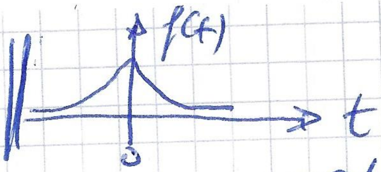
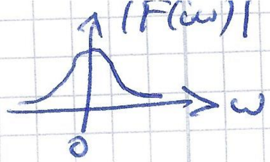
Gauss-Sinus Anregung

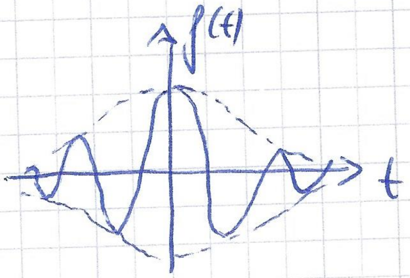


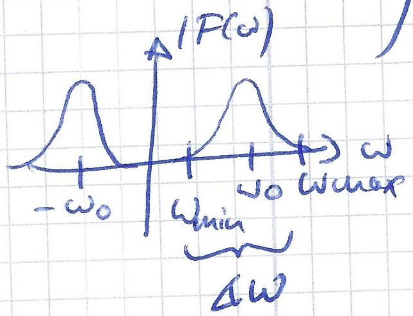


1) $$f(t) = e^{-at^2} \quad ○\!\!-\!\!● \quad F(i\omega) = \sqrt{\frac{\pi}{a}}\, e^{-\frac{\omega^2}{4a}}$$

Multiplikation mit $\cos(\omega_0 t)$:

$$\Rightarrow e^{-at^2} \cdot \cos(\omega_0 t) \quad ○\!\!-\!\!● \quad \frac{1}{2}\sqrt{\frac{\pi}{a}}\left(e^{-\frac{(\omega-\omega_0)^2}{4a}} + e^{-\frac{(\omega+\omega_0)^2}{4a}} \right)$$





$$\Rightarrow \frac{\omega_{min} + \omega_{max}}{2} = \omega_0$$

$$\frac{1}{2}\sqrt{\frac{\pi}{a}}\, e^{-\frac{(\omega_{max}-\omega_0)^2}{4a}} = \varepsilon \cdot \frac{1}{2}\sqrt{\frac{\pi}{a}}$$

$$\Rightarrow e^{-\frac{(\omega_{max}-\omega_0)^2}{4a}} = \varepsilon$$

$$-\frac{(\omega_{max}-\omega_0)^2}{4a} = \ln \varepsilon$$

$$(\omega_{max}-\omega_0)^2 = -4a \ln \varepsilon$$

$$a = -\frac{\left(\omega_{max} - \frac{\omega_{min}+\omega_{max}}{2}\right)^2}{4 \ln \varepsilon}$$

$$a = -\frac{(\omega_{max} - \omega_{min})^2}{16 \ln \varepsilon}$$

z.B. $\varepsilon = 1 \cdot 10^{-3}$

$$\omega_{max} = 2\pi f_{max}$$

$$\omega_{min} = 2\pi f_{min}$$

$$a = -\frac{\pi^2}{4 \ln \varepsilon} (f_{max} - f_{min})^2$$

$$f_0 = \frac{f_{min} + f_{max}}{2}$$

$$\Rightarrow f(t) = e^{-at^2} \cdot \cos(2\pi f_0 t)$$

Zeitverschiebung um t_0: (lineare Phase)

$$f(t) = e^{-a(t-t_0)^2} \cdot \cos(2\pi f_0 (t-t_0))$$

Ampl: $f(0) = \varepsilon \Rightarrow e^{-a(\cancel{t}-t_0)^2} = \varepsilon$

$$-a(\cancel{t} - t_0)^2 = \ln \varepsilon$$

$$t_0^2 = -\frac{\ln \varepsilon}{a} \Rightarrow t_0 = \sqrt{-\frac{\ln \varepsilon}{a}}$$

Also:

$$f(t) = e^{-a(t-t_0)^2} \cdot \cos(2\pi f_0 (t-t_0))$$

mit:

$$f_0 = \frac{f_{min} + f_{max}}{2}$$

$$a = -\frac{\pi^2}{4 \ln \varepsilon} (f_{max} - f_{min})^2$$

$$t_0 = \sqrt{-\frac{\ln \varepsilon}{a}}$$

z.B. $\varepsilon = 10^{-3}$