

Exercise 4.2.20

```
A = randn(8,4);
A(:,5:6)=A(:,1:2) + A(:,3:4);
[Q,R]=qr(randn(6));
A = A * Q;
A
```

```
A = 8×6
    1.2880e+00   -2.6749e+00   -7.1700e-01    1.2865e+00   -2.5589e+00    1.8635e+00
   -1.2273e+00    7.2965e-01   -3.7717e-02   -7.3116e-01    1.7739e+00   -2.4644e-01
    9.4414e-01    2.1948e+00    6.9207e-01   -1.2440e+00    3.1253e+00   -1.0307e+00
    2.1050e-02    1.9076e-01   -1.4278e-01   -1.7191e-01    2.6570e+00    6.0782e-01
    1.7269e+00   -1.3422e+00   -4.5538e-01    1.3914e-01    4.4244e-01    1.5767e+00
    2.4711e-01   -1.8232e+00   -9.4248e-01   -1.5740e+00   -1.4220e+00    1.4838e+00
   -6.4776e-01   -1.6829e+00   -4.1297e-01    1.9949e+00   -1.8353e+00    8.8628e-01
   -1.3360e+00    9.1636e-01    7.4043e-02   -3.0774e-01    2.1059e+00   -3.7049e-01
```

There's no good way to determine the numerical rank of this matrix by inspection of it's elements.

```
S = svd(A);
format short e;
S'
```

```
ans = 1×6
    8.0426e+00    3.6182e+00    2.5673e+00    2.4537e+00    4.3969e-16    2.9516e-16
```

We see there are two very small singular vaules - near machine precision in value. This indicates the numerical rank is 4.

```
tol =eps;
r=rank(A,tol)
```

```
r =
    6
```

Sometimes we get 5 using eps for tol. If we set $\text{tol}=\text{eps}^2$ we always get 6.