Bistable motif: full signalling cycle (simplified)

Finding the condition of multistationarity

We consider the following reactions:

$K + S \leftrightharpoons KS \to K + S_p$
$K_p + S \leftrightharpoons K_p\,S \to K_p + S_p$
$P + S_p \leftrightharpoons PS_p \to P + S$
$K \leftrightharpoons K_p$
$KS \leftrightharpoons K_p\,S$

Note: the following system is also bistable, we study this simplied one to avoid the complexity of too many parameters
$K + S \to KS \to K + S_p$
$K_p + S \to K_p\,S \to K_p + S_p$
$P + S_p \to PS_p \to P + S$
$K \to K_p$
$KS \leftarrow K_p\,S$

The species of the system are:

$\{S, S_p, K, K_p, KS, K_p\,S, P, PS_p\}$

In total, there are 13 reations and 10 species.

We firstly construct the ordinary differential equations based on mass-action kinetics. Then compute the determinant of Jacobian, using the solution at critical point (steady state) to calculate the determinant. The (necessary) condition for multistationarity is to make determinant equal to zero (non-zero determinant implys injectivity).

```
A = Table[0, {8}, {8}];
A[[1]][[1]] = -1; A[[1]][[3]] = -1; A[[1]][[5]] = 1;
A[[2]][[2]] = 1; A[[2]][[3]] = 1; A[[2]][[5]] = -1;
A[[3]][[1]] = -1; A[[3]][[4]] = -1; A[[3]][[6]] = 1;
A[[4]][[2]] = 1; A[[4]][[4]] = 1; A[[4]][[6]] = -1;
A[[5]][[2]] = -1; A[[5]][[7]] = -1; A[[5]][[8]] = 1;
A[[6]][[1]] = 1; A[[6]][[7]] = 1; A[[6]][[8]] = -1;
A[[7]][[3]] = -1; A[[7]][[4]] = 1;
A[[8]][[5]] = 1; A[[8]][[6]] = -1;
stoiM = Transpose[A]
```

```
{{-1, 0, -1, 0, 0, 1, 0, 0}, {0, 1, 0, 1, -1, 0, 0, 0},
 {-1, 1, 0, 0, 0, 0, -1, 0}, {0, 0, -1, 1, 0, 0, 1, 0}, {1, -1, 0, 0, 0, 0, 0, 1},
 {0, 0, 1, -1, 0, 0, 0, -1}, {0, 0, 0, 0, -1, 1, 0, 0}, {0, 0, 0, 0, 1, -1, 0, 0}}
```

$ks = \{k_1 \times x_3 \times x_1, k_2 \times x_5, k_3 \times x_4 * x_1, k_4 \times x_6, k_5 \times x_7 \times x_2, k_6 \times x_8, k_7 * x_3, k_8 * x_6\}$

$\{k_1\, x_1\, x_3, k_2\, x_5, k_3\, x_1\, x_4, k_4\, x_6, k_5\, x_2\, x_7, k_6\, x_8, k_7\, x_3, k_8\, x_6\}$

ssEqns = stoiM.ks

$\{-k_1 x_1 x_3 - k_3 x_1 x_4 + k_6 x_8,\ k_2 x_5 + k_4 x_6 - k_5 x_2 x_7,\ -k_7 x_3 - k_1 x_1 x_3 + k_2 x_5,\ k_7 x_3 - k_3 x_1 x_4 + k_4 x_6,\ k_1 x_1 x_3 - k_2 x_5 + k_8 x_6,\ k_3 x_1 x_4 - k_4 x_6 - k_8 x_6,\ -k_5 x_2 x_7 + k_6 x_8,\ k_5 x_2 x_7 - k_6 x_8\}$

mC = RowReduce[NullSpace[A]]

{{1, 1, 0, 0, 1, 1, 0, 1}, {0, 0, 1, 1, 1, 1, 0, 0}, {0, 0, 0, 0, 0, 0, 1, 1}}

cons = {$x_1 + x_2 + x_5 + x_6 + x_8 - T_1$, $x_3 + x_4 + x_5 + x_6 - T_2$, $x_7 + x_8 - T_3$};

subsEqns = {ssEqns[[2]], ssEqns[[4]], ssEqns[[5]], ssEqns[[6]], ssEqns[[8]], cons[[1]], cons[[2]], cons[[3]]}

$\{k_2 x_5 + k_4 x_6 - k_5 x_2 x_7,\ k_7 x_3 - k_3 x_1 x_4 + k_4 x_6,\ k_1 x_1 x_3 - k_2 x_5 + k_8 x_6,\ k_3 x_1 x_4 - k_4 x_6 - k_8 x_6,\ k_5 x_2 x_7 - k_6 x_8,\ -T_1 + x_1 + x_2 + x_5 + x_6 + x_8,\ -T_2 + x_3 + x_4 + x_5 + x_6,\ -T_3 + x_7 + x_8\}$

sol1 = Solve[{ssEqns[[8]], cons[[3]]} == 0, {x_7, x_8}]

$$\left\{\left\{x_7 \to \frac{k_6 T_3}{k_6 + k_5 x_2},\ x_8 \to \frac{k_5 T_3 x_2}{k_6 + k_5 x_2}\right\}\right\}$$

sol2 = Solve[{ssEqns[[4]], ssEqns[[5]], ssEqns[[6]], cons[[2]]} == 0, {x_3, x_4, x_5, x_6}]

$$\left\{\left\{x_3 \to \frac{k_2 k_3 k_8 T_2 x_1}{k_2 k_4 k_7 + k_2 k_7 k_8 + k_2 k_3 k_7 x_1 + k_2 k_3 k_8 x_1 + k_3 k_7 k_8 x_1 + k_1 k_3 k_8 x_1^2},\right.\right.$$
$$x_4 \to \frac{k_2 k_7 (k_4 + k_8) T_2}{k_2 k_4 k_7 + k_2 k_7 k_8 + k_2 k_3 k_7 x_1 + k_2 k_3 k_8 x_1 + k_3 k_7 k_8 x_1 + k_1 k_3 k_8 x_1^2},$$
$$x_5 \to \frac{k_3 T_2 \left(k_7 k_8 x_1 + k_1 k_8 x_1^2\right)}{k_2 k_4 k_7 + k_2 k_7 k_8 + k_2 k_3 k_7 x_1 + k_2 k_3 k_8 x_1 + k_3 k_7 k_8 x_1 + k_1 k_3 k_8 x_1^2},$$
$$\left.\left.x_6 \to \frac{k_2 k_3 k_7 T_2 x_1}{k_2 k_4 k_7 + k_2 k_7 k_8 + k_2 k_3 k_7 x_1 + k_2 k_3 k_8 x_1 + k_3 k_7 k_8 x_1 + k_1 k_3 k_8 x_1^2}\right\}\right\}$$

sol3 = x_2 /. Solve[{ssEqns[[2]]} == 0, {x_2}]

$$\left\{\frac{k_2 x_5 + k_4 x_6}{k_5 x_7}\right\}$$

sol4 = Solve[{x_2 == sol3[[1]]} /. Join[sol1[[1]], sol2[[1]]], {x_2}]

$$\left\{\left\{x_2 \to \left(\frac{k_2 k_3 k_4 k_7 T_2 x_1}{k_2 k_4 k_7 + k_2 k_7 k_8 + k_2 k_3 k_7 x_1 + k_2 k_3 k_8 x_1 + k_3 k_7 k_8 x_1 + k_1 k_3 k_8 x_1^2} + \frac{k_2 k_3 T_2 \left(k_7 k_8 x_1 + k_1 k_8 x_1^2\right)}{k_2 k_4 k_7 + k_2 k_7 k_8 + k_2 k_3 k_7 x_1 + k_2 k_3 k_8 x_1 + k_3 k_7 k_8 x_1 + k_1 k_3 k_8 x_1^2}\right) \Big/ \right.\right.$$
$$\left.\left.\left(k_5 T_3 \left(1 - \frac{1}{k_6 T_3}\left(\frac{k_2 k_3 k_4 k_7 T_2 x_1}{k_2 k_4 k_7 + k_2 k_7 k_8 + k_2 k_3 k_7 x_1 + k_2 k_3 k_8 x_1 + k_3 k_7 k_8 x_1 + k_1 k_3 k_8 x_1^2} + \frac{k_2 k_3 T_2 \left(k_7 k_8 x_1 + k_1 k_8 x_1^2\right)}{k_2 k_4 k_7 + k_2 k_7 k_8 + k_2 k_3 k_7 x_1 + k_2 k_3 k_8 x_1 + k_3 k_7 k_8 x_1 + k_1 k_3 k_8 x_1^2}\right)\right)\right)\right\}\right\}$$

sol5 = Solve[x_8 == {x_8 /. sol1[[1]]} /. sol4[[1]], x_8]

$$\left\{\left\{x_8 \to \left(\frac{k_2\,k_3\,k_4\,k_7\,T_2\,x_1}{k_2\,k_4\,k_7 + k_2\,k_7\,k_8 + k_2\,k_3\,k_7\,x_1 + k_2\,k_3\,k_8\,x_1 + k_3\,k_7\,k_8\,x_1 + k_1\,k_3\,k_8\,x_1^2} + \frac{k_2\,k_3\,T_2\,\left(k_7\,k_8\,x_1 + k_1\,k_8\,x_1^2\right)}{k_2\,k_4\,k_7 + k_2\,k_7\,k_8 + k_2\,k_3\,k_7\,x_1 + k_2\,k_3\,k_8\,x_1 + k_3\,k_7\,k_8\,x_1 + k_1\,k_3\,k_8\,x_1^2}\right) \Big/ \left(\left(1 - \frac{1}{k_6\,T_3}\left(\frac{k_2\,k_3\,k_4\,k_7\,T_2\,x_1}{k_2\,k_4\,k_7 + k_2\,k_7\,k_8 + k_2\,k_3\,k_7\,x_1 + k_2\,k_3\,k_8\,x_1 + k_3\,k_7\,k_8\,x_1 + k_1\,k_3\,k_8\,x_1^2} + \frac{k_2\,k_3\,T_2\,\left(k_7\,k_8\,x_1 + k_1\,k_8\,x_1^2\right)}{k_2\,k_4\,k_7 + k_2\,k_7\,k_8 + k_2\,k_3\,k_7\,x_1 + k_2\,k_3\,k_8\,x_1 + k_3\,k_7\,k_8\,x_1 + k_1\,k_3\,k_8\,x_1^2}\right)\right) \left(k_6 + \left(\frac{k_2\,k_3\,k_4\,k_7\,T_2\,x_1}{k_2\,k_4\,k_7 + k_2\,k_7\,k_8 + k_2\,k_3\,k_7\,x_1 + k_2\,k_3\,k_8\,x_1 + k_3\,k_7\,k_8\,x_1 + k_1\,k_3\,k_8\,x_1^2} + \frac{k_2\,k_3\,T_2\,\left(k_7\,k_8\,x_1 + k_1\,k_8\,x_1^2\right)}{k_2\,k_4\,k_7 + k_2\,k_7\,k_8 + k_2\,k_3\,k_7\,x_1 + k_2\,k_3\,k_8\,x_1 + k_3\,k_7\,k_8\,x_1 + k_1\,k_3\,k_8\,x_1^2}\right) \Big/ \left(T_3\,\left(1 - \frac{1}{k_6\,T_3}\left(\frac{k_2\,k_3\,k_4\,k_7\,T_2\,x_1}{k_2\,k_4\,k_7 + k_2\,k_7\,k_8 + k_2\,k_3\,k_7\,x_1 + k_2\,k_3\,k_8\,x_1 + k_3\,k_7\,k_8\,x_1 + k_1\,k_3\,k_8\,x_1^2} + \frac{k_2\,k_3\,T_2\,\left(k_7\,k_8\,x_1 + k_1\,k_8\,x_1^2\right)}{k_2\,k_4\,k_7 + k_2\,k_7\,k_8 + k_2\,k_3\,k_7\,x_1 + k_2\,k_3\,k_8\,x_1 + k_3\,k_7\,k_8\,x_1 + k_1\,k_3\,k_8\,x_1^2}\right)\right)\right)\right)\right\}\right\}$$

term = Simplify[cons[[1]] /. Join[sol5[[1]], sol2[[1]], sol4[[1]]]]

$$-T_1 + x_1 + \frac{k_2\,k_3\,k_7\,T_2\,x_1}{k_3\,k_8\,x_1\,(k_7 + k_1\,x_1) + k_2\,(k_4\,k_7 + k_3\,k_8\,x_1 + k_7\,(k_8 + k_3\,x_1))} + \frac{k_3\,k_8\,T_2\,x_1\,(k_7 + k_1\,x_1)}{k_3\,k_8\,x_1\,(k_7 + k_1\,x_1) + k_2\,(k_4\,k_7 + k_3\,k_8\,x_1 + k_7\,(k_8 + k_3\,x_1))} + \frac{k_2\,k_3\,T_2\,x_1\,(k_4\,k_7 + k_8\,(k_7 + k_1\,x_1))}{k_6\,(k_3\,k_8\,x_1\,(k_7 + k_1\,x_1) + k_2\,(k_4\,k_7 + k_3\,k_8\,x_1 + k_7\,(k_8 + k_3\,x_1)))} + (k_2\,k_3\,k_6\,T_2\,x_1\,(k_4\,k_7 + k_8\,(k_7 + k_1\,x_1)))\,/\,(k_5\,(k_3\,k_6\,k_8\,T_3\,x_1\,(k_7 + k_1\,x_1) + k_2\,(-k_3\,k_8\,T_2\,x_1\,(k_7 + k_1\,x_1) + k_4\,k_7\,(k_6\,T_3 - k_3\,T_2\,x_1) + k_6\,T_3\,(k_3\,k_8\,x_1 + k_7\,(k_8 + k_3\,x_1)))))$$

Together[term]

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polynomial = Collect[Numerator[Together[term]], x1]
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$$
\begin{aligned}
& k_2^2 k_4^2 k_5 k_6^2 k_7^2 T_1 T_3 + 2 k_2^2 k_4 k_5 k_6^2 k_7^2 k_8 T_1 T_3 + k_2^2 k_5 k_6^2 k_7^2 k_8^2 T_1 T_3 + \\
& \quad \big(-k_2^2 k_3 k_4^2 k_6^2 k_7^2 T_2 - 2 k_2^2 k_3 k_4 k_6^2 k_7^2 k_8 T_2 - k_2^2 k_3 k_6^2 k_7^2 k_8^2 T_2 - \\
& \qquad k_2^2 k_3 k_4^2 k_5 k_6 k_7^2 T_1 T_2 - 2 k_2^2 k_3 k_4 k_5 k_6 k_7^2 k_8 T_1 T_2 - k_2^2 k_3 k_5 k_6 k_7^2 k_8^2 T_1 T_2 - \\
& \qquad k_2^2 k_4^2 k_5 k_6^2 k_7^2 T_3 - 2 k_2^2 k_4 k_5 k_6^2 k_7^2 k_8 T_3 - k_2^2 k_5 k_6^2 k_7^2 k_8^2 T_3 + 2 k_2^2 k_3 k_4 k_5 k_6^2 k_7^2 T_1 T_3 + \\
& \qquad 2 k_2^2 k_3 k_4 k_5 k_6^2 k_7 k_8 T_1 T_3 + 2 k_2^2 k_3 k_5 k_6^2 k_7^2 k_8 T_1 T_3 + 2 k_2 k_3 k_4 k_5 k_6^2 k_7^2 k_8 T_1 T_3 + \\
& \qquad 2 k_2^2 k_3 k_5 k_6^2 k_7 k_8^2 T_1 T_3 + 2 k_2 k_3 k_5 k_6^2 k_7^2 k_8^2 T_1 T_3 - k_2^2 k_3 k_4^2 k_5 k_6 k_7^2 T_2 T_3 - \\
& \qquad k_2^2 k_3 k_4 k_5 k_6^2 k_7^2 T_2 T_3 - 2 k_2^2 k_3 k_4 k_5 k_6 k_7^2 k_8 T_2 T_3 - k_2^2 k_3 k_5 k_6^2 k_7^2 k_8 T_2 T_3 - \\
& \qquad k_2 k_3 k_4 k_5 k_6^2 k_7^2 k_8 T_2 T_3 - k_2^2 k_3 k_5 k_6 k_7^2 k_8^2 T_2 T_3 - k_2 k_3 k_5 k_6^2 k_7^2 k_8^2 T_2 T_3\big)\; x_1 + \\
& \quad \big(k_2^2 k_3 k_4^2 k_5 k_6 k_7^2 T_2 - k_2^2 k_3^2 k_4 k_6^2 k_7^2 T_2 - k_1 k_2^2 k_3 k_4 k_6^2 k_7 k_8 T_2 - k_2^2 k_3^2 k_4 k_6^2 k_7 k_8 T_2 + \\
& \qquad 2 k_2^2 k_3 k_4 k_5 k_6 k_7^2 k_8 T_2 - k_2^2 k_3^2 k_6^2 k_7^2 k_8 T_2 - k_2 k_3^2 k_4 k_6^2 k_7^2 k_8 T_2 - \\
& \qquad k_1 k_2^2 k_3 k_6^2 k_7 k_8^2 T_2 - k_2^2 k_3^2 k_6^2 k_7 k_8^2 T_2 + k_2^2 k_3 k_5 k_6 k_7^2 k_8^2 T_2 - k_2 k_3^2 k_6^2 k_7^2 k_8^2 T_2 - \\
& \qquad k_2^2 k_3^2 k_4 k_5 k_6 k_7^2 T_1 T_2 - k_1 k_2^2 k_3 k_4 k_5 k_6 k_7 k_8 T_1 T_2 - k_2^2 k_3^2 k_4 k_5 k_6 k_7 k_8 T_1 T_2 - \\
& \qquad k_2^2 k_3^2 k_5 k_6 k_7^2 k_8 T_1 T_2 - k_2 k_3^2 k_4 k_5 k_6 k_7^2 k_8 T_1 T_2 - k_1 k_2^2 k_3 k_5 k_6 k_7 k_8^2 T_1 T_2 - \\
& \qquad k_2^2 k_3^2 k_5 k_6 k_7 k_8^2 T_1 T_2 - k_2 k_3^2 k_5 k_6 k_7^2 k_8^2 T_1 T_2 + k_2^2 k_3^2 k_4^2 k_5 k_7^2 T_2^2 + k_2^2 k_3^2 k_4 k_5 k_6 k_7^2 T_2^2 + \\
& \qquad 2 k_2^2 k_3^2 k_4 k_5 k_7^2 k_8 T_2^2 + k_2^2 k_3^2 k_5 k_6 k_7^2 k_8 T_2^2 + k_2 k_3^2 k_4 k_5 k_6 k_7^2 k_8 T_2^2 + k_2^2 k_3^2 k_5 k_7^2 k_8^2 T_2^2 + \\
& \qquad k_2 k_3^2 k_5 k_6 k_7^2 k_8^2 T_2^2 - 2 k_2^2 k_3 k_4 k_5 k_6^2 k_7^2 T_3 - 2 k_2^2 k_3 k_4 k_5 k_6^2 k_7 k_8 T_3 - \\
& \qquad 2 k_2^2 k_3 k_5 k_6^2 k_7^2 k_8 T_3 - 2 k_2 k_3 k_4 k_5 k_6^2 k_7^2 k_8 T_3 - 2 k_2^2 k_3 k_5 k_6^2 k_7 k_8^2 T_3 - \\
& \qquad 2 k_2 k_3 k_5 k_6^2 k_7^2 k_8^2 T_3 + k_2^2 k_3^2 k_5 k_6^2 k_7^2 T_1 T_3 + 2 k_2^2 k_3^2 k_5 k_6^2 k_7 k_8 T_1 T_3 + \\
& \qquad 2 k_1 k_2 k_3 k_4 k_5 k_6^2 k_7 k_8 T_1 T_3 + 2 k_2 k_3^2 k_5 k_6^2 k_7^2 k_8 T_1 T_3 + k_2^2 k_3^2 k_5 k_6^2 k_8^2 T_1 T_3 + \\
& \qquad 2 k_1 k_2 k_3 k_5 k_6^2 k_7 k_8^2 T_1 T_3 + 2 k_2 k_3^2 k_5 k_6^2 k_7 k_8^2 T_1 T_3 + k_3^2 k_5 k_6^2 k_7^2 k_8^2 T_1 T_3 - \\
& \qquad k_2^2 k_3^2 k_4 k_5 k_6 k_7^2 T_2 T_3 - k_2^2 k_3^2 k_5 k_6^2 k_7^2 T_2 T_3 - k_1 k_2^2 k_3 k_4 k_5 k_6 k_7 k_8 T_2 T_3 - \\
& \qquad k_2^2 k_3^2 k_4 k_5 k_6 k_7 k_8 T_2 T_3 - k_2^2 k_3^2 k_5 k_6^2 k_7 k_8 T_2 T_3 - k_1 k_2 k_3 k_4 k_5 k_6^2 k_7 k_8 T_2 T_3 - \\
& \qquad k_2^2 k_3^2 k_5 k_6 k_7^2 k_8 T_2 T_3 - k_2 k_3^2 k_4 k_5 k_6 k_7^2 k_8 T_2 T_3 - 2 k_2 k_3^2 k_5 k_6^2 k_7^2 k_8 T_2 T_3 - \\
& \qquad k_1 k_2^2 k_3 k_5 k_6 k_7 k_8^2 T_2 T_3 - k_2^2 k_3^2 k_5 k_6 k_7 k_8^2 T_2 T_3 - k_1 k_2 k_3 k_5 k_6^2 k_7 k_8^2 T_2 T_3 - \\
& \qquad k_2 k_3^2 k_5 k_6^2 k_7 k_8^2 T_2 T_3 - k_2 k_3^2 k_5 k_6 k_7^2 k_8^2 T_2 T_3 - k_3^2 k_5 k_6^2 k_7^2 k_8^2 T_2 T_3\big)\; x_1^2 + \\
& \quad \big(k_2^2 k_3^2 k_4 k_5 k_6 k_7^2 T_2 + k_1 k_2^2 k_3 k_4 k_5 k_6 k_7 k_8 T_2 + k_2^2 k_3^2 k_4 k_5 k_6 k_7 k_8 T_2 - k_1 k_2^2 k_3^2 k_6^2 k_7 k_8 T_2 - \\
& \qquad k_1 k_2 k_3^2 k_4 k_6^2 k_7 k_8 T_2 + k_2^2 k_3^2 k_5 k_6 k_7^2 k_8 T_2 + k_2 k_3^2 k_4 k_5 k_6 k_7^2 k_8 T_2 - k_1 k_2^2 k_3^2 k_6^2 k_8^2 T_2 + \\
& \qquad k_1 k_2^2 k_3 k_5 k_6 k_7 k_8^2 T_2 + k_2^2 k_3^2 k_5 k_6 k_7 k_8^2 T_2 - 2 k_1 k_2 k_3^2 k_6^2 k_7 k_8^2 T_2 + k_2 k_3^2 k_5 k_6 k_7^2 k_8^2 T_2 - \\
& \qquad k_1 k_2^2 k_3^2 k_5 k_6 k_7 k_8 T_1 T_2 - k_1 k_2 k_3^2 k_4 k_5 k_6 k_7 k_8 T_1 T_2 - k_1 k_2^2 k_3^2 k_5 k_6 k_8^2 T_1 T_2 - \\
& \qquad 2 k_1 k_2 k_3^2 k_5 k_6 k_7 k_8^2 T_1 T_2 + 2 k_1 k_2^2 k_3^2 k_4 k_5 k_7 k_8 T_2^2 + k_1 k_2^2 k_3^2 k_5 k_6 k_7 k_8 T_2^2 + \\
& \qquad k_1 k_2 k_3^2 k_4 k_5 k_6 k_7 k_8 T_2^2 + 2 k_1 k_2^2 k_3^2 k_5 k_7 k_8^2 T_2^2 + 2 k_1 k_2 k_3^2 k_5 k_6 k_7 k_8^2 T_2^2 - \\
& \qquad k_2^2 k_3^2 k_5 k_6^2 k_7^2 T_3 - 2 k_2^2 k_3^2 k_5 k_6^2 k_7 k_8 T_3 - 2 k_1 k_2 k_3 k_4 k_5 k_6^2 k_7 k_8 T_3 - \\
& \qquad 2 k_2 k_3^2 k_5 k_6^2 k_7^2 k_8 T_3 - k_2^2 k_3^2 k_5 k_6^2 k_8^2 T_3 - 2 k_1 k_2 k_3 k_5 k_6^2 k_7 k_8^2 T_3 - \\
& \qquad 2 k_2 k_3^2 k_5 k_6^2 k_7 k_8^2 T_3 - k_3^2 k_5 k_6^2 k_7^2 k_8^2 T_3 + 2 k_1 k_2 k_3^2 k_5 k_6^2 k_7 k_8 T_1 T_3 + \\
& \qquad 2 k_1 k_2 k_3^2 k_5 k_6^2 k_8^2 T_1 T_3 + 2 k_1 k_3^2 k_5 k_6^2 k_7 k_8^2 T_1 T_3 - k_1 k_2^2 k_3^2 k_5 k_6 k_7 k_8 T_2 T_3 - \\
& \qquad k_1 k_2 k_3^2 k_4 k_5 k_6 k_7 k_8 T_2 T_3 - 2 k_1 k_2 k_3^2 k_5 k_6^2 k_7 k_8 T_2 T_3 - k_1 k_2^2 k_3^2 k_5 k_6 k_8^2 T_2 T_3 - \\
& \qquad k_1 k_2 k_3^2 k_5 k_6^2 k_8^2 T_2 T_3 - 2 k_1 k_2 k_3^2 k_5 k_6 k_7 k_8^2 T_2 T_3 - 2 k_1 k_3^2 k_5 k_6^2 k_7 k_8^2 T_2 T_3\big)\; x_1^3 + \\
& \quad \big(k_1 k_2^2 k_3^2 k_5 k_6 k_7 k_8 T_2 + k_1 k_2 k_3^2 k_4 k_5 k_6 k_7 k_8 T_2 + k_1 k_2^2 k_3^2 k_5 k_6 k_8^2 T_2 - k_1^2 k_2 k_3^2 k_6^2 k_8^2 T_2 + \\
& \qquad 2 k_1 k_2 k_3^2 k_5 k_6 k_7 k_8^2 T_2 - k_1^2 k_2 k_3^2 k_5 k_6 k_8^2 T_1 T_2 + k_1^2 k_2^2 k_3^2 k_5 k_8^2 T_2^2 + k_1^2 k_2 k_3^2 k_5 k_6 k_8^2 T_2^2 - \\
& \qquad 2 k_1 k_2 k_3^2 k_5 k_6^2 k_7 k_8 T_3 - 2 k_1 k_2 k_3^2 k_5 k_6^2 k_8^2 T_3 - 2 k_1 k_3^2 k_5 k_6^2 k_7 k_8^2 T_3 + \\
& \qquad k_1^2 k_3^2 k_5 k_6^2 k_8^2 T_1 T_3 - k_1^2 k_2 k_3^2 k_5 k_6 k_8^2 T_2 T_3 - k_1^2 k_3^2 k_5 k_6^2 k_8^2 T_2 T_3\big)\; x_1^4 + \\
& \quad \big(k_1^2 k_2 k_3^2 k_5 k_6 k_8^2 T_2 - k_1^2 k_3^2 k_5 k_6^2 k_8^2 T_3\big)\; x_1^5
\end{aligned}
$$

This is also a degree 5 polynomial. Now the polynomial is simple (comparing to the original model) enough for sampling the parameters to check if the polynomial has 5 roots of x_1.

Sampling the parameter to make the system display multistable dynamics

(Test) In[1]:=

```
ClearAll["Global`*"];
```

$$
\begin{aligned}
\mathrm{pol} = {} & k_2^2 k_4^2 k_5 k_6^2 k_7^2 T_1 T_3 + 2 k_2^2 k_4 k_5 k_6^2 k_7^2 k_8 T_1 T_3 + k_2^2 k_5 k_6^2 k_7^2 k_8^2 T_1 T_3 + \\
& \big(-k_2^2 k_3 k_4^2 k_6^2 k_7^2 T_2 - 2 k_2^2 k_3 k_4 k_6^2 k_7^2 k_8 T_2 - k_2^2 k_3 k_6^2 k_7^2 k_8^2 T_2 - k_2^2 k_3 k_4^2 k_5 k_6 k_7^2 T_1 T_2 - \\
& \quad 2 k_2^2 k_3 k_4 k_5 k_6 k_7^2 k_8 T_1 T_2 - k_2^2 k_3 k_5 k_6 k_7^2 k_8^2 T_1 T_2 - k_2^2 k_4^2 k_5 k_6^2 k_7^2 T_3 - \\
& \quad 2 k_2^2 k_4 k_5 k_6^2 k_7^2 k_8 T_3 - k_2^2 k_5 k_6^2 k_7^2 k_8^2 T_3 + 2 k_2^2 k_3 k_4 k_5 k_6^2 k_7^2 T_1 T_3 + \\
& \quad 2 k_2^2 k_3 k_4 k_5 k_6^2 k_7 k_8 T_1 T_3 + 2 k_2^2 k_3 k_5 k_6^2 k_7^2 k_8 T_1 T_3 + 2 k_2 k_3 k_4 k_5 k_6^2 k_7^2 k_8 T_1 T_3 + \\
& \quad 2 k_2^2 k_3 k_5 k_6^2 k_7 k_8^2 T_1 T_3 + 2 k_2 k_3 k_5 k_6^2 k_7^2 k_8^2 T_1 T_3 - k_2^2 k_3 k_4^2 k_5 k_6 k_7^2 T_2 T_3 - \\
& \quad k_2^2 k_3 k_4 k_5 k_6^2 k_7^2 T_2 T_3 - 2 k_2^2 k_3 k_4 k_5 k_6 k_7^2 k_8 T_2 T_3 - k_2^2 k_3 k_5 k_6^2 k_7^2 k_8 T_2 T_3 - \\
& \quad k_2 k_3 k_4 k_5 k_6^2 k_7^2 k_8 T_2 T_3 - k_2^2 k_3 k_5 k_6 k_7^2 k_8^2 T_2 T_3 - k_2 k_3 k_5 k_6^2 k_7^2 k_8^2 T_2 T_3\big) x_1 + \\
& \big(k_2^2 k_3 k_4^2 k_5 k_6 k_7^2 T_2 - k_2^2 k_3^2 k_4 k_6^2 k_7^2 T_2 - k_1 k_2^2 k_3 k_4 k_6^2 k_7 k_8 T_2 - k_2^2 k_3^2 k_4 k_6^2 k_7 k_8 T_2 + \\
& \quad 2 k_2^2 k_3 k_4 k_5 k_6 k_7^2 k_8 T_2 - k_2^2 k_3^2 k_6^2 k_7^2 k_8 T_2 - k_2 k_3^2 k_4 k_6^2 k_7^2 k_8 T_2 - \\
& \quad k_1 k_2^2 k_3 k_6^2 k_7 k_8^2 T_2 - k_2^2 k_3^2 k_6^2 k_7 k_8^2 T_2 + k_2^2 k_3 k_5 k_6 k_7^2 k_8^2 T_2 - k_2 k_3^2 k_6^2 k_7^2 k_8^2 T_2 - \\
& \quad k_2^2 k_3^2 k_4 k_5 k_6 k_7^2 T_1 T_2 - k_1 k_2^2 k_3 k_4 k_5 k_6 k_7 k_8 T_1 T_2 - k_2^2 k_3^2 k_4 k_5 k_6 k_7 k_8 T_1 T_2 - \\
& \quad k_2^2 k_3^2 k_5 k_6 k_7^2 k_8 T_1 T_2 - k_2 k_3^2 k_4 k_5 k_6 k_7^2 k_8 T_1 T_2 - k_1 k_2^2 k_3 k_5 k_6 k_7 k_8^2 T_1 T_2 - \\
& \quad k_2^2 k_3^2 k_5 k_6 k_7 k_8^2 T_1 T_2 - k_2 k_3^2 k_5 k_6 k_7^2 k_8^2 T_1 T_2 + k_2^2 k_3^2 k_4^2 k_5 k_7^2 T_2^2 + k_2^2 k_3^2 k_4 k_5 k_6 k_7^2 T_2^2 + \\
& \quad 2 k_2^2 k_3^2 k_4 k_5 k_7^2 k_8 T_2^2 + k_2^2 k_3^2 k_5 k_6 k_7^2 k_8 T_2^2 + k_2 k_3^2 k_4 k_5 k_6 k_7^2 k_8 T_2^2 + k_2^2 k_3^2 k_5 k_7^2 k_8^2 T_2^2 + \\
& \quad k_2 k_3^2 k_5 k_6 k_7^2 k_8^2 T_2^2 - 2 k_2^2 k_3 k_4 k_5 k_6^2 k_7^2 T_3 - 2 k_2^2 k_3 k_4 k_5 k_6^2 k_7 k_8 T_3 - \\
& \quad 2 k_2^2 k_3 k_5 k_6^2 k_7^2 k_8 T_3 - 2 k_2 k_3 k_4 k_5 k_6^2 k_7^2 k_8 T_3 - 2 k_2^2 k_3 k_5 k_6^2 k_7 k_8^2 T_3 - \\
& \quad 2 k_2 k_3 k_5 k_6^2 k_7^2 k_8^2 T_3 + k_2^2 k_3^2 k_5 k_6^2 k_7^2 T_1 T_3 + 2 k_2^2 k_3^2 k_5 k_6^2 k_7 k_8 T_1 T_3 + \\
& \quad 2 k_1 k_2 k_3 k_4 k_5 k_6^2 k_7 k_8 T_1 T_3 + 2 k_2 k_3^2 k_5 k_6^2 k_7^2 k_8 T_1 T_3 + k_2^2 k_3^2 k_5 k_6^2 k_8^2 T_1 T_3 + \\
& \quad 2 k_1 k_2 k_3 k_5 k_6^2 k_7 k_8^2 T_1 T_3 + 2 k_2 k_3^2 k_5 k_6^2 k_7 k_8^2 T_1 T_3 + k_3^2 k_5 k_6^2 k_7^2 k_8^2 T_1 T_3 - \\
& \quad k_2^2 k_3^2 k_4 k_5 k_6 k_7^2 T_2 T_3 - k_2^2 k_3^2 k_5 k_6^2 k_7^2 T_2 T_3 - k_1 k_2^2 k_3 k_4 k_5 k_6 k_7 k_8 T_2 T_3 - \\
& \quad k_2^2 k_3^2 k_4 k_5 k_6 k_7 k_8 T_2 T_3 - k_2^2 k_3^2 k_5 k_6^2 k_7 k_8 T_2 T_3 - k_1 k_2 k_3 k_4 k_5 k_6^2 k_7 k_8 T_2 T_3 - \\
& \quad k_2^2 k_3^2 k_5 k_6 k_7^2 k_8 T_2 T_3 - k_2 k_3^2 k_4 k_5 k_6 k_7^2 k_8 T_2 T_3 - 2 k_2 k_3^2 k_5 k_6^2 k_7^2 k_8 T_2 T_3 - \\
& \quad k_1 k_2^2 k_3 k_5 k_6 k_7 k_8^2 T_2 T_3 - k_2^2 k_3^2 k_5 k_6 k_7 k_8^2 T_2 T_3 - k_1 k_2 k_3 k_5 k_6^2 k_7 k_8^2 T_2 T_3 - \\
& \quad k_2 k_3^2 k_5 k_6^2 k_7 k_8^2 T_2 T_3 - k_2 k_3^2 k_5 k_6 k_7^2 k_8^2 T_2 T_3 - k_3^2 k_5 k_6^2 k_7^2 k_8^2 T_2 T_3\big) x_1^2 + \\
& \big(k_2^2 k_3^2 k_4 k_5 k_6 k_7^2 T_2 + k_1 k_2^2 k_3 k_4 k_5 k_6 k_7 k_8 T_2 + k_2^2 k_3^2 k_4 k_5 k_6 k_7 k_8 T_2 - k_1 k_2^2 k_3^2 k_6^2 k_7 k_8 T_2 - \\
& \quad k_1 k_2 k_3^2 k_4 k_6^2 k_7 k_8 T_2 + k_2^2 k_3^2 k_5 k_6 k_7^2 k_8 T_2 + k_2 k_3^2 k_4 k_5 k_6 k_7^2 k_8 T_2 - k_1 k_2^2 k_3^2 k_6^2 k_8^2 T_2 + \\
& \quad k_1 k_2^2 k_3 k_5 k_6 k_7 k_8^2 T_2 + k_2^2 k_3^2 k_5 k_6 k_7 k_8^2 T_2 - 2 k_1 k_2 k_3^2 k_6^2 k_7 k_8^2 T_2 + k_2 k_3^2 k_5 k_6 k_7^2 k_8^2 T_2 - \\
& \quad k_1 k_2^2 k_3^2 k_5 k_6 k_7 k_8 T_1 T_2 - k_1 k_2 k_3^2 k_4 k_5 k_6 k_7 k_8 T_1 T_2 - k_1 k_2^2 k_3^2 k_5 k_6 k_8^2 T_1 T_2 - \\
& \quad 2 k_1 k_2 k_3^2 k_5 k_6 k_7 k_8^2 T_1 T_2 + 2 k_1 k_2^2 k_3^2 k_4 k_5 k_7 k_8 T_2^2 + k_1 k_2^2 k_3^2 k_5 k_6 k_7 k_8 T_2^2 + \\
& \quad k_1 k_2 k_3^2 k_4 k_5 k_6 k_7 k_8 T_2^2 + 2 k_1 k_2^2 k_3^2 k_5 k_7 k_8^2 T_2^2 + 2 k_1 k_2 k_3^2 k_5 k_6 k_7 k_8^2 T_2^2 - \\
& \quad k_2^2 k_3^2 k_5 k_6^2 k_7^2 T_3 - 2 k_2^2 k_3^2 k_5 k_6^2 k_7 k_8 T_3 - 2 k_1 k_2 k_3 k_4 k_5 k_6^2 k_7 k_8 T_3 - \\
& \quad 2 k_2 k_3^2 k_5 k_6^2 k_7^2 k_8 T_3 - k_2^2 k_3^2 k_5 k_6^2 k_8^2 T_3 - 2 k_1 k_2 k_3 k_5 k_6^2 k_7 k_8^2 T_3 - \\
& \quad 2 k_2 k_3^2 k_5 k_6^2 k_7 k_8^2 T_3 - k_3^2 k_5 k_6^2 k_7^2 k_8^2 T_3 + 2 k_1 k_2 k_3^2 k_5 k_6^2 k_7 k_8 T_1 T_3 + \\
& \quad 2 k_1 k_2 k_3^2 k_5 k_6^2 k_8^2 T_1 T_3 + 2 k_1 k_3^2 k_5 k_6^2 k_7 k_8^2 T_1 T_3 - k_1 k_2^2 k_3^2 k_5 k_6 k_7 k_8 T_2 T_3 - \\
& \quad k_1 k_2 k_3^2 k_4 k_5 k_6 k_7 k_8 T_2 T_3 - 2 k_1 k_2 k_3^2 k_5 k_6^2 k_7 k_8 T_2 T_3 - k_1 k_2^2 k_3^2 k_5 k_6 k_8^2 T_2 T_3 - \\
& \quad k_1 k_2 k_3^2 k_5 k_6^2 k_8^2 T_2 T_3 - 2 k_1 k_2 k_3^2 k_5 k_6 k_7 k_8^2 T_2 T_3 - 2 k_1 k_3^2 k_5 k_6^2 k_7 k_8^2 T_2 T_3\big) x_1^3 + \\
& \big(k_1 k_2^2 k_3^2 k_5 k_6 k_7 k_8 T_2 + k_1 k_2 k_3^2 k_4 k_5 k_6 k_7 k_8 T_2 + k_1 k_2^2 k_3^2 k_5 k_6 k_8^2 T_2 - \\
& \quad k_1^2 k_2 k_3^2 k_6^2 k_8^2 T_2 + 2 k_1 k_2 k_3^2 k_5 k_6 k_7 k_8^2 T_2 - k_1^2 k_2 k_3^2 k_5 k_6 k_8^2 T_1 T_2 + k_1^2 k_2^2 k_3^2 k_5 k_8^2 T_2^2 + \\
& \quad k_1^2 k_2 k_3^2 k_5 k_6 k_8^2 T_2^2 - 2 k_1 k_2 k_3^2 k_5 k_6^2 k_7 k_8 T_3 - 2 k_1 k_2 k_3^2 k_5 k_6^2 k_8^2 T_3 - \\
& \quad 2 k_1 k_3^2 k_5 k_6^2 k_7 k_8^2 T_3 + k_1^2 k_3^2 k_5 k_6^2 k_8^2 T_1 T_3 - k_1^2 k_2 k_3^2 k_5 k_6 k_8^2 T_2 T_3 - k_1^2 k_3^2 k_5 k_6^2 k_8^2 T_2 T_3\big) \\
& x_1^4 + \big(k_1^2 k_2 k_3^2 k_5 k_6 k_8^2 T_2 - k_1^2 k_3^2 k_5 k_6^2 k_8^2 T_3\big) x_1^5;
\end{aligned}
$$

coeffs

Sampling

(Test) In[3]:=
```
multistableParSets = {};
multistablePolSets = {};
multistableSolSets = {};
bistableParSets = {};
bistablePolSets = {};
bistableSolSets = {};
biCount = 0;
multiCount = 0;
termCount = 0;
Timing[
 Do[{
    pars = Exp[-RandomVariate[
          ExponentialDistribution[Log[2] / (-Log[0.001])], 8]] * 1000;
    tots = Exp[-RandomVariate[ExponentialDistribution[
           Log[2] / (-Log[0.0001])], 3]] * 1000;
    (*pars=Exp[RandomReal[{Log[0.001],Log[1000.]},8]];*)
    (*tots=Exp[RandomReal[{Log[0.001],Log[10.]},3]];*)
    subs = {k1 → pars[[1]], k2 → pars[[2]], k3 → pars[[3]],
      k4 → pars[[4]], k5 → pars[[5]], k6 → pars[[6]], k7 → pars[[7]],
      k8 → pars[[8]], T1 → tots[[1]], T2 → tots[[2]], T3 → tots[[3]]};
    (*term4=coeff4/.subs;
    term3=coeff3/.subs;
    term2=coeff2/.subs;
    term1=coeff1/.subs;
    If[term4<0&&term3>0&&term2<0&&term1>0,{*)
    solution = NSolve[{pol == 0 && x1 > 0} /. subs, x1, Reals];
    If[Length[Flatten[solution]] > 1, {
      AppendTo[bistableParSets, Flatten[Join[pars, tots]]];
      AppendTo[bistablePolSets, pol /. subs];
      AppendTo[bistableSolSets, Flatten[solution]];
      biCount++;
      If[Length[Flatten[solution]] > 3, {
        AppendTo[multistableParSets, Flatten[Join[pars, tots]]];
        AppendTo[multistablePolSets, pol /. subs];
        AppendTo[multistableSolSets, Flatten[solution]];
        multiCount++;
       }];
     }];
    (*}];*)
   }, {i, 100000}];
]
```

(Test) Out[12]= {2537.84, Null}

(Test) In[16]:= `Length[bistableParSets]`

(Test) Out[16]= 52169

(Test) In[17]:= `Length[multistableParSets]`

(Test) Out[17]= 53

(Test) In[18]:= `Length[multistableSolSets]`

(Test) Out[18]= 53

(Test) In[20]:=
```
SetDirectory[NotebookDirectory[]];

Export["fullCycle_simplified_bistableParSets3.csv", bistableParSets, "Table"];
```

```
Export["fullCycle_simplified_multistableParSets3.csv",
  multistableParSets, "Table"];
```

```
InputForm[multistableParSets]
```

```
{{87.8252507051046, 0.06859978979268634, 47.85656645845265, 123.42698990061858, 0.00178
 0.5010244243796536, 0.07708098183076302, 0.005924687409234897, 0.1199944114126423, 0.0
 {57.61608551314746, 0.08771656907227311, 87.69045071228797, 71.91044050429956, 0.76583
 0.2557846694694169 7, 0.03434080609530512, 6.071028756177273, 0.8673325551444022, 0.021
 0.022498399086100792, 339.8611667443143, 0.7994210840523621, 0.03434315392741496, 0.00
 0.009580305153584377, 0.016429689638779867, 4.675785278981223, 0.1951704852212429 7, 3.
 {0.2974513709031221, 0.0015295832832042363, 4.271764506081529, 991.0811888264554, 0.00
 0.0023956989820281722, 0.008660261037543956, 0.0020142243202341324, 2.23142972887903 9,
 {2.6418581926054845, 0.0010890693111172806, 182.39348711326582, 61.42047877163915, 4.4
 0.04507952831274341, 0.0535558392276498, 9.657989391827222, 1.1405826441331537, 0.3627
 0.022053481278507054, 755.2319877127612, 865.9227494158074, 10.79858459465063, 0.06936
 4.47064017905331, 2.7510184248154608, 0.41532178389818114, 6.040741667482379}, {34.179
 37.80989038928277, 134.10531883202407, 0.03866320310995702, 0.005066137620037989, 0.00
 3.177412521303917, 1.91249378213722, 0.0023253076307465344}, {409.5122683616875, 0.687
 61.140406401383416, 0.025319021557519257, 0.4829948424997722, 0.6429332513538244, 0.00
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 128.76025772333844, 5.57588095950202, 0.004206949178975614 5, 0.09449920357902004, 1.11
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 11.65646170494112, 0.020509964906097064, 0.00834318316042656 5, 2.3785900729256317, 2.2
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 0.2958217992690478, 0.0024519878837389827, 2.6969920650890993, 0.012739669175784023, 9
 0.011936363998354849}, {1.6859190032420917, 0.0014919619452753003, 321.7874207887694,
 101.19957307358229, 0.003123705511777472, 0.001860516720804966, 0.03408569179 7296535,
 0.008658688468605723}, {21.179554758074016, 0.0037815450516138975, 11.506796707528743,
 0.051770678256908655, 0.014521590481172833, 1.447159013706113, 0.0029540738734454538,
 0.3553366868795438}, {10.687353532243206, 0.005248448898703257, 558.8770427765689, 336
 0.002474361749962581, 0.04126559154610191, 0.46733141728700456, 1.726234041659292 3, 0.
 0.0639863500951314}, {1.2975212320218454, 0.00832197142452612 7, 288.4125492451333, 0.0
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 0.016999407186649936}, {2.2881959838559194, 0.02909683944171355, 15.265900430297231, 6
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 0.007534704360826151}, {469.98298073777624, 0.002609507318442764, 17.209193725883765,
 0.1697673189053157, 0.006409191286742243, 4.222739606259007, 0.08716562039290107, 9.53
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 0.42648500380631954}, {4.627887096543039, 0.001204128735291439, 60.51090576637821, 578
 0.08768198092811469, 0.044467881403052356, 1.254354657448420 3, 0.3915202574340788, 3.5
 {0.14258437694603257, 0.001847259762118014, 2.181856751755701, 384.5427623466492, 0.00
 0.009538411810138142, 0.002942395323682531, 0.17380707249970337, 0.691113716900121, 0.
 0.00362327316641827 9}, {1.584771131857972, 0.0018973169469846882, 685.6925141906376, 0
 0.03522902823358874, 0.028349321351645137, 0.00326449712034173 5, 4.0582180013251445, 1
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multistablePolSets

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$0.0236851 - 294.153\, x_1 + 11\,081.3\, x_1^2 - 24\,867.7\, x_1^3 + 2308.05\, x_1^4 + 479.397\, x_1^5,$

$2.57687\times 10^{-13} - 1.52863\times 10^{-9}\, x_1 + 2.24381\times 10^{-6}\, x_1^2 - 0.000781859\, x_1^3 +$
$0.0634058\, x_1^4 - 0.0190919\, x_1^5,\ 6.1546\times 10^{-15} - 8.76933\times 10^{-7}\, x_1 +$
$5.49526\times 10^{-6}\, x_1^2 - 2.31826\times 10^{-6}\, x_1^3 + 1.4296\times 10^{-7}\, x_1^4 + 1.29855\times 10^{-10}\, x_1^5,$

$1.63625\times 10^{-7} - 0.00282682\, x_1 + 1.37527\, x_1^2 - 21.1987\, x_1^3 + 11.8515\, x_1^4 - 1.11358\, x_1^5,$

$0.168336 - 291.276\, x_1 + 96\,367.7\, x_1^2 - 5.94823\times 10^6\, x_1^3 + 6.28961\times 10^7\, x_1^4 - 2.8504\times 10^7\, x_1^5,$

$1.90734\times 10^{-15} - 1.21984\times 10^{-8}\, x_1 +$
$0.0000499543\, x_1^2 - 0.00487213\, x_1^3 - 0.442624\, x_1^4 + 0.44872\, x_1^5,$

$0.000894105 - 63\,770.1\, x_1 + 2.02292\times 10^6\, x_1^2 - 623\,308.\, x_1^3 + 29\,654.9\, x_1^4 + 550.474\, x_1^5,$

$1.21056\times 10^{-7} - 0.00326804\, x_1 + 0.153701\, x_1^2 -$
$0.0646671\, x_1^3 + 0.00273229\, x_1^4 - 0.0000213098\, x_1^5,$

$1.68371\times 10^{-6} - 0.0191338\, x_1 + 4.98081\, x_1^2 - 84.8017\, x_1^3 - 15.363\, x_1^4 + 7.58868\, x_1^5,$

$1.52396\times 10^{-8} - 0.000202273\, x_1 + 0.00328598\, x_1^2 - 0.00159079\, x_1^3 + 0.000103096\, x_1^4 +$
$0.0000175456\, x_1^5,\ 3.20562\times 10^{-17} - 7.90562\times 10^{-10}\, x_1 + 0.000017057\, x_1^2 -$
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$0.000614207\, x_1^2 - 0.000493835\, x_1^3 + 0.0000713777\, x_1^4 - 1.24595\times 10^{-6}\, x_1^5,$

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$0.825833\, x_1^4 - 0.346597\, x_1^5,\ 4.31479\times 10^{-18} - 2.15649\times 10^{-12}\, x_1 +$
$6.10047\times 10^{-9}\, x_1^2 - 1.02029\times 10^{-7}\, x_1^3 - 6.34668\times 10^{-7}\, x_1^4 + 7.88173\times 10^{-6}\, x_1^5,$

$0.00371709 - 87.3307\, x_1 + 58.181\, x_1^2 - 10.031\, x_1^3 + 0.332291\, x_1^4 + 0.0000631377\, x_1^5,$
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$6.75659 \times 10^{-11} - 0.00337044\, x_1 + 5.77961\, x_1^2 - 31.5569\, x_1^3 - 19.4083\, x_1^4 + 1.17834\, x_1^5,$
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$2.13391 \times 10^{-15} - 2.99959 \times 10^{-10}\, x_1 + 3.78353 \times 10^{-9}\, x_1^2 - 7.08488 \times 10^{-9}\, x_1^3 +$
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$0.000201172\, x_1^2 - 0.0100807\, x_1^3 + 0.00292006\, x_1^4 + 0.00014002\, x_1^5,\ 0.0252988 -$
$3.35376 \times 10^{6}\, x_1 + 1.74086 \times 10^{7}\, x_1^2 - 7.79216 \times 10^{6}\, x_1^3 + 167\,238.\, x_1^4 + 136.287\, x_1^5,$
$1.51619 \times 10^{-8} - 0.00550272\, x_1 + 0.562512\, x_1^2 - 2.69499\, x_1^3 - 0.0130301\, x_1^4 + 0.0674595\, x_1^5,$
$7.08845 \times 10^{-14} - 3.33531 \times 10^{-8}\, x_1 + 0.0000129216\, x_1^2 +$
$0.000456301\, x_1^3 - 0.0219676\, x_1^4 + 0.123809\, x_1^5,$
$1.87915 \times 10^{-6} - 0.176691\, x_1 + 15.8493\, x_1^2 - 3.04673\, x_1^3 + 0.114817\, x_1^4 + 0.00318554\, x_1^5,$
$2.04555 \times 10^{-6} - 0.0746466\, x_1 + 397.804\, x_1^2 -$
$254\,706.\, x_1^3 + 3.29852 \times 10^{7}\, x_1^4 - 1.67538 \times 10^{7}\, x_1^5,$
$4.20834 \times 10^{-6} - 0.339882\, x_1 + 28.8352\, x_1^2 - 46.4785\, x_1^3 - 319.067\, x_1^4 + 158.935\, x_1^5,$
$7.04479 \times 10^{-11} - 9.42958 \times 10^{-6}\, x_1 + 0.00232553\, x_1^2 -$
$0.0191517\, x_1^3 - 0.00617532\, x_1^4 + 0.0099982\, x_1^5,$
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$15\,593.7 - 1.63492 \times 10^{8}\, x_1 + 7.84976 \times 10^{9}\, x_1^2 +$
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$0.000184852 - 8852.12\, x_1 + 33\,351.6\, x_1^2 - 7637.59\, x_1^3 - 121.364\, x_1^4 + 24.4298\, x_1^5,$
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$110\,584.\, x_1^2 - 2.72649 \times 10^{6}\, x_1^3 - 1.95415 \times 10^{7}\, x_1^4 + 1.4245 \times 10^{7}\, x_1^5,$
$1.42487 \times 10^{-7} - 0.0090789\, x_1 + 2.52485\, x_1^2 - 101.933\, x_1^3 - 769.186\, x_1^4 + 1019.83\, x_1^5,$
$7.9131 \times 10^{-17} - 3.57754 \times 10^{-10}\, x_1 + 1.84267 \times 10^{-9}\, x_1^2 -$
$1.02949 \times 10^{-9}\, x_1^3 + 8.71107 \times 10^{-11}\, x_1^4 - 3.41861 \times 10^{-13}\, x_1^5,$
$0.0000116505 - 0.469318\, x_1 + 86.3545\, x_1^2 - 916.633\, x_1^3 - 13\,787.1\, x_1^4 + 43\,866.1\, x_1^5,$
$1.4607 \times 10^{-14} - 3.1896 \times 10^{-10}\, x_1 + 2.23687 \times 10^{-7}\, x_1^2 -$
$9.61354 \times 10^{-7}\, x_1^3 - 0.0000232105\, x_1^4 + 0.0000114772\, x_1^5,$
$7.75877 \times 10^{-10} - 0.00455349\, x_1 + 11.9456\, x_1^2 - 5.78444\, x_1^3 + 0.382369\, x_1^4 + 0.0766462\, x_1^5,$
$1.75088 \times 10^{-14} - 0.0000897341\, x_1 + 0.0104409\, x_1^2 +$
$0.0223439\, x_1^3 - 0.0820013\, x_1^4 + 0.0467038\, x_1^5,$
$1.22061 \times 10^{-8} - 0.453044\, x_1 + 16.8365\, x_1^2 - 14.8379\, x_1^3 - 0.0426511\, x_1^4 + 0.0825087\, x_1^5,$
$7.8399 \times 10^{-14} - 3.23044 \times 10^{-6}\, x_1 + 0.000738961\, x_1^2 -$
$0.0377643\, x_1^3 - 0.0185127\, x_1^4 + 0.00571236\, x_1^5,$
$2.82002 \times 10^{-8} - 7.14353\, x_1 + 18\,135.7\, x_1^2 - 51\,465.1\, x_1^3 - 31\,930.4\, x_1^4 + 3376.02\, x_1^5,$
$5.15211 \times 10^{-6} - 0.264371\, x_1 + 12.55\, x_1^2 - 151.506\, x_1^3 + 294.91\, x_1^4 - 15.1544\, x_1^5,$
$1.50727 \times 10^{-7} - 75.1104\, x_1 + 4094.79\, x_1^2 - 5875.46\, x_1^3 - 1248.86\, x_1^4 + 839.921\, x_1^5,$
$0.000101316 - 0.833382\, x_1 + 49.1395\, x_1^2 - 496.067\, x_1^3 + 183.892\, x_1^4 - 1.5452\, x_1^5,$
$1.74992 \times 10^{-8} - 0.1339\, x_1 + 34.9934\, x_1^2 - 76.9315\, x_1^3 + 13.9723\, x_1^4 + 0.036552\, x_1^5,$
$0.667989 - 286\,642.\, x_1 + 7.56776 \times 10^{6}\, x_1^2 - 1.1208 \times 10^{6}\, x_1^3 + 38\,157.6\, x_1^4 - 35.2843\, x_1^5\}$

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bistableSolSets[[1 ;; 50]]
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multistableSolSets
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$\{x_1 \to 0.000257067, x_1 \to 0.000630689, x_1 \to 0.00295021, x_1 \to 0.00852842,$
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$\{x_1 \to 0.0000596104, x_1 \to 0.00206335, x_1 \to 0.0651934, x_1 \to 2.18363, x_1 \to 8.39172\},$
$\{x_1 \to 0.000760242, x_1 \to 0.00295185, x_1 \to 0.0158737, x_1 \to 0.0786327, x_1 \to 2.10835\},$
$\{x_1 \to 1.56461 \times 10^{-7}, x_1 \to 0.000250287, x_1 \to 0.00628692, x_1 \to 0.99719\},$
$\{x_1 \to 1.40208 \times 10^{-8}, x_1 \to 0.0318356, x_1 \to 4.06638, x_1 \to 12.6683\},$
$\{x_1 \to 0.000037107, x_1 \to 0.0214184, x_1 \to 2.64783, x_1 \to 27.5441, x_1 \to 98.0041\},$
$\{x_1 \to 0.0000901068, x_1 \to 0.00403515, x_1 \to 0.0540521, x_1 \to 4.48399\},$
$\{x_1 \to 0.0000754341, x_1 \to 0.0634224, x_1 \to 2.72081, x_1 \to 4.91521\},$
$\{x_1 \to 4.05842 \times 10^{-8}, x_1 \to 0.0000466552, x_1 \to 0.00399646, x_1 \to 1.15444\},$
$\{x_1 \to 0.000838556, x_1 \to 0.0238136, x_1 \to 1.57201, x_1 \to 6.21267, x_1 \to 49.4784\},$
$\{x_1 \to 0.0000471981, x_1 \to 0.00185823, x_1 \to 0.0143931, x_1 \to 0.699542, x_1 \to 1.66685\},$
$\{x_1 \to 2.01229 \times 10^{-6}, x_1 \to 0.000353592, x_1 \to 0.053421, x_1 \to 0.134126\},$
$\{x_1 \to 0.0000425646, x_1 \to 2.45721, x_1 \to 4.63184, x_1 \to 22.9598\},$
$\{x_1 \to 5.6622 \times 10^{-6}, x_1 \to 0.00735114, x_1 \to 0.0273702, x_1 \to 8.4232\},$
$\{x_1 \to 2.00473 \times 10^{-8}, x_1 \to 0.000585011, x_1 \to 0.165773, x_1 \to 17.9478\},$
$\{x_1 \to 3.86827 \times 10^{-7}, x_1 \to 0.00464764, x_1 \to 0.0349088, x_1 \to 6.59998\},$
$\{x_1 \to 7.11462 \times 10^{-6}, x_1 \to 0.0963317, x_1 \to 0.496156, x_1 \to 4.18812\},$
$\{x_1 \to 2.45408 \times 10^{-6}, x_1 \to 0.000546531, x_1 \to 0.0195208, x_1 \to 2.99821\},$
$\{x_1 \to 7.54343 \times 10^{-9}, x_1 \to 0.212832, x_1 \to 2.1295, x_1 \to 42.6808\},$
$\{x_1 \to 2.75612 \times 10^{-6}, x_1 \to 0.0102868, x_1 \to 0.198441, x_1 \to 6.3127\},$
$\{x_1 \to 2.12703 \times 10^{-6}, x_1 \to 0.00239884, x_1 \to 0.0443476, x_1 \to 0.147802\},$
$\{x_1 \to 0.0000106454, x_1 \to 0.0111615, x_1 \to 9.18993, x_1 \to 9.81233\},$
$\{x_1 \to 0.0000331283, x_1 \to 0.000178934, x_1 \to 0.00184671, x_1 \to 0.00568738,$
$x_1 \to 1.96107\}, \{x_1 \to 0.0000123948, x_1 \to 0.0120272, x_1 \to 0.240499, x_1 \to 2.10572\},$
$\{x_1 \to 7.48477 \times 10^{-6}, x_1 \to 0.00419254, x_1 \to 0.113696, x_1 \to 1.67723\},$
$\{x_1 \to 0.000131351, x_1 \to 0.000188224, x_1 \to 0.186955, x_1 \to 0.267485, x_1 \to 0.787564\},$
$\{x_1 \to 3.94294 \times 10^{-9}, x_1 \to 0.00160339, x_1 \to 0.155258, x_1 \to 12.1218\},$
$\{x_1 \to 8.36833 \times 10^{-8}, x_1 \to 0.000923006, x_1 \to 0.00556707, x_1 \to 0.00809467\},$
$\{x_1 \to 1.31821 \times 10^{-7}, x_1 \to 0.00452533, x_1 \to 1.22819, x_1 \to 9.81907\},$
$\{x_1 \to 0.0000279485, x_1 \to 0.00128919, x_1 \to 0.00297476, x_1 \to 0.702371\},$
$\{x_1 \to 2.41771 \times 10^{-7}, x_1 \to 0.000201713, x_1 \to 0.119437, x_1 \to 0.319437\},$
$\{x_1 \to 3.00133 \times 10^{-7}, x_1 \to 0.420892, x_1 \to 1.37775, x_1 \to 5.26727\},$
$\{x_1 \to 0.0000958199, x_1 \to 0.0193919, x_1 \to 0.381383, x_1 \to 1.94272\},$
$\{x_1 \to 2.08823 \times 10^{-8}, x_1 \to 0.283962, x_1 \to 4.03049, x_1 \to 18.1229\},$
$\{x_1 \to 1.05486 \times 10^{-6}, x_1 \to 0.0381527, x_1 \to 0.155217, x_1 \to 5.66607, x_1 \to 72.2223\},$
$\{x_1 \to 0.0000237591, x_1 \to 0.00255206, x_1 \to 0.030755, x_1 \to 1.49628\},$
$\{x_1 \to 0.0000157633, x_1 \to 0.00438463, x_1 \to 0.0174245, x_1 \to 0.866321\},$
$\{x_1 \to 2.21189 \times 10^{-7}, x_1 \to 0.220903, x_1 \to 1.91738, x_1 \to 10.1891, x_1 \to 242.486\},$
$\{x_1 \to 0.0000249388, x_1 \to 0.00579904, x_1 \to 0.0512037, x_1 \to 0.357575\},$
$\{x_1 \to 0.0000473689, x_1 \to 0.0013874, x_1 \to 0.0800278, x_1 \to 2.05841\},$
$\{x_1 \to 1.70468 \times 10^{-7}, x_1 \to 0.000381084, x_1 \to 3.07587, x_1 \to 4.14807\},$
$\{x_1 \to 1.95119 \times 10^{-10}, x_1 \to 0.00844652, x_1 \to 0.781383, x_1 \to 1.20711\},$
$\{x_1 \to 2.69424 \times 10^{-8}, x_1 \to 0.0275788, x_1 \to 1.1113, x_1 \to 13.0776\},$
$\{x_1 \to 2.42689 \times 10^{-8}, x_1 \to 0.00661523, x_1 \to 0.0128096, x_1 \to 4.65503\},$
$\{x_1 \to 3.94769 \times 10^{-9}, x_1 \to 0.00039433, x_1 \to 0.298417, x_1 \to 10.8209\},$
$\{x_1 \to 0.0000195062, x_1 \to 0.0343378, x_1 \to 0.0625123, x_1 \to 0.428822, x_1 \to 18.9347\},$
$\{x_1 \to 2.00674 \times 10^{-9}, x_1 \to 0.018855, x_1 \to 0.628143, x_1 \to 3.19966\},$
$\{x_1 \to 0.000122455, x_1 \to 0.0214874, x_1 \to 0.0806774, x_1 \to 2.65699, x_1 \to 116.249\},$
$\{x_1 \to 1.30693 \times 10^{-7}, x_1 \to 0.00385901, x_1 \to 0.496116, x_1 \to 4.93519\},$
$\{x_1 \to 2.33053 \times 10^{-6}, x_1 \to 0.0380891, x_1 \to 10.3153, x_1 \to 19.6642, x_1 \to 1051.41\}\}$

Example 3

```
subpars = multistableParSets[[6]]
```

```
{2.8539, 0.0220535, 755.232, 865.923, 10.7986,
 0.0693642, 0.023003, 4.47064, 2.75102, 0.415322, 6.04074}
```

```
subs = {k1 → subpars[[1]], k2 → subpars[[2]], k3 → subpars[[3]],
   k4 → subpars[[4]], k5 → subpars[[5]], k6 → subpars[[6]], k7 → subpars[[7]],
   k8 → subpars[[8]], T1 → subpars[[9]], T2 → subpars[[10]], T3 → subpars[[11]]}
```

$\{k_1 \to 2.8539,\ k_2 \to 0.0220535,\ k_3 \to 755.232,\ k_4 \to 865.923,\ k_5 \to 10.7986,\ k_6 \to 0.0693642,\ k_7 \to 0.023003,\ k_8 \to 4.47064,\ T_1 \to 2.75102,\ T_2 \to 0.415322,\ T_3 \to 6.04074\}$

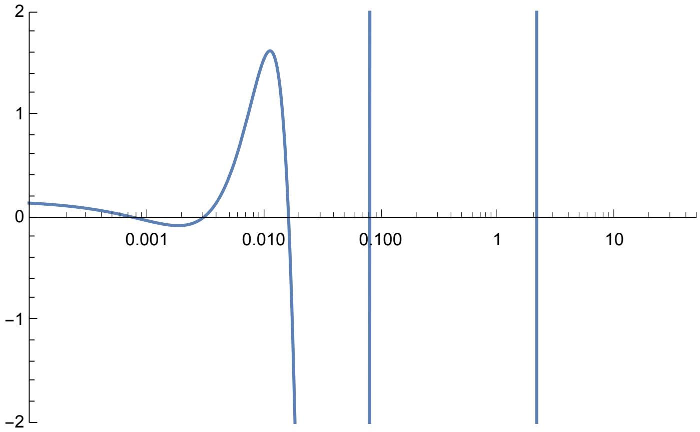
```
solution = NSolve[{pol == 0} /. subs, x1]
```

$\{\{x_1 \to 0.000760242\},\ \{x_1 \to 0.00295185\},\ \{x_1 \to 0.0158737\},\ \{x_1 \to 0.0786327\},\ \{x_1 \to 2.10835\}\}$

```
subpol = pol /. subs
```

$0.168336 - 291.276\, x_1 + 96\,367.7\, x_1^2 - 5.94823 \times 10^6\, x_1^3 + 6.28961 \times 10^7\, x_1^4 - 2.8504 \times 10^7\, x_1^5$

```
LogLinearPlot[subpol, {x1, 0.000001, 10}, PlotRange → {{0.0001, 50}, {-2, 2}}]
```



Example 2

```
subpars = multistableParSets[[5]]
```

```
{2.64186, 0.00108907, 182.393, 61.4205, 4.46069,
 0.0339027, 0.0450795, 0.0535558, 9.65799, 1.14058, 0.362777}
```

```
subpars
```

```
{2.64186, 0.00108907, 182.393, 61.4205, 4.46069,
 0.0339027, 0.0450795, 0.0535558, 9.65799, 1.14058, 0.362777}
```

```
subs = {k1 → subpars[[1]], k2 → subpars[[2]], k3 → subpars[[3]],
   k4 → subpars[[4]], k5 → subpars[[5]], k6 → subpars[[6]], k7 → subpars[[7]],
   k8 → subpars[[8]], T1 → subpars[[9]], T2 → subpars[[10]], T3 → subpars[[11]]}
```

$\{k_1 \to 2.64186,\ k_2 \to 0.00108907,\ k_3 \to 182.393,\ k_4 \to 61.4205,\ k_5 \to 4.46069,\ k_6 \to 0.0339027,\ k_7 \to 0.0450795,\ k_8 \to 0.0535558,\ T_1 \to 9.65799,\ T_2 \to 1.14058,\ T_3 \to 0.362777\}$

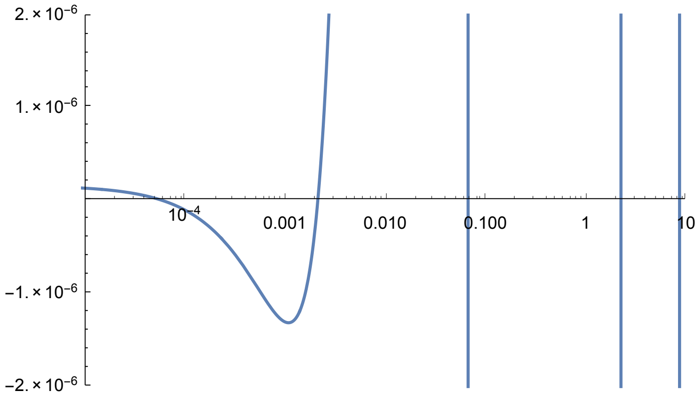
```
solution = NSolve[{pol == 0} /. subs, x1]
```

$\{\{x_1 \to 0.0000596104\}, \{x_1 \to 0.00206335\},$
$\{x_1 \to 0.0651934\}, \{x_1 \to 2.18363\}, \{x_1 \to 8.39172\}\}$

```
subpol = pol /. subs
```

$1.63625\times10^{-7} - 0.00282682\, x_1 + 1.37527\, x_1^2 - 21.1987\, x_1^3 + 11.8515\, x_1^4 - 1.11358\, x_1^5$

```
LogLinearPlot[subpol, {x1, 0.000001, 10},
 PlotRange → {{0.00001, 10}, {-0.000002, 0.000002}}]
```



Example I

```
subpars = multistableParSets[[3]]
```

{22.7426, 0.0224984, 339.861, 0.799421, 0.0343432,
0.00396755, 0.00958031, 0.0164297, 4.67579, 0.19517, 3.29662}

```
subpars
```

{22.7426, 0.0224984, 339.861, 0.799421, 0.0343432,
0.00396755, 0.00958031, 0.0164297, 4.67579, 0.19517, 3.29662}

```
subs = {k1 → subpars[[1]], k2 → subpars[[2]], k3 → subpars[[3]],
  k4 → subpars[[4]], k5 → subpars[[5]], k6 → subpars[[6]], k7 → subpars[[7]],
  k8 → subpars[[8]], T1 → subpars[[9]], T2 → subpars[[10]], T3 → subpars[[11]]}
```

$\{k_1 \to 22.7426, k_2 \to 0.0224984, k_3 \to 339.861,$
$k_4 \to 0.799421, k_5 \to 0.0343432, k_6 \to 0.00396755, k_7 \to 0.00958031,$
$k_8 \to 0.0164297, T_1 \to 4.67579, T_2 \to 0.19517, T_3 \to 3.29662\}$

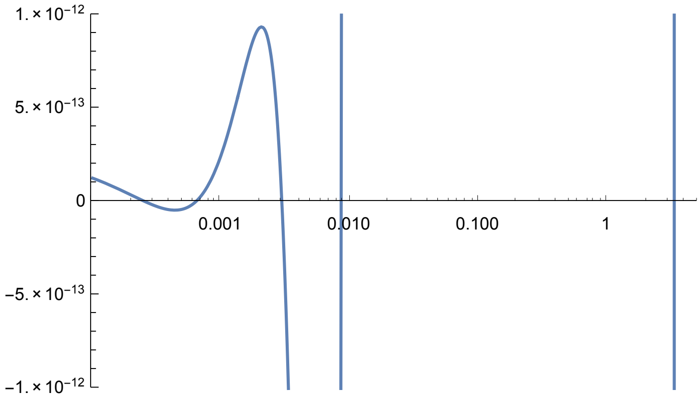
```
solution = NSolve[{pol == 0} /. subs, x1]
```

$\{\{x_1 \to 0.000257067\}, \{x_1 \to 0.000630689\},$
$\{x_1 \to 0.00295021\}, \{x_1 \to 0.00852842\}, \{x_1 \to 3.30872\}\}$

```
subpol = pol /. subs
```

$2.57687\times10^{-13} - 1.52863\times10^{-9}\, x_1 +$
$2.24381\times10^{-6}\, x_1^2 - 0.000781859\, x_1^3 + 0.0634058\, x_1^4 - 0.0190919\, x_1^5$

```
LogLinearPlot[subpol, {x1, 0.0001, 10},
 PlotRange → {{0.0001, 5}, {-0.000000000001, 0.000000000001}}]
```



Test