

Numeical trial

```
Clear["Global`*"];

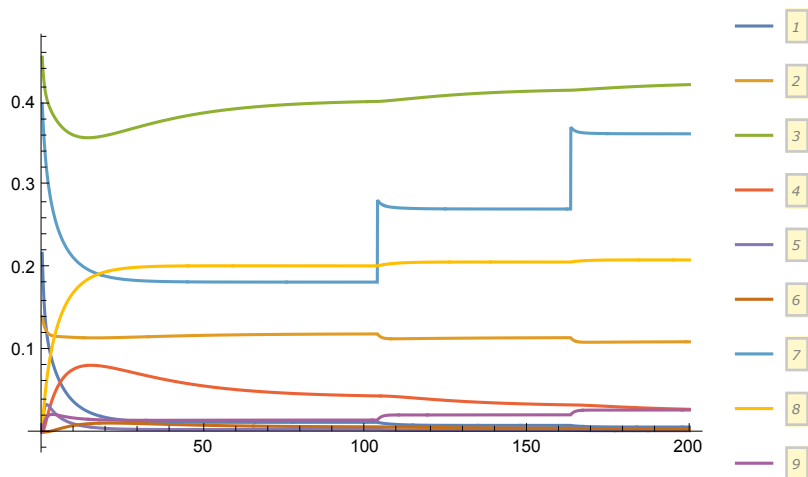
des = {x[1]'[t], x[2]'[t], x[3]'[t], x[4]'[t],
      x[5]'[t], x[6]'[t], x[7]'[t], x[8]'[t], x[9]'[t]} ==
{-k[1] x[1][t] x[3][t] + k[2] x[5][t] + k[3] x[5][t] - k[7] x[1][t] x[7][t] +
 k[8] x[8][t], -k[4] x[2][t] x[4][t] + k[5] x[6][t] + k[6] x[6][t] -
 k[9] x[2][t] x[7][t] + k[10] x[9][t], -k[1] x[1][t] x[3][t] + k[2] x[5][t] +
 k[6] x[6][t], -k[4] x[2][t] x[4][t] + k[3] x[5][t] + k[5] x[6][t],
 k[1] x[1][t] x[3][t] - k[2] x[5][t] - k[3] x[5][t],
 k[4] x[2][t] x[4][t] - k[5] x[6][t] - k[6] x[6][t],
 -k[7] x[1][t] x[7][t] - k[9] x[2][t] x[7][t] + k[8] x[8][t] + k[10] x[9][t],
 k[7] x[1][t] x[7][t] - k[8] x[8][t], k[9] x[2][t] x[7][t] - k[10] x[9][t]};

init = {T[1], T[2], T[3], 0, 0, 0, T[4], 0, 0};

vars = Array[x, 9];
dvars = Thread[Derivative[1][vars]];

Block[{k, T, ssthreshold}, k[n_] := k[n] = (SeedRandom[n];
RandomReal[]);
T[n_] := T[n] = (SeedRandom[n + 10]; RandomReal[]);
ssthreshold = 1.*^-4;
(*Print[des];*)(*to see the ODE*)
{sol} = NDSolve[{des, Through[vars[0]] == init, With[{df = Through[dvars[t]]},
WhenEvent[Norm[df] < ssthreshold, x[7][t] → x[7][t] + 0.1]}],
vars, {t, 0, 200}, MaxSteps → 100 000];

Plot@@{Through[vars[t]] /. sol,
Flatten@{t, x[1][ "Domain"] /. sol}, PlotLegends → Automatic}
```



Analytical solution trial

```
Solve[Table[0, {i, Length[des[[1]]]}] == des[[2]],
Table[x[i][t], {i, Length[des[[1]]]}]]
```

Solve::vars: Equations may not give solutions for all "solve" variables»

$$\left\{ \begin{aligned} x[3][t] &\rightarrow \frac{(k[2] + k[3]) x[5][t]}{k[1] x[1][t]}, x[4][t] \rightarrow \frac{k[3] (k[5] + k[6]) x[5][t]}{k[4] k[6] x[2][t]}, \\ x[6][t] &\rightarrow \frac{k[3] x[5][t]}{k[6]}, x[8][t] \rightarrow \frac{k[7] x[1][t] x[7][t]}{k[8]}, \\ x[9][t] &\rightarrow \frac{k[9] x[2][t] x[7][t]}{k[10]}, \{x[1][t] \rightarrow 0, x[4][t] \rightarrow 0, \\ x[5][t] &\rightarrow 0, x[6][t] \rightarrow 0, x[8][t] \rightarrow 0, x[9][t] \rightarrow \frac{k[9] x[2][t] x[7][t]}{k[10]}, \\ \{x[1][t] &\rightarrow 0, x[2][t] \rightarrow 0, x[5][t] \rightarrow 0, x[6][t] \rightarrow 0, x[8][t] \rightarrow 0, x[9][t] \rightarrow 0\}, \\ \{x[2][t] &\rightarrow 0, x[3][t] \rightarrow 0, x[5][t] \rightarrow 0, x[6][t] \rightarrow 0, \\ x[8][t] &\rightarrow \frac{k[7] x[1][t] x[7][t]}{k[8]}, x[9][t] \rightarrow 0\} \end{aligned} \right\}$$

Here we have substitution:

$$\left\{ \begin{aligned} \frac{k[2] + k[3]}{k[1]} &\rightarrow km[1], \frac{k[5] + k[6]}{k[4]} \rightarrow km[2], \\ \frac{k[3]}{k[6]} &\rightarrow kcr, \frac{k[7]}{k[8]} \rightarrow kd[1], \frac{k[9]}{k[10]} \rightarrow kd[2] \end{aligned} \right\}$$

$$\text{solution} = \left\{ \begin{aligned} x[3] &\rightarrow \frac{km[1] x[5]}{x[1]}, x[4] \rightarrow \frac{kcr * km[2] x[5]}{x[2]}, \\ x[6] &\rightarrow kcr x[5], x[8] \rightarrow kd[1] x[1] x[7], x[9] \rightarrow kd[2] x[2] x[7] \end{aligned} \right\}$$

$$\left\{ \begin{aligned} x[3] &\rightarrow \frac{km[1] x[5]}{x[1]}, x[4] \rightarrow \frac{kcr km[2] x[5]}{x[2]}, \\ x[6] &\rightarrow kcr x[5], x[8] \rightarrow kd[1] x[1] x[7], x[9] \rightarrow kd[2] x[2] x[7] \end{aligned} \right\}$$

Here we have

$$\begin{aligned} T[1] &= x[1] + x[5] + x[8], T[2] = x[2] + x[6] + x[9], \\ T[3] &= x[3] + x[4] + x[5] + x[6], T[4] = x[7] + x[8] + x[9] \end{aligned}$$

Continue

$$t4 = \left\{ \begin{aligned} T[4] &= x[7] + \frac{kd[2] km[2] T[2] x[7]}{km[2] + x[4] + kd[2] km[2] x[7]} - \\ &\frac{(kd[1] x[7] (-km[2] T[1] - T[1] x[4] + kcr T[2] x[4] - kd[2] km[2] T[1] x[7]))}{((1 + kd[1] x[7]) (km[2] + x[4] + kd[2] km[2] x[7]))} \end{aligned} \right\}$$

$$\left\{ \begin{aligned} T[4] &= x[7] + \frac{kd[2] km[2] T[2] x[7]}{km[2] + x[4] + kd[2] km[2] x[7]} - \\ &\frac{(kd[1] x[7] (-km[2] T[1] - T[1] x[4] + kcr T[2] x[4] - kd[2] km[2] T[1] x[7]))}{((1 + kd[1] x[7]) (km[2] + x[4] + kd[2] km[2] x[7]))} \end{aligned} \right\}$$

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t42 = Collect[ExpandAll[{((1 + kd[1] x[7]) (km[2] + x[4] + kd[2] km[2] x[7])) *
(km[2] + x[4] + kd[2] km[2] x[7]) * T[4] ==
((1 + kd[1] x[7]) (km[2] + x[4] + kd[2] km[2] x[7])) *
(km[2] + x[4] + kd[2] km[2] x[7]) * x[7] +
((1 + kd[1] x[7]) (km[2] + x[4] + kd[2] km[2] x[7])) * kd[2] km[2] T[2] x[7] -
((km[2] + x[4] + kd[2] km[2] x[7]) * kd[1] x[7] (-km[2] T[1] - T[1] x[4] +
kcr T[2] x[4] - kd[2] km[2] T[1] x[7]))}], {x[4], x[7]}]

{km[2]^2 T[4] + (kd[1] km[2]^2 T[4] + 2 kd[2] km[2]^2 T[4]) x[7] +
(2 kd[1] kd[2] km[2]^2 T[4] + kd[2]^2 km[2]^2 T[4]) x[7]^2 +
kd[1] kd[2]^2 km[2]^2 T[4] x[7]^3 + x[4]^2 (T[4] + kd[1] T[4] x[7]) + x[4] (2 km[2] T[4] +
(2 kd[1] km[2] T[4] + 2 kd[2] km[2] T[4]) x[7] + 2 kd[1] kd[2] km[2] T[4] x[7]^2) ==
(km[2]^2 + kd[1] km[2]^2 T[1] + kd[2] km[2]^2 T[2]) x[7] +
(kd[1] km[2]^2 + 2 kd[2] km[2]^2 + 2 kd[1] kd[2] km[2]^2 T[1] +
kd[1] kd[2] km[2]^2 T[2] + kd[2]^2 km[2]^2 T[2]) x[7]^2 +
(2 kd[1] kd[2] km[2]^2 + kd[2]^2 km[2]^2 + kd[1] kd[2]^2 km[2]^2 T[1] +
kd[1] kd[2]^2 km[2]^2 T[2]) x[7]^3 + kd[1] kd[2]^2 km[2]^2 x[7]^4 +
x[4]^2 ((1 + kd[1] T[1] - kcr kd[1] T[2]) x[7] + kd[1] x[7]^2) +
x[4] ((2 km[2] + 2 kd[1] km[2] T[1] - kcr kd[1] km[2] T[2] + kd[2] km[2] T[2]) x[7] +
(2 kd[1] km[2] + 2 kd[2] km[2] + 2 kd[1] kd[2] km[2] T[1] + kd[1] kd[2] km[2]
T[2] - kcr kd[1] kd[2] km[2] T[2]) x[7]^2 + 2 kd[1] kd[2] km[2] x[7]^3)}

Collect[km[2]^2 T[4] + (kd[1] km[2]^2 T[4] + 2 kd[2] km[2]^2 T[4]) x[7] +
(2 kd[1] kd[2] km[2]^2 T[4] + kd[2]^2 km[2]^2 T[4]) x[7]^2 +
kd[1] kd[2]^2 km[2]^2 T[4] x[7]^3 + x[4]^2 (T[4] + kd[1] T[4] x[7]) + x[4] (2 km[2] T[4] +
(2 kd[1] km[2] T[4] + 2 kd[2] km[2] T[4]) x[7] + 2 kd[1] kd[2] km[2] T[4] x[7]^2) -
((km[2]^2 + kd[1] km[2]^2 T[1] + kd[2] km[2]^2 T[2]) x[7] +
(kd[1] km[2]^2 + 2 kd[2] km[2]^2 + 2 kd[1] kd[2] km[2]^2 T[1] +
kd[1] kd[2] km[2]^2 T[2] + kd[2]^2 km[2]^2 T[2]) x[7]^2 +
(2 kd[1] kd[2] km[2]^2 + kd[2]^2 km[2]^2 + kd[1] kd[2]^2 km[2]^2 T[1] +
kd[1] kd[2]^2 km[2]^2 T[2]) x[7]^3 + kd[1] kd[2]^2 km[2]^2 x[7]^4 +
x[4]^2 ((1 + kd[1] T[1] - kcr kd[1] T[2]) x[7] + kd[1] x[7]^2) +
x[4] ((2 km[2] + 2 kd[1] km[2] T[1] - kcr kd[1] km[2] T[2] + kd[2] km[2] T[2]) x[7] +
(2 kd[1] km[2] + 2 kd[2] km[2] + 2 kd[1] kd[2] km[2] T[1] +
kd[1] kd[2] km[2] T[2] - kcr kd[1] kd[2] km[2] T[2])
x[7]^2 + 2 kd[1] kd[2] km[2] x[7]^3)], {x[4], x[7]}]

km[2]^2 T[4] + (-km[2]^2 - kd[1] km[2]^2 T[1] -
kd[2] km[2]^2 T[2] + kd[1] km[2]^2 T[4] + 2 kd[2] km[2]^2 T[4]) x[7] +
(-kd[1] km[2]^2 - 2 kd[2] km[2]^2 - 2 kd[1] kd[2] km[2]^2 T[1] - kd[1] kd[2] km[2]^2 T[2] -
kd[2]^2 km[2]^2 T[2] + 2 kd[1] kd[2] km[2]^2 T[4] + kd[2]^2 km[2]^2 T[4]) x[7]^2 +
(-2 kd[1] kd[2] km[2]^2 - kd[2]^2 km[2]^2 - kd[1] kd[2]^2 km[2]^2 T[1] - kd[1] kd[2]^2
km[2]^2 T[2] + kd[1] kd[2]^2 km[2]^2 T[4]) x[7]^3 - kd[1] kd[2]^2 km[2]^2 x[7]^4 +
x[4]^2 (T[4] + (-1 - kd[1] T[1] + kcr kd[1] T[2] + kd[1] T[4]) x[7] - kd[1] x[7]^2) +
x[4] (2 km[2] T[4] + (-2 km[2] - 2 kd[1] km[2] T[1] + kcr kd[1] km[2] T[2] -
kd[2] km[2] T[2] + 2 kd[1] km[2] T[4] + 2 kd[2] km[2] T[4]) x[7] +
(-2 kd[1] km[2] - 2 kd[2] km[2] - 2 kd[1] kd[2] km[2] T[1] -
kd[1] kd[2] km[2] T[2] + kcr kd[1] kd[2] km[2] T[2] +
2 kd[1] kd[2] km[2] T[4]) x[7]^2 - 2 kd[1] kd[2] km[2] x[7]^3)

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Simplify[km[2]^2 T[4] + (-km[2]^2 - kd[1] km[2]^2 T[1] -
  kd[2] km[2]^2 T[2] + kd[1] km[2]^2 T[4] + 2 kd[2] km[2]^2 T[4]) x[7] +
  (-kd[1] km[2]^2 - 2 kd[2] km[2]^2 - 2 kd[1] kd[2] km[2]^2 T[1] - kd[1] kd[2] km[2]^2 T[2] -
  kd[2]^2 km[2]^2 T[2] + 2 kd[1] kd[2] km[2]^2 T[4] + kd[2]^2 km[2]^2 T[4]) x[7]^2 +
  (-2 kd[1] kd[2] km[2]^2 - kd[2]^2 km[2]^2 - kd[1] kd[2]^2 km[2]^2 T[1] - kd[1] kd[2]^2
  km[2]^2 T[2] + kd[1] kd[2]^2 km[2]^2 T[4]) x[7]^3 - kd[1] kd[2]^2 km[2]^2 x[7]^4 +
  x[4]^2 (T[4] + (-1 - kd[1] T[1] + kcr kd[1] T[2] + kd[1] T[4]) x[7] - kd[1] x[7]^2) +
  x[4] (2 km[2] T[4] + (-2 km[2] - 2 kd[1] km[2] T[1] + kcr kd[1] km[2] T[2] -
  kd[2] km[2] T[2] + 2 kd[1] km[2] T[4] + 2 kd[2] km[2] T[4]) x[7] +
  (-2 kd[1] km[2] - 2 kd[2] km[2] - 2 kd[1] kd[2] km[2] T[1] -
  kd[1] kd[2] km[2] T[2] + kcr kd[1] kd[2] km[2] T[2] +
  2 kd[1] kd[2] km[2] T[4]) x[7]^2 - 2 kd[1] kd[2] km[2] x[7]^3)]
- (km[2] + x[4] + kd[2] km[2] x[7])
(x[4] (-T[4] (1 + kd[1] x[7]) + x[7] (1 + kd[1] (T[1] - kcr T[2] + x[7]))) +
  km[2] (-T[4] (1 + kd[1] x[7]) (1 + kd[2] x[7]) + x[7] (1 + kd[2] (T[2] + x[7]) +
  kd[1] (T[1] (1 + kd[2] x[7]) + x[7] (1 + kd[2] (T[2] + x[7]))))))))

FullSimplify[x[4] (-T[4] (1 + kd[1] x[7]) + x[7] (1 + kd[1] (T[1] - kcr T[2] + x[7]))) +
  km[2] (-T[4] (1 + kd[1] x[7]) (1 + kd[2] x[7]) + x[7] (1 + kd[2] (T[2] + x[7]) +
  kd[1] (T[1] (1 + kd[2] x[7]) + x[7] (1 + kd[2] (T[2] + x[7])))))))]
x[4] (-T[4] (1 + kd[1] x[7]) + x[7] (1 + kd[1] (T[1] - kcr T[2] + x[7]))) +
  km[2] (-T[4] (1 + kd[1] x[7]) (1 + kd[2] x[7]) + x[7] (1 + kd[2] (T[2] + x[7]) +
  kd[1] (T[1] + x[7] + kd[2] (T[1] + T[2]) x[7] + kd[2] x[7]^2)))))]

t3 = {T[3] == x[4] +  $\frac{T[2] x[4]}{km[2] + x[4] + kd[2] km[2] x[7]}$  +
 $\frac{kcr T[2] x[4]}{km[2] + x[4] + kd[2] km[2] x[7]}$  - (kcr km[1] T[2] x[4] (1 + kd[1] x[7])) /
  (km[2] + x[4] + kd[2] km[2] x[7]
  (-km[2] T[1] - T[1] x[4] + kcr T[2] x[4] - kd[2] km[2] T[1] x[7]))}

{T[3] == x[4] +  $\frac{T[2] x[4]}{km[2] + x[4] + kd[2] km[2] x[7]}$  +
 $\frac{kcr T[2] x[4]}{km[2] + x[4] + kd[2] km[2] x[7]}$  - (kcr km[1] T[2] x[4] (1 + kd[1] x[7])) /
  (km[2] + x[4] + kd[2] km[2] x[7]
  (-km[2] T[1] - T[1] x[4] + kcr T[2] x[4] - kd[2] km[2] T[1] x[7]))}

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t32 = Collect[
  ExpandAll[{(km[2] + x[4] + kd[2] km[2] x[7]) * (km[2] + x[4] + kd[2] km[2] x[7]) *
    (-km[2] T[1] - T[1] x[4] + kcr T[2] x[4] - kd[2] km[2] T[1] x[7]) * T[3] ==
    (-km[2] T[1] - T[1] x[4] + kcr T[2] x[4] - kd[2] km[2] T[1] x[7]) *
    (km[2] + x[4] + kd[2] km[2] x[7]) * (km[2] + x[4] + kd[2] km[2] x[7]) * x[4] +
    (-km[2] T[1] - T[1] x[4] + kcr T[2] x[4] - kd[2] km[2] T[1] x[7]) *
    (km[2] + x[4] + kd[2] km[2] x[7]) * T[2] x[4] +
    (-km[2] T[1] - T[1] x[4] + kcr T[2] x[4] - kd[2] km[2] T[1] x[7]) *
    (km[2] + x[4] + kd[2] km[2] x[7]) * kcr T[2] x[4] -
    (km[2] + x[4] + kd[2] km[2] x[7]) * (km[2] + x[4] + kd[2] km[2] x[7]) *
    kcr km[1] T[2] x[4] (1 + kd[1] x[7]))}, {x[4], x[7]}]
  { -km[2]^3 T[1] T[3] + (-T[1] T[3] + kcr T[2] T[3]) x[4]^3 -
    3 kd[2] km[2]^3 T[1] T[3] x[7] - 3 kd[2]^2 km[2]^3 T[1] T[3] x[7]^2 -
    kd[2]^3 km[2]^3 T[1] T[3] x[7]^3 + x[4]^2 (-3 km[2] T[1] T[3] + 2 kcr km[2] T[2] T[3] +
    (-3 kd[2] km[2] T[1] T[3] + 2 kcr kd[2] km[2] T[2] T[3]) x[7]) +
    x[4] (-3 km[2]^2 T[1] T[3] + kcr km[2]^2 T[2] T[3] +
    (-6 kd[2] km[2]^2 T[1] T[3] + 2 kcr kd[2] km[2]^2 T[2] T[3]) x[7] +
    (-3 kd[2]^2 km[2]^2 T[1] T[3] + kcr kd[2]^2 km[2]^2 T[2] T[3]) x[7]^2) ==
    (-T[1] + kcr T[2]) x[4]^4 + x[4]^3 (-3 km[2] T[1] - kcr km[1] T[2] +
    2 kcr km[2] T[2] - T[1] T[2] - kcr T[1] T[2] + kcr T[2]^2 + kcr^2 T[2]^2 +
    (-3 kd[2] km[2] T[1] - kcr kd[1] km[1] T[2] + 2 kcr kd[2] km[2] T[2]) x[7]) +
    x[4]^2 (-3 km[2]^2 T[1] - 2 kcr km[1] km[2] T[2] + kcr km[2]^2 T[2] -
    2 km[2] T[1] T[2] - 2 kcr km[2] T[1] T[2] + kcr km[2] T[2]^2 + kcr^2 km[2] T[2]^2 +
    (-6 kd[2] km[2]^2 T[1] - 2 kcr kd[1] km[1] km[2] T[2] - 2 kcr kd[2] km[1]
    km[2] T[2] + 2 kcr kd[2] km[2]^2 T[2] - 2 kd[2] km[2] T[1] T[2] -
    2 kcr kd[2] km[2] T[1] T[2] + kcr kd[2] km[2] T[2]^2 + kcr^2 kd[2] km[2] T[2]^2)
    x[7] + (-3 kd[2]^2 km[2]^2 T[1] - 2 kcr kd[1] kd[2] km[1] km[2] T[2] +
    kcr kd[2]^2 km[2]^2 T[2]) x[7]^2) +
    x[4] (-km[2]^3 T[1] - kcr km[1] km[2]^2 T[2] - km[2]^2 T[1] T[2] - kcr km[2]^2 T[1] T[2] +
    (-3 kd[2] km[2]^3 T[1] - kcr kd[1] km[1] km[2]^2 T[2] - 2 kcr kd[2] km[1] km[2]^2
    T[2] - 2 kd[2] km[2]^2 T[1] T[2] - 2 kcr kd[2] km[2]^2 T[1] T[2]) x[7] +
    (-3 kd[2]^2 km[2]^3 T[1] - 2 kcr kd[1] kd[2] km[1] km[2]^2 T[2] - kcr kd[2]^2 km[1]
    km[2]^2 T[2] - kd[2]^2 km[2]^2 T[1] T[2] - kcr kd[2]^2 km[2]^2 T[1] T[2]) x[7]^2 +
    (-kd[2]^3 km[2]^3 T[1] - kcr kd[1] kd[2]^2 km[1] km[2]^2 T[2]) x[7]^3 }

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Collect[-km[2]^3 T[1] T[3] + (-T[1] T[3] + kcr T[2] T[3]) x[4]^3 -
  3 kd[2] km[2]^3 T[1] T[3] x[7] - 3 kd[2]^2 km[2]^3 T[1] T[3] x[7]^2 -
  kd[2]^3 km[2]^3 T[1] T[3] x[7]^3 + x[4]^2 (-3 km[2] T[1] T[3] + 2 kcr km[2] T[2] T[3] +
    (-3 kd[2] km[2] T[1] T[3] + 2 kcr kd[2] km[2] T[2] T[3]) x[7]) +
  x[4] (-3 km[2]^2 T[1] T[3] + kcr km[2]^2 T[2] T[3] +
    (-6 kd[2] km[2]^2 T[1] T[3] + 2 kcr kd[2] km[2]^2 T[2] T[3]) x[7] +
    (-3 kd[2]^2 km[2]^2 T[1] T[3] + kcr kd[2]^2 km[2]^2 T[2] T[3]) x[7]^2) -
  ((-T[1] + kcr T[2]) x[4]^4 + x[4]^3 (-3 km[2] T[1] - kcr km[1] T[2] +
    2 kcr km[2] T[2] - T[1] T[2] - kcr T[1] T[2] + kcr T[2]^2 + kcr^2 T[2]^2 +
    (-3 kd[2] km[2] T[1] - kcr kd[1] km[1] T[2] + 2 kcr kd[2] km[2] T[2]) x[7]) +
  x[4]^2 (-3 km[2]^2 T[1] - 2 kcr km[1] km[2] T[2] + kcr km[2]^2 T[2] -
    2 km[2] T[1] T[2] - 2 kcr km[2] T[1] T[2] + kcr km[2] T[2]^2 + kcr^2 km[2] T[2]^2 +
    (-6 kd[2] km[2]^2 T[1] - 2 kcr kd[1] km[1] km[2] T[2] - 2 kcr kd[2] km[1] km[2]
      T[2] + 2 kcr kd[2] km[2]^2 T[2] - 2 kd[2] km[2] T[1] T[2] - 2 kcr kd[2]
      km[2] T[1] T[2] + kcr kd[2] km[2] T[2]^2 + kcr^2 kd[2] km[2] T[2]^2) x[7] +
    (-3 kd[2]^2 km[2]^2 T[1] - 2 kcr kd[1] kd[2] km[1] km[2] T[2] +
      kcr kd[2]^2 km[2]^2 T[2]) x[7]^2) + x[4]
    (-km[2]^3 T[1] - kcr km[1] km[2]^2 T[2] - km[2]^2 T[1] T[2] - kcr km[2]^2 T[1] T[2] +
      (-3 kd[2] km[2]^3 T[1] - kcr kd[1] km[1] km[2]^2 T[2] - 2 kcr kd[2] km[1] km[2]^2
        T[2] - 2 kd[2] km[2]^2 T[1] T[2] - 2 kcr kd[2] km[2]^2 T[1] T[2]) x[7] +
      (-3 kd[2]^2 km[2]^3 T[1] - 2 kcr kd[1] kd[2] km[1] km[2]^2 T[2] - kcr kd[2]^2 km[1]
        km[2]^2 T[2] - kd[2]^2 km[2]^2 T[1] T[2] - kcr kd[2]^2 km[2]^2 T[1] T[2]) x[7]^2 +
      (-kd[2]^3 km[2]^3 T[1] - kcr kd[1] kd[2]^2 km[1] km[2]^2 T[2])
      x[7]^3)), {x[4], x[7]}]

-km[2]^3 T[1] T[3] + (T[1] - kcr T[2]) x[4]^4 - 3 kd[2] km[2]^3 T[1] T[3] x[7] -
  3 kd[2]^2 km[2]^3 T[1] T[3] x[7]^2 - kd[2]^3 km[2]^3 T[1] T[3] x[7]^3 +
  x[4]^3 (3 km[2] T[1] + kcr km[1] T[2] - 2 kcr km[2] T[2] + T[1] T[2] +
    kcr T[1] T[2] - kcr T[2]^2 - kcr^2 T[2]^2 - T[1] T[3] + kcr T[2] T[3] +
    (3 kd[2] km[2] T[1] + kcr kd[1] km[1] T[2] - 2 kcr kd[2] km[2] T[2]) x[7]) +
  x[4]^2 (3 km[2]^2 T[1] + 2 kcr km[1] km[2] T[2] - kcr km[2]^2 T[2] +
    2 km[2] T[1] T[2] + 2 kcr km[2] T[1] T[2] - kcr km[2] T[2]^2 -
    kcr^2 km[2] T[2]^2 - 3 km[2] T[1] T[3] + 2 kcr km[2] T[2] T[3] +
    (6 kd[2] km[2]^2 T[1] + 2 kcr kd[1] km[1] km[2] T[2] + 2 kcr kd[2] km[1] km[2] T[2] -
      2 kcr kd[2] km[2]^2 T[2] + 2 kd[2] km[2] T[1] T[2] + 2 kcr kd[2] km[2] T[1] T[2] -
      kcr kd[2] km[2] T[2]^2 - kcr^2 kd[2] km[2] T[2]^2 - 3 kd[2] km[2] T[1] T[3] +
      2 kcr kd[2] km[2] T[2] T[3]) x[7] + (3 kd[2]^2 km[2]^2 T[1] +
      2 kcr kd[1] kd[2] km[1] km[2] T[2] - kcr kd[2]^2 km[2]^2 T[2]) x[7]^2) +
  x[4] (km[2]^3 T[1] + kcr km[1] km[2]^2 T[2] + km[2]^2 T[1] T[2] +
    kcr km[2]^2 T[1] T[2] - 3 km[2]^2 T[1] T[3] + kcr km[2]^2 T[2] T[3] +
    (3 kd[2] km[2]^3 T[1] + kcr kd[1] km[1] km[2]^2 T[2] + 2 kcr kd[2] km[1] km[2]^2 T[2] +
      2 kd[2] km[2]^2 T[1] T[2] + 2 kcr kd[2] km[2]^2 T[1] T[2] -
      6 kd[2] km[2]^2 T[1] T[3] + 2 kcr kd[2] km[2]^2 T[2] T[3]) x[7] +
    (3 kd[2]^2 km[2]^3 T[1] + 2 kcr kd[1] kd[2] km[1] km[2]^2 T[2] + kcr kd[2]^2
      km[1] km[2]^2 T[2] + kd[2]^2 km[2]^2 T[1] T[2] + kcr kd[2]^2 km[2]^2 T[1] T[2] -
      3 kd[2]^2 km[2]^2 T[1] T[3] + kcr kd[2]^2 km[2]^2 T[2] T[3]) x[7]^2 +
    (kd[2]^3 km[2]^3 T[1] + kcr kd[1] kd[2]^2 km[1] km[2]^2 T[2]) x[7]^3)

```

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Simplify[-km[2]^3 T[1] T[3] + (T[1] - kcr T[2]) x[4]^4 - 3 kd[2] km[2]^3 T[1] T[3] x[7] -
  3 kd[2]^2 km[2]^3 T[1] T[3] x[7]^2 - kd[2]^3 km[2]^3 T[1] T[3] x[7]^3 +
  x[4]^3 (3 km[2] T[1] + kcr km[1] T[2] - 2 kcr km[2] T[2] + T[1] T[2] +
    kcr T[1] T[2] - kcr T[2]^2 - kcr^2 T[2]^2 - T[1] T[3] + kcr T[2] T[3] +
    (3 kd[2] km[2] T[1] + kcr kd[1] km[1] T[2] - 2 kcr kd[2] km[2] T[2]) x[7]) +
  x[4]^2 (3 km[2]^2 T[1] + 2 kcr km[1] km[2] T[2] - kcr km[2]^2 T[2] +
    2 km[2] T[1] T[2] + 2 kcr km[2] T[1] T[2] - kcr km[2] T[2]^2 -
    kcr^2 km[2] T[2]^2 - 3 km[2] T[1] T[3] + 2 kcr km[2] T[2] T[3] +
    (6 kd[2] km[2]^2 T[1] + 2 kcr kd[1] km[1] km[2] T[2] + 2 kcr kd[2] km[1] km[2] T[2] -
    2 kcr kd[2] km[2]^2 T[2] + 2 kd[2] km[2] T[1] T[2] + 2 kcr kd[2] km[2] T[1] T[2] -
    kcr kd[2] km[2] T[2]^2 - kcr^2 kd[2] km[2] T[2]^2 - 3 kd[2] km[2] T[1] T[3] +
    2 kcr kd[2] km[2] T[2] T[3]) x[7] + (3 kd[2]^2 km[2]^2 T[1] +
    2 kcr kd[1] kd[2] km[1] km[2] T[2] - kcr kd[2]^2 km[2]^2 T[2]) x[7]^2) +
  x[4] (km[2]^3 T[1] + kcr km[1] km[2]^2 T[2] + km[2]^2 T[1] T[2] +
    kcr km[2]^2 T[1] T[2] - 3 km[2]^2 T[1] T[3] + kcr km[2]^2 T[2] T[3] +
    (3 kd[2] km[2]^3 T[1] + kcr kd[1] km[1] km[2]^2 T[2] + 2 kcr kd[2] km[1] km[2]^2 T[2] +
    2 kd[2] km[2]^2 T[1] T[2] + 2 kcr kd[2] km[2]^2 T[1] T[2] -
    6 kd[2] km[2]^2 T[1] T[3] + 2 kcr kd[2] km[2]^2 T[2] T[3]) x[7] +
    (3 kd[2]^2 km[2]^3 T[1] + 2 kcr kd[1] kd[2] km[1] km[2]^2 T[2] + kcr kd[2]^2
    km[1] km[2]^2 T[2] + kd[2]^2 km[2]^2 T[1] T[2] + kcr kd[2]^2 km[2]^2 T[1] T[2] -
    3 kd[2]^2 km[2]^2 T[1] T[3] + kcr kd[2]^2 km[2]^2 T[2] T[3]) x[7]^2 +
    (kd[2]^3 km[2]^3 T[1] + kcr kd[1] kd[2]^2 km[1] km[2]^2 T[2]) x[7]^3)]
(km[2] + x[4] + kd[2] km[2] x[7]) (-km[2]^2 T[1] (T[3] - x[4]) (1 + kd[2] x[7])^2 +
  km[2] x[4] (1 + kd[2] x[7]) (T[1] (T[2] - 2 T[3] + 2 x[4]) +
    kcr T[2] (km[1] + T[1] + T[3] - x[4] + kd[1] km[1] x[7])) +
  x[4]^2 (-kcr^2 T[2]^2 + T[1] (T[2] - T[3] + x[4]) +
    kcr T[2] (km[1] + T[1] - T[2] + T[3] - x[4] + kd[1] km[1] x[7])))

FullSimplify[-km[2]^2 T[1] (T[3] - x[4]) (1 + kd[2] x[7])^2 +
  km[2] x[4] (1 + kd[2] x[7]) (T[1] (T[2] - 2 T[3] + 2 x[4]) +
    kcr T[2] (km[1] + T[1] + T[3] - x[4] + kd[1] km[1] x[7])) +
  x[4]^2 (-kcr^2 T[2]^2 + T[1] (T[2] - T[3] + x[4]) +
    kcr T[2] (km[1] + T[1] - T[2] + T[3] - x[4] + kd[1] km[1] x[7]))]
-km[2]^2 T[1] (T[3] - x[4]) (1 + kd[2] x[7])^2 +
  km[2] x[4] (1 + kd[2] x[7]) (T[1] (T[2] - 2 T[3] + 2 x[4]) +
    kcr T[2] (km[1] + T[1] + T[3] - x[4] + kd[1] km[1] x[7])) +
  x[4]^2 (-kcr^2 T[2]^2 + T[1] (T[2] - T[3] + x[4]) +
    kcr T[2] (km[1] + T[1] - T[2] + T[3] - x[4] + kd[1] km[1] x[7]))

simpT4 =
Collect[{x[4] (-T[4] (1 + kd[1] x[7]) + x[7] (1 + kd[1] (T[1] - kcr T[2] + x[7]))) +
  km[2] (-T[4] (1 + kd[1] x[7]) (1 + kd[2] x[7]) +
    x[7] (1 + kd[2] (T[2] + x[7]) + kd[1] (T[1] + x[7] +
      kd[2] (T[1] + T[2]) x[7] + kd[2] x[7]^2)))} == 0], {x[4], x[7]}]

{-km[2] T[4] +
  (km[2] + kd[1] km[2] T[1] + kd[2] km[2] T[2] - kd[1] km[2] T[4] - kd[2] km[2] T[4])
  x[7] + (kd[1] km[2] + kd[2] km[2] + kd[1] kd[2] km[2] (T[1] + T[2]) -
    kd[1] kd[2] km[2] T[4]) x[7]^2 + kd[1] kd[2] km[2] x[7]^3 +
  x[4] (-T[4] + (1 + kd[1] T[1] - kcr kd[1] T[2] - kd[1] T[4]) x[7] + kd[1] x[7]^2) == 0}

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subSimpT4 = {-c[1] + c[2] x[7] + c[3] x[7]^2 +
  c[4] x[7]^3 - c[5] x[4] + c[6] x[4] x[7] + c[7] x[4] x[7]^2 == 0}

{-c[1] - c[5] x[4] + c[2] x[7] +
  c[6] x[4] x[7] + c[3] x[7]^2 + c[7] x[4] x[7]^2 + c[4] x[7]^3 == 0}

simpT3 = Collect[{-km[2]^2 T[1] (T[3] - x[4]) (1 + kd[2] x[7])^2 +
  km[2] x[4] (1 + kd[2] x[7]) (T[1] (T[2] - 2 T[3] + 2 x[4]) +
  kcr T[2] (km[1] + T[1] + T[3] - x[4] + kd[1] km[1] x[7])) +
  x[4]^2 (-kcr^2 T[2]^2 + T[1] (T[2] - T[3] + x[4]) + kcr T[2]
  (km[1] + T[1] - T[2] + T[3] - x[4] + kd[1] km[1] x[7])) == 0}, {x[4], x[7]}]

{-km[2]^2 T[1] T[3] + (T[1] - kcr T[2]) x[4]^3 -
  2 kd[2] km[2]^2 T[1] T[3] x[7] - kd[2]^2 km[2]^2 T[1] T[3] x[7]^2 +
  x[4]^2 (2 km[2] T[1] + kcr km[1] T[2] - kcr km[2] T[2] + T[1] T[2] +
  kcr T[1] T[2] - kcr T[2]^2 - kcr^2 T[2]^2 - T[1] T[3] + kcr T[2] T[3] +
  (2 kd[2] km[2] T[1] + kcr kd[1] km[1] T[2] - kcr kd[2] km[2] T[2]) x[7]) +
  x[4] (km[2]^2 T[1] + kcr km[1] km[2] T[2] + km[2] T[1] T[2] +
  kcr km[2] T[1] T[2] - 2 km[2] T[1] T[3] + kcr km[2] T[2] T[3] +
  (2 kd[2] km[2]^2 T[1] + kcr kd[1] km[1] km[2] T[2] + kcr kd[2] km[1] km[2] T[2] +
  kd[2] km[2] T[1] T[2] + kcr kd[2] km[2] T[1] T[2] -
  2 kd[2] km[2] T[1] T[3] + kcr kd[2] km[2] T[2] T[3]) x[7] +
  (kd[2]^2 km[2]^2 T[1] + kcr kd[1] kd[2] km[1] km[2] T[2]) x[7]^2) == 0}

subSimpT3 = {-t[1] + t[2] * t[4]^3 - t[3] x[7] - t[4] x[7]^2 + t[5] x[4]^2 +
  t[6] x[4]^2 x[7] + t[7] x[4] + t[8] x[4] x[7] + t[9] x[7]^2 == 0}

{-t[1] + t[2] t[4]^3 + t[7] x[4] + t[5] x[4]^2 - t[3] x[7] +
  t[8] x[4] x[7] + t[6] x[4]^2 x[7] - t[4] x[7]^2 + t[9] x[7]^2 == 0}

Solve[simpT4, x[4]]

{{x[4] -> (km[2] T[4] - km[2] x[7] - kd[1] km[2] T[1] x[7] - kd[2] km[2] T[2] x[7] +
  kd[1] km[2] T[4] x[7] + kd[2] km[2] T[4] x[7] - kd[1] km[2] x[7]^2 -
  kd[2] km[2] x[7]^2 - kd[1] kd[2] km[2] T[1] x[7]^2 - kd[1] kd[2] km[2] T[2] x[7]^2 +
  kd[1] kd[2] km[2] T[4] x[7]^2 - kd[1] kd[2] km[2] x[7]^3) / (-T[4] + x[7] +
  kd[1] T[1] x[7] - kcr kd[1] T[2] x[7] - kd[1] T[4] x[7] + kd[1] x[7]^2)}}

FullSimplify[(km[2] T[4] - km[2] x[7] - kd[1] km[2] T[1] x[7] - kd[2] km[2] T[2] x[7] +
  kd[1] km[2] T[4] x[7] + kd[2] km[2] T[4] x[7] - kd[1] km[2] x[7]^2 -
  kd[2] km[2] x[7]^2 - kd[1] kd[2] km[2] T[1] x[7]^2 - kd[1] kd[2] km[2] T[2] x[7]^2 +
  kd[1] kd[2] km[2] T[4] x[7]^2 - kd[1] kd[2] km[2] x[7]^3) /
  (-T[4] + x[7] + kd[1] T[1] x[7] - kcr kd[1] T[2] x[7] - kd[1] T[4] x[7] + kd[1] x[7]^2)]

(km[2] (-T[4] (1 + kd[1] x[7]) (1 + kd[2] x[7]) + x[7] (1 + kd[2] (T[2] + x[7]) +
  kd[1] (T[1] + x[7] + kd[2] (T[1] + T[2]) x[7] + kd[2] x[7]^2)))) /
  (T[4] (1 + kd[1] x[7]) - x[7] (1 + kd[1] (T[1] - kcr T[2] + x[7])))

```


t3Sol = Solve[t3, x[7]]

$$\left\{ \left\{ x[7] \rightarrow \frac{\left(2 \, kd[2] \, km[2]^2 \, T[1] \, T[3] - 2 \, kd[2] \, km[2]^2 \, T[1] \, x[4] - kcr \, kd[1] \, km[1] \, km[2] \, T[2] \, x[4] - kcr \, kd[2] \, km[1] \, km[2] \, T[2] \, x[4] - kd[2] \, km[2] \, T[1] \, T[2] \, x[4] - kcr \, kd[2] \, km[2] \, T[1] \, T[2] \, x[4] + 2 \, kd[2] \, km[2] \, T[1] \, T[3] \, x[4] - kcr \, kd[2] \, km[2] \, T[2] \, T[3] \, x[4] - 2 \, kd[2] \, km[2] \, T[1] \, x[4]^2 - kcr \, kd[1] \, km[1] \, T[2] \, x[4]^2 + kcr \, kd[2] \, km[2] \, T[2] \, x[4]^2 - \sqrt{\left((-2 \, kd[2] \, km[2]^2 \, T[1] \, T[3] + 2 \, kd[2] \, km[2]^2 \, T[1] \, x[4] + kcr \, kd[1] \, km[1] \, km[2] \, T[2] \, x[4] + kcr \, kd[2] \, km[1] \, km[2] \, T[2] \, x[4] + kd[2] \, km[2] \, T[1] \, T[2] \, x[4] + kcr \, kd[2] \, km[2] \, T[1] \, T[2] \, x[4] - 2 \, kd[2] \, km[2] \, T[1] \, T[3] \, x[4] + kcr \, kd[2] \, km[2] \, T[2] \, T[3] \, x[4] + 2 \, kd[2] \, km[2] \, T[1] \, x[4]^2 + kcr \, kd[1] \, km[1] \, T[2] \, x[4]^2 - kcr \, kd[2] \, km[2] \, T[2] \, x[4]^2 \right)^2 - 4 \left(-kd[2]^2 \, km[2]^2 \, T[1] \, T[3] + kd[2]^2 \, km[2]^2 \, T[1] \, x[4] + kcr \, kd[1] \, kd[2] \, km[1] \, km[2] \, T[2] \, x[4] \right) \left(-km[2]^2 \, T[1] \, T[3] + km[2]^2 \, T[1] \, x[4] + kcr \, km[1] \, km[2] \, T[2] \, x[4] + km[2] \, T[1] \, T[2] \, x[4] + kcr \, km[2] \, T[1] \, T[2] \, x[4] - 2 \, km[2] \, T[1] \, T[3] \, x[4] + kcr \, km[2] \, T[2] \, T[3] \, x[4] + 2 \, km[2] \, T[1] \, x[4]^2 + kcr \, km[1] \, T[2] \, x[4]^2 - kcr \, km[2] \, T[2] \, x[4]^2 + T[1] \, T[2] \, x[4]^2 + kcr \, T[1] \, T[2] \, x[4]^2 - kcr \, T[2]^2 \, x[4]^2 - kcr^2 \, T[2]^2 \, x[4]^2 - T[1] \, T[3] \, x[4]^2 + kcr \, T[2] \, T[3] \, x[4]^2 + T[1] \, x[4]^3 - kcr \, T[2] \, x[4]^3 \right)} \right) \right) / \left(2 \left(-kd[2]^2 \, km[2]^2 \, T[1] \, T[3] + kd[2]^2 \, km[2]^2 \, T[1] \, x[4] + kcr \, kd[1] \, kd[2] \, km[1] \, km[2] \, T[2] \, x[4] \right) \right) \right\}, \left\{ x[7] \rightarrow \left(2 \, kd[2] \, km[2]^2 \, T[1] \, T[3] - 2 \, kd[2] \, km[2]^2 \, T[1] \, x[4] - kcr \, kd[1] \, km[1] \, km[2] \, T[2] \, x[4] - kcr \, kd[2] \, km[1] \, km[2] \, T[2] \, x[4] - kd[2] \, km[2] \, T[1] \, T[2] \, x[4] - kcr \, kd[2] \, km[2] \, T[1] \, T[2] \, x[4] + 2 \, kd[2] \, km[2] \, T[1] \, T[3] \, x[4] - kcr \, kd[2] \, km[2] \, T[2] \, T[3] \, x[4] - 2 \, kd[2] \, km[2] \, T[1] \, x[4]^2 - kcr \, kd[1] \, km[1] \, T[2] \, x[4]^2 + kcr \, kd[2] \, km[2] \, T[2] \, x[4]^2 - \sqrt{\left((-2 \, kd[2] \, km[2]^2 \, T[1] \, T[3] + 2 \, kd[2] \, km[2]^2 \, T[1] \, x[4] + kcr \, kd[1] \, km[1] \, km[2] \, T[2] \, x[4] + kcr \, kd[2] \, km[1] \, km[2] \, T[2] \, x[4] + kd[2] \, km[2] \, T[1] \, T[2] \, x[4] + kcr \, kd[2] \, km[2] \, T[1] \, T[2] \, x[4] - 2 \, kd[2] \, km[2] \, T[1] \, T[3] \, x[4] + kcr \, kd[2] \, km[2] \, T[2] \, T[3] \, x[4] + 2 \, kd[2] \, km[2] \, T[1] \, x[4]^2 + kcr \, kd[1] \, km[1] \, T[2] \, x[4]^2 - kcr \, kd[2] \, km[2] \, T[2] \, x[4]^2 \right)^2 - 4 \left(-kd[2]^2 \, km[2]^2 \, T[1] \, T[3] + kd[2]^2 \, km[2]^2 \, T[1] \, x[4] + kcr \, kd[1] \, kd[2] \, km[1] \, km[2] \, T[2] \, x[4] \right) \left(-km[2]^2 \, T[1] \, T[3] + km[2]^2 \, T[1] \, x[4] + kcr \, km[1] \, km[2] \, T[2] \, x[4] + km[2] \, T[1] \, T[2] \, x[4] + kcr \, km[2] \, T[1] \, T[2] \, x[4] - 2 \, km[2] \, T[1] \, T[3] \, x[4] + kcr \, km[2] \, T[2] \, T[3] \, x[4] + 2 \, km[2] \, T[1] \, x[4]^2 + kcr \, km[1] \, T[2] \, x[4]^2 - kcr \, km[2] \, T[2] \, x[4]^2 + T[1] \, T[2] \, x[4]^2 + kcr \, T[1] \, T[2] \, x[4]^2 - kcr \, T[2]^2 \, x[4]^2 - kcr^2 \, T[2]^2 \, x[4]^2 - T[1] \, T[3] \, x[4]^2 + kcr \, T[2] \, T[3] \, x[4]^2 + T[1] \, x[4]^3 - kcr \, T[2] \, x[4]^3 \right)} \right) \right) / \left(2 \left(-kd[2]^2 \, km[2]^2 \, T[1] \, T[3] + kd[2]^2 \, km[2]^2 \, T[1] \, x[4] + kcr \, kd[1] \, kd[2] \, km[1] \, km[2] \, T[2] \, x[4] \right) \right) \right\} \right\}$$

The following is substituting the first solution of x[7]: $\frac{-b - \sqrt{b^2 - 4ac}}{2a}$

t4Inv1 = t4 /. t3Sol[[1]][[1]]

$$\{ T[4] == \left(2 \, kd[2] \, km[2]^2 \, T[1] \, T[3] - 2 \, kd[2] \, km[2]^2 \, T[1] \, x[4] - kcr \, kd[1] \, km[1] \, km[2] \, T[2] \, x[4] - kcr \, kd[2] \, km[1] \, km[2] \, T[2] \, x[4] - kd[2] \, km[2] \, T[1] \, T[2] \, x[4] - \right.$$

$$\begin{aligned}
& \left(\frac{kcr km[1] T[2] x[4]^2 - kcr km[2] T[2] x[4]^2 + T[1] T[2] x[4]^2 + kcr T[1] T[2] x[4]^2 - kcr T[2]^2 x[4]^2 - kcr^2 T[2]^2 x[4]^2 - T[1] T[3] x[4]^2 + kcr T[2] T[3] x[4]^2 + T[1] x[4]^3 - kcr T[2] x[4]^3}{2 \left(-kd[2]^2 km[2]^2 T[1] T[3] + kd[2]^2 km[2]^2 T[1] x[4] + kcr kd[1] kd[2] km[1] km[2] T[2] x[4] \right)} \right) - \\
& \left(kd[1] \left(2 kd[2] km[2]^2 T[1] T[3] - 2 kd[2] km[2]^2 T[1] x[4] - kcr kd[1] km[1] km[2] T[2] x[4] - kcr kd[2] km[1] km[2] T[2] x[4] - kd[2] km[2] T[1] T[2] x[4] - kcr kd[2] km[2] T[1] T[2] x[4] + 2 kd[2] km[2] T[1] T[3] x[4] - kcr kd[2] km[2] T[2] T[3] x[4] - 2 kd[2] km[2] T[1] x[4]^2 - kcr kd[1] km[1] T[2] x[4]^2 + kcr kd[2] km[2] T[2] x[4]^2 - \sqrt{\left(-2 kd[2] km[2]^2 T[1] T[3] + 2 kd[2] km[2]^2 T[1] x[4] + kcr kd[1] km[1] km[2] T[2] x[4] + kcr kd[2] km[1] km[2] T[2] x[4] + kd[2] km[2] T[1] T[2] x[4] + kcr kd[2] km[2] T[1] T[2] x[4] - 2 kd[2] km[2] T[1] T[3] x[4] + kcr kd[2] km[2] T[2] T[3] x[4] + 2 kd[2] km[2] T[1] x[4]^2 + kcr kd[1] km[1] T[2] x[4]^2 - kcr kd[2] km[2] T[2] x[4]^2 \right)^2} - 4 \left(-kd[2]^2 km[2]^2 T[1] T[3] + kd[2]^2 km[2]^2 T[1] x[4] + kcr kd[1] kd[2] km[1] km[2] T[2] x[4] \right) \left(-km[2]^2 T[1] T[3] + km[2]^2 T[1] x[4] + kcr km[1] km[2] T[2] x[4] + km[2] T[1] T[2] x[4] + kcr km[2] T[1] T[2] x[4] - 2 km[2] T[1] T[3] x[4] + kcr km[2] T[2] T[3] x[4] + 2 km[2] T[1] x[4]^2 + kcr km[1] T[2] x[4]^2 - kcr km[2] T[2] x[4]^2 + T[1] T[2] x[4]^2 + kcr T[1] T[2] x[4]^2 - kcr T[2]^2 x[4]^2 - kcr^2 T[2]^2 x[4]^2 - T[1] T[3] x[4]^2 + kcr T[2] T[3] x[4]^2 + T[1] x[4]^3 - kcr T[2] x[4]^3 \right) \right) \right) \\
& \left(-km[2] T[1] - T[1] x[4] + kcr T[2] x[4] - \left(kd[2] km[2] T[1] \left(2 kd[2] km[2]^2 T[1] T[3] - 2 kd[2] km[2]^2 T[1] x[4] - kcr kd[1] km[1] km[2] T[2] x[4] - kcr kd[2] km[1] km[2] T[2] x[4] - kd[2] km[2] T[1] T[2] x[4] - kcr kd[2] km[2] T[1] T[2] x[4] + 2 kd[2] km[2] T[1] T[3] x[4] - kcr kd[2] km[2] T[2] T[3] x[4] - 2 kd[2] km[2] T[1] x[4]^2 - kcr kd[1] km[1] T[2] x[4]^2 + kcr kd[2] km[2] T[2] x[4]^2 - \sqrt{\left(-2 kd[2] km[2]^2 T[1] T[3] + 2 kd[2] km[2]^2 T[1] x[4] + kcr kd[1] km[1] km[2] T[2] x[4] + kcr kd[2] km[1] km[2] T[2] x[4] + kd[2] km[2] T[1] T[2] x[4] + kcr kd[2] km[2] T[1] T[2] x[4] - 2 kd[2] km[2] T[1] T[3] x[4] + kcr kd[2] km[2] T[2] T[3] x[4] + 2 kd[2] km[2] T[1] x[4]^2 + kcr kd[1] km[1] T[2] x[4]^2 - kcr kd[2] km[2] T[2] x[4]^2 \right)^2} - 4 \left(-kd[2]^2 km[2]^2 T[1] T[3] + kd[2]^2 km[2]^2 T[1] x[4] + kcr kd[1] kd[2] km[1] km[2] T[2] x[4] \right) \left(-km[2]^2 T[1] T[3] + km[2]^2 T[1] x[4] + kcr km[1] km[2] T[2] x[4] + km[2] T[1] T[2] x[4] + kcr km[2] T[1] T[2] x[4] - 2 km[2] T[1] T[3] x[4] + kcr km[2] T[2] T[3] x[4] + 2 km[2] T[1] x[4]^2 + kcr km[1] T[2] x[4]^2 - kcr km[2] T[2] x[4]^2 + T[1] T[2] x[4]^2 + kcr T[1] T[2] x[4]^2 - kcr T[2]^2 x[4]^2 - kcr^2 T[2]^2 x[4]^2 - T[1] T[3] x[4]^2 + kcr T[2] T[3] x[4]^2 + T[1] x[4]^3 - kcr T[2] x[4]^3 \right) \right) \right) \right) / \\
& \left(2 \left(-kd[2]^2 km[2]^2 T[1] T[3] + kd[2]^2 km[2]^2 T[1] x[4] + kcr kd[1] kd[2] km[1] km[2] T[2] x[4] \right) \right) / \\
& \left(2 \left(-kd[2]^2 km[2]^2 T[1] T[3] + kd[2]^2 km[2]^2 T[1] x[4] + kcr kd[1] kd[2] km[1] km[2] T[2] x[4] \right) \right) / \\
& \left(1 + \left(kd[1] \left(2 kd[2] km[2]^2 T[1] T[3] - 2 kd[2] km[2]^2 T[1] x[4] - \right. \right. \right.
\end{aligned}$$


```

Simplify[2 kd[2] km[2]^2 T[1] T[3] - 2 kd[2] km[2]^2 T[1] x[4] -
  kcr kd[1] km[1] km[2] T[2] x[4] - kcr kd[2] km[1] km[2] T[2] x[4] -
  kd[2] km[2] T[1] T[2] x[4] - kcr kd[2] km[2] T[1] T[2] x[4] +
  2 kd[2] km[2] T[1] T[3] x[4] - kcr kd[2] km[2] T[2] T[3] x[4] -
  2 kd[2] km[2] T[1] x[4]^2 - kcr kd[1] km[1] T[2] x[4]^2 + kcr kd[2] km[2] T[2] x[4]^2]
-kcr kd[1] km[1] T[2] x[4] (km[2] + x[4]) + kd[2] km[2] (2 km[2] T[1] (T[3] - x[4]) +
  x[4] (-kcr T[2] (km[1] + T[1] + T[3] - x[4]) - T[1] (T[2] - 2 T[3] + 2 x[4])))

```

And we have the non-trivial solution of inverse function of solution x[4].

```
t4Inv2 = t4 /. t3Sol[[2]]
```

```

{T[4] ==
  (2 kd[2] km[2]^2 T[1] T[3] - 2 kd[2] km[2]^2 T[1] x[4] - kcr kd[1] km[1] km[2] T[2] x[4] -
    kcr kd[2] km[1] km[2] T[2] x[4] - kd[2] km[2] T[1] T[2] x[4] -
    kcr kd[2] km[2] T[1] T[2] x[4] + 2 kd[2] km[2] T[1] T[3] x[4] -
    kcr kd[2] km[2] T[2] T[3] x[4] - 2 kd[2] km[2] T[1] x[4]^2 -
    kcr kd[1] km[1] T[2] x[4]^2 + kcr kd[2] km[2] T[2] x[4]^2 +
    Sqrt[(-2 kd[2] km[2]^2 T[1] T[3] + 2 kd[2] km[2]^2 T[1] x[4] + kcr kd[1] km[1] km[2]
      T[2] x[4] + kcr kd[2] km[1] km[2] T[2] x[4] + kd[2] km[2] T[1] T[2]
      x[4] + kcr kd[2] km[2] T[1] T[2] x[4] - 2 kd[2] km[2] T[1] T[3] x[4] +
      kcr kd[2] km[2] T[2] T[3] x[4] + 2 kd[2] km[2] T[1] x[4]^2 +
      kcr kd[1] km[1] T[2] x[4]^2 - kcr kd[2] km[2] T[2] x[4]^2)^2 -
      4 (-kd[2]^2 km[2]^2 T[1] T[3] + kd[2]^2 km[2]^2 T[1] x[4] +
        kcr kd[1] kd[2] km[1] km[2] T[2] x[4]) (-km[2]^2 T[1] T[3] +
        km[2]^2 T[1] x[4] + kcr km[1] km[2] T[2] x[4] + km[2] T[1] T[2] x[4] +
        kcr km[2] T[1] T[2] x[4] - 2 km[2] T[1] T[3] x[4] + kcr km[2] T[2] T[3]
        x[4] + 2 km[2] T[1] x[4]^2 + kcr km[1] T[2] x[4]^2 - kcr km[2] T[2] x[4]^2 +
        T[1] T[2] x[4]^2 + kcr T[1] T[2] x[4]^2 - kcr T[2]^2 x[4]^2 - kcr^2 T[2]^2 x[4]^2 -
        T[1] T[3] x[4]^2 + kcr T[2] T[3] x[4]^2 + T[1] x[4]^3 - kcr T[2] x[4]^3)]]) /
    (2 (-kd[2]^2 km[2]^2 T[1] T[3] + kd[2]^2 km[2]^2 T[1] x[4] +
      kcr kd[1] kd[2] km[1] km[2] T[2] x[4])) +
  (kd[2] km[2] T[2] (2 kd[2] km[2]^2 T[1] T[3] - 2 kd[2] km[2]^2 T[1] x[4] -
    kcr kd[1] km[1] km[2] T[2] x[4] -
    kcr kd[2] km[1] km[2] T[2] x[4] - kd[2] km[2] T[1] T[2] x[4] -
    kcr kd[2] km[2] T[1] T[2] x[4] + 2 kd[2] km[2] T[1] T[3] x[4] -
    kcr kd[2] km[2] T[2] T[3] x[4] - 2 kd[2] km[2] T[1] x[4]^2 -
    kcr kd[1] km[1] T[2] x[4]^2 + kcr kd[2] km[2] T[2] x[4]^2 +
    Sqrt[(-2 kd[2] km[2]^2 T[1] T[3] + 2 kd[2] km[2]^2 T[1] x[4] + kcr kd[1] km[1]
      km[2] T[2] x[4] + kcr kd[2] km[1] km[2] T[2] x[4] + kd[2] km[2]
      T[1] T[2] x[4] + kcr kd[2] km[2] T[1] T[2] x[4] - 2 kd[2] km[2] T[1]
      T[3] x[4] + kcr kd[2] km[2] T[2] T[3] x[4] + 2 kd[2] km[2] T[1]
      x[4]^2 + kcr kd[1] km[1] T[2] x[4]^2 - kcr kd[2] km[2] T[2] x[4]^2)^2 -
      4 (-kd[2]^2 km[2]^2 T[1] T[3] + kd[2]^2 km[2]^2 T[1] x[4] + kcr kd[1] kd[2]
        km[1] km[2] T[2] x[4]) (-km[2]^2 T[1] T[3] + km[2]^2 T[1] x[4] + kcr km[1]
        km[2] T[2] x[4] + km[2] T[1] T[2] x[4] + kcr km[2] T[1] T[2] x[4] -
        2 km[2] T[1] T[3] x[4] + kcr km[2] T[2] T[3] x[4] + 2 km[2] T[1] x[4]^2 +
        kcr km[1] T[2] x[4]^2 - kcr km[2] T[2] x[4]^2 + T[1] T[2] x[4]^2 +
        kcr T[1] T[2] x[4]^2 - kcr T[2]^2 x[4]^2 - kcr^2 T[2]^2 x[4]^2 -
        T[1] T[3] x[4]^2 + kcr T[2] T[3] x[4]^2 + T[1] x[4]^3 - kcr T[2] x[4]^3)]]) /
    (2 (-kd[2]^2 km[2]^2 T[1] T[3] + kd[2]^2 km[2]^2 T[1] x[4] +
      kcr kd[1] kd[2] km[1] km[2] T[2] x[4])
    (km[2] + x[4] + (kd[2] km[2] (2 kd[2] km[2]^2 T[1] T[3] - 2 kd[2] km[2]^2 T[1] x[4] -

```


$$\begin{aligned} & \left(2 \left(-kd[2]^2 km[2]^2 T[1] T[3] + kd[2]^2 km[2]^2 T[1] x[4] + \right. \right. \\ & \quad \left. \left. kcr kd[1] kd[2] km[1] km[2] T[2] x[4] \right) \right) \Big/ \\ & \left(2 \left(-kd[2]^2 km[2]^2 T[1] T[3] + kd[2]^2 km[2]^2 T[1] x[4] + \right. \right. \\ & \quad \left. \left. kcr kd[1] kd[2] km[1] km[2] T[2] x[4] \right) \right. \\ & \quad \left(1 + \left(kd[1] \left(2 kd[2] km[2]^2 T[1] T[3] - 2 kd[2] km[2]^2 T[1] x[4] - \right. \right. \right. \\ & \quad \left. \left. kcr kd[1] km[1] km[2] T[2] x[4] - kcr kd[2] km[1] km[2] T[2] x[4] - \right. \right. \\ & \quad \left. \left. kd[2] km[2] T[1] T[2] x[4] - kcr kd[2] km[2] T[1] T[2] x[4] + \right. \right. \\ & \quad \left. \left. 2 kd[2] km[2] T[1] T[3] x[4] - kcr kd[2] km[2] T[2] T[3] x[4] - \right. \right. \\ & \quad \left. \left. 2 kd[2] km[2] T[1] x[4]^2 - kcr kd[1] km[1] T[2] x[4]^2 + kcr kd[2] km[2] \right. \right. \\ & \quad \left. \left. T[2] x[4]^2 + \sqrt{\left((-2 kd[2] km[2]^2 T[1] T[3] + 2 kd[2] km[2]^2 T[1] x[4] + \right. \right. \right. \\ & \quad \left. \left. kcr kd[1] km[1] km[2] T[2] x[4] + kcr kd[2] km[1] km[2] T[2] \right. \right. \\ & \quad \left. \left. x[4] + kd[2] km[2] T[1] T[2] x[4] + kcr kd[2] km[2] T[1] T[2] \right. \right. \\ & \quad \left. \left. x[4] - 2 kd[2] km[2] T[1] T[3] x[4] + kcr kd[2] km[2] T[2] T[3] \right. \right. \\ & \quad \left. \left. x[4] + 2 kd[2] km[2] T[1] x[4]^2 + kcr kd[1] km[1] T[2] x[4]^2 - \right. \right. \\ & \quad \left. \left. kcr kd[2] km[2] T[2] x[4]^2 \right)^2 - 4 \left(-kd[2]^2 km[2]^2 T[1] T[3] + \right. \right. \\ & \quad \left. \left. kd[2]^2 km[2]^2 T[1] x[4] + kcr kd[1] kd[2] km[1] km[2] T[2] x[4] \right) \right) \\ & \quad \left(-km[2]^2 T[1] T[3] + km[2]^2 T[1] x[4] + kcr km[1] km[2] T[2] x[4] + \right. \\ & \quad \left. km[2] T[1] T[2] x[4] + kcr km[2] T[1] T[2] x[4] - 2 km[2] T[1] \right. \\ & \quad \left. T[3] x[4] + kcr km[2] T[2] T[3] x[4] + 2 km[2] T[1] x[4]^2 + \right. \\ & \quad \left. kcr km[1] T[2] x[4]^2 - kcr km[2] T[2] x[4]^2 + T[1] T[2] x[4]^2 + kcr \right. \\ & \quad \left. T[1] T[2] x[4]^2 - kcr T[2]^2 x[4]^2 - kcr^2 T[2]^2 x[4]^2 - T[1] T[3] \right. \\ & \quad \left. x[4]^2 + kcr T[2] T[3] x[4]^2 + T[1] x[4]^3 - kcr T[2] x[4]^3 \right) \Big) \Big/ \\ & \left(2 \left(-kd[2]^2 km[2]^2 T[1] T[3] + kd[2]^2 km[2]^2 T[1] x[4] + \right. \right. \\ & \quad \left. \left. kcr kd[1] kd[2] km[1] km[2] T[2] x[4] \right) \right) \Big) \\ & \left(km[2] + x[4] + \left(kd[2] km[2] \left(2 kd[2] km[2]^2 T[1] T[3] - \right. \right. \right. \\ & \quad \left. \left. 2 kd[2] km[2]^2 T[1] x[4] - kcr kd[1] km[1] km[2] T[2] x[4] - \right. \right. \\ & \quad \left. \left. kcr kd[2] km[1] km[2] T[2] x[4] - kd[2] km[2] T[1] T[2] x[4] - \right. \right. \\ & \quad \left. \left. kcr kd[2] km[2] T[1] T[2] x[4] + 2 kd[2] km[2] T[1] T[3] x[4] - \right. \right. \\ & \quad \left. \left. kcr kd[2] km[2] T[2] T[3] x[4] - 2 kd[2] km[2] T[1] x[4]^2 - \right. \right. \\ & \quad \left. \left. kcr kd[1] km[1] T[2] x[4]^2 + kcr kd[2] km[2] T[2] x[4]^2 + \right. \right. \\ & \quad \left. \left. \sqrt{\left((-2 kd[2] km[2]^2 T[1] T[3] + 2 kd[2] km[2]^2 T[1] x[4] + kcr kd[1] \right. \right. \right. \\ & \quad \left. \left. km[1] km[2] T[2] x[4] + kcr kd[2] km[1] km[2] T[2] x[4] + \right. \right. \\ & \quad \left. \left. kd[2] km[2] T[1] T[2] x[4] + kcr kd[2] km[2] T[1] T[2] x[4] - \right. \right. \\ & \quad \left. \left. 2 kd[2] km[2] T[1] T[3] x[4] + kcr kd[2] km[2] T[2] T[3] x[4] + \right. \right. \\ & \quad \left. \left. 2 kd[2] km[2] T[1] x[4]^2 + kcr kd[1] km[1] T[2] x[4]^2 - \right. \right. \\ & \quad \left. \left. kcr kd[2] km[2] T[2] x[4]^2 \right)^2 - 4 \left(-kd[2]^2 km[2]^2 T[1] T[3] + \right. \right. \\ & \quad \left. \left. kd[2]^2 km[2]^2 T[1] x[4] + kcr kd[1] kd[2] km[1] km[2] T[2] x[4] \right) \right) \\ & \quad \left(-km[2]^2 T[1] T[3] + km[2]^2 T[1] x[4] + kcr km[1] km[2] T[2] x[4] + \right. \\ & \quad \left. km[2] T[1] T[2] x[4] + kcr km[2] T[1] T[2] x[4] - 2 km[2] T[1] \right. \\ & \quad \left. T[3] x[4] + kcr km[2] T[2] T[3] x[4] + 2 km[2] T[1] x[4]^2 + \right. \\ & \quad \left. kcr km[1] T[2] x[4]^2 - kcr km[2] T[2] x[4]^2 + T[1] T[2] x[4]^2 + kcr \right. \\ & \quad \left. T[1] T[2] x[4]^2 - kcr T[2]^2 x[4]^2 - kcr^2 T[2]^2 x[4]^2 - T[1] T[3] \right. \\ & \quad \left. x[4]^2 + kcr T[2] T[3] x[4]^2 + T[1] x[4]^3 - kcr T[2] x[4]^3 \right) \Big) \Big) \Big/ \end{aligned}$$

$$\begin{aligned}
& \left(\begin{aligned} & x[4] - 2 \, kd[2] \, km[2] \, T[1] \, T[3] \, x[4] + kcr \, kd[2] \, km[2] \, T[2] \, T[3] \\ & x[4] + 2 \, kd[2] \, km[2] \, T[1] \, x[4]^2 + kcr \, kd[1] \, km[1] \, T[2] \, x[4]^2 - \\ & kcr \, kd[2] \, km[2] \, T[2] \, x[4]^2 \Big)^2 - 4 \left(-kd[2]^2 \, km[2]^2 \, T[1] \, T[3] + \right. \\ & kd[2]^2 \, km[2]^2 \, T[1] \, x[4] + kcr \, kd[1] \, kd[2] \, km[1] \, km[2] \, T[2] \, x[4] \Big) \\ & \left(-km[2]^2 \, T[1] \, T[3] + km[2]^2 \, T[1] \, x[4] + kcr \, km[1] \, km[2] \, T[2] \, x[4] + \right. \\ & km[2] \, T[1] \, T[2] \, x[4] + kcr \, km[2] \, T[1] \, T[2] \, x[4] - 2 \, km[2] \, T[1] \\ & T[3] \, x[4] + kcr \, km[2] \, T[2] \, T[3] \, x[4] + 2 \, km[2] \, T[1] \, x[4]^2 + \\ & kcr \, km[1] \, T[2] \, x[4]^2 - kcr \, km[2] \, T[2] \, x[4]^2 + T[1] \, T[2] \, x[4]^2 + kcr \\ & T[1] \, T[2] \, x[4]^2 - kcr \, T[2]^2 \, x[4]^2 - kcr^2 \, T[2]^2 \, x[4]^2 - T[1] \, T[3] \\ & x[4]^2 + kcr \, T[2] \, T[3] \, x[4]^2 + T[1] \, x[4]^3 - kcr \, T[2] \, x[4]^3 \Big) \Big) \Big) / \\ & \left(2 \left(-kd[2]^2 \, km[2]^2 \, T[1] \, T[3] + kd[2]^2 \, km[2]^2 \, T[1] \, x[4] + \right. \right. \\ & \left. \left. kcr \, kd[1] \, kd[2] \, km[1] \, km[2] \, T[2] \, x[4] \right) \right) \Big) - \\ & \left(kd[1] \left(2 \, kd[2] \, km[2]^2 \, T[1] \, T[3] - 2 \, kd[2] \, km[2]^2 \, T[1] \, x[4] - \right. \right. \\ & kcr \, kd[1] \, km[1] \, km[2] \, T[2] \, x[4] - \\ & kcr \, kd[2] \, km[1] \, km[2] \, T[2] \, x[4] - \\ & kd[2] \, km[2] \, T[1] \, T[2] \, x[4] - \\ & kcr \, kd[2] \, km[2] \, T[1] \, T[2] \, x[4] + \\ & 2 \, kd[2] \, km[2] \, T[1] \, T[3] \, x[4] - \\ & kcr \, kd[2] \, km[2] \, T[2] \, T[3] \, x[4] - \\ & 2 \, kd[2] \, km[2] \, T[1] \, x[4]^2 - kcr \, kd[1] \, km[1] \, T[2] \, x[4]^2 + \\ & kcr \, kd[2] \, km[2] \, T[2] \, x[4]^2 + \\ & \sqrt{\left(\left(-2 \, kd[2] \, km[2]^2 \, T[1] \, T[3] + 2 \, kd[2] \, km[2]^2 \, T[1] \, x[4] + kcr \, kd[1] \, km[1] \right. \right. \\ & km[2] \, T[2] \, x[4] + kcr \, kd[2] \, km[1] \, km[2] \, T[2] \, x[4] + kd[2] \, km[2] \\ & T[1] \, T[2] \, x[4] + kcr \, kd[2] \, km[2] \, T[1] \, T[2] \, x[4] - 2 \, kd[2] \, km[2] \, T[1] \\ & T[3] \, x[4] + kcr \, kd[2] \, km[2] \, T[2] \, T[3] \, x[4] + 2 \, kd[2] \, km[2] \, T[1] \\ & x[4]^2 + kcr \, kd[1] \, km[1] \, T[2] \, x[4]^2 - kcr \, kd[2] \, km[2] \, T[2] \, x[4]^2 \Big)^2 - \\ & 4 \left(-kd[2]^2 \, km[2]^2 \, T[1] \, T[3] + kd[2]^2 \, km[2]^2 \, T[1] \, x[4] + kcr \, kd[1] \, kd[2] \right. \\ & km[1] \, km[2] \, T[2] \, x[4] \Big) \left(-km[2]^2 \, T[1] \, T[3] + km[2]^2 \, T[1] \, x[4] + kcr \, km[1] \right. \\ & km[2] \, T[2] \, x[4] + km[2] \, T[1] \, T[2] \, x[4] + kcr \, km[2] \, T[1] \, T[2] \, x[4] - \\ & 2 \, km[2] \, T[1] \, T[3] \, x[4] + kcr \, km[2] \, T[2] \, T[3] \, x[4] + 2 \, km[2] \, T[1] \, x[4]^2 + \\ & kcr \, km[1] \, T[2] \, x[4]^2 - kcr \, km[2] \, T[2] \, x[4]^2 + T[1] \, T[2] \, x[4]^2 + \\ & kcr \, T[1] \, T[2] \, x[4]^2 - kcr \, T[2]^2 \, x[4]^2 - kcr^2 \, T[2]^2 \, x[4]^2 - \\ & T[1] \, T[3] \, x[4]^2 + kcr \, T[2] \, T[3] \, x[4]^2 + T[1] \, x[4]^3 - kcr \, T[2] \, x[4]^3 \Big) \Big) \Big) \\ & \left(-km[2] \, T[1] - T[1] \, x[4] + kcr \, T[2] \, x[4] - \left(kd[2] \, km[2] \, T[1] \right. \right. \\ & \left. \left(2 \, kd[2] \, km[2]^2 \, T[1] \, T[3] - 2 \, kd[2] \, km[2]^2 \, T[1] \, x[4] - \right. \right. \\ & kcr \, kd[1] \, km[1] \, km[2] \, T[2] \, x[4] - kcr \, kd[2] \, km[1] \, km[2] \, T[2] \, x[4] - \\ & kd[2] \, km[2] \, T[1] \, T[2] \, x[4] - kcr \, kd[2] \, km[2] \, T[1] \, T[2] \, x[4] + \\ & 2 \, kd[2] \, km[2] \, T[1] \, T[3] \, x[4] - kcr \, kd[2] \, km[2] \, T[2] \, T[3] \, x[4] - \\ & 2 \, kd[2] \, km[2] \, T[1] \, x[4]^2 - kcr \, kd[1] \, km[1] \, T[2] \, x[4]^2 + kcr \, kd[2] \, km[2] \\ & T[2] \, x[4]^2 + \sqrt{\left(\left(-2 \, kd[2] \, km[2]^2 \, T[1] \, T[3] + 2 \, kd[2] \, km[2]^2 \, T[1] \, x[4] + \right. \right. \\ & kcr \, kd[1] \, km[1] \, km[2] \, T[2] \, x[4] + kcr \, kd[2] \, km[1] \, km[2] \, T[2] \\ & x[4] + kd[2] \, km[2] \, T[1] \, T[2] \, x[4] + kcr \, kd[2] \, km[2] \, T[1] \, T[2] \\ & x[4] - 2 \, kd[2] \, km[2] \, T[1] \, T[3] \, x[4] + kcr \, kd[2] \, km[2] \, T[2] \, T[3] \\ & x[4] + 2 \, kd[2] \, km[2] \, T[1] \, x[4]^2 + kcr \, kd[1] \, km[1] \, T[2] \, x[4]^2 - \\ & kcr \, kd[2] \, km[2] \, T[2] \, x[4]^2 \Big)^2 - 4 \left(-kd[2]^2 \, km[2]^2 \, T[1] \, T[3] + \right. \\ & kd[2]^2 \, km[2]^2 \, T[1] \, x[4] + kcr \, kd[1] \, kd[2] \, km[1] \, km[2] \, T[2] \, x[4] \Big) \\ & \left(-km[2]^2 \, T[1] \, T[3] + km[2]^2 \, T[1] \, x[4] + kcr \, km[1] \, km[2] \, T[2] \, x[4] + \right. \\ & km[2] \, T[1] \, T[2] \, x[4] + kcr \, km[2] \, T[1] \, T[2] \, x[4] - 2 \, km[2] \, T[1] \\ & T[3] \, x[4] + kcr \, km[2] \, T[2] \, T[3] \, x[4] + 2 \, km[2] \, T[1] \, x[4]^2 + \\ & kcr \, km[1] \, T[2] \, x[4]^2 - kcr \, km[2] \, T[2] \, x[4]^2 + T[1] \, T[2] \, x[4]^2 + kcr \\ & T[1] \, T[2] \, x[4]^2 - kcr \, T[2]^2 \, x[4]^2 - kcr^2 \, T[2]^2 \, x[4]^2 - T[1] \, T[3] \\ & x[4]^2 + kcr \, T[2] \, T[3] \, x[4]^2 + T[1] \, x[4]^3 - kcr \, T[2] \, x[4]^3 \Big) \Big) \Big) / \end{aligned}
\end{aligned}$$

$$\begin{aligned}
& \left(2 \left(-kd[2]^2 km[2]^2 T[1] T[3] + kd[2]^2 km[2]^2 T[1] x[4] + \right. \right. \\
& \quad \left. \left. kcr kd[1] kd[2] km[1] km[2] T[2] x[4] \right) \right) \Big/ \\
& \left(2 \left(-kd[2]^2 km[2]^2 T[1] T[3] + kd[2]^2 km[2]^2 T[1] x[4] + \right. \right. \\
& \quad \left. \left. kcr kd[1] kd[2] km[1] km[2] T[2] x[4] \right) \right. \\
& \left(1 + \left(kd[1] \left(2 kd[2] km[2]^2 T[1] T[3] - 2 kd[2] km[2]^2 T[1] x[4] - \right. \right. \right. \\
& \quad kcr kd[1] km[1] km[2] T[2] x[4] - kcr kd[2] km[1] km[2] T[2] x[4] - \\
& \quad kd[2] km[2] T[1] T[2] x[4] - kcr kd[2] km[2] T[1] T[2] x[4] + \\
& \quad 2 kd[2] km[2] T[1] T[3] x[4] - kcr kd[2] km[2] T[2] T[3] x[4] - \\
& \quad 2 kd[2] km[2] T[1] x[4]^2 - kcr kd[1] km[1] T[2] x[4]^2 + \\
& \quad kcr kd[2] km[2] T[2] x[4]^2 + \sqrt{\left((-2 kd[2] km[2]^2 T[1] T[3] + \right.} \\
& \quad 2 kd[2] km[2]^2 T[1] x[4] + kcr kd[1] km[1] km[2] T[2] x[4] + \\
& \quad kcr kd[2] km[1] km[2] T[2] x[4] + kd[2] km[2] T[1] T[2] x[4] + \\
& \quad kcr kd[2] km[2] T[1] T[2] x[4] - 2 kd[2] km[2] T[1] T[3] x[4] + \\
& \quad kcr kd[2] km[2] T[2] T[3] x[4] + 2 kd[2] km[2] T[1] x[4]^2 + \\
& \quad kcr kd[1] km[1] T[2] x[4]^2 - kcr kd[2] km[2] T[2] x[4]^2 \Big)^2 - \\
& \quad 4 \left(-kd[2]^2 km[2]^2 T[1] T[3] + kd[2]^2 km[2]^2 T[1] x[4] + \right. \\
& \quad \left. kcr kd[1] kd[2] km[1] km[2] T[2] x[4] \right) \left(-km[2]^2 T[1] T[3] + \right. \\
& \quad km[2]^2 T[1] x[4] + kcr km[1] km[2] T[2] x[4] + km[2] T[1] T[2] \\
& \quad x[4] + kcr km[2] T[1] T[2] x[4] - 2 km[2] T[1] T[3] x[4] + kcr \\
& \quad km[2] T[2] T[3] x[4] + 2 km[2] T[1] x[4]^2 + kcr km[1] T[2] x[4]^2 - \\
& \quad kcr km[2] T[2] x[4]^2 + T[1] T[2] x[4]^2 + kcr T[1] T[2] x[4]^2 - \\
& \quad kcr T[2]^2 x[4]^2 - kcr^2 T[2]^2 x[4]^2 - T[1] T[3] x[4]^2 + \\
& \quad kcr T[2] T[3] x[4]^2 + T[1] x[4]^3 - kcr T[2] x[4]^3 \Big) \Big) \Big/ \\
& \left(2 \left(-kd[2]^2 km[2]^2 T[1] T[3] + kd[2]^2 km[2]^2 T[1] x[4] + \right. \right. \\
& \quad \left. \left. kcr kd[1] kd[2] km[1] km[2] T[2] x[4] \right) \right) \Big) \\
& \left(km[2] + x[4] + \left(kd[2] km[2] \left(2 kd[2] km[2]^2 T[1] T[3] - 2 kd[2] \right. \right. \right. \\
& \quad km[2]^2 T[1] x[4] - kcr kd[1] km[1] km[2] T[2] x[4] - \\
& \quad kcr kd[2] km[1] km[2] T[2] x[4] - kd[2] km[2] T[1] T[2] x[4] - \\
& \quad kcr kd[2] km[2] T[1] T[2] x[4] + 2 kd[2] km[2] T[1] T[3] x[4] - \\
& \quad kcr kd[2] km[2] T[2] T[3] x[4] - 2 kd[2] km[2] T[1] x[4]^2 - \\
& \quad kcr kd[1] km[1] T[2] x[4]^2 + kcr kd[2] km[2] T[2] x[4]^2 + \\
& \quad \sqrt{\left((-2 kd[2] km[2]^2 T[1] T[3] + 2 kd[2] km[2]^2 T[1] x[4] + kcr kd[1] \\
& \quad km[1] km[2] T[2] x[4] + kcr kd[2] km[1] km[2] T[2] x[4] + \\
& \quad kd[2] km[2] T[1] T[2] x[4] + kcr kd[2] km[2] T[1] T[2] x[4] - \\
& \quad 2 kd[2] km[2] T[1] T[3] x[4] + kcr kd[2] km[2] T[2] T[3] x[4] + \\
& \quad 2 kd[2] km[2] T[1] x[4]^2 + kcr kd[1] km[1] T[2] x[4]^2 - \\
& \quad kcr kd[2] km[2] T[2] x[4]^2 \Big)^2 - 4 \left(-kd[2]^2 km[2]^2 T[1] T[3] + \right. \\
& \quad kd[2]^2 km[2]^2 T[1] x[4] + kcr kd[1] kd[2] km[1] km[2] T[2] x[4] \right) \\
& \quad \left(-km[2]^2 T[1] T[3] + km[2]^2 T[1] x[4] + kcr km[1] km[2] T[2] x[4] + \right. \\
& \quad km[2] T[1] T[2] x[4] + kcr km[2] T[1] T[2] x[4] - 2 km[2] T[1] \\
& \quad T[3] x[4] + kcr km[2] T[2] T[3] x[4] + 2 km[2] T[1] x[4]^2 + \\
& \quad kcr km[1] T[2] x[4]^2 - kcr km[2] T[2] x[4]^2 + T[1] T[2] x[4]^2 + kcr \\
& \quad T[1] T[2] x[4]^2 - kcr T[2]^2 x[4]^2 - kcr^2 T[2]^2 x[4]^2 - T[1] T[3] \\
& \quad x[4]^2 + kcr T[2] T[3] x[4]^2 + T[1] x[4]^3 - kcr T[2] x[4]^3 \Big) \Big) \Big) \Big/ \\
& \left(2 \left(-kd[2]^2 km[2]^2 T[1] T[3] + kd[2]^2 km[2]^2 T[1] x[4] + \right. \right. \\
& \quad \left. \left. kcr kd[1] kd[2] km[1] km[2] T[2] x[4] \right) \right) \Big) \Big] \\
& \left(\left(2 kd[2] km[2]^2 T[1] T[3] - 2 kd[2] km[2]^2 T[1] x[4] - \right. \right. \\
& \quad kcr kd[1] km[1] km[2] T[2] x[4] - kcr kd[2] km[1] km[2] T[2] x[4] - \\
& \quad kd[2] km[2] T[1] T[2] x[4] - kcr kd[2] km[2] T[1] T[2] x[4] + \\
& \quad 2 kd[2] km[2] T[1] T[3] x[4] - kcr kd[2] km[2] T[2] T[3] x[4] - \\
& \quad 2 kd[2] km[2] T[1] x[4]^2 - kcr kd[1] km[1] T[2] x[4]^2 + kcr kd[2] km[2] T[2] x[4]^2 +
\end{aligned}$$

$$\frac{(\text{kr kd}[1] \text{ km}[1] \text{ T}[2] \text{ x}[4] (\text{km}[2] + \text{x}[4]) + \text{kd}[2] \text{ km}[2] (2 \text{ km}[2] \text{ T}[1] (-\text{T}[3] + \text{x}[4]) + \text{x}[4] (\text{kr T}[2] (\text{km}[1] + \text{T}[1] + \text{T}[3] - \text{x}[4]) + \text{T}[1] (\text{T}[2] - 2 \text{ T}[3] + 2 \text{ x}[4]))^2))))}{(2 \text{ kd}[2] \text{ km}[2] (\text{kr kd}[1] \text{ km}[1] \text{ T}[2] \text{ x}[4] + \text{kd}[2] \text{ km}[2] \text{ T}[1] (-\text{T}[3] + \text{x}[4]))})}$$

Here we get the second derivative of the inverse function:

```
ddfinverse = D[finverse, {x[4], 2}];
```

```
Simplify[ddfinverse]
```

\$Aborted

If we don't substitute T_4 only consider relationship between x_4 and x_7 . Then we will have `ts3Sol[[2]]` as the inverse function of solution x_4 .

t3Sol[[2]]

finversex7 =

$$\begin{aligned} & (2 \, kd[2] \, km[2]^2 \, T[1] \, T[3] - 2 \, kd[2] \, km[2]^2 \, T[1] \, x[4] - kcr \, kd[1] \, km[1] \, km[2] \, T[2] \, x[4] - \\ & \quad kcr \, kd[2] \, km[1] \, km[2] \, T[2] \, x[4] - kd[2] \, km[2] \, T[1] \, T[2] \, x[4] - \\ & \quad kcr \, kd[2] \, km[2] \, T[1] \, T[2] \, x[4] + 2 \, kd[2] \, km[2] \, T[1] \, T[3] \, x[4] - \\ & \quad kcr \, kd[2] \, km[2] \, T[2] \, T[3] \, x[4] - 2 \, kd[2] \, km[2] \, T[1] \, x[4]^2 - \\ & \quad kcr \, kd[1] \, km[1] \, T[2] \, x[4]^2 + kcr \, kd[2] \, km[2] \, T[2] \, x[4]^2 + \\ & \quad \sqrt{\left((-2 \, kd[2] \, km[2]^2 \, T[1] \, T[3] + 2 \, kd[2] \, km[2]^2 \, T[1] \, x[4] + kcr \, kd[1] \, km[1] \, km[2] \right. \\ & \quad \quad T[2] \, x[4] + kcr \, kd[2] \, km[1] \, km[2] \, T[2] \, x[4] + kd[2] \, km[2] \, T[1] \, T[2] \, x[4] + \\ & \quad \quad kcr \, kd[2] \, km[2] \, T[1] \, T[2] \, x[4] - 2 \, kd[2] \, km[2] \, T[1] \, T[3] \, x[4] + \\ & \quad \quad kcr \, kd[2] \, km[2] \, T[2] \, T[3] \, x[4] + 2 \, kd[2] \, km[2] \, T[1] \, x[4]^2 + \\ & \quad \quad kcr \, kd[1] \, km[1] \, T[2] \, x[4]^2 - kcr \, kd[2] \, km[2] \, T[2] \, x[4]^2)^2 - \\ & \quad 4 \left(-kd[2]^2 \, km[2]^2 \, T[1] \, T[3] + kd[2]^2 \, km[2]^2 \, T[1] \, x[4] + \right. \\ & \quad \quad kcr \, kd[1] \, kd[2] \, km[1] \, km[2] \, T[2] \, x[4] \left. \right) \left(-km[2]^2 \, T[1] \, T[3] + \right. \\ & \quad \quad km[2]^2 \, T[1] \, x[4] + kcr \, km[1] \, km[2] \, T[2] \, x[4] + km[2] \, T[1] \, T[2] \, x[4] + \\ & \quad \quad kcr \, km[2] \, T[1] \, T[2] \, x[4] - 2 \, km[2] \, T[1] \, T[3] \, x[4] + kcr \, km[2] \, T[2] \, T[3] \\ & \quad \quad x[4] + 2 \, km[2] \, T[1] \, x[4]^2 + kcr \, km[1] \, T[2] \, x[4]^2 - kcr \, km[2] \, T[2] \, x[4]^2 + \\ & \quad \quad T[1] \, T[2] \, x[4]^2 + kcr \, T[1] \, T[2] \, x[4]^2 - kcr \, T[2]^2 \, x[4]^2 - kcr^2 \, T[2]^2 \, x[4]^2 - \\ & \quad \quad T[1] \, T[3] \, x[4]^2 + kcr \, T[2] \, T[3] \, x[4]^2 + T[1] \, x[4]^3 - kcr \, T[2] \, x[4]^3 \left. \right) \Big) \Big) / \\ & \quad \left(2 \left(-kd[2]^2 \, km[2]^2 \, T[1] \, T[3] + kd[2]^2 \, km[2]^2 \, T[1] \, x[4] + \right. \right. \\ & \quad \quad \left. \left. kcr \, kd[1] \, kd[2] \, km[1] \, km[2] \, T[2] \, x[4] \right) \right); \end{aligned}$$

```
Simplify[D[finversex7, {x[4], 2}]]
```