MATH 5603 Homework 3

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Problem 9

```
dimensions=[8,16,32,64,128,256,512,1024,2048];
num_trials = size(dimensions,2);
A_cond=zeros(num_trials,1);
Lcond=zeros(num_trials,1);
Ucond=zeros(num_trials,1);
residaul_no_pivot=zeros(num_trials,1);
Lpivot_cond=zeros(num_trials,1);
Upivot_cond=zeros(num_trials,1);
residual_pivot=zeros(num_trials,1);

for k = 1:num_trials
    n=dimensions(k);
    A = randn(n);
    [Lnp,Unp] = GENP(A);
    A_cond(k)=cond(A,"inf");
    Lcond(k)=cond(Lnp,"inf");
    Ucond(k) = cond(Unp,"inf");
    r = Lnp*Unp-A;
    residaul_no_pivot(k)=norm(r,"inf");

    [Lp,Up] = lu(A);
    Lpivot_cond(k)=cond(Lp,"inf");
    Upivot_cond(k) = cond(Up,"inf");
    rp = Lp*Up-A;
    residual_pivot(k)=norm(rp,"inf");
end
format short
Data = [dimensions(:),A_cond(:), Lcond(:), Ucond(:),residaul_no_pivot(:), Lpivot_cond(:),Upivot
colnames={'dim', 'cond(A)','cond(L_np)', 'cond(U_np)','residual_np','cond(L_p)', 'cond(U_p)','r
Table = array2table(Data,'VariableNames',colnames);
disp(Table)
```

dim	cond(A)	cond(L_np)	cond(U_np)	residual_np	cond(L_p)	cond(U_p)	residual_p
8	233.3	699.45	432.89	1.3323e-15	11.406	118.61	6.1062e-16
16	385.71	5331.3	5630.9	3.6693e-14	114.37	184.34	5.0238e-15
32	1777.6	9411.6	24639	1.8548e-13	284.73	411.1	1.0422e-14
64	1047.6	8.1799e+05	8.6883e+05	4.5298e-12	797.36	1106.3	3.2127e-14
128	18605	9.3653e+06	9.1035e+07	3.2861e-11	2839.9	4721.6	1.0195e-13
256	31838	1.6653e+08	1.9402e+08	2.2465e-10	13558	9661.2	4.8344e-13
512	1.1654e+05	1.8502e+08	1.0054e+08	7.8591e-10	59228	36829	1.9488e-12
1024	52863	7.6215e+09	9.6424e+09	5.9718e-09	1.9228e+05	1.1036e+05	6.9684e-12
2048	5.0261e+05	5.0539e+10	2.9858e+11	1.3527e-07	7.6958e+05	5.0821e+05	2.8135e-11

We see that pivoting does reduce the error, and that the residuals for the no pivot cases are not that bad given the high condition numbers

Now let's see what happens when we make one of the pivots very small.

```
dimensions=[8,16,32,64,128,256];
num_trials = size(dimensions,2);
A_cond=zeros(num_trials,1);
Lcond=zeros(num_trials,1);
Ucond=zeros(num_trials,1);
residaul_no_pivot=zeros(num_trials,1);
Lpivot_cond=zeros(num_trials,1);
Upivot_cond=zeros(num_trials,1);
residual_pivot=zeros(num_trials,1);

for k = 1:num_trials
    n=dimensions(k);
    A = randn(n);
    A(1,1) = 50*eps*A(1,1);
    [Lnp,Unp] = GENP(A);
    A_cond(k)=cond(A,"inf");
    Lcond(k)=cond(Lnp,"inf");
    Ucond(k) = cond(Unp,"inf");
    r = Lnp*Unp-A;
    residaul_no_pivot(k)=norm(r,"inf");
    [Lp,Up] = lu(A);
    Lpivot_cond(k)=cond(Lp,"inf");
    Upivot_cond(k) = cond(Up,"inf");
    rp = Lp*Up-A;
    residual_pivot(k)=norm(rp,"inf");
end
format short
Data = [dimensions(:),A_cond(:), Lcond(:), Ucond(:),residaul_no_pivot(:), Lpivot_cond(:),Upivot
colnames={'dim', 'cond(A)','cond(L_np)', 'cond(U_np)','residual_np','cond(L_p)', 'cond(U_p)','r
Table = array2table(Data,'VariableNames',colnames);
disp(Table)
```

dim	cond(A)	cond(L_np)	cond(U_np)	residual_np	cond(L_p)	cond(U_p)	residual_p
8	951.22	4.5992e+27	2.2973e+28	0.027289	15.535	326.32	1.0131e-15
16	87.894	7.628e+28	6.2891e+29	0.22749	87.055	155.77	5.1625e-15
32	10236	7.5431e+27	7.7588e+28	0.10977	183.01	2823.9	9.3675e-15
64	802.53	5.1691e+29	1.0178e+31	3.5627	1016.5	950.24	2.8666e-14
128	2659.1	1.0251e+28	3.9135e+29	1.5973	3805.1	2900.5	1.0515e-13
256	8220	1.8377e+29	1.2435e+31	11.776	9895.6	9445.3	4.824e-13

We see the condition numbers of Lnp and Unp are much higher and the residual for the no pivot cases are also larger.

```matlab
%Gaussian Elimination No Pivoting

function [L,U] = GENP(A)
    n=size(A,2);
    L = zeros(n); U = zeros(n);
    for k = 1:n
        U(k,k:n) = A(k,k:n) - L(k,1:k-1)*U(1:k-1,k:n);
        L(k+1:n,k)=(A(k+1:n,k) - L(k+1:n,1:k-1)*U(1:k-1,k)) / U(k,k);
    end
    L = L + eye(n);
end
```