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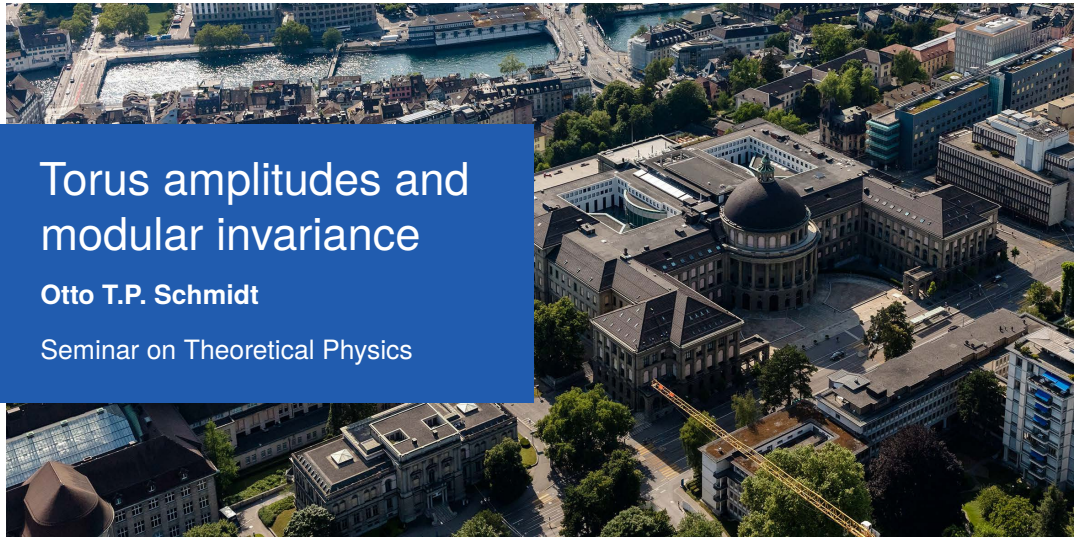


Torus amplitudes and modular invariance

Otto T.P. Schmidt

Seminar on Theoretical Physics

Outline

1. Motivation
2. The moduli space of tori
3. Torus partition function
4. Modular invariance
5. URLs and links

Outline

Interactions and observables

In the study of string interactions, the ultimate goal will be the assignment of a probability for a certain process and the prediction of a physical cross section.

As outlined in Section 22, the computation of an observable cross section involves a series of steps:

1. Canonical representation of string diagram through moduli space
2. Compute scattering amplitude by means of conformal field theory
3. Convert scattering amplitude into a cross section

Loop amplitudes in string theory

In order to obtain accurate scattering amplitudes of processes, one needs to include contributions from loops in string diagrams.

These loops can be seen as contributions from the next higher order pertubation. Graphically we consider the following processes:

Ultraviolet divergence

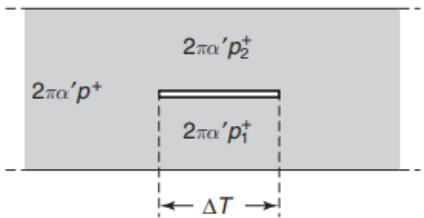
Amplitudes from virtual processes as depicted before can lead to ultraviolet (UV) divergences in quantum field theory (QFT).

Whereas QFT must employ complex renormalizations to deal with these UV divergences, we do not encounter these problems in string theory.

Outline

One-loop open strings

Before approaching the moduli space of tori, lets consider a one-loop open string with light-cone momentum p^+. This will serve as an intuitive analogon.
The light-cone diagram is:

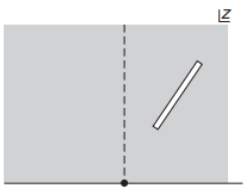


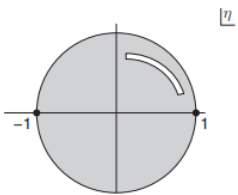
For fixed external momentum p^+ we find the two parameters: $\Delta T \in (0, \infty)$ and $p_1^+ \in (0, p^+)$.
$\rightarrow$ The class of Riemann surfaces of this process has two moduli.

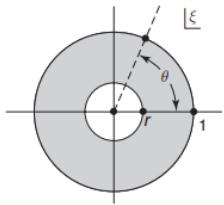
Canonical annulus

Use $w = \tau + i\sigma$ and apply conformal transformations:

1. Exponential map: $z = exp[\frac{w}{2\alpha' p^+}]$
2. Linear fractional transformation: $\eta = \frac{1+iz}{1-iz}$
3. Canonical annulus: *A region in $\mathbb{C}$ that is topologically an annulus can be mapped conformally to a canonical annulus*

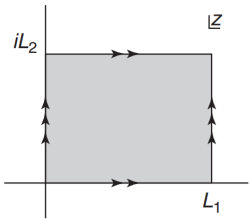


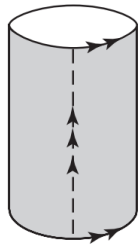


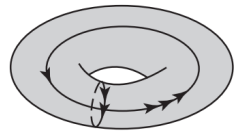


Rectangular torus

In order to apply the concept of moduli spaces to a torus, we need to assure that a torus is indeed a Riemann surface.
Consider a rectangular region of $\mathbb{C}$. By applying the analytic identifications $z \sim z + L_1$ and $z \sim z + iL_2$ we obtain a torus. This shows that the region remains a Riemann surface.
Graphically:







Colors

You need to pick these colors

- `titlefgcolor` (the box on the title page)
- `titlebgcolor` (the background on the title page, in case you don't use an image)
- `accentcolor` (alert text, blocks)

Use these commands at the beginning of the document

```
\colorlet{titlefgcolor}{ETHblue}
\colorlet{titlebgcolor}{ETHblue!60!black} % Use only multiples of 20%
\colorlet{accentcolor}{ETHred}
```

ETHblue
ETHgreen
ETHred
ETHgray
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Old ETH colors (`ETH1`, ..., `ETH9`) are deprecated and should not be used.
They are available as `oldETH1`, ..., `oldETH9` for backward compatibility.

Title
Subtitle

Text and some **alert text**

$$m_a^\top h(\cdot)$$

- list one
- list another one
  - test 1
  - test 2

Outline

Title with no subtitle

Large box

Notice that blocks are a bit larger than the text, that's intended.

Column environments also eat some margins. Use the option `[onlytextwidth]` if you want to align columns to the wide blocks.

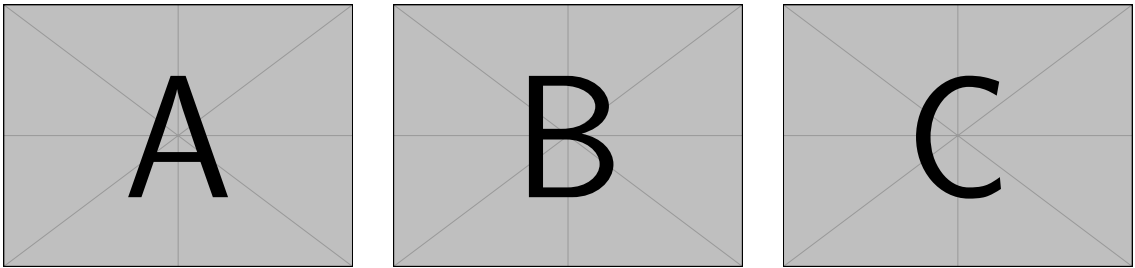
Small box

With some more text

Think outside the box!

Outline

And, of course, figures!



Free overlay

The package `textpos` is also enabled in case you want to overlay content freely in the slide.

This text is located at position (1,3):
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(1 unit equals to the left text margin)

The upper left corner of this image is at the slide center point:

```
\begin{textblock*}{40mm}(0.5\paperwidth,0.5\paperheight)
   \includegraphics[width=20mm]{example-image-a}
\end{textblock*}
```

Tables

Don't use vanilla LaTeX tables please

Item		
Animal	Description	Price ($)
Gnat	per gram	13.65
	each	0.01
Gnu	stuffed	92.50
Emu	stuffed	33.33
Armadillo	frozen	8.99

Outline

Clickable links

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