OTP and AES: A Historical Transition Between two Systems of Cryptography

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Linguistic supervision by Ms. Margrit Oetiker

Kantonsschule Zug

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Overview

OTP

AES: The Advanced Encryption standard

- High Level Structure
- Rounds

A Historical Transition

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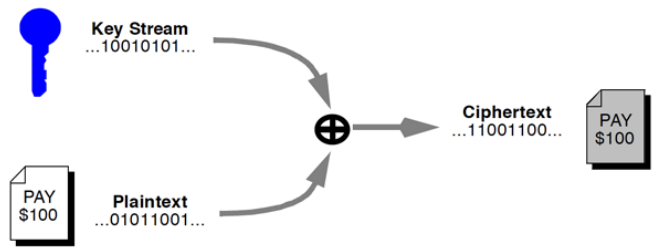
OTP: The Cipher

- Stream Cipher



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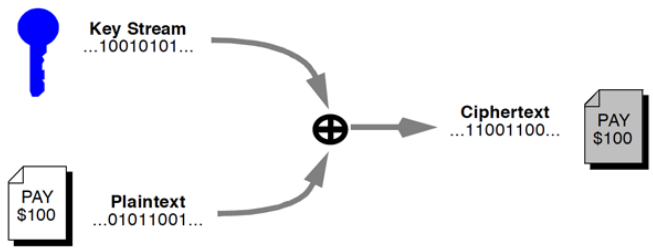

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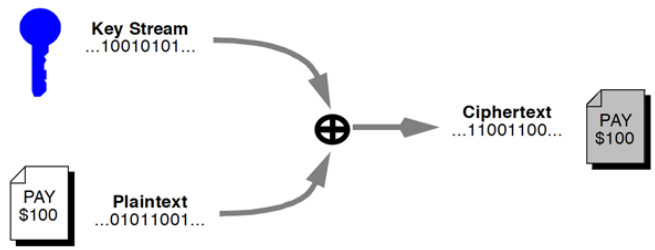

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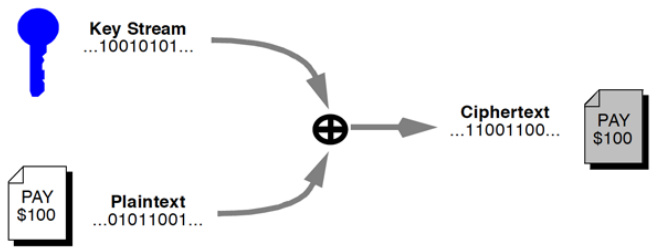
$j + t = 9 + 19 = 28$ A letter can't be assigned to 28!

$(9 + 19) \mod 26 = 2 = c$

OTP: The Cipher

- Stream Cipher
- Key length $\geq$ Message length
- Based on modular addition
- Perfect (forward) secrecy



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OTP: A Precursor to modern Computer-aided Cryptography

AES: Terminology

$$0 \vee 1$$

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- Bit: Boolean value
- Byte: 8 Bits; can represent any number from 0-255

$$0 \vee 1$$

$$(2^7 + 2^6 + 2^5 + 2^4 + 2^3 + 2^2 + 2^1 + 2^0)_b$$
$$(00000011)_b = 1 \cdot 2^1 + 1 \cdot 2^0 = 3$$
$$(16 + 1)_h \; 1 - 9; A; B; C; D; E; F$$

$$(B4)_h = 16 \cdot 11 + 4 \cdot 1 = 180$$

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- Diffusion: The "avalanche effect"
- Two different implementations: Computationally or memory efficient

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- Block Cipher

$$\begin{pmatrix} a_0 & a_4 & a_8 & a_{12} \\ a_1 & a_5 & a_9 & a_{13} \\ a_2 & a_6 & a_{10} & a_{14} \\ a_3 & a_7 & a_{11} & a_{15} \end{pmatrix}$$

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- The current N.I.S.T standard for SECRET and TOP-SECRET designated files

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- Block Cipher
- The current N.I.S.T standard for SECRET and TOP-SECRET designated files
- Original name: Rijndael; was selected as the successor to DES.

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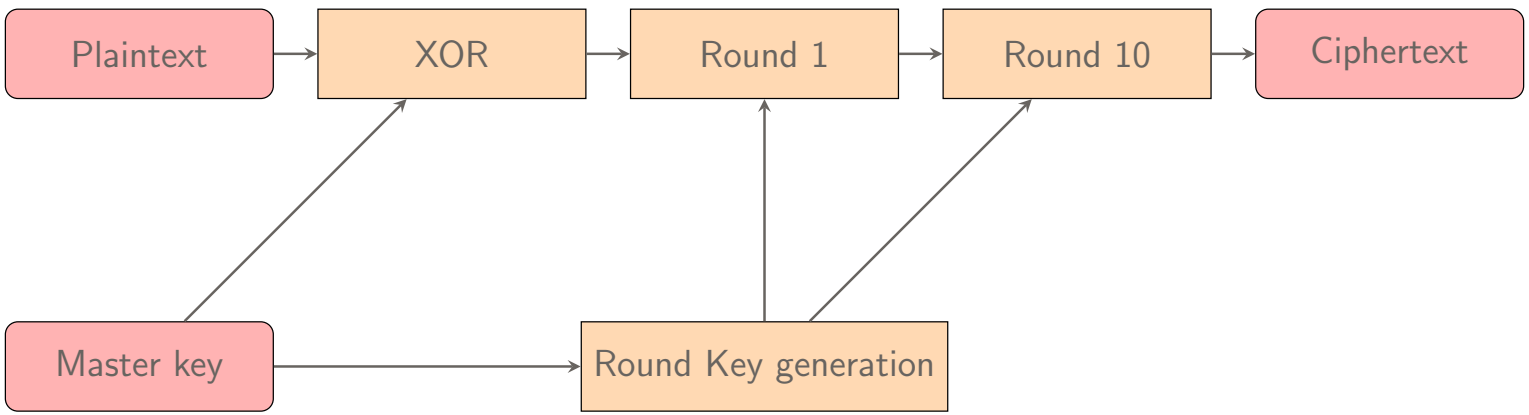
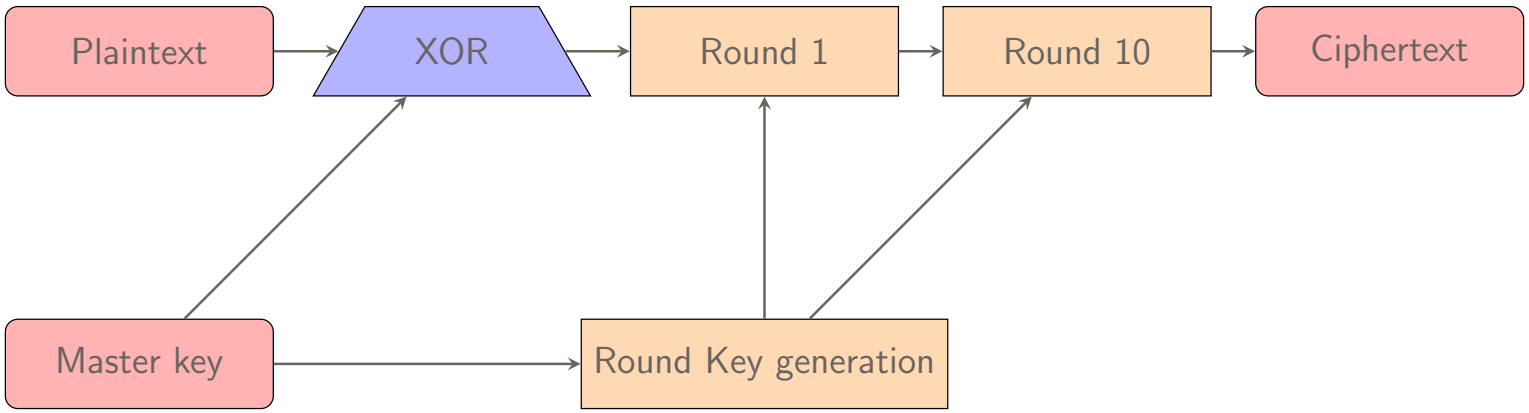
AES: High-Level Structure



AES: Rounds


Plaintext
XOR
Round 1
Round 10
Ciphertext
Master key
Round Key generation

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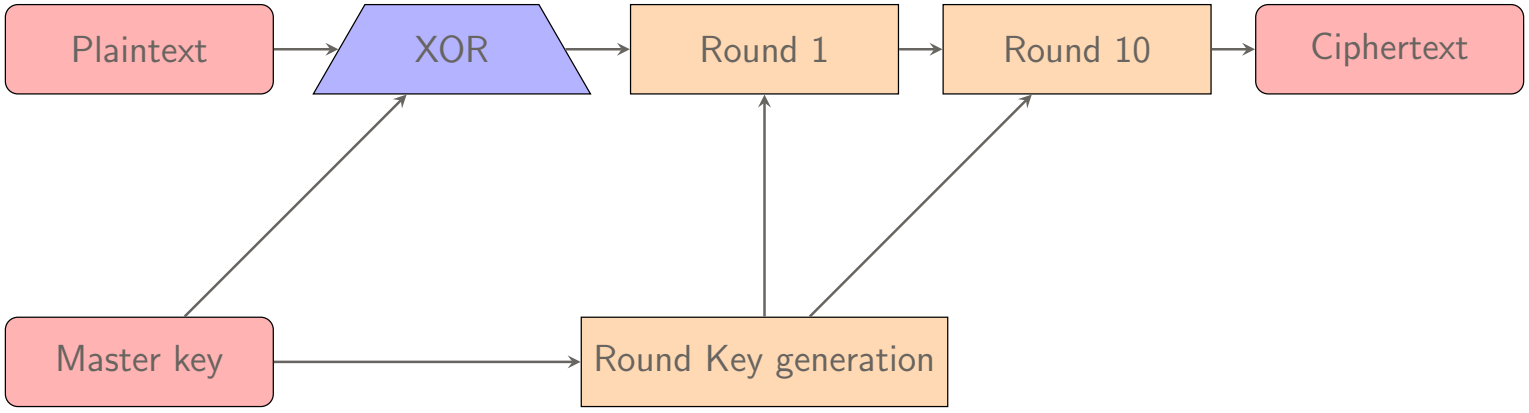

$$\begin{pmatrix} 1 & 0 & 1 & 0 \\ \oplus & \oplus & \oplus & \oplus \\ 0 & 1 & 1 & 0 \\ = & = & = & = \\ 1 & 1 & 0 & 0 \end{pmatrix}$$

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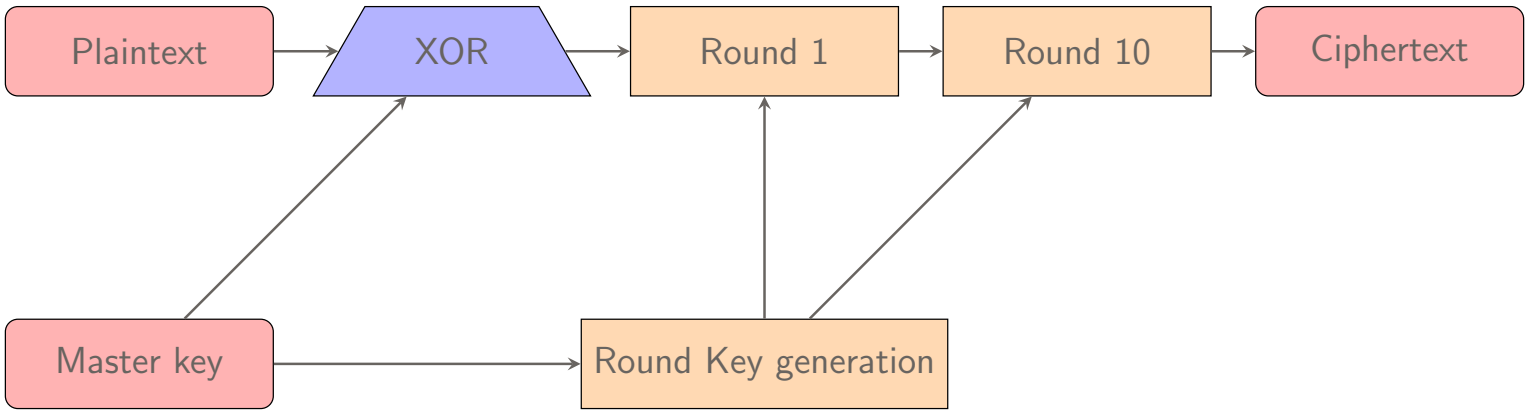

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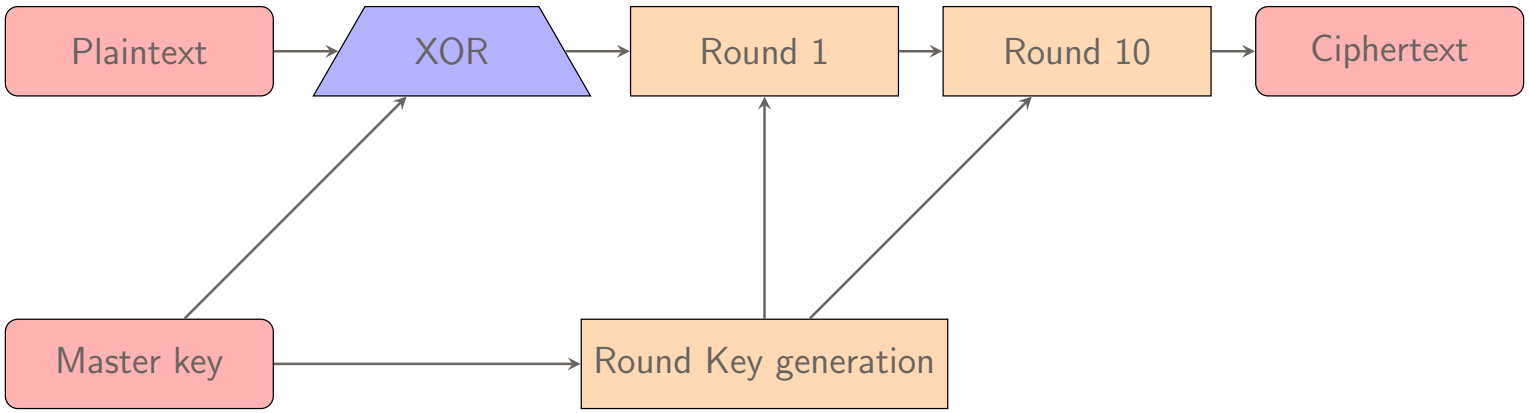

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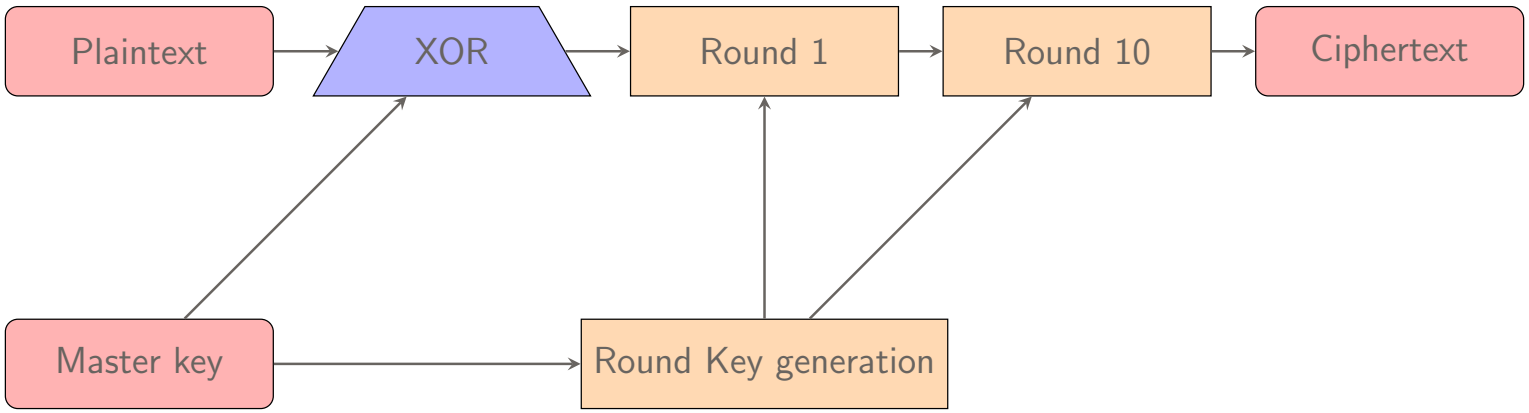

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- addition mod 2
- can randomize biased input

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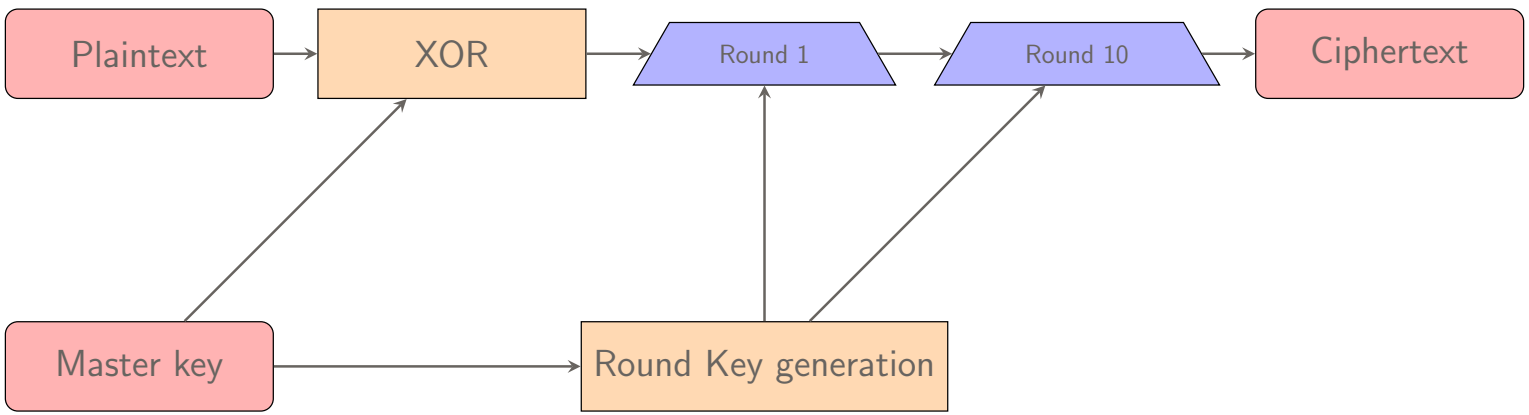
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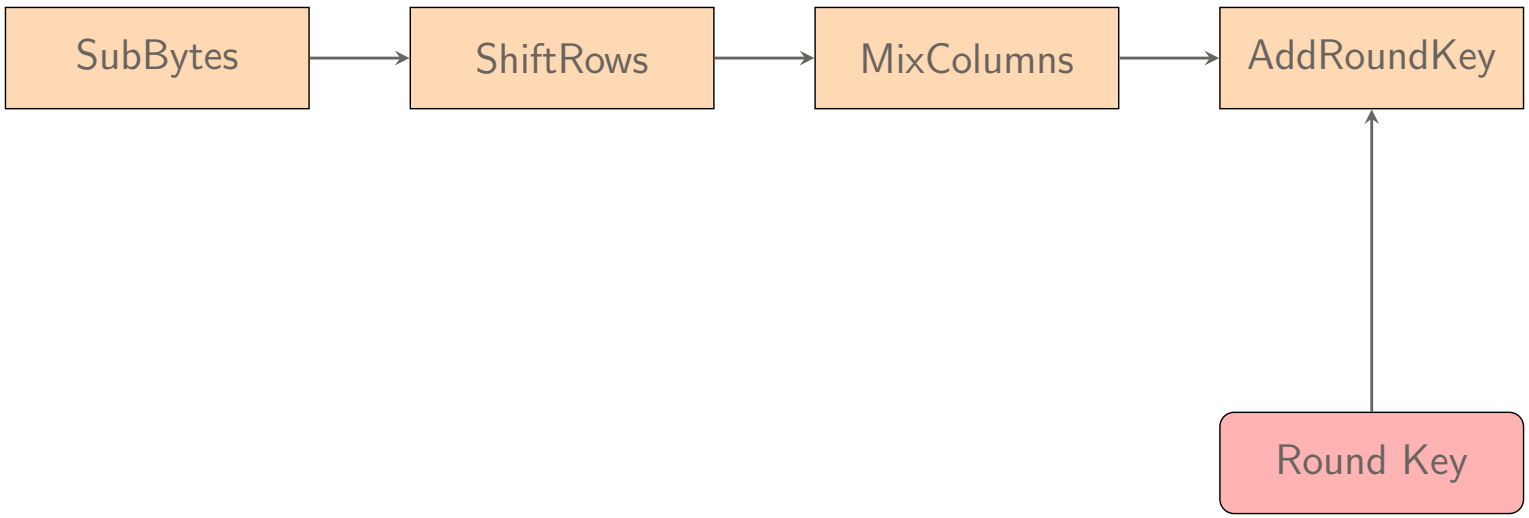

AES: SubBytes

S-Box Values																
S(rs)	s															
	0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
r 0	63	7c	77	7b	f2	6b	6f	c5	30	01	67	2b	fe	d7	ab	76
1	ca	82	c9	7d	fa	59	47	f0	ad	d4	a2	af	9c	a4	72	c0
2	b7	fd	93	26	36	3f	f7	cc	34	a5	e5	f1	71	d8	31	15
3	04	c7	23	c3	18	96	05	9a	07	12	80	e2	eb	27	b2	75
4	09	83	2c	1a	1b	6e	5a	a0	52	3b	d6	b3	29	e3	2f	84
5	53	d1	00	ed	20	fc	b1	5b	6a	cb	be	39	4a	4c	58	cf
6	d0	ef	aa	fb	43	4d	33	85	45	f9	02	7f	50	3c	9f	a8
7	51	a3	40	8f	92	9d	38	f5	bc	b6	da	21	10	ff	f3	d2
8	cd	0c	13	ec	5f	97	44	17	c4	a7	7e	3d	64	5d	19	73
9	60	81	4f	dc	22	2a	90	88	46	ee	b8	14	de	5e	0b	db
a	e0	32	3a	0a	49	06	24	5c	c2	d3	ac	62	91	95	e4	79
b	e7	c8	37	6d	8d	d5	4e	a9	6c	56	f4	ea	65	7a	ae	08
c	ba	78	25	2e	1c	a6	b4	c6	e8	dd	74	1f	4b	bd	8b	8a
d	70	3e	b5	66	48	03	f6	0e	61	35	57	b9	86	c1	1d	9e
e	e1	f8	98	11	69	d9	8e	94	9b	1e	87	e9	ce	55	28	df
f	8c	a1	89	0d	bf	e6	42	68	41	99	2d	0f	b0	54	bb	16

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- Sole source of Confusion; the only non-linear operation in AES (affine transformation)
- Key-independence is accepted in return for non-linearity; this eliminates one of DES' major weaknesses
- Utilization of the multiplicative inverse maximizes non-linearity, but negatively impacts diffusion: $0^{-1} = 0$ and $1^{-1} = 1$

AES: ShiftRows

$$\begin{pmatrix} a_{0,0} & a_{0,1} & a_{0,2} & a_{0,3} \\ a_{1,0} & a_{1,1} & a_{1,2} & a_{1,3} \\ a_{2,0} & a_{2,1} & a_{2,2} & a_{2,3} \\ a_{3,0} & a_{3,1} & a_{3,2} & a_{3,3} \end{pmatrix} \xrightarrow{\text{ShiftRows}} \begin{pmatrix} a_{0,0} & a_{0,1} & a_{0,2} & a_{0,3} \\ a_{1,1} & a_{1,2} & a_{1,3} & a_{1,0} \\ a_{2,2} & a_{2,3} & a_{2,0} & a_{2,1} \\ a_{3,3} & a_{3,0} & a_{3,1} & a_{3,2} \end{pmatrix}$$

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- One of the two primary sources of diffusion

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- One of the two primary sources of diffusion
- One small change to the plaintext should result in a large change to the ciphertext
- Bytes are placed into the state in column order, but shifted across rows

AES: MixColumns

$$\begin{pmatrix} 02 & 03 & 01 & 01 \\ 01 & 02 & 03 & 01 \\ 01 & 01 & 02 & 03 \\ 03 & 01 & 01 & 02 \end{pmatrix} \cdot \begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{pmatrix} = \begin{pmatrix} s_0 \\ s_1 \\ s_2 \\ s_3 \end{pmatrix}$$

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$$s_0 = 02a_0 + 03a_1 + 01a_2 + 01a_3$$
$$s_1 = 01a_0 + 02a_1 + 03a_2 + 01a_3$$
$$s_2 = 01a_0 + 01a_1 + 02a_2 + 03a_3$$
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- Each new byte is dependent on an entire column of four old bytes
- Second source of diffusion

AES: AddRoundKey

OTP

AES: The Advanced Encryption standard

- High Level Structure
- Rounds

A Historical Transition

Historical Impact of Cryptography

Motivator for more Powerful Computing

Today's Issues

The Modern War