MTH 231 Mock Exam Season 2 Part 1 **Name:** _____
Fall 2018

10/27/2018

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1. Consider a bit string of length 10. In the box below, answer the following questions:

   - How many bit strings of length 10 both begin and end with a 1?
   - How many bit strings with length 10 that consists entirely of 1s, not counting the empty string?
   - How many bit strings of length 10 contain at least three 1s and at least three 0s?

2. Use the pigeonhole principle to show that whenever 25 girls and 25 boys are seated around a circular table there is always a person both whose neighbors are boys.

3. How many solutions are there to the equation:

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 67 \tag{1}$$

where x_1, x_3, and x_5 have to be odd and x_2 and x_4 are even?

4. Using the Pigeonhole Principle, prove that in any graph with two or more vertices there must exist two vertices that have the same degree. (Note: the problem does not assume that the graph is connected. Your proof must work for any graph, even those in which some vertices are isolated.)