \\ k_{16}\, x_4 \\ k_{17}\, x_7 \\ k_{18}\, x_8 \\ k_{19}\, x_9 \\ k_{20}\, x_{10} \end{bmatrix} \tag{2}$$

> $ssEqs := A.ks$

(3)

$$ssEqs := \left[\begin{array}{c} -k_1 x_1 x_5 + k_2 x_7 + k_9 x_9 - k_{13} x_1 + k_{14} x_3 \\ -k_7 x_2 x_6 + k_3 x_7 + k_8 x_9 - k_{15} x_2 + k_{16} x_4 \\ -k_4 x_3 x_5 - k_{10} x_4 x_6 + k_5 x_8 + k_{11} x_{10} + k_{12} x_{10} + k_{13} x_1 - k_{14} x_3 \\ k_6 x_8 + k_{15} x_2 - k_{16} x_4 \\ -k_1 x_1 x_5 - k_4 x_3 x_5 + k_2 x_7 + k_3 x_7 + k_5 x_8 + k_6 x_8 \\ -k_7 x_2 x_6 - k_{10} x_4 x_6 + k_8 x_9 + k_9 x_9 + k_{11} x_{10} + k_{12} x_{10} \\ k_1 x_1 x_5 - k_2 x_7 - k_3 x_7 - k_{17} x_7 + k_{18} x_8 \\ k_4 x_3 x_5 - k_5 x_8 - k_6 x_8 + k_{17} x_7 - k_{18} x_8 \\ k_7 x_2 x_6 - k_8 x_9 - k_9 x_9 - k_{19} x_9 + k_{20} x_{10} \\ k_{10} x_4 x_6 - k_{11} x_{10} - k_{12} x_{10} + k_{19} x_9 - k_{20} x_{10} \end{array}\right] \quad \textbf{(3)}$$

> $C := ReducedRowEchelonForm(Transpose(Matrix([op(NullSpace(Transpose(A)))])))$

$$C := \left[\begin{array}{cccccccccc} 1 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 \end{array}\right] \quad \textbf{(4)}$$

> $subsEqs := [ssEqs[1], ssEqs[2], ssEqs[4], ssEqs[7], ssEqs[8], ssEqs[9], ssEqs[10], x_1 + x_2 + x_3 + x_4 + x_7 + x_8 + x_9 + x_{10} - T_1, x_5 + x_7 + x_8 - T_2, x_6 + x_9 + x_{10} - T_3]$

$$subsEqs := [-k_1 x_1 x_5 + k_2 x_7 + k_9 x_9 - k_{13} x_1 + k_{14} x_3, -k_7 x_2 x_6 + k_3 x_7 + k_8 x_9 - k_{15} x_2 + k_{16} x_4, k_6 x_8 + k_{15} x_2 - k_{16} x_4, k_1 x_1 x_5 - k_2 x_7 - k_3 x_7 - k_{17} x_7 + k_{18} x_8, k_4 x_3 x_5 - k_5 x_8 - k_6 x_8 + k_{17} x_7 - k_{18} x_8, k_7 x_2 x_6 - k_8 x_9 - k_9 x_9 - k_{19} x_9 + k_{20} x_{10}, k_{10} x_4 x_6 - k_{11} x_{10} - k_{12} x_{10} + k_{19} x_9 - k_{20} x_{10}, x_1 + x_2 + x_3 + x_4 + x_7 + x_8 + x_9 + x_{10} - T_1, x_5 + x_7 + x_8 - T_2, x_6 + x_9 + x_{10} - T_3] \quad \textbf{(5)}$$

>

> #calculate the Jacobian of subsEqs

> $J := VectorCalculus[Jacobian](subsEqs, [seq(x_i, i = 1..10)])$

$$J := [[-k_1 x_5 - k_{13}, 0, k_{14}, 0, -k_1 x_1, 0, k_2, 0, k_9, 0], \quad \textbf{(6)}$$
$$[0, -k_7 x_6 - k_{15}, 0, k_{16}, 0, -k_7 x_2, k_3, 0, k_8, 0],$$
$$[0, k_{15}, 0, -k_{16}, 0, 0, 0, k_6, 0, 0],$$
$$[k_1 x_5, 0, 0, 0, k_1 x_1, 0, -k_2 - k_3 - k_{17}, k_{18}, 0, 0],$$
$$[0, 0, k_4 x_5, 0, k_4 x_3, 0, k_{17}, -k_5 - k_6 - k_{18}, 0, 0],$$
$$[0, k_7 x_6, 0, 0, 0, k_7 x_2, 0, 0, -k_8 - k_9 - k_{19}, k_{20}],$$
$$[0, 0, 0, k_{10} x_6, 0, k_{10} x_4, 0, 0, k_{19}, -k_{11} - k_{12} - k_{20}],$$

$[1, 1, 1, 1, 0, 0, 1, 1, 1, 1],$

$[0, 0, 0, 0, 1, 0, 1, 1, 0, 0],$

$[0, 0, 0, 0, 0, 1, 0, 0, 1, 1]]$

```
> Determinant(J) :
> detJ := collect(%, {seq(x[i], i = 1 ..10)},'distributed') :
> solution := solve([subsEqs[1], subsEqs[2], subsEqs[3], subsEqs[4], subsEqs[5],
      subsEqs[6], subsEqs[7]], [x[1], x[2], x[4], x[7], x[8], x[9], x[10]]) :
>
> detSubs := subs(solution[1], detJ) :
> polSubs := numer(detSubs) :
> myPol := collect(polSubs, {x[4], x[5], x[6]},'distributed')
```

[Length of output exceeds limit of 1000000] **(7)**

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>
```