1976250 유희정 ⑧ Due date: Tuesday, 4.5

Problem 1

one-to-one : for all $x_1, x_2 \in X$

(a) $f(n) = n^2 - 1$, if $f(x_1) = f(x_2)$ then $x_1 = x_2$

① $f(n_1) = f(n_2)$

$n_1^2 - 1 = n_2^2 - 1$

$n_1^2 = n_2^2$ $\quad (n_1 - n_2)(n_1 + n_2) = 0$

Therefore $n_1 = \pm n_2$, $n_1 \neq n_2$ while $f(n_1) = f(n_2)$

Thus, it isn't one-to-one

② Find a counterexample

-100 is in the codomain but for all n in the domain, $n^2 \geq 0$ $\quad n^2 - 1 = f(n) \geq -1$

Therefore $-100 \neq f(n)$ for all n in the domain.

Thus, given function is neither one-to-one nor onto function

(b) $f(n) = \lceil n/3 \rceil$

① Find a counterexample.

$f(1) = \lceil 1/3 \rceil = 1$, $f(2) = \lceil 2/3 \rceil = 1$

② is onto function.

(for every $y \in Y$, there exists $x \in X$ such that $f(x) = y$)

for every integers y, there exists $n \in \mathbb{Z}$ such that $f(n) = y$

Thus $f(n) = \lceil n/3 \rceil$ is an onto function

Problem 2

(a) $f(x) = 4x^2 - 3x + 2$

① $x_1, x_2 \in \mathbb{R}$

$f(x_1) = f(x_2)$

$4x_1^2 - 3x_1 + 2 = 4x_2^2 - 3x_2 + 2$

$4(x_1 - x_2)(x_1 + x_2) - 3(x_1 - x_2) = 0$

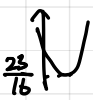
$(x_1 - x_2)(4x_1 + 4x_2 - 3) = 0$

if $x_1 + x_2 = \frac{3}{4}$, then $f(x_1) = f(x_2)$

counterexample If $x_1 = \frac{1}{2}$, $x_2 = \frac{1}{4}$

then $f(x_1) = f(x_2)$ while $x_1 \neq x_2$

② $f(x) \geq \frac{23}{16}$



Thus, $f(x)$ is neither one-to-one neither onto function

(b) $f(x) = 3^x - 2$

① $x_1, x_2 \in \mathbb{R}$

show if $f(x_1) = f(x_2)$, then $x_1 = x_2$

$3^{x_1} - 2 = 3^{x_2} - 2$

$3^{x_1} = 3^{x_2}$

let $3^{x_2} = 3^{x_1} = y$ $\quad x_1 = \log_3 y$, $x_2 = \log_3 y$

by the definition of logarithm

② since $3^x - 2 > -2$,

$f(x) \neq -10$ for all x in $\mathbb{R}$.

Thus $f(x)$ is one-to-one function

Problem 3

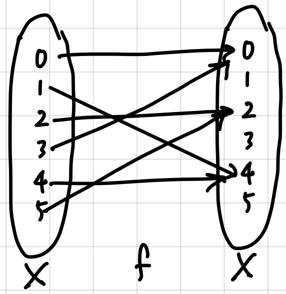
$f \circ f = 3(3n-2) - 2 = 9n - 8$

$g \circ g = 4(4n+3) + 3 = 16n + 15$

$f \circ g = 3(4n+3) - 2 = 12n + 7$

Problem 4

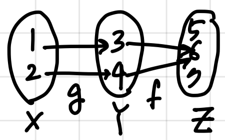
$f(x) = \{(0,0), (1,4), (2,2), (3,0), (4,4), (5,2)\}$



f is neither one-to-one nor onto.

Problem 5

(a) False. There exists a counterexample below.



(b) let $z \in Z$. Since $f \circ g$ (X to Y to Z) is one-to-one, $x_1, x_2 \in X$, if $f(g(x_1)) = f(g(x_2))$, then $x_1 \neq x_2$ let $y_1 = g(x_1)$, $y_2 = g(x_2)$. $y_1, y_2 \in Y$.

$y_1 \neq y_2$ 여기 하겠…