Problem 1 Part A Problem Statement

Determine the number of refrigerators to be shipped from plants to warehouses, and then warehouses to retailers to minimize the cost.

Problem 1 Part A Solution

- In all 1000 units will travel through the network at a minimum cost of $17100.

- Ship 150 units from plant #1 to warehouse #1 at a cost of $1500.
- Ship 200 units from plant #2 to warehouse #1 at a cost of $2200.
- Ship 250 units from plant #2 to warehouse #2 at a cost of $2000.
- Ship 150 units from plant #3 to warehouse #2 at a cost of $1200.
- Ship 100 units from plant #3 to warehouse #3 at a cost of $ 900.
- Ship 150 units from plant #4 to warehouse #3 at a cost of $1200.

- Ship 100 units from warehouse #1 to retailer #1 at a cost of $ 500.
- Ship 150 units from warehouse #1 to retailer #2 at a cost of $ 900.
- Ship 100 units from warehouse #1 to retailer #3 at a cost of $ 700.
- Ship 200 units from warehouse #2 to retailer #4 at a cost of $1600.
- Ship 200 units from warehouse #2 to retailer #5 at a cost of $2000.
- Ship 150 units from warehouse #3 to retailer #6 at a cost of $1800.
- Ship 100 units from warehouse #3 to retailer #7 at a cost of $ 600.

- 150 total units will leave plant #1 (capacity is 150).
- 450 total units will leave plant #2 (capacity is 450).
- 250 total units will leave plant #3 (capacity is 250).

- 150 total units will leave plant #4 (capacity is 150).

- 350 total units will enter warehouse #1, 350 units will leave.
- 400 total units will enter warehouse #2, 400 units will leave.
- 250 total units will enter warehouse #3, 250 units will leave.

- 100 total units will enter retailer #1 (demand is 100).
- 150 total units will enter retailer #2 (demand is 150).
- 100 total units will enter retailer #3 (demand is 100).
- 200 total units will enter retailer #4 (demand is 200).
- 200 total units will enter retailer #5 (demand is 200).
- 150 total units will enter retailer #6 (demand is 150).
- 100 total units will enter retailer #7 (demand is 100).

Problem 1 Part A Linear Program Formulation

1. Overall idea of problem
   - Refrigerators moving from $n = 4$ plants to $q = 3$ warehouses to $m = 7$ retailers.
   - Not all plants deliver to all warehouses.
   - Not all warehouses deliver to all retailers.
   - Costs of shipping from plants to warehouses vary by pair.
   - Costs of shipping from warehouses to retailers vary by pair.
   - Each plant has a capacity in terms of number of refrigerators it can supply.
   - Each retailer has a capacity in terms of number of refrigerators it demands.
2. What is the goal? What are you trying to achieve?

- Determine optimal shipping routes (n to q and q to m).
- Determine number of refrigerators moving along each route (n to q and q to m).
- Satisfy the demand of the retailers.
- Minimize the cost.

3. Identify variables

- cp_{ij} = cost of moving a refrigerator between plant i and warehouse j
  - ex. $cp_{32} = 8$ = cost of moving from plant 3 to warehouse 2
  - 9 variables
- cw_{jk} = cost of moving a refrigerator between warehouse j and retailer k
  - ex. $cp_{14} = 10$ = cost of moving from warehouse 1 to retailer 4
  - 12 variables
- s_i = capacity (supply) of each plant
  - ex. $s_2 = 450$ = number of refrigerators that plant 2 can supply
  - 4 variables
- d_k = capacity (demand) of each retailer
  - ex. $d_6 = 150$ = number of refrigerators that plant 6 demands
  - 7 variables
- np_{ij} = number of refrigerators shipped from plant i to warehouse j
  - 9 variables
- nw_{jk} = number of refrigerators shipped from warehouse j to retailer k
  - 12 variables

4. Identify constraints

- $s_1 \leq 150$
- $s_2 \leq 450$
- $s_3 \leq 250$
- $s_4 \leq 150$

- $d_1 \geq 100$
- $d_2 \geq 150$
- $d_3 \geq 100$
- $d_4 \geq 200$
- $d_5 \geq 200$
- $d_6 \geq 150$
- $d_7 \geq 100$
- $np_{11} + np_{21} + np_{31} = nw_{11} + nw_{12} + nw_{13} + nw_{14}$
- $np_{12} + np_{22} + np_{32} + np_{42} = nw_{23} + nw_{24} + nw_{25} + nw_{26}$
- $np_{33} + np_{43} = nw_{34} + nw_{35} + nw_{36} + nw_{37}$
- $s_1 = np_{11} + np_{12}$
- $s_2 = np_{21} + np_{22}$
- $s_3 = np_{31} + np_{32} + np_{33}$
- $s_4 = np_{42} + np_{43}$
- $d_1 = nw_{11}$
- $d_2 = nw_{12}$
- $d_3 = nw_{13} + nw_{23}$
- $d_4 = nw_{14} + nw_{24} + nw_{34}$
- $d_5 = nw_{25} + nw_{35}$
- $d_6 = nw_{26} + nw_{36}$
- $d_7 = nw_{37}$
- $np_{11} \geq 0$
- $np_{12} \geq 0$
- $np_{21} \geq 0$
- $np_{22} \geq 0$
- $np_{31} \geq 0$
- $np_{32} \geq 0$
- $np_{33} \geq 0$
- $np_{42} \geq 0$

- $np_{43} \geq 0$
- $nw_{11} \geq 0$
- $nw_{12} \geq 0$
- $nw_{13} \geq 0$
- $nw_{14} \geq 0$
- $nw_{23} \geq 0$
- $nw_{24} \geq 0$
- $nw_{25} \geq 0$
- $nw_{26} \geq 0$
- $nw_{34} \geq 0$
- $nw_{35} \geq 0$
- $nw_{36} \geq 0$
- $nw_{37} \geq 0$

5. Identify inputs and outputs that you can control

   - np_{ij}
   - nw_{jk}
   - $cost$

6. Specify all quantities mathematically

   - Many have been defined above already. A few more will be added here.
   - $cost$ = [sum of $(np_{ij} * cp_{ij})$ for all routes between plants and warehouses] + [sum of $(nw_{jk} * cw_{jk})$ for all routes between warehoues and retailers]
   - $cp_{11} = 10$
   - $cp_{12} = 15$
   - $cp_{21} = 11$
   - $cp_{22} = 8$
   - $cp_{31} = 13$

- $cp_{32} = 8$
- $cp_{33} = 9$
- $cp_{42} = 14$
- $cp_{43} = 8$
- $cw_{11} = 5$
- $cw_{12} = 6$
- $cw_{13} = 7$
- $cw_{14} = 10$
- $cw_{23} = 12$
- $cw_{24} = 8$
- $cw_{25} = 10$
- $cw_{26} = 14$
- $cw_{34} = 14$
- $cw_{35} = 12$
- $cw_{36} = 12$
- $cw_{37} = 6$

7. Check the model for completeness and correctness

   - All variables are positive.

Problem 1 Part A Matlab Code

```
% ------------------------------------------------------------
% reference: array index to variable mapping
% ------------------------------------------------------------
% (1) np11
% (2) np12
% (3) np21
% (4) np22
% (5) np31
% (6) np32
% (7) np33
% (8) np42
```

```
% (9) np43
% (10) nw11
% (11) nw12
% (12) nw13
% (13) nw14
% (14) nw23
% (15) nw24
% (16) nw25
% (17) nw26
% (18) nw34
% (19) nw35
% (20) nw36
% (21) nw37
% (22) s1
% (23) s2
% (24) s3
% (25) s4
% (26) d1
% (27) d2
% (28) d3
% (29) d4
% (30) d5
% (31) d6
% (32) d7

% -------------------------------------------------------------
% lower bounds vector
%   note matlab arrays/vectors start at index 1 (not 0)
% -------------------------------------------------------------
lb = zeros(32,1);
lb(26) = 100;   % d1
lb(27) = 150;   % d2
lb(28) = 100;   % d3
lb(29) = 200;   % d4
lb(30) = 200;   % d5
lb(31) = 150;   % d6
lb(32) = 100;   % d7

% -------------------------------------------------------------
% upper bounds vector
%   note matlab arrays/vectors start at index 1 (not 0)
% -------------------------------------------------------------
ub = Inf(32,1);
ub(1)  = 150;   % np11
ub(2)  = 150;   % np12
```

```
ub(3)  = 450;   % np21
ub(4)  = 450;   % np22
ub(5)  = 250;   % np31
ub(6)  = 250;   % np32
ub(7)  = 250;   % np33
ub(8)  = 150;   % np42
ub(9)  = 150;   % np43
ub(22) = 150;   % s1
ub(23) = 450;   % s2
ub(24) = 250;   % s3
ub(25) = 150;   % s4

% ------------------------------------------------------------
% linear inequality matrix and vector
%   note matlab arrays/vectors start at index 1 (not 0)
% ------------------------------------------------------------
A = [];
b = [];

% ------------------------------------------------------------
% linear equality matrix and vector
%   note matlab arrays/vectors start at index 1 (not 0)
%   14 equations in 32 variables
% ------------------------------------------------------------
Aeq = zeros(14, 32);
beq = zeros(14, 1);
%np11 + np21 + np31        = nw11 + nw12 + nw13 + nw14
%np11 + np21 + np31 - nw11 - nw12 - nw13 - nw14 = 0
Aeq(1,[1,3,5,10,11,12,13]) = [1,1,1,-1,-1,-1,-1];
%np12 + np22 + np32 + np42 =               nw23 + nw24 + nw25 + nw26
%np12 + np22 + np32 + np42 - nw23 - nw24 - nw25 - nw26 = 0
Aeq(2,[2,4,6,8,14,15,16,17]) = [1,1,1,1,-1,-1,-1,-1];
%              np33 + np43 =                      nw34 + nw35 + nw36 + nw37
%np33 + np43 - nw34 - nw35 - nw36 - nw37 = 0
Aeq(3,[7,9,18,19,20,21]) = [1,1,-1,-1,-1,-1];
%s1 = np11 + np12
%s1 - np11 - np12 = 0
Aeq(4,[22,1,2]) = [1,-1,-1];
%s2 = np21 + np22
%s2 - np21 - np22 = 0
Aeq(5,[23,3,4]) = [1,-1,-1];
%s3 = np31 + np32 + np33
%s3 - np31 - np32 - np33 = 0
Aeq(6,[24,5,6,7]) = [1,-1,-1,-1];
%s4 =        np42 + np43
```

```
%s4 - np42 - np43 = 0
Aeq(7,[25,8,9]) = [1,-1,-1];
%d1 = nw11
%d1 - nw11 = 0
Aeq(8,[26,10]) = [1,-1];
%d2 = nw12
%d2 - nw12 = 0
Aeq(9,[27,11]) = [1,-1];
%d3 = nw13 + nw23
%d3 - nw13 - nw23 = 0
Aeq(10,[28,12,14]) = [1,-1,-1];
%d4 = nw14 + nw24 + nw34
%d4 - nw14 - nw24 - nw34 = 0
Aeq(11,[29,13,15,18]) = [1,-1,-1,-1];
%d5 =        nw25 + nw35
%d5 - nw25 - nw35 = 0
Aeq(12,[30,16,19]) = [1,-1,-1];
%d6 =        nw26 + nw36
%d6 - nw26 - nw36 = 0
Aeq(13,[31,17,20]) = [1,-1,-1];
%d7 =                      nw37
%d7 - nw37 = 0
Aeq(14,[32,21]) = [1,-1];

% -------------------------------------------------------------
% objective function vector
%   note matlab arrays/vectors start at index 1 (not 0)
% -------------------------------------------------------------
f = zeros(32,1);
f(1)  = 10;    % np11(value in f is cp11)
f(2)  = 15;    % np12(value in f is cp12)
f(3)  = 11;    % np21(value in f is cp21)
f(4)  = 8;     % np22(value in f is cp22)
f(5)  = 13;    % np31(value in f is cp31)
f(6)  = 8;     % np32(value in f is cp32)
f(7)  = 9;     % np33(value in f is cp33)
f(8)  = 14;    % np42(value in f is cp42)
f(9)  = 8;     % np43(value in f is cp43)
f(10) = 5;     % nw11(value in f is cw11)
f(11) = 6;     % nw12(value in f is cw12)
f(12) = 7;     % nw13(value in f is cw13)
f(13) = 10;    % nw14(value in f is cw14)
f(14) = 12;    % nw23(value in f is cw23)
f(15) = 8;     % nw24(value in f is cw24)
f(16) = 10;    % nw25(value in f is cw25)
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```
f(17) = 14;     % nw26(value in f is cw26)
f(18) = 14;     % nw34(value in f is cw34)
f(19) = 12;     % nw35(value in f is cw35)
f(20) = 12;     % nw36(value in f is cw36)
f(21) = 6;      % nw37(value in f is cw37)

% ------------------------------------------------------------
% call solver and obtain solution
% ------------------------------------------------------------
[x fval] = linprog(f,A,b,Aeq,beq,lb,ub);

% ------------------------------------------------------------
% print the optimum shipping routes and min cost
% ------------------------------------------------------------
fileID = fopen('partA.out','w');
fprintf(fileID, '----------------------------------------------------------\n');
fprintf(fileID, 'Project 3 Problem 1 Part A Solution\n');
fprintf(fileID, '----------------------------------------------------------\n');
fprintf(fileID, '\n');
fprintf(fileID, 'Ship %3.0f units from plant #%d to warehouse #%d at ', ...
    'a cost of $%4.0f.\n', x(1), 1, 1, x(1) * f(1));
fprintf(fileID, 'Ship %3.0f units from plant #%d to warehouse #%d at ', ...
    'a cost of $%4.0f.\n', x(2), 1, 2, x(2) * f(2));
fprintf(fileID, 'Ship %3.0f units from plant #%d to warehouse #%d at ', ...
    'a cost of $%4.0f.\n', x(3), 2, 1, x(3) * f(3));
fprintf(fileID, 'Ship %3.0f units from plant #%d to warehouse #%d at ', ...
    'a cost of $%4.0f.\n', x(4), 2, 2, x(4) * f(4));
fprintf(fileID, 'Ship %3.0f units from plant #%d to warehouse #%d at ', ...
    'a cost of $%4.0f.\n', x(5), 3, 1, x(5) * f(5));
fprintf(fileID, 'Ship %3.0f units from plant #%d to warehouse #%d at ', ...
    'a cost of $%4.0f.\n', x(6), 3, 2, x(6) * f(6));
fprintf(fileID, 'Ship %3.0f units from plant #%d to warehouse #%d at ', ...
    'a cost of $%4.0f.\n', x(7), 3, 3, x(7) * f(7));
fprintf(fileID, 'Ship %3.0f units from plant #%d to warehouse #%d at ', ...
    'a cost of $%4.0f.\n', x(8), 4, 2, x(8) * f(8));
fprintf(fileID, 'Ship %3.0f units from plant #%d to warehouse #%d at ', ...
    'a cost of $%4.0f.\n', x(9), 4, 3, x(9) * f(9));
fprintf(fileID, '\n');
fprintf(fileID, 'Ship %3.0f units from warehouse #%d to retailer #%d ', ...
    'at a cost of $%4.0f.\n', x(10), 1, 1, x(10) * f(10));
fprintf(fileID, 'Ship %3.0f units from warehouse #%d to retailer #%d ', ...
    'at a cost of $%4.0f.\n', x(11), 1, 2, x(11) * f(11));
fprintf(fileID, 'Ship %3.0f units from warehouse #%d to retailer #%d ', ...
    'at a cost of $%4.0f.\n', x(12), 1, 3, x(12) * f(12));
fprintf(fileID, 'Ship %3.0f units from warehouse #%d to retailer #%d ', ...
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    'at a cost of $%4.0f.\n', x(13), 1, 4, x(13) * f(13));
fprintf(fileID, 'Ship %3.0f units from warehouse #%d to retailer #%d ', ...
    'at a cost of $%4.0f.\n', x(14), 2, 3, x(14) * f(14));
fprintf(fileID, 'Ship %3.0f units from warehouse #%d to retailer #%d ', ...
    'at a cost of $%4.0f.\n', x(15), 2, 4, x(15) * f(15));
fprintf(fileID, 'Ship %3.0f units from warehouse #%d to retailer #%d ', ...
    'at a cost of $%4.0f.\n', x(16), 2, 5, x(16) * f(16));
fprintf(fileID, 'Ship %3.0f units from warehouse #%d to retailer #%d ', ...
    'at a cost of $%4.0f.\n', x(17), 2, 6, x(17) * f(17));
fprintf(fileID, 'Ship %3.0f units from warehouse #%d to retailer #%d ', ...
    'at a cost of $%4.0f.\n', x(18), 3, 4, x(18) * f(18));
fprintf(fileID, 'Ship %3.0f units from warehouse #%d to retailer #%d ', ...
    'at a cost of $%4.0f.\n', x(19), 3, 5, x(19) * f(19));
fprintf(fileID, 'Ship %3.0f units from warehouse #%d to retailer #%d ', ...
    'at a cost of $%4.0f.\n', x(20), 3, 6, x(20) * f(20));
fprintf(fileID, 'Ship %3.0f units from warehouse #%d to retailer #%d ', ...
    'at a cost of $%4.0f.\n', x(21), 3, 7, x(21) * f(21));
fprintf(fileID, '\n');
fprintf(fileID, '%3.0f total units will leave plant #%d (capacity is ', ...
    '%3.0f).\n', x(1) + x(2), 1, x(22));
fprintf(fileID, '%3.0f total units will leave plant #%d (capacity is ', ...
    '%3.0f).\n', x(3) + x(4), 2, x(23));
fprintf(fileID, '%3.0f total units will leave plant #%d (capacity is ', ...
    '%3.0f).\n', x(5) + x(6) + x(7), 3, x(24));
fprintf(fileID, '%3.0f total units will leave plant #%d (capacity is ', ...
    '%3.0f).\n', x(8) + x(9), 4, x(25));
fprintf(fileID, '\n');
fprintf(fileID, '%3.0f total units will enter warehouse #%d, %3.0f ', ...
    'units will leave.\n', x(1) + x(3) + x(5), 1, ...
    x(10) + x(11) + x(12) + x(13));
fprintf(fileID, '%3.0f total units will enter warehouse #%d, %3.0f ', ...
    'units will leave.\n', x(2) + x(4) + x(6) + x(8), 2, ...
    x(14) + x(15) + x(16) + x(17));
fprintf(fileID, '%3.0f total units will enter warehouse #%d, %3.0f ', ...
    'units will leave.\n', x(7) + x(9), 3, ...
    x(18) + x(19) + x(20) + x(21));
fprintf(fileID, '\n')
fprintf(fileID, '%3.0f total units will enter retailer #%d (demand ', ...
    'is %3.0f).\n', x(10), 1, x(26));
fprintf(fileID, '%3.0f total units will enter retailer #%d (demand ', ...
    'is %3.0f).\n', x(11), 2, x(27));
fprintf(fileID, '%3.0f total units will enter retailer #%d (demand ', ...
    'is %3.0f).\n', x(12) + x(14), 3, x(28));
fprintf(fileID, '%3.0f total units will enter retailer #%d (demand ', ...
    'is %3.0f).\n', x(13) + x(15) + x(18), 4, x(29));
```

```
fprintf(fileID, '%3.0f total units will enter retailer #%d (demand ', ...
    'is %3.0f).\n', x(16) + x(19), 5, x(30));
fprintf(fileID, '%3.0f total units will enter retailer #%d (demand ', ...
    'is %3.0f).\n', x(17) + x(20), 6, x(31));
fprintf(fileID, '%3.0f total units will enter retailer #%d (demand ', ...
    'is %3.0f).\n', x(21), 7, x(32));
fprintf(fileID, '\n')
total = x(22) + x(23) + x(24) + x(25);
fprintf(fileID, 'In all %3.0f units will travel through the network ', ...
    'at a minimum cost of $%5.0f.\n', total, fval);
fclose(fileID);
```