Introductory physics: A question library

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1 Units, Dimensions, Estimates

1.1 Multiple Choice

1. A motor is rotating at 420 rpm (rotations per minute). What does this correspond to in Hz (rotations per second)?

   ◯ 7 Hz

   ◯ 17.5 Hz

   ◯ 25 200 Hz

2. Which of the following are examples of systematic error?

   ◯ You measured the length of an object in inches instead of centimeters, the unit required for your lab

   ◯ The ramps you were using in an experiment with physics carts were old and rough, causing the calculated force of friction to be higher than expected

   ◯ On one trial, you accidentally stopped the timer in a lab trial 1.5 seconds too late as you were distracted by a pretty butterfly by the lab window

   ◯ The mass you were told weighed 1.000 kg actually weighed 1.008 kg and affected your results in a lab

3. In solving a physics problem you end up with units of m in the numerator and units of m/s in the denominator. The units for your answer are:

   ◯ m^2/s

   ◯ m^2

   ◯ m

   ◯ 1/s

   ◯ s

4. The density of a solid object is defined as the ratio of the mass of the object to its volume. The dimension of density is:

   ◯ M/L

   ◯ M/L^3

   ◯ L^3/M

   ◯ ML

5. Power is defined as the rate of work per time, power = work/time. If the dimensions of power are ML^2T^{-3}, what are the dimensions of work?

   ◯ MLT^{-3}

- ○ ML^2T^{-1}
- ○ ML^3T^{-3}
- ○ ML^2T^{-2}
- ○ ML^2T^{-4}

6. The position, $x(t)$, of an object is given by the equation $x(t) = A + Bt + Ct^2$, where t refers to time. Note that the dimension of position is length ($[x] = L$). What are the dimensions of A, B, and C?
   - ○ [A]=L, [B]=L, [C]=L
   - ○ [A]=L, [B]=T, [C]=T
   - ○ [A]=L, [B]=T, [C]=T^2
   - ○ [A]=L, [B]=L/T, [C]=LT^{-2}
   - ○ [A]=L/T, [B]=LT^{-2}, [C]=LT^{-3}
7. By repeating a measurement many times, you can reduce:
   - ○ the systematic error in the value
   - ○ the random (statistical) error in the value

1.2 Long answers

8. While observing a particularly fast guanaco in a herd, you measured that it covered a distance of $d = (100.0 \pm 5.0)\,\mathrm{m}$ in a time $t = (7.0 \pm 1.0)\,\mathrm{s}$. What is the speed of the guanco, with uncertainty?

   Note that speed is defined as distance over time. You are free to use whichever method that you like to calculate the speed and its uncertainty, but you must explain what you did in your answer.
9. A light year is defined as the distance that light travels in one year, at a speed of 2.998×10^8 m/s.
   (a) How many metres are in a light year?
   (b) An astronomical unit (AU) is defined as the average distance between the Earth and the Sun, and is given by $1\,\mathrm{AU} = 1.50 \times 10^8\,\mathrm{km}$. Express the distance light travels in one year in AU.
   (c) An officer of the United Federation of Planets pulls your spaceship over. The officer claims that you were speeding, and that the speed limit in this section of the galaxy is 5.0 AU/h. Unfortunately, your spaceship's speedometer measures speed in m/s, and it reads that you were travelling at 3.2×10^8 m/s. Was the officer correct to pull you over?

10. A watchmaker wants to test the accuracy of their watches. The watchmaker finds that their watch loses 6 s in a year. How precise are the watches (expressed as a percentage)?

11. In 1899, Max Plank suggested that there was a measurement of time that could be defined by only physical constants. Show that the following combination gives a quantity of time:

$$t_P = \sqrt{\frac{Gh}{c^5}}$$

where, G, is Newton's gravitational constant, h is Planck's constant, and c is the speed of light. This combination of physical constants gives what is called Plank time, which is potentially the smallest possible measureable time interval.

12. Estimate the combined volume of all of the students in the PHYS 104/106 class.

The numerical answer that you get, as long as it is reasonable, is not as important as the process. Describe in detail what assumptions you make in order to arrive at your answer, and comment on whether your answer is reasonable.

13. Estimate the number of dentists in Toronto.

The numerical answer that you get, as long as it is reasonable, is not as important as the process. Describe in detail what assumptions you make in order to arrive at your answer, and comment on whether your answer is reasonable.

14. You decide to measure's Newton's universal constant of gravity, G, which you determine by measuring the force of attraction between two spheres of mass $m = (10.000 \pm 0.001)$ kg placed a distance $r = (1.00 \pm 0.05)$ cm apart. You determine that the force of attraction is $F = (6.5 \pm 1.0) \times 10^{-5}$ N. G is given by:

$$G = \frac{Fr^2}{m^2}$$

Based on these measurements, what is the value of G that you determine, with uncertainty?

You are encouraged to determine G using QExpy and a Jupyter notebook. Save the notebook to pdf and upload it as your answer to show your work. You may need to copy the answer from the notebook to format it for significant figures and units.

15. The period of oscillation, T, of a pendulum (how long it takes to swing back and forth), depends on the length of the pendulum, L, and the acceleration due to gravity, g. Use dimensional analysis to:

    (a) determine a formula for the period of oscillation, T, in terms of L and g.

    (b) show (or argue) that the period cannot depend on the mass of the pendulum

2 Math: Vectors, Derivatives, Integrals

2.1 Multiple Choice

16. What is the angle (in degrees) between the 3-dimensional vectors $\vec{a} = (4, 6, 7)$ and $\vec{b} = (-5, 3, 17)$ when put tail to tail?

    - ◯ 45°
    - ◯ 43.7°
    - ◯ 49.6°
    - ◯ 90°

17. $\vec{a} \times \vec{b} = \vec{b} \times \vec{a}$

    - ◯ True
    - ◯ False

18. Jimmy is doing a problem for his physics homework and must determine the cross product of two parralel vectors a and b (I'm not sure how to create the vector error) however he forgot his calculator at home. Should Jimmy be worrying?

    - ◯ No because the vectors are parralel therefore the cross product is zero (Jimmy should still learn how to use the cross product)
    - ◯ Yes, cross product is a fundmental tool in physics
    - ◯ No because all Jimmy needs to do is multiply each of the values together, he can use mental math
    - ◯ Jimmy can just ask a friend to borrow their calculator!

19. A function, $f(x)$, is plotted and found to have a maximal value at $x = 3.5$. What is $f'(x = 3.5)$?

    - ◯ Negative
    - ◯ 0
    - ◯ Positive
    - ◯ Need more information

20. Which of the following would result in a scalar?

    - ◯ The cross product of two vectors
    - ◯ A vector multiplied by a scalar
    - ◯ The addition of two vectors
    - ◯ The dot product of two vectors

21. You find that a rocket is burning fuel at a rate of 100 kg/s, and is thus losing mass as it burns the fuel. If $M(t)$ is the mass of the rocket as a function of time, which of the following is correct?

   - ○ $\int M dt$+C=100 kg/s
   - ○ $\int M dt$+C=-100 kg/s
   - ○ $\frac{dM}{dt} = -100\,\mathrm{kg/s}$
   - ○ $\frac{dM}{dt} = 100\,\mathrm{kg/s}$
   - ○ $M(t) = 100\,\mathrm{kg/s}$
   - ○ $M(t) = -100\,\mathrm{kg/s}$

22. You have opened up a coffee shop and found that you can model the rate of money coming into your store (in dollars per hour) as $f(t) = 2t^2$, where t is the time in hours measured from when the store opened. The total amount of money that your store will make in the first 6 hours after opening, M, is given by:

   - ○ $M = \frac{df}{dt}\Big|_{t=6h}$
   - ○ $M = \int_0^{6h} 4t dt$
   - ○ $M = \int_0^{6h} 2t^2 dt$
   - ○ $M = 2(6\,\mathrm{h})^2$

23. Physical (measurable) quantities can be given by:

   - ○ Anti-derivatives
   - ○ Indefinite integrals
   - ○ Definite integrals
   - ○ All of the above

24. What is the magnitude of the vector $\vec{a} = (5, 5, 1)$?

   - ○ 51
   - ○ 7.14
   - ○ 3.32
   - ○ 2.54

25. Given two vectors, $\vec{a} = (5, -7)$, and $\vec{b} = (9, 2)$, and knowing that the angle between them is $67°$ when they are placed tail to tail, what is their scalar product?

   - ○ 29
   - ○ 30
   - ○ 31

- ◯ 32

26. You have decided that you dislike the choice of x and y axes of a Cartesian coordinate system given to you by the professor, and decide to define your own coordinate system, with axes v and w. If your v axis points in the direction $2\hat{x} + 3\hat{y}$, and you would like the w axis to be perpendicular to the v axis, in which direction should the w axis point?
    - ◯ $-2\hat{x} - 3\hat{y}$
    - ◯ $3\hat{x} - 2\hat{y}$
    - ◯ $-3\hat{x} - 2\hat{y}$
    - ◯ $2\hat{x} - 3\hat{y}$

27. The derivative of a function $f(x)$ with respect to x
    - ◯ is always positive if $f(x)$ is always positive.
    - ◯ is always zero if $f(x)$ increases at a constant rate.
    - ◯ is always positive if $f(x)$ increases at a constant rate.
    - ◯ must be zero at some point if $f(x)$ increases at a constant rate.

28. You would like to know the total amount of water that has leaked out of a bucket with a hole in it in the five minutes since the bucket was filled. You have measured that the bucket leaks water at a rate of $r(t) = 0.5\,\text{litres/min}$
    - ◯ You need to take the derivative of $r(t)$ evaluated at $t = 5$
    - ◯ You need to take the definite integral of $r(t)$ from $t = 0$ to $t = 5\,\text{min}$
    - ◯ You need to take the indefinite integral of $r(t)$ and evaluate it at $t = 5\,\text{min}$ with the constant of integration evaluated at $t = 0$.

29. A function $f(x)$ is plotted and found to have a minimum value when $x = 5$. The derivative of $f(x)$, $f'(x)$, evaluated at $x = 5$ is
    - ◯ positive.
    - ◯ negative.
    - ◯ zero.
    - ◯ Need more information.

2.2 Long answers

30. The summit of Mount Kilimanjaro lies 5895 m above sea level. Suppose you are currently staying at a camp which is located 2411 m above sea level. Your GPS says that the summit is 3470 m horizontally from the base camp in a direction of 21.1° West of North.
    (a) What are the components for the displacement vector from base camp to the summit?

(b) What is the magnitude of this vector?

(c) What angle does the vector make with the horizontal?

Choose the x axis to be East, the y axis to be North, and the z axis to be up

31. As a summer job during your physics studies, you get hired by a pharmaceutical company in Fiji that is interested in your strong ability to model physical situations. The company is testing a new drug that increases the number of spines on sea urchins from the South Pacific. This is to reduce the number of tourists that spoil their nice beaches. Through careful observation, you have determined that you can model the number of spines on a sea urchin, as $N(t) = \frac{1}{8}t^2 + 2t + 2$, where t is the time, measured in days, from when the drug was administered. The drug is always administered on sea urchins when they are 42 days old.

    (a) How many total spines are on a sea urchin 10 days after administering the drug?

    (b) How many spines per day are growing on a sea urchin 10 days after administering the drug?

32. Your 300 l bathtub is draining water, and you find that you can model the rate of water draining out as a function of time to be $r(t) = 65 - 2t$, where $r(t)$ is expressed in litres per minute, and t is the time in minutes since you pulled the plug from a full baththub. As times goes by and the water level goes down, the rate of water coming out of the bathtub decreases.

    (a) At what rate is water draining out of your bathtub, 5 minutes after pulling the plug, if your bathtub was full?

    (b) How many litres drain out in the first 5 minutes after pulling the plug, if your bathtub was full?

    (c) At what time does your model break down?

33. In physics, we often make approximations to model physical situations. A common approximation that is made is to approximate a cow as a sphere (the "spherical cow"). Suppose that a flea is walking along the surface of a spherical cow of radius $r = 1.2\,\text{m}$ and that the position of the flea is given with respect to a coordinate system that is at the centre of the spherical cow. The flea starts where the positive part of the x axis intersects the cow and walks to a position where the x, y, and z coordinates of its position are all positive and equal. What is the net displacement vector of the flea for this path?

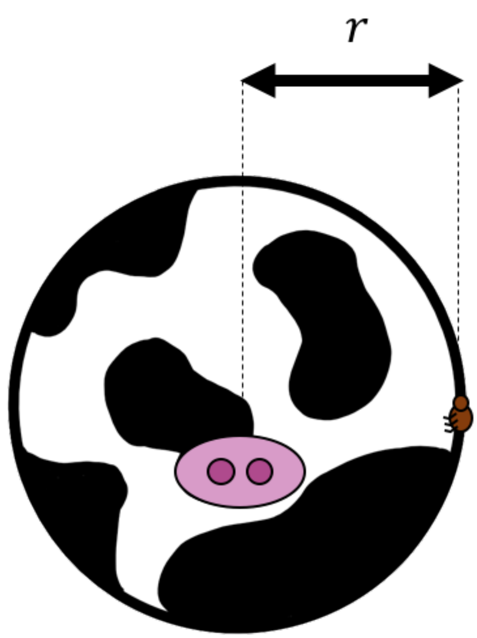

Figure 1: A spherical cow

34. Let $\vec{v}_1 = 2.0\hat{\imath} - 9\hat{\jmath}$ and $\vec{v}_2 = 1.5\hat{\imath} + 7.0\hat{\jmath}$. Determine the magnitude and direction (angle that the vector makes with the x axis) of:

    (a) $\vec{v}_1$

    (b) $\vec{v}_2$

    (c) $\vec{v}_1 + \vec{v}_2$

    (d) $\vec{v}_2 - \vec{v}_1$

35. Figure 2 shows two vectors, $\vec{G}$ and $\vec{H}$, along with their magnitudes ($G = 10.0$, $H = 15.0$).

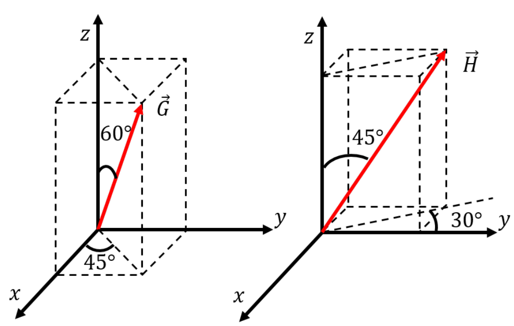

Figure 2: Two vectors

    (a) Write out the two vectors in component form.

    (b) What is the angle between the two vectors, if these are placed with their tails at the same point?

    (c) Write out the component form of the vector product, $\vec{G} \times \vec{H}$, of these two vectors.

36. Determine the following:

(a) $\frac{\partial}{\partial x}(x^4 + 3x^3 + ax + y)$

(b) $\frac{\partial}{\partial y}(x^4 + 3x^3 + ax + y)$

(c) $\frac{d}{dx}\sin(10^x)$

(d) $\frac{\partial}{\partial y}\sqrt{x^2 + xy + ay^3}$

(e) $\int(ax^2 + bx + c)dx$

(f) $\int_0^\pi \cos(x)dx$

37. Figure 3 shows a skateboarder as she is about to jump off a parabolic ramp (she is moving towards the right in the figure). With the shown coordinate system, and for $x > 0$, the ramp can be modelled by the function $y(x) = (2.0\,\text{m}) + (0.5)x + (0.04\,\text{m}^{-1})x^2$, where y is in the vertical direction. The ramp ends at $x = 5.0\,\text{m}$. Assuming that she is going fast enough to make it to the end of the ramp, which angle will her trajectory make with the horizontal just at the point that she leaves the ramp?

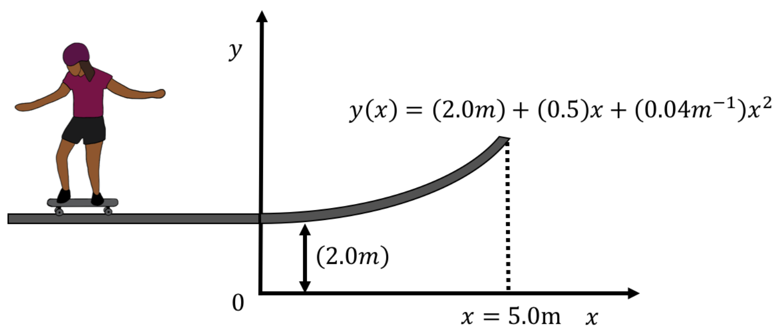

Figure 3: A skateboarder on a parabolic ramp.

38. The work, W, done by a force with magnitude, F, exerted on an object, along a distance, Δx, is defined to be:

$$W = F\Delta x$$

and corresponds to the amount of (kinetic) energy that the force gives to the object. If the force is exerted over a longer distance, it will give more energy to the object, so it makes some intuitive sense. For example, a force of $F = 15\,\text{N}$ exerted on an object over a distance of $\Delta x = 5.0\,\text{m}$, will give the object a kinetic energy of $W = 75\,\text{Nm}$ (note that Nm are units of energy and equivalent to J).

If the force depends on position, $F = F(x)$, then to calculate the work done in going from $x = x_a$ to $x = x_b$, we have to divide up the interval between x_a and x_b into many little intervals, of infinitely small length, dx. We can then calculate the small amount of work on that interval, dW, assuming that the force is constant. We can then sum all of the little works, dW, to obtain the total work, W, of the force over the interval from $x = x_a$ to $x = x_b$.

If the force is given by $F(x) = \frac{1}{x^2}$, what is the work done in going from from $x = x_a$ to $x = x_b$?

39. Take the following partial derivative:

$$\frac{\partial}{\partial y}\left(\frac{-kz}{(x^2+y^2+z^2)^{\frac{3}{2}}}\right)$$

40. Draw the vector sum $\vec{a} - 2\vec{b}$ on the graph.

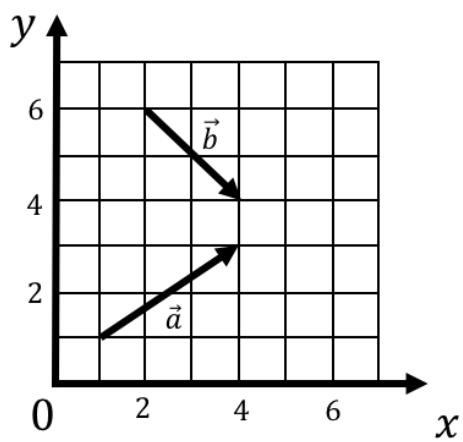

Figure 4: Two vectors, $\vec{a}$ and $\vec{b}$.

41. You find that you can model the rate at which a guanaco eats as

$$r(t) = at + b$$

where $r(t)$ is the amount of kilograms per minute that the guanaco eats and a and b are constants. t is the amount of time in minutes since the guanaco started eating.

(a) What are the units of a and b?

(b) Suppose that $a = -0.2$ and $b = 2$ (in the units that you determined in part a)), how many kilograms does the guanaco eat in the first 2 min?

(c) After how much time does the guanaco stop eating?

42. You find that the number of customers in a store at a certain time is given by:

$$N(t) = a + bt^2 - ct^3$$

where t is the time in hours since the store opened, and, N, is the number of customers in the store at that time. a, b and c are constants.

(a) What are the units of a, b and c?

(b) If $a = 5$, $b = 6$, $c = 1$, (in the units found in part a)) at what time is the number of customers in the store the largest (express your answer in the number of hours since the store opened)?

(c) If $a = 5$, $b = 6$, $c = 1$, at what time are the most customers entering the store (express your answer in the number of hours since the store opened)?

43. You spot a balloon in the distance, and estimate that it is at an altitude of $500\,\mathrm{m}$ with respect to you, in the North direction from your position. You estimate that the point directly below the balloon, is a distance of $350\,\mathrm{m}$ from you. Your friend is located on the ground, a distance of $200\,\mathrm{m}$ from you, in the direction that is 35° to the East from the North direction.

    (a) What is the displacement vector from your friend to the balloon? Be very clear in your choice of coordinate system.

    (b) What angle does the displacement vector make with the horizontal?

3 Kinematics

3.1 Multiple Choice

44. You bring a concrete brick and a feather to the moon with you in order to conduct an experiment. You drop the brick and the feather from the same height at the same time, which one hits the ground first?

    ◯ The Brick

    ◯ The Feather

    ◯ They land at the same time

    ◯ Both the brick and the feather don't hit the ground because they floated away

45. Person 1 is riding on the back of a horse that is travelling at a constant speed of 18 m/s. Their friend, Person 2, is riding on a guanaco that is travelling at a constant speed of 15 m/s. When the two are at the same point, X_0, the guanaco has a sudden burst of energy and begins to accelerate at a rate of $0.5\,\mathrm{m/s^2}$. Which of the following statements is true about this situation?

    ◯ The guanaco will always be ahead of the horse beyond X_0 because the guanaco is accelerating and the horse is not.

    ◯ The guanaco will never catch up with the horse because it is not accelerating fast enough to make up for the extra distance covered by the horse, which had a larger initial speed at X_0.

    ◯ The guanaco's speed will be exactly 18 m/s when it catches up to the horse.

    ◯ The guanaco's speed will be greater than 18 m/s of the horse when it catches up to the horse.

46. The acceleration vector must point in the same direction as the velocity vector.

    ◯ True

    ◯ False

47. A motor is rotating at 420 rpm (rotations per minute). What does this correspond to in Hz (rotations per second)?

    ◯ 1.1 Hz

    ◯ 7 Hz

    ◯ 17.5 Hz

    ◯ 25 200 Hz

48. A block is moving at 3 m/s westward. It begins accelerating at $0.75\,\mathrm{m/s^2}$ northward. There is a wall 16 m to the west, and a wall 8 m to the north. Which wall does it reach first?

- ◯ Western wall
- ◯ Northern wall
- ◯ Same time for both walls

49. A child, Alice, is dropped from rest at from a height of 10 m while an identical child, Brittany, is thrown horizontally (from the same height) with a velocity of $v = (15\,\mathrm{m/s})\hat{\imath} + (24\,\mathrm{m/s})\hat{\jmath}$, where the positive y direction is defined as North, the positive x direction is defined as East, and the positive z direction is upward. Which child will hit the ground first?

    - ◯ Alice
    - ◯ Brittany
    - ◯ Neither, they will hit the ground at the same time
    - ◯ I should not have children

50. Consider an object moving in 3 dimensions, whose acceleration in x is given by $a_x(t)$. Integration of $a_x(t)$ with respect to t yields (choose all that apply)

    - ◯ The x-component of the object's velocity.
    - ◯ The x-distance covered by the object.
    - ◯ The change in the x-component of the object's velocity.
    - ◯ The time elapsed during the object's motion.
    - ◯ The x-velocity of the object, to within a constant of integration.

51. The position vector of an object under uniform circular motion is given by $\vec{r}(t) = \cos(\omega t)\hat{\imath} + \sin(\omega t)\hat{\jmath}$. If it takes 5 s to complete a single rotation and the object is along the $\hat{\imath}$ axis at $t = 0\,\mathrm{s}$, what is its position after 13.75 s?

    - ◯ $-\hat{\imath} + 0\hat{\jmath}$
    - ◯ $0\hat{x} - \hat{y}$
    - ◯ $-\hat{\imath} - \hat{\jmath}$
    - ◯ $0\hat{\imath} + \hat{\jmath}$

52. A ball is thrown vertically upwards, then falls back down and hits the ground. The acceleration of the ball:

    - ◯ Points upwards when the ball is released from the hand throwing it, then points downwards as it falls to the ground.
    - ◯ Always points downwards.
    - ◯ Always points downwards, except at the vertex of the ball's trajectory where there is no acceleration because the ball is not moving.

53. An archer shoots an arrow straight upwards. He observes that it takes the arrow 10.2 s to reach its maximum height before it begins to fall back down. How much time does he now have to move out of the way before the arrow hits him? Assume acceleration due to gravity is constant at 9.8 m/s and that air resistance is negligible.

   ◯ More than 10.2 seconds

   ◯ 10.2 seconds

   ◯ Less than 10.2 seconds

54. A swimmer is standing on the South shore of a wide river that is flowing East at 1 m/s. She wants to swim across the river and land directly across from where she started. She can swim with a speed of 2 m/s. In which direction does she have to swim?

   ◯ 60° West of North

   ◯ 45° West of North

   ◯ 30° West of North

   ◯ 60° West of South

55. A feather and a hammer are dropped at the same time from the same height above the surface of a planet. They land on the surface of the planet at the same time.

   ◯ The hammer had a larger acceleration than the feather.

   ◯ The feather had a larger acceleration than the hammer.

   ◯ The hammer and feather had the same acceleration.

   ◯ Not enough information to tell.

56. A feather and a hammer are dropped at the same time from the same height above the surface of a planet. The hammer lands on the surface of the planet before the feather.

   ◯ The hammer had a larger acceleration than the feather.

   ◯ The feather had a larger acceleration than the hammer.

   ◯ The hammer and feather had the same acceleration.

   ◯ Not enough information to tell.

57. A rock and a feather are dropped at the same time from a height of 1 m from the ground. The rock is assumed to fall with the acceleration due to gravity ($g = 9.8\,\mathrm{m/s^2}$), whereas the feather has half of that acceleration. How much time elapses between when the rock and feather hit the ground?

   A. 0.187 s

   B. 0.204 s

   C. 0.408 s

D. 0.639 s

58. Two bugs are sitting on a rotating disk (the disk is rotating about its axis of symmetry), at different distances from the center of the disk.

    - ◯ Both bugs have the same velocity.
    - ◯ Both bugs have the same speed.
    - ◯ Both bugs have the same angular velocity.
    - ◯ Both bugs have the same centripetal acceleration.

3.2 Long answers

59. An airplane is flying at a speed of 100 kn (knots) relative to the air and pointing in the North-West direction (a heading of 315° if North is defined 0°). A strong wind, blowing towards the east at a speed of 30 kn (knots), changes the trajectory of the plane, with respect to the ground.

    (a) What is the magnitude (in knots) and direction of the plane's velocity vector relative to the ground?

    (b) Given that 1 kn corresponds to 1.852 km/h, how much distance does the plane cover on the ground in 10 min.

    (c) A second plane flies with the same speed relative to the air, but with a heading of 45° (pointing North - East). The angle between the velocity vectors of the two planes as seen in the air is 90°. What is the angle between the velocity vectors of the two planes when the velocity is measured from the ground?

60. Chloë is out attempting to take a picture of an elusive and shy vicuña. Chloë is jogging along a path in the North direction at a speed of 10 km/h, while the vicuña is walking in the East direction at a speed of 5 km/h. At one instant in time, $t = 0\,\text{s}$, the vicuña is directly North of Chloë, at a distance of 1 km. Assume that both Chloë and the vicuña maintain constant velocity vectors.

    (a) At which time will the distance between Chloë and the vicuña be minimal?

    (b) What will be the distance between Chloë and the vicuña at that time?

61. A child, who is 45 m from the bank of a river is being carried by the river's current at 1.0 m/s. A lifeguard jumps into the water just as the child passes by (at this point in time, the child is 45 m from the life guard). The life guard can swim at a speed of 2.0 m/s relative to the water.

    (a) How long does it take the lifeguard to reach the child?

    (b) What is the magnitude of the displacement vector of the lifeguard once she reaches the child?

62. Curious about how quickly you can cause water to spray from your garden hose, you decide to conduct an experiment to find out. You place your thumb over the hose, then hold it vertically upwards 0.5 m above the ground. You shut off the water instantaneously (such that the stream does not slow before stopping) and count 2.5 s seconds before you hear the water stop hitting the ground. Given these measurements, determine the speed of the water as it left the nozzle of the hose.

63. A young banjo player is trying to get from Texas to Tennessee. She decides to jump onto the cart of a freight train that travels at a constant speed of 5.0 m/s. The banjo player notices an empty cart just as it begins to pass her and she begins to accelerate from rest at 1.4 $\mathrm{m/s^2}$ (in the direction of the train) to a maximum speed of 5.6 m/s.

    (a) How long does it take the banjo player to reach the box car?

    (b) What distance did the banjo player travel to reach the box car?

64. A projectile is shot from the edge of a cliff 115 m above ground level with an initial speed of 65 m/s at an angle of 35.0° from the horizontal. Determine:

    (a) The distance at which the projectile lands from the base of the cliff

    (b) The maximum height above the ground that the projectile reaches

    (c) The velocity vector of the projectile just before it hits the ground (give the components and the magnitude)

65. Chloë is riding a toy guanaco on a merry-go-round (carousel). The guanaco that Chloë is riding is located 2.5 m from the centre of the merry-go-round, which rotates twenty times per minute. You decide to model Chloë's position using a coordinate system whose origin coincides with the centre of the merry-go-round, the x axis points East, the y axis points North, and the z axis points upwards. At time $t = 0$, Chloë's guanaco crosses the x axis.

    (a) What are Chloë's angular velocity (in rad/s) and centripetal acceleration?

    (b) What is Chloë's velocity vector $\vec{v}(t)$ (as a function of time) and its magnitude? What is the velocity vector at $t = 5.0\,\mathrm{s}$? (give the x and y components of the vector)

    (c) What is Chloë's acceleration vector $\vec{a}(t)$ (as a function of time) and its magnitude? What is the acceleration vector at $t = 5.0\,\mathrm{s}$? (give the x and y components of the vector)

    Suddenly, to everyone's surprise, you realize that the whole merry-go-round is on wheels and that Steve, the merry-go-round thief, has attached the merry-go-round to his truck and started driving South at a constant speed of 10 m/s, taking the whole thing with him. Assume that Steve started driving away at $t = 0$, when Chloë's guanaco crossed the x axis, and that he accelerated in a negligible amount of time. The merry-go-round, being powered by batteries, continues to rotate.

    (d) What is Chloë's velocity vector (as a function of time) relative to the coordinate system that you originally defined (i.e. relative to a fixed coordinate system whose origin is where the centre of the merry-go-round used to be)? What is the velocity vector at $t = 5.0\,\text{s}$? (give the x and y components of the vector)

66. You try to wake your friend up by tossing pebbles at her window. The pebbles leave your hand hand $4.0\,\text{m}$ away from the side of your friend's house and $10.0\,\text{m}$ below her window. At what speed do the pebbles hit her window if they are only moving with a horizontal component of velocity?

67. Model a long jump. Make precise sketches of position, velocity, and acceleration as a function of time (x and y components) for a long jumper, from when she starts at rest to when she lands and stops in the sand pit. Make sure that the six graphs are well labeled and that the various features are explained in your answer. You should come up with reasonable numerical estimates for all variables that you need to describe the motion (how fast she can run, how long she runs, how far she jumps, how high she jumps, her acceleration when running, etc). Although the graphs do not need to be exact, they should be close sketches to what you would get if you plotted your model with a computer (which of course, you are welcome to do). For example, the axes should be labelled and show the correct maximum height of the jump, even if they are not perfectly to scale.

68. Answer the following:

    (a) A projectile is launched from a horizontal surface and lands a distance R away. What is the **minimum** speed with which the projectile should be launched in order to land at the distance R?

    (b) A child wishes to throw a ball to a friend on the other side of a building of height h and width d as shown in Figure 7. They experiment by trying different launch speeds, angles of projection, and starting points of the launch. What is the **minimum speed** with which the ball can be thrown to clear the building and reach the other side? **Hint:** You may find the solution to part a) useful in this question.

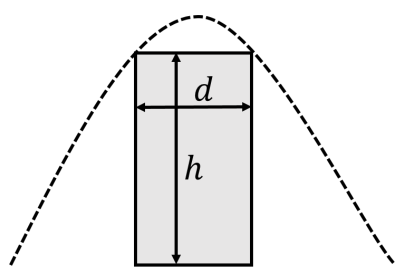

Figure 7: Trajectory of ball thrown over the building.

69. A satellite is in a circular orbit around a spherical planet. The satellite is at a height H from the surface of the planet and executes one full orbit around the planet at constant speed in a time T. The planet has a radius R and does not rotate about itself. You wish to intercept the satellite by launching a rocket straight up from the surface of the planet. The rocket has a vertical acceleration given by:

$$a(t) = jt^2$$

where j is a constant and t is the amount of time since the launch. At some time, $t = 0$, the satellite passes right above the launch point of the rocket, as illustrated in Figure 8. Assume that the rocket's acceleration is given by the above equation throughout its trajectory.

(a) How much time should you wait before launching the rocket so that it intercepts the satellite during its next orbit? Give your answer in terms of T, j, H, and R.

(b) What will be the speed of the rocket when it intercepts the satellite?

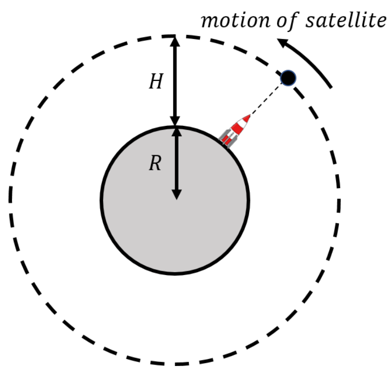

Figure 8: At time $t = 0$, a satellite is just above a rocket at the surface of a planet. How much time should one wait in order to fire the rocket such that it will intercept the satellite on its next orbit?

70. You are standing at a train station in Toronto saying good-bye to your little cousin. As the train carrying your cousin begins to pull away at 2.00 m/s towards the East, your cousin runs 4.20 m/s in the opposite direction, up the aisle of the train.

(a) What is your cousin's velocity relative to the ground?

(b) If you now begin to walk away at 1.50 m/s West, and your cousin continues to run as before, what is his relative velocity to you?

(c) Your cousin realizes that he forgot to give you the chocolate egg he bought for you. He quickly throws it horizontally out the train window at 5.00 m/s towards the South, landing on the ground 0.75 s later. From how high did the cousin throw the egg?

4 Newton's Laws

4.1 Multiple Choice

71. Two identical blocks of mass m are at rest on a horizontal surface, as shown in Figure 9. The coefficients of static friction, μ, between the two blocks and between the bottom block and the ground are the same. What is the minimum magnitude of a horizontal force $\vec{F}$ applied to the top block that will make the bottom block move?

    A. $F \geq \mu mg$

    B. $F \geq 2\mu mg$

    C. $F \geq 3\mu mg$

    D. It is not possible to make the bottom block move by pushing on the top block.

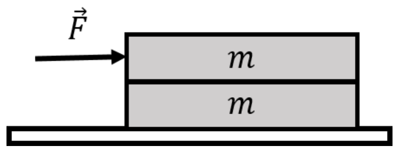

Figure 9: Two blocks (Question 71).

72. A block rests on an incline plane such that it has no movement. Which of the following forces are acting on the block?

    ◯ Gravity.

    ◯ Normal Force.

    ◯ Tension Force.

    ◯ Static Friction Force

    ◯ Applied Force

73. You push a heavy crate along a horizontal surface, moving it in the North direction at constant speed. Which statement is true?

    ◯ You exert a force on the ground in the North direction.

    ◯ The ground exerts a force on you in the North direction.

    ◯ The crate exerts a force on you in the North direction.

    ◯ The crate exerts no force on you.

74. If you punch a wall and break your hand but not the wall, what force is responsible for the damage to your hand?

    ◯ The force you exert on the wall

    ◯ The force the wall exerts on your hand, equal to the force you exert on the wall

- ◯ The force the wall exerts on your hand, greater than the force you exert on the wall
- ◯ It depends on the material of the wall

.

75. Name all the forces acting on an object that is sitting at rest on an inclined plane.
    - ◯ The object is at rest, therefore no forces are acting on the object
    - ◯ The force of gravity
    - ◯ The force of tension, the normal force, and the force of gravity
    - ◯ The force of friction, the normal force, and the force of gravity
76. Which of the following statements is true about weight?
    - ◯ Weight and mass describe the same intrinsic property of an object.
    - ◯ Weight is an intrinsic property of an object.
    - ◯ The weight of an object can change depending on an object's location on Earth.
    - ◯ Two of the above statements are correct.
    - ◯ None of the above.
77. Which of the following could be the mass of a tea cup in space?
    - ◯ 0 g
    - ◯ 0 N
    - ◯ 200 g
    - ◯ 1.9 N
78. A dog with a mass of 20 kg, as measured in the inertial reference frame of the Earth, is in an elevator standing on a standard bathroom scale. The elevator starts off stationary, then accelerates to a constant velocity, and then slows to a stop when it reaches the top floor. When will the scale display 20 kg? Select all that apply:
    - ◯ Before the elevator starts moving
    - ◯ While the elevator is accelerating upwards
    - ◯ While the elevator is moving upwards at its constant velocity
    - ◯ While the elevator is decelerating
    - ◯ After the elevator has stopped
79. A car having a mass of 900 kg collides head-on with a truck having a mass of 1800 kg. In this collision, the magnitude of the force exerted on the car by the truck, F_{CT} and the force exerted on the truck by the car, F_{TC} are such that

- ◯ $F_{CT} > F_{TC}$
- ◯ $F_{CT} = F_{TC}$
- ◯ $F_{CT} < F_{TC}$
- ◯ Not enough information given to determine the answer

80. Excited at the idea of your first university physics exam, you decide to study for the exam by creating little demonstrations to showcase your knowledge to your friends. You derive an equation to convince your friends that a feather and a ballpoint pen will take the same amount of time to fall a distance h, even when the ballpoint pen is much heavier than the feather. You then perform your experiment, and the ballpoint pen hits the floor significantly earlier than the feather. You correctly explain to your friends that:

    - ◯ The feather and pen both had the same acceleration, but the feather had more drag
    - ◯ The feather and pen both had the same force from gravity, but the feather had more drag
    - ◯ The feather and pen both had the same net force on them, but the feather is lighter, so it had less acceleration
    - ◯ The feather and pen had different accelerations $1\,\mathrm{s}$ after being released.

81. You are at the supermarket pushing a cart full of groceries. To keep the cart moving at constant speed, you notice that you have to keep applying a force to the cart. You conclude that a continuous force must be needed to sustain continuous motion. This statement is:

    - ◯ True, since the natural state of all objects is to be at rest. Eventually, all objects will be at rest, so to keep an object moving, a force needs to be applied.
    - ◯ True, as this can be tested with other objects and vehicles such as cars and boats (when the motor/propeller on the boat is turned off, the boat slows down to a stop).
    - ◯ False. The force you apply to keep the cart moving at constant speed is only to counteract a frictional force.

82. Which of the following options would not affect the drag force acting on an object moving through a fluid?

- ◯ The velocity of the object
- ◯ The density of the fluid
- ◯ The size of the object
- ◯ The mass of the object

83. You push a crate that is twice as heavy as you along a horizontal surface.

- ◯ The crate exerts a force on you that is twice the magnitude of the force that you exert on the crate.
- ◯ The crate exerts a force on you that is half the magnitude of the force that you exert on the crate.
- ◯ The crate exerts a force on you that is the same magnitude as the force that you exert on the crate.
- ◯ None of the above, as it will depend whether the crate is moving at constant speed or accelerating.

84. A person is standing on a bathroom scale that indicates their mass in kg. The scale reading depends on the amount that a spring inside the scale is compressed as a result of the person standing on the scale. If that person stands on that scale in an elevator that is accelerating upwards in a building, what happens to the reading on the scale relative to the reading outside of the elevator?

- ◯ It increases.
- ◯ It stays the same.
- ◯ It decreases.

85. If a bowling ball and a chocolate chip cookie collide,

- ◯ The bowling ball exerts a greater force on the cookie.
- ◯ The cookie exerts a greater force on the bowling ball.
- ◯ They both exert equal and opposite forces on each other.

4.2 Long answers

86. Four boxes connected by ropes are lying on a frictionless surface, as in Figure 10. The boxes have masses $m_1 = 6.0\,\mathrm{kg}$, $m_2 = 4.0\,\mathrm{kg}$, $m_3 = 3.0\,\mathrm{kg}$, $m_4 = 1.0\,\mathrm{kg}$. Box 4 begins to accelerate to the right with an acceleration of $2.0\,\mathrm{m/s^2}$.

    (a) Find the tension in each rope and determine the rope with the highest tension

    (b) If the boxes all have the same mass, would the tensions be the same in each rope?

(c) Box 4 stops accelerating when it hits 3.0 m/s. What is the tension in the rope between box 1 and 2 a moment later?

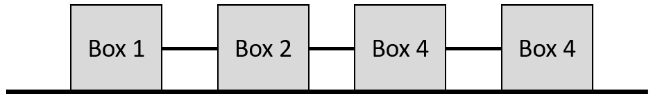

Figure 10: Four boxes on a frictionless surface.

87. Two boxes sit in a frictionless V-shaped pit with a 90° angle as shown in Figure 11.

(a) Draw a free-body diagram for box 1.

(b) Find the magnitude of each force on box 1, assuming that the mass of each box is 15 kg.

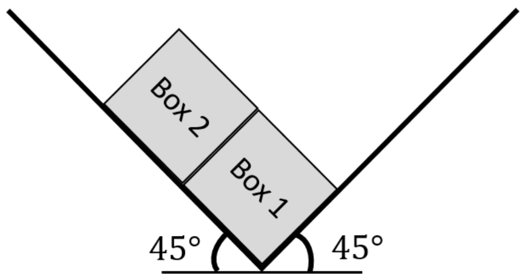

Figure 11: Two boxes sitting in a V-shaped pit.

88. Three blocks are connected by inextensible strings to a system of 2 massless and frictionless pulleys, as shown in Figure 14. Block 3 is on a frictionless horizontal surface.

(a) If block 1 has mass $m_1 = 3.0\,\text{kg}$, block 2 has mass $m_2 = 1.0\,\text{kg}$, and block 3 has mass $m_3 = 3.0\,\text{kg}$, draw the free body diagram for each block and determine the acceleration of block 2 (magnitude and direction).

(b) If block 1 has mass $m_1 = 1.0\,\text{kg}$, block 2 has mass $m_2 = 3.0\,\text{kg}$, and block 3 has mass $m_3 = 3.0\,\text{kg}$, draw the free body diagram for each block and determine the acceleration of block 2 (magnitude and direction).

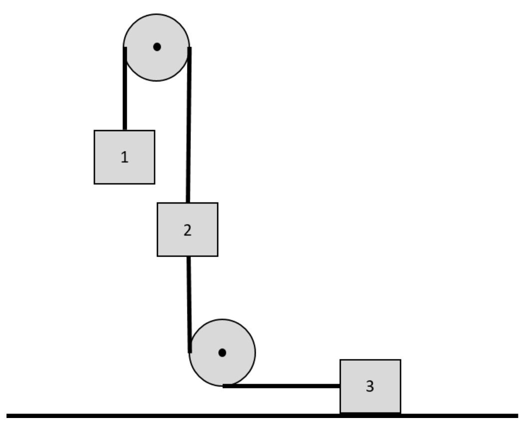

Figure 14: System of 3 blocks connected with two pulleys.

89. A fireman with mass $m = 90\,\text{kg}$ slides down the pole at the fire station with acceleration

$a = 5.0\,\text{m/s}^2$.

(a) What force (magnitude and direction) does the pole exert on the fireman?

(b) What force (magnitude and direction) does the fireman exert on the pole?

(c) If the pole is 10.0 m long, how much time does it take the fireman to reach the bottom?

(d) What speed will the fireman have when she hits the ground?

90. A 10.0 kg object is initially moving east with a speed of 15 m/s. A force then acts on it for 2.0 s, after which it moves north-west, also with a speed of 15 m/s. What are the magnitude and direction of the average force that acted on the object during that 2.0 s interval?

91. A little souvenir vicuña figurine with a mass of $m = 50\,\text{g}$ hangs by a thin string from the rear view mirror of your car. As you accelerate, you notice that the string makes an angle of 10.0° from the vertical. What is the acceleration of your car?

92. You are called in to investigate a car accident, in which a car struck a llama on a dry road. The driver claims that they were going below the speed limit of 50 km/h and hit the breaks, locking the cars' wheels, as soon as they struck the animal. You notice that there are skid marks from the car, which you measure to be 25 m in length. You also look up the coefficients of static friction and kinetic friction between the tires and the dry road, and find that they are $\mu_s = 0.9$ and $\mu_k = 0.6$, respectively.

(a) At what speed can you say that the driver was going before the car started skidding?

(b) If the road had been wet and the coefficients of friction correspondingly lower, would the skid marks be shorter or longer?

93. Two blocks with masses, m_1 (block 1), and m_2 (block 2), are placed on an incline, as shown in Figure 18. The coefficients of kinetic friction between block 1 and the incline is μ_{k1} and the coefficient of kinetic friction between block 2 and the incline is μ_{k2}. Block 1 is placed a distance L from the bottom of the incline, while block 2 is placed a distance D from block 1. You can neglect the dimensions of each block.

(a) Find an expression for the distance D that will result in the blocks colliding exactly at the bottom of the incline, if they are both released at time $t = 0$. Give your answer in terms of L, θ, μ_{k1}, μ_{k2}.

(b) How much time goes by between the blocks being released and the blocks reaching the bottom of the incline?

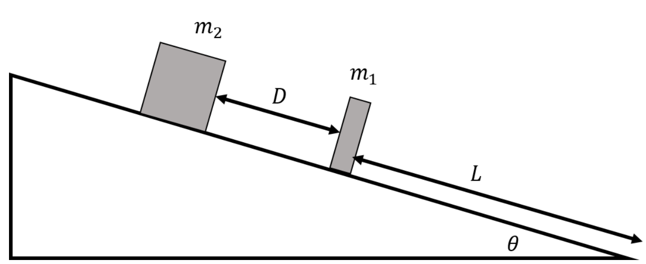

Figure 18: Two blocks on an incline at $t = 0$.

5 Applying Newton's Laws

5.1 Multiple Choice

94. A race car is travelling on a circular track, and completes one lap every 5 minutes. What is the race car's angular velocity?

    ◯ $\frac{3\pi}{2}$ rad/min

    ◯ $\frac{\pi}{4}$ rad/min

    ◯ $\frac{\pi}{2}$ rad/min

    ◯ π rad/min

95. A conical pendulum is made by swinging a mass, m, attached to a mass-less string of length, L, so that the mass moves in a horizontal circle of radius, R, with a constant speed, v, as shown in Figure 20. Which of the following describes half of the opening angle of the cone (the angle θ in Figure 20)?

    ◯ $\tan\theta = \sqrt{\frac{v^2}{gR}}$

    ◯ $\tan\theta = \frac{v^2}{gR}$

    ◯ $\tan\theta = \frac{gR}{v^2}$

    ◯ $\tan\theta = \sqrt{\frac{gR}{v^2}}$

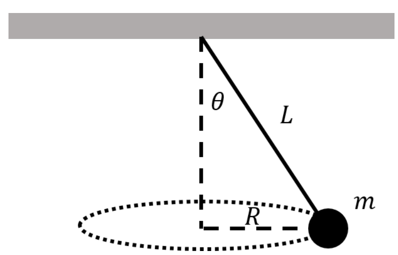

Figure 20: A conical pendulum executing uniform circular motion in a horizontal plane (Question 95).

96. You are sitting on a playground merry-go-round spinning at a constant speed and you remain in the same spot on the platform. Based on Newton's Laws, what statement best describes your situation?

    ◯ You must apply an equal and opposite force to the one trying to fling you off in order to hold yourself on the merry-go-round.

    ◯ If the merry-go-round doesn't slow down or speed up, you are able to easily remain on the platform as you are moving at constant speed.

    ◯ A force must be applied to keep you on the merry-go-round.

97. A block of mass 5 kg is on a slope held in place by a rope. The angle of the slope incline is $\alpha = 60°$ and the acceleration due to gravity is $a_g = 9.8\,\mathrm{m/s^2}$. Assuming no frictional forces, what is the tension of the rope, T, keeping the block from sliding down?

- ◯ 40.3 N
- ◯ 37.5 N
- ◯ 78.6 N
- ◯ 53.8 N
- ◯ 42.4 N

98. Two boxes of mass $m_1 = 5\,\mathrm{kg}$ and $m_2 = 3\,\mathrm{kg}$ are connected by a rope and rest on a frictionless surface. You pull on the box of mass $m_2 = 3\,\mathrm{kg}$ with a force $F = 32\,\mathrm{N}$ so that the rope between the boxes is taught. What is the tension in the rope connecting the boxes?

- ◯ 20 N
- ◯ 23 N
- ◯ 25 N
- ◯ 32 N

99. A block of mass $M = 10\,\mathrm{kg}$ is held by a force $\vec{F}$ on a surface inclined at angle of 45° with respect to the horizontal. Given that the coefficient of static friction between the block and the surface is 0.5, what is the minimum force F necessary to prevent the box from slipping?

- ◯ 34.6 N
- ◯ 49.0 N
- ◯ 69.3 N
- ◯ 138.6 N

100. A car goes around a curve of radius R at a constant speed v. It then goes around a second curve of radius $2R$ at speed $2v$. What is the centripetal force on the car as it goes around the second curve, compared to the first?

- ◯ The centripetal forces in both curves are equal
- ◯ The centripetal force in the second curve is half as big as in the first curve
- ◯ The centripetal force in the second curve is twice as big as in the first curve
- ◯ The centripetal force in the second curve is four times as big as in the first curve

101. You design a banked curve on a highway for trucks to safely navigate a 90° turn in the winter. The turn can be approximated by a quarter of a circle of radius $R = 500\,\mathrm{m}$. Which bank angle should you use so that trucks going $110\,\mathrm{km/h}$ will be able to go around the

curve even when the road is covered in ice (and thus frictionless to a good approximation)? *The bank angle is defined as the angle between the plane of the curve and the horizontal.*

- ◯ 68°
- ◯ 18°
- ◯ 11°
- ◯ 0.2°

102. Ozzy is riding in a clown car with a mass of 10 000 kg at a speed of 100 km/h around a curve with a radius of 400 m. What is the minimum coefficient of static friction between the car's four tires and the road to prevent the clown car from flying off the road?

- ◯ 0.049
- ◯ 0.197
- ◯ 0.420
- ◯ 0.787

103. What should the speed limit be on a curve with a radius of 100 m to prevent cars from slipping if the static coefficient of friction between the road and wheels of cars is 0.70? Assume that the road is flat.

- ◯ 26 km/h
- ◯ 50 km/h
- ◯ 94 km/h
- ◯ Not enough information to determine

104. You are planning your descent onto planet Camelid with your crew by deploying parachutes from the command module where you are all strapped in. Currently, the projected terminal velocity of 80 km/h is too high and will destroy your cool physics experiments on-board. You need to cut this velocity in half. Assuming all the drag force comes from the parachute, your crew come up with the following in order to cut your speed in half:

- ◯ Quadruple the size of the parachute
- ◯ Halve the size of the parachute
- ◯ Double the size of the parachute
- ◯ Increase the size of the parachute by 50 %

105. Which statement(s) is correct? [Select all that apply]

- ◯ The centripetal force and centrifugal force are the same force, but named differently.
- ◯ Centrifugal force is equal and opposite to the centripetal force.

- ◯ The centrifugal force is actually an apparent force.
- ◯ The centripetal force is an inertial force.

106. An American man at a gun range is practising his marksmanship. Using his high-powered rifle, he attempts to hit a target 1200 yards away, but notices all of his shots roll to the left. After several attempts, he gets frustrated and leaves. What was happening to the bullet at those long ranges?
    - ◯ Voodoo
    - ◯ Coriolis Effect
    - ◯ Drag Force
    - ◯ Stokes' Law
107. Why does a banked curve allow a car to go through a turn at a higher speed? [Select all that apply]
    - ◯ The centripetal force necessary is lessened by the bank.
    - ◯ The bank forces a component of the normal force to make up part of the centripetal force.
    - ◯ The bank is more fun to drive around.
    - ◯ The friction force necessary to keep the tires from slipping is lessened.
108. A mass on a string rotates in a vertical circle around a point. What can be said about the tension in the string when the object is at the top of the circle (T_{top}) and when the mass is at the bottom of the circle (T_{bottom})?
    - ◯ $T_{top} = T_{bottom}$
    - ◯ $T_{top} < T_{bottom}$
    - ◯ $T_{top} > T_{bottom}$
    - ◯ Not enough information to tell.
109. A roller coaster cart of mass m is going through a vertical loop with a radius R. What is the minimum speed that the cart must have at the top of the loop for the cart to stay on the track?
    - ◯ $v_{top} \geq m\sqrt{gR}$
    - ◯ $v_{top} \geq \sqrt{mgR}$
    - ◯ $v_{top} \geq \sqrt{gR}$
    - ◯ $v_{top} \geq gR$
110. What is the ideal bank angle (relative to the horizontal) for a road that goes around a curve of radius $R = 20\,\mathrm{m}$ if vehicles drive at $60\,\mathrm{km/h}$. In other words, if the road is perfectly frictionless (e.g. covered with ice), at what angle should the curve be banked?

◯ 5°

◯ 17°

◯ 55°

◯ 87°

5.2 Long answers

111. At how many revolutions per minute must a 22 m diameter Ferris wheel turn for passengers to feel weightless at the top?

112. A funnel is rotating with an angular speed ω about a vertical axis. A block of mass m sits inside of the funnel, at a distance r from the axis of rotation (Figure 21). The opening angle of the funnel is θ. The coefficient of static friction between the funnel and the block is μ. What are the minimum and maximum values of ω such that the block will not slide?

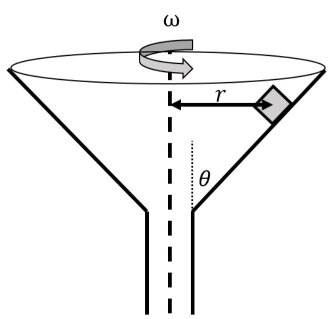

Figure 21: A block in a rotating funnel.

113. A child is playing with wooden blocks of mass, m and M, on a frictionless horizontal surface, as illustrated in Figure 24. The coefficient of static friction between the two blocks is μ. What is the minimum force that the child must apply to the wedge-shaped block for the small cube resting on top of it to accelerate upwards relative to the Wedge-shaped block?

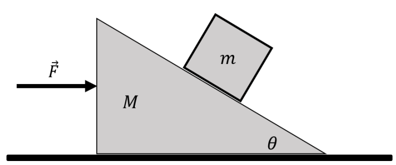

Figure 24: Block m placed on a wedge-shaped block of mass M.

114. Two blocks made of different materials and with masses m_A and m_B, respectively, are connected by a rope and slide down an inclined slope as shown in Figure 26. The incline makes an angle θ with the horizontal. The (different) coefficients of kinetic friction between the blocks and the incline are μ_A and μ_B for block A and B, respectively.

    (a) Given the masses of the blocks, the angle θ, and the coefficients of kinetic friction, what is the acceleration of the two blocks?

(b) Given the masses of the blocks, the angle θ, and the coefficients of kinetic friction, what is the tension in the rope?

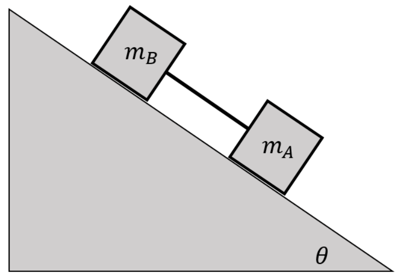

Figure 26: Two connected blocks sliding down an incline.

115. A square block, m_A, is placed on top of a rectangular block, m_B. A wire is connected to both blocks through a pulley which is attached to a nearby wall, as shown in Figure 28. Suppose the coefficient of static friction between block A and block B and the coefficient of static friction between block B and the ground are both equal to μ. A force F pulls block B away from the pulley. What is the maximum force that can be applied such that block A and block B remain at rest?

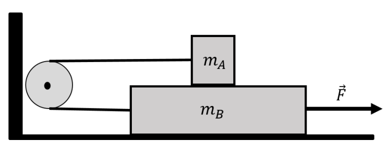

Figure 28: Two blocks connected by a pulley.

116. An air plane is flying at 200 m/s in a holding pattern above the Cusco airport. The airplane flies around in horizontal circles, such that each circle takes 5 min to complete.

(a) What bank angle must the pilot fly so that each circle takes 5 min to complete?

(b) What is the percentage increase in the perceived weight of the passengers? (This is given by the new perceived weight minus the old perceived weight divided by the old perceived weight)

117. A rocket of mass m burns fuel producing an upwards force (thrust) on the rocket, $F(t)$, that varies with time:

$$F(t) = A + Bt$$

The rocket starts at rest and travels in the vertical direction. Assume that the only two forces on the rocket are its weight and the thrust (no drag).

(a) Assuming that the mass of the rocket is constant with time, and that the acceleration due Earth's gravity does not change as the rocket's altitude increases, what is the rocket's altitude as a function of time (in terms of A, B, m, g, and time)?

(b) If the rocket loses mass as it burns fuel, would it gain altitude at a higher or at a lower rate than in part a)? Justify your answer!

118. Three blocks with masses m_1, m_2, and m_3 are stacked upon each other as shown in Figure 31. The coefficients of static friction between blocks is μ_s, and the coefficient of kinetic friction between the bottom block and the ground is μ_k. The blocks are moving in unison towards the right. Show that the maximum magnitude of the force that can be applied to the bottom block before the blocks slide with respect to each other is given by:

$$F = (\mu_s + \mu_k)(m_1 + m_2 + m_3)g$$

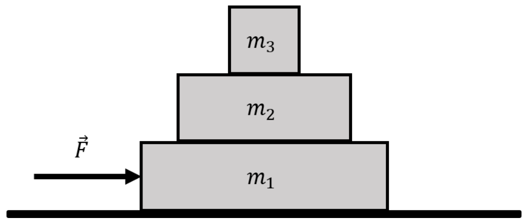

Figure 31: Three blocks sliding with friction.

119. Block A and B are connected by a string and massless frictionless pulleys as shown in Figure 33. The blocks have masses of $m_A = 2.0\,\text{kg}$ and $m_B = 4.0\,\text{kg}$. If the blocks are in motion (A is falling), and the coefficient of kinetic friction between block B and the surface of the table-top is $\mu = 0.5$, what are the accelerations of blocks A and B?

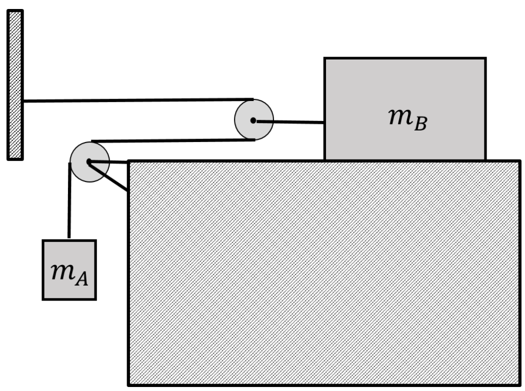

Figure 33: A system of blocks and pulleys.

120. You push with an unknown horizontal force, $\vec{F}$, against a 4 kg crate that is located on an inclined plane that makes a 30° angle with respect to the horizontal, as shown in Figure 35. The coefficient of kinetic friction between the crate and the incline is 0.2.

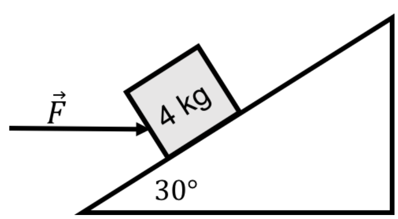

Figure 35: A crate being pushed up an incline.

(a) What is the magnitude of the force $\vec{F}$ that must be applied in order for the box to move at a constant speed?

(b) What is the value of the normal force if the box instead accelerates at $2\,\mathrm{m/s^2}$ up the incline? How does this compare to the magnitude of the normal force when the box moves at a constant speed?

121. An object is released from an airplane that is flying in the horizontal direction with speed v_A, as depicted in Figure 37. The force of drag on the object is given by:

$$\vec{F}_d = -b\vec{v}$$

where b is a positive coefficient, and $\vec{v}$ is the velocity of the object.

(a) Find expressions for the x and y components of the velocity of the object as a function of time. **Hint:** You should first show that you can obtain the two following equations:

$$\frac{dv_x}{dt} = -\frac{b}{m}v_x$$
$$\frac{dv_y}{dt} = g - \frac{b}{m}v_y$$

by applying Newton's Second Law at some point on the trajectory of the object, when the velocity vector of the object makes some angle θ with the horizontal. Note that the velocity vector is anti-parallel to the drag force, and use that to write out the two components of Newton's Second Law to obtain the equations above.

(b) Plot the x and y components of the velocity as a function of time.

(c) Show that the terminal velocity of the object is vertical, and determine its magnitude.

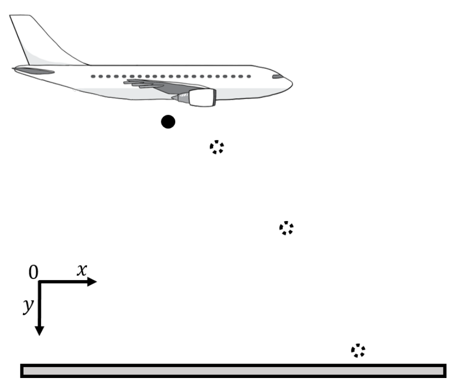

Figure 37: An object dropped from a moving airplane.

122. Two blocks, with masses m_a and m_b, are attached to a mass-less string and move in uniform circular motion on a horizontal friction-less table, as shown in Figure 40. The blocks describe circles of radius R_a and R_b, respectively. Find an expression for the ratio of the tension in the two section of the string in terms of m_a, m_b, R_a and R_b.

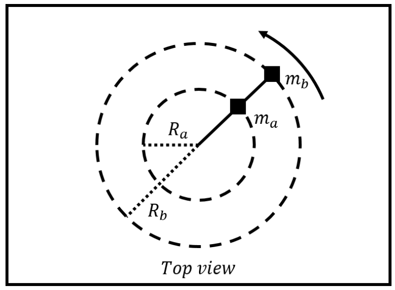

Figure 40: Two blocks attached to a string, rotating on a friction-less horizontal table, as seen from above.

123. A vertical hoop of radius, R, rotates with constant angular velocity, ω, about a vertical axis through its centre, as shown in Figure 42. The hoop has a track in it which allows a marble to roll inside the hoop. When the hoop rotates at a particular angular velocity, the marble is in equilibrium and rotates with the hoop at a position that makes an angle θ from the vertical, as shown.

    (a) Give an expression for the angle θ in terms of the radius of the hoop and the angular velocity.

    (b) Explain or show why one can never have $\theta = 90°$.

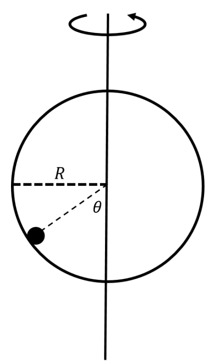

Figure 42: A vertical hoop with a marble in it rotating about a vertical axis.

6 Work and Energy

6.1 Multiple Choice

124. A constant force applied by a person is pushing a 200 kg box to the right. Before the person starts to push, the initial velocity of the box is 2 m/s to the right. After 200 s the velocity of the box is 20 m/s to the right. What is the average power exerted by the person, assuming that the box moves on a frictionless surface?

    - ◯ 100 W
    - ◯ 198 W
    - ◯ 186 W
    - ◯ 200 W

125. Two students push identical boxes across the same floor, but student A pushed the box twice as far in the same amount of time. Student A ...

    - ◯ Did twice as much work as student B
    - ◯ Did four times the amount of work as student B
    - ◯ Did the same amount of work as student B
    - ◯ Did half as much work as student B

126. In which of the following scenarios is work being done?

    - ◯ Carrying a briefcase horizontally above the ground with a constant velocity
    - ◯ A normal force from the floor on the second level of a building stops you from falling into the floor below
    - ◯ The tension in a string pulling on a rubber stopper when you spin it around at a constant speed
    - ◯ Both 1 and 3
    - ◯ None of the above

127. On Halloween, a clown enters a pumpkin carving contest. Part of the contest involves the participants lifting up their pumpkin after it has been carved to declare they are finished with their design. This also allows the audience of the contest to see the funky designs carved into the pumpkins. The clown completes his design and lifts up his pumpkin. What is the work done by the clown on the pumpkin if the clown lifts his pumpkin a total of 2 m and the pumpkin's mass is 6 kg?

    - ◯ 117.6 J
    - ◯ 102.3 J
    - ◯ 120.0 J

128. What are the SI units for power?

- ◯ $kgms^{-2}$
- ◯ kgm^2s^{-2}
- ◯ kgm^2s^{-3}
- ◯ kgm^3s^{-3}

129. A worker horizontally pushes a sled with a force of 40 N over a distance of 6.0 m, while a frictional force of 24 N acts on the wheelbarrow in a direction opposite to that of the worker. What net work is done on the wheelbarrow?

- ◯ 96 J
- ◯ 144 J
- ◯ −144 J
- ◯ −96 J

130. A horizontal conveyor belt is moving a 1 m tall guanaco at constant speed. The guanaco has a mass of 90 kg and moves 15 m along the conveyor belt, in 10 s. What is the work done on the guanaco, assuming the conveyor belt has a 100% efficiency?

- ◯ 13 000 J
- ◯ 0 J
- ◯ 90 J
- ◯ Not enough information.
- ◯ Gaunacos are immune to work.

131. You are using an electric crane to lift a 400 kg box onto the roof of a house. The crane has a power rating of 1000 W, and is 50% efficient at converting electrical power into mechanical work. If the house is 10 m tall, what is the smallest time in which the crane can lift the box from the ground up to the roof?

- ◯ 0.013 s
- ◯ 7.8 s
- ◯ 78.4 s
- ◯ 156.8 s

132. If 500 J of net work is done to a 20 kg mass with an initial speed of 5 m/s, then what is the final speed of the mass?

- ◯ 7.64 m/s
- ◯ 8.67 m/s
- ◯ 25.00 m/s

- ◯ $30.00\,\mathrm{m/s}$

133. The work done by a person that climbed up a staircase of height h:
    - ◯ is negative.
    - ◯ is 0.
    - ◯ is positive.
    - ◯ depends on our choice of whether positive is up or down.

134. You push a crate a distance d up an incline (measured parallel to the incline). The crate started at rest and ended at rest, and there is friction between the incline and the crate. The work that you did:
    - ◯ is negative.
    - ◯ is 0.
    - ◯ is positive.
    - ◯ depends on how fast you pushed the crate up the incline.

6.2 Long answers

135. A sled and passenger with a combined mass of $50\,\mathrm{kg}$ are pulled $20\,\mathrm{m}$ across snowy flat ground ($\mu_k = 0.20$) at constant velocity, by a force, $\vec{F}$, directed $25°$ above the horizontal.
    (a) Draw a free body diagram of the sled and passenger. Be sure to show your coordinate system, show the velocity and acceleration of the sled, and have accurate lengths of force vectors.
    (b) What is the work done by the applied force?
    (c) What is the work done by the frictional force?
    (d) What is the total work done? Calculate the total work and explain your result.

136. A $7.0\,\mathrm{kg}$ body has three times the kinetic energy of a $20.0\,\mathrm{kg}$ body. Calculate the ratio of the speeds of these bodies.

137. Not all springs exhibit a linear restoring force ($F(x) = -kx$). If you stretch any spring far enough, it will exhibit *non-linearity*, where terms in the force go with higher powers of x (e.g. $F(x) = -kx - ax^3 - bx^5 - ...$).

    Consider a bungee cord that exerts a non-linear elastic force of magnitude:

    $$F(x) = -k_1 x - k_3 x^3$$

    where x is the distance the cord is stretched, and $k_1 = 200\,\mathrm{N/m}$ is the linear spring coefficient. Suppose that the cord is stretched a distance of 19 m, and it is found that this requires $24\,\mathrm{kJ}$. What is the non-linear spring coefficient k_3 of the bungee cord (and its units)?

138. A person with a mass of 80 kg runs up a flight of stairs 20 m high in 10 s at constant speed.

(a) Assuming the person's body is only 25% efficient in converting energy to work, how much power do they expend lifting themselves up the stairs?

(b) The person's daily energy intake from food is 10.5 MJ (2500 food calories) while maintaining a constant weight. What is the average power they produce over a day? How does this compare with their power production when running up the stairs?

139. A 60 kg meteorite strikes the earth, colliding with a patch of mud. The meteorite stops after travelling 4.0 m into the mud. The force between the meteorite and the mud can be modelled as:

$$F(x) = (590\,\mathrm{N/m^3})x^3$$

where x is the depth in the mud. Assuming that the meteorite is at rest once it has travelled 4.0 m, what was the initial speed of the meteorite?

140. A skateboarder of mass m is riding along a frictionless ramp, as show in Figure 44. Using the coordinate system shown in the Figure, the ramp can be modelled as being horizontal for negative values of x, and then a parabola, $y(x) = a + bx + cx^2$, for positive values of x. The ramp ends at $x = x_0$. Show that the work done by gravity on the skateboarder over the path of the ramp (between $x = 0$ and $x = x_0$) is given by:

$$W = -mg(bx_0 + cx_0^2)$$

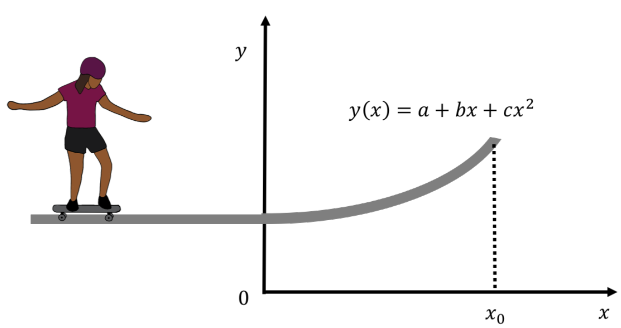

Figure 44: A skateboarder on a ramp.

141. A toy rocket of mass m is tied to a string and forced to go around in a horizontal circle of radius R, as shown in Figure 45. The thrust of the rocket has a constant magnitude F and is always directed at an angle θ with respect to the tangent of circle.

(a) If the rocket starts at rest, what will its speed be after 1 revolution (in terms of the given variables)?

(b) How long will it take to complete the first revolution?

(c) What is the tension in the string after 1 revolution?

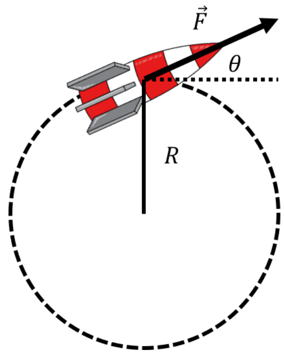

Figure 45: A rocket going around in a circle.

142. A mass m starts from rest and slides down an inclined plane that makes an angle θ with the horizontal (see Figure 46). The mass starts at a height h above the ground. At the bottom of the ramp, the mass slides for a distance d along the ground, before running into a spring and compressing it. The spring is non-linear and the restoring force is given by $F(x) = -k_1 x - k_3 x^3$, where x is the amount that the spring is compressed relative to its rest position. The coefficient of kinetic friction between the mass and the inclined plane, and between the mass and the ground is μ_k.

   (a) Write an expression for the speed of the mass just before it makes contact with the spring.

   (b) What is the maximum value of the coefficient of kinetic friction that will allow the mass to make contact with the spring?

   (c) If instead, the coefficient of kinetic friction is zero everywhere, and the maximum compression of the spring is found to be X, what was the initial height of the block?

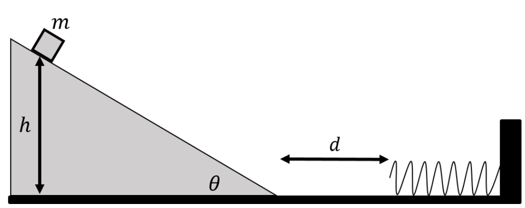

Figure 46: A mass sliding down a slope.

143. A constant force $\vec{F}$ is applied to an object. The object moves a total displacement $\vec{d}$ in the same direction as the force vector (Figure 47).

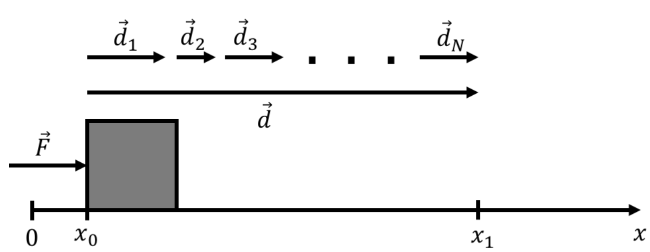

Figure 47: A force is applied to an object. The object's displacement $\vec{d}$ is divided into N segments.

Show that if you divided the displacement into N segments of length d_i (where $i = 1, 2, 3, ..., N$), the sum of the work done over each segment would be equal to the work done over the whole displacement $\vec{d}$. i.e. Show that

$$W = \sum_{i=1}^{N} \vec{F} \cdot \vec{d_i}$$

144. The vertical displacement of the centre of a trampoline can be modelled as a spring with spring constant $k = 1 \times 10^4\,\mathrm{N/m}$ that follows Hooke's Law ($F(x) = -kx$). If you ($m = 50\,\mathrm{kg}$) leap (i.e. initial velocity of zero) onto the trampoline from a height $H = 1.5\,\mathrm{m}$ above the surface of the trampoline, what is your total vertical displacement when you (momentarily) come to rest, when the trampoline is maximally compressed?

145. Oh no, you've created a positive feedback loop! You've synthesized a material with a coefficient of friction $\mu = -0.2$ and constructed a system with two springs of $k = (100 + 60x)\mathrm{N/m}$ which are a distance $d = 0.5\,\mathrm{m}$ from one another when uncompressed. If the springs are compressed a distance greater than $x_{break} = 1.5\,\mathrm{m}$, they will snap and the system will break. The space between the two springs is covered in your $\mu = -0.2$ substance, but the space underneath the springs is frictionless, as shown in Figure 48. Motion begins in the system when a block of mass $m = 12\,\mathrm{kg}$ is fixed at a compressed distance of $x = 0.5\,\mathrm{m}$ on one of the springs, then released. How many times will the block move from one spring to the other before the system breaks?

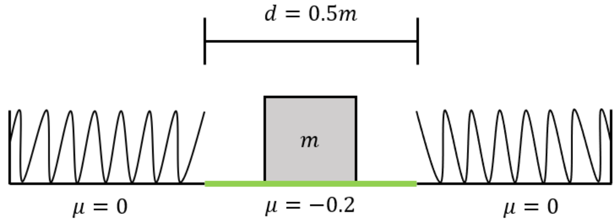

Figure 48: A positive feedback loop.

7 Potential energy and conservation of energy

7.1 Multiple Choice

146. The Darlington Nuclear Generating station in Ontario converts nuclear binding energy into electrical energy. What electrical power does the Darlington Nuclear Generating station produce?

  ◯ $2.512 \times 10^6\,\mathrm{kg \cdot m^3/s^2}$

  ◯ $3.512 \times 10^6\,\mathrm{kg \cdot m^2/s^3}$

  ◯ $4.512 \times 10^6\,\mathrm{kg^2 \cdot m^3/s^2}$

  ◯ $5.512 \times 10^6\,\mathrm{kg \cdot m^3/s^3}$

147. There exists a closed system with a mechanical energy of 65 J. When a conservative force acts in this closed system, the mechanical energy...

  ◯ increases

  ◯ decreases

  ◯ could increase or decrease, depending on what the force does

  ◯ will remain at 65 J regardless of what the force does

148. In an effort to create a cheerleading robot, an engineer has attached a pom-pom of mass $m = 100\,\mathrm{g}$ to a sping of $k = 1.5\,\mathrm{N/m}$. The pom-pom is compressed 7 cm, then released. What is the maximum speed of the pom-pom?

  ◯ 0.86 m/s

  ◯ 1.5 m/s

  ◯ 0.27 m/s

  ◯ 0.86 cm/s

149. The Force from Star Wars is found to be conservative, so that a potential energy function, $U(\vec{r})$ (aka “The Potential”), can be defined. Captain Jean-Luc Picard uses The Force to create a Force Field around his ship in order to repel an incoming rebel ship full of jedis, klingons and other rebellious space people. As the incoming rebel ship powers through the repelling force field, the rebel ship’s Potential Energy due to The Force:

  ◯ decreases.

  ◯ remains the same.

  ◯ increases.

  ◯ not enough information to tell.

150. A bird with a mass of 200 g is resting in a tree. It has a potential energy of 10 J with zero potential energy defined to be where the ground is. What is the height of the bird relative to the ground?

- ◯ 19.6 m
- ◯ 9.8 m
- ◯ 7.2 m
- ◯ 5.1 m

151. If a lightning bolt were to discharge all of its (roughly) one billion Joules into a 2.0 g bullet, and we assume that 100% of that energy is converted into horizontal kinetic energy, how fast would the bullet travel immediately after being hit?

- ◯ 7.1×10^5 m/s
- ◯ 1.0×10^6 m/s
- ◯ 1.0×10^{12} m/s
- ◯ 3.0×10^8 m/s

152. A vicuña with a mass of 35 kg is launched through the air in a parabolic arc. It has an initial kinetic energy of 28 kJ and travels a total horizontal distance of 100 m, reaching its max height of 150 m at a distance of 50 m from where it was launched. Ignoring air resistance, what is the vicuña's speed when it hits the ground?

- ◯ 1.3 m/s
- ◯ 40 m/s
- ◯ 36.6 m/s
- ◯ 20 m/s

153. What is true about the kinetic energy of a one-dimensional, vertical mass-spring system?

- ◯ It is at a maximum at the highest and lowest point and zero at equilibrium point
- ◯ It is zero at the highest and lowest point and at a maximum at the equilibrium point
- ◯ It is constant for the entire motion cycle
- ◯ There is no kinetic energy, only a balance between elastic potential energy and gravitational potential energy

154. Two children are on a playground competing to see who can throw a lawn dart the highest. Unfortunately, they fail to notice that they are standing under a tree. As soon as the game begins, they each throw their lawn dart straight upwards, but neither one comes down. One is stuck 20 m high in the tree and the other one is stuck 30 m high in the tree. Which of the following statements is true? Select all that apply:

- ◯ The lawn darts both currently have the same kinetic energy
- ◯ The lawn darts both currently have the same gravitational potential energy
- ◯ The lawn darts both experienced the same amount of net work since they were thrown
- ◯ The path of each lawn dart is needed to determine the net work done on each dart
- ◯ The tree is currently doing work on the lawn darts to keep them from falling

155. An object is released from a height and falls under the influence of gravity. Which of the following statements is true in this situation if you neglect air resistance? [Select all that apply]

- ◯ The mechanical energy of the object is decreasing
- ◯ The kinetic energy of the object is increasing and the potential energy is increasing
- ◯ The kinetic energy of the object is increasing and the potential energy is decreasing
- ◯ The mechanical energy of the object is increasing
- ◯ The mechanical energy of the object is constant

156. The net force acting on a particle is conservative and increases the kinetic energy by 500 J. What is the change in the potential energy, ΔU, and the total energy, ΔE, of the particle?

- ◯ $\Delta U = -500\,\mathrm{J} \,|\, \Delta E = 500\,\mathrm{J}$
- ◯ $\Delta U = -500\,\mathrm{J} \,|\, \Delta E = 0$
- ◯ $\Delta U = -250\,\mathrm{J} \,|\, \Delta E = 0$
- ◯ Not enough information

157. A 500 g squirrel climbs up a tree 2.3 m high. What is its change in gravitational potential energy?

- ◯ 11 270 J
- ◯ 9.8 J
- ◯ 11.3 J
- ◯ 112.7 J

158. A child is sliding down a frictionless water slide. The slope is very small at the top of the slide, then the slide gets steeper and steeper towards the middle and finally begins to level out again near the bottom. At what point on the slide will the child be moving the fastest?

- ◯ at the top of the slide

- ◯ at the steepest part of the waterslide, in the middle
- ◯ at the bottom of the slide
- ◯ the child moves at a constant speed

159. Can a particle's mechanical energy equal its potential energy?

- ◯ No, because these are two different types of energy
- ◯ Yes, as long as it has no kinetic energy
- ◯ Yes, as long as it has no gravitational energy
- ◯ No, because mechanical energy has different dimensions than potential energy

160. A particle of mass m is constrained to move along the x axis. The only net force acting on the particle is conservative and can be modelled by a potential energy function $U(x) = \frac{1}{4}k_3x^4 - \frac{1}{2}k_1x^2$, where x is the particle's position expressed in metres, and k_3 and k_1 are positive numbers. Which statement is correct?

- ◯ The force is zero when $x = \frac{k_1}{k_3}$ and in the positive x direction when $x > \frac{k_1}{k_3}$
- ◯ The force is zero when $x = \frac{k_1}{k_3}$ and in the positive x direction when $x < \frac{k_1}{k_3}$
- ◯ The force is zero when $x = \sqrt{\frac{k_1}{k_3}}$ and in the positive x direction when $x > \sqrt{\frac{k_1}{k_3}}$
- ◯ The force is zero when $x = \sqrt{\frac{k_1}{k_3}}$ and in the positive x direction when $x < \sqrt{\frac{k_1}{k_3}}$

161. Is the force of drag on an accelerating car conservative?

- ◯ Yes
- ◯ No

162. As you walk on a windy beach, you find that the force exerted on you by the wind is constant in magnitude and direction. Is this force conservative?

- ◯ Yes
- ◯ No

163. The work done by a non-conservative force on an object that moves from position A to position B

- ◯ depends on the path taken between A and B
- ◯ does not depend on the path taken between A and B

7.2 Long answers

164. A 140 g baseball is dropped from a tree 15.0 m above the ground.

   (a) With what speed would it hit the ground if air resistance could be ignored?

(b) If it actually hits the ground with a speed of 7.5 m/s, what is the average value of the drag force that was exerted on the ball by the air?

165. A 4.0 kg block slides along a horizontal surface with a coefficient of kinetic friction $\mu_k = 0.30$. The block has a speed $v = 3.0$ m/s when it strikes a massless spring head-on.

(a) If the spring has force constant $k = 200$ N/m, what is the maximum compression of the spring?

(b) What minimum value of the coefficient of static friction, μ_s, will assure that the spring remains compressed at the maximum compressed position?

(c) If μ_s is less than this, what is the speed of the block when it detaches from the decompressing spring?

Hint: Detachment occurs when the spring reaches its uncompressed length

166. A small mass m starts at rest the top of a sphere, and slides down the frictionless surface of the sphere. At what angle θ will the mass fall off of the sphere (see Figure 49)?

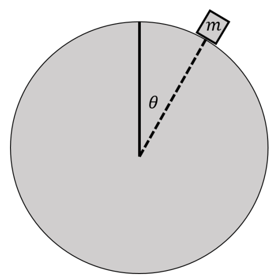

Figure 49: A small mass sliding down a frictionless sphere.

167. We can model protons inside of a nucleus as being repelled by an electrostatic force, while being attracted by a nuclear force. If we place one proton at the origin of a coordinate system ($r = 0$), we can model the potential energy of a second proton, a distance r from the first proton. The potential energy from the electric force can be written as:

$$U^E(r) = \frac{k}{r}$$

where $k = 1.0 \times 10^{-28}$ Jm. The potential energy from the nuclear force can be modelled as:

$$U^N(r) = -U_0 \frac{r_0}{r} e^{-\frac{r}{r_0}}$$

where $U_0 = 1 \times 10^{-13}$ J, and $r_0 = 1.5 \times 10^{-15}$ m

(a) Write an expression for the total force on one proton as a function of the distance between protons

(b) Show that the nuclear force is attractive, whereas the electric force is repulsive

(c) Make a plot of the two potential energy functions, and their sum, between $r = 0.5 \times 10^{-15}$ m and $r = 5 \times 10^{-15}$ m.

(d) Is there an equilibrium point where the proton feels no net force from the proton at the origin? If yes, what is the corresponding distance between protons and is it a stable equilibrium? (Note that you may need to find this point numerically, rather than analytically).

Note that this is a very poor model of a nucleus, as you will see.

168. The top floor of your neighbour's house is on fire, and you offer to help the firemen by using the water from your fancy swimming pool to put out the fire. You happen to have an old pump in your garage, but the label indicating its electrical power consumption is illegible, so you're not sure if the pump is powerful enough to pump the water to the required height of 20 m, through the 2.0 cm diameter of the fire hose. If the pump is 50% efficient in converting electrical power to kinetic energy of the water being pumped, what electrical power must the pump be rated for in order to help your neighbour?

    Assume that the temperature of the water is $0\,^{\circ}\mathrm{C}$ *and that the density of water is* $\rho = 1\,\mathrm{g/cm^3}$. *Also assume that there are no guanacos in the burning house.*

169. In the "advanced" approach to classical physics, we use only scalar quantities to determine the motion of a particle, instead of Newton's Second Law, which relies on vectors. For a particle of mass, m, moving in one dimension, we can define the Lagrangian, L, given by the difference between the kinetic energy of a particle, K, and its potential energy, U:

$$L(x, v) = K - U$$

    where, in general, the Lagrangian depends on the velocity of the particle (v, through the kinetic energy), and its position (x, through the potential energy). The equation of motion for a particle in one dimension (x) is given by the Euler-Lagrange equation:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial v}\right) - \frac{\partial L}{\partial x} = 0$$

    Show that the Euler-Lagrange equation is equivalent to Newton's Second Law in the case of a particle of mass m in vertical free fall near the surface of the Earth.

    Hint: Write the kinetic and potential energies for the particle at some point in time, when it has a speed v *and is at a position* x *above the ground. This will give you the Lagrangian, to which you can apply the Euler-Lagrange equation.*

170. A child has a mass M and a height h. They stand on a trampoline which, under their weight, compresses by a distance, d, equal to one tenth of their height (their feet are a distance d below where the trampoline would rest if nobody stood on it), as shown in the left panel of Figure 52. You may treat the trampoline as a spring that obeys Hooke's Law ($F(x) = -kx$).

    By how much should the trampoline be compressed (with respect to its rest position, x on the middle panel figure), so that it would launch the child a distance h above the trampoline's rest position (your feet end up a distance h above the rest position of the trampoline, as shown in the right panel)?

    Give your answer in terms of h.

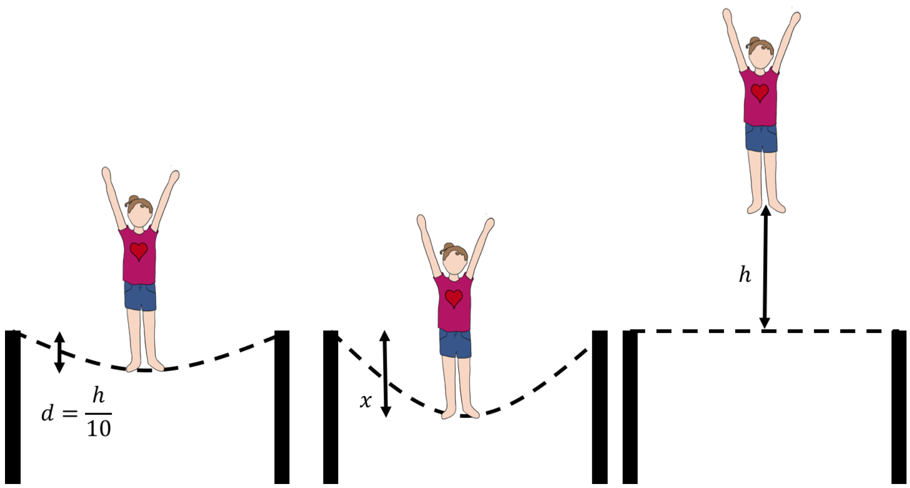

Figure 52: Determine the compression x to launch you at a height h.

171. A non-linear spring exerts a restoring force given by $\vec{F}(x) = (-k_1 x - k_3 x^3)\hat{x}$.
    (a) Show that the force is conservative.
    (b) Find the potential energy function for this spring.
    (c) Check your answer for part b) by recovering the force function from the potential energy function.

172. You need to design a horizontal spring to stop a car with a mass $m = 1 \times 10^3$ kg moving with a speed of $v = 100$ km/h. The spring is attached to a vertical wall on one end, and the car will collide with the other end of the spring. What should the spring constant, k, be if the occupants of the car are to experience an acceleration (deceleration) no greater than $4g$, where $g = 9.8\,\text{m/s}^2$? **Hint:** Think about the location at which the force from the spring will be the greatest, as that will correspond to the greatest acceleration.

173. The "Les Attelas" chairlift in the Swiss ski resort of Verbier can carry 2400 people per hour from an altitude of 2227 m to an altitude of 2722 m. Estimate the total electrical power required by the chairlift if it is 50% efficient at converting electrical power into mechanical work. **Hint:** You are not given enough data, you will have to estimate (and justify) values for any missing data.

174. In the "advanced" approach to classical physics, we use only scalar quantities to determine the motion of a particle, instead of Newton's Second Law, which relies on vectors. For a particle of mass, m, moving in one dimension, we can define the Lagrangian, L, given by the difference between the kinetic energy of a particle, K, and its potential energy, U:

$$L(x, v) = K - U$$

where, in general, the Lagrangian depends on the velocity of the particle (v, through the kinetic energy), and its position (x, through the potential energy). The equation of motion for a particle in one dimension (x) is given by the Euler-Lagrange equation:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial v}\right) - \frac{\partial L}{\partial x} = 0$$

Show that the Euler-Lagrange equation is equivalent to Newton's Second Law in the case of a particle of mass m attached to a horizontal spring with spring constant k. The mass can move on a frictionless horizontal surface.

Hint: Write the kinetic and potential energies for the particle at some point in time, when it has a speed v and is at a position x relative to the rest position of the spring. This will give you the Lagrangian, to which you can apply the Euler-Lagrange equation.

175. A mass-less spring with a linear restoring force has a spring constant $k = 100\,\text{N/m}$ and a rest length of $L = 1.0\,\text{m}$. The spring is placed at the bottom of an incline that makes an angle $\theta = 30°$ with respect to the horizontal, as shown in Figure 53. A block of mass $m = 0.5\,\text{kg}$ is then placed on the spring, and the spring is compressed by a distance $d = 0.5\,\text{m}$ (so that the block is a distance $L - d$ from the bottom of the incline). The block is then released and slides up the incline to a distance D from the bottom before sliding back down. The coefficient of kinetic friction between the block and the incline is $\mu_k = 0.3$.

    (a) (2 points) What will be the speed of the block when it leaves the spring?

    (b) (2 points) At what distance, D, measured from the bottom of the incline, will the block stop?

    (c) (2 points) The amount of thermal energy, ΔE, that is required to heat a block of mass m so that it changes temperature by an amount ΔT (in degrees Kelvin) is given by:

    $$\Delta E = mC\Delta T$$

    where C is called the heat capacity of the block and depends on the material with which the block is made. Instead of modelling friction as a non-conservative force, we can model it as a change in the thermal energy of the block, if we account for this type of energy. We will assume that all of the thermal energy from friction will heat up the block (rather than the incline).

    Again, we launch the block by placing it on the spring so that the spring is compressed by a distance $d = 0.5\,\text{m}$. If the block has a heat capacity of $C = 897\,\text{J/kg/°K}$ (that for aluminium) and stops at a distance $D = 5\,\text{m}$ from the bottom of the incline, what will be its change in temperature? Comment on whether this would be a practical experiment to perform.

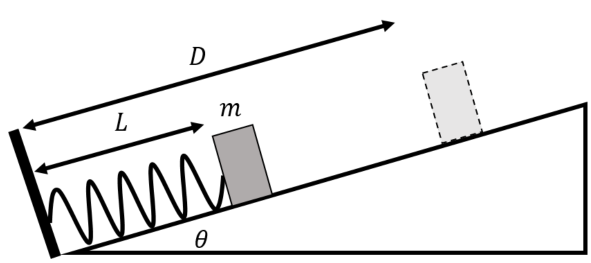

Figure 53: A block is placed on spring of rest length L, which is then compressed and released so that the block moves up the incline.

8 Gravity

8.1 Multiple Choice

176. An object of mass m has an escape velocity of v. What is the escape velocity of an object with a mass of $3m$?

    ◯ $\sqrt{3v}$

    ◯ $3v$

    ◯ $\sqrt{\frac{1}{3}v}$

    ◯ $\frac{1}{3}v$

177. The Earth is closest to the Sun in December. When is the Earth's gravitational potential energy with respect to the Sun the greatest?

    ◯ When the Northern Hemisphere has Summer.

    ◯ When the Northern Hemisphere has Winter.

    ◯ Not enough information.

178. Which of the following planets has the lowest value of the acceleration from gravity at its surface?

    ◯ A planet with mass $2M$ and radius $1/2\ R$

    ◯ A planet with mass $4M$ and radius $1/\sqrt{2}\ R$

    ◯ A planet with mass M and radius $1/3\ R$

    ◯ A planet with mass $6M$ and radius R

179. Jack is standing on top of a tall ladder, and Jill is on the ground. The gravitational acceleration felt by Jack is ____ the gravitational acceleration acting on Jill.

    ◯ less than

    ◯ greater than

    ◯ the same as

180. A satellite of mass m is in a circular orbit of radius r about a spherical planet of mass M, which can be considered fixed in space. If the gravitational potential energy, U, of m is defined as $U = -G\frac{Mm}{r}$, what can you say about the kinetic energy, K, of m?

    A. $K = \frac{1}{2}U$

    B. $K = -\frac{1}{2}U$

    C. $K = U$

    D. $K = -U$

181. Two astronauts are floating freely in space with no means of propulsion. Each astronaut, with their suit on, has a mass of 100 kg. They are holding onto each other with their arms extended keeping them 0.1 m apart, so that they don't drift apart. They let go for a second, and push off on each other a little. What is the maximum speed they can move apart if their bodies are ever to be reunited under their mutual gravitational attraction?

- ◯ 0.37 m/s
- ◯ 0.037 m/s
- ◯ 0.0037 m/s
- ◯ 0.000 37 m/s

182. The Earth experiences tidal forces from both the Sun and the Moon. Which of the following statements is true about the tidal forces acting on the Earth?

- ◯ The Sun is responsible for the larger of the two forces because of its much larger mass
- ◯ The Moon is responsible for the larger of the two forces because of its much closer proximity
- ◯ The Sun is responsible for the larger of the two forces because it has a larger difference between forces on the near and far side of the Earth
- ◯ The Moon is responsible for the larger of the two forces because it has a larger difference between forces on the near and far side of the Earth
- ◯ Two of the above are correct

183. A guanaco and a vicuña are trying to escape Earth's gravity. The guanaco has mass of 85 kg, and the mass of the vicuña is 60 kg. Which animal will require a greater initial velocity to escape Earth's gravitational pull, never to return? (Neglect air resistance.)

- ◯ The guanaco requires a greater initial velocity.
- ◯ The vicuña requires a greater initial velocity.
- ◯ Both animals will require the same initial velocity.

184. If a planet has a mass 10 times that of the Earth, and a radius 2 times that of the Earth, the acceleration due to gravity on the surface of the planet in terms of g would be:

- ◯ 2.5g
- ◯ g
- ◯ g/5
- ◯ 5g
- ◯ g/2.5

185. You are on an airplane travelling with a constant velocity at an altitude of 20 000 m above the surface of the Earth. What is the acceleration of gravity at that altitude? (The radius of Earth is 6.37×10^6 m, and its mass is 5.97×10^{24} kg).

◯ $9.70\,\mathrm{m/s^2}$

◯ $9.75\,\mathrm{m/s^2}$

◯ $9.80\,\mathrm{m/s^2}$

◯ $9.85\,\mathrm{m/s^2}$

186. A rocketship follows a circular orbit of radius r around Earth. The rocket quickly accelerates to twice its initial speed (so the acceleration is essentially instantaneous). What will happen to the period of the rocket's orbit?

◯ It will be the same as its initial period.

◯ It will be greater than its initial period.

◯ It will be less than its initial period.

187. In general, the gravitational potential energy of a mass m a distance r away from a mass M

◯ is equal to the work done by gravity in moving m from an infinite distance away from M to the current position.

◯ is equal to the work done by a person in moving m from an infinite distance away from M to the current position.

◯ can be positive or negative.

◯ is none of the above.

188. A mass m is located in vacuum a distance r away from another mass M which can be considered fixed in space. If we move mass m so that it is infinitely far away from M, what is the work done by the force of gravity on m?

◯ $W = -G\frac{Mm}{r}$

◯ $W = G\frac{Mm}{r}$

◯ not enough information to tell

189. A mass m is located in vacuum a distance r away from another mass M which can be considered fixed in space. If we move mass m so that it is closer to mass M (decrease r), what can you say about the gravitational potential energy of m?

◯ It decreases

◯ It increases

◯ not enough information to tell

8.2 Long answers

190. Answer the following:

   (a) Show that the escape velocity a distance R from the centre of an object of mass M is given by:

$$v_{esc} = \sqrt{\frac{2GM}{R}}$$

   (b) Determine the escape velocity from the Sun for an object at the Sun's surface ($R_\odot = 7.0 \times 10^5\,\text{km}$, $M_\odot = 2.0 \times 10^{30}\,\text{kg}$).

   (c) Determine the escape velocity from the Sun for an object at the average distance of the Earth ($d_\oplus = 1.5 \times 10^8\,\text{km}$). Compare this with the average orbital speed of the Earth around the Sun (assume a circular orbit to determine the Earth's orbital speed).

191. A sphere of radius R_2 has a concentric spherical cavity of radius R_1, resulting in a spherical shell. This spherical has a uniform density throughout and has a total mass of M. Show that the force experienced by a mass m at a distance r from the centre of the shell is given by:

$$F(r) = \begin{cases} 0 & r \leq R_1 \\ -\frac{GMm}{r^2}\frac{(r^3-R_1^3)}{(R_2^3-R_1^3)} & R_1 < r \leq R_2 \\ -G\frac{Mm}{r^2} & R_2 < r \end{cases}$$

where a minus sign indicates that the force points towards the centre of the shell.

192. A sphere of radius R_2 has a concentric spherical cavity of radius R_1, resulting in a spherical shell. This spherical shell has a uniform density throughout and has a total mass of M. Show that the gravitational potential energy of a mass m at a distance r from the centre of the shell is given by:

$$U(r) = \begin{cases} \frac{GMm}{(R_2^3-R_1^3)}\left(\frac{3}{2}R_1^2\right) - G\frac{Mm}{R_2} - \frac{GMm}{(R_2^3-R_1^3)}\left(\frac{1}{2}R_2^2 + \frac{R_1^3}{R_2}\right) & r \leq R_1 \\ \frac{GMm}{(R_2^3-R_1^3)}\left(\frac{1}{2}r^2 + \frac{R_1^3}{r}\right) - G\frac{Mm}{R_2} - \frac{GMm}{(R_2^3-R_1^3)}\left(\frac{1}{2}R_2^2 + \frac{R_1^3}{R_2}\right) & R_1 < r \leq R_2 \\ -G\frac{Mm}{r} & R_2 < r \end{cases}$$

where we have chosen 0 potential energy when $r \to \infty$.

193. A robotic lander with an Earth weight of $3430\,\text{N}$ is sent to Mars, which has a radius of $R_M = 3.40 \times 10^6\,\text{m}$ and a mass of $M_M = 6.42 \times 10^{23}\,\text{kg}$.

   (a) Find the acceleration due to gravity on the surface of Mars, g_M

   (b) Find the weight F_g of the lander on the Martian surface.

   (c) Since the mass of Mars is about 10 times less than that of Earth, comment on whether the results in parts a) and b) make sense.

194. You wish to put a 1000 kg satellite into a circular orbit 300 km above the Earth's surface.

   (a) What speed, period, and radial acceleration will it have?

   (b) What is the minimum amount of work that needs to be done to the satellite to put it in orbit if launched from the Earth's equator? What if it is launched from the North pole?

   (c) How much additional work would have to be done to make the satellite escape the Earth's gravity?

   Assume that the Earth has a mass of $M_{\oplus} = 5.97 \times 10^{24}$ kg *and a radius of* $R_{\oplus} = 6.38 \times 10^{6}$ m.

195. Comet Halley moves in an elliptical orbit around the Sun ($M_{\odot} = 1.99 \times 10^{30}$ kg). It's distances from the sun at perihelion and aphelion are $d_p = 8.75 \times 10^{7}$ km and $d_a = 5.26 \times 10^{9}$ km, respectively.

   (a) Find the orbital semi-major axis.

   (b) Find the eccentricity of the orbit (defined as the distance between the foci of the ellipse divided by the length of the major axis).

   (c) Find the period of the orbit.

196. A thin ring of mass M and radius R lies in the yz plane, as shown in Figure 55. A mass m is placed at a distance x away from the origin at point P, along the x axis. Give an expression for the gravitational force vector exerted by the ring on the small mass m.

   Hint: break the ring up into small mass elements dM and sum the forces, $d\vec{F}$, from each mass element. Think about the symmetry when summing the contributions from each mass element.

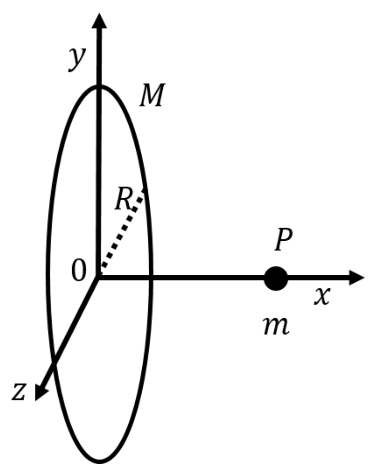

Figure 55: A ring of mass M.

197. You are interested in exploring planet Camelid, but are worried that the force of gravity on this gigantic planet would crush you. You plan to visit the planet at the equator, as you will have the greatest chance to observe a space llama there, as indicated by a previous un-manned probe that visited the planet. The planet rotates about itself once every 12 hr,

has a radius of $R = 7 \times 10^4$ km, and a mass of $M = 2 \times 10^{27}$ kg. What is the value of your apparent weight at the equator on the surface of the planet? Give your answer as a multiple of mg, your weight at the Earth's surface ($g = 9.8\,\mathrm{m/s^2}$).

Remember that the apparent weight is the value of the normal force that you feel. You need to take the centripetal acceleration from being at the equator into consideration. Newton's Universal Constant for Gravity is $G = 6.67 \times 10^{-11}\,\mathrm{Nm^2/kg^2}$.

198. Observations of a galaxy far far away seem to imply that there is a black hole at its centre. By using spectroscopic measurements, astronomers were able to measure the speed of gases orbiting 50 ly from the centre of the galaxy (1 ly is the distance that light travels in a year, at a speed of 3×10^8 m/s). Astronomers measured that the speed of those gases was approximately 750 km/s.

   (a) Estimate the mass of the black hole at the centre of the galaxy in terms of the number of solar masses (the mass of the Sun is 1.98×10^{30} kg). Clearly state any assumption that you make.

   (b) Find the radius of circular orbits around the black hole whose orbital speed is the speed of light.

   Newton's Universal Constant for Gravity is $G = 6.67 \times 10^{-11}\,\mathrm{Nm^2/kg^2}$.

199. A satellite of mass $m = 1 \times 10^3$ kg is in a circular orbit of radius $2R$, where $R = 6.23 \times 10^6$ m is the radius of the Earth, and $M = 5.97 \times 10^{24}$ kg is the mass of the Earth.

   (a) What is the speed of the satellite in this orbit?

   (b) What is the minimum amount of net work that was done to place the satellite in this orbit if the satellite started from the surface of the Earth?

200. It is possible for a small satellite of mass m to orbit the Sun with the same period T as the Earth. This is only possible if the satellite is located at one of five points that are called "Lagrange points." Assume that the Earth follows a circular orbit of radius r_E. The first Lagrange point is located between the Earth and the Sun. As the satellite orbits the Sun, it will always be a distance d from the Earth's centre. Hint: You can use the binomial expansion $(1 + x)^n \approx 1 + nx$ if $x << 1$.

   (a) Kepler's third law says that for an object orbiting the Sun, the period will change with the distance to the Sun. How is it possible for the satellite to orbit the Sun with the same period as the Earth if they don't have the same orbital radius?

   (b) Find d, the distance from the Earth's centre to the satellite, in terms of the mass of the Earth M_E, the mass of the sun M_S, and the radius of the Earth's orbit, r_E.

201. What is the field due to a thin uniform rod of mass M at a point P located a distance d along the axis of the rod (Figure 56)?

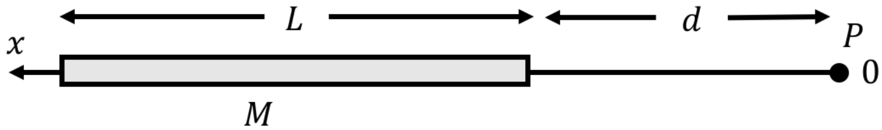

Figure 56: A thin rod of mass M and length L produces a gravitational field at a point P located along the axis of the rod.

202. An object of mass m orbits a much larger spherical object of mass M in a circular orbit. Show that the kinetic energy, K, of m is given by:

$$K = -\frac{1}{2}U$$

where $U = U(r)$, is the gravitational potential energy of m due to M:

$$U(r) = -G\frac{Mm}{r}$$

This is a special case of the "Virial Theorem".

203. You are designing the landing rockets for a probe of mass $m = 1 \times 10^3$ kg to land on a spherical planet of mass $M = 1 \times 10^{24}$ kg and radius $R = 1 \times 10^6$ m with no atmosphere (i.e. no drag on the probe as it lands). The rockets will fire and exert a total constant force $F = 1 \times 10^8$ N in the direction opposite to the velocity of the probe. At what altitude from the surface of the planet do the landing rockets have to fire for the probe to land with a speed of zero relative to the surface of the planet? Assume that the probe originated at an infinite distance from the planet, where it had a speed of zero, and that it is moving directly towards the centre of the planet.

204. The Falcon Heavy rocket was first launched on February 6, 2018, as the world's most powerful operational rocket. It has a mass of 1.421×10^6 kg, and additionally, carried a payload (Elon Musk's Tesla Roadster) of approximately 1300 kg during its maiden flight which brought it to a low Earth orbit of nearly 2000 km in altitude.

    (a) Knowing that the Earth has a mass of 5.972×10^{24} kg and a radius of 6371 km, determine the work done by gravity as the Falcon Heavy traveled from sea level to its low Earth orbit. Recall that the gravitational constant is $G = 6.674 \times 10^{-11}$ m^{3}kg^{-1}s^{-2}.

    (b) An object in a circular geosynchronous orbit remains above the same position on Earth at all times. Based on this information, determine the work done by gravity if the Falcon Heavy were to reach geosynchronous orbit starting from sea level while carrying a payload of 2.5×10^4 kg.

9 Centre of Mass and Momentum

9.1 Multiple Choice

205. Which of the following can best be described as an elastic collision?

- ◯ A cake landing in someone's face.
- ◯ A golf club hitting a golf ball.
- ◯ A meteorite striking the ground.
- ◯ A car crashing into a wall.

206. The USS Enterprise is travelling across the galaxy at a speed $v = \frac{c}{8}$. A Romulan vessel is detected behind the starship and prepares an attack. Captain Jean-Luc Picard asks Commander Data to increase the speed of the USS Enterprise to $v = \frac{c}{4}$. What was the impulse necessary to change the speed of the starship? (Use classical mechanics.)

- ◯ $\frac{mc}{8}$
- ◯ $\frac{c}{8}$
- ◯ $\frac{3m}{8}$
- ◯ $\frac{-mc}{8}$

207. A hockey puck is placed in the centre of a hockey rink, just before a game between Team Canada and Team Switzerland (which Team Switzerland obviously won). The puck suddenly explodes into 2 pieces. One piece lands 2 m from the centre line in the direction of Team Canada's goal. Where does the second piece land, given that the second piece is twice as heavy as the first piece?

- ◯ 0.5 m in the direction of Team Switzerland's goal
- ◯ 1 m in the direction of Team Switzerland's goal
- ◯ 2 m in the direction of Team Switzerland's goal
- ◯ 4 m in the direction of Team Switzerland's goal

208. Two sumo wrestlers are preparing to attack each other. The first weighs 833 N, and the second weighs 784 N. The first charges at the second at 3 m/s, while the other comes from the other direction at 2.5 m/s. The second one, though smaller, holds ground and remains stationary after impact and the first bounces back off. At what speed does the first sumo wrestler bounce off the second?

- ◯ 5.5 m/s
- ◯ 0.55 m/s
- ◯ 1.12 m/s
- ◯ 0.64 m/s

209. Two objects with mass m are moving in opposite directions with the same speed. What can we say about the total kinetic energy, K, and total momentum, p, of this system?

- ◯ $K = 0$, $p = 0$
- ◯ $K > 0$, $p > 0$
- ◯ $K < 0$, $p = 0$
- ◯ $K > 0$, $p = 0$

210. Did you know that a young guanaco is called a chulengo? Betcha didn't! Two chulengos are wearing rollerblades. A stationary chulengo A has a mass of 50 kg. She pushes chulengo B who has a mass of 75 kg and is also stationary. After the push, chulengo B moves with a velocity of 2 m/s to the right. What is the velocity of chulengo A?

- ◯ 2 m/s to the right
- ◯ 1 m/s to the right
- ◯ 3 m/s to the left
- ◯ 2 m/s to the left
- ◯ 1 m/s to the left

211. A mine cart with a mass of 250 kg is stationary on a frictionless track. The cart is filled with 1000 kg of water. To drain the water, a hole is drilled in the back of the cart near the bottom. Assuming the water flows out of the hole with a constant velocity of 1.0 m/s until the cart is empty, what will be the final speed of the mine cart?

- ◯ 1.6 m/s
- ◯ 1.0 m/s
- ◯ 0.22 m/s
- ◯ 0 m/s

212. Two friends are moving from residence to a house in Kingston. They collectively exert a force (resulting in a net force of 150 N) on a trolley with several of their boxes (the entire system weighs 75 kg) for 5 seconds. Assuming the trolley is initially at rest, what is the final speed of the trolley system?

- ◯ 10 m/s
- ◯ 16.67 m/s
- ◯ 30 m/s

213. A large truck of mass M collides head on with a car of mass m, causing significantly more damage to the car than to the truck. Which statement is true?

- ◯ The collision was elastic and the truck gave the same impulse to the car as the car did to the truck

- ◯ The collision was inelastic and the truck gave the same impulse to the car as the car did to the truck
- ◯ The collision was elastic and the truck gave a larger impulse to the car than the car did to the truck
- ◯ The collision was inelastic and the truck gave a larger impulse to the car than the car did to the truck
- ◯ Not enough information to tell

214. Rhonda and John are bowling with large watermelons. By mistake, Rhonda lets go of her $3\,\mathrm{kg}$ watermelon at a speed of $10\,\mathrm{m/s}$ and it charges at John's $1\,\mathrm{kg}$ watermelon that was travelling at $2\,\mathrm{m/s}$. If the watermelons hit and start rolling together, at what speed will they be travelling?
    - ◯ $2\,\mathrm{m/s}$
    - ◯ $4\,\mathrm{m/s}$
    - ◯ $8\,\mathrm{m/s}$
    - ◯ $-6\,\mathrm{m/s}$

215. A cannon recoils after shooting a cannonball. If one assumes that the momentum of the cannon and cannonball system is conserved, and that the cannon has 100 times the mass of the cannonball, what can you say about their relative velocities?
    - ◯ The cannon ball will have have 10 times the speed of the cannon.
    - ◯ The cannon ball will have have 25 times the speed of the cannon.
    - ◯ The cannon ball will have have 50 times the speed of the cannon.
    - ◯ The cannon ball will have have 100 times the speed of the cannon.

216. Which of the following could not be considered an elastic collision? (Choose the one that would be the least elastic).
    - ◯ A train crashing into another train and the two trains sticking together and moving in unison after the collision.
    - ◯ A billiard ball striking another.
    - ◯ The collision between balls in Newton's Cradle.
    - ◯ Two masses colliding with a spring in between them.

217. Two cars collide. If we consider the two cars and their contents to be a system, which force is external to the system?
    - ◯ The force of gravity exerted by one car on the other.
    - ◯ The normal force exerted by the road on one of the cars.
    - ◯ The normal force exerted by the seat on one of the passengers in one of the cars.

◯ The force exerted by the bumper of one car onto the bumper of the other car.

9.2 Long answers

218. Three astronauts, Alice, Bob, and Caroline, with masses $m_a = 75\,\text{kg}$, $m_b = 62\,\text{kg}$, and $m_c = 98\,\text{kg}$, respectively, are floating in space. I look out of my space station window and notice that they are located at the corners of a right triangle whose sides are about 12.00 m and 18.00 m long, as shown by Figure 58.

    (a) Locate the centre of mass of the systems of three astronauts using the coordinate system provided.

    (b) Caroline throws a lifeline to Bob. Caroline then pulls Bob towards her. At what position do Bob and Caroline meet? (Assume that the mass of the lifeline was included in Caroline's mass quoted above).

    (c) Bob and Caroline attach themselves together and then throw the lifeline to Alice to real her in. Where do the three of them finally meet?

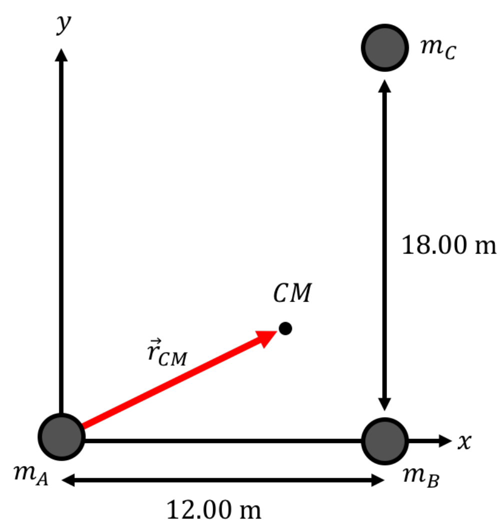

Figure 58: Astronaut positions.

219. A bullet of mass m is fired vertically into a block of mass M and embeds itself into the block (Figure 59).

    (a) If the block with the bullet embedded in it rises by a height h, what was the speed of the bullet just before hitting the block?

    (b) What is the ratio of the kinetic energy after the collision over the kinetic energy before the collision?

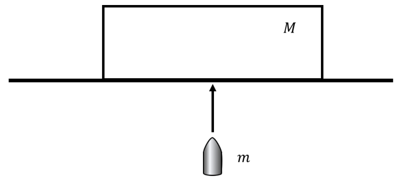

Figure 59: Bullet fired vertically into a block.

220. Although I am not very skilled in the game of pool, my friends still convince me to join them in a match. If I break, the pool balls will go flying off the table so my partner, Elizabeth, and I make the collective decision to let her break.

   (a) I observe Elizabeth line up her pool cue to hit the 0.165 kg cue ball. I estimate that the cue ball moves at about 10 m/s after being hit. Elizabeth is a skilled player, so she can stop the motion of the cue stick after it has moved the short distance of 2 cm. What is the impulse that Elizabeth delivers to the cue ball during her break?

   (b) What is the average force associated with the impulse calculated in part (a) if I estimate the impulse to occur over the course of roughly 4 ms?

   (c) After a few rounds, Elizabeth and I are in the lead! Elizabeth lines up a difficult shot, and I watch her carefully as she hits the cue ball in the direction of her target (the solid yellow ball, with the same mass as the cue ball). She causes the 165 g cue ball to move directly towards her target, hitting it at a speed of 11 m/s. After the collision, both balls move in the same direction as the initial velocity of the cue ball; however, the cue ball moves much more slowly than the yellow ball, at a speed of about 3 m/s. What is the speed of the yellow ball after the collision? Was this collision elastic or inelastic?

221. A pendulum of length l has a wooden block of mass 1.20 kg attached at its end. A bullet with a mass of 7.4 g is fired at the block. The bullet embeds itself into the block and stops. Consequently, the wooden block (now with the bullet inside of it) swings up to a height of 0.62 m as shown in Figure 60. Given this information, what is the initial velocity of the bullet?

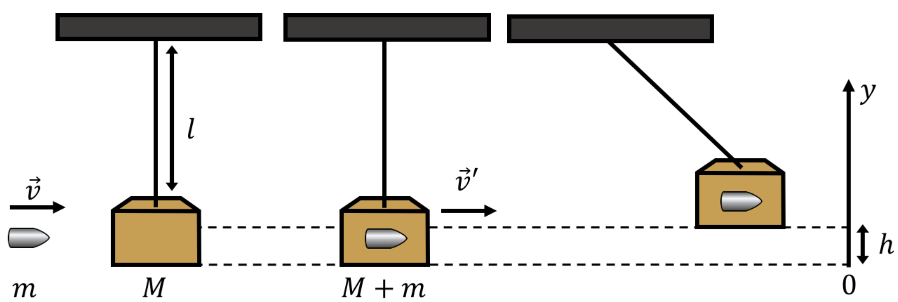

Figure 60: Ballistic pendulum.

222. Suppose there is a rod of length l and mass M.

   (a) Show that the centre of mass of this rod is at its centre if the mass of the rod is uniformly distributed.

(b) Determine the centre of mass of the rod if its mass is not uniformly distributed, but instead its linear mass density begins as λ_0 at the bottom of the rod and increases linearly to a maximum of $\lambda = 2\lambda_0$ at the top of the rod.

223. A basketball of mass m_b is dropped from a height h above the Earth. Assume that the collision between the basketball and the Earth is elastic. Show that the basketball will rebound to a height h.

Note that you have to model this as a collision with the Earth, and show that because the mass of the basketball is much smaller than the mass of the Earth, m_E, the final height of the basketball will be h. When you want to show that something is true when one quantity is much bigger than the other, it is useful to write the quantities as a ratio (if $m_E >> m_b$, then $\frac{m_b}{m_E} \sim 0$).

224. A baby guanaco, with a mass of 30 kg, is in the middle of a $L = 10$ m long wooden plank that weighs 100 kg and is floating on lake Titicaca, on the Peruvian side. The plank is lined up so that it is perpendicular to the shore line (i.e. the plank points to the shore). You whistle from the shore, and the guanaco, intrigued, starts to walk towards you, until it reaches the end of the plank. How much closer is the guanaco to you?

225. Figure 62 shows a shot that you are trying to make in pool. You are trying to sink the red ball into the pocket in the upper left corner by hitting it with the cue ball which sits directly below. Both balls have the same mass and dimensions, the collision is elastic, and you can assume that the initial velocity of the cue ball is parallel to the long side of the pool table. You are worried that if you sink the red ball, the cue ball will go into the upper right pocket, and your opponent will then make fun of you for not realizing this. Should you be concerned? Show us why or why not!

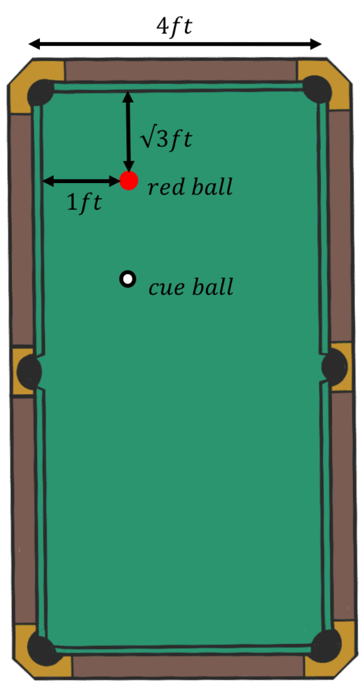

Figure 62: A pool shot. Dimensions are in feet.

226. A proton (mass m) makes an elastic head-on collision with a nucleus at rest and rebounds with a speed that is nine-tenths of its initial speed. What is the mass, M, of the nucleus?

227. You have always wanted to be a ninja turtle. You are thus interested in spinning a slice of pizza on the tip of your finger (the slice of pizza is in the horizontal plane and balanced on your vertical finger). The slice of pizza can be modelled as an isosceles triangle of uniform density and thickness with a base length of b, and a height of h, as in Figure 65.

    (a) Where is the centre of mass of the slice of pizza?

    (b) Where is the centre of mass of the slice of pizza if, instead, it can be modelled as an equilateral triangle of side b?

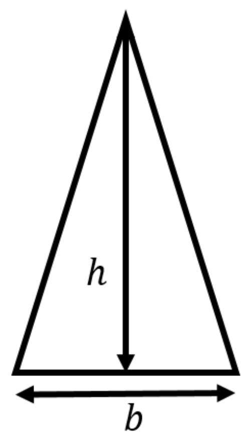

Figure 65: An isosceles triangle.

228. Find the centre of mass of a uniform half disk of radius R.

229. A building that is 10 m tall and 5 m wide is being struck by a 120 km/h wind head on. If you assume that the air comes to rest upon striking the building, determine the force exerted by the wind on the building, and show that it scales with the square of the wind speed. The density of air is $1.2\,\mathrm{kg/m^3}$.

230. A cannonball is launched with a speed $v = 100\,\mathrm{m/s}$ by a cannon that aims at an angle of 45° from the horizontal. At some point during its trajectory the cannonball breaks up into two pieces of equal mass. The first piece then lands at a distance $D = 800\,\mathrm{m}$ from the cannon, in the same direction that the original cannonball was fired. How far from the cannon does the second piece land? Assume that drag is negligible and that the fragments land on the ground at the same height as the cannonball was fired.

231. A bullet of mass m is moving horizontally just before it impact the bob of mass M of a pendulum that consists of the mass M attached to a mass-less string of length L. The bullet embeds itself in the bob. What is the minimum speed of the bullet required for the pendulum to make a complete circle after the impact? Assume that drag and friction are negligible.

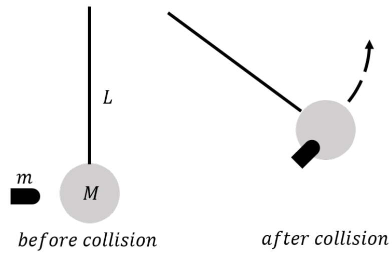

Figure 68: A bullet colliding and embedding itself in a pendulum.

232. A mass m is attached to a mass-less string of length $L = 4.20\,\text{m}$ and can swing in the vertical plane. The mass m is released from a horizontal position as shown by the dotted lines in Figure 69. At the bottom of the trajectory, it collides elastically head-on with a block of mass $M = 2m$ which lies on a frictionless surface and is constrained to move in one dimension (the horizontal direction in the diagram).

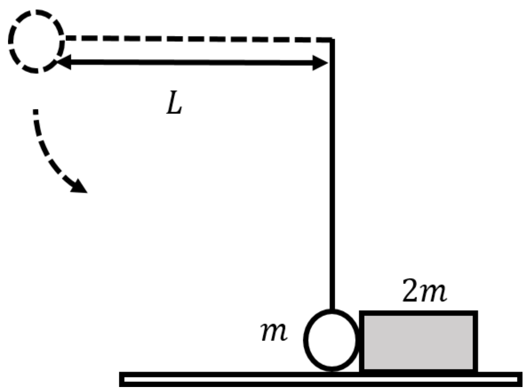

Figure 69: A mass on a pendulum hits a block elastically.

(a) Show that the speed of mass m just before the collision is $v_m = 9.07\,\text{m/s}$

(b) What is the velocity of the block just after the collision?

(c) To what height above the ground does the mass m rebound after the collision?

233. A bullet of mass m_b strikes a block of wood of mass m_w which is at rest. The bullet is embedded in the wood, causing the bullet and the block of wood to move together. Use conservation of momentum to prove that this collision is inelastic.

10 Rotational dynamics

10.1 Multiple Choice

234. A bug sits on a horizontal vinyl record at a distance r from the centre of the record. The record starts playing (accelerating), and the bug is accelerated by a torque τ. A second bug with the same mass as the first is located at a distance $\frac{r}{2}$ from the centre of the record. What is the net torque exerted on the second bug? Assume that both bugs do not move relative to the record as it accelerates.

   ◯ $\frac{1}{4}\tau$

   ◯ $\frac{1}{2}\tau$

   ◯ 2τ

   ◯ 4τ

235. You approach a closed door with a door handle to your left. You need to push the door handle to open the door, and thus you exert a torque on the door (relative to an axis through the hinges on the right of the door) that is directed

   ◯ Upwards.

   ◯ Downwards.

   ◯ Away from you.

   ◯ Towards you.

236. A mass-less ladder has a total length L and is placed against a wall such that it makes an angle θ with the horizontal, as shown in Figure 70. The coefficient of static friction between the ladder and the ground is μ_s, while there is no friction between the ladder and the vertical wall. What is the maximum distance, D, along the ladder (as measured from the ground) that a person can climb up the ladder before the ladder starts to slide?

   ◯ $D \leq L(1 - \mu_s \sin\theta)$

   ◯ $D \leq L(1 - \mu_s \tan\theta)$

   ◯ $D \leq L\mu_s \cos\theta$

   ◯ $D \leq L\mu_s \tan\theta$

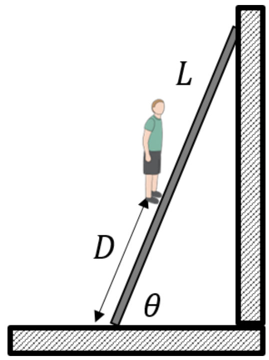

Figure 70: A person climbing a ladder (Question 236).

237. A travelling amusement park came to town, and your little brother decides that you both need to test the new spinning saucer ride. Your brother wants to go “as fast as possible”, in attempt to take-off into outer-space. There are seats near the centre of the saucer, and seats on the outer-edge of the saucer. Where should you sit, in order to go the fastest?

   ◯ Near the centre, because you will increase linear speed and angular velocity.

   ◯ Near the centre, because you will increase linear speed.

   ◯ Near the outer-edge, because you will increase linear speed.

   ◯ Near the outer-edge, because you will increase linear speed and angular velocity.

238. A construction worker has to turn a lever at an angle of 55°. The length of the lever is 0.61 m long and is a force of 25 N is exerted to turn it. What is the net torque on the lever?

   ◯ 9.0 Nm

   ◯ 34 Nm

   ◯ 12 Nm

   ◯ 22 Nm

239. You are fed up with your non-physics course and decide to throw the book in the air. If you want the book to spin as fast as possible, which of the following axes should you spin it about? All of the axes pass through the centre of mass of the book. Ignore the thickness of the book and assume it is longer than it is wide.

   ◯ The axis along the length of the book, parallel to its surface

   ◯ The axis along the width of the book, parallel to its surface

   ◯ The axis through the center of the book, perpendicular to its surface

   ◯ Both the length and width axes will work

240. A vicuña with a mass of 50 kg stands on the corner of a giant spinning chocolate bar with dimensions 50 m by 100 m (assume uniform density and no thickness). The axis of rotation is in the center of the chocolate bar and perpendicular to the plane it forms. If the

chocolate bar has a mass of 1000 kg, what is the moment of inertia of the whole system? (The vicuña can be treated as a point mass)

- ◯ 1.20×10^6 kgm
- ◯ 1.67×10^6 kgm
- ◯ 6.41×10^6 kgm
- ◯ 1.20×10^6 kg^{2}m

241. A sphere with a radius of 2 m and a mass of 1 kg and uniform density begins spinning from rest about its centre. With uniform angular acceleration it reaches a rotational speed of 2 revolutions per second over the course of 10 seconds. If the applied force was tangential to the surface of the sphere, what was the magnitude of the applied force?

- ◯ 1.01 N
- ◯ 1.21 N
- ◯ 0.96 N
- ◯ 2.01 N

242. While sitting in class, you find yourself watching the clock very closely. By watching the ticking of the second hand, you determine that it spends 0.9 s stationary and then spends 0.1 s to move to the next position. It does this 60 times in one minute for a full rotation. What is the average and maximum angular velocity of the second hand during one full rotation?

- ◯ $\omega_{AVG} = 0.11$ rad/s, $\omega_{MAX} = 1.1$ rad/s
- ◯ $\omega_{AVG} = 0.002$ rad/s, $\omega_{MAX} = 0.02$ rad/s
- ◯ $\omega_{AVG} = 0$, $\omega_{MAX} = 1.1$ rad/s
- ◯ $\omega_{AVG} = \omega_{MAX} = 1.1$ rad/s

243. All parts of a rigid object rotating about a fixed axis have the same angular velocity ω and the same angular acceleration α at any instant.

- ◯ True
- ◯ False

244. A moment of inertia : [select all that apply]

- ◯ Has units of m^2/kg
- ◯ Is the counterpart of mass in the formula for rotational kinetic energy
- ◯ Is the quantitative measure of rotational inertia

245. You have two circular frisbees of the same mass and radius. One of them is a ring, and the other is the more traditional disk shape. You throw them exactly the same way. Which one will spin the fastest?

- ◯ The ring
- ◯ The disk
- ◯ They spin at the same rate

246. Two forces of different magnitude are exerted at the same point on an object. Is it possible for them to produce the same torque about an axis that does not go through the point where the forces are applied?

- ◯ Yes
- ◯ No

247. You ride a bicycle forward in a straight line. In which direction does the angular velocity of the wheels point

- ◯ Forward
- ◯ Backward
- ◯ To your left
- ◯ To your right

248. If a point mass A is double the distance from a rotation axis than another point mass B that is half of the mass of A, how do their moments or inertia about that axis relate?

- ◯ $I_A = I_B$
- ◯ $I_A = 2I_B$
- ◯ $I_B = 2I_A$
- ◯ $I_A = 4I_B$

249. You have designed an engine that provides a constant torque regardless of its rotational speed. You place different shapes on the axle and measure their angular velocity after a fixed period of time. Which one will have the highest angular velocity?

- ◯ A solid disk of mass M and radius R (the axle is perpendicular to the plane of the disk and goes through its centre)
- ◯ A solid disk of mass M and radius $R/2$ (the axle is perpendicular to the plane of the disk and goes through its centre)
- ◯ A ring of mass M and radius $R/2$ (the axle is perpendicular to the plane of the ring and goes through its centre)
- ◯ A ring of mass M and radius R (the axle is perpendicular to the plane of the ring and goes through its centre)

250. You wish to accelerate a uniform solid disk of mass $M = 20\,\text{kg}$ and radius $R = 0.5\,\text{m}$ to an angular velocity of $100\,\text{rad/s}$ in $2\,\text{s}$. The disk rotates about a fixed axis that goes through its centre and is perpendicular to the plane of the disk. How much power is required?

- ◯ 3125 W
- ◯ 6250 W
- ◯ 12 500 W
- ◯ 25 000 W

251. Suppose that you spin a ball that is resting on a desk. As time passes by, the angular velocity of the ball relative to some axis decreases. In what direction is the angular acceleration of the ball, relative to that same axis?

- ◯ The angular velocity is zero.
- ◯ Anti-parallel to the angular velocity.
- ◯ Parallel to the angular velocity.
- ◯ Not enough information to tell.

252. You push on a door located in front of you to open it and exert a downwards torque on the door relative to its hinges. Relative to you, on which side are the door's hinges?

- ◯ On your right.
- ◯ On your left.
- ◯ Above you.
- ◯ At your feet.

253. Which is easier (i.e. requires less effort on your part)?

- ◯ Rotating a bar about its centre.
- ◯ Rotating a bar about its end.

254. You are driving in your car and go around a left turn. You notice that the shocks (springs) in your car compress more on one side than the other.

- ◯ The right side of the car is lower in the turn
- ◯ The left side of the car is lower in the turn

255. Why are the breaks on a motorcycle (and cars) bigger on the front wheel(s)?

- ◯ Because there is more room on those wheels, as the back wheels are near the engine.
- ◯ Because the normal force from the ground is bigger on the front wheel(s) than on the back wheel(s) when breaking
- ◯ Because the normal force from the ground is bigger on the back wheel(s) than on the front wheel(s) when breaking

256. A speed skater leans towards the centre of the track as they go around it. If you consider the torques on the skater about the point where their skates contact the ice, which forces create a torque?

- ◯ Gravity
- ◯ Gravity and the normal force from the ice
- ◯ Gravity, an inertial (centrifugal) force, and the normal force from the ice
- ◯ Gravity and an inertial (centrifugal) force

257. A guanaco with a mass of m is standing on a horizontal plank that is suspended on the side of Huayna Picchu (a mountain near Machu Picchu). The plank has a length L and a mass M. The plank is anchored to the side of the mountain with a hinge and would swing freely if there were no rope attached to the plank. A rope is attached to the end of the plank that is away from the mountain and the rope is anchored into the mountain, as shown in Figure 71. The rope makes an angle θ with the plank. The guanaco is standing a distance $\frac{3}{4}L$ from the mountain. What is the tension in the rope?

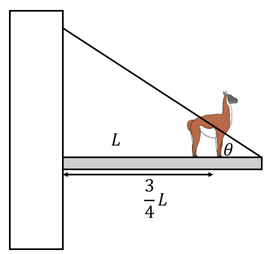

Figure 71: A guanaco on a plank.

- ◯ $T = \frac{\left(\frac{1}{2}m+\frac{3}{4}M\right)g}{\cos\theta}$
- ◯ $T = \frac{mg}{\sin\theta}$
- ◯ $T = \frac{\left(\frac{3}{4}m+\frac{1}{2}M\right)g}{\cos\theta}$
- ◯ $T = \frac{\left(\frac{3}{4}m+\frac{1}{2}M\right)g}{\sin\theta}$

10.2 Long answers

258. Four equal point masses m are located at the corners of a square of length a. The masses are connected by rigid mass-less rods. The square is in the vertical plane, as shown in Figure 72.

   (a) What is the moment of inertia of the system about an axis that is perpendicular to the page and goes through the bottom left mass?

   (b) If the system is free to rotate under the influence of gravity about the same axis in part (a), what is its initial angular acceleration about that axis, just as it is released?

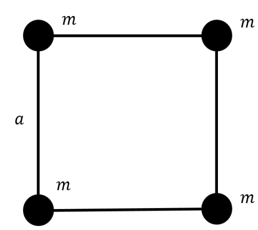

Figure 72: Four equal masses on the side of a square.

259. A ceiling fan is powered on and rotating with constant angular velocity. You switch it off and measure that it takes 36 revolutions to reduce its angular velocity to half its original value. How many more revolutions will it take before it comes to rest? Assume constant angular acceleration.

260. You are working on a physics assignment the night before it is due, but you realize that you do not have enough time to complete it. You start to wonder if you could stop the Earth from rotating and stop the sun from rising the next day. Hopefully the ensuing confusion would give you more time to work on the assignment. Instead of working on your physics assignment, you decide to explore the feasibility of this scenario. Model the Earth as a perfect solid sphere rotating constantly without any precession.

    (a) What is the angular velocity of the Earth?

    (b) If your assignment is due in 12 hours, what is the minimum constant angular acceleration that you would need to stop the Earth's rotation by then?

    (c) Calculate the torque required to produce the angular acceleration you calculated in part (b).

    (d) To achieve the torque from part (c), you decide to install rocket engines onto the Earth. The Rocketdyne F-1 is the most powerful single-nozzle liquid-fuelled rocket engine ever flown. It has a thrust of approximately 6800 kN. What is the minimum number of F-1 engines that you need to stop the Earth?

    Note: The mass of the Earth is $5.972 \times 10^{24}\,\mathrm{kg}$*, the radius of the Earth is* $6378\,\mathrm{km}$*, the moment of inertia of a solid sphere of radius* r *is* $I = \frac{2}{5}mr^2$*.*

261. Two different blocks of mass m_1 and m_2, respectively, are on different sides of an incline, where the slopes of the two sides make angles of α and β with the horizontal, respectively, as depicted in Figure 73. The blocks are connected by a mass-less inextensible rope that goes around a frictionless pulley of mass M and radius R without slipping. The pulley can be modelled as a solid disk with moment of inertia $I = \frac{1}{2}MR^2$. The coefficient of kinetic friction between the blocks and the incline is μ_k. Show that when the system is accelerating such that m_1 is moving downwards, the acceleration is given by:

$$a = \frac{m_1(\sin\alpha - \mu_k \cos\alpha) - m_2(\sin\beta + \mu_k \cos\beta)}{m_1 + m_2 + \frac{1}{2}M} g$$

    Hint: The tension in the rope is not the same on either side of the pulley if the pulley has a moment of inertia. If the tensions were the same, the pulley would not rotate, as the net torque on it would be zero.

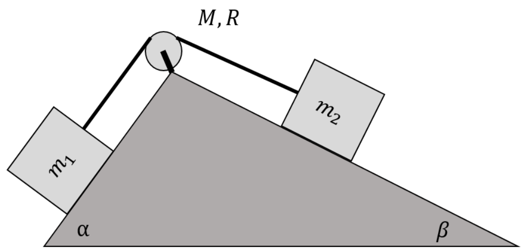

Figure 73: Two masses on an inclined plane with a pulley.

262. One can model the blades of a helicopter as 3 identical uniform thin rods of length $L = 4.0\,\mathrm{m}$ and mass $M = 120\,\mathrm{kg}$, as shown in Figure 75. The helicopter's blades accelerate from rest to 450 rpm in 10 s, and the helicopter's engine is 85% efficient at delivering power to the rotor blades.

    (a) What is the minimum required power for the helicopter's engine? Give your answer in horsepower (1 hp=745.7 W)

    (b) If a small bird of mass $m = 500\,\mathrm{g}$ managed to hold on to the end of one of the blades as they accelerated from rest, what speed would the blades have after 10 s if you assume that power was delivered by the engine at the same rate as in part a)? Express your answer in rpms.

    (c) What is the centripetal acceleration felt by the bird in part b after 10 s? Express your answer in gs.

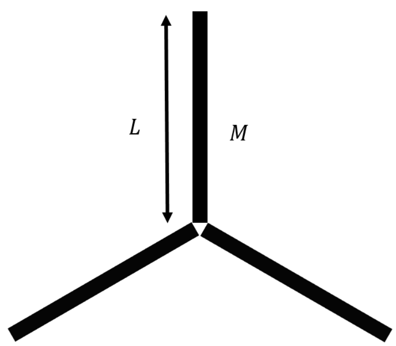

Figure 75: A model for helicopter blades.

263. A solid cone of mass M and height H has a circular base of radius R. The cone is rotated about its axis of symmetry as shown in Figure 76. Derive a formula for its moment of inertia about its axis of symmetry.

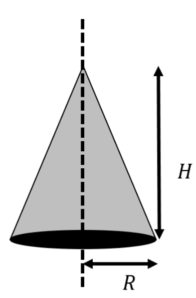

Figure 76: A cone.

264. Two solid spheres of masses $M = 4\,\text{kg}$ and $m = 2\,\text{kg}$, with radii $R = 20\,\text{cm}$ and $r = 10\,\text{cm}$, respectively, are attached at either end of a rod of mass $m_r = 5\,\text{kg}$ and length $L = 50\,\text{cm}$ to form a dumbbell (see Figure 78). Note that the spheres are attached to the ends of the rod (the rod does not penetrate the spheres).

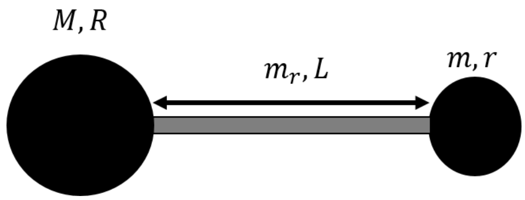

Figure 78: A dumbbell.

(a) What is the moment of inertia of the system about its centre of mass?

(b) A coordinate system is set up so that the origin is at the centre of mass of the dumbbell, and the x axis is parallel to the rod, with the positive direction from the M to m. A force:

$$\vec{F} = (0.8\,\text{N})\hat{\imath} + (1.0\,\text{N})\hat{\jmath} + (1.2\,\text{N})\hat{k}$$

is applied to the outer edge of the small mass m (the edge of the sphere opposite of where it meets the rod). If $\vec{F}$ is the only force applied to the dumbbell, what is the angular acceleration vector of the dumbbell with respect to the origin of the coordinate system that is specified? What is its magnitude?

265. Derive a formula for the moment of inertia of a solid Double Chocolate Donut of radius r rotated about its axis of symmetry, as illustrated in Figure 79.

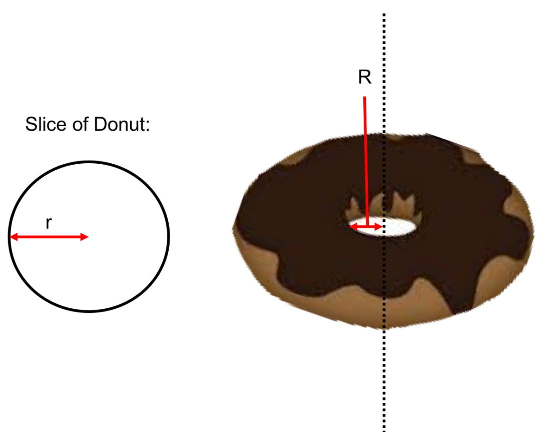

Figure 79: This is a Double Chocolate Donut.

266. The system in Figure 80 is in static equilibrium. A mass of $m = 225\,\text{kg}$ hangs from the end of the uniform strut whose mass is $M = 45.0\,\text{kg}$.

(a) Find the tension, T, in the cable.

(b) Find the horizontal and vertical force components exerted on the sturt by the hinge.

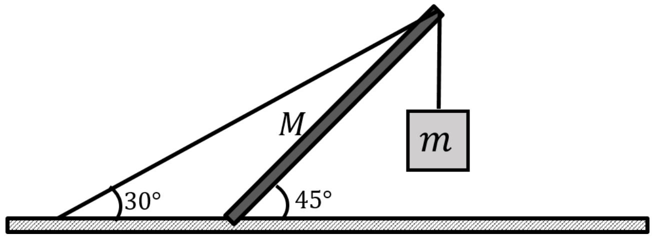

Figure 80: The geometry of the system.

267. A $L = 3.0\,\text{m}$ long rigid uniform beam with a mass of $M = 100\,\text{kg}$ is supported at each end, as shown in Figure 82. A $m = 80\,\text{kg}$ student stands a distance $d = 2.0\,\text{m}$ away from support 1. How much upward force does each support exert on the beam if the system is at rest?

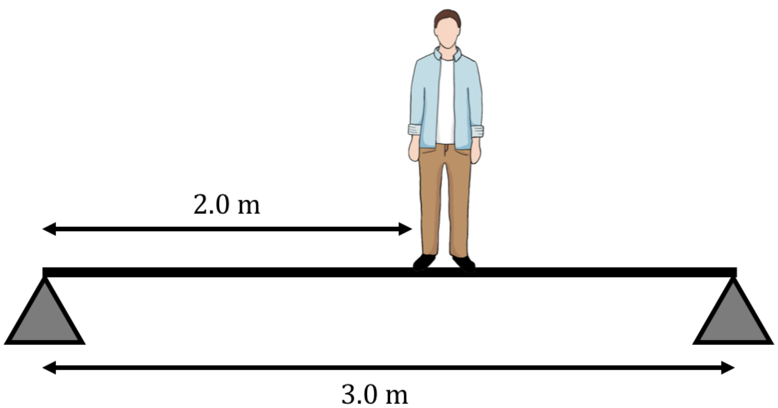

Figure 82

268. You wish to pull a sphere of radius R and mass M over a step of height h, by exerting a tangential force $\vec{F}$ at the top of the sphere as shown in Figure 83. What is the minimum magnitude of the force that is required?

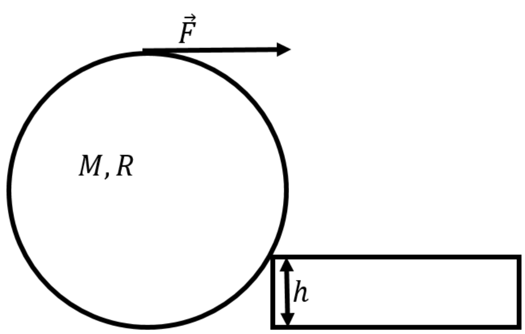

Figure 83

269. A uniform ladder with a length of $L = 6.0\,\text{m}$ and a mass of $M = 20\,\text{kg}$ leans against a frictionless wall (so that the wall can only exert a horizontal force on the ladder). The ladder

makes an angle $\theta = 20°$ with the wall, as shown in Figure 85. Determine the minimum value of the coefficient of static friction between the ladder and the ground for the ladder not to slip when a person weighing $m = 75\,\text{kg}$ stands two-thirds of the way up the ladder.

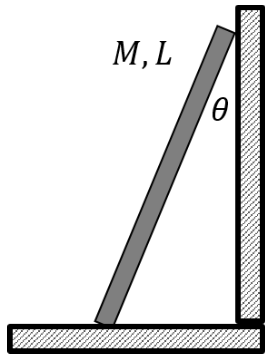

Figure 85: A ladder leaning against a frictionless wall.

270. A thin rod of length $L = 0.5\,\text{m}$ and mass $M = 1.0\,\text{kg}$ makes an angle $\theta = 60°$ with the horizontal. It is held in position by a light horizontal string attached at the top end of the rod, as shown in Figure 87. A mass m is suspended from the top of the rod by a second string. If the coefficient of static friction between the rod and the ground at the point of contact is $\mu = 0.50$, what is the largest mass m that can be suspended such that the rod does not slip?

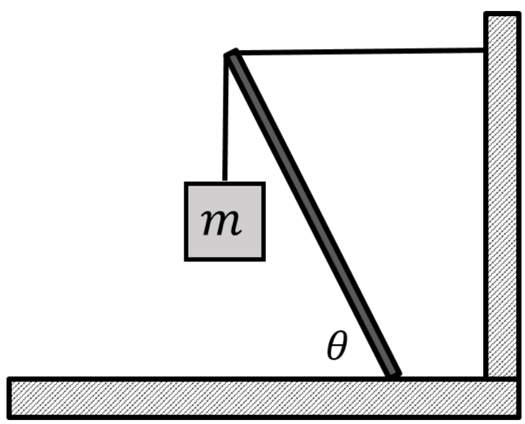

Figure 87: A mass hanging from a rod.

271. Chloë loves her rocket shoes! I investigated whether it would be safe to buy her some *actual* rocket shoes. To do this, I modelled Chloë as a vertical uniform bar of length $L = 80\,\text{cm}$ and mass $M = 13\,\text{kg}$ with rocket shoes at the bottom, as in Figure 89. I modelled the rocket shoes as being massless and having negligible dimensions (assumed to be a point at the end of the rod). The shoes provide a combined constant thrust of $F = 20\,\text{N}$ parallel to the ground for a duration of $10\,\text{s}$. Although the shoes have wheels on them, I assumed that it would be equivalent to sliding on the ground with a kinetic friction coefficient of $\mu_k = 0.1$.

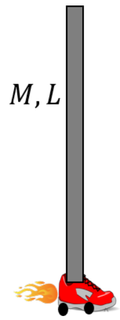

Figure 89: A model for Chloë and her rocket shoes.

(a) If starting from rest, how fast will Chloë be travelling after 10 s?

(b) Since I do not want to get too much exercise chasing after her, what total distance will she have travelled when she comes to rest? Remember, the rockets only burn for the first 10 s; after that, she will slow back down and come to rest.

(c) In order to not tip over while the rockets are on, Chloë will need to lean forwards or backwards. What angle will she need to make with the vertical to remain stable as the rocket shoes push her forward? Make a diagram and indicate whether she needs to lean forwards or backwards.

272. A majestic llama of mass $m = 500\,\text{kg}$ is walking on a horizontal plank of mass $M = 200\,\text{kg}$ and length $L = 5\,\text{m}$ that is attached to the vertical face of Huyana Picchu (a mountain in Peru). The plank is attached to the mountain with a hinge that allows the plank to rotate in the vertical plane. The other end of the plank is secured by a mass-less rope that makes an angle of $\theta = 30°$ with respect to the plank. The rope can sustain a maximum tension of 5000 N. As the llama walks away from the mountain, the tension in the rope increases until the rope suddenly breaks, and the plank with the llama on it starts to rotate about the hinge. What is the (linear) acceleration of the llama the moment just after the rope breaks?

Note that the moment of inertial of plank of mass M and length L rotated about one of its ends is $I = \frac{1}{3}ML^2$.

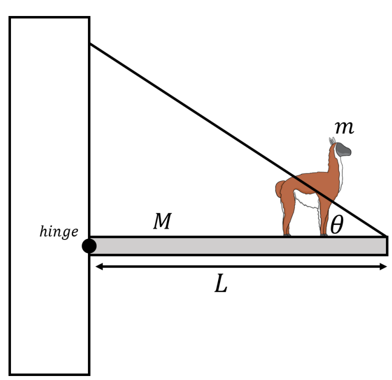

Figure 91: An adventurous llama walking out on a plank.

273. A uniform wire of mass M is bent into a closed semi-circle with radius R (the wire has a

curved section and a straight section), as shown in Figure 92.

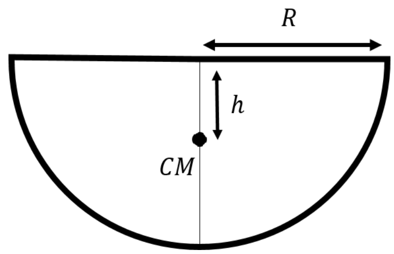

Figure 92: A wire bent into a closed semi-circle of radius R.

(a) Show that the centre of mass of the wire is located at a distance:

$$h = \frac{2R}{2+\pi}$$

from the centre of the semi-circle.

(b) The semi-circular wire from above is held with a pin that is placed at the centre of the straight section of wire (point P in Figure 93), so that the wire can rotate in the vertical plane about a horizontal axis that goes through the pin and that is perpendicular to the plane of the wire. What is the moment of inertia of the wire about this axis of rotation?

(c) A small bug of mass $m = 5\,\text{g}$ lands on the wire, directly below the point where the wire is suspended (left panel of Figure 93), and starts to walk up the wire. When the bug is at a position such that a line from P to the bug makes an angle $\theta = 30°$ with respect to the vertical (right panel), what angle, ϕ, does the straight part of the wire make with the horizontal? Assume that the wire has a mass $M = 100\,\text{g}$.

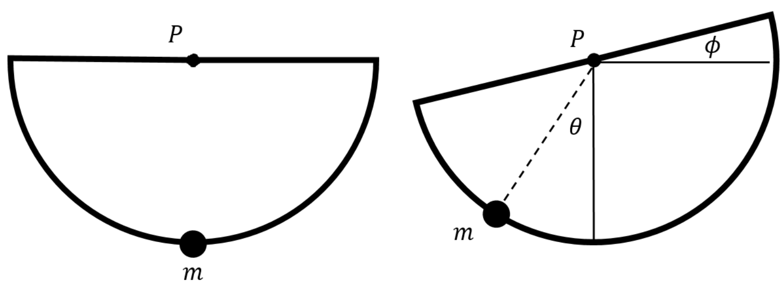

Figure 93: The wire is suspended at point P (midpoint of the straight section) so that it may rotate in the vertical plane. A bug of mass m starts below point P (left panel) and starts to walk up the wire, causing the wire to rotate (right panel).

11 Rotational energy and momentum

11.1 Multiple Choice

274. If you could carefully mould a solid sphere of clay that is rotating on a frictionless Lazy Susan into a solid cylinder, what would you expect to happen to the angular velocity?

    - ◯ It will increase.
    - ◯ It will decrease.
    - ◯ It will not change.
    - ◯ It depends on the dimensions of the cylinder.

275. A solid disk of mass M and radius R rolls without slipping along a horizontal surface, so that its centre of mass has a speed v relative to the ground. What is the total kinetic energy of the disk?

    - ◯ $K_{tot} = \frac{1}{2}mv^2$
    - ◯ $K_{tot} = \frac{1}{4}mv^2$
    - ◯ $K_{tot} = \frac{3}{2}mv^2$
    - ◯ $K_{tot} = \frac{3}{4}mv^2$

276. A figure skater is spinning with her arms extended. As she brings her arms in close to her body, she begins spinning faster because

    - ◯ Her arms do work on her body, and thus increases her kinetic energy.
    - ◯ Her moment of inertia decreases, which increases her angular momentum.
    - ◯ Her moment of inertia decreases, which increases her angular velocity.
    - ◯ It only looks like she's spinning faster because she is more compact.

277. If a figure skater starts to spin twice as fast when he brings his arms in during a spin, he

    A. increased his moment of inertia by a factor of 2.

    B. decreased his moment of inertial by a factor of 2.

    C. increased his moment of inertia by a factor of 4.

    D. decreased his moment of inertial by a factor of 4.

278. Two identical solid spheres roll down two different inclined planes A and B. Both A and B have the same height, but different angles of inclination. If you release the balls at the same time, then they will reach the bottom:

    - ◯ With different speeds at different times
    - ◯ With different speeds at the same time

- ◯ With the same speed at different times
- ◯ With the same speed at the same time

279. A particle is travelling along a straight line. If you measured the angular momentum of the particle about each of the following points, for which would the result be zero?

- ◯ Any point along its axis of travel
- ◯ Any point, because it is travelling in a straight line
- ◯ There are no points where the angular momentum will measured to be zero

280. A pinball ($m = 10\,\mathrm{g}$) is shot at with a speed $v = 100\,\mathrm{m/s}$ into the end of a small rod of length $L = 2\,\mathrm{m}$, and mass $M = 2\,\mathrm{kg}$, in the direction perpendicular to the length of the rod. The rod has an axis of rotation at the centre. The rod is initially at rest. What is the angular velocity of the rod after impact?

- ◯ 2.91 rad/s
- ◯ 1.47 rad/s
- ◯ 2.01 rad/s
- ◯ 1.78 rad/s

281. (1 point) For your next demonstration, you wish to demonstrate how figure skaters can increase their angular velocity when doing spins. You place one of your friends on your desk chair (which can spin with virtually no friction) and ask them to cross their arms tightly against their chest. You give the chair a good spin and then ask your friend to spread their arms out.

- ◯ The chair starts to spin faster because the moment of inertia of your friend about the rotation axis increased
- ◯ The chair starts to spin faster because the angular momentum of your friend about the rotation axis increased
- ◯ The chair starts to spin slower because the moment of inertia of your friend about the rotation axis increased
- ◯ The chair starts to spin slower because the angular momentum of your friend about the rotation axis increased

11.2 Long answers

282. A large meteorite of mass m strikes the Earth. It hits with a velocity perpendicular to the ground. It causes the Earth to start spinning slightly slower such that each day is slightly longer. Assume that the Earth is a solid sphere of radius $R_\oplus$ and mass $M_\oplus$, with uniform density.

(a) If each day before impact lasts exactly $d = 24\,\text{hr}$, find the Earth's rotational (not orbital) angular momentum before the meteorite hits. Express your answer in terms of the variables above.

(b) Express the angular momentum of the Earth after the meteorite hits in terms of the variables above, and in terms of Δt, the extra time in seconds that it takes the Earth to complete one day.

(c) Find an expression for Δt, in terms of the mass of the meteorite and the mass of the Earth, expressed as a fraction of a day.

(d) Hoba, the largest single meteorite ever found, has a mass of about $60\,000\,\text{kg}$. Based on our assumptions here, how much longer is each day since it hit? The mass of the Earth is $5.972 \times 10^{24}\,\text{kg}$.

283. A bar of mass, M, and length, L, sits at rest on a frictionless horizontal table. A piece of putty of mass, m, and velocity, $\vec{v}$, perpendicular to the bar, strikes the bar at a distance $\frac{L}{4}$ above its centre of mass and remains stuck to the bar, as illustrated in Figure 95.

(a) What is the velocity of the centre of mass of the bar with the putty attached?

(b) Show that the angular speed of the bar after the collision is given by:

$$\omega = \frac{12mv}{L(4M + 7m)}$$

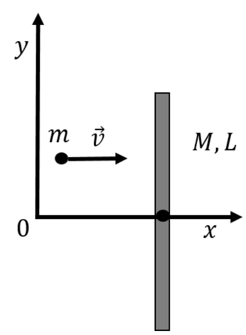

Figure 95: A piece of putty about to strike a bar, $\frac{L}{4}$ above the bar's CM.

284. A bullet of mass m is moving horizontally with velocity v when it strikes the edge of a uniform horizontal disk with mass M and radius R. The disk is anchored at its center but free to rotate about a vertical axis. Assume that the bullet struck the disk tangentially at its edge.

(a) Which of energy, momentum, and angular momentum are conserved for the bullet+disk system? Explain.

(b) Find ω, the angular velocity of the disk with respect to its centre, in terms of the given variables. Assume that the disk started at rest.

285. A solid cylinder of mass M and radius R is placed on an incline that makes and angle $\theta = 45^\circ$ with the horizontal, as shown at the top of Figure 96.

(a) What is the minimum coefficient of static friction between the cylinder and the incline that will allow the cylinder to roll down the incline without slipping?

(b) What is the acceleration vector of the centre of mass of the cylinder as it rolls down the incline?

(c) If the cylinder was released from a height $h = 3.0\,\mathrm{m}$ above the ground (as shown), and it rolls down the incline without slipping, what is the velocity vector of point A on the cylinder as shown in the bottom of Figure 96?

(d) If the cylinder was released from a height $h = 3.0\,\mathrm{m}$ above the ground (as shown), and it rolls down the incline without slipping, what is the velocity vector of point B on the cylinder as shown in the bottom of Figure 96?

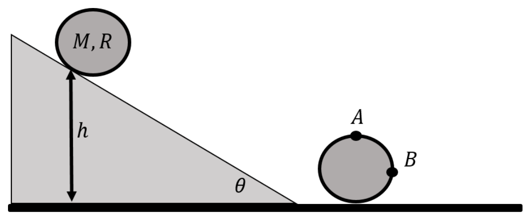

Figure 96: A cylinder rolling without slipping down an incline, before being released, and at the bottom.

286. A flat circular disk of radius R and mass M has a string wrapped around its rim and is laid flat on a horizontal frictionless table, as in Figure 98. The string passes over a frictionless massless pulley and a mass m is suspended from the end.

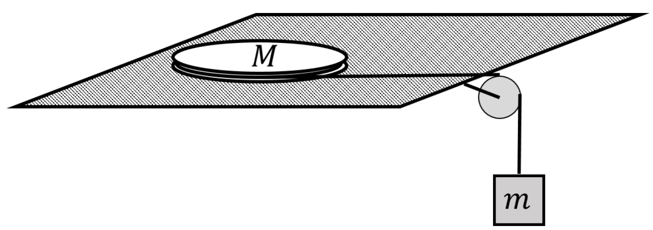

Figure 98: A disk on frictionless table being pulled on by a falling mass

(a) What are the linear accelerations of the centre of mass of the disk and of the mass m, and the angular acceleration of the disk?

(b) What is the angular speed of the disk after it has completed one revolution (assuming the system was released from rest)?

287. A uniform disk of mass M and radius R has an angular velocity ω_0 with its axis of rotation in the horizontal plane through its centre of mass. The disk is gently placed on a horizontal surface. The coefficient of kinetic friction between the surface and the disk is μ_k.

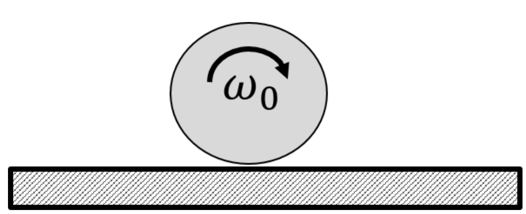

Figure 100: A vertical rotating disk gently lowered onto a surface.

(a) What is the linear acceleration of the centre of mass of the disk, and what is its angular acceleration about its centre of mass?

(b) How far along the surface does the disk travel before it stops sliding?

288. You are in a spaceship of mass $m = 4 \times 10^3$ kg in a circular orbit around planet Camelid. The planet is spherical with mass $M = 3 \times 10^{24}$ kg and radius $R = 6 \times 10^6$ m. You are in a circular orbit of radius $2R$. You wish to take a closer look at the inhabitants of the planet which are able to live with zero atmosphere. You briefly fire the rockets on your ship in the direction opposite to your motion so that you slow down and place yourself on an elliptical orbit that will skim the planet (the perigee will be a distance R from the planet's centre), as shown in Figure 102. How much work do the rockets need to do in order to place your ship in this new orbit?

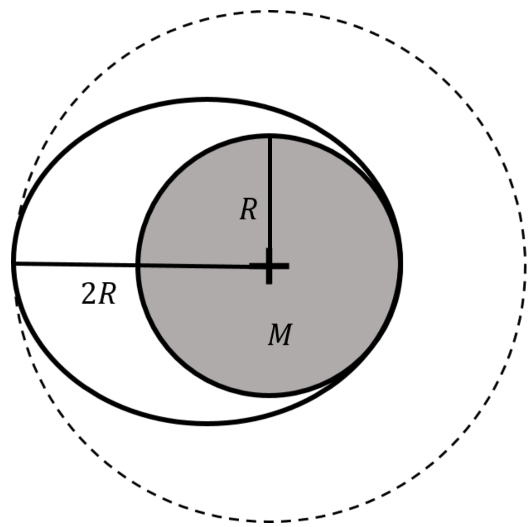

Figure 102: An elliptical orbit skimming the planet.

289. A cube of mass $m = 8$ kg is fixed at a position along a frictionless surface where it compresses a spring of $k = 60 + 15x + 5x^2$ a distance of $x = 0.8$ m, as shown in Figure 103.

(a) If the cube is released from its fixed position, what will its final velocity be?

(b) Suppose that instead of a cube, a sphere is fixed at the position, as shown in Figure 104. Now, assume that the surface is not frictionless, but instead has a μ_s sufficient for rolling without slipping. If the sphere is released from its fixed position, what will its final velocity be?

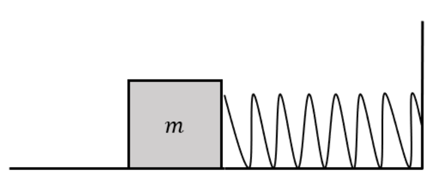

Figure 103: A cube compressing a spring.

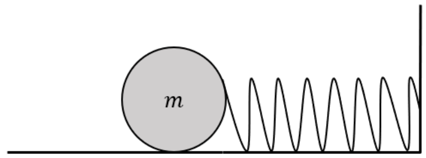

Figure 104: A sphere compressing a spring.

In parts a) and b), you will notice a difference in velocity for the cube and sphere. Suppose you are tasked with moving an object which can either be spherical or cubic across a surface. Determine which shape of the two would move across the following surfaces most quickly, and justify your answer:

(c) The surface is a distance d, with $\mu_s = 0$ and $\mu_k = 0$.

(d) The surface is a very large distance d, with $\mu_s \neq 0$ and $\mu_k \neq 0$.

(e) The surface is a distance d, with a very large μ_s and μ_k.

(f) The surface is a distance d, with a very large μ_s and $\mu_k = 0$

12 Simple Harmonic Motion

12.1 Multiple Choice

290. You wish to double the period of a simple pendulum (a mass at the end of a string), which should you do?

- ◯ Double the mass of the bob
- ◯ Quadruple the length of the string
- ◯ Double the length of the string
- ◯ Quadruple the mass

291. You have constructed a clock that is based on the oscillations of a simple pendulum of length L. You bring your clock to a planet where gravity at the surface is double the strength of gravity at the surface of the Earth (where you designed the clock). What should the length of the pendulum be in order for your clock to still tick at the same rate on the new planet?

- ◯ $2L$
- ◯ $4L$
- ◯ $\frac{1}{2}L$
- ◯ $\frac{1}{4}L$

292. Which of the following simple harmonic oscillators has the lowest frequency?

- ◯ a mass M on a horizontal frictionless surface attached to a horizontal spring with spring constant $2k$.
- ◯ a mass M attached to a vertical spring with spring constant $2k$.
- ◯ a mass M on a horizontal frictionless surface attached to two horizontal springs in series, each with spring constant k.
- ◯ a mass M on a horizontal frictionless surface attached to two horizontal springs in parallel, each with spring constant k.
- ◯ None of the above, they all oscillate with the same frequency.

293. The position of a frictionless horizontal spring mass-system is described by $x(t) = A\cos(\omega t + \phi)$, where $x = 0$ corresponds to the rest length of the spring. At time $t = 0$ the mass has its maximal speed and is moving in the positive x direction. What is the value of ϕ?

- ◯ 0
- ◯ $\frac{\pi}{4}$
- ◯ $\frac{\pi}{2}$
- ◯ $\frac{3\pi}{2}$

294. The position of a frictionless horizontal spring mass-system is described by $x(t) = A\cos(\omega t + \phi)$, where $x = 0$ corresponds to the rest length of the spring. At time $t = 0$ the mass has its maximal speed and is moving in the negative x direction. What is the value of ϕ?

- ◯ $\frac{\pi}{4}$
- ◯ $\frac{\pi}{2}$
- ◯ $\frac{3\pi}{2}$
- ◯ π

295. The position of a frictionless horizontal spring mass-system is described by $x(t) = A\cos(\omega t + \phi)$, where $x = 0$ corresponds to the rest length of the spring. At time $t = 0$ the mass has a speed of zero and its position is along the negative x axis. What is the value of ϕ?

- ◯ $\frac{\pi}{4}$
- ◯ $\frac{\pi}{2}$
- ◯ $\frac{3\pi}{2}$
- ◯ π

296. Which one of the following increases the period of motion of a horizontal frictionless mass-spring system?

- ◯ Increasing the mass
- ◯ Increasing the spring constant
- ◯ Increasing the amplitude of the motion
- ◯ Increasing the maximum speed of the mass

297. A basketball is bouncing up and down in a straight line. Assume that the collision with the ground is elastic and that there is no air resistance. Is this an example of simple harmonic motion?

- ◯ Yes
- ◯ No

298. A mass attached to a horizontal spring is undergoing simple harmonic motion. If the maximum amplitude of the displacement from the equilibrium position of the spring is increased by a factor of 3, by what factor does the maximal acceleration of the mass increase?

- ◯ $\sqrt{3}$
- ◯ 3
- ◯ 9
- ◯ it does not change

299. A mass attached to a horizontal spring is undergoing simple harmonic motion. If the maximum amplitude of the displacement from the equilibrium position of the spring is increased by a factor of 3, by what factor does the period of the oscillations change?

- ○ $\sqrt{3}$
- ○ 3
- ○ 9
- ○ it does not change

300. A mass attached to a horizontal spring is undergoing simple harmonic motion. If the maximum amplitude of the displacement from the equilibrium position of the spring is increased by a factor of 3, by what factor does the mechanical energy of the mass change (assuming zero potential energy when the spring is at its equilibrium length)?

- ○ $\sqrt{3}$
- ○ 3
- ○ 9
- ○ it does not change

301. A one-dimensional system is at rest. If the potential energy graph on both the left and right side has a negative slope, what type of equilibrium is it currently in?

- ○ Stable Equilibrium
- ○ Unstable Equilibrium
- ○ Cannot be answered with information given

302. A mass attached to a horizontal spring is undergoing simple harmonic motion. If the maximum amplitude of the displacement from the equilibrium position of the spring is increased by a factor of 3, by what factor does the maximal acceleration of the mass increase?

- ○ $\sqrt{3}$
- ○ 3
- ○ 9
- ○ it does not change

303. A mass attached to a horizontal spring is undergoing simple harmonic motion. If the maximum amplitude of the displacement from the equilibrium position of the spring is increased by a factor of 3, by what factor does the period of the oscillations change?

- ○ $\sqrt{3}$
- ○ 3
- ○ 9

- ◯ it does not change

304. A mass attached to a horizontal spring is undergoing simple harmonic motion. If the maximum amplitude of the displacement from the equilibrium position of the spring is increased by a factor of 3, by what factor does the mechanical energy of the mass change (assuming zero potential energy when the spring is at its equilibrium length)?
    - ◯ $\sqrt{3}$
    - ◯ 3
    - ◯ 9
    - ◯ it does not change

305. A pendulum of length 1 m in Mars' gravitational potential ($g = 3.7\,\mathrm{m/s^2}$) swings from its maximum amplitude of $\theta_0 = 4°$. The bob of the pendulum has mass of 2 kg, and the rest of the rod is massless. At the bottom of the arc, the pendulum collides with a horizontal spring at equilibrium with $k = 100\,\mathrm{kg/s^2}$. The bob detaches from the pendulum and attaches to the spring and starts oscillating in the horizontal plane. How much time does it take from the maximum amplitude of the pendulum, until the maximum compression of the spring?
    - ◯ 1.04 s
    - ◯ 1.26 s
    - ◯ 2.08 s
    - ◯ 0.76 s

306. A torsional spring is brought from Earth to a another planet with a smaller gravitational acceleration. How will its period on the new planet compare to its period on Earth?
    - ◯ It will be larger
    - ◯ It will be smaller
    - ◯ It will be the same

307. A mass attached to a horizontal spring is undergoing simple harmonic motion. If the maximum amplitude of the displacement from the equilibrium position of the spring is increased by a factor of 3, by what factor does the maximal acceleration of the mass increase?
    - ◯ $\sqrt{3}$
    - ◯ 3
    - ◯ 9
    - ◯ it does not change

308. A mass attached to a horizontal spring is undergoing simple harmonic motion. If the maximum amplitude of the displacement from the equilibrium position of the spring is increased by a factor of 3, by what factor does the period of the oscillations change?

- ○ $\sqrt{3}$
- ○ 3
- ○ 9
- ○ it does not change

309. A mass attached to a horizontal spring is undergoing simple harmonic motion. If the maximum amplitude of the displacement from the equilibrium position of the spring is increased by a factor of 3, by what factor does the mechanical energy of the mass change (assuming zero potential energy when the spring is at its equilibrium length)?

    - ○ $\sqrt{3}$
    - ○ 3
    - ○ 9
    - ○ it does not change

310. A simple pendulum is observed to have a period of 1 s when undergoing simple harmonic motion in a vertical plane. How far from the axis of rotation is the mass at the end of the pendulum?

    A. 12 cm

    B. 25 cm

    C. 50 cm

    D. 78 cm

311. A simple pendulum is made by suspending a mass m at the end of a string of length L. One measures the angle, θ, between the vertical and the position of the mass ($\theta = 0$ when the mass is at the lowest point of the trajectory). What can you say about the motion of the mass, if it released in such a way that it swings in a plane?

    - ○ It is simple harmonic motion only if the amplitude of the oscillations is small
    - ○ It is simple harmonic motion regardless of the amplitude of the oscillations
    - ○ It is never simple harmonic motion
    - ○ It is uniform circular motion

312. In a simple harmonic oscillator, if we double the mass and halve the spring constant, what will happen to the period of the oscillations?

    - ○ It will stay the same
    - ○ It will double
    - ○ It will be halved
    - ○ It will be multiplied by $\sqrt{2}$

12.2 Long answers

313. A simple pendulum is constructed by suspending a mass m from a massless inextensible string of length l. The angle between the the vertical and the string is θ (where $\theta = 0$ is the equilibrium position of the pendulum). Use the Lagrangian method to show that the equation of motion of the system is given by:

$$\alpha = -\frac{g}{l}\sin\theta$$

where α is the angular acceleration. The Lagrangian is given by the difference between the kinetic energy, K, and potential energy, U, of the pendulum:

$$L(\theta, \omega) = K(\omega) - U(\theta) \tag{3}$$

where the kinetic energy of the pendulum depends on its angular velocity, ω, and its potential energy depends on its angular position, θ. From the Lagrangian, the equation of motion for the pendulum is given by the Euler-Lagrange equation:

$$\frac{\partial L}{\partial \theta} - \frac{d}{dt}\left(\frac{\partial L}{\partial \omega}\right) = 0 \tag{4}$$

314. Consider two simple pendula, A and B, made from a mass at the end of a massless inextensible string. The string of pendulum A has a length of $l_A = 1\,\text{m}$, while the length of the string of pendulum B is unknown. Both pendula are released at the same time. After pendulum A has completed 12 complete oscillations, pendulum B has completed 10. What is the length of the string on pendulum B?

315. A uniform metre-stick of mass M is attached to a wall by a pivot. The free end of the metre stick is then attached to a spring with a spring constant k, as shown in Figure 105. If the end of the metre stick makes small oscillations, what is the frequency of the oscillations? [Hint: Use torques!]

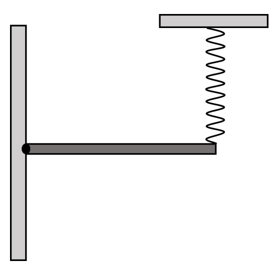

Figure 105: A horizontal uniform metre stick of mass M attached to a hinge (on the left) and a vertical spring on the right.

316. You have been hired to design an efficient long-distance transport system to compete with the HyperLoop. You propose to dig a straight tunnel to connect two points across the Earth, as illustrated in Figure 106. The tunnel is under vacuum (so that there is no friction), and a train on a frictionless track is used to move in the tunnel. The train is only powered by gravity, thus making the system very energy efficient. You assume that the Earth has a uniform density.

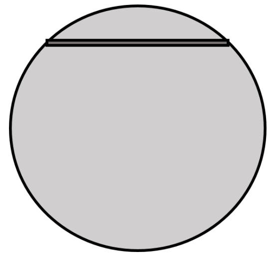

Figure 106: A tunnel connecting two points on Earth.

(a) Show that the motion of the train in the tunnel is simple harmonic motion.

(b) If you had such a tunnel constructed between Kingston, ON, Canada, and Coinsins, Switzerland, how long would the one-way journey take? How does this compare to a commercial airline flight?

Hint: Remember to use Gauss' Law to take into account the fact that the force of gravity is less nearer the centre of the Earth.

317. A mass $m = 300\,\text{g}$ lies on a frictionless horizontal surface and is attached to a horizontal spring with a spring constant $k = 3\,\text{N/m}$. A coordinate system is given such that the x axis is parallel to the motion of the mass under the action of the spring, and the origin is located at the un-stretched position of the spring. The position of the mass is given by:

$$x(t) = A\cos(\omega t + \phi)$$

At time $t = 2\,\text{s}$, the position of the mass m is $x = 0.2\,\text{m}$, and its velocity is $v = -0.5\,\text{m/s}$.

(a) Determine A, ω, and ϕ, and make a plot of $x(t)$ and $v(t)$ between $t = 0\,\text{s}$ and $t = 10\,\text{s}$.

(b) What is the mechanical energy of the mass?

(c) What is the speed of the mass at position $x = \frac{A}{2}$?

318. You accidentally left your phone on the end of a diving board. You watch in horror as your friend jumps off the diving board, causing it to oscillate with simple harmonic motion at a frequency of 3.1 Hz. If the phone loses contact with the diving board, it will fall into the water. What is the maximum possible amplitude of the diving board oscillations for your phone not to lose contact with the diving board?

319. A mass $m = 2.0\,\text{kg}$ is placed at the end of a massless inextensible string of length $L = 1.0\,\text{m}$ to form a simple pendulum. The mass is released from an angle $\theta = 30^\circ$ from the vertical (assume that the small angle approximation is valid to describe the pendulum). When the mass arrives at the bottom of its trajectory, it detaches from the string and attaches to a horizontal spring with spring constant $k = 3\,\text{N/m}$. The spring is positioned such that it is at rest exactly at the position that the mass makes contact.

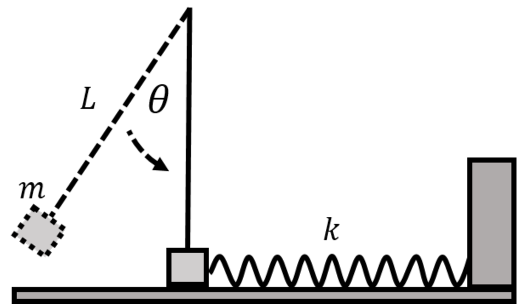

Figure 110: A mass swinging from a pendulum and attaching to a spring.

(a) How much time elapses between when the mass is released and when the spring is maximally compressed?

(b) What is the maximum compression of the spring?

320. You are designing a ride for crias (baby llamas) consisting of a platform that is attached to a spring with spring constant $k = 420\,\mathrm{N/m}$ and that slides along a frictionless surface. The platform has a mass of $M = 45\,\mathrm{kg}$. The coefficient of static friction between a cria and the platform is $\mu_s = 0.4$. You wish for the amplitude of the oscillations to be $A = 1.2\,\mathrm{m}$. You realize that for the ride to be safe, the crias must have a minimum mass or they will slide off. What is that minimum mass?

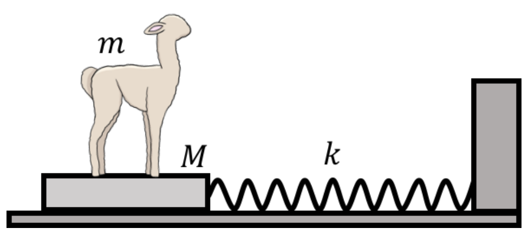

Figure 111: A cria on a platform that oscillates with SHM.

13 Waves

13.1 Multiple Choice

321. If a wave has a velocity of 7 m/s and a period of 3 s, what is its wavenumber?

- ◯ 0.299 m
- ◯ 2.693 m^{-1}
- ◯ 2.693 m
- ◯ 0.299 m^{-1}

322. A guitar string with uniform mass density has a length of 1 m and a mass of 3.0 g. The string is installed such that there is a distance of 0.7 m over which the string is taught (the remaining length of the string is used to attach it). Under what tension should the guitar string be so that its fundamental frequency corresponds to the A note for a guitar (110 Hz)?

- ◯ 57 N
- ◯ 71 N
- ◯ 145 N
- ◯ 924 N

323. If a wave has frequency 10 Hz and a wavelength of 5 m, with what speed is the wave travelling?

- ◯ 0.5 m/s
- ◯ 1.0 m/s
- ◯ 10.0 m/s
- ◯ 50.0 m/s

324. A rope acting as a leash for a Guanaco is chewed on by a fellow Guanaco trying to aid his friend escape to freedom. While trying to save his friend, the Guanaco moves the string up and then down (the string is fixed to wall), creating a series of pulses. The reflected pulses (after hitting the wall), compared to the original are:

- ◯ inverted.
- ◯ in the same orientation.

325. (1 point) A wave passes from a section of heavy rope to a section of lighter rope; what will happen to the speed of the wave?

- ◯ It will not transfer to the lighter rope.
- ◯ It will slow down.

- ◯ It will speed up.
- ◯ There will be twice as many waves in the lighter rope.

326. Which of the following parameters of a wave do not change when a wave transfers mediums?

- ◯ Velocity
- ◯ Frequency
- ◯ Period
- ◯ Wavelength

327. Wave motion in a medium transfers:

- ◯ Energy
- ◯ Mass
- ◯ Both energy and mass
- ◯ Neither mass or energy

328. Richard plays his guitar. The length of the strings is 64.8 cm, and the mass of one of his strings is 1.2 g. He plays a note on this string, which can be heard as a G note, at 196 Hz. To do this he places his finger on one of the strings, shortening its length to 50 cm. Assume the note is the fundamental frequency. What is the tension in the string?

- ◯ 1.32 N
- ◯ 71.1 N
- ◯ 91.2 N
- ◯ 541 N

329. Two otherwise identical standing waves are out of phase by 180 degrees. What will happen if you allow these waves to interfere?

- ◯ Entirely constructive interference
- ◯ Entirely destructive interference
- ◯ A mix of constructive and destructive interence
- ◯ No interference, both waves will continue on their separate ways

330. How does the energy in one wavelength of a standing wave, E_s, compare to the energy in one wavelength of a travelling wave, E_T, if both waves have the same maximum amplitude?

- ◯ $E_s = 1/4E_T$
- ◯ $E_s = 1/2E_T$
- ◯ $E_s = E_T$

- ◯ $E_s = 2E_T$
- ◯ $E_s = 4E_T$

331. Which of the following waves have the same velocity? Note that this refers to the velocity of the wave itself and not the medium in which it travels.

$$\begin{aligned} &1)\ y = 2\sin(3x - 6t + 5) \\ &2)\ y = 4\sin(3x - 4t - 5) \\ &3)\ y = 2\cos(2x - 6t + 5) \\ &4)\ y = 4\cos(2x - 4t - 5) \end{aligned}$$

- ◯ 1 & 3
- ◯ 3 & 4
- ◯ 1 & 2
- ◯ 1 & 4

332. Two guitar strings made of the same material have the same (volume) mass density, the same tension and the same length (between points that are held fixed). String 1 has twice the radius of string 2. What is the ratio, $\frac{f_1}{f_2}$, of the fundamental frequency of string 1 divided by the fundamental frequency of string 2?

A. 0.25

B. 0.5

C. 2.0

D. 4.0

333. Two guitar strings are made of the same material. In order for the strings to make different sounds, their diameters are different, but their volume density is the same. In addition, the tension in both strings is the same, and the distance between points where the strings are held fixed on the guitar is the same as well. The low E string has a fundamental frequency of 82 Hz and the high E string a fundamental frequency of 330 Hz. If the diameter of the low E string is 1.2 mm what is the diameter of the high E string?

A. 0.15 mm

B. 0.3 mm

C. 0.6 mm

D. 4.8 mm

334. The winter holidays came to an end and the Grinch found himself so happy that he actually made a new year's resolution. His resolution was to tone his arms a little more. Instead of going out and buying a new workout toy, the Grinch attached a rope to a fixed point on

a wall on one end and held the other end in his hand. Playing his favourite 80's workout tunes, he moved his arm up and down creating a wave on the rope. The mass of the rope was 2.00 kg and he pulled on the rope with a tension of 550 N. If the waves travelled with a speed of 20 m/s, what was the length of the rope?

A. 1.45 m

B. 2.25 m

C. 2.75 m

D. 55 m

335. A string on a cello is twice as long as a string on a violin (between points where the string is held fixed). If the strings have the same mass per unit length and are under the same tension, what can you say about their fundamental frequency?

A. The fundamental frequency of the violin string is a quarter of that of the cello

B. The fundamental frequency of the violin string is half of that of the cello

C. The fundamental frequency of the violin string is double that of the cello

D. The fundamental frequency of the violin string is quadruple that of the cello

336. If we double the tension in a stretched rope, waves that propagate along the rope will travel:

◯ 2 times faster

◯ 4 times faster

◯ $\sqrt{2}$ times faster

◯ 2 times slower

337. Two identical loudspeakers face each other and emit a constant tone with a frequency of 500 Hz (the sound waves travel at a speed of 343 m/s). The speakers are turned on at exactly the same time (such that they would be in phase if placed next to each other and facing the same direction). You wish to set the distance between the speakers such that one cannot hear the tone from either speaker anywhere along the line joining them. How far away should you place the speakers?

◯ Any multiple of 0.686 m

◯ 0.686 m

◯ 0.343 m

◯ Any multiple of 0.343 m

13.2 Long answers

338. Figure 112 shows two graphs, both describing the same wave. One graph shows the displacement of the wave at some position as a function of time and the other graph shows the displacement of the wave at some time as a function of position. What is the speed of the wave?

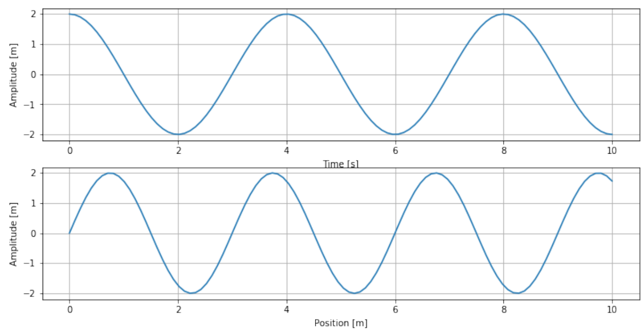

Figure 112: Displacement as a function of position and time for a wave.

339. When an earthquake strikes an area, the type of wave that typically does the most damage is called a secondary wave. Secondary waves are named as such because a quicker, less damaging wave called a primary wave will move through the area as a precursor to the secondary wave. Secondary waves travel transversely by shearing the Earth, while primary waves travel longitudinally as a pressure wave. Primary waves are often undetectable by human senses, but many animals can sense a primary wave. Suppose an earthquake strikes an area. The density of the material the primary and secondary waves move through is $\rho = 3.2 \times 10^3\,\mathrm{kg/m^3}$, the Young's bulk modulus is $8.0 \times 10^{10}\,\mathrm{N/m^2}$, and the shear modulus is $2.0 \times 10^{10}\,\mathrm{N/m^2}$. A woman is walking her dog, and notices that her dog begins barking 20 seconds before she felt the earthquake. Determine the approximate distance between the dog walker and the earthquake.

340. A seismologist measures the intensity of a spherical Earthquake to be $1.6 \times 10^5\,\mathrm{J/m^2/s}$. The measurement of intensity is taken 95 km from the epicentre of the Earthquake.

    (a) What was the intensity of the earthquake when it passed a point 2.0 km from the epicentre?

    (b) At what rate did energy pass through an area of $5.0\,\mathrm{m^2}$ if the area was 2.0 km from the epicentre?

341. A guitarist plucks the string of their guitar and produces a note. The string is 92.5 cm long, has a mass of 5.42 g, the distance from the bridge to the support post is 64.8 cm, and the string is under a tension of 65.9 N. What are the frequencies of the fundamental and first two overtones of this string? Which guitar string is this?

342. Sine waves propagate along a rope that is split into two sections, one with a lighter linear mass density, μ_L, and one with a heavier linear mass density, μ_H as in Figure 113.

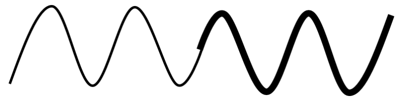

Figure 113: A wave propagating along a rope that has two different sections with different linear mass densities.

(a) Determine the ratio of the speeds of the wave in the two sections, $\frac{v_H}{v_L}$

(b) Determine the ratio of the wavelengths of the wave in the two section$\frac{\lambda_H}{\lambda_L}$

(c) Which section has a longer wavelength?

343. A sailor wishes to test the depth of the water below his ship. The sailor strikes the side of his ship just below the surface of the water and measures how long it takes to hear the echo of his ship. The echo of the wave reflected from the ocean floor is heard 3.4 s later. How deep is the ocean at this point?

344. Show that if D_1 and D_2 satisfy the one dimensional wave equation, so does $a_1D_1 + a_2D_2$, where a_1 and a_2 are constants and D_1 and D_2 are functions that depend on x and t.

345. When the source of a wave and an observer move either towards or away from each other, the observed frequency of the wave will be different from the frequency at which the wave is emitted. For example, if you are standing by the side of the road and a police car with a siren passes by, the frequency of the sound you hear will be higher when the car is moving towards you and lower when it is moving away from you. This is called the Doppler Effect.

Consider a stationary source that emits waves at some constant frequency f_s. The waves move at a constant speed v through the medium. If that source starts to move at a constant speed v_s (relative to the medium) towards an observer, it will "catch up" with the wave crests that it emits, so that the wave crests will be "closer together" (the wavelength will be shorter) and the observed frequency, f_o, will be higher.

(a) Show that the frequency observed when a wave source moves either towards or away from a stationary observer with speed v_s relative to the medium is given by:

$$f_o = \frac{v}{v \pm v_s} f_s$$

Determine when the sign will be positive or negative.

(b) An observer moves towards/away from a source with speed v_o relative to the medium. Show that the observed frequency is given by:

$$f_o = \frac{v \pm v_o}{v} f_s$$

Determine when the sign will be positive or negative.
Hint: Think relative velocities!

346. Show that the function $D(x,t) = A \sin(kx - \omega t + \phi)$ satisfies the one-dimensional wave equation:

$$\frac{\partial^2 D}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 D}{\partial t^2}$$

347. You have decided to learn a song on the guitar. Unfortunately, you know nothing about guitars, and don't have one. You know that the fundamental frequency of an E note is $f = 123.6\,\mathrm{Hz}$, and that the length of a string on a guitar is $60\,\mathrm{cm}$. You have a string of mass $m = 4.2\,\mathrm{g}$. If the tension acting on the string is $F = 500\,\mathrm{N}$, to what length must the string be shortened in order for it to produce an E note?

14 Fluid Mechanics

14.1 Multiple Choice

348. Which of the following is the most correct explanation of how airplanes fly?

- ◯ The wings or airfoils are designed so that there is lower pressure above the wings which then generates lift provided the plane is moving fast enough
- ◯ The propellers in the jets on the wings or on the front pull the plane fast enough to counteract gravity and then it lifts off the groud
- ◯ Pilots are wizards
- ◯ The thrust from the propellers is at an upward angle, so it counteracts gravity while pushing the plane forward.

349. A reservoir's surface is 500 m above an output pipe. Given this information (and neglecting pipe friction), what is the velocity of the water as it exits the pipe?

- ◯ 49 m/s
- ◯ 25 m/s
- ◯ 99 m/s
- ◯ 200 m/s

350. A cork floats such that three quarters of its volume is below water. If the density of water is taken to be 1.0, what is the density of the cork?

- ◯ 0.25
- ◯ 0.50
- ◯ 0.75
- ◯ 0.83
- ◯ None of the above

351. Icebergs have a density of $0.9167\,\mathrm{g/cm^3}$ and ocean water has a density of $1.029\,\mathrm{g/cm^3}$. What percentage of the average iceberg is underwater?

- ◯ 80
- ◯ 94
- ◯ 89
- ◯ 0.83

352. The wings of an Airbus 320 airplane have a total useful area of $123\,\mathrm{m^2}$, while the average mass of the airplane at take-off when carrying passengers is $6 \times 10^4\,\mathrm{kg}$. If the difference in

air pressure above and below the wings is solely responsible for the airplane's lift, what is that minimum pressure difference in order for the plane to have enough lift to take-off?

- ◯ 488 Pa
- ◯ 4780 Pa
- ◯ 7.38×10^6 Pa
- ◯ 72.3×10^6 Pa

353. You would like to double the water flow rate in your shower, so you replace the horizontal pipe from the main water supply to your shower with a wider pipe that has a lower resistance to water flow. The original pipe has radius of 1 cm, what radius should the new pipe have? (Assume a horizontal pipe and incompressible laminar flow of water with non-negligible viscosity, so that the flow is described by the Poiseuille equation).

- ◯ 1.19 cm
- ◯ 1.41 cm
- ◯ 2.00 cm
- ◯ 4.00 cm

354. An object made of an unknown material is observed to float in water. Does the object still float in water on the moon, where the force of gravity is about six times less?

- ◯ Yes.
- ◯ No.
- ◯ It depends on the density of the object.

355. When is the pressure inside an incompressible fluid the highest, if the fluid is undergoing laminar flow?

- ◯ When the fluid is moving slow.
- ◯ When the fluid is moving fast.
- ◯ The pressure does not depend on the speed of the fluid.

the speed has no effect on the pressure inside the fluid.

356. A 60 kg woman stands on a square piston of side 2 m, which pushes down on a hydraulic fluid. How much force does a second piston (a circle of radius 2 m, which is at the same height) feel?

- ◯ 60 N
- ◯ 188.5 N
- ◯ 588 N
- ◯ 1847 N

357. Suppose there are three glasses, all filled with the same amount of water. Glass A in shaped such that the top has a larger radius than the bottom, glass B is cylindrical, and glass C is shaped such that the bottom has a larger radius than the top. Which of the three glasses have the greatest liquid pressure at the bottom?

- ◯ Glass A
- ◯ Glass B
- ◯ Glass C
- ◯ All three have equal non-zero pressure at the bottom
- ◯ All three have zero pressure at the bottom.

358. A 50 kg pig floats in fresh water with 27% of its volume submerged. What is its density?

- ◯ $420\,\mathrm{kg/m^3}$
- ◯ $380\,\mathrm{kg/m^3}$
- ◯ $270\,\mathrm{kg/m^3}$
- ◯ $150\,\mathrm{kg/m^3}$

359. Which principle describes the lift produced by an airplanes wing?

- ◯ Pascal's principle
- ◯ Bernouilli's principle
- ◯ Archimedes' principle
- ◯ Poiseuille's principle

360. An incompressible fluid flows with negligible resistance through a pipe that gets progressively narrower in the direction of the fluid flow as shown in Figure 114. Which one of the following is correct about the pressure and speed of the fluid?

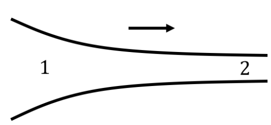

Figure 114: Fluid flowing in the direction of the arrow through a narrowing pipe.

A. The pressure and speed are highest at point 1

B. The pressure and speed are highest at point 2

C. The pressure is highest at point 1 and the speed highest at point 2

D. The pressure is highest at point 2 and the speed highest at point 1

361. The water level in a reservoir is 500 m above the output of a pipe. If one neglects pipe friction and the difference in atmospheric pressure, what is the velocity of the water as it exits the pipe?

A. 50 m/s

B. 70 m/s

C. 99 m/s

D. 9800 m/s

14.2 Long answers

362. Your friend decides that they want to go for the Guinness World Record for the longest drinking straw used in a vertical setting. That is, they wish to place a very long vertical drinking straw into a glass of water on the ground (at sea level) and drink from the top of the straw by leaning out of the balcony of a nearby building. You tell your friend that there is a physical limit to how long the straw can be and for them to still be able to drink the water. What is that length? How would your answer differ if your friend used a glass of olive oil instead?

363. A vertical U-shaped tube with open ends is filled with two immiscible fluids (e.g. water and oil), as shown in Figure 115. Because the two fluids have different densities, they do not reach the same height on either side of the tube. If the fluid on the right is a height h_1 below the the fluid on the left, and the fluid on the left has a height of h_2, what is the density, ρ_2 of the fluid on the left in terms of ρ_1, the density of the fluid on the right?

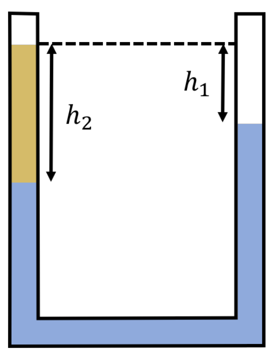

Figure 115: A U-shaped tube filled with two different fluids.

364. A tank of diameter D is filled with water up to a height h above the bottom of the tank (Figure 116). At the bottom of the tank is a hole of diameter d. Assume that the water flows out of the hole with a laminar flow and that the difference in atmospheric pressure between the top and the bottom of the tank is negligible.

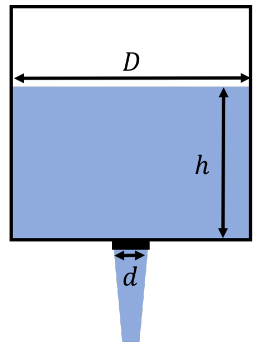

Figure 116: A tank draining.

(a) What speed will the water have just as it exits the bottom of the tank? Assume that the hole is small enough so that the water level in the tank is constant.

(b) As the stream of water gets further from the bottom of the tank, it will narrow to a smaller diameter than d. You can observe the same process for water flowing out of a tap. What will be the diameter of the (laminar) water stream a distance x below the bottom of the tank?

(c) If you no longer assume that the level in the tank is constant, but rather that the top of the water has a speed downwards as the tank drains, show that the speed of the water exiting the bottom of the tank is given by:

$$v = \sqrt{\frac{2gh}{(1 - \frac{d^4}{D^4})}}$$

at an instant when the water level in the tank is h.

365. A cylindrical cork with density $\rho_c = 0.2 \times 10^3\,\mathrm{kg/m^3}$ and length L is held submerged under water ($\rho_w = 1.0 \times 10^3\,\mathrm{kg/m^3}$) such that the cork is vertical and its top is just below the surface of the water (Figure 117). When the cork is released, how far above the surface of the water will the top of the cork reach before it falls back down? Express your answer in terms of L and assume that there are no drag forces on the cork as it moves through the water or air.

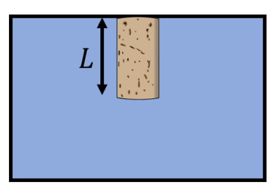

Figure 117: A cork held under water.

366. An airplane has a wing with an area of $35\,\mathrm{m^2}$ which has air flowing along the top at $215\,\mathrm{m/s}$ and air flowing along the bottom at $180\,\mathrm{m/s}$. If the density of air is $1.29\,\mathrm{kg/m^3}$, what is the lift on the wing in newtons?

367. A vertical dam is constructed between two mountains that are a distance b apart. The dam can be approximated to be uniform and rectangular in cross-section.

(a) If water fills the dam to a height h above the bottom of the dam, what is the net force exerted on the side of the dam?

(b) If water fills the dam to a height h above the bottom of the dam, what is the net torque exerted on the dam about the bottom of the dam?

368. You measure that water comes out of your bathtub faucet at a rate 10 litres/min. The diameter of the bathtub faucet is 2 cm and the faucet is 2.5 m above ground level. At what rate does water come out of the 1 cm diameter faucet in your basement bathroom, located 1.5 m below ground level, in litres/min?

Assume that the atmospheric pressure does not change appreciably between your basement and first floor, that water flows without turbulence or viscosity in your plumbing, and that opening or closing one faucet does not affect the other.

369. You measure that water comes out of your kitchen faucet at a rate of 6 litres/min. The faucet has a diameter of 1 cm. If you try to plug the running faucet by pressing your thumb against the opening to close it off, how much force must you exert?

The density of water is $1000\,\text{kg/m}^3$ and you should assume that water flows without turbulence or viscosity in your pipes.

370. A hydraulic press is designed such that a force applied to a lever of length $2l$ is carried through hydraulic fluid and puts pressure on a target sample. The hydraulic press has a small cylinder of diameter $d = 1.5\,\text{cm}$, which carries fluid that is pressed down by a device attached to the lever. The pressure on the small cylinder is carried through the hydraulic fluid, which causes a larger cylinder of diameter $D = 15\,\text{cm}$ to press up on the sample, as shown in Figure **??**. If the sample has an area $A = 2.0\,\text{cm}^2$ and $F = 280\,\text{N}$ newtons are applied to the lever, what pressure is applied to the sample?

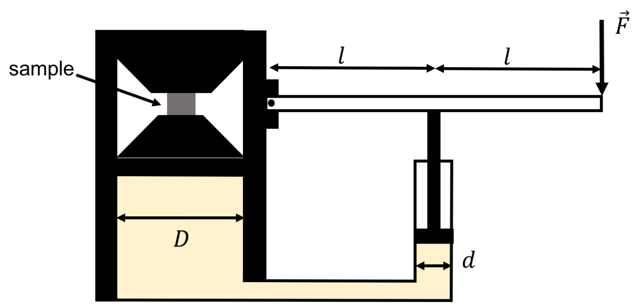

Figure 118: A schematic of a press.

371. A cylindrical cork of radius R and length L is floating in a liquid such that its axis of symmetry is vertical and half of the length of the cork is below the fluid (the density of the fluid is ρ_f), as in Figure 119. For this problem, ignore any effects from friction, drag and the viscosity of the fluid.

(a) Show that the density of the cork, ρ_c, is half that of the fluid ($\rho_c = \frac{1}{2}\rho_f$)

(b) If the cork is displaced vertically by a distance x ($x < \frac{L}{2}$) from its equilibrium (either pushed down or pulled up slightly compared to the depiction in the figure), show that the net force on the cork is given by:

$$F = -\pi R^2 \rho_f g x$$

where the net force is in the opposite direction of the displacement, x.

(c) If you press down on the cork slightly (so that part of the cork is still above the fluid) and release it, the cork will oscillate up and down. What should the length of the cork be for it to oscillate with a frequency of 5 Hz?

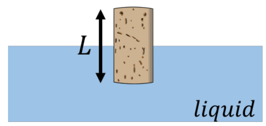

Figure 119: A cork floating in a liquid.

372. Divers can travel deep below the surface of the Earth. One of the challenges that divers face is great changes in pressure. Show that, if you take into account that g changes as you get closer to the centre of the Earth, the pressure difference in going from y_1 to y_2 is:

$$P(y_2) - P(y_1) = -\frac{\rho G M}{2R^3}(y_2^2 - y_1^2)$$

where y is the distance from the centre of the Earth, ρ is the density of the water, M is the mass of the Earth, R is the radius of the Earth, and G is the gravitational constant. Assume the Earth has a uniform density, and that the water is incompressible.

373. What is the pressure difference between the Challenger Deep and a point just below the surface of the water?

374. A water-based heating system is shown in Figure 120. A pump exerts a constant pressure difference between its inlet and outlet, $\Delta P = 12\,\text{psi}$, to circulate hot water across 2 distinct paths. One path contains a single radiator that can be modelled as a pipe with an effective resistance to water flow of $R_1 = 4 \times 10^7\,\text{kg} \cdot \text{m}^{-4} \cdot \text{s}^{-1}$. The other path has two radiators with effective resistances to water flow given by $R_2 = 5 \times 10^7\,\text{kg} \cdot \text{m}^{-4} \cdot \text{s}^{-1}$ and $R_3 = 3 \times 10^7\,\text{kg} \cdot \text{m}^{-4} \cdot \text{s}^{-1}$.

All of the pipes are at the same height so that Figure 120 is a top view. Assume that the pipes and the pump have effectively no resistance to the water flow (as their diameters are much larger than that of the radiators which are the only components that have a resistance to the flow of water).

(a) What is the pressure difference across each radiator?

(b) How many kilograms of water per second go through the pump and through each radiator?

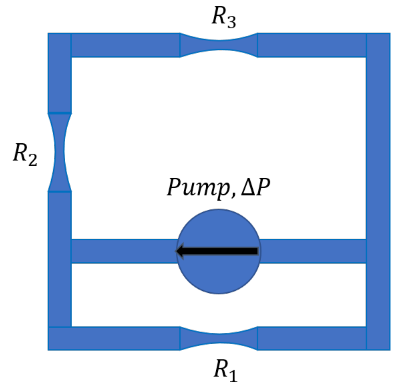

Figure 120: A constant pressure pump feeding a small network of radiators in a house, as seen from above.

375. A cork in the shape of a cylinder of radius $r = 1\,\mathrm{cm}$ and length $L = 5\,\mathrm{cm}$ is floating upright on water (density $\rho_W = 1\,\mathrm{g/cm^3}$) such that one quarter of the cork's height is above water (Figure 122). What is the minimum amount of that work must be done by a person pressing down on the cork in order to fully submerge the cork?

   Assume that there is no friction or drag, and that the cork always remains upright.

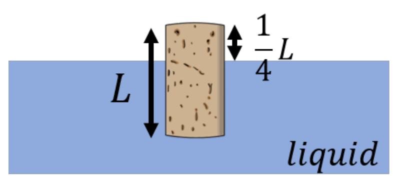

Figure 122: Cork floating in water.

376. The density of the ice in an iceberg is about $\rho_I = 0.92\,\mathrm{g/ml}$. The density of salt water is about $\rho_W = 1.03\,\mathrm{g/ml}$.

   (a) If such an iceberg is floating on salt water, what fraction of the iceberg's volume is below the surface of the water (assume that the iceberg has a uniform density)?

   (b) If a polar guanaco swims up to an iceberg shaped as a cube of side $L = 10\,\mathrm{m}$ (same density as above), climbs to the top, and then jumps off into the salt water, with what frequency will the iceberg oscillate once the guanaco has jumped off? Assume that there is no friction or drag on the iceberg, and that it oscillates up and down in the vertical direction.

377. You connect a straight horizontal water hose (radius $r = 1\,\mathrm{cm}$) to the bottom of a water tank, where the water is at a height of $H = 5\,\mathrm{m}$ from the location where the hose is connected (Figure 123). The hose has a length of $L = 10\,\mathrm{m}$ and water has a viscosity of $\eta = 0.001\,\mathrm{kg/(m \cdot s)}$. Assume that the flow through the hose is laminar and incompressible, that the level in the tank is constant, and that atmospheric pressure does not change appreciably between the top of the tank and the horizontal hose. The density of water is $1\,\mathrm{kg/litre}$.

   (a) What is the speed of the water as it exits the hose?

(b) What is the speed of the water exiting the hose if you ignore its viscosity?

Hint: Bernoulli's equation holds in the tank which allows you to determine the pressure in the water as it enters the hose (through which it does not change speed, by continuity). In particular, the pressure difference across the hose is **not** $\rho g H$.

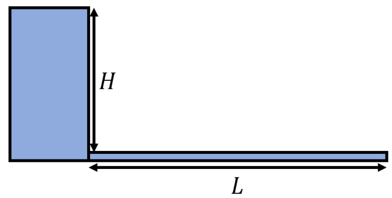

Figure 123: Water flowing from a tank through a horizontal hose.

378. A tank with a total height h is filled to the top with water. A hole is placed a distance s from the top of the tank, as shown in Figure 124 so that water leaks out of the hole and lands a distance l from the bottom of the tank. At what distance s from the top of the tank should the hole be placed in order to maximize the distance l at which the water lands?

Assume incompressible flow with no viscosity, neglect air friction, and assume that atmospheric pressure does not change appreciably over the height h. Furthermore, assume that the level in the tank is constant.

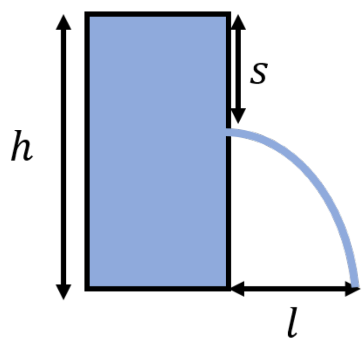

Figure 124: A tank leaking water.

15 Charges and the electric field

15.1 Multiple Choice

379. The electric field at the centre of an electric quadrupole is...

- ◯ Positive to the left.
- ◯ Positive to the right.
- ◯ it depends on the magnitude of the charges.
- ◯ zero

380. An object with a net charge of $1.4\,\mathrm{C}$ has...

- ◯ $8.75e18$ protons.
- ◯ $8.75e18$ more protons than electrons.
- ◯ $8.75e18$ electrons.
- ◯ 1.4 more protons than electrons.

381. A net electric charge is placed on a solid conducting sphere. How does the charge distribute itself?

- ◯ It remains where it was placed.
- ◯ It migrates to the centre of the sphere.
- ◯ It distributes itself uniformly on the surface of the sphere.
- ◯ None of the above.

382. An electric dipole is made of two charges, $+Q$ and $-Q$, separated by a distance $l = 1\,\mathrm{cm}$. When placed in an electric field with magnitude $E = 100\,\mathrm{V/m}$, the dipole experiences a maximal torque of magnitude $\tau = 1.3 \times 10^{-3}\,\mathrm{N \cdot m}$. What is the magnitude of Q?

- ◯ $Q = 1.3 \times 10^{-6}\,\mathrm{C}$
- ◯ $Q = 1.3 \times 10^{-5}\,\mathrm{C}$
- ◯ $Q = 1.3 \times 10^{-4}\,\mathrm{C}$
- ◯ $Q = 1.3 \times 10^{-3}\,\mathrm{C}$

383. A glass rod is rubbed with a piece of fabric, and the glass rod gains a negative charge. What can be said?

- ◯ Protons were transferred from the glass to the piece of fabric.
- ◯ Electrons were transferred from the glass to the piece of fabric.
- ◯ Protons were transferred from the piece of fabric to the glass.
- ◯ Electrons were transferred from the piece of fabric to the glass.

384. A sphere of radius R_1 made of an insulating material has a total charge $+Q$. A spherical shell (inner radius $R_2 > R_1$) made of an insulating material is concentric and completely envelopes the sphere. The shell has a net positive charge $+2Q$. An electron is placed at rest exactly between the sphere and the shell. In which direction does the electron move?

- ◯ Towards the sphere.
- ◯ Towards the shell.
- ◯ The electrons stays at rest.
- ◯ Not enough information to tell.

385. Which of the following is true?

- ◯ Electric field vectors point in the opposite direction as the electric force that a positive test charge would undergo.
- ◯ Electric fields are scalar quantities and thus do not have a vector direction.
- ◯ Electric field vectors point in the same direction as the force a positive test charge would experience.
- ◯ None of the Above

386. A sphere of radius r has a net charge $+Q$ distributed uniformly through its volume. Which material could the sphere be made of?

- ◯ Copper
- ◯ Gold
- ◯ Superconducting aluminium
- ◯ Plastic

387. Materials in which the electrons are bound very tightly to the nuclei are referred to as?

- ◯ conductors
- ◯ insulators
- ◯ semi-conductors
- ◯ solid

388. In a region of space, the electric field is given by $\vec{E}(x) = -ax\hat{x}$, where a is a constant. A charge q is released near $x = 0$, and the only force on the charge is that from the electric field. Which statement is true?

- ◯ If q is positive, it will undergo simple harmonic motion
- ◯ If q is negative, it will undergo simple harmonic motion
- ◯ Regardless of the sign of q, it will undergo simple harmonic motion
- ◯ Regardless of the sign of q, it will not undergo simple harmonic motion

389. A positively charged rod, A, is brought close to (but not touching) one end of a neutral conducting rod, B. If a positive charge q is brought close to the other end of rod B (opposite the side with rod A), what force will q feel?

- ○ A force attracting it to rod B
- ○ A force repelling it from rod B
- ○ No force since rod B is neutral

390. Two point charges of equal magnitude but unknown sign are placed close to each other. The force between them will be:

- ○ Repulsive
- ○ Attractive
- ○ Zero
- ○ Not enough information to answer

391. What happens when a molecule with a permanent electric dipole moment is placed in an electric field?

- ○ The molecule gains an additional induced dipole
- ○ Torque generated from the electric field aligns the molecule's dipole with the external field
- ○ Nothing happens

392. Which particles are responsible for excess charge in material?

- ○ protons
- ○ electrons
- ○ neutrons
- ○ photons

393. Consider a 2D system in which you have a positive charge $+2Q$ located at (-3,0) and another positive charge $+4Q$ located at an unknown position (x,y). If a third charge B is placed at the origin (0,0), where does the $+4Q$ charge have to be located for B to remain stationary?

- ○ $(-3\sqrt{2}, 0)$
- ○ $(0, 6)$
- ○ $(\sqrt{2}, 6)$
- ○ $(\ 3\sqrt{2}, 0)$

394. A positive charge is free to move. Another positive charge is fixed in place one metre to the West of the charge. A negative charge is fixed in place one metre to the South of the first charge. In what direction will the first charge move?

- ◯ North
- ◯ South
- ◯ East
- ◯ West
- ◯ SW
- ◯ SE
- ◯ NW
- ◯ NE

395. A positively charged sphere is brought near a neutral conducting metallic rod, as shown in Figure 126.

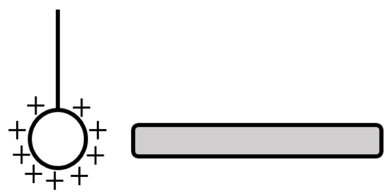

Figure 125

A. There is no electric field in the rod

B. The total electric field in the rod is to the right

C. The total electric field in the rod is to the left

396. A positively charged sphere is brought near a neutral conducting metallic rod, as shown in Figure 126. What can you say about charges on the rod?

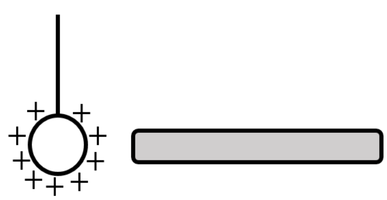

Figure 126: A charged sphere brought close to a neutral conducting rod.

A. The right side of the rod is positively charged, but the net charge on the rod is zero.

B. The left side of the rod is positively charged, but the net charge on the rod is zero.

C. Both sides of the rod are positively charged.

D. Neither side of the rod is charged.

397. Three charges (q, $2q$, and $-q$) are held fixed at the corners of a right angle isosceles triangle as shown in Figure 127. Which statement is true?

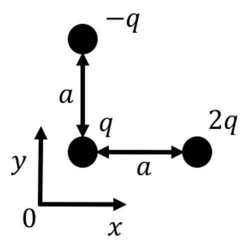

Figure 127: Three charges at the corners of a right angle triangle.

A. The net electric force vector is the same on each charge

B. If released, q would move in the positive x and negative y direction

C. The x component of the net electric force on q is larger in magnitude than the y component of the net electric force on q

D. If released, $-q$ would move in the positive y direction

398. A charge $q_1 = 1\,\text{nC}$ ($1\,\text{nC}=10^{-9}\,\text{C}$) is placed at the origin of a coordinate system. A charge $q_2 = 0.5\,\text{nC}$ is placed at a position $x = 0.5\,\text{m}$ and $y = 0.5\,\text{m}$. What is the force vector on q_1?

- ○ $\vec{F} = (9.00, 9.00, 0) \times 10^{-9}\,\text{N}$
- ○ $\vec{F} = (-9.00, -9.00, 0) \times 10^{-9}\,\text{N}$
- ○ $\vec{F} = (6.36, 6.36, 0) \times 10^{-9}\,\text{N}$
- ○ $\vec{F} = (-6.36, -6.36, 0) \times 10^{-9}\,\text{N}$

399. An electron ($m = 9.11 \times 10^{-31}\,\text{kg}$, $q = -1.6 \times 10^{-19}\,\text{C}$) is in a circular orbit with radius $r = 1.0 \times 10^{-10}\,\text{m}$ around a proton ($m = 1.67 \times 10^{-27}\,\text{kg}$, $q = 1.6 \times 10^{-19}\,\text{C}$), much like in a hydrogen atom. Assuming that only the Coulomb force from the proton is acting on the electron and that the proton is held fixed in space, what is the frequency of the electron's orbit?

- ○ 2.53×10^{10} Hz
- ○ 5.91×10^{13} Hz
- ○ 2.53×10^{15} Hz
- ○ 3.58×10^{15} Hz

400. An electric dipole is made from two (opposite) charges with magnitude, Q, separated by a distance, l, and placed such that the centre of the dipole is at the origin of a coordinate

system. At a distance $R >> l$ from the origin, what can be said of the magnitude of the electric field from the dipole?

- ◯ It is weaker than that from a single positive charge Q placed at the origin
- ◯ It is stronger than that from a single positive charge Q placed at the origin
- ◯ It is zero, because no net charge is enclosed by a gaussian sphere of radius R
- ◯ It is the same strength as the field from a single charge $2Q$ placed at the origin

401. An electric dipole is made from two (opposite) charges with magnitude $Q = 42.0\,\mathrm{C}$ separated by a distance $l = 2.0\,\mathrm{m}$, and placed at the origin of a coordinate system. The origin is half-way between the two charges, the dipole is aligned along the x-axis such that the positive charge is at $x = +1.0\,\mathrm{m}$. How much work is required to bring a charge $q = -1.6 \times 10^{-19}\,\mathrm{C}$ from infinity to the origin?

- ◯ 1.344 $\times 10^{-17}$ J
- ◯ 0 J
- ◯ -1.344 $\times 10^{-17}$ J
- ◯ -1.344 $\times 10^{-17}$ eV

15.2 Long answers

402. Four charges of magnitude $+q$ are located at each corner of a square which has sides of length d. A charge Q is placed at a distance b orthogonal to the plane which the square lies on and at an equal distance from each charge at the corners of the square. What is the magnitude of the net force exerted by the four charges on the charge Q?

403. A thin glass rod is bent into a semicircle of radius a. A charge $+Q$ is uniformly distributed along the upper half, and a charge $-Q$ is uniformly distributed along the lower half, as shown in Figure 129. Find the electric field $\vec{E}$ (magnitude and direction) at point P (the centre of the circle).

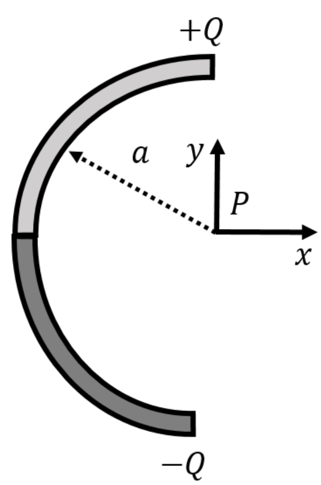

Figure 129: A glass rod bent into a half circle with charges distributed on it.

404. An electron is constrained to move along the central axis of a ring which has a radius R and a net charge Q that is uniformly distributed along the ring.

   (a) Show that the electron will experience a force of the form $F_z = -kz$ when it is near the centre of the ring (where z is the distance from the centre of ring and is small compared to R).

   (b) Derive a formula for the frequency of the oscillation that occurs in part (a)

   Hint: Note that for $z << R$:

$$\frac{1}{z^2 + R^2} = \frac{1}{R^2\left(\frac{z^2}{R^2} + 1\right)} \sim \frac{1}{R^2}$$

   since $\frac{z^2}{R^2} << 1$

405. A charge $-q$ is held fixed in space. An electron is placed a distance, d, from the fixed charge, and released. The electron then accelerates away from the charge. What speed does the electron have:

   (a) when it reaches a distance D ($D > d$)?

   (b) when it reaches infinity?

406. Consider the square charge distribution shown in Figure 132, with $q = 2.6 \times 10^{-8}$ C and $a = 1.5$ m.

   (a) What is the force on the charge q due to the other three charges (magnitude and direction)?

   (b) What is the electric field at the centre of the square?

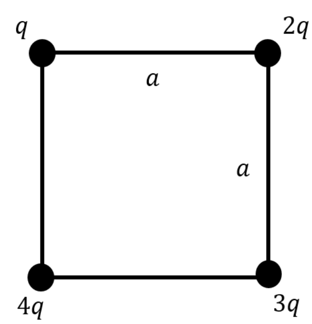

Figure 132: Charges arranged on a square.

407. Determine the electric field at a point which is a distance, z, above an infinite plane with surface charge density σ (in C/m^2).

   Hint: You may think of the plane as a sum of line charges. You can use the result that the magnitude of the electric field a distance r away from an infinite line of charge with linear charge density λ is given by:

$$E(r) = \frac{\lambda}{2\pi\epsilon_0 r}$$

You may also need the following anti-derivative:

$$\int \frac{1}{x^2+a^2}dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$$

408. A dipole consisting of charges $+e$ and $-e$ separated by 0.85 nm is in an electric field with strength $E = 4.2 \times 10^4$ N/C.
   (a) What is the magnitude of the dipole moment?
   (b) What is the torque on the dipole when its dipole moment is perpendicular to the electric field?
   (c) What is the torque on the dipole when its dipole moment makes an angle of 45 degrees with respect to the electric field?
   (d) What is the work required to rotate the dipole from being oriented parallel to the field to being antiparallel to the field?
409. A dipole has two charges seperated by a distance l. Show that at a point along the axis of the dipole (the line which passes through $+Q$ and $-Q$) the electric field has the magnitude:

$$E = \frac{2p}{4\pi\epsilon_0 d^3}$$

Where p is the dipole moment, d is the distance from a point along the axis of the dipole to the centre of the dipole, and $d >> l$

410. A dipole is made up of two positive charges Q.
   (a) Show that the electric field on the **perpendicular bisector** of the dipole is given by:

$$E = \frac{2Q}{4\pi\epsilon_0 r^2}$$

   Where r is the distance from the centre of the dipole, l is the distance between the two charges, and $r >> l$.
   (b) Why does the electric field in this dipole decrease by a factor of r^2? (The electric field decreases by a factor of r^3 in a typical dipole)
411. A dipole is made from two small oppositely charged spheres each carrying a charge with magnitude $Q = 2 \times 10^{-2}$ C, and separated by a distance of $l = 1$ cm. The horizontal dipole is held at its centre of mass and is free to rotate in the horizontal plane about a vertical axis through its centre of mass. The dipole has a moment of inertia $I = 0.001\,\mathrm{kgm^2}$. The dipole is placed in a uniform horizontal electric field of magnitude $E = 1000$ N/C.

(a) Show that for small angles between the electric field vector and the dipole vector, the dipole will experience simple harmonic motion.

(b) What is the frequency of the simple harmonic oscillations?

412. A rod made of an insulating material has a length, L, and carries a total charge $+Q$. The rod is placed along the x-axis, such that one end is at the origin and the other end is at a position $x = -L$ (Figure 135). Give an expression for the electric field vector at a point $x > 0$ along the x-axis, as shown.

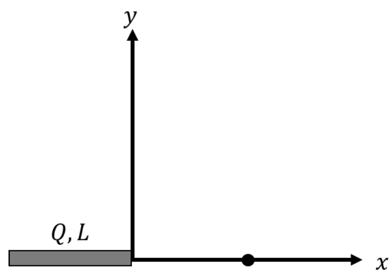

Figure 135: Charged rod of length L carrying charge Q.

Hint: If you need to integrate $\int_a^b (u+c)^n du$, where c is a constant, you can use substitution: for example, let $v = u+c$ (so that $dv = du$), change the limits of the integral, and integrate $\int_{a+c}^{b+c} v^n dv$.

413. Four uniformly electrically charged rods of length $2a$ are arranged in a square as shown in Figure 136. Two of the rods carry a total positive charge Q, while two of the rods carry a negative charge $-Q$. What is the electric field vector at the centre of the square (use the given coordinate system)?

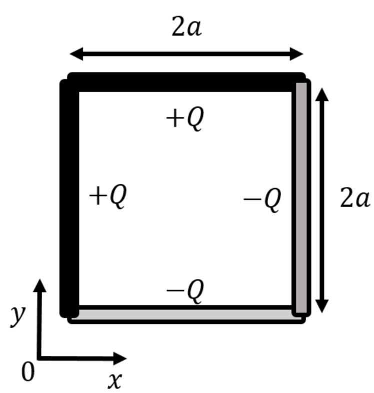

Figure 136: Four uniformly charged rods arranged in a square.

414. Two uniformly electrically charged rods of length $2a$ are arranged perpendicular to each other with their ends touching, as shown in Figure 138. One rod carries a positive charge Q, while the other rod carries a negative charge $-Q$. What is the electric field vector at point P (shown) which is a distance a from the midpoint of each of the two rods?

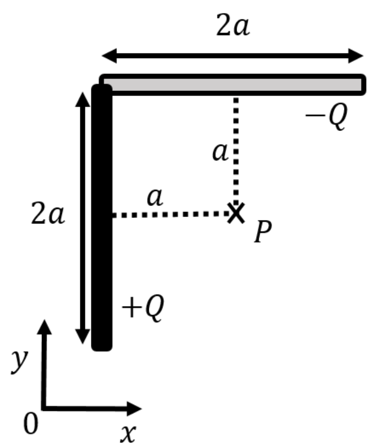

Figure 138: Two uniformly charged rods that are perpendicular to each other.

415. Four uniformly electrically charged rods of length $2a$ are arranged in a square as shown in Figure 140. Three of the rods carry a total positive charge Q, while one of the rods carries a negative charge $-Q$. What is the electric field vector at the centre of the square (use the given coordinate system)?

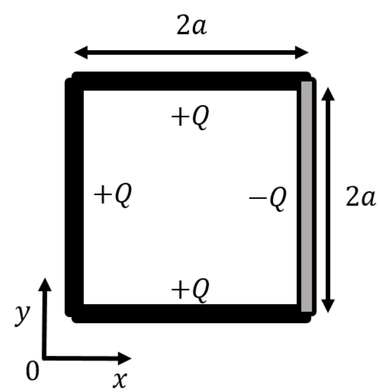

Figure 140: Four uniformly charged rods arranged in a square.

16 Gauss' Law

16.1 Multiple Choice

416. When is flux negative?

- ◯ When the field and normal are in same direction.
- ◯ When the field is perpendicular to normal.
- ◯ When the field and normal are in different direction.
- ◯ When the gaussian surface doesn't enclose any charge.

417. Which of the following closed surfaces will have the greatest net electric flux coming out of it?

- ◯ A sphere of radius R with a point charge Q at its centre.
- ◯ A sphere of radius R that is concentric with a uniformly charged sphere of radius $r < R$ carrying charge Q
- ◯ A sphere of radius $2R$ with a point charge $\frac{Q}{2}$ at its centre.
- ◯ A sphere of radius $\frac{R}{2}$ with a point charge $2Q$ at its centre.

418. A spherical metallic shell with outer radius R encloses a positive point charge $+Q$ at its centre. Which one of the following statements is true?

A. The outer surface of the shell has a net positive charge $+Q$.

B. The electric field inside the shell is zero.

C. The electric field at the surface of the shell has magnitude $E = \frac{kQ}{R^2}$.

D. All of the above are true.

E. None of the above are true.

419. A sphere of radius r has a net charge $+Q$. A spherical metallic shell with no net charge on it and of inner radius $R_1 > r$ and outer radius R_2 is concentric with the small sphere. The outer surface of the metallic shell

- ◯ is neutral
- ◯ is negatively charged
- ◯ is positively charged

420. A positive charge $+Q$ and a negative charge $-2Q$ are placed a certain distance apart from each other. A spherical gaussian surface is centred about the positive charge such that the negative charge is outside of the sphere. What is the total flux of the electric field through the surface?

- ◯ Negative

- ◯ Zero
- ◯ Positive
- ◯ Not enough information to tell

421. A positive charge $+2Q$ and a negative charge $-Q$ are placed a certain distance apart from each other. A spherical gaussian surface is centred about the negative charge such that the positive charge is outside of the sphere. What is the total flux of the electric field through the surface?

    - ◯ Negative
    - ◯ Zero
    - ◯ Positive
    - ◯ Not enough information to tell

422. An amount of charge q is placed on a solid conducting sphere of radius r. What is the magnitude of the electric field at the centre of the sphere?

    - ◯ $\frac{q}{4\pi\epsilon_0 r}$
    - ◯ $\frac{q}{4\pi\epsilon_0 r^2}$
    - ◯ Zero
    - ◯ None of the above

423. A cylindrical Gaussian surface with length L and radius r surrounds an infinite line of positive charge, such that the axis of cylinder is colinear with the liner of charge. The flux Φ through the surface of the cylinder is:

    - ◯ $2\pi r L E$
    - ◯ $\pi r^2 L E$
    - ◯ $2\pi r(L + r)E$
    - ◯ 0
    - ◯ None of the above

424. What statement is not true of conductors in electrostatic equilibrium?

    - ◯ The electric field inside a conductor is zero
    - ◯ The electric field is parallel to the surface of a conductor everywhere on that surface
    - ◯ Any excess charge placed on a conductor resides entirely on the surface of the conductor
    - ◯ All of these statements are true

425. An electric dipole is enclosed by a Gaussian sphere. Under what conditions will the electric flux be zero?

- ○ When the Gaussian sphere is centred on the dipole
- ○ When the Gaussian sphere is not centred on the dipole
- ○ When the Gaussian sphere is very large
- ○ When the Gaussian sphere is very small (but still encloses the dipole)
- ○ All of the above

426. If a charge q is placed in the corner of a cube, how many of the six cube faces have a non-zero flux?

- ○ All Six
- ○ Two
- ○ Three
- ○ All are zero

427. Suppose you have a uniform electric field going through a plane surface of $8.0\,\mathrm{m}^2$. If it has a magnitude of $14\,\mathrm{N/C}$ and the plane in inclined $20°$ relative to the electric field what would be the electric flux through the plane surface?

- ○ $22.7\,\mathrm{Nm^2/C}$
- ○ $105\,\mathrm{Nm^2/C}$
- ○ $98.3\,\mathrm{Nm^2/C}$
- ○ $120\,\mathrm{Nm^2/C}$

428. A point particle with charge q is placed inside a cube but not at its centre. There are no other charges near the cube. The electric flux through any one side of the cube is:

A. Zero

B. $\frac{q}{\epsilon_0}$

C. $\frac{q}{6\epsilon_0}$

D. None of the above

429. An electric dipole is placed at the origin of a coordinate system. A spherical surface of radius R is such that it encloses the dipole completely. Which statement is true?

- ○ There is a net electric flux into the surface
- ○ The electric field magnitude is constant along the surface.
- ○ The net charge enclosed by the surface is zero.
- ○ There is a net electric flux out of the surface

16.2 Long answers

430. A cube with a side length of L lies in a region with a uniform electric field with magnitude, E, such that the electric field is perpendicular to one of the faces of the cube.

    (a) What is the net flux through the cube?

    (b) What is the flux through each of its six faces?

    (c) Now, imagine that we shut off the uniform field, and instead place a charge Q at the center of the cube. What is the flux through one face of the cube?

431. A long cylindrical rod of radius a is made from an insulating material with a uniform charge density ρ. Find the electric field a distance r from the centre of the rod for

    (a) $0 < r < a$

    (b) $r > a$

432. Consider a sphere of radius r_0 which has a sphere of radius $\frac{r_0}{2}$ hollowed out as shown in Figure 142. Suppose the sphere of radius r has a volume charge density of ρ, while the hollowed out section of the sphere carries no charge density. Points A and C are located in the centres of their respective spheres.

    (a) What is the magnitude and direction of the electric field at point A?

    (b) What is the magnitude and direction of the electric field at point B?

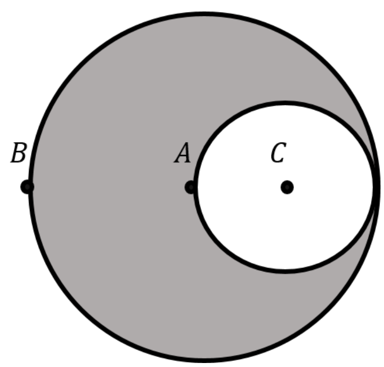

Figure 142: A uniformly charged sphere with a hollowed out cavity.

433. A cube with a side length l is placed such that one corner is at the origin of a coordinate system and extends along the positive x, y, and z axes. What is the charge enclosed by the cube if the electric field in the region is given by $\vec{E}(x, y, z) = (ay + b)\hat{y}$, where a and b are positive constants.

434. A very large plate carries a uniform surface charge density of $\sigma = 0.5 \times 10^{-6}\,\mathrm{C/m^2}$. If you draw equipotentials around the plate, how far must they be spaced so that there is 100 V between equipotentials?

    Hint: Use Gauss' Law to find the electric field above the plate!

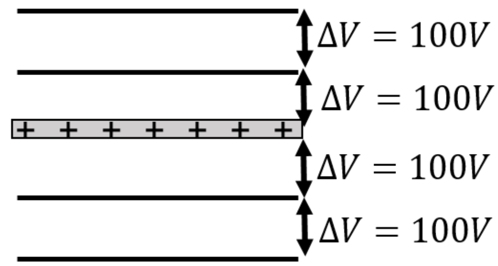

Figure 143: Equipotentials spaced by 100 V *above a uniformly charged plate.*

17 Electric Potential

17.1 Multiple Choice

435. A proton is moving away from a fixed positive charge. Which of the following statements is true?

- ◯ its kinetic energy is decreasing.
- ◯ its potential energy is increasing.
- ◯ its electric potential is decreasing.
- ◯ its electric potential is increasing.

436. An electron is placed at $x = 1\,\text{m}$ in a region where the electric potential is given by $V(x) = (10\,\text{V/m})x$. Which statement is true?

- ◯ The electric field in this region is constant as a function of x and points in the positive x direction
- ◯ The electric field in this region increases linearly with x and points in the negative x direction
- ◯ The electron will move with constant velocity in the negative x direction
- ◯ The electron will move with constant acceleration in the positive x direction

437. Which will have the highest final speed?

- ◯ An electron accelerated from rest across a potential difference of 200 V.
- ◯ A proton accelerated from rest across a potential difference of 200 V.
- ◯ A single ionized Helium atom accelerated from rest across a potential difference of 200 V.
- ◯ All three will have the same speed.

438. A potential difference is applied along a thin metallic strip with a width of $w = 2\,\text{cm}$ and a cross sectional area of $A = 0.1\,\text{cm}^2$, so that a current $I = 2.5\,\text{A}$ goes through the strip, as shown in Figure 144. The strip is made of copper (with a free electron density, $n = 8.48 \times 10^{22}\,\text{cm}^{-3}$) and is immersed in a uniform magnetic field. The maximum Hall voltage across the strip is measured to be $\Delta V_{Hall} = 0.1\,\text{V}$. What is the strength of the magnetic field?

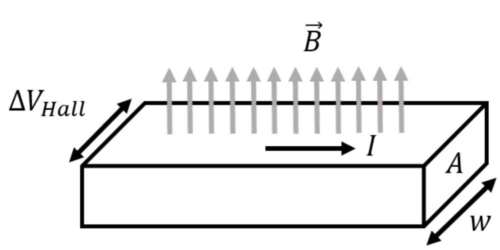

Figure 144: The Hall potential across a Hall probe in a magnetic field.

A. $2.7 \times 10^{-1}\,\mathrm{T}$

B. $2.7 \times 10^{1}\,\mathrm{T}$

C. $2.7 \times 10^{3}\,\mathrm{T}$

D. $2.7 \times 10^{5}\,\mathrm{T}$

439. Which of the following is a valid definition for the electric potential difference between two points in space?

- ◯ The change in potential energy of a charge q moved between those two points and then divided by the charge.
- ◯ The potential energy stored in a system of charged objects due to the charges at those points.
- ◯ The potential energy per one unit charge at that point in space.
- ◯ The difference in electrical potential energy of a charge q between those two points.

440. Two uniformly charged plates placed 1 m apart have a potential difference of 120 V between them. An electron (charge $q_e = -e$, mass m_e) is released from the negative plate. What is the speed of the electron right as it reaches the positive plate?

- ◯ $v = \sqrt{\frac{q_e(120\,\mathrm{V})}{m_e}}$
- ◯ $v = \sqrt{\frac{-q_e(120\,\mathrm{V})}{m_e}}$
- ◯ $v = \sqrt{\frac{-2q_e(120\,\mathrm{V})}{m_e}}$
- ◯ $v = \sqrt{\frac{2q_e(120\,\mathrm{V})}{m_e}}$

441. The voltage between two large parallel plates separated by a distance of 0.5 m is 200 V. The magnitude of the electric field between the plates is:

- ◯ 50 N/C
- ◯ 100 N/C
- ◯ 200 N/C
- ◯ 400 N/C

442. Which statement is true of an electron or proton in a region of space where the only force on the particle is from the electric field?

- ◯ An electron will reduce its electrical potential energy by moving to a region of high electric potential

- ◯ An electron will increase its electrical potential energy by moving to a region of high electric potential
- ◯ Both a proton and an electron will reduce their electrical potential energy by moving to a region of lower electric potential
- ◯ A proton will increase its electrical potential energy by moving to a region of lower electric potential

443. A capacitor stores a charge of 1.25 C and stores a total of 3.5×10^4 J of electric potential energy. What is the electric potential difference between the plates of the capacitor?
   - ◯ 0 V
   - ◯ 5.6×10^4 V
   - ◯ 2.8×10^4 V
   - ◯ 1.12×10^5 V

444. What potential difference is needed to stop an electron that has an initial velocity $v = 4.2 \times 10^6$ m/s?
   - ◯ $\Delta V = 30.1V$
   - ◯ $\Delta V = -50.2V$
   - ◯ $\Delta V = -10.0V$
   - ◯ $\Delta V = 50.2V$

445. The electric field between two parallel plates connected to a 65 V battery is 1750 V/m. How far apart are the plates?
   - ◯ $d = 3.71 \times 10^{-2}$ m
   - ◯ $d = 3.5 \times 10^{-2}$ m
   - ◯ $d = 4.71 \times 10^{-2}$ m
   - ◯ $d = 3.21 \times 10^{-2}$ m

446. How far away from the center of a 1 cm diameter solid metal sphere (which has a -3.00 nC static charge) will the voltage be -270 V if 0 V is defined to be at infinity?
   - ◯ 0.1 cm
   - ◯ 1.0 cm
   - ◯ 10 cm
   - ◯ 100 cm

447. What causes lightning to strike the ground during a thunderstorm?
   - ◯ The electric field generated by the Earth's molten metal core

- ◯ A large potential difference between the Earth's surface and the cloud
- ◯ A high local electron flux
- ◯ Global warming

448. How much energy is given to an electron accelerated through a potential difference of 1000 V?

- ◯ 1000 J
- ◯ 1.6×10^{-16} J
- ◯ 1.6×10^{19} J
- ◯ 1.6×10^{-19} J

449. Consider eight equally spaced locations along the circumference of a circle of radius R. If a charge q is then placed at seven of the eight locations, what is the electric potential at the center of the circle?

- ◯ $k\frac{7q}{R}$
- ◯ $k\frac{q}{8R}$
- ◯ $k\frac{7q}{8R}$
- ◯ $k\frac{q}{7R}$

450. A uniform electric field with strength 100 V/m points to the right. Two points, A and B, are separated by a distance of 10 cm along a line that makes a 60° angle with the field, as shown in Figure 145. What is the electric potential difference, $\Delta V = V_B - V_A$.

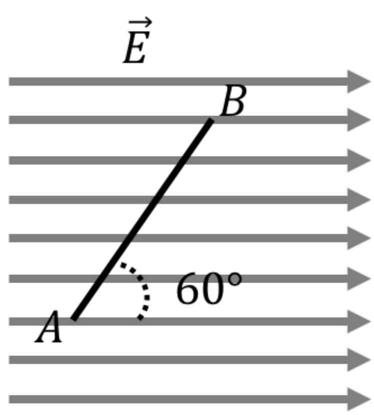

Figure 145: Two points in a uniform electric field.

A. $\Delta V = 5$ V

B. $\Delta V = -5$ V

C. $\Delta V = 10$ V

D. $\Delta V = -10$ V

451. Two charges, q_1 and q_2, are held at a distance d apart. Which statement is correct?

- ○ The magnitude of the stored electrical potential energy is bigger if the charges have the same sign
- ○ The magnitude of the stored electrical potential energy is bigger if the charges have opposite signs
- ○ The magnitude of the stored electrical potential energy is independent of the relative sign of the charges

452. A charge $q_1 = 1.5 \times 10^{-9}\,\mathrm{C}$ is placed on the x-axis at a position $x = 30\,\mathrm{cm}$ and a second charge, $q_2 = -2.5 \times 10^{-9}\,\mathrm{C}$, is placed on the y-axis at $y = 40\,\mathrm{cm}$. If we define electric potential at infinity to be $V = 1\,\mathrm{V}$, what is the electric potential at point $P = (30, 40, 0)\,\mathrm{cm}$?

- ○ -42.25 V
- ○ -40.25 V
- ○ 107.75 V
- ○ 109.75 V

17.2 Long answers

453. A charge, $q = -4.20 \times 10^{-6}\,\mathrm{C}$, is placed at rest at point a. An external force, F, does work, $W_F = 6.00 \times 10^{-4}\,\mathrm{J}$, to accelerate the charge from point a to point b. At point b, the charge has a kinetic energy of $1.7 \times 10^{-4}\,\mathrm{J}$. What is the electric potential difference between points a and b, $\Delta V = V_b - V_a$, if there are no other forces acting on the charge other than then external force and the electric field?

454. The inner conductor of two coaxial cylindrical conductors carries a net positive charge per unit of length λ. Let the radius of the inner conductor equal a and the inner radius of the outer conductor equal b, as shown in Figure 146.

(a) Give an expression for the electric field as a function of radius between the two cylinders, $E(r)$.

(b) Give an expression for the for the potential difference between the two conductors, $V_b - V_a$.

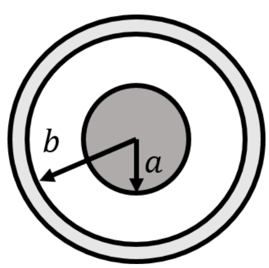

Figure 146: Cross-section of two coaxial cylindrical conductors.

455. A cathode-ray tube is commonly used in television sets to display images. The cathode-ray tube steers electrons (mass $m_e = 9.11 \times 10^{-31}$kg, charge $e = 1.6 \times 10^{-19}$C) towards the screen. Consider a cathode-ray tube with a potential difference of $\Delta V_1 = 5500$V that accelerates an electron from rest. The electron moves through two charged plates with a potential difference of $\Delta V_2 = 250$V. What is the angle θ of the velocity vector once the electron leaves the space between the two plates?

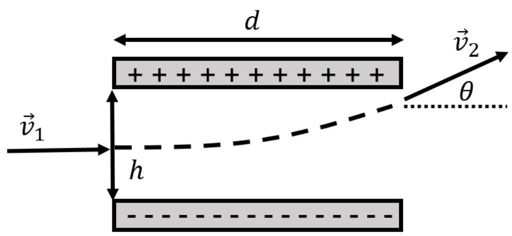

Figure 147: An electron deflected by charged plates.

456. Suppose that the end of your finger is charged after dragging your feet along the carpet.

(a) Estimate the breakdown voltage in air for your finger. Air will breakdown and conduct current if the electric field exceeds 3×10^6 N/C

(b) About what surface charge density would have to be on your finger at this estimated voltage?

457. A Geiger counter is used to detect x-rays and gamma rays. The counter is a gas-filled metal cylinder with a diameter $D = 2.0$ cm. Along the axis of the cylinder, there is a thin wire that is stretched so that it is taut. The wire has a diameter $d = 0.013$ cm. When an x-ray or gamma ray enters the tube, passing through the walls, and ionizes a gas molecule, the electron moves towards the wire, which is biased at a positive potential relative to the cylinder, and the molecule moves towards the cylinder. The electron is accelerated by the field and it ionizes more gas molecules on its way to the wire, causing a pulse that can easily be measured.

(a) If we apply a positive voltage difference ΔV between the wire and the cylinder (the wire is at higher potential, with positive charge on it, the cylinder is at ground, 0 V), show that the electric field between the wire and cylinder, at a radial distance r from the centre of the wire, is given by:

$$\vec{E}(r) = \frac{\Delta V}{r \ln\left(\frac{D}{d}\right)}\hat{r}$$

(b) If 850 V are applied between the wire and the cylinder, what is the magnitude of the electric field just above the surface of the wire?

(c) If 850 V are applied between the wire and the cylinder, what is the charge per unit length on the wire?

458. A solid sphere of radius R_1 carries a total positive charge $+Q$. A thin metallic spherical shell of radius R_2 ($R_2 = 2R_1$) surrounds the sphere ($R_2 > R_1$) and carries a total positive

charge $+2Q$. An electron is placed halfway between the sphere and the shell. What speed will the electron have when it collides with the sphere or the shell, and which object will it collide with?

459. The volume charge density $\rho(r)$ within a sphere of radius R is distributed in accordance with the following spherically symmetric relation:

$$\rho(r) = \rho_0 \left(1 - \frac{r^2}{R^2}\right)$$

where r is measured from the centre of the sphere and ρ_0 is a constant.

(a) Find the electric field as a function of distance r from the centre of the sphere inside and outside of the sphere.

(b) Find the electric potential as a function of distance r from the centre of the sphere inside and outside of the sphere, assuming that the surface of the sphere is at 100 V.

460. A proton makes a head-on elastic collision with a nucleus at rest and rebounds with a speed which is nine-tenths of its initial speed. The mass of the proton is $m = 1.673 \times 10^{-27}$ kg and its charge is $e = 1.6 \times 10^{-19}$ C.

(a) What is the mass of the nucleus?

(b) If the nucleus has a charge +9e and the proton has an initial energy of 1 MeV, what is the distance of closest approach between the proton and nucleus?

461. The electric potential in a region of space is given as:

$$V(x, y, z) = \frac{by}{a^2 + y^2}$$

where a and b are constants.

(a) Give an expression for the electric field vector in that region of space.

(b) If $a = 1$ m and $b = 1$ Vm, plot the magnitude of the electric field and the value of the electric potential between $y = 0$ and $y = 10$ m. Make sure that your plots have labels and units on the axes.

(c) If a charge is placed in this field ($a = 1$ m and $b = 1$ Vm), will there be a point where it is in equilibrium? If yes, where is the equilibrium point and what type of equilibrium is it?

462. A rod made of an insulating material has a length, L, and carries a total charge $+Q$. The rod is placed along the x-axis, such that one end is at the origin and the other end is at a position $x = -L$ (Figure 149).

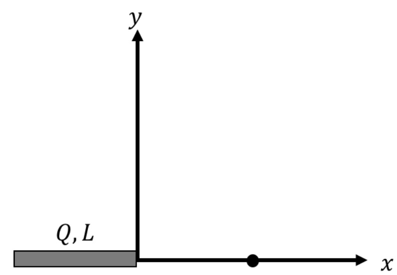

Figure 149: Charged rod of length L carrying charge Q, Question 464.

(a) Give an expression for the electric field vector at a point $x > 0$ along the x-axis, as shown.

(b) How much work is required to bring a positive charge q from infinity and place it at a position $x > 0$ along the x-axis?

Hint: If you need to integrate $\int_a^b (u+c)^n du$, where c is a constant, you can use substitution: for example, let $v = u+c$ (so that $dv = du$), change the limits of the integral, and integrate $\int_{a+c}^{b+c} v^n dv$.

463. You need to build a demonstration of an electric dipole oscillating in a uniform electric field.

(a) In order to create a uniform electric field, you use two very large parallel metal plates that are $d = 5\,\text{cm}$ apart and connect them to a DC generator that can provide $\Delta V = 110\,\text{V}$. How strong will the electric field be between the plates?

(b) You build the dipole using two spherical masses (each with mass $m_s = 1\,\text{g}$ and radius $R = 2.5\,\text{mm}$) connected by a thin rod of mass $m_r = 2\,\text{g}$ and length $L = 1\,\text{cm}$ (the rod is inserted into the spheres, so that the centres of the spheres are 1 cm apart). What is the moment of inertia, I, of the dipole about an axis that goes through the middle of the rod and is perpendicular to the rod?

(c) The centre of the rod connecting the two spheres is held fixed, allowing the dipole to rotate freely about the centre of the rod. For small angles between the dipole vector and the electric field, the electric dipole will undergo simple harmonic motion. What is the magnitude of the charge on each sphere for the frequency of the small angle oscillations of the dipole to be 5 Hz?

464. A semi-infinite rod made of an insulating material has a charge per unit length λ. The rod is placed along the x-axis, such that one end is at the origin and the other end is at negative infinity (Figure 150).

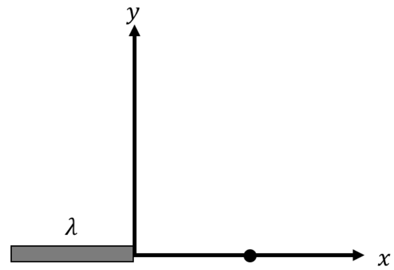

Figure 150: Semi-infinite charged rod, Question 464.

(a) Give an expression for the electric field vector at a point $x > 0$ along the x-axis, as shown.

(b) How much work is required to move a positive charge, q, from a position $x = a$ to a position $x = b$ along the x-axis? Note that both a and b are positive.

Hint: If you need to integrate $\int_a^b (u+c)^n du$, where c is a constant, you can use substitution: for example, let $v = u+c$ (so that $dv = du$), change the limits of the integral, and integrate $\int_{a+c}^{b+c} v^n dv$.

465. An electron (mass, m_e, charge, $-e$) is placed at rest at a distance d from the midpoint of a uniformly negatively charged rod of length L holding total charge $-Q$, as shown in Figure 151. What will be the speed of the electron once it reaches infinity (i.e. an infinite distance away from the rod)?

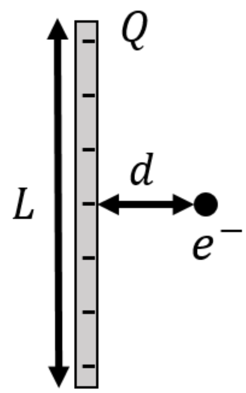

Figure 151: An electron about to be accelerated by a uniformly charged negative rod.

466. An electron is accelerated horizontally from rest by a potential difference of $\Delta V_1 = 5000\,\mathrm{V}$. It then passes between two horizontal plates with a length, $L = 20\,\mathrm{cm}$, and a distance, $d = 2.0\,\mathrm{cm}$, apart that have a potential difference of $\Delta V_2 = 100\,\mathrm{V}$ between them, as shown in Figure 153. What is the vertical displacement of the electron, h, as it exits the region of electric field? Ignore gravity.

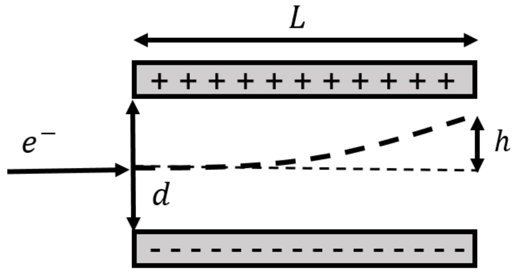

Figure 153: An electron deflected by charged plates.

467. A conducting sphere of radius R_1 carries a total charge Q on its surface. A second, neutral, metallic sphere of radius R_2 is then connected to the first sphere using a conducting wire of negligible dimensions, so that some charge is transferred to the second sphere, as shown in Figure 154. How much charge is left on each sphere after charges have transferred from the sphere of radius R_1 to that or radius R_2? Assume that the two spheres are far enough from each other that the charges on each sphere are distributed uniformly and that only negligible charge is left on the wire. **Hint:** The spheres form an equipotential when connected together.

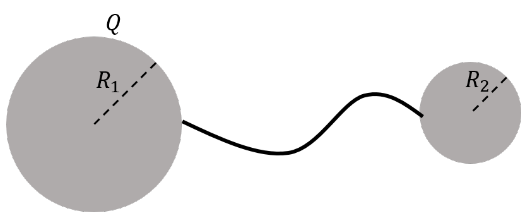

Figure 154: A sphere of radius R_1 initially carrying charge Q is connected to a sphere of radius R_2.

468. A capacitor is made up of one sphere which is enclosed by a thin spherical shell, as shown in Figure 155. The sphere holds a charge of $+Q$ and a radius of r_a, while the spherical shell holds a charge of $-Q$ and has a radius of r_b. What is the capacitance of this capacitor?

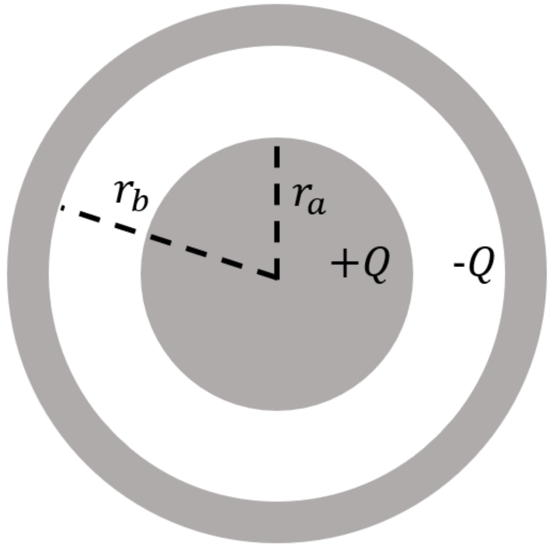

Figure 155: A spherical capacitor

469. A point charge, $Q_1 = 3\,\mathrm{C}$, is fixed in space. Another point charge, $Q_2 = 0.2\,\mathrm{C}$, is held at a distance of $r = 0.5\,\mathrm{m}$ from Q_1. Electric potential is defined to be $0\,\mathrm{V}$ at infinity.

    (a) If Q_1 had no charge, what would the electric potential be at the position of Q_1?

    (b) If Q_2 had no charge, what would the electric potential be at the position of Q_2?

    (c) If Q_2 is released, in what direction will it travel, and what will be the maximum kinetic energy that it has?

18 Electric current

18.1 Multiple Choice

470. If a current carrying wire is replaced with a thicker wire, what will happen?

- ◯ Resistance will decrease.
- ◯ Resistance will increase
- ◯ It will make no difference.
- ◯ Voltage and current will not change.

471. When you stick a fork in an electrical outlet, what is it that can kill you?

- ◯ The voltage drop
- ◯ Your resistivity
- ◯ The current flowing through you
- ◯ The power delivered

472. If resistance of an electric bulb is $500\,\Omega$ and voltage across its ends is $250\,\mathrm{V}$ then power consumed by it is:

- ◯ $130\,\mathrm{W}$
- ◯ $125\,\mathrm{W}$
- ◯ $120\,\mathrm{W}$
- ◯ $200\,\mathrm{W}$

473. What is the name of a material in which electrons are free to move?

- ◯ Resistor
- ◯ Conductor
- ◯ Transistor
- ◯ Insulator

474. What is the resistance of a $60\,\mathrm{W}$ light designed for your $120\,\mathrm{V}$ outlet?

- ◯ $2\,\Omega$
- ◯ $60\,\Omega$
- ◯ $120\,\Omega$
- ◯ $240\,\Omega$

475. A copper wire conducts a current of 1 A when a potential difference of 120 V is applied across its length. If the wire is cut so that it is half as long, how much current will it conduct with the same potential difference across of it?

- ◯ 0.5 A
- ◯ 1 A
- ◯ 2 A
- ◯ 4 A

476. A copper wire consumes 10 W when a potential difference of 120 V is applied across its length. If the wire is cut so that it is half as long, how much power will it consume with the same potential difference across of it?

- ◯ 2.5 W
- ◯ 5 W
- ◯ 10 W
- ◯ 20 W

477. A cylindrical conductor is placed in series with a source of constant current, I, so that current flows along the axis of symmetry of the cylinder. If the radius of the cylinder is reduced by half of its original value, what happens to the current density in the conductor?

- ◯ It increases by a factor of 4.
- ◯ It increases by a factor of 2.
- ◯ It remains the same.
- ◯ It is reduced by a factor of 2.
- ◯ It is reduced by a factor of 4.

18.2 Long answers

478. A time-dependent current in a section of wire is found to be given by:

$$I = a\frac{\ln(bt)}{t^2}$$

where a and b are constants that have a value of 2.

(a) What are the SI units of a and b?

(b) How much charge passes through that section of wire between $t = 1\,\text{s}$ and $t = 5\,\text{s}$

479. Figure 157 shows a 1 A current entering a conical section of wire made of a conducting metal. The electron drift speed at the 3.0 mm diameter end of the cone is measured to be 4.0×10^{-4} m/s. What is the electron drift speed at the 1.0 mm diameter end of the wire?

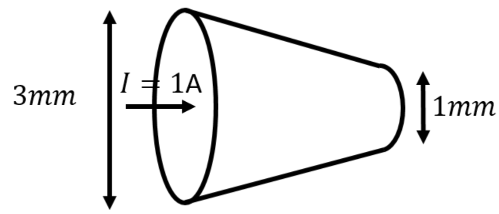

Figure 157: Current flowing through a conical section of wire.

480. A 5.0 mm diameter proton beam carries a total current of 1.0 mA. The current density in the proton beam, which increases with distance from the center, r, is given by:

$$j(r) = J_0 \frac{r}{R}$$

where R is the radius of the beam. What is the value J_0?

481. You have been tasked with designing a resistor has a constant resistance of 2.88 kΩ at all temperatures. You combine a resistor made of carbon ($\alpha = -0.0005$) in series with a resistor made if nichrome ($\alpha = 0.0004$), where α is specified for a reference temperature of $T_0 = 0\,°\mathrm{C}$. What must be the resistance of each resistor be (at 0 °C) to maintain the combined resistance of 2.88 kΩ ?

482. Consider a hollow cylinder with inner radius, r_1, and outer radius, r_2, as shown in Figure 158. The cylinder is constructed with a material that has a resistivity of ρ

   (a) find an expression for the resistance from one end to the other (current flowing axially, along the axis of symmetry)

   (b) find an expression for the resistance between the inner and outer surface (current flowing radially)

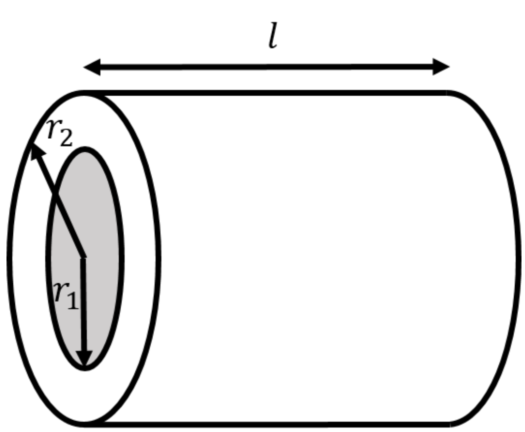

Figure 158: A hollow cylinder of length l.

483. An innovative witch is tired of attempting to boil her cauldron with an open fire. She creates a machine which draws power from a battery to produce heat. The witch is attempting to heat 800 ml of water from 14 °C to 110 °C in 17 minutes. The machine the witch created is 85% efficient at converting electrical power to heat.

(a) How much current does the machine draw from a 12 V battery?

(b) What is the resistance of the witch's machine?

19 Circuits

19.1 Multiple Choice

484. Circuit A has three identical resistors connected in series, whereas circuit B has three resistors (identical to those in circuit A) connected in parallel. Which circuit uses the most power if connected to identical batteries?

- ◯ Circuit A
- ◯ Circuit B
- ◯ Both circuits have the same power
- ◯ Cannot be determined with the information given

485. Referring to Figure 159, which circuit will result in the highest current through the battery?

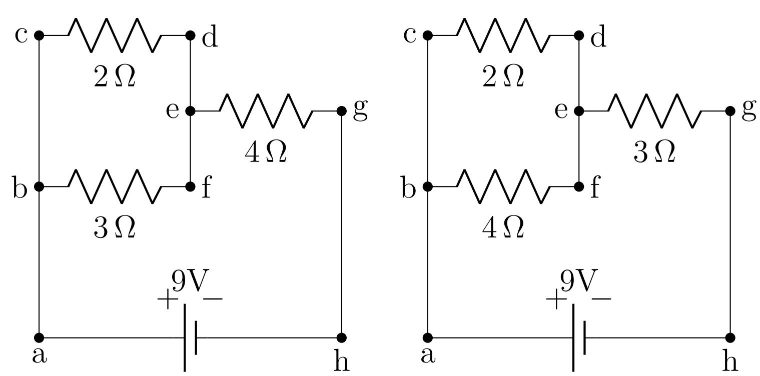

Figure 159: Two different configurations of resistors connected to a battery.

- ◯ The circuit on the left.
- ◯ The circuit on the right.
- ◯ They will both lead to the same current in the battery.
- ◯ It depends on whether this is an ideal battery.

486. Which dissipates the most power?

- ◯ N identical $1\,\Omega$ resistors connected in parallel to a $9\,\text{V}$ battery.
- ◯ N identical $1\,\Omega$ resistors connected in series with a $9\,\text{V}$ battery.
- ◯ Both will dissipate the same power.

487. Which of the following wires has the smallest resistance (all made of the same material)?

- ◯ a wire of length L and diameter $d/2$
- ◯ a wire of length $2L$ and diameter $d/2$
- ◯ a wire of length $2L$ and diameter d

- ◯ a wire of length L and diameter d

488. If a metal wire with radius 0.5 cm and length $L = 2$ m has a resistance of 10 Ω, what is the material's conductivity?

- ◯ $3.9 \times 10^{-4}\,\Omega^{-1}\mathrm{m}^{-1}$
- ◯ $0.25\,\Omega^{-1}\mathrm{m}^{-1}$
- ◯ $3.9\,\Omega^{-1}\mathrm{m}^{-1}$
- ◯ $2.5 \times 10^{3}\,\Omega^{-1}\mathrm{m}^{-1}$

489. A 2 Ω resistor in series with a 4 Ω resistor are in parallel with a 3 Ω resistor. What is the equivalent resistance of these three resistors?

- ◯ 1 Ω
- ◯ 2 Ω
- ◯ 3 Ω
- ◯ 4 Ω

490. Three identical resistors are connected in series to a 12 V battery. What is the voltage across the middle resistor?

- ◯ 3 V
- ◯ 4 V
- ◯ 5 V
- ◯ 6 V

491. A 15 V battery is connected to a 30 Ω resistor in series with a 15 Ω resistor. How much power is dissipated in the resistors?

- ◯ 0.3 W
- ◯ 3 W
- ◯ 5 W
- ◯ 30 W

492. Circuit A has three identical resistors connected in series, whereas circuit B has three resistors (identical to those in circuit A) connected in parallel. Which circuit draws the most current from if connected to identical batteries?

- ◯ Circuit A
- ◯ Circuit B
- ◯ Both circuits draw the same current
- ◯ Cannot be determined with the information given

493. Circuit A has three identical resistors connected in series, whereas circuit B has three resistors (identical to those in circuit A) connected in parallel. In which circuit is the potential difference across any given resistor the greatest, if the circuits are connected to identical batteries?

- ◯ Circuit A
- ◯ Circuit B
- ◯ The potential difference across each resistor is the same in both circuits.
- ◯ Cannot be determined with the information given

494. A 30 W and a 50 W light bulb are designed for use with the same voltage. What is the ratio of the resistance of the 50 W bulb to the resistance of the 30 W bulb ($\frac{R_{50}}{R_{30}}$)?

A. 0.36

B. 0.6

C. 1.7

D. 2.8

495. If two resistors are connected in parallel and the equivalent resistance of those two resistors is $0.5\,\Omega$, which of the following values could not possibly be the resistance of one of the individual resistors?

A. $0.25\,\Omega$

B. $1\,\Omega$

C. $1.5\,\Omega$

D. $2.0\,\Omega$

496. Referring to the circuit in Figure 160, what is the current through the $6\,\Omega$ resistor?

A. 0 A

B. 0.56 A

C. 1.11 A

D. 2.22 A

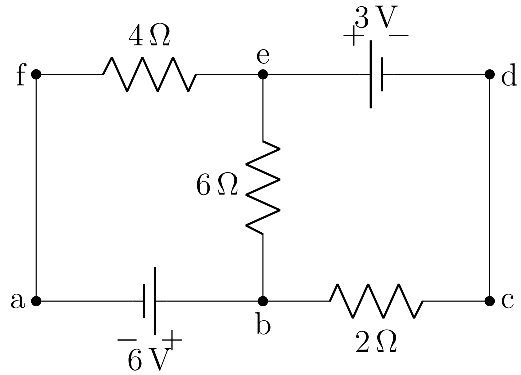

Figure 160: A circuit

497. Referring to Figure 162, what is the resistance between points A and B?

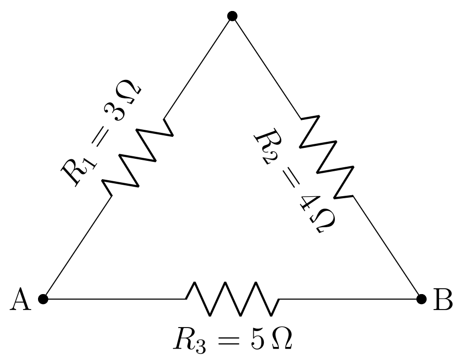

Figure 162: Circuit for Question 497

- ◯ $0.34\,\Omega$
- ◯ $0.44\,\Omega$
- ◯ $2.3\,\Omega$
- ◯ $2.9\,\Omega$

19.2 Long answers

498. In the circuit that is shown in Figure **??**, what is the power dissipated by the $6.0\,\Omega$ resistor?

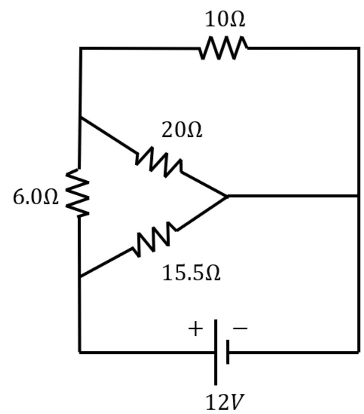

Figure 163: A circuit.

499. In the circuit that is shown in Figure 164, what is the power dissipated by the 4.5 Ω resistor?

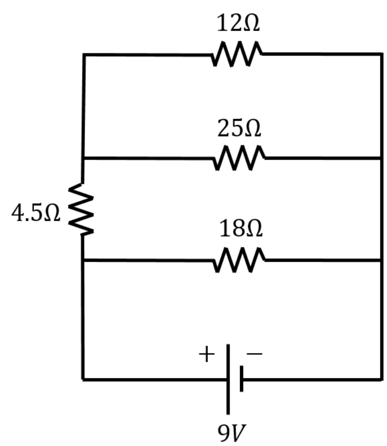

Figure 164: A circuit.

500. In the circuit that is shown in Figure 165, the 4.5 Ω resistor is found to dissipate 2 W. What is the battery voltage, ΔV?

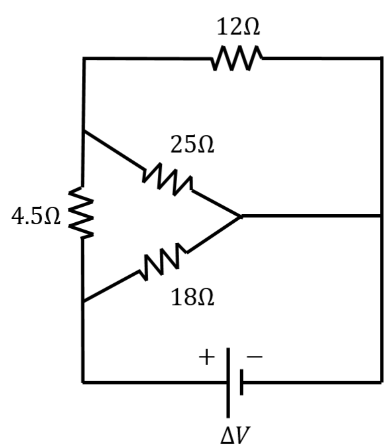

Figure 165: A circuit.

501. Find the currents I_1,I_2, and I_3 in the circuit shown in Figure 166.

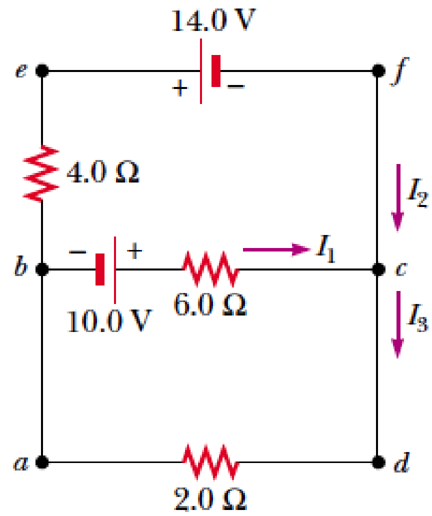

Figure 166: A circuit containing different branches.

502. You are given three identical resistors and each of the resistors has a resistance of $4\,\Omega$. Moreover, each resistor can dissipate at most $20\,\mathrm{W}$. Consider, in turn, the four possible ways of connecting the three resistors together and determine the maximum power that can be dissipated in each configuration without blowing one of the resistors.

503. For the circuit that is shown in Figure 168

    (a) What is the potential difference between points a and d, $V_d - V_a$?

    (b) What is the terminal voltage at each battery? (Note that the lower case r, represent the internal resistance of the batteries)

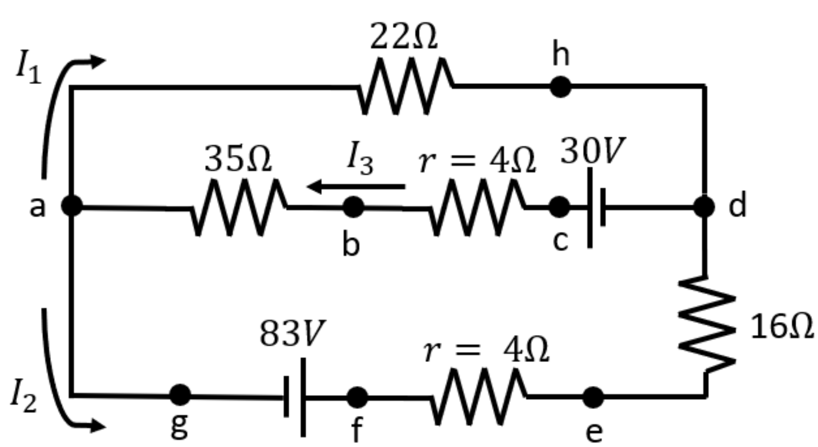

Figure 168: A circuit.

504. You are tasked with changing a milliammeter into a voltmeter. The milliammeter reads 18 mA full scale, and consists of a $0.3\,\Omega$ resistor in parallel with a $41\,\Omega$ galvanometer. You must change the ammeter to a voltmeter which gives a full scale reading of 18 V without disassembling the ammeter. How should you do this?

505. Measurements made on circuits that contain large resistances can be confusing. Consider a circuit powered by a 12 V battery (of negligible internal resistance) with an $8\,\mathrm{M\Omega}$ resistor in series with an unknown resistor, R. As shown in Figure 170, a particular voltmeter reads $V_1 = 0.621\,\mathrm{V}$ when connected across the $8\,\mathrm{M\Omega}$ resistor and $V_2 = 8.552\,\mathrm{V}$ when connected across the unknown resistor. Determine the value of R.

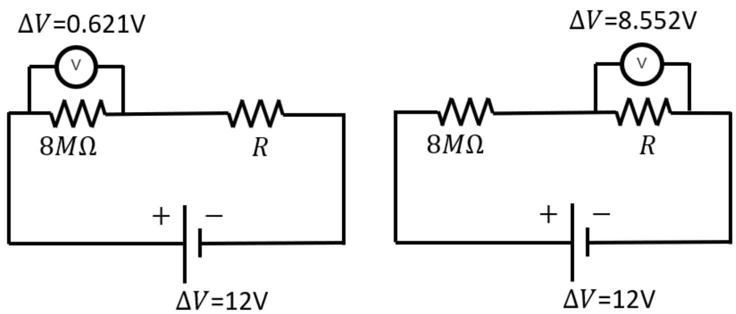

Figure 170: A voltmeter affecting a circuit.

506. Referring to figure 172, what potential difference ΔV is required in order for there to be no current through the $6\,\Omega$ resistor?

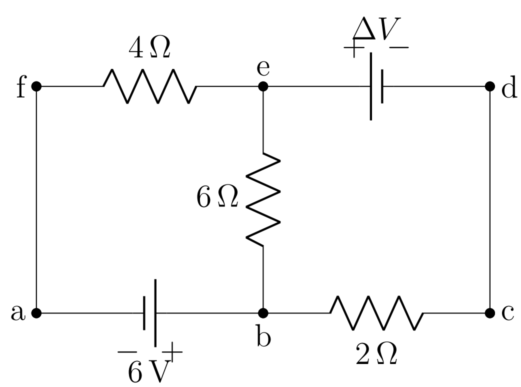

Figure 172: A circuit

507. Referring to figure 174, what is the power dissipated by the $2\,\Omega$ resistor?

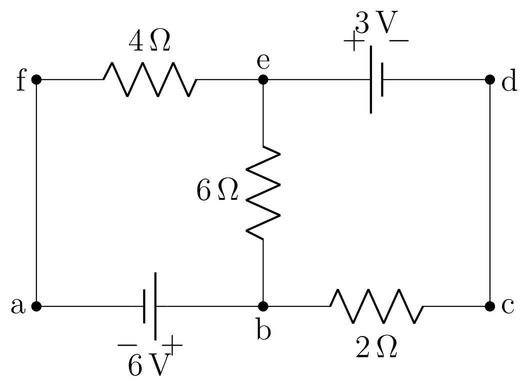

Figure 174: A circuit

508. Referring to figure 176, what is the potential difference between points a and c on the circuit?

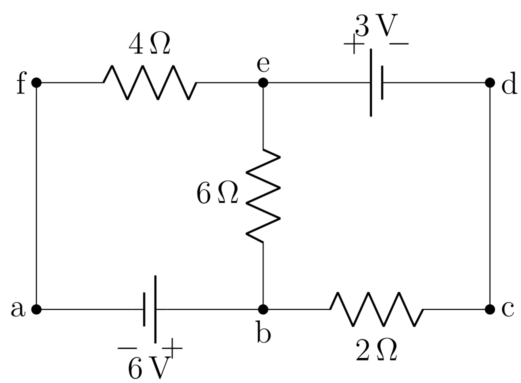

Figure 176: A circuit

509. Answer the following:

    (a) A beam of electrons with a kinetic energy of 10 keV (the electrons were accelerated through a potential difference of 1×10^4 V) is directed at the surface of a conducting metallic sphere of radius $R = 15.0$ cm. The sphere is initially neutral, but acquires charge as the electrons collect on the sphere. Eventually, the charge on the sphere will be strong enough to repel any further electrons. What is the maximum charge that can be deposited on the sphere by the electron beam? Assume that the charge on the sphere is uniform on the surface and that the beam of electrons effectively originates from infinitely far away.

    (b) Referring to the circuit in Figure 178, determine the readings (absolute value) of the voltmeter and ammeter when the **switch S is open**. You may assume that the voltmeter and ammeter are "ideal", in that connecting them to the circuit does not affect the circuit (thus, the voltmeter can be treated as having infinite resistance and the ammeter as having zero resistance).

    (c) Referring to the circuit in Figure 178, determine the readings (absolute value) of the voltmeter and ammeter when the **switch S is closed**. You may again assume that the voltmeter and ammeter are "ideal".

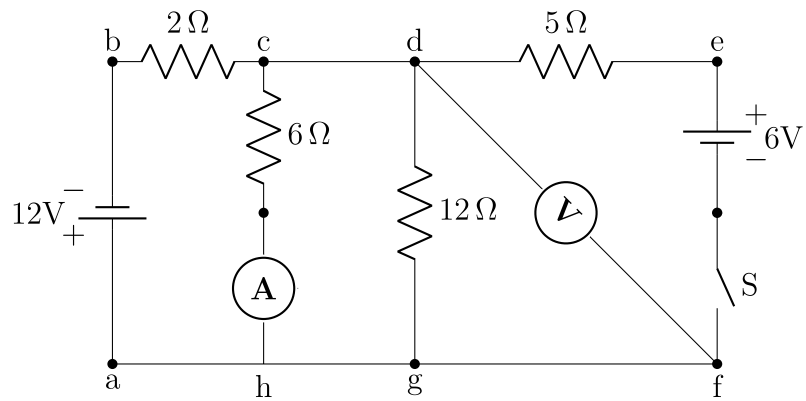

Figure 178: A circuit with an ideal voltmeter and ammeter.

20 Magnetic fields and forces

20.1 Multiple Choice

510. An electron in a magnetic field experiences uniform circular motion. The magnetic field...

- ◯ is zero.
- ◯ points towards the centre of the circle.
- ◯ is perpendicular to the velocity of the electron and the plane which the circle lies on.
- ◯ is perpendicular to the velocty of the electron.

511. Which of the following is FALSE concerning the properties of magnetic field lines?

- ◯ The field lines cannot cross
- ◯ The field strength is proportional to the line density
- ◯ The field is tangent to the magnetic field line
- ◯ The field lines can form open loops

512. Two vertical infinitely long wires are separated by a distance of 1 m. One wire carries an upwards current of 2 A, whereas the other wire caries a downward current of 3 A. What is the strength of the magnetic field midway between the wires?

- ◯ 9.6×10^{-13} T
- ◯ 4.0×10^{-7} T
- ◯ 2.0×10^{-6} T
- ◯ 0.7 T

513. What is the minimum magnitude of the magnetic field required to maintain electrons (mass, $m_e = 9.11 \times 10^{-31}$ kg, and charge $q = 1.6 \times 10^{-19}$ C) with a speed of 1.5×10^{8} m/s in a circular orbit of radius 1.0 m?

- ◯ 1.7×10^{-3} T
- ◯ 8.5×10^{-4} T
- ◯ 5.9×10^{3} T
- ◯ 1.3×10^{5} T

514. An electron is moving West through a magnetic field that points North. The direction of the force on the electron due to that field is:

- ◯ Up
- ◯ Down

- ◯ East
- ◯ South

515. A charged particle in a magnetic field will undergo helical motion if

- ◯ it has component of velocity parallel to the field
- ◯ it has component of velocity perpendicular to the field
- ◯ All of the above
- ◯ None of the above

516. If a moving charged particle enters a region with a magnetic field perpendicular to the direction of motion and begins to follow a curved path, which of the following sets of conditions will result in the largest radius?

- ◯ Mass m, speed v, charge q, magnetic field B
- ◯ Mass $2m$, speed v, charge q, magnetic field $4B$
- ◯ Mass $2m$, speed v, charge $4q$, magnetic field B
- ◯ Mass m, speed $2v$, charge $2q$, magnetic field $2B$

517. Which of the three following statements about magnetic fields is false?

- ◯ Magnetic field lines can never cross one another
- ◯ Magnetic field lines, unlike electric field lines, never end
- ◯ A charged particle travelling anti-parallel to a magnetic field does not experience a force from that field.
- ◯ None of the three statements are false
- ◯ All three statements are false

518. A segment of wire carries current, $I = 0.7\,\mathrm{A}$. One end of the segment is at the origin of the coordinate system and the other end is at a point $P = (20, 30, 10)\,\mathrm{cm}$. The current is flowing from the origin towards point P. A uniform magnetic field, $\vec{B} = (0, 0.5, 0.6)\,\mathrm{T}$, is present. What is the force vector on the wire?

- ◯ $\vec{F} = (0.084, -0.091, 0.07)\,N$
- ◯ $\vec{F} = (0.091, -0.084, 0.07)\,N$
- ◯ $\vec{F} = (-0.084, 0.091, 0.07)\,N$
- ◯ $\vec{F} = (0.091, -0.084, -0.07)\,N$

519. What velocity vector must an electron ($m = 9.11 \times 10^{-31}\,\mathrm{kg}$, $q = -1.6 \times 10^{-19}\,\mathrm{C}$) have in order to perform uniform circular motion with a diameter $D = 0.4\,\mathrm{m}$ after entering a region where the magnetic field is uniform and given by $B = 1.0 \times 10^{-4}\,\mathrm{T}$ in the positive z-direction?

- ◯ $(0, 0, 3.51) \times 10^6\,\mathrm{m/s}$
- ◯ $(0, 7.03, 0) \times 10^6\,\mathrm{m/s}$
- ◯ $(3.51, 0, 0) \times 10^6\,\mathrm{m/s}$
- ◯ $(0, 0, 7.03) \times 10^6\,\mathrm{m/s}$

520. An electron is initially moving in the positive x-direction, when suddenly a uniform magnetic field is turned on. The electron is observed to be deflected in the negative y-direction. In which direction is the magnetic field?

- ◯ positive x-direction
- ◯ negative x-direction
- ◯ positive y-direction
- ◯ negative y-direction
- ◯ positive z-direction
- ◯ negative z-direction

20.2 Long answers

521. A particle with a charge of $q = -1.24 \times 10^{-8}\,\mathrm{C}$ is moving with instantaneous velocity $\vec{v} = (4.19 \times 10^4\,\mathrm{m/s})\hat{x} + (-3.85 \times 10^4\,\mathrm{m/s})\hat{y}$. What is the force vector exerted on this particle by a magnetic field at that instant, if:

    (a) $\vec{B} = (1.40\,\mathrm{T})\hat{x}$

    (b) $\vec{B} = (1.40\,\mathrm{T})\hat{x} + (1.40\,\mathrm{T})\hat{y} + (1.40\,\mathrm{T})\hat{z}$?

522. The magnetic flux through one face of a cube is $0.120\,\mathrm{Wb}$.

    (a) What must be the total magnetic flux through the other five faces of the cube?

    (b) Would this change if it was not a cube, but a rectangular box?

    (c) What is the equivalent of Gauss' Law for the magnetic field?

523. An electron moves at $2.50 \times 10^6\,\mathrm{m/s}$ through a region in which there is a magnetic field of unspecified direction and magnitude $8.40 \times 10^{-2}\,\mathrm{T}$.

    (a) What are the largest and smallest possible magnitude of the acceleration of the electron due to the magnetic field?

    (b) If the actual acceleration of the electron is one-fourth of the largest possible, what is the angle between the direction the electron is moving and the magnetic field?

    (c) For the situation described in part (b), the electron will travel in a helix. What is the radius of the helix, and what is the pitch of the helix (the distance between the "coils" in the helix)?

    Note: The charge and mass of an electron are $q = 1.6 \times 10^{-19}\,\mathrm{C}$ and $m_e = 9.11 \times 10^{-31}\,\mathrm{kg}$.

524. A magician's trick requires the creation of a magnetic dipole moment. The magician decides to create a magic wand of length, d, which has a uniformly distributed static charge, Q, as shown in Figure 181. The magician holds the wand by one of its tips and begins twirling it with an angular velocity of ω. Show that the magnetic dipole moment of the magician's wand is given by:

$$\mu = \frac{1}{6}Q\omega d^2$$

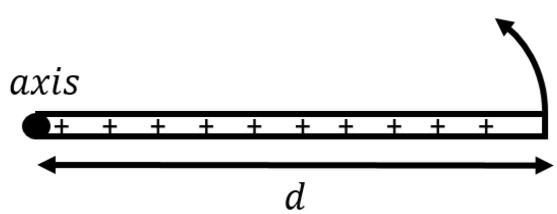

Figure 181: A charged rod rotating about an axis perpendicular to the page.

525. A circular loop of radius r carries a current I. The centre of the circular loop is placed a distance d above a point from which a magnetic field diverges, as shown in Figure 182.

Show that the magnitude of the net force on the circular loop is given by:

$$F = 2\pi IB\frac{r^2}{\sqrt{r^2 + d^2}}$$

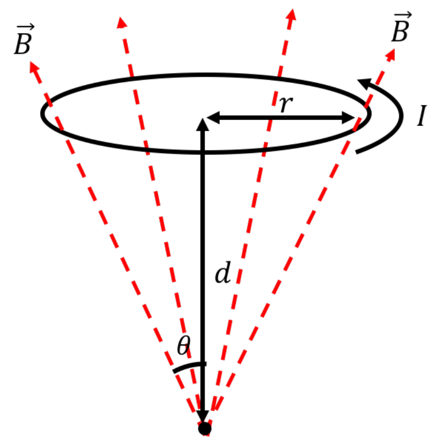

Figure 182: A current carrying ring in a diverging magnetic field.

526. You are tasked with identifying a mysterious particle. In order to narrow down your choices, you decide to determine its mass. To do this, you place the particle in a magnetic field of amplitude $B = 0.034\,\mathrm{T}$ and an electric field of magnitude $E = 1.5\,\mathrm{kV/m}$. When the particle is in this region, it moves in a straight line. You shut the electric field off, and find that the particle begins to move in a circle of radius $r = 2.7\,\mathrm{cm}$. Given this information, what possible particles could this mystery particle be?

527. You have been hired by a hair importer to search for trace amounts of arsenic in guanaco hair destined for the Canadian market. The importer has an old mass spectrometer lying around which you propose to modify in order to measure the presence of arsenic. A schematic of the mass spectrometer is shown in Figure 184.

    A magnet creates a uniform magnetic field of $1\,\mathrm{T}$ in the region shown by the dashed box. A sensor to detect ions is placed at a location to detect only those ions that have a circular path with radius $R = 10\,\mathrm{cm}$. Ions are initially accelerated by two parallel plates through a potential difference ΔV_1. The ions then enter a velocity selector where two plates separated by a distance $D = 1\,\mathrm{cm}$ are held at a potential difference ΔV_2 in the region of magnetic field. After the velocity selector, the ions are forced along a semi-circular path of radius R where they will hit the sensor directly below the point where they exit the velocity selector.

    Arsenic ions have a charge of $-3e$ and a mass of $75\,\mathrm{amu}$ ($1\,\mathrm{amu}=1.66 \times 10^{-27}\,\mathrm{kg}$). Ignore gravity and assume that there are no energy losses due to friction and drag.

    (a) Using the given coordinate system, in which direction must the magnetic field point in order for the arsenic ions to reach the sensor?

    (b) Show that the speed of the ions exiting the velocity selector must be $3.9 \times 10^5\,\mathrm{m/s}$ in order to be detected at the sensor. Calculate the corresponding relativistic factor, γ, and comment on whether effects from special relativity are relevant in this situation.

(c) What is the potential difference, ΔV_2, required so that arsenic ions pass through the velocity selector un-deflected if they are moving with the speed identified in part (b)?

(d) What is the potential difference, ΔV_1, required in order to accelerate the arsenic ions to the speed identified in part (b)?

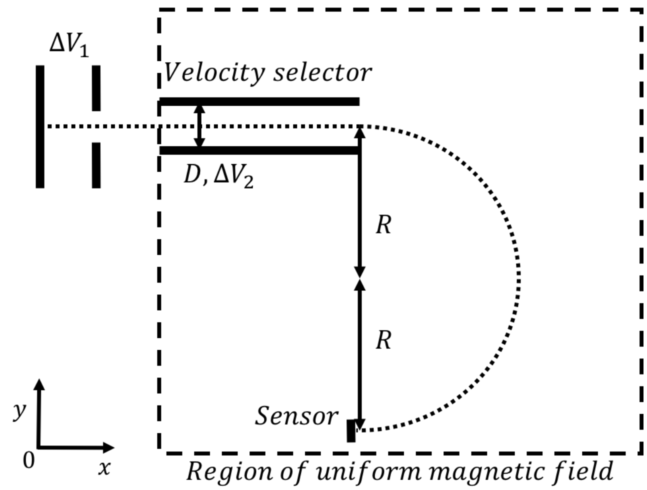

Figure 184: A mass spectrometer

21 Sources of magnetic fields

21.1 Multiple Choice

528. A long, thick wire has a radius of 5 cm and a current of 500 A that runs from West to East along a relatively straight shoreline. If you are walking North towards the beach, what is the strength of the magnetic field that you feel when you are 8 m away from the wire?

- ◯ 2.00×10^{-3} T
- ◯ 1.25×10^{-5} T
- ◯ 2.00×10^{-5} T
- ◯ 1.25×10^{-3} T

529. A generator is made of a single coil with resistance $R = 5\,\Omega$ and area $A = 0.1\,\mathrm{m}^2$ rotated at a frequency of $f = 60$ Hz in a magnetic field of $B = 5.0$ T. What is the maximum torque that needs to be applied to the coil for the generator to rotate at constant speed? Note that the torque required to rotate the generator varies with time, we want to know its maximal value.

A. $\tau = 3.77\,\mathrm{N \cdot m}$

B. $\tau = 18.8\,\mathrm{N \cdot m}$

C. $\tau = 188\,\mathrm{N \cdot m}$

D. $\tau = 471\,\mathrm{N \cdot m}$

530. A rectangular loop of current is moved from a region with no magnetic field into a region where the magnetic field is constant and uniform out of the page, as shown in Figure 185. In which way is the current in the loop induced?

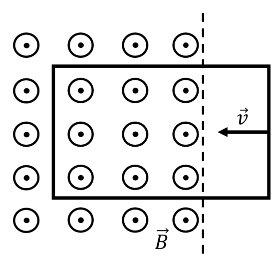

Figure 185: A rectangular loop moving into a region of uniform magnetic field out of the page.

- ◯ counter-clockwise.
- ◯ clockwise.
- ◯ no current is induced.

531. A rectangular loop of current is moved from a region where the magnetic field is constant and uniform out of the page, as shown in Figure 186, to a region with no magnetic field. In which way is the current in the loop induced?

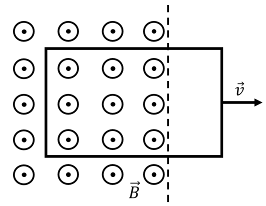

Figure 186: A rectangular loop moving out of a region of uniform magnetic field out of the page.

◯ counter-clockwise.

◯ clockwise.

◯ no current is induced.

532. A rectangular loop of current is located in a region of uniform magnetic field, as shown in Figure 187. If the magnetic field increases with time, in which way is the current in the loop induced?

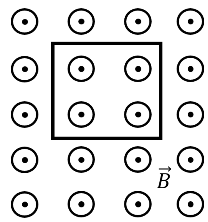

Figure 187: A rectangular loop in a regio of uniform magnetic field out of the page.

◯ counter-clockwise.

◯ clockwise.

◯ no current is induced.

533. Two infinitely long vertical wires are separated by a distance of 1 m. One wire carries an upwards going current of 2 A, while the other carries the same current in the opposite direction. What is the magnitude of the magnetic field half way between the two wires?

◯ 0 T

◯ 8×10^{-7} T

◯ 1.6×10^{-6} T

◯ 3.2×10^{-6} T

534. Two infinitely long vertical wires are separated by a distance of 1 m. Both wires carry an upwards going current of 2 A. What is the magnitude of the magnetic field half way between the two wires?

◯ 0 T

◯ 8×10^{-7} T

◯ 1.6×10^{-6} T

- ◯ 3.2×10^{-6} T

535. Two infinitely long vertical wires are separated by a distance of 1 m. One wire carries an upwards going current of 1 A, while the other carries the same current in the opposite direction. What is the magnitude of the magnetic field half way between the two wires?

- ◯ 0 T
- ◯ 4×10^{-7} T
- ◯ 8×10^{-7} T
- ◯ 1.6×10^{-6} T

536. Two parallel identical wires are beside each other with currents going in opposite directions. The magnetic force between the wires is

- ◯ Attractive
- ◯ Repulsive
- ◯ Zero
- ◯ Cannot be determined

537. Two parallel identical wires are beside each other with currents going in same direction. The magnetic force between the wires is

- ◯ Attractive
- ◯ Repulsive
- ◯ Zero
- ◯ Cannot be determined

538. A straight wire carries current I and results in a magnetic field at a distance r from the wire. What happens to the direction of the magnetic field if the current in the wire is reversed?

- ◯ It is reversed
- ◯ It stays the same
- ◯ It goes to zero

539. A uniform magnetic field B is created by an electric current I flowing through a long solenoid with n turns per unit length and radius R. What is the magnitude of the magnetic field inside the solenoid?

- ◯ $\mu_0 n I R$
- ◯ $\mu_0 n I$
- ◯ $\mu_0 n I R^2$
- ◯ $\mu_0 n I / R$

○ $\mu_0 n I / R^2$

540. A long solenoid of radius R is made of N loops. When a current I goes through the solenoid, the magnetic field at the centre the solenoid has magnitude B. If one doubles the number of loops in the solenoid, what will be the magnetic field at the centre of the solenoid?

A. $\frac{B}{2}$

B. B

C. $2B$

D. or not $2B$, that is the question

21.2 Long answers

541. Two vertical infinite wires are separated by a distance r.

(a) If both wires carry the same positive charge per unit length, λ, show that one wire will feel a repulsive electric force per unit length:

$$\frac{F_E}{l} = \frac{\lambda^2}{2\pi\epsilon_0 r}$$

from the other wire.

(b) Now, suppose that the two wires each carry an upwards current I. Show that the attractive magnetic force per unit length between the wires is:

$$\frac{F_B}{l} = \frac{\mu_0 I^2}{2\pi r}$$

(c) For the case in part (a), assume that you are moving downwards with such a speed that it appears that the charge density on the wire is moving upwards with the same current as in part (b). In part (a), you argued that the wires repelled each other, but now, it would appear that because of the current they might attract (although, the wires still have a net charge on them, so there is also an electrostatic repulsion). Is the force between the wires still the same as in part (a) when you are moving downwards? Explain what is going on! **Hint**: look up special relativity.

542. Two identical rectangular loops, with width 3 m, and height 2 m, as shown in Figure 188, carry a clockwise current, $I = 1$ A. One loop is 1 m directly to the right of the other. Point P is located 1 m away from the left edge of the right loop midway between the horizontal sections. What is the magnetic field vector at point P?

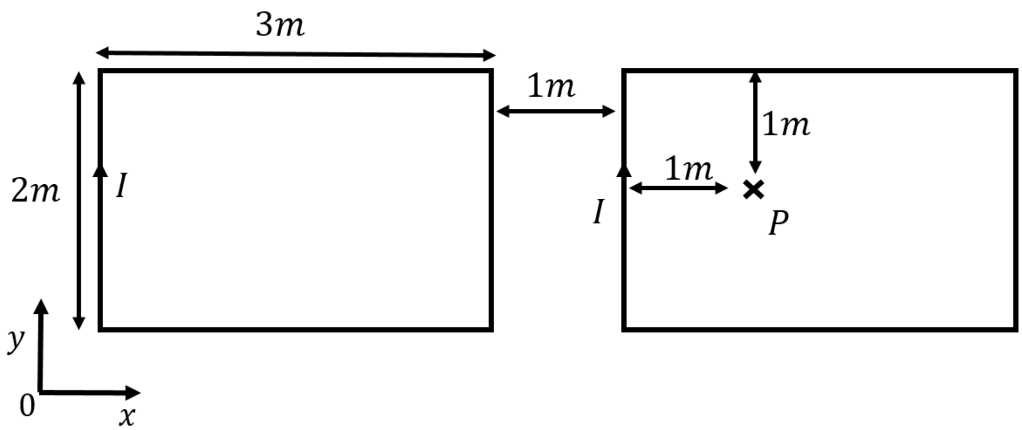

Figure 188: Two identical rectangular wires carrying current I.

543. A wire is shaped into a regular polygon with N sides ($N \geq 3$), that is inscribed in a circle of radius R. If the wire carries current I, show that the magnetic field at the centre of the inscribing circle is:

$$B = N\frac{\mu_0 I}{2\pi R}\tan\left(\frac{\pi}{N}\right)$$

Furthermore, show that in the limit of $N \to \infty$, one recovers the expression for the magnetic field at the centre of a circular loop with current I.

544. Two concentric semi-circles of radii R_1 and R_2 carry currents I_1 and I_2, respectively, as shown in Figure 191. Find an expression for the magnitude and direction of the magnetic field at the centre of the two circles (point P in Figure 191).

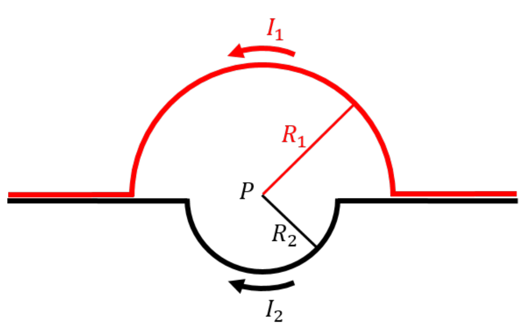

Figure 191: Two current carrying semicircles

545. A 20 cm long and 3 cm in diameter solenoid with 600 turns is aligned along the x axis. The current in its coils is 40 A. A straight wire aligned with the y axis passes directly through the center of the solenoid. The current in the wire is 20 A and travels in the $-y$ direction. What is the force on the section of wire inside the solenoid assuming the magnetic field of the solenoid points in the $+x$ direction?

546. Two very long wires are placed perpendicular to one another. When the wires are closest, they are 15 cm apart, as shown in Figure 193. If the top wire in the figure has a current of 14 A and the bottom wire has a current of 18 A, what is the magnetic field midway between them at their closest point.

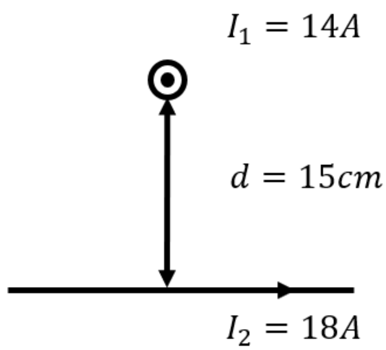

Figure 193: Two perpendicular current carrying wires.

547. A coaxial cable is made up of three shells of radius R_1, R_2, and R_3 respectively. The outermost shell does not carry a current, but the inner most shell and the middle shell carry equal and opposite currents which are uniformly distributed, as shown in Figure 195. Determine the magnetic field a distance r from the axis of the cable for each of the following cases:

   (a) $r < R_1$

   (b) $R_1 < r < R_2$

   (c) $R_2 < r < R_3$

   (d) $R_3 < r$

   (e) Plot the magnitude of the magnetic field as a function of r (choose values for the various quantities).

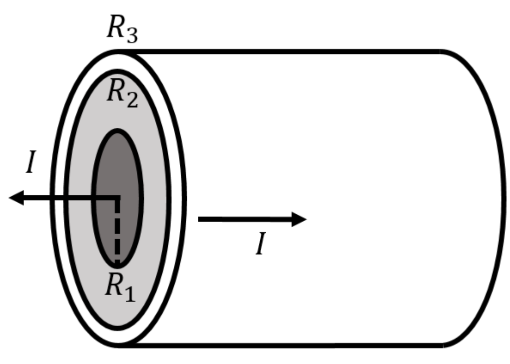

Figure 195: A coaxial wire.

548. The magnetic field at the surface of the Earth is approximately uniform with a magnitude of $B = 25 \times 10^{-6}$ T and points North. If you have a circular loop of wire with radius $r = 20$ cm, what is the minimum frequency with which you must rotate it at the surface of the Earth in order to induce a maximal voltage (emf) of $\Delta = V5$ V across the loop?

549. The magnetic field at the surface of the Earth is approximately uniform with a magnitude of $B = 25 \times 10^{-6}$ T and points North. You have a piece of wire that you shape into a coil

containing N concentric circular loops of radius $r = 20\,\mathrm{cm}$. If you rotate the coil with a frequency of $f = 100\,\mathrm{Hz}$, what is the minimum number of loops, N, required in the coil in order to have a maximal induced voltage (emf) across the coil of $\Delta V = 5\,\mathrm{V}$?

550. Answer the following:

(a) Two infinitely long wires separated by a distance R carry currents I_1 and I_2 as shown in Figure 197. The wire carrying current I_1 has a section that is bent into a semi-circle of radius R and centred at point P (a distance R from the wire carrying current I_2). What is the total magnetic field vector at point P?

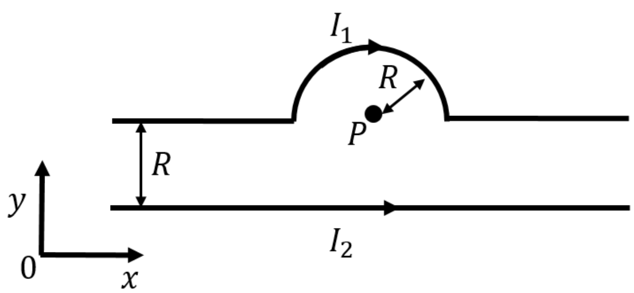

Figure 197: Two infinitely long wires carrying current.

(b) A thin insulating charged disk of radius R carries a total charge $+Q$ spread uniformly over its surface. The disk rotates with a constant angular velocity, ω, as shown in Figure 198, and has a small hole (of negligible dimensions) right at its centre. What is the magnitude of the magnetic field created by the disk inside the centre of the hole? Assume that the thickness of the disk is negligible and that the charge can be treated as a uniform surface charge density $\sigma = \frac{Q}{\pi R^2}$.

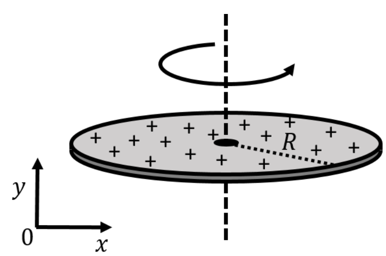

Figure 198: A thin charged rotating disk with a hole at its centre.

22 Electromagnetic induction

22.1 Multiple Choice

551. A loop of wire lies in the $x - y$ plane in a region with a constant magnetic field, with magnitude B, in the positive z direction. Which of the following situations will cause the magnetic flux through the loop to decrease?

   ◯ The loop of wire is moved upwards (positive z direction) at a constant velocity.

   ◯ The loop of wire is moved to the right (positive x direction) at a constant velocity

   ◯ More wire is added to the loop, causing the circumference of the loop to double.

   ◯ The loop of wire is rotated such that it becomes oriented in the $y - z$ plane.

552. Which statement best describes Faraday's law?

   ◯ A change in the magnetic flux across a surface induces a voltage along the loop enclosing that surface

   ◯ A magnetically-induced electric current always creates an opposite force to the one inducing it

   ◯ The total electric flux out of a surface is proportional to the charge enclosed by that surface.

553. A circular loop of radius r and resistance R lies in the xy-plane in a region of uniform time-varying magnetic field given by $\vec{B}(t) = (-2.5\,\mathrm{T/s})t\hat{z}$. In which direction does the magnetic moment of the current induced in the loop point at some time $t > 0$?

   ◯ Positive z

   ◯ Negative z

   ◯ Other

554. A circular loop of radius r and resistance R lies in the xy-plane in a region of uniform time-varying magnetic field given by $\vec{B}(t) = (2.5\,\mathrm{T/s})t\hat{z}$. In which direction does the magnetic moment of the current induced in the loop point at some time $t > 0$?

   ◯ Positive z

   ◯ Negative z

   ◯ Other

555. A circular loop of radius r and resistance R lies in the xy-plane in a region of uniform time-varying magnetic field given by $\vec{B}(t) = (20\,\mathrm{T}) + (2.5\,\mathrm{T/s})t\hat{z}$. In which direction does the magnetic moment of the current induced in the loop point at some time $t > 0$?

   ◯ Positive z

   ◯ Negative z

◯ Other

556. Chloë bought herself a pair of rocket shoes (TODO:reference other example). We want to see if we can use Chloë to charge a cell phone while she rides her rocket shoes. We model Chloë as a thin rigid bar of length $L = 85\,\text{cm}$ that is moving at constant speed $v = 20\,\text{km/h}$ due East (perpendicular to the Earth's magnetic field, assumed to be horizontal in the North direction, $B = 5 \times 10^{-5}\,\text{T}$), as in Figure 200. Since she is moving at constant speed, she does not need to lean forward, so we assume the "Chloë-bar" is vertical. I've attached a wire that runs from her head to her feet from which I will use the generated emf for my cell phone charger. What is the electric potential difference between the ends of the wire?

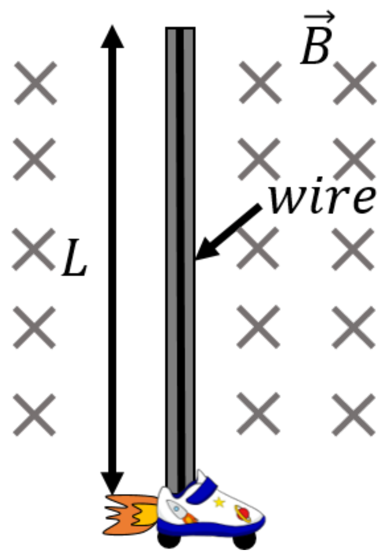

Figure 200: Chloë with a vertical wire.

A. $2.4 \times 10^{-4}\,\text{V}$

B. $4.3 \times 10^{-4}\,\text{V}$

C. $8.5 \times 10^{-4}\,\text{V}$

D. $12.3\,\text{V}$

557. A galvanometer is constructed using a rectangular coil with dimension $2\,\text{cm} \times 3\,\text{cm}$ with $N = 100$ turns. The coil can rotate about an axis that goes through its centre (as in Figure 201). The coil is immersed in a magnetic field that is shaped in such a way that the flux of the field through the coil is constant and has a density per unit area $\sigma_\Phi = 0.02\,\text{Wb/m}^2$ (recall, $1\,\text{Wb} = 1\ \text{T}{\cdot}\text{m}^2$, the unit of magnetic flux). If the coil is rotated by an angle θ, a coil spring provides a restoring torque of $\tau = -\kappa\theta$, where κ is the torsional spring constant of the spring. What should the value of κ be so that a current $I = 1.0 \times 10^{-6}\,\text{A}$ leads to a deflection of $\theta = 45°$?

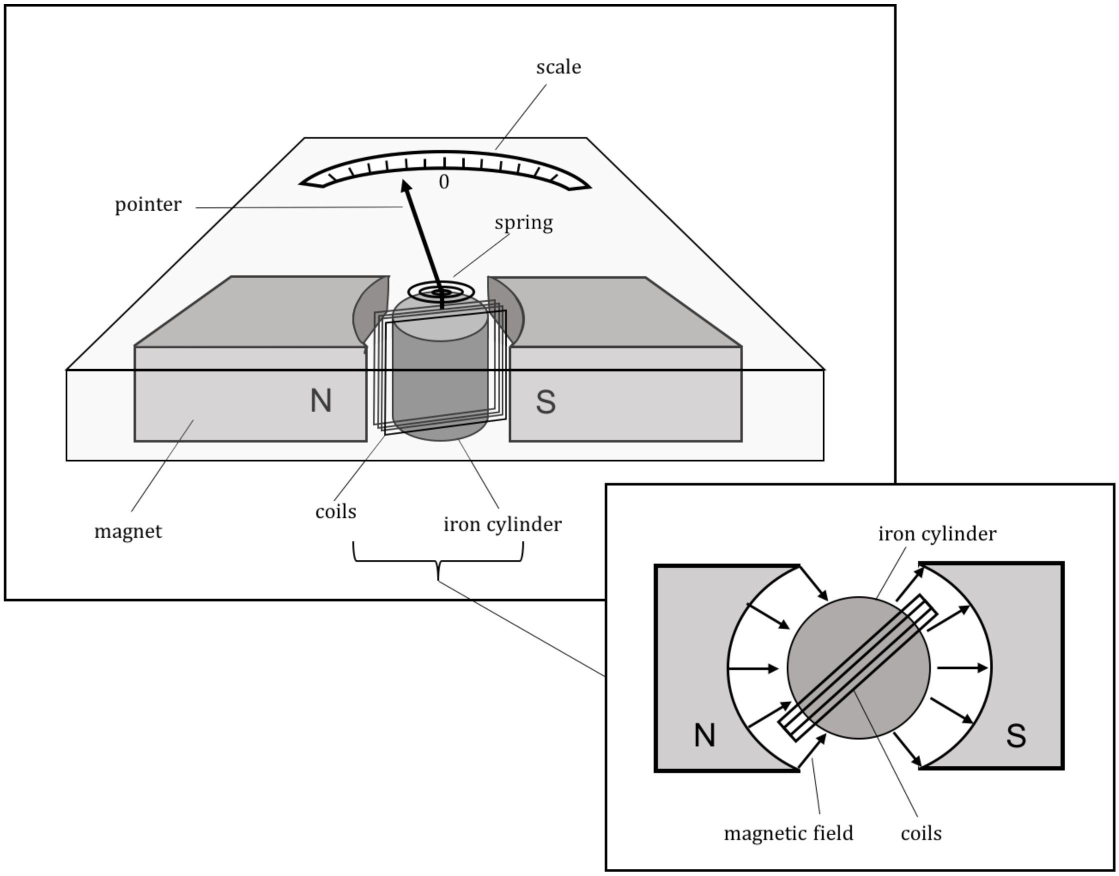

Figure 201: Galvanometer in question 557.

- ◯ 1.53×10^{-9} Nm/rad
- ◯ 1.53×10^{-11} Nm/rad
- ◯ 2.67×10^{-11} Nm/rad
- ◯ 2.67×10^{-13} Nm/rad

558. A magnetic braking system for a car is constructed by attaching a conducting disk (radius R and thickness h) to each car wheel. A portion of each disk is near an electromagnet that can be turned on when one needs to brake. The magnetic field from each electromagnet is perpendicular to the surface of each corresponding disk. For a given car speed and a given set of electromagnets, what happens if we make the disks thicker (h bigger)?

    - ◯ Nothing, the braking torque from the magnetic brakes will be the same.
    - ◯ The braking torque from the magnetic brakes will be reduced.
    - ◯ The braking torque from the magnetic brakes will be increased.

559. An electric generator is made of $N = 100$ coils wrapped around a circle of radius $r = 0.2$ m, and immersed in a magnetic field $B = 0.2$ T. The coils are rotated along an axis that passes through their centre and is perpendicular to the magnetic field (Figure 202). The coils

have negligible resistance and are connected to a heater with total resistance $R = 500\,\Omega$ (the connection to the heater is non-restoring; that is, the current in the heater from the generator is alternating). At what frequency must the coils rotate so that the average power dissipated in the heater is 100 W?

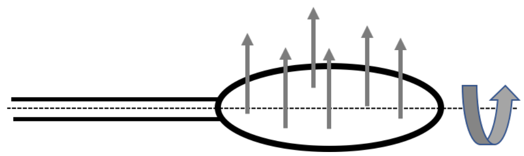

Figure 202: Coils rotating in a magnetic field, question 559.

- ◯ 20 Hz
- ◯ 126 Hz
- ◯ 2002 Hz
- ◯ 12,582 Hz

22.2 Long answers

560. A coil is made of N circular loops of radius $r = 5.0\,\text{cm}$ and has a total resistance $R = 5\,\Omega$. The axis of the coil is in the z direction such that each loop lies in the xy-plane. The coil is in a region of uniform time-varying magnetic field given by:

$$\vec{B}(t) = B_0 + at\hat{z}$$

where $B_0 = 20\,\text{T}$ and $a = 2.5\,\text{T/s}$.

(a) How many loops, N, should the coil have such that the induced voltage across the coil is 120 V (round N to the nearest integer, e.g. $1.5 \to 2$)?

(b) What is the magnitude of the current induced in the coil?

561. A square loop of side a is placed a distance b away from an infinitely long wire carrying current I, as shown in Figure 203

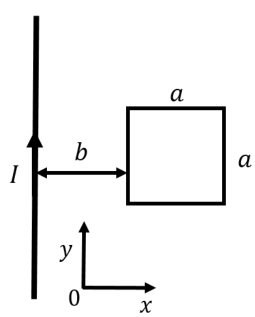

Figure 203: A loop of wire next to a current carrying wire.

(a) Find the magnetic flux through the loop.

(b) If the loop is pulled away from the wire at speed v, what voltage difference is induced across the loop? Give a formula for the voltage as a function of time, $V(t)$, given that $b(t) = b_0 + vt$.

(c) If the loop has total resistance, R, what force, $F(t)$, is required to keep the loop moving at constant speed? Note that the force must do work to pull the loop; that work gets converted into the electric power dissipated in the loop.

562. A right-angle triangular loop of wire is constructed with a base of b and a height h, as shown in Figure 205. The loop is immersed in a uniform magnetic field B which points in the positive z-direction. At time $t = 0$, the loop is in the $x - y$ plane, as shown. The loop is rotated with a constant angular speed $\vec{\omega} = -\omega\hat{y}$ about an axis that is co-linear with the side that lies in the y direction (its height). The loop of wire has a total resistance R.

(a) Write an expression for the current in the loop as a function of time (indicate if the equation is for clockwise or counter-clockwise current, when the loop is viewed at $t = 0$, as in the figure)

(b) What is the maximum value of the current in the loop?

(c) What is the average power dissipated by current in the loop?

(d) What (time-dependent) torque must one exert on the loop in order to keep the loop rotating at constant angular speed? Specify the torque vector that one must exert on the loop and show that it is always in the direction of the angular velocity.

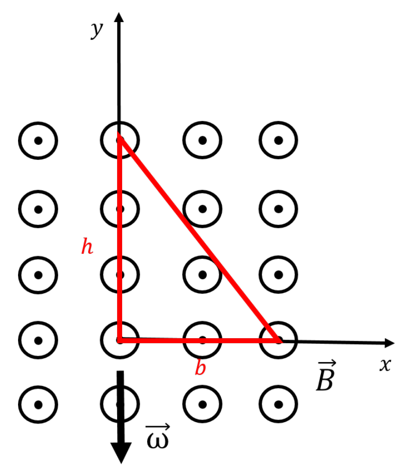

Figure 205: A triangular loop rotating in a magnetic field.

563. A rectangular loop of length $l = 0.60\,\text{m}$, width $w = 0.40\,\text{m}$, and resistance $R = 0.49\,\Omega$ is pulled by a force $\vec{F}$ from a uniform magnetic field $B = 0.80\,\text{T}$, as shown in Figure 207. What force must be applied to the rectangular loop in order for it to maintain a constant speed of $v = 2.15\,\text{m/s}$ while it exits the magnetic field?

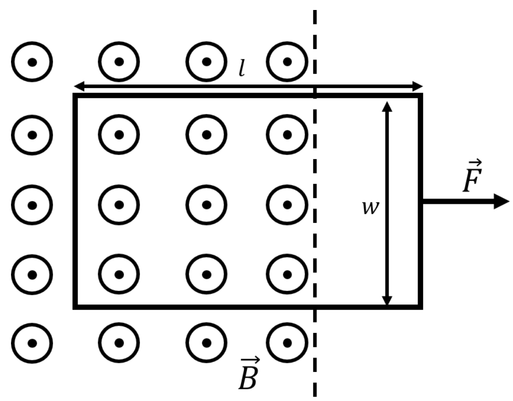

Figure 207: A rectangular loop being pulled out of a magnetic field.

564. A point charge q $= 15\,\mu$C is fixed on the x axis, at a distance of 12 cm from the centre of a region with a uniform magnetic field $\vec{B}$, as shown in Figure 208. The magnetic field begins to decrease at a rate of 0.08 T/s. What is the resulting force vector on the charge?

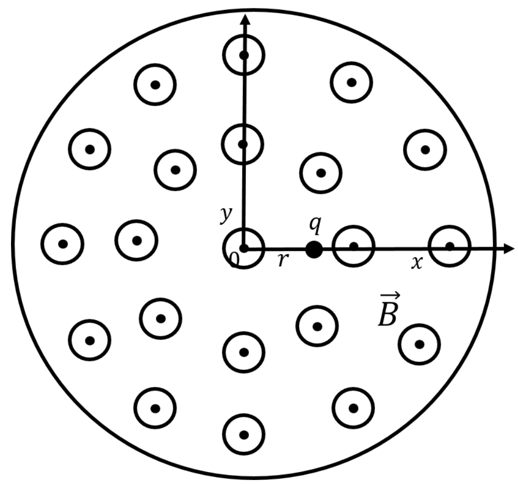

Figure 208: A charge in a region of changing magnetic field.

565. Two superconducting and frictionless rails a distance l apart are connected by a superconducting wire, as shown in Figure 209. A platform of mass m connects the two rails and can slide without friction along the rails. The platform has a resistance R between the points where it makes contact with the rails. The rails are infinitely long and a uniform magnetic field is perpendicular to the plane of the rails and the platform at all points in space. If the platform is put in motion with a speed v in the direction opposite of the superconducting wire that connects the rails, show that it will take a time:

$$t = \ln(2)\frac{mR}{l^2B^2}$$

for the platform to slow down to half of its initial speed.

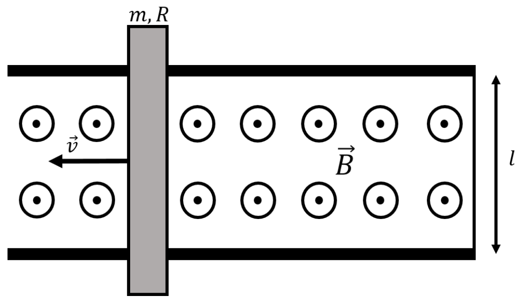

Figure 209: A platform moving along frictionless rails.

566. A generator is made up of a square coil with a 35 cm side and 600 loops. The coil is set to generate a 100 V peak output. If the coil is in a 0.73 T magnetic field, at what frequency must the coil rotate to generate the peak output?

567. A llama farmer has decided to attempt to steal electrical power from the high voltage line that passes above his property by building an induction coil underneath the power line, as in Figure 210. The horizontal power line is a height $d = 20\,\mathrm{m}$ above the ground and carries a sinusoidally time-varying current:

$$I(t) = I_0 \sin(\omega t)$$

with a frequency of 60 Hz and a peak current of $I_0 = 10\,\mathrm{A}$. The farmer builds a vertical induction coil that has a length $L = 15\,\mathrm{m}$ and a height $h = 2\,\mathrm{m}$ with N loops, and that is located directly below the power line, as shown.

(a) Assuming that the power line is very long, show that the magnetic field a distance r from the line is given by:

$$B(r,t) = (2 \times 10^{-6}\,\mathrm{Tm})\frac{1}{r}\sin(\omega t)$$

(b) Show that the magnetic flux from the power line through one loop of the induction coil is given by:

$$\Phi(t) = (3.161 \times 10^{-6}\,\mathrm{Wb})\sin(\omega t)$$

(c) How many loops, N, would the farmer need to put in his coil if he is to generate a maximal potential difference of 170 V in his induction coil (the voltage in the coil will vary sinusoidally and we want the maximum value to be 170 V)? Comment on whether this would be practical.

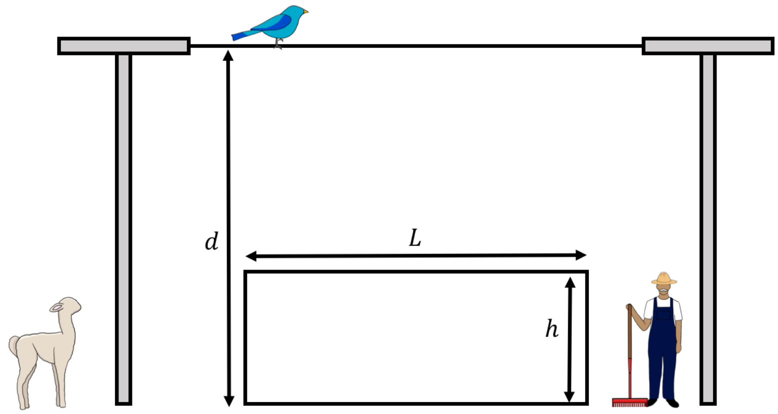

Figure 210: A farmer stealing power.

568. In order to create a region of uniform magnetic field, you use two identical circular coils in the Helmholtz configuration (Figure 211). The coils each have $N = 100$ turns and a radius $R = 20\,\text{cm}$. The centres of the coils are on the z-axis at positions $z = \pm\frac{R}{2}$ and both coils are parallel to the xy-plane. The coils each carry a current, $I = 2\,\text{A}$, such that the magnetic dipole moment of each coil is in the positive z-direction as shown.

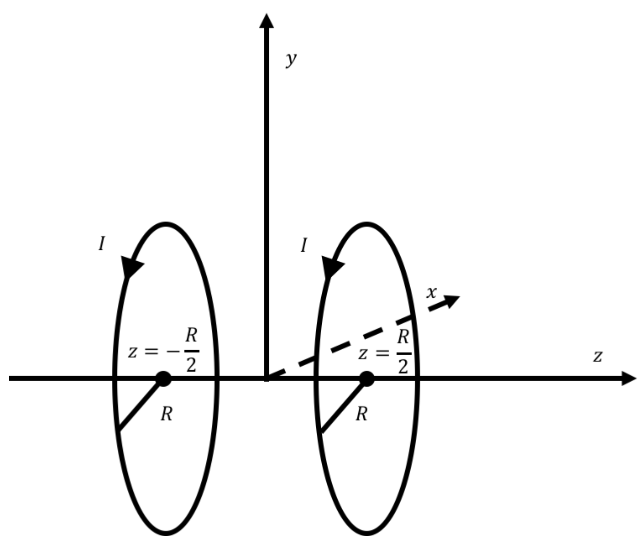

Figure 211: Helmholtz coils (the x-axis is into the page), Question 568a.

(a) Give an expression for the magnetic field along the z-axis and evaluate the strength of the magnetic field at $z = 0$.

(b) A small circular pickup coil with a single turn and a radius $r = 1\,\text{cm}$ is placed with its centre at the origin of the coordinate system parallel to the Helmoltz coils. The pickup coil has a total resistance, $R_{pickup} = 1\,\Omega$, and is rotated at a frequency of 10 Hz about the x-axis (Figure 212, the angular velocity is in the positive x direction). At time $t = 0$, the pickup coil is in the xy-plane. Assume that the magnetic field from the Helmholtz coils is uniform near the pickup coil, with magnitude $B_0 = 9.0 \times 10^{-4}\,\text{T}$.

Make a graph showing the torque that is required to rotate the pickup coil as a function of time. Make your graph as precise as possible (quantitative labels) and make sure it covers at least one rotation period in time.

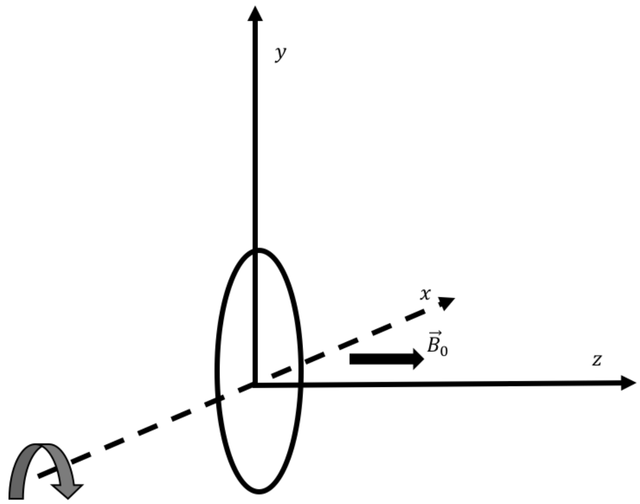

Figure 212: Pickup coil in uniform magnetic field $B_0\hat{z}$ (the x-axis is into the page). The coil rotates about the x-axis, with angular velocity in the positive x direction. Question 568b.

569. You wish to build a little generator for your house using the little stream of water that runs behind it. You construct a coil with $N = 1 \times 10^4$ circular loops of radius $r = 20\,\text{cm}$, which you attach to a paddle wheel so that it can rotate. You then go on ebay to search for a suitable magnet. Assuming that you want to generate AC voltage with a frequency of $60\,\text{Hz}$ and a maximum amplitude of $\Delta V = 110\,\text{V}$, how strong a magnetic field do you need? Assume that the magnetic field is uniform through the volume of the coil.

570. You are part of a research team which is measuring spikes in a magnetic field by measuring the current induced in a circular loop of wire with radius, r, and resistance, R, placed such that the magnetic field is perpendicular to the plane of the loop. A spike in the strength of the magnetic field results in an induced current:

$$I(t) = (-50t + 30)\frac{\pi r^2 e^{-(5t-3)^2}}{R}$$

Given that the magnetic field typically has a constant value of B_0, what time-dependant function best describes the magnetic field during the spike in strength?

23 Special Relativity

23.1 Multiple Choice

571. A train is moving at a speed close to the speed of light relative to the Earth. An observer not moving relative to the Earth will see the train:

 ◯ Become shorter parallel to the direction it is moving.

 ◯ Become longer parallel to the direction it is moving.

 ◯ Become wider perpendicular to the direction it is moving.

 ◯ Become narrower perpendicular to the direction it is moving.

 ◯ No changes will be observed.

572. How fast does a spaceship need to move relative to a reference frame for the spaceship to appear half as long in that reference frame as it does when at rest (we define length to be parallel to the direction of motion)?

 ◯ $\frac{\sqrt{3}}{2}c$

 ◯ $\frac{1}{2}c$

 ◯ $\frac{2}{3}c$

 ◯ $\frac{3}{2}c$

573. In a stationary reference frame on Earth, a 2 m rod is moving at half the speed of light. By how much does its length contract?

 ◯ 27 cm

 ◯ 30 cm

 ◯ 25 cm

 ◯ It does not contract

574. You measure your car to be 5 m long. How fast must you travel in your car for an outside observer to measure your car as being 4 m long?

 A. 0.4 c

 B. 0.6 c

 C. 0.75 c

 D. 0.8 c

575. You are on a platform at a train station, standing halfway between two clocks (A and B) that you observe to be synchronized. The clocks each emit a pulse of light when they strike 9:00am (in your reference frame). Since you are equidistant from the clocks, the pulses of light arrive at your location at the same time. At precisely 9:00am in your reference frame,

a speeding train goes by at nearly the speed of light. The train travels in a direction parallel to the line joining the two clocks, and you observe that the train first passes in front of clock A, then in front of clock B (i.e. the train goes in the direction from A to B). Your physics professor, riding on the train, waves hello. What does your physics professor see?

- ◯ Clock A emitted a pulse of light at the same time as clock B
- ◯ Clock A emitted a pulse of light before clock B
- ◯ Clock A emitted a pulse of light after clock B

23.2 Long answers

576. After getting your spaceship back from the Galactic Federation of Planets' impound lot, you decide to start driving your spaceship at the galactic speed limit. You pass by a Galactic Federation police officer, and they measure that you are travelling at $0.81c$. The Galactic Federation officer measures your spacecraft to have a length of $L = 3.58\,\mathrm{m}$ and a height of $H = 2.40\,\mathrm{m}$. Given that the spaceship is travelling parallel to its length, determine the following:
   (a) What is your spacecraft's length and height at rest?
   (b) How many seconds elapsed on your watch when $15.0\,\mathrm{s}$ passed on the officer's watch?
   (c) How fast did the officer appear to be travelling according you?
   (d) How many seconds elapsed on the officer's watch when they saw $20.0\,\mathrm{s}$ pass on yours?

577. Suppose that an imprecise news report stated that a spaceship from planet Guanaco had just arrived on Earth after travelling for five years at a speed of 0.74c.
   (a) If the report meant 5.0 years of Earth time, how much time elapsed on the ship?
   (b) If the report meant 5.0 years of ship time, how much time elapsed on Earth?

578. A muon has a life time of $2 \times 10^{-6}\,\mathrm{s}$ in its rest frame. It is created in the atmosphere, $100\,\mathrm{km}$ above the Earth and moves down towards the Earth at a speed of $2.97 \times 10^{8}\,\mathrm{m/s}$.
   (a) At what altitude does it decay, if in its rest frame it decayed $2 \times 10^{-6}\,\mathrm{s}$ after being created?
   (b) In the frame of the muon, how far did it travel before decaying?

579. Two rockets of rest length L_0 are approaching the Earth from opposite directions at velocities $\pm$ c/2. How long does one rocket appear to the other?

580. In a frame at rest with respect to a billiard table, two billiard balls (A and B) of the same mass, m, are moving toward each other with the same, very large speed, v. After the collision, the two balls come to rest.
   (a) Show that classical momentum, $\vec{p} = m\vec{v}$, is conserved in the frame of reference of the billiard table.

(b) Now, use relativistic velocity addition to describe the speeds of the billiard balls from the perspective of a frame that is moving with speed v in the direction of the motion of the first ball (ball A). That is, use relativistic velocity addition to place yourself in a frame, S', where ball A appears to be at rest, and transform the velocities (before and after the collision, from part (a)) for each ball before and after the collision, to determine those velocities in frame S'.

(c) Is classical momentum conserved in this moving frame, S'? Use the velocities that you found in part (b) to verify whether classical momentum is conserved, and if not, use your result to describe under what condition it is conserved?

581. The total energy of a proton is found to be three times its rest energy. The mass of a proton is $m_p = 1.67 \times 10^{-27}$ kg.

(a) Express the proton's rest energy in electron volts.

(b) With what speed is the proton moving?

(c) Determine the kinetic energy of the proton in units of MeV.

(d) What is the proton's momentum, in units of MeV/c?