

Chapter 2:

(1) Find

$$\lim_{x \rightarrow 1} \sqrt{\frac{19 - x}{x + 1}}.$$

- (a) 1,
- (b) 2,
- (c) 3,
- (d) d.n.e.

(2) Find

$$\lim_{x \rightarrow -3} \frac{2x^2 - 5x - 33}{x^2 - 9}.$$

- (a) $-\frac{17}{6}$,
- (b) $\frac{17}{6}$,
- (c) ∞ ,
- (d) d.n.e.

(3) Find

$$\lim_{x \rightarrow \frac{3\pi}{2}^+} \sec x.$$

- (a) ∞ ,
- (b) $-\infty$,
- (c) 0,
- (d) d.n.e.

(4) Find

$$\lim_{x \rightarrow \infty} \frac{(2-x)(3x^2+1)}{1-x+x^2-x^3}.$$

- (a) 0,
- (b) 3,
- (c) -3,
- (d) d.n.e.

(5) Find

$$\lim_{x \rightarrow 2} \frac{x^4 - 16}{2x^2 - 8x + 8}.$$

- (a) ∞ ,
- (b) $-\infty$,
- (c) 0,
- (d) d.n.e.

(6) Find

$$\lim_{x \rightarrow 1^-} \frac{x^2 + 1}{1 - x^3}.$$

- (a) ∞ ,
- (b) $-\infty$,
- (c) 0,
- (d) d.n.e.

(7) Find

$$\lim_{x \rightarrow -\infty} \sqrt{x^2 + 1} + x.$$

Hint: Rationalize!

- (a) 0,
- (b) ∞ ,
- (c) $-\infty$,
- (d) d.n.e.

(8) Find the following limit:

$$\lim_{x \rightarrow 7} \frac{\frac{1}{x} - \frac{1}{7}}{x - 7}.$$

- (a) 0,
- (b) ∞ ,
- (c) $-\frac{1}{49}$,
- (d) d.n.e.

(9) Let

$$f(x) = \begin{cases} x^2 - 6x + 7 & \text{if } x < 3 \\ 1 - x & \text{if } x \geq 3 \end{cases}$$

Which of the following statements is true.

- (a) $f(x)$ is discontinuous at $x = 3$,
- (b) $f(x)$ is continuous and differentiable at $x = 3$,
- (c) $f(x)$ is differentiable but not continuous at $x = 3$,
- (d) None of the above statements are true.

(10) Find the following limit:

$$\lim_{x \rightarrow -\infty} \frac{1 - 8x^9}{x^5 - 3x^3 + 1}.$$

- (a) $-\infty$,
- (b) ∞ ,
- (c) -8 ,
- (d) 0 .

(11) Find the following limit:

$$\lim_{x \rightarrow \frac{\pi}{4}^+} \left(\frac{\tan x}{\sin x - \cos x} \right)$$

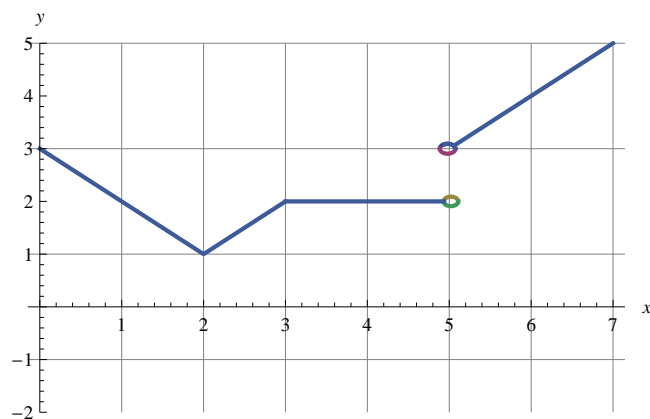
- (a) 0,
- (b) ∞ ,
- (c) $-\infty$,
- (d) d.n.e.

(12) Find the following limit:

$$\lim_{x \rightarrow -\infty} \frac{\sqrt{4x^4 - 88x + 484}}{1 - x^2}.$$

- (a) ∞ ,
- (b) $-\infty$,
- (c) -2 ,
- (d) 2.

Problems 13 and 14 are based on a function f with the graph below.



(13) Where is f continuous? Select the best answer.

- (a) f is continuous on $(0, 2) \cup (2, 3) \cup (3, 5) \cup (5, 7)$.
- (b) f is continuous on $(0, 7)$.
- (c) f is continuous on $(1, 3) \cup (3, 5)$.
- (d) f is continuous on $(0, 5) \cup (5, 7)$.

(14) Find the following limit:

$$\lim_{x \rightarrow 5} f(x)$$

- (a) 10,
- (b) 12,
- (c) 14,
- (d) d.n.e.

(15) Find

$$\lim_{x \rightarrow 2} \ln \left(\frac{x-1}{5-x^2} \right).$$

- (a) 0,
- (b) 1,
- (c) 2,
- (d) d.n.e.

(16) Find

$$\lim_{x \rightarrow -2} \frac{3x^2 + 7x + 2}{4 - x^2}.$$

- (a) $\frac{5}{4}$,
- (b) $-\frac{5}{4}$,
- (c) $-\infty$,
- (d) d.n.e.

(17) Find

$$\lim_{x \rightarrow \frac{\pi}{2}} \cot x.$$

- (a) ∞ ,
- (b) $-\infty$,
- (c) 0,
- (d) d.n.e.

(18) Find

$$\lim_{x \rightarrow -\infty} \frac{(x+1)(x+2)(x+3)}{1-x^4}.$$

- (a) 1,
- (b) -1 ,
- (c) 0,
- (d) d.n.e.

(19) Find

$$\lim_{x \rightarrow -3^-} \frac{x^2 - 9}{x^2 + 6x + 9}.$$

- (a) ∞ ,
- (b) $-\infty$,
- (c) 0,
- (d) d.n.e.

(20) Find

$$\lim_{x \rightarrow 25} \frac{2x - 50}{\sqrt{x} - 5}.$$

Hint: Rationalize!

- (a) 0,
- (b) 10,
- (c) 20,
- (d) d.n.e.

(21) Find the following limit:

$$\lim_{x \rightarrow 9} \frac{\sqrt{x} - 3}{x^2 - 9x}.$$

- (a) 0,
- (b) 6,
- (c) $\frac{1}{54}$,
- (d) d.n.e.

(22) Find

$$\lim_{x \rightarrow -1} \frac{-9x^2}{\sqrt{x+10}}.$$

- (a) 1,
- (b) -1 ,
- (c) 3,
- (d) -3 .

(23) Find

$$\lim_{x \rightarrow 4} \frac{2 - \sqrt{x}}{x - 4}.$$

- (a) $\frac{1}{4}$,
- (b) $-\frac{1}{4}$,
- (c) ∞ ,
- (d) d.n.e.

(24) Find

$$\lim_{x \rightarrow -\infty} \frac{3x - 7}{x + \sqrt{4x^2 + 1}}.$$

- (a) -3 ,
- (b) 1 ,
- (c) 0 ,
- (d) d.n.e.

(25) Find

$$\lim_{x \rightarrow \infty} \frac{5x^3 + 4x - 11}{x^4 + 7x^2 - 8}.$$

- (a) ∞ ,
- (b) 0,
- (c) 5,
- (d) d.n.e.

(26) Find

$$\lim_{x \rightarrow 1} \frac{x^2 - 1}{x^2 - 2x + 1}.$$

- (a) 1,
- (b) ∞ ,
- (c) 0,
- (d) d.n.e.

(27) For what value of c is the function,

$$f(x) = \begin{cases} x^2 + c & \text{if } x \leq 2 \\ cx + 1 & \text{if } x > 2 \end{cases}$$

continuous?

- (a) 0,
- (b) 1,
- (c) 2,
- (d) 3.

If a function $f(t)$ models a system that varies in time, the existence of the limit,

$$\lim_{t \rightarrow \infty} f(t),$$

indicates that the system reaches an equilibrium, with the limit value as the equilibrium value.

(28) Would the population of a species that is modeled by a function of the form,

$$P(t) = \frac{A}{1 + Be^{-kt}} \text{ where } A, B, k \text{ are positive constants and } t \geq 0 \text{ is time,}$$

reach an equilibrium? If so, report the equilibrium value in terms of the constants.

- (a) No
- (b) Yes; 0
- (c) Yes; A
- (d) Yes; $\frac{A}{1+B}$

Problems on this page are based on the function $f(x)$ defined below:

$$f(x) = \frac{4x^2(1-x)(1+x)}{(2x-1)^2(x^2+1)}.$$

(29) Determine the following limit:

$$\lim_{x \rightarrow -\infty} f(x).$$

- (a) 0
- (b) -1
- (c) 1
- (d) $-\infty$

(30) Determine the following limit:

$$\lim_{x \rightarrow 2} f(x).$$

- (a) $-\infty$
- (b) ∞
- (c) $-\frac{16}{15}$
- (d) $\frac{16}{15}$

(31) Determine the following limit:

$$\lim_{x \rightarrow -1^+} \frac{x^2 + x}{(x^2 - 3x - 4)^2}.$$

(a) $-\infty$

(b) ∞

(c) 0

(d) 1

Chapter 3:

(1) If $f'(a)$ exists then f is continuous at $x = a$.

(a) True,

(b) False.

(2) Which of the following will correctly calculate $f'(-5)$ for $f(x) = \frac{4}{x+1}$?

(a)

$$\lim_{h \rightarrow 0} \frac{\frac{4}{-4+h} - 1}{h},$$

(b)

$$\lim_{h \rightarrow 0} \frac{\frac{4}{-5+h} + 1}{h},$$

(c)

$$\lim_{h \rightarrow 0} \frac{\frac{4}{-4+h} + 1}{h},$$

(d)

None of the above.

(3) Which of the following will correctly calculate $f'(-1)$ for $f(x) = \sqrt{3-x}$?

(a)

$$\lim_{h \rightarrow 0} \frac{\sqrt{3-h} - 2}{h},$$

(b)

$$\lim_{h \rightarrow 0} \frac{\sqrt{4-h} - 2}{h},$$

(c)

$$\lim_{h \rightarrow 0} \frac{\sqrt{-4+h} - 2}{h},$$

(d)

None of the above.

(4) Given that the derivative of the function $f(x) = \tan^{-1}(x)$ is

$$f'(x) = \frac{1}{1+x^2},$$

determine all the points in the xy -plane where the tangent line to $y = f(x)$ is parallel to $x - 4y = -5$.

(a) $(0, 0)$

(b) $(\pm \frac{\sqrt{3}}{3}, \pm \frac{\pi}{6})$

(c) $(\pm 1, \pm \frac{\pi}{4})$

(d) $(\pm \sqrt{3}, \pm \frac{\pi}{3})$

- (5) Suppose $f(x) = 3g(x^2)$. Given that $g(-1) = 5$, $g(1)=1$, $g'(-1) = 0$, and $g'(1) = 2$ determine $f'(-1)$.

- (a) 0,
- (b) 6,
- (c) -12 ,
- (d) None of the above.

- (6) Find

$$\frac{d}{dx} (\log_2(x^2 + 1)) .$$

- (a)

$$\frac{1}{x^2 + 1} ,$$

- (b)

$$\frac{1}{(\ln(2))(x^2 + 1)} ,$$

- (c)

$$\frac{2x}{x^2 + 1} ,$$

- (d)

$$\frac{2x}{(\ln(2))(x^2 + 1)} .$$

- (7) Suppose the radius of a circular puddle increases at 2 centimeters per hour. Determine the rate at which the puddle's area increases at the instant when the puddle reaches an area of 100π square centimeters.

Formula for the area of a circle: $A = \pi r^2$.

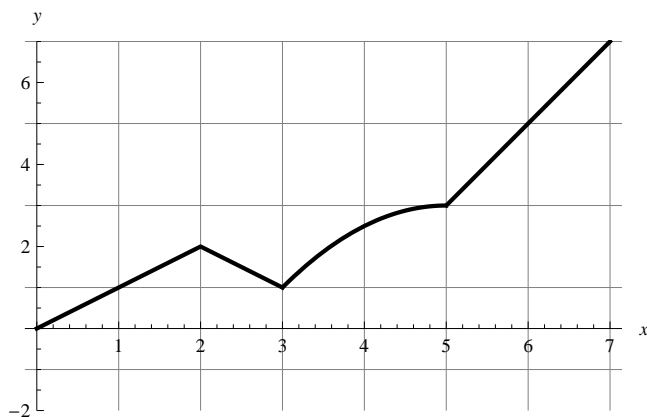
- (a) 0 square centimeters per hour,
- (b) 2π square centimeters per hour,
- (c) 20π square centimeters per hour,
- (d) 40π square centimeters per hour.

- (8) Determine the following limit:

$$\lim_{x \rightarrow 0} \frac{\tan 2x}{x}.$$

- (a) 0,
- (b) 1,
- (c) 2,
- (d) d.n.e.

Problem 9 is based on the function f on the domain $(0, 7)$ with the following graph:



(9) Where is f differentiable?

- (a) f is differentiable on $(0, 7)$,
- (b) f is differentiable on $(0, 2) \cup (2, 3) \cup (3, 5) \cup (5, 7)$,
- (c) f is differentiable on $(0, 3) \cup (3, 7)$,
- (d) f is differentiable on $(0, 2) \cup (5, 7)$.

(10) Find the derivative of the function:

$$f(x) = \sqrt{x^2 + 1}.$$

(a)

$$f'(x) = \frac{1}{2\sqrt{x^2 + 1}},$$

(b)

$$f'(x) = \frac{2x}{\sqrt{x^2 + 1}},$$

(c)

$$f'(x) = \frac{1}{\sqrt{x^2 + 1}},$$

(d)

$$f'(x) = \frac{x}{\sqrt{x^2 + 1}},$$

- (11) Let $f(x) = x^3 + 4x + 1$. Which of the following is an equation of a tangent line to $y = f(x)$ that is parallel to $7x - y = 14$? Select the best answer.

- (a) $y - 5 = 7(x - 1)$,
- (b) $y + 4 = 7(x + 1)$,
- (c) Both of the above,
- (d) None of the above.

- (12) Find the derivative of the function:

$$f(x) = \ln \left(\frac{x+1}{x} \right).$$

- (a)

$$f'(x) = -\frac{1}{x^2 + x},$$

- (b)

$$f'(x) = \frac{1}{x^2 + x},$$

- (c)

$$f'(x) = -\frac{x}{x+1},$$

- (d)

$$f'(x) = \frac{x}{x+1},$$

(13) Find the derivative of the function:

$$f(x) = e^{x^2+1}.$$

(a)

$$f'(x) = e^{x^2+1},$$

(b)

$$f'(x) = 2xe^{x^2+1},$$

(c)

$$f'(x) = (x^2 + 1)e^{x^2},$$

(d)

$$f'(x) = 2e^{x^2+1},$$

(14) Find

$$\frac{d}{dx} (\tan(x) \csc(x)).$$

(a) $-\csc(x) \tan^2(x),$

(b) $\csc^2(x) \tan(x),$

(c) $-\sec(x) \tan(x),$

(d) $\sec(x) \tan(x).$

(15) Find an equation for the tangent line to the ellipse $x^2 - 6x + 4y^2 - 8y = 7$ at the point $(7, 2)$.

(a) $y = x - 5,$

(b) $y = -x + 9,$

(c) $y = -x - 5,$

(d) $y = x - 9,$

(16) Find the **second derivative** of the function:

$$f(x) = x^2 e^{1/x}.$$

(a)

$$f''(x) = e^{1/x} \left[2 + \frac{2}{x} + \frac{1}{x^2} \right],$$

(b)

$$f''(x) = e^{1/x} \left[2 - \frac{2}{x} - \frac{1}{x^2} \right],$$

(c)

$$f''(x) = e^{1/x} \left[2 + \frac{2}{x} - \frac{1}{x^2} \right],$$

(d)

$$f''(x) = e^{1/x} \left[2 - \frac{2}{x} + \frac{1}{x^2} \right].$$

(17) The derivative of $f(x) = \tan^{-1} x$ is sometimes negative.

(a) True,

(b) False.

(18) Find the derivative of the function:

$$f(x) = \frac{x^2 + 4x - 44}{x - 5}.$$

(a)

$$f'(x) = \frac{x^2 - 10x + 24}{x - 5},$$

(b)

$$f'(x) = \frac{x^2 - 10x + 24}{(x - 5)^2},$$

(c)

$$f'(x) = \frac{x^2 - 10x + 26}{(x - 5)^2},$$

(d)

$$f'(x) = \frac{x^2 - 10x + 26}{x - 5}.$$

(19) Suppose that $h(x) = f(g(x))$. Determine $h'(1)$ given $f(2) = 3$, $f'(2) = -4$, $g(1) = 2$, $g'(1) = -7$.

(a) $h'(1) = 28$,

(b) $h'(1) = -28$,

(c) $h'(1) = -21$,

(d) $h'(1)$ cannot be determined with the given information.

- (20) Let $f(x) = x^2 + x$. Which of the following is an equation of a tangent line to $y = f(x)$ that is perpendicular to $x + 3y = 3$? Select the best answer.

- (a) $y - 2 = 3(x - 1)$,
- (b) $y = 3(x + 1)$,
- (c) Both of the above,
- (d) None of the above.

- (21) Find the derivative of the function:

$$f(x) = \tan^{-1}\left(\frac{1}{x}\right).$$

- (a)

$$f'(x) = \frac{1}{1 + x^2},$$

- (b)

$$f'(x) = -\frac{1}{1 + x^2},$$

- (c)

$$f'(x) = \frac{1}{1 - x^2},$$

- (d)

$$f'(x) = -\frac{1}{1 - x^2},$$

(22) Find

$$\frac{d}{dx} (\sin(x) \cot(x)).$$

(a) $-\csc x$,

(b) $\csc x$,

(c) $\sin x$,

(d) $-\sin x$.

(23) Find an equation for the tangent line to the ellipse $(x + 3)^2 + 4(y + 1)^2 = 20$ at the point $(-7, -2)$.

(a) $y = x + 5$,

(b) $y = x + 9$,

(c) $y = -x - 5$,

(d) $y = -x - 9$,

(24) Find the **second derivative** of the function:

$$f(x) = x^2 \ln x.$$

(a)

$$f''(x) = 2 \ln x,$$

(b)

$$f''(x) = 2 \ln x + 1,$$

(c)

$$f''(x) = 2 \ln x + 2,$$

(d)

$$f''(x) = 2 \ln x + 3.$$

(25) Find the derivative of the function:

$$f(x) = \frac{x^2 + 3x - 33}{x - 4}.$$

(a)

$$f'(x) = \frac{x^2 - 8x + 21}{x - 4},$$

(b)

$$f'(x) = \frac{x^2 - 8x + 21}{(x - 4)^2},$$

(c)

$$f'(x) = \frac{x^2 - 8x + 11}{(x - 4)^2},$$

(d)

$$f'(x) = \frac{x^2 - 8x + 11}{x - 4}.$$

(26) The derivative of $f(x) = e^{-x}$ is always negative.

(a) True,

(b) False.

(27) The function $f(t) = (1 + e^{-2t})^{-1}$ sometimes decreases.

- (a) True,
- (b) False.

(28) Suppose $f(x) = 3 + g(1 - x)$. Given that $g(0) = 5$, $g(1) = 1$, $g'(0) = -1$, and $g'(1) = 7$ determine $f'(1)$.

- (a) 0,
- (b) 1,
- (c) -1 ,
- (d) None of the above.

(29) Find

$$\frac{d}{dx} (2^{\tan x}).$$

(a)

$$2^{\tan(x)-1} \tan x,$$

(b)

$$(\ln(2))2^{\tan x},$$

(c)

$$2^{\sec^2 x},$$

(d)

$$(\ln(2))2^{\tan x} \sec^2 x.$$

(30) Find the derivative of the function:

$$f(x) = (x^3 + 1)^2.$$

(a) $f'(x) = 6x^5,$

(b) $f'(x) = 6x^5 + 6x^2,$

(c) $f'(x) = 6x^5 + 6x^3,$

(d) $f'(x) = 5x^4.$

(31) Let $f(x) = (x - 1)(x - 2)$. Find the value of x for which the slope of the curve $y = f(x)$ is 3.

(a) 0,

(b) 1,

(c) 2,

(d) 3.

(32) Find the derivative of the function:

$$f(x) = \frac{2x + 1}{2\sqrt{x^3} + 1}.$$

(a)

$$f'(x) = \frac{2(2\sqrt{x^3} + 1) - 3\sqrt{x}(2x + 1)}{(2\sqrt{x^3} + 1)^2},$$

(b)

$$f'(x) = \frac{2(2\sqrt{x^3} + 1) + 3\sqrt{x}(2x + 1)}{(2\sqrt{x^3} + 1)^2},$$

(c)

$$f'(x) = \frac{2(2\sqrt{x^3} + 1) - \sqrt{x}(2x + 1)}{(2\sqrt{x^3} + 1)^2},$$

(d)

$$f'(x) = \frac{2(2\sqrt{x^3} + 1) - 3\sqrt{x}(2x + 1)}{2\sqrt{x^3} + 1}.$$

(33) Find

$$\frac{d}{dx} (\sin x \cos x) .$$

(a) $-\cos x \sin x$,

(b) 1 ,

(c) $\cos^2 x - \sin^2 x$,

(d) $\sin 2x$.

(34) Find

$$\frac{d}{dx} ((\sin x - \cos x)^4) .$$

(a) $4(\sin x - \cos x)^3$,

(b) $(\sin x + \cos x)^4$,

(c) $4(\sin x - \cos x)^4$,

(d) $4(\sin x - \cos x)^3(\cos x + \sin x)$.

(35) Find the **second derivative** for the function:

$$f(x) = x \sin x^2.$$

(a) $f''(x) = \sin x^2 + 2x^2 \cos x^2,$

(b) $f''(x) = 6x \cos x^2 - 4x^3 \sin x^2,$

(c) $f''(x) = 6x \cos x^2 + 4x^3 \sin x^2,$

(d) $f''(x) = 2x \cos x^2 - 4x^3 \sin x^2.$

(36) Find an equation for the tangent line to the curve $x^2 + xy + y^2 = 13$ at the point $(3, 1)$.

(a)

$$y = -\frac{7}{5}x + \frac{26}{5},$$

(b)

$$y = \frac{7}{5}x - \frac{16}{5},$$

(c)

$$y = -\frac{5}{7}x + \frac{22}{7},$$

(d)

$$y = \frac{5}{7}x - \frac{8}{7},$$

(37) Find the derivative of the function:

$$f(x) = \ln(\ln x).$$

(a)

$$f'(x) = \frac{1}{x},$$

(b)

$$f'(x) = \frac{1}{\ln x},$$

(c)

$$f'(x) = \frac{1}{x \ln x},$$

(d)

$$f'(x) = \ln \frac{1}{x}.$$

(38) You are in charge of setting the price of airfare from Portland to Las Vegas. When you charge 200 dollars, you sell 3,000 seats daily. For each dollar increase in airfare, ten fewer customers per day purchase the tickets. Which of the following represents the **derivative of the revenue function** (aka marginal revenue) for x seats sold on a daily basis?

(a) $-\frac{1}{10},$

(b) $500 - 0.2x,$

(c) $3020 - 0.2x,$

(d) 0.

(39) If $f(x)$ is infinitely-many times differentiable at $x = a$ then $|f(x)|$ is differentiable at $x = a$.

(a) True

(b) False

(40) Which of the following proposed derivatives is **incorrect**?

(a)

$$\frac{d}{dx} (\sin (x^2)) = 2x \cos (x^2).$$

(b)

$$\frac{d}{dx} (\sin^2 (x)) = 2 \cos (x).$$

(c)

$$\frac{d}{dx} [(1 + \sin (x))^3] = 3 (1 + \sin (x))^2 \cos (x).$$

(d)

$$\frac{d}{dx} \left[\frac{1}{1 + \sin (x)} \right] = - \frac{\cos (x)}{(1 + \sin (x))^2}.$$

- (41) (4 pts) Two cars leave the center of town at the same time. One travels at v_1 km/hr on a straight road heading north. The other travels at v_2 km/hr on a straight road heading west. Which expression determines how fast the cars are moving apart (in km/hr) after an hour?

(a)

$$v_1 + v_2$$

(b)

$$\frac{v_1^2 + v_2^2}{v_1 + v_2}$$

(c)

$$v_1^2 + v_2^2$$

(d)

$$\sqrt{v_1^2 + v_2^2}$$

- (42) Suppose $P(t)$ models the number of fish in a lake at t years from 2000. What units does $P''(t)$ have?

(a) fish

(b) fish/yr

(c) fish/yr²

(d) fish/yr³

Suppose that $f(x)$ and $g(x)$ are differentiable functions on $(-\infty, \infty)$. Answer questions on this page given the information in the table below:

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
-1	1	3	0	2
1	2	2	3	-1
2	-5	4	5	-2
4	5	-1	0	1
8	2	0	3	-3

(43) Let $h(x) = \frac{f(x^2)}{g(x)}$. Determine $h'(2)$ (if it exists).

(a) $\frac{74}{25}$

(b) $\frac{14}{5}$

(c) $-\frac{2}{5}$

(d) $h(x)$ is not differentiable at $x = 2$.

(44) Let $h(x) = e^{f(x)g(x)}$. Determine $h'(-1)$ (if it exists).

(a) 2

(b) $2e$

(c) 6

(d) $h(x)$ is not differentiable at $x = -1$.

(45) Determine the following limit:

$$\lim_{x \rightarrow 0} \frac{\tan(3x)}{2x}.$$

(a) 0

(b) $\frac{2}{3}$

(c) 1

(d) $\frac{3}{2}$

Chapter 4:

(1) Let $f(x) = x^2e^{-x}$. Find and classify the extremes of $f(x)$.

(a) $f(x)$ has a local minimum at $x = -2$ and a global maximum at $x = 0$,

(b) $f(x)$ has a local minimum at $x = 0$ and a global maximum at $x = 2$,

(c) $f(x)$ has a global minimum at $x = -2$ and a local maximum at $x = 0$,

(d) $f(x)$ has a global minimum at $x = 0$ and a local maximum at $x = 2$.

(2) Which of the following represents a linear approximation (aka tangent line approximation) of $f(x) = \ln(x^2)$ about $x = 1$?

(a)

$$f(x) \approx x - 1,$$

(b)

$$f(x) \approx 2x - 2,$$

(c)

$$f(x) \approx 2x - 1,$$

(d)

$$f(x) \approx x - 2.$$

(3) Let

$$f(x) = e^{-0.5x^2}.$$

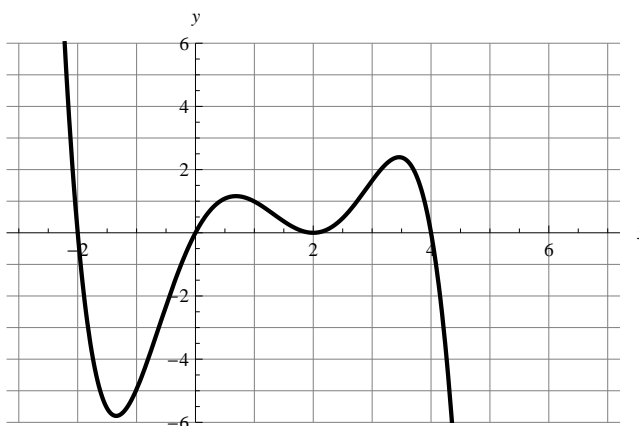
On what interval(s) is $f(x)$ concave down?

- (a) $(-1, 1)$,
- (b) $(-\infty, -1) \cup (1, \infty)$,
- (c) $(-\infty, \infty)$,
- (d) This function is never concave down.

(4) The function $f(t) = 30(1 - e^{-2t})$ is always increasing and concave up.

- (a) True,
- (b) False.

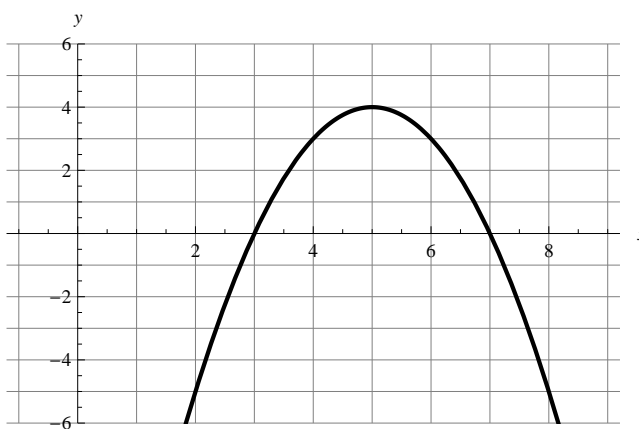
Problem 5 is based on the function f with the following graph:



(5) Which of the following are true? Select the best answer.

- (a) f has two critical points on $(0, 3)$,
- (b) f has one absolute minimum and one absolute maximum on $(-2, 2)$,
- (c) f has no absolute minimum on $(-1, 4)$.
- (d) All of the above.

(6) Given that below is the graph of the **SECOND DERIVATIVE** of $f(x)$, on what interval(s) is the function $f(x)$ concave up?



- (a) $(-\infty, \infty)$,
- (b) $(3, 7)$,
- (c) $(-\infty, 3) \cup (7, \infty)$,
- (d) $f(x)$ is never concave up.

(7) Find all critical points for $f(x) = 4x^3 + 27x^2 - 30x + 71$.

(a) $x = -\frac{1}{2}, 5$,

(b) $x = \frac{1}{2}, -5$,

(c) $x = \frac{1}{2}, 5$,

(d) $f(x)$ has no critical points.

(8) John owns a small craft brewery. In dollars, his yearly revenue and cost are respectively given by $R(x) = 2x$ and by $C(x) = 5000 + .05x + .000005x^2$, where x is the number of pints produced by the brewery each year. Find his maximum yearly profit.

(a) 185,125 dollars, (b) 105,625 dollars, (c) 95,225 dollars, (d) 55,725 dollars.

(9) Find all relative extremes for

$$f(x) = \frac{4x^3 + 1}{x}.$$

- (a) $f(x)$ has a relative minimum at $(\frac{1}{2}, 3)$,
- (b) $f(x)$ has a relative maximum at $(\frac{1}{2}, 3)$,
- (c) $f(x)$ has a relative minimum at $(-\frac{1}{2}, -1)$ and a relative maximum at $(\frac{1}{2}, 3)$,
- (d) $f(x)$ has a relative maximum at $(-\frac{1}{2}, -1)$ and a relative minimum at $(\frac{1}{2}, 3)$,

(10) Which of the following represents a linear approximation (aka tangent line approximation) of $f(x) = e^{-x}$ about $x = 0$?

- (a)
$$f(x) \approx -x - 1,$$
- (b)
$$f(x) \approx x - 1,$$
- (c)
$$f(x) \approx -x + 1,$$
- (d)
$$f(x) \approx x + 1.$$

(11) The function $g(x) = x^3 - 4x^2 + 4x + 5$ has...

- (a) No relative extremes and no inflections,
- (b) No relative extremes and one inflection,
- (c) A local minimum, a local maximum, and no inflections,
- (d) A local minimum, a local maximum, and one inflection.

(12) Find the limit:

$$\lim_{x \rightarrow 0} \frac{e^x - 1 - x}{x^2}.$$

- (a) 2,
- (b) 0.5,
- (c) ∞ ,
- (d) d.n.e.

- (13) A ball is dropped off a tall building. It hits the ground below at 192 feet per second. Use $a = -32 \text{ ft}/s^2$. How tall is the building?

- (a) 556 ft,
- (b) 576 ft,
- (c) 596 ft,
- (d) It cannot be determined with the given information.

- (14) Which of the following approximates $\sqrt{26}$ using a **linear approximation** for $f(x) = \sqrt{x}$ at $x = 25$?

- (a) $\sqrt{26} \approx 5.11,$
- (b) $\sqrt{26} \approx 5.1,$
- (c) $\sqrt{26} \approx 5.099,$
- (d) $\sqrt{26} \approx 5.09.$

(15) Find the location of the global maximum of $f(x) = x^2e^{-x}$ on $[1, 3]$.

- (a) $x = 1$,
- (b) $x = 2$,
- (c) $x = 3$,
- (d) This function does not have a global maximum on $[1, 3]$.

(16) Which of the following statements are true?

(a) If a function is not differentiable at $x = a$ then it is not continuous at $x = a$.

(b) If a function f has a local extreme at $x = a$ then $f'(a) = 0$.

(c)

$$\frac{d}{dx}(\cos(\pi)) = \tan(\pi).$$

(d) All of the above.

(17) For what value of c does the average rate of change in $f(x) = x^{-1}$ over $[3, 12]$ equal $f'(c)$, that is, the instantaneous rate of change at $x = c$?

- (a) $c = 4$,
- (b) $c = 6$,
- (c) $c = 7.5$,
- (d) $c = 9$.

(18) The function $f(t) = (1 - e^{-t})^{-1}$ has no critical points on its domain.

- (a) True,
- (b) False.

(19) Which of the following is the **differential** of $y = \ln |x|$?

- (a) $dy = \frac{1}{x}$,
- (b) $dy = \frac{1}{|x|}$,
- (c) $dy = \frac{dx}{x}$,
- (d) $dy = \frac{dx}{|x|}$.

(20) Let

$$f(x) = \frac{3x^2 - 7x + 1}{x - 2}.$$

Which of the following are true? Select the best answer.

- (a) $f(x)$ always decreases on its domain,
- (b) $f(x)$ increases on $(-\infty, 2)$ and decreases on $(2, \infty)$,
- (c) $f(x)$ decreases on $(-\infty, 2)$ and increases on $(2, \infty)$,
- (d) $f(x)$ always increases on its domain.

- (21) Let $f(x)$ and $g(x)$ be differentiable functions. Suppose that $L_1(x) = 2x+5$ is the linear approximation of $f(x)$ at $x = -1$ and $L_2(x) = -3x+2$ is the linear approximation of $g(x)$ at $x = 1$. Which of the following is the linear approximation to $h(x) = f(g(x))$ at $x = 1$?

- (a) $L(x) = -x + 3$
- (b) $L(x) = -x + 4$
- (c) $L(x) = -6x + 3$
- (d) $L(x) = -6x + 9$

- (22) 2000 ft of fencing is going to be used to create 4 identical rectangular pens as shown in the figure below. What are the **dimensions of each pen** that maximize the area?



- (a) 125 ft by 200 ft
- (b) 500 ft by 200 ft
- (c) 250 ft by 100 ft
- (d) 1000 ft by 100 ft

- (23) (6 pts) Find the point $3 < c < 5$ where the instantaneous rate of change is equal to the average rate of change in $f(x) = x^2 - 2x + 3$ on the interval $[3, 5]$.

Recall: The Mean Value Theorem states that if f is continuous on $[a, b]$ and differentiable on (a, b) then there exists at point c in (a, b) such that $f'(c) = \frac{f(b) - f(a)}{b - a}$.

- (a) 3.5
 - (b) 4.5
 - (c) 4
 - (d) No such point exists.
- (24) The yellow felt on a 6.6 cm diameter tennis ball is approximately 1 mm thick. Which of the following is a differential estimate of the volume of felt (in cubic centimeters) used on the tennis ball?

Hint: The volume of a sphere is $V = \frac{4}{3}\pi r^3$, where r is the radius.

- (a) $dV = \frac{4}{3}\pi(6.6)^3 \text{ cm}^3$
- (b) $dV = \frac{4}{3}\pi(3.3)^3 \text{ cm}^3$
- (c) $dV = \frac{2}{5}\pi(6.6)^2 \text{ cm}^3$
- (d) $dV = \frac{2}{5}\pi(3.3)^2 \text{ cm}^3$

Key:

Chapter 2:

- (1) (c)
- (2) (b)
- (3) (a)
- (4) (b)
- (5) (d)
- (6) (a)
- (7) (a)
- (8) (c)
- (9) (d)
- (10) (a)
- (11) (b)
- (12) (c)
- (13) (d)
- (14) (d)
- (15) (a)
- (16) (b)
- (17) (c)
- (18) (c)
- (19) (a)
- (20) (c)
- (21) (c)
- (22) (d)
- (23) (b)
- (24) (a)
- (25) (b)
- (26) (d)
- (27) (d)
- (28) (d)
- (29) (b)
- (30) (c)
- (31) (a)

Chapter 3:

- (1) (a)
- (2) (c)
- (3) (b)
- (4) (d)
- (5) (c)
- (6) (d)
- (7) (d)
- (8) (c)
- (9) (b)
- (10) (d)
- (11) (b)
- (12) (a)
- (13) (b)
- (14) (d)
- (15) (b)
- (16) (d)
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- (28) (b)
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- (30) (b)
- (31) (d)
- (32) (a)
- (33) (c)
- (34) (d)
- (35) (b)
- (36) (a)
- (37) (c)
- (38) (b)
- (39) (b)
- (40) (b)
- (41) (d)
- (42) (c)
- (43) (c)
- (44) (a)
- (45) (d)

Chapter 4:

- (1) (d)
- (2) (b)
- (3) (a)
- (4) (b)
- (5) (d)
- (6) (b)
- (7) (b)
- (8) (a)
- (9) (a)
- (10) (c)
- (11) (d)
- (12) (b)
- (13) (b)
- (14) (b)
- (15) (b)
- (16) (c)
- (17) (b)
- (18) (a)
- (19) (c)
- (20) (d)
- (21) (d)
- (22) (a)
- (23) (c)
- (24) (d)