Bruce Campbell OSU MATH 5603 HW #6

Problem 3

5.3.10

a)

```
clear all;
A = [1 1 1 ; -1 9 2 ; 0 -1 -2];
q= [1 1 1]';
iterate(:,1)=q;
for j=1:10
    q=A*q;
    [bgst,index] = max(abs(q));
    scale_factor(j+1) = q(index(1));
    q = q/scale_factor(j+1);
    iterate(:,j+1 ) =q;
end
```

b)

```
[V,D] =eig(A)
```

```
V = 3×3
  -1.1649e-01   9.8968e-01  -2.6661e-01
  -9.8888e-01   1.3632e-01  -1.9952e-01
   9.2460e-02  -4.4070e-02   9.4293e-01
D = 3×3
   8.6952e+00            0            0
            0   1.0932e+00            0
            0            0  -1.7884e+00
```

```
format long e;
D(1,1)
```

```
ans =
     8.695199486633525e+00
```

```
scale_factor(10)
```

```
ans =
     8.695207013366158e+00
```

```
D(1,1) -scale_factor(10)
```

```
ans =
    -7.526732632712196e-06
```

We see that after 10 iterations we have agreement to five significant digits.

c)

```
format shortE;
v = V(:,1)'
```

```
v = 1×3
  -1.1649e-01  -9.8888e-01   9.2460e-02
```

```
[bgst,index] = max(abs(v));
scale_factor=v(index(1));
v = v /scale_factor'
```

```
v = 1×3
   1.1780e-01   1.0000e+00  -9.3500e-02
```

```
ratios=zeros(9,1);
for j=1:9
    ratios(j) = norm( iterate(:,j+1) -v) / norm(iterate(:,j) - v);
end
table(ratios')
```

```
ans = 1×1 table
```

...

	Var1							
1	6.5528e-01	8.9831e-01	9.9681e-01	9.9527e-01	9.9999e-01	9.9977e-01	1.0000e+00	9.9999e-01

d)

```
abs(D(2,2)/D(1,1))
```

```
ans =
   1.2573e-01
```

```
errors=zeros(9,1);
for j=1:9
    errors(j) = norm( iterate(:,j) -v) ;
end
ediff = diff(errors)';
table(ediff)
```

```
ans = 1×1 table
```

	ediff							
1	-8.3889e-01	-1.6216e-01	-4.5690e-03	-6.7608e-03	-9.1246e-06	-3.2540e-04	2.2841e-06	-1.4437e-05

5.3.11

We're going to run the same code and have coommentary after all the calculations

a)

```
clear all;
A = [1 1 1 ; -1 9 2 ; -4 -1 -2];
q= [1 1 1]';
iterate(:,1)=q;
for j=1:10
    q=A*q;
    [bgst,index] = max(abs(q));
    scale_factor(j+1) = q(index(1));
    q = q/scale_factor(j+1);
    iterate(:,j+1 ) =q;
end
```

b)

```
[V,D] =eig(A)
```

```
V = 3×3 complex
   3.2197e-01 + 2.2953e-01i   3.2197e-01 - 2.2953e-01i ···
   2.2005e-01 + 5.0385e-02i   2.2005e-01 - 5.0385e-02i
  -8.9033e-01 + 0.0000e+00i  -8.9033e-01 + 0.0000e+00i
D = 3×3 complex
  -3.0636e-01 + 1.0878e+00i   0.0000e+00 + 0.0000e+00i ···
   0.0000e+00 + 0.0000e+00i  -3.0636e-01 - 1.0878e+00i
   0.0000e+00 + 0.0000e+00i   0.0000e+00 + 0.0000e+00i
```

```
format long e;
D(1,1)
```

```
ans =
     -3.063597295441190e-01 + 1.087807209189638e+00i
```

```
scale_factor(10)
```

```
ans =
     8.612718552069152e+00
```

```
D(1,1) -scale_factor(10)
```

```
ans =
     -8.919078281613272e+00 + 1.087807209189638e+00i
```

c)

```
v = V(:,1)'
```

```
v = 1×3 complex
      3.219650285568484e-01 - 2.295317833465273e-01i ···
```

```
[bgst,index] = max(abs(v));
scale_factor=v(index(1));
v = v /scale_factor'
```

```
v = 1×3 complex
     -3.616226778672104e-01 + 2.578040805284286e-01i ···
```

```
ratios=zeros(9,1);
for j=1:9
    ratios(j) = norm( iterate(:,j+1) -v) / norm(iterate(:,j) - v);
end
format shortE;table(ratios')
```

```
ans = 1×1 table
```

...

	Var1							
1	6.8465e-01	9.4578e-01	1.0028e+00	9.9913e-01	1.0000e+00	1.0000e+00	1.0000e+00	1.0000e+00

d)

```
abs(D(2,2)/D(1,1))
```

```
ans =
     1
```

```
errors=zeros(9,1);
for j=1:9
    errors(j) = norm( iterate(:,j) -v) ;
end
ediff = diff(errors)';
table(ediff)
```

```
ans = 1×1 table
```

	ediff							
1	-1.0188e+00	-1.1992e-01	5.8826e-03	-1.8187e-03	1.9755e-05	2.9393e-05	-2.4392e-06	-3.3260e-07

We notice that we have non-real eigenvalues, and that $| \lambda_2 / \lambda_1 |$ is 1 meaning we will have slower convergence than in the first case. Now we calculate the square root of the ratios

```
for j=1:9
    ratios(j) = sqrt(norm( iterate(:,j+1) -v) / norm(iterate(:,j) - v));
end
format shortE;table(ratios')
```

```
ans = 1×1 table
```

...

	Var1							
1	8.2744e-01	9.7251e-01	1.0014e+00	9.9957e-01	1.0000e+00	1.0000e+00	1.0000e+00	1.0000e+00

5.3.12

We're going to run the same code and have coommentary after all the calculations.

a)

```
clear all;
A = [1 1 1 ; -1 3 2 ; -4 -1 -2];
q= [1 1 1]';
```

```
iterate(:,1)=q;
for j=1:10
    q=A*q;
    [bgst,index] = max(abs(q));
    scale_factor(j+1) = q(index(1));
    q = q/scale_factor(j+1);
    iterate(:,j+1 ) =q;
end
```

b)

```
[V,D] =eig(A)
```

```
V = 3×3 complex
  -1.9902e-01 + 0.0000e+00i  -3.1945e-01 + 1.3825e-01i ···
  -5.4510e-01 + 0.0000e+00i  -7.1325e-01 + 0.0000e+00i
   8.1441e-01 + 0.0000e+00i   4.9054e-01 - 3.5983e-01i
D = 3×3 complex
  -3.5321e-01 + 0.0000e+00i   0.0000e+00 + 0.0000e+00i ···
   0.0000e+00 + 0.0000e+00i   1.1766e+00 + 1.2028e+00i
   0.0000e+00 + 0.0000e+00i   0.0000e+00 + 0.0000e+00i
```

```
format long e;
D(1,1)
```

```
ans =
    -3.532099641993243e-01
```

```
scale_factor(10)
```

```
ans =
    -1.302788844621514e+00
```

```
D(1,1) -scale_factor(10)
```

```
ans =
     9.495788804221896e-01
```

c)

```
v = V(:,1)'
```

```
v = 1×3
    -1.990150061876847e-01    -5.450988423816862e-01     8.144079317800519e-01
```

```
[bgst,index] = max(abs(v));
scale_factor=v(index(1));
v = v /scale_factor'
```

```
v = 1×3
    -2.443677160077475e-01    -6.693191717696846e-01     1.000000000000000e+00
```

```
ratios=zeros(9,1);
for j=1:9
    ratios(j) = norm( iterate(:,j+1) -v) / norm(iterate(:,j) - v);
```

```
end
format shortE;table(ratios')
```

ans = 1×1 table

...

	Var1							
1	6.0184e-01	1.1965e+00	9.4065e-01	1.0665e+00	8.2449e-01	1.2173e+00	9.3451e-01	1.0756e+00

d)

```
abs(D(2,2)/D(1,1))
```

```
ans =
   4.7638e+00
```

```
errors=zeros(9,1);
for j=1:9
    errors(j) = norm( iterate(:,j) -v) ;
end
ediff = diff(errors)';
table(ediff)
```

ans = 1×1 table

	ediff							
1	-1.4359e+00	4.2645e-01	-1.5412e-01	1.6247e-01	-4.5725e-01	4.6676e-01	-1.7124e-01	1.8472e-01

We see the ratio $| \lambda_2 / \lambda_1 |$ is increased and we have faster convergence.