



Sergei Bernstein
(1880-1968)



Paul de Casteljau
(1930-1968)



Pierre Bezier
(1910-1999)



3D Object Representation

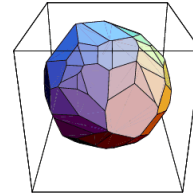
Young J. Kim

EWHA,
THE FUTURE
WE CREATE

Polyhedron

Use a set of polygons to represent the surface of a polyhedron

Consisting of Vertex, Edge, Face (facet)



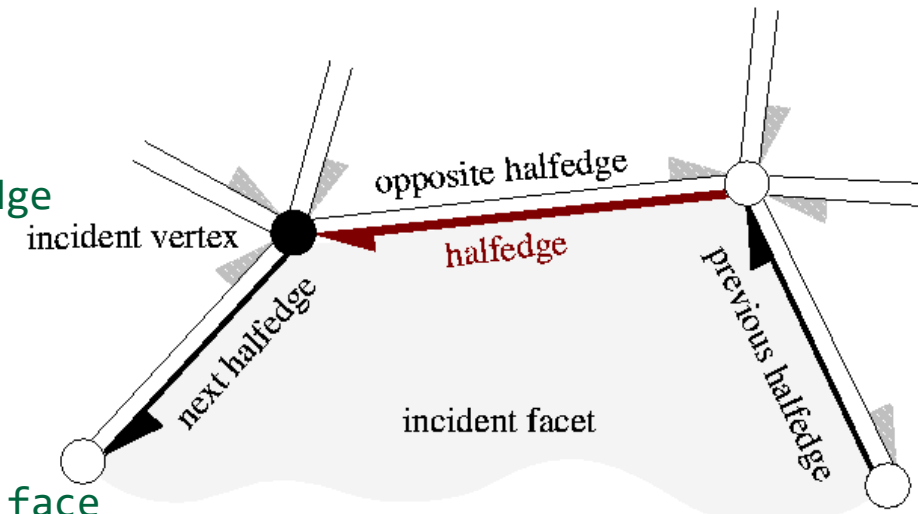
Data structure

- Half edge or doubly-connected edge list (DCEL)

```
struct HE_vert {  
    float x; float y; float z;  
    HE_edge* edge; // one of the half-edges emanating from the vertex  
};
```

```
struct HE_edge {  
    HE_vert* vert; // vertex at the end of the half-edge  
    HE_edge* pair; // oppositely oriented adjacent half-edge  
    HE_face* face; // face the half-edge borders  
    HE_edge* next; // next half-edge around the face  
};
```

```
struct HE_face {  
    HE_edge* edge; // one of the half-edges bordering the face  
};
```



Adjacency Queries using Half Edge

1. Finding vertices and face adjacent to an edge

```
HE_vert* vert_e = edge->vert;
```

```
HE_vert* vert_b = edge->pair->vert;  
HE_face* face1 = edge->face;
```

```
HE_face* face2 = edge->pair->face;
```

2. Find all edges adjacent to a face

```
HE_edge* edge = face->edge;  
do {
```

```
    edge = edge->next;
```

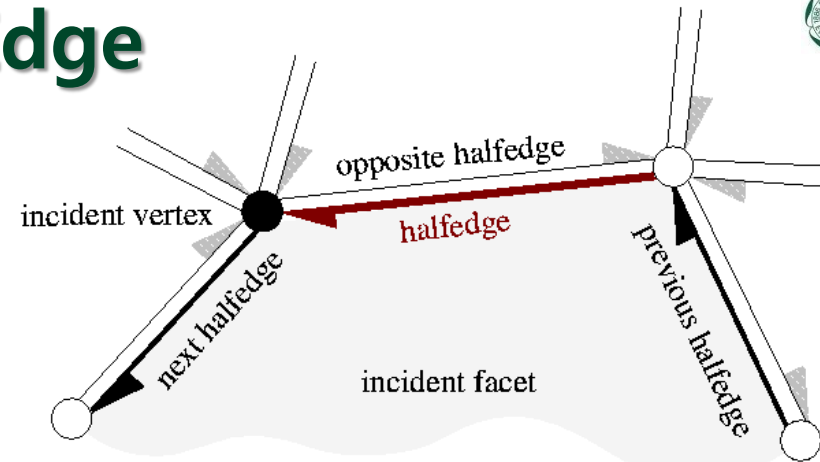
```
} while (edge != face->edge);
```

3. Find all edges adjacent to a vertex

```
HE_edge* edge = vert->edge;  
do {
```

```
    edge = edge->pair->next;
```

```
} while (edge != vert->edge);
```



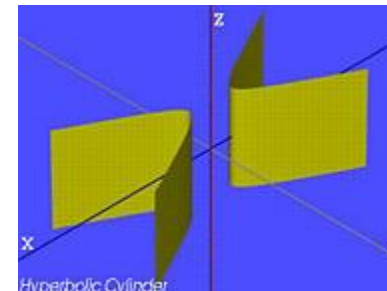
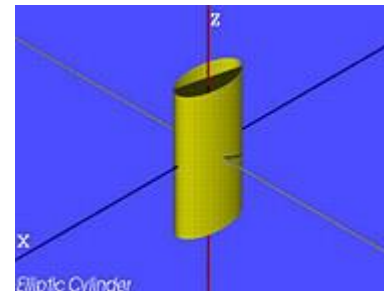
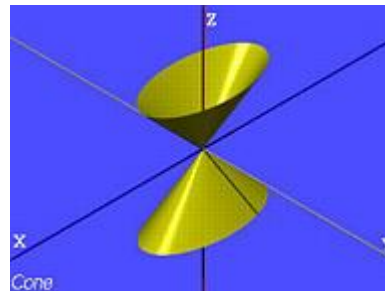
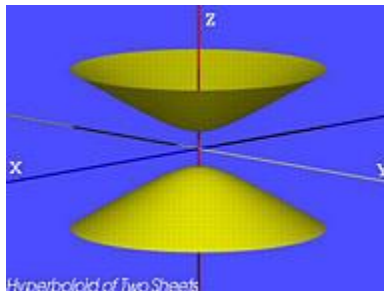
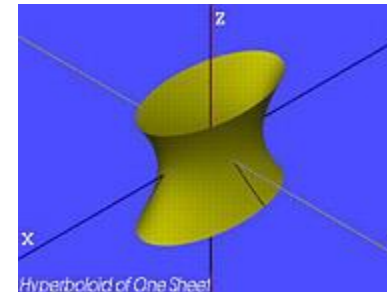
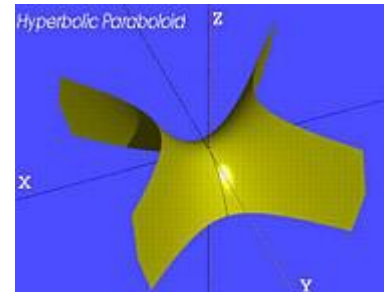
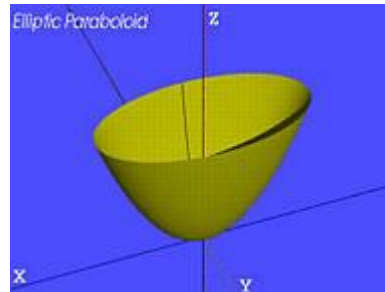
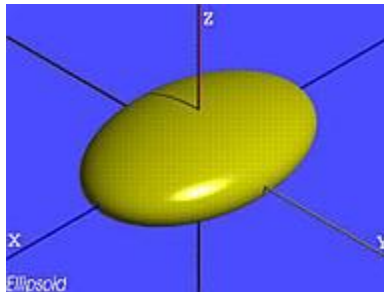
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};
```

Quadric Surfaces

General form

$$\sum_{i,j=1}^D Q_{i,j}x_i x_j + \sum_{i=1}^D P_i x_i + R = 0$$

$$ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2hxy + 2px + 2qy + 2rz + d = 0.$$

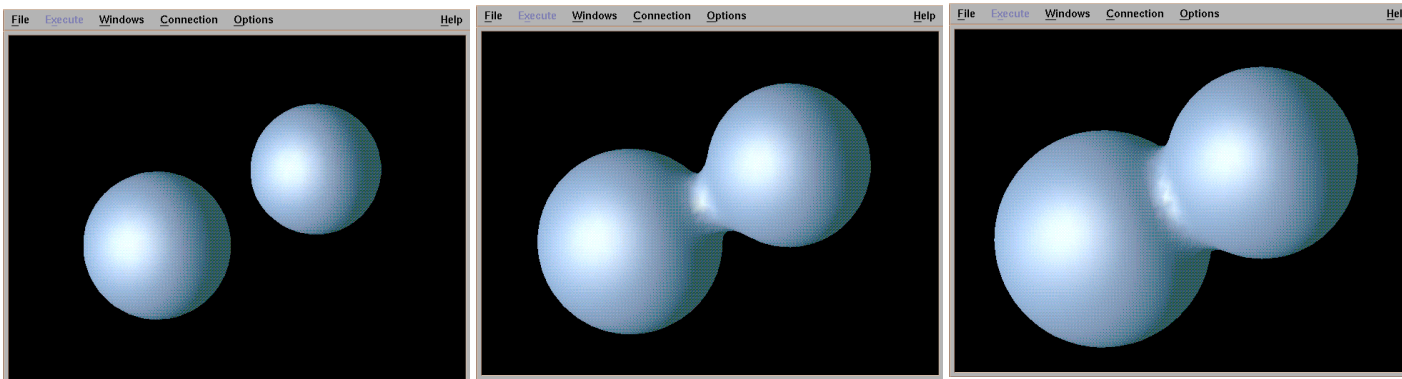


Blobby Object

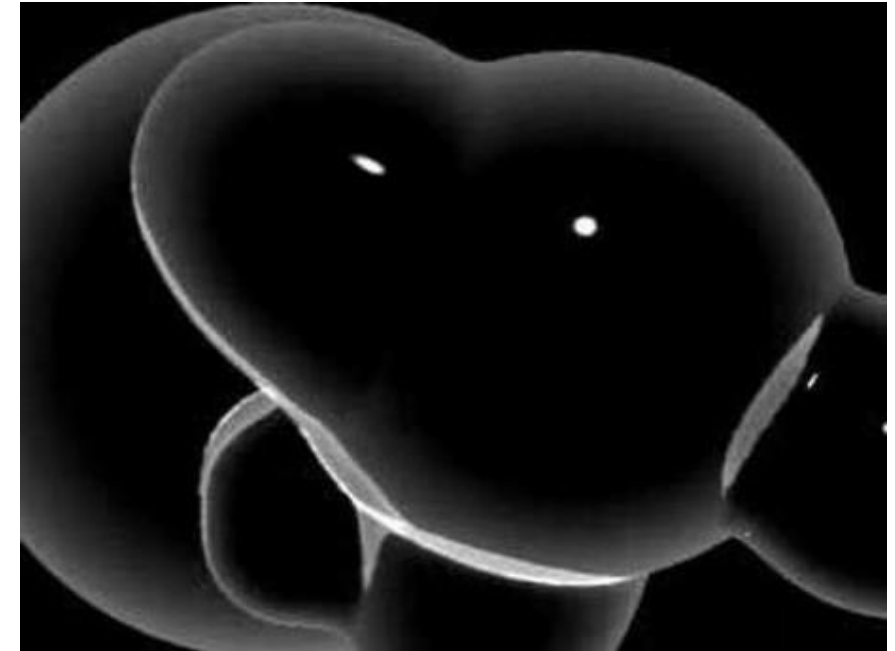
Used to model non-rigid objects:

- Cloth, rubber, molecules, liquids, droplets

$$f(x, y, z) = \sum_k b_k e^{-a_k r_k^2} - T = 0 \quad r_k^2 = x_k^2 + y_k^2 + z_k^2$$



[1D Illustration](#) [2D Illustration](#)

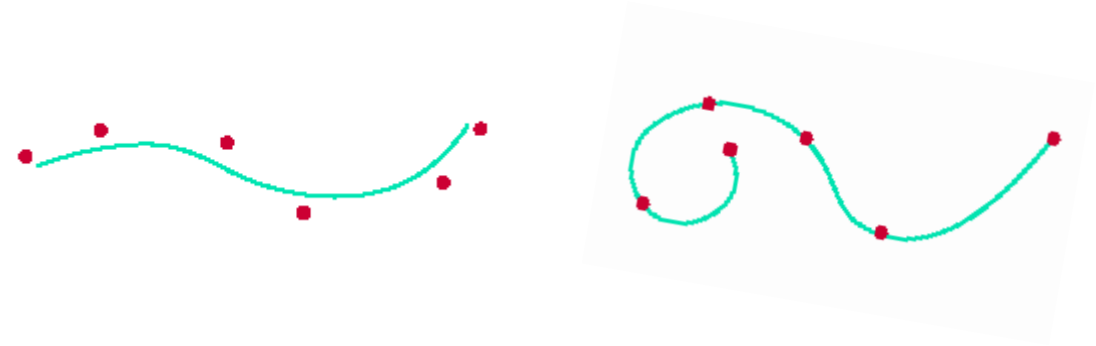


Splines

Controls points

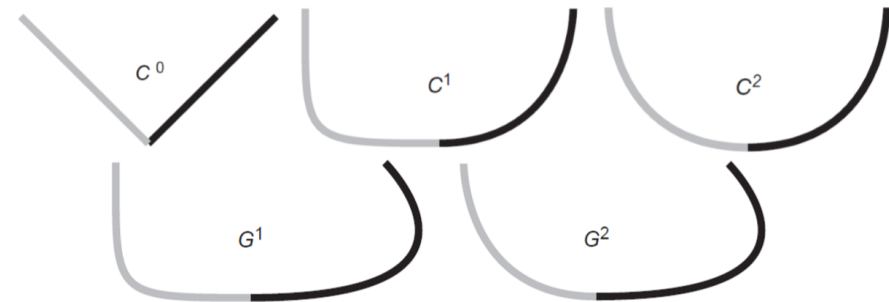


Approximation vs. interpolation



Piecewise construction

- Continuity (C^n vs G^n)



Bezier, b-spline, NURBS

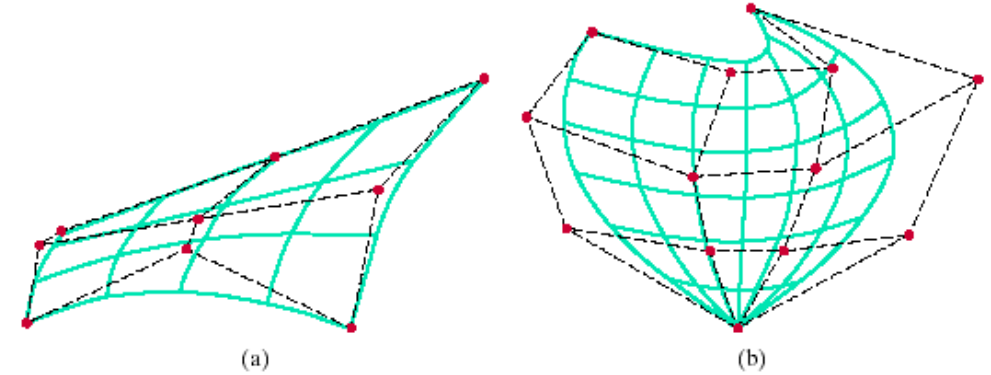
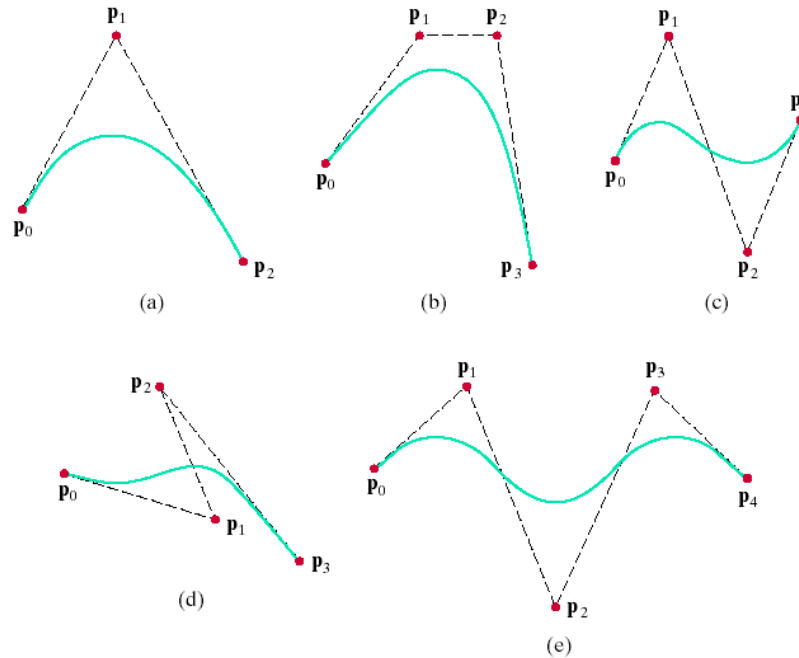
Bezier Curves and Surfaces

$$\begin{cases} b_{0,0}(u) = 1 \\ b_{0,1}(u) = 1 - u \\ b_{1,1}(u) = u \end{cases}$$

$$\begin{cases} b_{0,2}(u) = (1 - u)^2 \\ b_{1,2}(u) = 2(1 - u)u \\ b_{2,2}(u) = u^2 \end{cases}$$

$$\begin{cases} b_{0,3}(u) = (1 - u)^3 \\ b_{1,3}(u) = 3(1 - u)^2u \\ b_{2,3}(u) = 3(1 - u)u^2 \\ b_{3,3}(u) = u^3 \end{cases}$$

Bernstein Polynomials



$$P(u) = \sum_{k=0}^n p_k b_{k,n}(u), \quad 0 \leq u \leq 1$$

$$b_{k,n}(u) = {}_n C_k u^k (1 - u)^{n-k}, \quad {}_n C_k = \frac{n!}{k! (n - k)!}$$

Since $\sum_{k=0}^n b_{k,n}(u) = 1$, Bezier curves/surfaces have the convex hull property.

de Castlejau's Algorithm

$$P(u) = \sum_{k=0}^n p_k {}_nC_k u^k (1-u)^{n-k}, \quad 0 \leq u \leq 1$$

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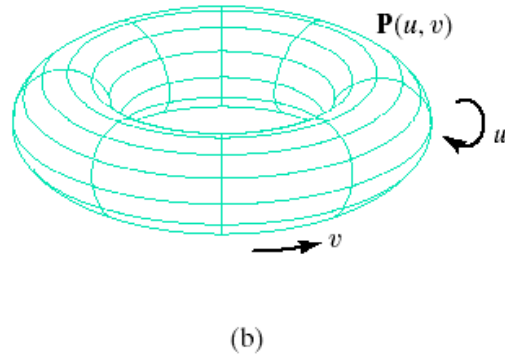
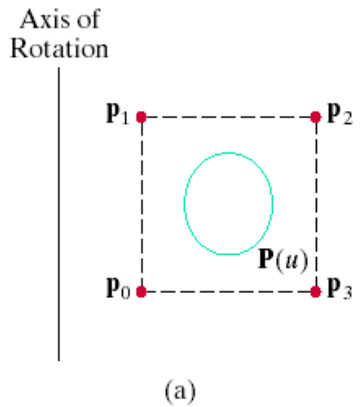
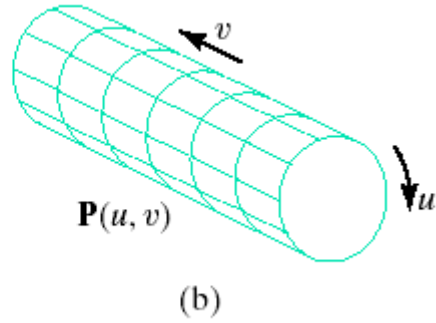
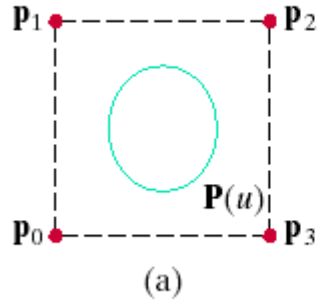
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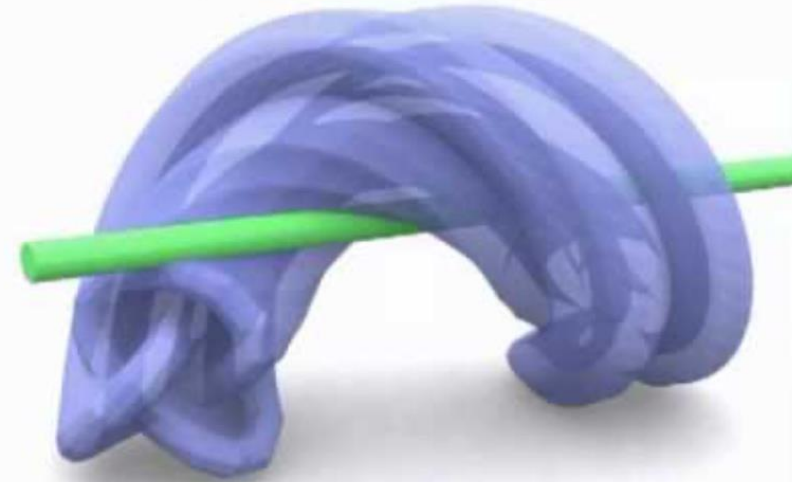
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Sweep Representation



Solid Modeling #4



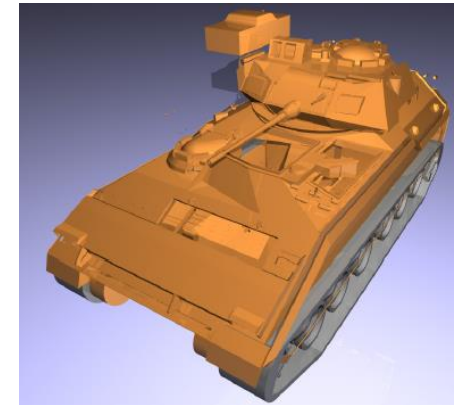
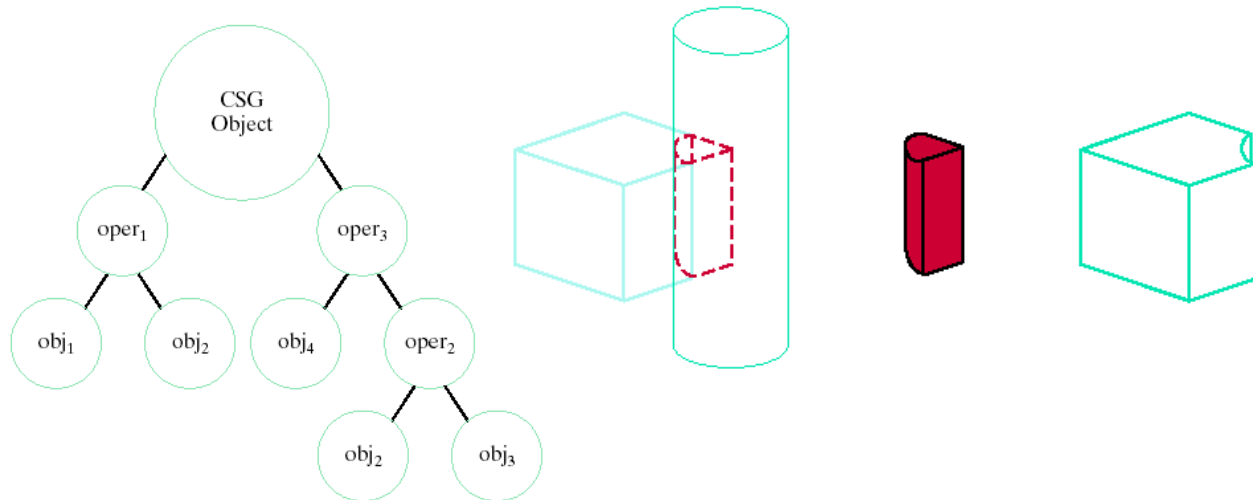
Swept Volume Result (Translucent)

Constructive Solid Geometry

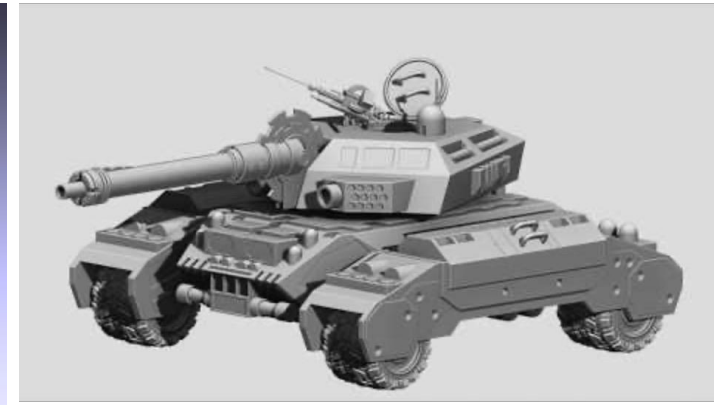
CSG

- Create a new object by performing a series of Boolean operations (union, intersection, difference)

CSG tree



8456 CSG operations



Zbrush