CS 325 Project 3: Linear Programming

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Problem 1

Problem 1 Part A Problem Statement

Determine the number of refrigerators to be shipped from plants to warehouses, and then warehouses to retailers to minimize the cost.

Problem 1 Part A Solution

- In all 1000 units will travel through the network at a minimum cost of $17100.

- Ship 150 units from plant #1 to warehouse #1 at a cost of $1500.
- Ship 200 units from plant #2 to warehouse #1 at a cost of $2200.
- Ship 250 units from plant #2 to warehouse #2 at a cost of $2000.
- Ship 150 units from plant #3 to warehouse #2 at a cost of $1200.
- Ship 100 units from plant #3 to warehouse #3 at a cost of $ 900.
- Ship 150 units from plant #4 to warehouse #3 at a cost of $1200.

- Ship 100 units from warehouse #1 to retailer #1 at a cost of $ 500.

- Ship 150 units from warehouse #1 to retailer #2 at a cost of \$ 900.
- Ship 100 units from warehouse #1 to retailer #3 at a cost of \$ 700.
- Ship 200 units from warehouse #2 to retailer #4 at a cost of \$1600.
- Ship 200 units from warehouse #2 to retailer #5 at a cost of \$2000.
- Ship 150 units from warehouse #3 to retailer #6 at a cost of \$1800.
- Ship 100 units from warehouse #3 to retailer #7 at a cost of \$ 600.

- 150 total units will leave plant #1 (capacity is 150).
- 450 total units will leave plant #2 (capacity is 450).
- 250 total units will leave plant #3 (capacity is 250).
- 150 total units will leave plant #4 (capacity is 150).

- 350 total units will enter warehouse #1, 350 units will leave.
- 400 total units will enter warehouse #2, 400 units will leave.
- 250 total units will enter warehouse #3, 250 units will leave.

- 100 total units will enter retailer #1 (demand is 100).
- 150 total units will enter retailer #2 (demand is 150).
- 100 total units will enter retailer #3 (demand is 100).
- 200 total units will enter retailer #4 (demand is 200).
- 200 total units will enter retailer #5 (demand is 200).
- 150 total units will enter retailer #6 (demand is 150).
- 100 total units will enter retailer #7 (demand is 100).

Problem 1 Part A Linear Program Formulation

1. Overall idea of problem

    - Refrigerators moving from $n = 4$ plants to $q = 3$ warehouses to $m = 7$ retailers.
    - Not all plants deliver to all warehouses.
    - Not all warehouses deliver to all retailers.
    - Costs of shipping from plants to warehouses vary by pair.
    - Costs of shipping from warehouses to retailers vary by pair.
    - Each plant has a capacity in terms of number of refrigerators it can supply.
    - Each retailer has a capacity in terms of number of refrigerators it demands.

2. What is the goal? What are you trying to achieve?

    - Determine optimal shipping routes (n to q and q to m).
    - Determine number of refrigerators moving along each route (n to q and q to m).
    - Satisfy the demand of the retailers.
    - Minimize the cost.

3. Identify variables

    - cp_{ij} = cost of moving a refrigerator between plant i and warehouse j
        - ex. $cp_{32} = 8$ = cost of moving from plant 3 to warehouse 2
        - 9 variables
    - cw_{jk} = cost of moving a refrigerator between warehouse j and retailer k
        - ex. $cp_{14} = 10$ = cost of moving from warehouse 1 to retailer 4
        - 12 variables
    - s_i = capacity (supply) of each plant
        - ex. $s_2 = 450$ = number of refrigerators that plant 2 can supply
        - 4 variables

- d_k = capacity (demand) of each retailer
  - ex. $d_6 = 150$ = number of refrigerators that plant 6 demands
  - 7 variables
- np_{ij} = number of refrigerators shipped from plant i to warehouse j
  - 9 variables
- nw_{jk} = number of refrigerators shipped from warehouse j to retailer k
  - 12 variables

4. Identify constraints

- $s_1 \leq 150$
- $s_2 \leq 450$
- $s_3 \leq 250$
- $s_4 \leq 150$
- $d_1 \geq 100$
- $d_2 \geq 150$
- $d_3 \geq 100$
- $d_4 \geq 200$
- $d_5 \geq 200$
- $d_6 \geq 150$
- $d_7 \geq 100$
- $np_{11} + np_{21} + np_{31} = nw_{11} + nw_{12} + nw_{13} + nw_{14}$
- $np_{12} + np_{22} + np_{32} + np_{42} = nw_{23} + nw_{24} + nw_{25} + nw_{26}$
- $np_{33} + np_{43} = nw_{34} + nw_{35} + nw_{36} + nw_{37}$
- $s_1 = np_{11} + np_{12}$
- $s_2 = np_{21} + np_{22}$
- $s_3 = np_{31} + np_{32} + np_{33}$
- $s_4 = np_{42} + np_{43}$
- $d_1 = nw_{11}$

- $d_2 = nw_{12}$
- $d_3 = nw_{13} + nw_{23}$
- $d_4 = nw_{14} + nw_{24} + nw_{34}$
- $d_5 = nw_{25} + nw_{35}$
- $d_6 = nw_{26} + nw_{36}$
- $d_7 = nw_{37}$
- $np_{11} \geq 0$
- $np_{12} \geq 0$
- $np_{21} \geq 0$
- $np_{22} \geq 0$
- $np_{31} \geq 0$
- $np_{32} \geq 0$
- $np_{33} \geq 0$
- $np_{42} \geq 0$
- $np_{43} \geq 0$
- $nw_{11} \geq 0$
- $nw_{12} \geq 0$
- $nw_{13} \geq 0$
- $nw_{14} \geq 0$
- $nw_{23} \geq 0$
- $nw_{24} \geq 0$
- $nw_{25} \geq 0$
- $nw_{26} \geq 0$
- $nw_{34} \geq 0$
- $nw_{35} \geq 0$
- $nw_{36} \geq 0$
- $nw_{37} \geq 0$

5. Identify inputs and outputs that you can control

- np_{ij}
- nw_{jk}
- $cost$

6. Specify all quantities mathematically

   - Many have been defined above already. A few more will be added here.
   - $cost$ = [sum of $(np_{ij} * cp_{ij})$ for all routes between plants and warehouses] + [sum of $(nw_{jk} * cw_{jk})$ for all routes between warehoues and retailers]
   - $cp_{11} = 10$
   - $cp_{12} = 15$
   - $cp_{21} = 11$
   - $cp_{22} = 8$
   - $cp_{31} = 13$
   - $cp_{32} = 8$
   - $cp_{33} = 9$
   - $cp_{42} = 14$
   - $cp_{43} = 8$
   - $cw_{11} = 5$
   - $cw_{12} = 6$
   - $cw_{13} = 7$
   - $cw_{14} = 10$
   - $cw_{23} = 12$
   - $cw_{24} = 8$
   - $cw_{25} = 10$
   - $cw_{26} = 14$
   - $cw_{34} = 14$
   - $cw_{35} = 12$
   - $cw_{36} = 12$

- $cw_{37} = 6$

7. Check the model for completeness and correctness

   - All variables are positive.

Problem 1 Part A Matlab Code

```
% ------------------------------------------------------------
% reference: array index to variable mapping
% ------------------------------------------------------------
% (1) np11
% (2) np12
% (3) np21
% (4) np22
% (5) np31
% (6) np32
% (7) np33
% (8) np42
% (9) np43
% (10) nw11
% (11) nw12
% (12) nw13
% (13) nw14
% (14) nw23
% (15) nw24
% (16) nw25
% (17) nw26
% (18) nw34
% (19) nw35
% (20) nw36
% (21) nw37
% (22) s1
% (23) s2
% (24) s3
% (25) s4
% (26) d1
% (27) d2
% (28) d3
% (29) d4
% (30) d5
% (31) d6
% (32) d7
```

```
% ------------------------------------------------------------
% lower bounds vector
%   note matlab arrays/vectors start at index 1 (not 0)
% ------------------------------------------------------------
lb = zeros(32,1);
lb(26) = 100;   % d1
lb(27) = 150;   % d2
lb(28) = 100;   % d3
lb(29) = 200;   % d4
lb(30) = 200;   % d5
lb(31) = 150;   % d6
lb(32) = 100;   % d7

% ------------------------------------------------------------
% upper bounds vector
%   note matlab arrays/vectors start at index 1 (not 0)
% ------------------------------------------------------------
ub = Inf(32,1);
ub(1)  = 150;   % np11
ub(2)  = 150;   % np12
ub(3)  = 450;   % np21
ub(4)  = 450;   % np22
ub(5)  = 250;   % np31
ub(6)  = 250;   % np32
ub(7)  = 250;   % np33
ub(8)  = 150;   % np42
ub(9)  = 150;   % np43
ub(22) = 150;   % s1
ub(23) = 450;   % s2
ub(24) = 250;   % s3
ub(25) = 150;   % s4

% ------------------------------------------------------------
% linear inequality matrix and vector
%   note matlab arrays/vectors start at index 1 (not 0)
% ------------------------------------------------------------
A = [];
b = [];

% ------------------------------------------------------------
% linear equality matrix and vector
%   note matlab arrays/vectors start at index 1 (not 0)
%   14 equations in 32 variables
% ------------------------------------------------------------
```

```
Aeq = zeros(14, 32);
beq = zeros(14, 1);
%np11 + np21 + np31        = nw11 + nw12 + nw13 + nw14
%np11 + np21 + np31 - nw11 - nw12 - nw13 - nw14 = 0
Aeq(1,[1,3,5,10,11,12,13]) = [1,1,1,-1,-1,-1,-1];
%np12 + np22 + np32 + np42 =               nw23 + nw24 + nw25 + nw26
%np12 + np22 + np32 + np42 - nw23 - nw24 - nw25 - nw26 = 0
Aeq(2,[2,4,6,8,14,15,16,17]) = [1,1,1,1,-1,-1,-1,-1];
%              np33 + np43 =                      nw34 + nw35 + nw36 + nw37
%np33 + np43 - nw34 - nw35 - nw36 - nw37 = 0
Aeq(3,[7,9,18,19,20,21]) = [1,1,-1,-1,-1,-1];
%s1 = np11 + np12
%s1 - np11 - np12 = 0
Aeq(4,[22,1,2]) = [1,-1,-1];
%s2 = np21 + np22
%s2 - np21 - np22 = 0
Aeq(5,[23,3,4]) = [1,-1,-1];
%s3 = np31 + np32 + np33
%s3 - np31 - np32 - np33 = 0
Aeq(6,[24,5,6,7]) = [1,-1,-1,-1];
%s4 =        np42 + np43
%s4 - np42 - np43 = 0
Aeq(7,[25,8,9]) = [1,-1,-1];
%d1 = nw11
%d1 - nw11 = 0
Aeq(8,[26,10]) = [1,-1];
%d2 = nw12
%d2 - nw12 = 0
Aeq(9,[27,11]) = [1,-1];
%d3 = nw13 + nw23
%d3 - nw13 - nw23 = 0
Aeq(10,[28,12,14]) = [1,-1,-1];
%d4 = nw14 + nw24 + nw34
%d4 - nw14 - nw24 - nw34 = 0
Aeq(11,[29,13,15,18]) = [1,-1,-1,-1];
%d5 =        nw25 + nw35
%d5 - nw25 - nw35 = 0
Aeq(12,[30,16,19]) = [1,-1,-1];
%d6 =        nw26 + nw36
%d6 - nw26 - nw36 = 0
Aeq(13,[31,17,20]) = [1,-1,-1];
%d7 =                      nw37
%d7 - nw37 = 0
Aeq(14,[32,21]) = [1,-1];

```

```matlab
% ------------------------------------------------------------
% objective function vector
%   note matlab arrays/vectors start at index 1 (not 0)
% ------------------------------------------------------------
f = zeros(32,1);
f(1)  = 10;    % np11(value in f is cp11)
f(2)  = 15;    % np12(value in f is cp12)
f(3)  = 11;    % np21(value in f is cp21)
f(4)  = 8;     % np22(value in f is cp22)
f(5)  = 13;    % np31(value in f is cp31)
f(6)  = 8;     % np32(value in f is cp32)
f(7)  = 9;     % np33(value in f is cp33)
f(8)  = 14;    % np42(value in f is cp42)
f(9)  = 8;     % np43(value in f is cp43)
f(10) = 5;     % nw11(value in f is cw11)
f(11) = 6;     % nw12(value in f is cw12)
f(12) = 7;     % nw13(value in f is cw13)
f(13) = 10;    % nw14(value in f is cw14)
f(14) = 12;    % nw23(value in f is cw23)
f(15) = 8;     % nw24(value in f is cw24)
f(16) = 10;    % nw25(value in f is cw25)
f(17) = 14;    % nw26(value in f is cw26)
f(18) = 14;    % nw34(value in f is cw34)
f(19) = 12;    % nw35(value in f is cw35)
f(20) = 12;    % nw36(value in f is cw36)
f(21) = 6;     % nw37(value in f is cw37)

% ------------------------------------------------------------
% call solver and obtain solution
% ------------------------------------------------------------
[x fval] = linprog(f,A,b,Aeq,beq,lb,ub);

% ------------------------------------------------------------
% print the optimum shipping routes and min cost
% ------------------------------------------------------------
fileID = fopen('partA.out','w');
fprintf(fileID, '-----------------------------------------------------\n');
fprintf(fileID, 'Project 3 Problem 1 Part A Solution\n');
fprintf(fileID, '-----------------------------------------------------\n');
fprintf(fileID, '\n');
fprintf(fileID, 'Ship %3.0f units from plant #%d to warehouse #%d at ', ...
    'a cost of $%4.0f.\n', x(1), 1, 1, x(1) * f(1));
fprintf(fileID, 'Ship %3.0f units from plant #%d to warehouse #%d at ', ...
    'a cost of $%4.0f.\n', x(2), 1, 2, x(2) * f(2));
fprintf(fileID, 'Ship %3.0f units from plant #%d to warehouse #%d at ', ...
```

```
    'a cost of $%4.0f.\n', x(3), 2, 1, x(3) * f(3));
fprintf(fileID, 'Ship %3.0f units from plant #%d to warehouse #%d at ', ...
    'a cost of $%4.0f.\n', x(4), 2, 2, x(4) * f(4));
fprintf(fileID, 'Ship %3.0f units from plant #%d to warehouse #%d at ', ...
    'a cost of $%4.0f.\n', x(5), 3, 1, x(5) * f(5));
fprintf(fileID, 'Ship %3.0f units from plant #%d to warehouse #%d at ', ...
    'a cost of $%4.0f.\n', x(6), 3, 2, x(6) * f(6));
fprintf(fileID, 'Ship %3.0f units from plant #%d to warehouse #%d at ', ...
    'a cost of $%4.0f.\n', x(7), 3, 3, x(7) * f(7));
fprintf(fileID, 'Ship %3.0f units from plant #%d to warehouse #%d at ', ...
    'a cost of $%4.0f.\n', x(8), 4, 2, x(8) * f(8));
fprintf(fileID, 'Ship %3.0f units from plant #%d to warehouse #%d at ', ...
    'a cost of $%4.0f.\n', x(9), 4, 3, x(9) * f(9));
fprintf(fileID, '\n');
fprintf(fileID, 'Ship %3.0f units from warehouse #%d to retailer #%d ', ...
    'at a cost of $%4.0f.\n', x(10), 1, 1, x(10) * f(10));
fprintf(fileID, 'Ship %3.0f units from warehouse #%d to retailer #%d ', ...
    'at a cost of $%4.0f.\n', x(11), 1, 2, x(11) * f(11));
fprintf(fileID, 'Ship %3.0f units from warehouse #%d to retailer #%d ', ...
    'at a cost of $%4.0f.\n', x(12), 1, 3, x(12) * f(12));
fprintf(fileID, 'Ship %3.0f units from warehouse #%d to retailer #%d ', ...
    'at a cost of $%4.0f.\n', x(13), 1, 4, x(13) * f(13));
fprintf(fileID, 'Ship %3.0f units from warehouse #%d to retailer #%d ', ...
    'at a cost of $%4.0f.\n', x(14), 2, 3, x(14) * f(14));
fprintf(fileID, 'Ship %3.0f units from warehouse #%d to retailer #%d ', ...
    'at a cost of $%4.0f.\n', x(15), 2, 4, x(15) * f(15));
fprintf(fileID, 'Ship %3.0f units from warehouse #%d to retailer #%d ', ...
    'at a cost of $%4.0f.\n', x(16), 2, 5, x(16) * f(16));
fprintf(fileID, 'Ship %3.0f units from warehouse #%d to retailer #%d ', ...
    'at a cost of $%4.0f.\n', x(17), 2, 6, x(17) * f(17));
fprintf(fileID, 'Ship %3.0f units from warehouse #%d to retailer #%d ', ...
    'at a cost of $%4.0f.\n', x(18), 3, 4, x(18) * f(18));
fprintf(fileID, 'Ship %3.0f units from warehouse #%d to retailer #%d ', ...
    'at a cost of $%4.0f.\n', x(19), 3, 5, x(19) * f(19));
fprintf(fileID, 'Ship %3.0f units from warehouse #%d to retailer #%d ', ...
    'at a cost of $%4.0f.\n', x(20), 3, 6, x(20) * f(20));
fprintf(fileID, 'Ship %3.0f units from warehouse #%d to retailer #%d ', ...
    'at a cost of $%4.0f.\n', x(21), 3, 7, x(21) * f(21));
fprintf(fileID, '\n');
fprintf(fileID, '%3.0f total units will leave plant #%d (capacity is ', ...
    '%3.0f).\n', x(1) + x(2), 1, x(22));
fprintf(fileID, '%3.0f total units will leave plant #%d (capacity is ', ...
    '%3.0f).\n', x(3) + x(4), 2, x(23));
fprintf(fileID, '%3.0f total units will leave plant #%d (capacity is ', ...
    '%3.0f).\n', x(5) + x(6) + x(7), 3, x(24));
```

```
fprintf(fileID, '%3.0f total units will leave plant #%d (capacity is ', ...
    '%3.0f).\n', x(8) + x(9), 4, x(25));
fprintf(fileID, '\n');
fprintf(fileID, '%3.0f total units will enter warehouse #%d, %3.0f ', ...
    'units will leave.\n', x(1) + x(3) + x(5), 1, ...
    x(10) + x(11) + x(12) + x(13));
fprintf(fileID, '%3.0f total units will enter warehouse #%d, %3.0f ', ...
    'units will leave.\n', x(2) + x(4) + x(6) + x(8), 2, ...
    x(14) + x(15) + x(16) + x(17));
fprintf(fileID, '%3.0f total units will enter warehouse #%d, %3.0f ', ...
    'units will leave.\n', x(7) + x(9), 3, ...
    x(18) + x(19) + x(20) + x(21));
fprintf(fileID, '\n')
fprintf(fileID, '%3.0f total units will enter retailer #%d (demand ', ...
    'is %3.0f).\n', x(10), 1, x(26));
fprintf(fileID, '%3.0f total units will enter retailer #%d (demand ', ...
    'is %3.0f).\n', x(11), 2, x(27));
fprintf(fileID, '%3.0f total units will enter retailer #%d (demand ', ...
    'is %3.0f).\n', x(12) + x(14), 3, x(28));
fprintf(fileID, '%3.0f total units will enter retailer #%d (demand ', ...
    'is %3.0f).\n', x(13) + x(15) + x(18), 4, x(29));
fprintf(fileID, '%3.0f total units will enter retailer #%d (demand ', ...
    'is %3.0f).\n', x(16) + x(19), 5, x(30));
fprintf(fileID, '%3.0f total units will enter retailer #%d (demand ', ...
    'is %3.0f).\n', x(17) + x(20), 6, x(31));
fprintf(fileID, '%3.0f total units will enter retailer #%d (demand ', ...
    'is %3.0f).\n', x(21), 7, x(32));
fprintf(fileID, '\n')
total = x(22) + x(23) + x(24) + x(25);
fprintf(fileID, 'In all %3.0f units will travel through the network ', ...
    'at a minimum cost of $%5.0f.\n', total, fval);
fclose(fileID);
```

Problem 2

Part A

Determine the combination of ingredients that minimizes calories but meets all nutritional requirements.

i. Formulate the problem as a linear program with an objective function and all constraints.

ii. Determine the optimal solution for the linear program using any software you want. Include a copy of the code/file in the report.

iii. What is the cost of the low calorie salad?

Part B

Veronica realizes that it is also important to minimize the cost associated with the new salad. Unfortunately some of the ingredients can be expensive. Determine the combination of ingredients that minimizes cost.

i. Formulate the problem as a linear program with an objective function and all constraints.

ii. Determine the optimal solution for the linear program using any software you want. Include a copy of the code/file in the report.

iii. How many calories are in the low cost salad?

Part C

Compare the results from part A and B. Veronicas goal is to create a Very Veggie Salad that is both low calorie and low cost. She would like to sell the salad for $5.00 and still have a profit of at least $3.00. However if she can advertise that the salad has under 250 calories then she may be able to sell more.

i. Suggest some possible ways that she select a combination of ingredients that is near optimal for both objectives. This is a type of multi-objective optimization.

ii. What combination of ingredient would you suggest and what is the associated cost and calorie.

iii. Note: There is not one right answer. Discuss how you derived your solution.

Problem 3

a) What are the lengths of the shortest paths from vertex a to all other vertices?

b) If a vertex z is added to the graph for which there is no path from vertex a to vertex z, what will be the result when you attempt to find the lengths of shortest paths as in part a).

c) What are the lengths of the shortest paths from each vertex to vertex m? How can you solve this problem with just one linear program?

d) Suppose that all paths must pass through vertex i. How can you calculate the length of the shortest path from any vertex x to vertex y that pass through vertex i (for all $x, y \in V$)? Calculate the lengths of these paths for the given graph. (Note: for some vertices x and y, it may be impossible to pass through vertex i).